International Perspectives on the Teaching and Learning of Mathematical Modelling

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Heiner Klock
Hans-Stefan Siller
Gilbert Greefrath

Measuring Professional Competence for the Teaching of Mathematical Modelling

A Test Instrument

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Series Preface

Applications and modelling and their learning and teaching in school and university have become a prominent topic for many decades now in view of the growing worldwide relevance of the usage of mathematics in science, technology and everyday life. There is consensus that modelling should play an important role in mathematics education, and the situation in schools and university is slowly changing to include real-world aspects, frequently with modelling as real world problem solving, in several educational jurisdictions. Given the worldwide continuing shortage of students who are interested in mathematics and science, it is essential to discuss changes of mathematics education in school and tertiary education towards the inclusion of real world examples and the competencies to use mathematics to solve real world problems.

This innovative book series established by Springer *International Perspectives on the Teaching and Learning of Mathematical Modelling*, aims at promoting academic discussion on the teaching and learning of mathematical modelling at various educational levels all over the world. The series will publish books from different theoretical perspectives from around the world dealing with Teaching and Learning of Mathematical Modelling in Schooling and at Tertiary level. This series will also enable the *International Community of Teachers of Mathematical Modelling and Applications* (ICTMA), an International Commission on Mathematical Instruction affiliated Study Group, to publish books arising from its biennial conference series. ICTMA is a unique worldwide educational research group where not only mathematics educators dealing with education at school level are included but also applied mathematicians interested in teaching and learning modelling at tertiary level are represented as well. Six of these books published by Springer have already appeared.

The planned books display the worldwide state-of-the-art in this field, most recent educational research results and new theoretical developments and will be of interest for a wide audience. Themes dealt with in the books focus on the teaching and learning of mathematical modelling in schooling from the early years and at tertiary level including the usage of technology in modelling, psychological, social, historical and cultural aspects of modelling and its teaching, learning and assessment, modelling competencies, curricular aspects, teacher education and teacher education courses.

The book series aims to support the discussion on mathematical modelling and its teaching internationally and will promote the teaching and learning of mathematical modelling and research of this field all over the world in schools and universities.

The series is supported by an editorial board of internationally well-known scholars, who bring in their long experience in the field as well as their expertise to this series. The members of the editorial board are: Maria Salett Biembengut (Brazil), Werner Blum (Germany), Helen Doerr (USA), Peter Galbraith (Australia), Toshikazu Ikeda (Japan), Mogens Niss (Denmark), and Jinxing Xie (China).

We hope this book series will inspire readers in the present and the future to promote the teaching and learning of mathematical modelling all over the world.

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Introduction

From a mathematics education perspective, initiating and evaluating modelling processes offers a great potential for the acquisition of competences by students. Through the choice of appropriate tasks and interventions, allow, inter alia, a close connection to reality as well as a high degree of independence and openness, thus allowing individual access to mathematics and multiple solution approaches at different levels (Schukajlow and Krug, 2013). At the same time, the great potential of these mathematical modelling processes poses a challenge for (pre-service) teachers (Kuntze, Siller and Vogel, 2013). For this reason, a close examination of possible support measures to develop professional competence for teaching mathematical modelling in the sense of quality development in teacher education is necessary (Blum, 2015).

In this context, the operationalisation of modelling-specific professional competence is an important and relatively new field of research (Borromeo Ferri, 2019) to which we wish to contribute with this book. In doing so, we understand competences as context-specific, cognitive dispositions for achievement that are functionally related to specific situations and requirements (Klieme, Hartig and Rauch, 2008), so that not only declarative but also procedural, situation-related knowledge facets are focused.

Models describing the requirements for pre-service teachers are needed to measure the above competences. A structural model of professional competence to teach mathematical modelling was developed as part of a cooperation[1] between several German universities and was empirically confirmed to a great extent (Klock, Wess, Greefrath and Siller, 2019; Wess, Klock, Greefrath and Siller, 2021). Among other things, this model forms the basis for the conceptualisation and operationalisation of specific didactic knowledge facets as well as other affective-motivational components of modelling-specific teacher professionalism.

[1]This project is part of the "Qualitätsoffensive Lehrerbildung", a joint initiative of the Federal Government and the Länder which aims to improve the quality of teacher training. The programme is funded by the Federal Ministry of Education and Research. The authors are responsible for the content of this publication.

In order to present this structure and the related test instrument in a comprehensive manner, the first part of the book gives an overview of selected concepts and theoretical backgrounds of mathematical modelling. This is followed by explanations on the concept of competence used here and of different competence models before specific competence dimensions for teaching mathematical modelling are considered. Finally, these are used to interpret a structural model of modelling-specific professional competence and serve as a basis for test construction.

The second part presents operationalisation and the test instrument as the product of this process. In this context, the primary focus is on the analysis of the quality with which the quantitative test instrument measures the aspects of modelling-specific professional competence.

In the final part, the possibilities of use, but also limitations of the instrument, are discussed and further implications for research and practical applications are highlighted.

Contents

Part I
Theoretical Background

Chapter 1
Mathematical Modelling

The integration of applications and mathematical modelling into mathematics education plays an important role in many national curricula (Kaiser, 2020; Niss, Blum and Galbraith, 2007), and thus an increasing role in teacher training. Theoretical contributions from mathematics education form the basis of practical-related teacher education. In the following, selected concepts and theoretical backgrounds for modelling are presented and modelling tasks are examined regarding types, categories and criteria. Then, aspects of teaching mathematical modelling are discussed from a theoretical perspective and exemplary results of empirical studies are presented.

1.1 Terms and Definitions Used in Mathematical Modelling

Applications and modelling are indisputably regarded in the international discussion as a relevant part of mathematics education. For example, the *International Conference on the Teaching and Learning of Mathematical Modelling and Applications* (ICTMA) presents the current state of the international debate every two years. Applications and modelling are all aspects of relationships between mathematics and reality, including nature, culture, society and everyday life. In applications, the focus is on the transition from mathematics to reality and primarily on products, while modelling is more about the complementary transition from reality to mathematics and the processes (Niss et al., 2007). Only the term modelling is mentioned hereinafter, but it is always applicable for the transitions in both directions, namely mathematics to reality and from reality to mathematics.

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1.1.1 Mathematical Modelling and Mathematical Model

Over the past few years, the discussion in mathematics education of reality-based teaching has given rise to numerous conceptions about mathematical modelling and the associated translation processes. In preparation for the ICME-3 conference, Werner Blum conducted intensive research of the literature on mathematical modelling. This work later resulted in two-volume documentation of selected literature on application-oriented mathematics education (Kaiser, 2020). This distinguishes between two directions: the so-called *scientifically humanistic* stream, with representatives such as Freudenthal (1973), focuses more on the mathematics processes, while the *pragmatic* stream, with representatives such as Pollak (1968), is characterised more by a utilitarian aim.

In the context of the modelling debate in German, Blum's position (1985) is central, which regards the entire modelling process and the related distinction between mathematics and reality as fundamental to the notion of modelling. For a more in-depth look at the modelling discussion in German, refer to the book by Greefrath and Vorhölter (2016).

The construction of a mathematical model is a characteristic step for modelling. Niss et al. (2007) define the mathematical model term as a mapping: From a realm D of reality, translation processes are made into a subset of the mathematical world M. If the matching mapping rule is called f, a mathematical model can be described by the triple (D, M, f) (Blum & Niss, 1991). Thus, a mathematical model generally consists of defined objects (points, vectors, functions, etc.) that correspond to the elements essential for the initial situation in the real model and certain relationships between these objects that represent the real-world relationships of these elements with each other.

The reason for constructing and using a mathematical model is to understand or process problems from the part of reality. The term "problem" is used here in a broader sense. Thus, the focus is not only on pragmatic application-related problems but also on problems of a more intellectual nature, which are partly aimed at describing, understanding, explaining or even designing parts of the world with their questions (also of a scientific nature) (Niss et al., 2007).

However, the treatment of these problems has natural limits due to the usually inadequate mapping of complex reality with a mathematical model. Since the main focus in the construction of mathematical models is precisely on the possibility of a reduced form of representation in its complexity and a mathematical processing of real data, this incomplete representation is usually quite desirable. Only a certain amount of reality is translated into the mathematical world. Stachowiak (1973) summarised these aspects of the general model concept in three features:

- The mapping feature specifies that a model is a representation of a natural or artificial original.
- The shortening feature is that a model describes only the relevant features of the original, so the model is a reduction.

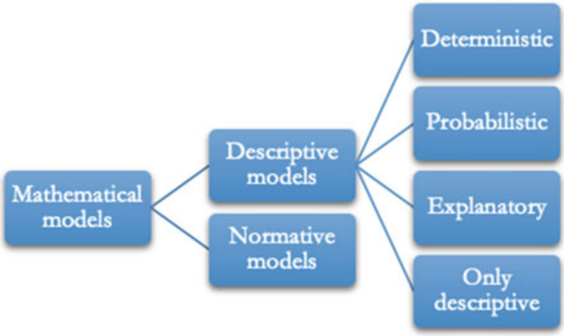


Fig. 1.1 Classification of models by Greefrath and Vorhölter (2016, p. 9)

- The pragmatic feature is that a model always has a specific purpose for certain subjects for a certain period.

Since such simplifications and formalisations are possible in different ways, the corresponding mathematical models also differ (see Fig. 1.1).

For example, prescriptive models are also called *normative* models. An example of this is tax models that set a payroll tax rate at a given gross annual wage. In addition, models can be used as images. These are called *descriptive* models (Freudenthal, 1978). *Explicative* models also provide an explanation for the data or facts and are therefore more meaningful. For example, a model that relates measurements of two variables to one another using linear regression can provide information about the strength of the relationship. *Probabilistic* models, on the other hand, make a prediction. As an example, the Rasch model provides the probability of solving an item with a given personal capability. If the model can determine a future event not only by a probability but by a clear prediction, it is known as a *deterministic* model.

Niss et al. (2007) emphasise that a distinction between mathematical models and the modelling process is particularly important since one or more mathematical models can be constructed during the modelling process. They are therefore integral parts of a larger whole, which is explained in more detail in the following section.

1.1.2 Modelling Processes and Modelling Cycles

The entire process of mathematical modelling can be ideally presented as a cycle, which in turn is formed as a model of the modelling process (Greefrath & Vorhölter, 2016). Until now, a variety of modelling cycles have existed that focus on different aspects (Borromeo Ferri, 2006). The different models are suitable for specific purposes. For example, some are used to illustrate modelling or help learners to work on modelling tasks. Through their extensive theoretical foundations, they represent their own learning content (Greefrath et al., 2013) and serve as a basis for empirical research. An idealised modelling process is described below. The designs are based

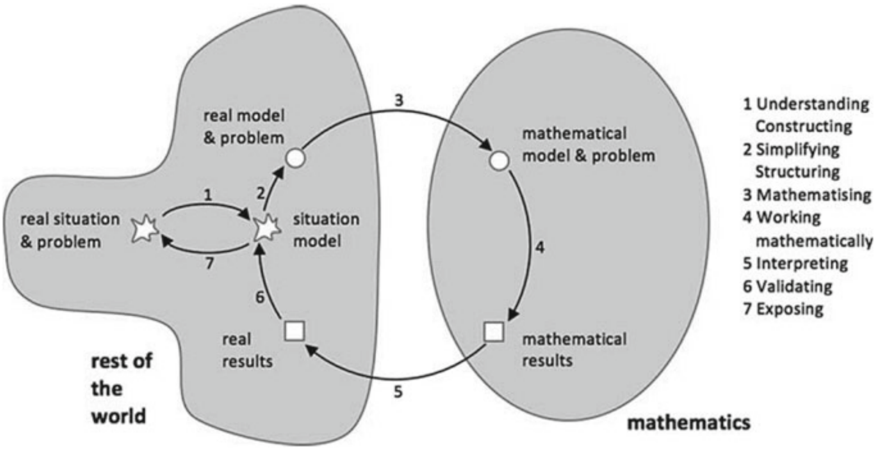


Fig. 1.2 7-Step modelling cycle as per Blum and Leiss (2007, p. 225)

on the 7-step modelling cycle according to Blum and Leiss (2007; see Fig. 1.2), which serves as a basis for the further theoretical considerations in this work.

The starting point for modelling processes is therefore a real-world situation, which involves an authentic problem situation that is processed with mathematical aids. This situation is transferred to a cognitive model according to the knowledge, aims and interests of the modellers. Simplification, structuring and clarification of the resulting mental representation lead to a real model and/or specification of the problem; assumptions must be made and central correlations must be derived. A mathematisation process translates the relevant objects, relationships and assumptions from the real model into mathematics, resulting in a mathematical model that can be used to solve the identified problem (Blum, 2015). Now mathematical methods are used to solve the mathematical problem within the framework of the model created and to obtain a result. The mathematical results thus obtained must then be interpreted in relation to the original real problem context (Greefrath & Vorhölter, 2016). The entire process is then validated. If the solution or the chosen procedure is considered unsatisfactory, individual steps or the entire process must be repeated using a modified or completely different model. Finally, the solution to the original problem of the real world will be outlined and, if necessary, passed on to others (Blum, 2015).

Other idealisations of the modelling process are also conceivable. For example, data acquisition could be considered separately or intermediate steps in the design of the mathematical model could be avoided. Thus, the above cycle is just one of many existing representations of the modelling process. The manifold idealisations of this process can be divided into three groups, which can be characterised by a different number of mathematical steps (Borromeo Ferri, 2006; Greefrath & Vorhölter, 2016). For example, cycles that require only one step from the situation to the mathematical model are assigned to the “direct mathematisation” category. On the other hand, the group of "two-step mathematisation" includes cycles that consider the simplifications in reality, the so-called real model, as an intermediate step from the real situation to the mathematical model.

Since a new perspective that emphasises cognitive analysis, Blum and colleagues developed the modelling cycle shown in Fig. 1.2, which is used to describe the modelling processes of learners as accurately as possible (Blum, 2011). This includes an additional third phase in mathematisation, an individual situation model that is formed from the understanding of the situation by the modellers (Blum & Leiss, 2007).

Real modelling processes of students rarely present the idealised processing steps in linear form. Rather, there are "mini-cycles" or frequent changes between the different stages of the modelling cycle, so-called "individual modelling routes" (Borromeo Ferri, 2018; Galbraith & Stillman, 2006).

All these idealisations have their specific strengths and weaknesses depending on their purpose (Blum, 2015). For cognitive analysis and as a diagnostic tool for (pre-service) teachers, the 7-step modelling cycle depicted seems to be particularly suitable and serves as a basis for further theoretical considerations in the context of the present work (Borromeo Ferri, 2018).

Adequate execution of the modelling processes presented requires certain skills and abilities of the modellers. These modelling competencies are examined in more detail in the following section.

1.1.3 Modelling Competencies

Students should be able to translate between reality and mathematics in both directions and work in the mathematical model. Niss et al. (2007) define modelling competence as follows:

> mathematical modelling competency means the ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation, as well as the ability to analyse or compare given models by investigating the assumptions being made, checking properties and scope of a given model etc. (Niss et al., 2007, p. 12)

Promoting the ability to process real-world problems using mathematical aids is thus a central goal of modelling in school.

The above definition describes a so-called global modelling competence in which certain partial processes can be identified by means of an atomistic perspective. Thus, Blum (2015) understands modelling competence as the ability to construct, use or adapt mathematical models by performing the process steps adequately and problem-appropriately, as well as analysing or comparing given models. Modelling competence is therefore not a one-dimensional construct but can be interpreted as a combination of different sub-competencies.

Table 1.1 Sub-competencies of modelling (Greefrath & Vorhölter, 2016, p. 19)

Sub-competency	Description
Understanding	The students construct their own mental model for a given problem situation and thus understand the question
Simplifying	The students separate important and unimportant information about a real situation
Mathematising	The students translate suitably simplified real situations into mathematical models (e.g. term, equation, figure, diagram, function)
Working Mathematically	Students apply heuristic strategies and mathematical knowledge to solve the mathematical problem
Interpreting	The students refer the results obtained in the model to the real situation and thus achieve real results
Validating	The students check the real results in the situation model for adequacy
Exposing	The students refer the answers found in the situation model to the real situation and thus answer the question

The examination of modelling cycles shows a different accentuation of these process steps. In doing so, the ability to execute such sub-processes can be considered as a sub-competency of modelling (Kaiser, 2007; Maaß, 2006; Niss, 2003). These sub-competencies can be characterised in accordance with the 7-step modelling cycle in Fig. 1.2, as shown in Table 1.1.

In addition, metacognitive competences are required for an adequate execution of modelling processes (Stillman, 2011). The absence of metacognition, such as the control of the solution process (Kaiser, 2007) or the reflexion of the adequacy of the solution process (Blomhøj & Højgaard, 2003), can lead to problems during the modelling process.

The question of how modelling processes can be shaped is closely linked to the perspectives on mathematical modelling as well as to the aims pursued with the integration of mathematical modelling into mathematics education. These are considered in more detail in the subsequent section.

1.2 Aims and Perspectives of Mathematical Modelling

The modelling debate showed that different perspectives are taken up—essentially the *scientifically humanistic* as well as the *pragmatic* mainstream (see Sect. 1.1.1). Although these directions have been recognised as the most important currents of debate, the perspectives for mathematical modelling can be more differentiated (Kaiser & Sriraman, 2006), so that great diversity in terms of their understanding of the goal in the field of applications and modelling becomes apparent.

Niss (1996) noted the need to address a discussion of mathematical education and the ways and means of improving its quality, primarily on the basis of a precise and comprehensive formulation of the valid aims and purposes of such education. Only on this basis can the problems of selecting and organising material, teaching methods and qualifications and training of teachers be adequately addressed. Thus, mathematics is

- a powerful tool to understand and master current or future real situations;
- a tool to train general mathematical competences;
- an important part of culture and society as well as the world itself.

Greefrath and Vorhölter (2016) design these general characteristics of mathematics as modelling-specific aims by differentiating between

- *content aims* which take into account the ability of students to recognise and understand real world phenomena;
- *process-oriented aims* that focus on the training of problem solving skills and a general mathematics interest;
- *general aims* that are aimed at building a balanced image of mathematics as a science, responsible participation in society and critical assessment of everyday models, as well as the development of social competences.

Based on comparable considerations, Blum (2015) examines the following justifications for the integration of mathematical modelling into teaching, which he also describes as the aims of teaching and learning applications and modelling:

1. *Pragmatic justification*: Understanding and mastering real-world situations require an explicit engagement with the appropriate application and modelling examples. In these cases, an adequate transfer from intra-mathematical activities cannot be expected.
2. *Formal justification*: General mathematical competences can also be trained through modelling activities. This way, for example mathematical reasoning is further developed by plausibility checks. However, modelling competence can only be acquired by examining the suitable application and modelling examples.
3. *Cultural justification*: Treating real-world phenomena with the aid of mathematics is essential for building a balanced picture of mathematics as a science in a comprehensive sense.
4. *Psychological justification*: Addressing examples from the rest of the world can help to stimulate students' interest in mathematics, demonstrate the relevance of mathematical content, and structure it in a way that promotes understanding.

These justifications or aims of teaching and learning applications and modelling require specific types of appropriate modelling examples. Kaiser and Sriraman (2006) distinguish in research different perspectives on mathematical modelling. The starting point of this identification of different theoretical directions in the current modelling discussion was an analysis of historical and current developments in applications and modelling in mathematics education:

- *Realistic* or *applied* modelling focuses on solving real problems and promoting modelling competence. Theoretically, this direction is based on pragmatic approaches to modelling and thus pursues utilitarian goals, in other words, a better understanding of the rest of the world through the application of mathematics (Kaiser & Sriraman, 2006). It focuses on ostensibly authentic, insignificantly simplified problems, for which holistic approaches are usually chosen, which leads to a comprehensive examination of these problems (Greefrath & Vorhölter, 2016).

- *Educational* modelling is a tradition of the so-called integrated approach and thus emphasizes not only content-related but also process-related aims. It is possible to distinguish more precisely between didactic and conceptual modelling (Kaiser et al., 2015). On the one hand, the *didactic* approach is to promote, on the other hand, to structure the learning processes in modelling. The *conceptual* approach focuses on the understanding and development of concepts. Both are focused on teaching didactic and learning-theoretical meta-knowledge (Kaiser & Sriraman, 2006).
- *Contextual* modelling is largely shaped by the "Model-Eliciting Activities" (MEA) approach developed by Lesh and Doerr (2003) in the USA. This stimulates mathematical activities through challenging real-life situations to stimulate modelling activities (Kaiser et al., 2015). The focused subjective and psychological goals are usually pursued by solving text problems (Kaiser and Sriraman, 2006).
- *Epistemological* or *theoretical* modelling is based on the previously described scientific-humanistic approach and thus focuses on theory-oriented aims. In other words, the application of mathematics, in reality, should contribute to the further development of the same (Kaiser & Sriraman, 2006). Thus, the focus is not so much on translation processes between mathematics and the rest of the world, but on real-life situations as intermediaries are used to address inner-mathematical issues and thus to achieve a science-oriented knowledge gain (Kaiser et al., 2015).
- *Socio-critical* or *socio-cultural* modelling pursues educational goals, such as a critical understanding of the surrounding world (Kaiser and Sriraman, 2006). In this perspective, the role of mathematical models or mathematics in society, in general, is emphasised and critically analysed (Kaiser et al., 2015). Thus, neither the modelling process itself nor the corresponding visualisations are in the foreground (Greefrath & Vorhölter, 2016).
- *Cognitive* modelling can be described as a kind of meta-perspective (Kaiser & Sriraman, 2006). It emphasizes the analysis and understanding of cognitive processes during modelling (Greefrath & Vorhölter, 2016). The development of mathematical thought processes through the use of models as mental or even physical images and the emphasis on modelling as a mental process also plays a role (Kaiser & Sriraman, 2006).

Blum (2015) shows that all the goals of learning theoretical considerations for mathematical modelling can be achieved only through high-quality teaching. Applications and modelling are central to the acquisition of mathematical competences, and a major effort must be made to make mathematical modelling accessible to students. However, it turns out that not only learning, but also teaching mathematical modelling in the classroom is cognitively demanding (Burkhardt, 2004; Freudenthal, 1973; Pollak, 1968). Thus, teachers need different skills, mathematical and non-mathematical knowledge, ideas for tasks and for teaching, and appropriate attitudes and beliefs to teach modelling adequately. In addition, overall teaching becomes more open and evaluation more complex. In view of the explained aims and perspectives of mathematical modelling, various task characteristics can be identified, which are used to stimulate the planned modelling processes in the classroom. There is a wide range of more artificial, less realistic tasks, some of which address only a sub-competency of modelling, to comprehensive, authentic modelling projects with a holistic approach. A detailed discussion of the modelling-specific task types, categories and criteria is contained in the following section.

1.3 Modelling Tasks

What is meant by a modelling task can vary greatly depending on the school or research context. To define other types of tasks and to describe the modelling tasks used in this book, criteria and category systems must be formulated to allow the classification of these tasks. However, the types, categories and criteria presented here are not all assignments that can be clearly defined. This allows classifying the tasks into multiple categories or identifying them as mixed forms. Furthermore, the type of processing in the specific teaching situation, as well as the individual requirements of the students, can influence the type of task. In addition, there are different names and different classification systems in the relevant discussion for the analysis of tasks based on criteria. Therefore, the following section is initially limited to selected, more general categories of mathematical tasks that can be considered relevant for the classification of modelling tasks.

1.3.1 General Categories of Tasks

There are several categories that focus on didactic principles or cognitive processes to examine the properties of tasks of designing the learning processes in mathematics teaching in detail. Among other things, a classification scheme, whose categories primarily cover the potential of tasks for cognitive activation of students, was developed (Neubrand et al., 2013). The dimensions are differentiated between

- *mathematical material areas as a content framework* (contents of geometry, arithmetic, algebra and stochastics; level in the curriculum),
- *types of mathematical work as a cognitive framework* (technical, computational, conceptual task);

- *cognitive elements of the modelling cycle* (non-mathematical modelling, internal mathematical work, basic concepts, mathematical text handling, mathematical thinking, mathematical presentation handling and mathematical reasoning); and
- *Solution room* (direction of processing; multiple solutions)

Moreover, the degree of openness is a feature of mathematics task. Open tasks are those that allow multiple approaches (at different levels) or solutions. In this way, considering tasks with different degrees of openness not only allows students to have their own access to the problems (Greefrath e al., 2017), it also supports students in the development of competences and thus leads to a better understanding of and flexibility in the handling of mathematical content (Borromeo Ferri, 2018). There are several categories of open tasks (see, e.g. Bruder, 2003; Maaß, 2010). In this work, the focus is on the classification of openness according to the initial state, transformation, and target state. For example, a task in which the initial state and transformation are unclear, but the target state is clear, can be called a *reversal problem* (Maaß, 2010).

While the classifications that have been considered so far are of a very general nature, the following focuses on categories that are primarily tailored to reality-related tasks.

1.3.2 Task Categories for Realistic Tasks

When dealing with the properties of modelling tasks, you can formulate a variety of special features that they should fulfil. Such criteria can support both the development and the selection of tasks, and teachers with appropriate classification schemes can gain an overview of modelling tasks. For example, Burkhardt (1989) distinguishes between illustrations of mathematical content and reality-related situations, as well as the latter, whether it can be used to process these standard models or whether new models need to be developed. On the other hand, Galbraith (1995) classifies according to the degree of structuring of the application problem at hand as well as the help provided to solve the problem. A segmentation that is widely used in the German-speaking discussion was developed by Kaiser (1995). In its extensive classification scheme, Maaß (2010) considers, in particular, the nature of the relationship with reality and the didactic intentions of modelling activity as specific criteria for modelling tasks.

Regarding the reality of tasks—in addition to a classification within the framework of the classical task types—a more precise characterisation can be made by the categories of authenticity, relevance to life, closeness to life and relevance to students. The concept of authenticity as well as its contribution to the development of modelling competence is an important area of studies, including the creation of a unified and meaningful meaning for the term “authenticity” itself. This challenge has implications on teaching as well as research (Niss et al., 2007). Greefrath et al.

(2017) therefore focus their attention on the prerequisites for problems to be considered authentic. After this, authenticity in the field of mathematical modelling refers to the non-mathematical context as well as the use of mathematics in the corresponding situation. The non-mathematical context must be real and must not have been specifically designed for the mathematics task. However, Vos (2015) points out that authenticity in this sense does not necessarily mean that a situation exists in the original, but authentic tasks can also represent a good replica of a real situation. The use of mathematics in this situation must also be sensible and realistic, and should not be confined to mathematics education. Authentic modelling tasks are therefore problems that belong exactly to an existing field or problem area and are accepted as such by people working there (Niss, 1992). However, the authenticity of tasks does not yet mean that these tasks are relevant to the present or future lives of students.

Blum (1996) focuses on the context of modelling tasks, like classical task types and considers a task-relevant for mathematics education when certain didactic goals can be achieved. In contrast, Burkhardt (1989) classifies tasks according to the interest that students can have in the context. It also distinguishes between problems arising from students' daily lives, those that may be relevant for students in the future, and those that are only close to the students' lives and whose core focus is on mathematics. The question of whether learners actually consider a context to be interesting, closely linked to, or relevant to their daily lives—as mentioned in the introduction—depends not only on the task itself but also on the specific teaching situation and the individual requirements of the learners. For this reason, PISA (OECD, 2003) distinguishes between tasks relating to the area from which their context comes:

> the situation is the part of the students' world in which the tasks are placed. It is located at a certain distance from the students. For OECD/PISA the closest situation is the student's personal life; next school life, work life and leisure, followed by the local community and society as encountered in daily life. Furthest away are scientific situations. Four situation-types will be defined and used for problems to be solved: personal, educational/occupational, public, and scientific. (OECD, 2003, p. 32)

Regarding the focus of the didactic intention pursued with the modelling activity, the modelling process can always be used to analyse real problems. Blomhøj and Højgaard (2003) distinguish between a holistic and an atomistic approach. According to the first approach, all phases of the modelling cycle will be followed in the modelling process. In an atomistic approach, the modelling task addresses individual phases of the modelling process (e.g. mathematisation). Since students can encounter difficulties in many parts of the solution process when performing modelling tasks, increasing the complexity and the level of demands of the processing, a reduction of the task in the atomistic sense can be useful, and can help to specifically promote or accurately diagnose the partial competencies of modelling. Therefore, the sub-stages of the modelling cycle are often also examined and used to categorize modelling tasks corresponding to the sub-competencies shown in Table 1.1 as was done by (Czocher, 2017).

Based on the previous general categories of tasks as well as the literature of frequently mentioned key properties for tasks with a life relevance, a catalogue of criteria for modelling tasks is compiled below, which serves as a basis for further conceptual considerations of this book.

1.3.3 Selected Criteria for Modelling Tasks

Looking back at the modelling-specific task categories, it can be seen together with Maaß (2010) that the nature of the relationship with reality—more precisely the context of the situation, the authenticity and the relevance for students—seems to be very important for an adequate analysis of reality-related tasks. At the interface of the special and general task criteria, the dimension of the cognitive elements of the modelling cycle—in particular the partial steps of modelling—is highlighted as a characteristic examination feature. Further information can be found in Bruder (2003), Maaß (2010) and Greefrath et al. (2017) clear evidence that the openness of a task, in the sense of multiple approaches and solutions (Schukajlow & Krug, 2013), is an essential feature of modelling tasks. Criteria for the development and analysis of modelling tasks are summarised and specified in Table 1.2.

In particular, it can be noted that through their authenticity and close connection to reality, modelling tasks enable students to access mathematics individually and affectively, and through their openness, with different solution approaches at different levels.

In addition to the selection and development of modelling tasks, appropriate support for modelling processes plays an important role in providing an appropriate learning environment. The task of teachers is primarily to diagnose difficulties and to eventually intervene if needed. The following sections provide more detailed insights into selected theoretical aspects of mathematical modelling regarding these two requirements.

Table 1.2 Catalogue of criteria for modelling tasks (Siller & Greefrath, 2020; Wess & Greefrath, 2019)

Criterion	Specification
Reality relation	The problem definition has a non-mathematical factual reference
Relevance	The problem definition is considered by students to be interesting, closely linked to or relevant to their daily lives
Authenticity	The problem definition is authentic with regard to the non-mathematical aspect The problem definition is authentic with regard to the use of mathematics in concrete situation
Openness	The problem definition allows different solutions The problem definition allows approaches at different levels
Promoting sub-competencies	The problem definition promotes cognitive elements in the form of partial competencies of mathematical modelling

1.4 Difficulties in the Modelling Process

In modelling, it is advisable to make it possible to work independently in cooperative learning environments (Maaß, 2005). Further, working in small groups is an appropriate social form for modelling tasks (Clohessy & Johnson, 2017; Ikeda & Stephens, 2001). However, modelling activities are generally cognitively demanding. In particular, studies have shown that the difficulty of modelling tasks can be explained mainly by the inherent complexity of these tasks, measured by the necessary sub-competencies. Thus, every step in the modelling process of students represents a potential cognitive barrier (Galbraith & Stillman, 2006; Stillman, 2011). Taking these sub-steps into account (see Table 1.1), some typical examples of difficulties and errors encountered by students in the processing of modelling tasks are given below:

- Many students already have problems reading and *understanding* the task. This is not only due to a lack of reading skills (Plath & Leiss, 2018), but students have learnt that they can also work on contextual tasks without carefully reading them and understanding the context (Blum, 2015).
- The *simplifying/structuring* and the associated setting of a real model can be identified as a common source of error (Blum, 2015). Not only do students find it difficult when they meet their own assumptions, but also, in part, from a misunderstanding of the question, errors in the structural contexts implied by them (Schukajlow et al., 2012) are shown.
- In particular, the distinction between real and mathematical models is difficult when *mathematising*. This is not always clear, because the processes of developing a real model and a mathematical model are intertwined. In addition, the change from the real world to the mathematical world is a barrier for the learners, especially since this requires mathematical background knowledge (Galbraith & Stillman, 2006).
- Despite a suitable mathematical result, difficulties can arise in *interpreting*, the translation process from mathematics to the rest of the world. Thus, learners often forget what their calculations actually mean and thus have problems identifying the mathematical results with their real counterparts (Galbraith & Stillman, 2006).
- It seems that it is particularly difficult for students to *validate* (Galbraith & Stillman, 2006). For example, some learners believe that validation is the same for each modelling task, or feel that validation is a depreciation of the result. In addition, interpreting and validating terms often cannot be separated from each other in terms of content. However, students do not usually validate their solutions (Blum, 2015).
- The main difficulties encountered in *communicating* are when students try to reconcile unexpected results with the real situation. These unexpected results are often the result of a previous error, which is easily visible by comparing results with others. They are therefore usually no barrier for attentive learners, although this may be the case for less attentive students (Galbraith & Stillman, 2006).

There are a large number of studies on errors, blockages or difficulties in modelling processes (Galbraith & Stillman, 2006; Galbraith et al., 2007; Stillman et al., 2013; Maaß, 2005; Schaap et al., 2011). An overview of potential difficulties in each modelling phase is given in Table 1.3.

Table 1.3 Difficulties in the modelling process (Klock & Siller, 2020)

Categories (modelling steps)	Subcategories (modelling sub-processes)
1. Forming a real-world model	1.1 Fail to understand the context[a, d]
	1.2 Fail to understand the task[b, d]
	1.3 Fail to search for missing information[c, d]
	1.4 Fail to identify relevant variables[a, b]
	1.5 Fail to make meaningful and simplifying assumptions[a, c]
	1.6 Fail to understand a foreign language[e]
	1.7 Calculating without including the context[d]
2. Forming a mathematical model	2.1 Fail to define variables[a]
	2.2 Fail to realise dependencies between variables[a]
	2.3 Fail to use adequate mathematical methods to mathematise[a]
	2.4 Fail to use technologies[a]
	2.5 Fail to understand the situation/mathematics conceptually[d]
	2.6 Fail to understand mathematical contents[e]
3. Working mathematically	3.1 Fail to use adequate formulae[a]
	3.2 Fail to use adequate solution strategies and algorithms[a, d]
	3.3 Fail to use technologies[a]
	3.4 Fail to understand the situation/mathematics conceptually[c]
	3.5 Fail to solve the model due to an excessive complexity[d]
	3.6 Fail to convert units[d]
4. Interpreting	4.1 Fail to identify the correct meaning of aspects of the mathematical results/model[a, d]
	4.2 Fail to answer the question with the help of the mathematical results[d]
5. Validating	5.1 Fail to reconcile (interim-)results with the real situation[a]
	5.2 Fail to identify the influence of constraints/real world aspects on the mathematical results[a]
	5.3 Fail to find opportunities to improve the model[d]
	5.4 Fail to validate including all relevant aspects[d]

(continued)

Table 1.3 (continued)

Categories (modelling steps)	Subcategories (modelling sub-processes)
	5.5 Fail to improve the model, so it fits to the real-world situation[d]

[a]Galbraith and Stillman (2006)
[b]Schaap et al. (2011)
[c]Stillman et al. (2013)
[d]Maaß (2005)
[e]Added by the authors

In addition to difficulties encountered during the modelling phases, the modelling process also involves metacognitive, affective, social or organizational difficulties, for example: the loss of an overview of your own work, a lack of mathematical self-confidence, a disturbed communication in the working group or an unclear formulated modelling task. Teachers need the knowledge about typical difficulties in modelling processes to be able to react quickly and adequately to them in modelling processes. How adequate intervention in modelling processes looks remains an integral part of current research. The following section deals with this aspect.

1.5 Interventions in the Modelling Process

Within mathematics education, there is an intensive discussion of which teaching behaviour is suitable for the most effective teaching of modelling among students (see, e.g. Tropper et al., 2015). Burkhardt (2006) emphasises that, unlike traditional treatment of the rest of the curriculum, the development of modelling competence entails a change in the role of teachers and related new requirements for teachers. Among other things, discussions must be conducted in a non-direct but supportive manner, students must be given sufficient time and confidence to thoroughly explore individual problems and, if necessary, strategic assistance without detailed proposals. Doerr (2007) also points to a changing role for teachers. In this context, she emphasises that teachers must have a broad and deep understanding of the diversity of approaches that students could pursue in the process of modelling. In addition, the teacher's task is to enable students to interpret, explain, justify and evaluate their models. Teacher intervention is an important element of learning process control in mathematical modelling processes, because modelling tasks often have a high degree of openness. One study suggests that an "operational-strategic" teaching that focuses students' independent work in groups could significantly increase students' modelling competence relative to a "direct" or instructional approach (Schukajlow et al., 2012).

De Jong and Lazonder (2014) classify the help they provide to support research-discovering learning processes according to their specificity. This specificity increases with increasing numbering:

1. *Process constraints*: The complexity of the learning process that is being discovered is reduced by reducing the number of possible options that students must include. An example is the division of tasks into manageable sub-tasks.
2. *Performance Dashboard*: The help gives students an overview of their own work process and its quality. The topics will focus on what has been done and how this contributes to the solution of the task. This aid requires the ability of the learner to continue working with this information.
3. *Prompts*: There are time-appropriate hints that remind students to perform a particular action. They tell us what to do, but not how to do it. This aid requires that students be able to perform the action.
4. *Heuristics*: Compared to Prompts, both the indication that an action is to be performed and how it is to be performed is given. This help is used when students do not know when and how to apply an action in the process.
5. *Scaffolds*: Scaffolds structure the solution process by providing all the components necessary to solve the task. This kind of help is used when the learners cannot manage the solution process independently or it is too complicated for the learners.
6. *Direct Presentation of Information*: The assistance consists of a direct instruction on the content. It makes sense to help learners who have a lack of knowledge or are unable to obtain information themselves.

The range of assistance is consistently oriented towards the competences of the learners. If students are able to control their own learning process, less specificity can be provided. Taxonomy should provide orientation on the selection of suitable aids. Lazonder and Harmsen (2016) showed in a meta-study that none of the categories of help described above promises greater learning success. However, providing help has had a median impact on learning success compared to no help (Lazonder and Harmsen, 2016). This research suggests that not so much the nature of assistance as their individual adaptation to the difficulty of the learner is crucial to greater learning success. Leiss (2007, 2010) also describes this aspect in his work.

Leiss (2007) has developed a general model for teacher interventions (see Fig. 1.3) in which he differentiates between three aspects: the basic knowledge, the area and the characteristics of an intervention (Tropper et al., 2015). A diagnosis of the situation (trigger of the intervention, previous interventions, knowledge required to solve the task, students' competence level, time available) and a diagnosis of difficulties (type

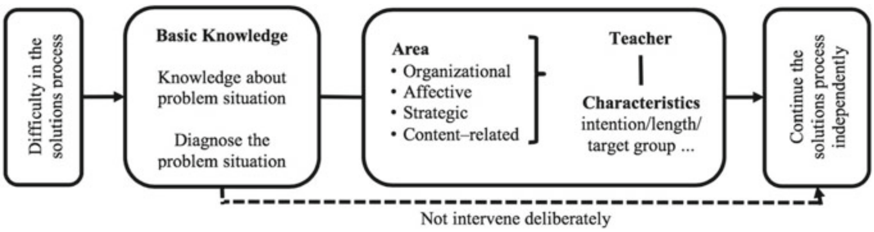


Fig. 1.3 Process model for general teacher interventions (Leiss, 2007; translation by authors)

of difficulty, area and cause of the difficulty, assignment in a theoretical model—here: the modelling cycle) is necessary to create a *basic knowledge* which is crucial for the selection of an adequate intervention.

The *area of intervention* describes to which aspect the intervention refers. Compared to the classification system according to De Jong and Lazonder (2014), these types of intervention have no hierarchy. *Organizational interventions* concern the design of the learning environment ("Watch the time!"). *Affective interventions* influence students' emotional aspects extrinsically ("You can do this!"). *Strategic interventions* are helpful on a meta-level ("What is still missing?" (Blum and Borromeo Ferri, 2010, p. 52)). *Content-related interventions* are related to the concrete contents of the task ("A car consumes 7 L per kilometre."). The area is the central feature of an intervention (Leiss, 2007). In particular, strategic interventions are considered to have a high potential to support students to overcome difficulties in the modelling process (Stender and Kaiser, 2015).

Interventions can be classified by different *characteristics* like the intention of the intervention (statement, question, request), its duration and the addressee (single student, group, whole class) (Leiss, 2007). The aspects of basic knowledge, area and characteristics of an intervention describe a general and idealized intervention. Based on that, Tropper et al. (2015) defined the notion of adaptive teacher interventions which

> are based on a diagnosis of the situation and can be described as an independence-preserving form of support, adapted in form and content to students' learning process, in order to enable them to overcome a (potential) barrier in the process and to continue the process as independently as possible. (Tropper et al., 2015, p. 1226)

Five essential characteristics of adaptive interventions can be identified from this and further definitions (Leiss, 2007; Stender & Kaiser, 2015). In our work adaptive interventions (Klock & Siller, 2019) …

- are based on a diagnosis,
- are adapted in form and content to students' learning process,
- provide minimal help,
- preserve independence,
- have a positive effect on the learning process by overcoming a difficulty.

These aspects are crucial for supporting students in mathematical modelling processes. They are the basis for assessing interventions in terms of adaptivity in our work. Adaptive intervention criteria show that good diagnosis is the basis for adaptive intervention. This is also emphasised in the scaffolding discussion. Van de Pol et al. (2014) distinguish in their process model between a diagnostic part and an intervention part, much like Leiss (2007). As with Leiss, the diagnostic part is preceded by the intervention part (see Fig. 1.4).

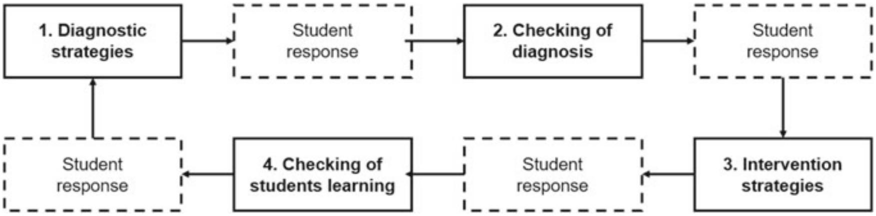


Fig. 1.4 Model of contingent teaching (Van de Pol et al., 2014)

In principle, the examination of the preceding sections shows that teachers play an important role in the development of students' competences and have a decisive influence on the progress of learning (Blum, 2015). To do this, they need different skills, knowledge facets and ideas for tasks and for teaching, as well as appropriate attitudes and beliefs.

Chapter 2
Professional Competence for Teaching Mathematical Modelling

Professional competence is a widely discussed topic (see, e.g. Cochran-Smith & Fries, 2001; Darling-Hammond & Bransford, 2005) and was measured globally in various large-scale studies (see, e.g. Blömeke et al., 2014; Kunter et al., 2013). The dimensions for the subject of mathematics range from knowledge to mathematical content to pedagogical and didactic knowledge of teachers with the aim of bringing them together. In the context of the professionalisation of mathematics teaching education students, the question of the existence and structure of specific professional competence is also raised in order to verify skills gains in specific areas. Due to the numerous requirements in the care of cooperative modelling processes and "the strong implantation of real-world problem solving […] into the curricula" (Schwarz et al., 2008, p. 788), it makes sense to differentiate professional competence in the field of mathematical modelling (Borromeo Ferri, 2018; Borromeo Ferri & Blum, 2010). A structural model describing and relating professional competence for teaching mathematical modelling was developed and empirically confirmed in cooperation between several German universities within the framework of the "Qualitätsoffensive Lehrerbildung" (Klock et al., 2019; Wess et al., 2021)—funded by the Federal Ministry of Education and Research (FKZ 01JA1605, FKZ 01JA1621). The conceptualisation of the model and empirical results for the structure are described in this section.

The first part of the section deals with the general concept of competence and the concept of professional competence of teachers. For this purpose, a clarification of the concept of profession and professional competence will first be made. Two conceptualised models of the professional competence of teachers are presented. With the help of a catalogue of didactic skills for teaching mathematical modelling (Borromeo Ferri & Blum, 2010), an interpretation of the COACTIV model (Baumert & Kunter, 2013) is made to conceptualise a structural model of professional competence for teaching mathematical modelling that can be verified empirically. Results for the empirical structure of the construct are reported in conclusion.

R. Wess et al., *Measuring Professional Competence for the Teaching of Mathematical Modelling*, International Perspectives on the Teaching and Learning of Mathematical Modelling, https://doi.org/10.1007/978-3-030-78071-5_2

2.1 The Concept of Competence

The basic theoretical reference point of output orientation is the conceptualisation of the concept of *competence*. This is central to empirical studies that address the quality development and productivity of the education system (Klieme et al., 2008). While qualitative studies often use generative models of competence that distinguish between the actual competence and the performance, in the context of quantitative considerations, the functional pragmatic concept of competence, which "conceives of competencies as context-specific dispositions for achievement that can be acquired through learning. Furthermore, they functionally relate to situations and demands in specific domains" (Klieme et al., 2008, p. 8).

This concept is not explicitly interested in the generative, cognitive, situation-independent system, detached from normative educational goals, but focuses on a person's ability to cope with challenges in certain situations. This work, therefore, relates—especially against the background of context dependence—to the extended concept of competence of Weinert (2001), which is frequently used in German-speaking countries, which defines competences as "intellectual abilities, content-specific knowledge, cognitive skills, domain-specific strategies, routines and subroutines, motivational tendencies, volitional control systems, personal value orientations, and social behaviors." (Weinert 2001, p. 51).

In this sense, competence is perceived as a complex construct that addresses key aspects of the professional debate. Among other things, skills are assumed to be employable, as they are based on declarative and procedural knowledge. Weinert (2001) also names motivational, volitional, and social readiness; broadening the definition as context-specific cognitive performance management (Klieme et al., 2008). However, he points out that the motivational aspects must be considered as a separate construct in addition to the cognitive aspects, otherwise a lack of motivation is tantamount to a lack of competence. The concept of competence presented is used beyond the German-speaking countries. For example, Blomhøj and Højgaard (2003, p. 126) describe competence as "someone's insightful readiness to act in a way that meets the challenges of a given situation." This is in line with the Danish KOM project. A person is described as competent when he or she is able to master essential aspects of this field effectively, succinctly and accurately (Niss & Højgaard, 2011).

2.2 Professional Competence of Teachers

The professional competence of a teacher in his or her profession is to be understood as the aforementioned concept of competence based on different occupational requirements since motivational, volitional and social aspects play a role in addition to cognitive dispositions for achievement (Weinert, 2001). In order to clarify the second part of the concept, the conditions under which competence can be described as “professional” are first specified.

2.2.1 Professional Competence

The concept of professional competence is used to describe the skills of teachers needed to meet their professional requirements. Shulman (1998) assigns six attributes to the concept of a profession:

- The obligations of service to others, as in a “calling”
- Understanding of a scholarly or theoretical kind
- A domain of skilled performance or practice
- The exercise of judgment under conditions of unavoidable uncertainty
- The need for learning from experience as theory and practice interact
- A professional community to monitor quality and aggregate knowledge

(Shulman, 1998, p. 516).

In a course, a pre-service teacher acquires basic scientific knowledge in his/her subjects. He/She serves society in the relevant field of education by carrying out his/her activities, and in doing so, through his/her evaluations, has a significant influence on the individuals to be formed. He/She sees himself/herself as a life-long learner and works professionally with colleagues to ensure the quality of school education. According to these characteristics, the profession of the teacher can be clearly described as a profession and professional competence can be regarded as a combination of the following factors with regard to the above concept of competence:

- “Specific declarative and procedural knowledge (competence in the narrow sense: knowledge and skills)
- Professional values, beliefs, and goals
- Motivational orientations
- Professional self-regulation skills”

(Baumert & Kunter, 2013, p. 28).

The specific competences of the above-mentioned aspects have been described differently in different conceptualisations. These conceptualisations are addressed in the following section.

2.2.2 *Conceptualisations of Professional Competence of Mathematics Teachers*

Shulman (1986, 1987) initiated an international discussion with his proposals to conceptualise the teacher knowledge. He developed and distinguished the following categories of professional knowledge as components of professional competence:

- *Content Knowledge*. This is pure expertise in the respective field. This includes knowledge about the systematic of the subject to organise the material according to the abilities of students. In this context, contents must be selected with regard to their importance for the subject.
- *Pedagogical Content Knowledge*. It contains content with regard to teaching. These include useful forms of representation of teaching content, analogies, examples and explanations. The teacher must have different approaches and forms of representation and be able to choose between them. The focus is on the knowledge about how students can learn content. For this purpose, the teacher must be aware of the difficulties of certain subjects and involve foresight, prior experience and misconceptions in the learning process.
- *Curricular Knowledge*. Knowledge about the educational plan that contains and arranges the topics in the different class levels is found in this knowledge dimension. It also includes the knowledge and understanding of various methods and materials for teaching instruction. A lateral curricular knowledge characterised by knowledge about current topics in other subjects and a vertical curriculum containing knowledge about topics and content that have been dealt with in the past and will be dealt with in the future can be distinguished.
- *Pedagogical Knowledge*. This is no subject-specific knowledge of a teacher, such as knowledge about effective class management and dealing with disciplinary problems. Shulman (1986, 1987) did not further differentiate this dimension since the focus is on content knowledge.

The teacher's perspectives, however, are not assigned to professional knowledge in the currently discussed conceptualisations, but to a specific construct, beliefs, attitudes or values (Baumert & Kunter, 2013).

There are different concepts of professional competence of mathematics teachers. In particular, the pedagogical content knowledge for mathematics teachers is conceived in a variety of ways, including in part content knowledge and pedagogical knowledge (Depaepe et al., 2013). Following the work of Shulman (1986, 1987), the professional competence of teachers was also examined in the German-speaking region using appropriate competence models. The studies "Mathematics Teaching in the 21st Century" (MT21; Schmidt et al., 2011; Tatto, Schwille, Senk, Ingvarson, Peck and Rowley, 2008) and their follow-up studies "Teacher Education and Development Study—Mathematics" (TEDS-M; Blömeke et al., 2014) and TEDS—Follow Up (TEDS-FU; Kaiser et al., 2015). Other studies include "Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers" (COACTIV) and its follow-up studies "COACTIV Internship" (COACTIV-R) and "COACTIV

Table 2.1 Conceptualisations of professional competence of (pre-service) teachers

MT21 (Tatto et al., 2008)	COACTIV (Kunter et al., 2013)
Professional knowledge Content knowledge Pedagogical content knowledge Teaching-related requirements (curricular and planning-related knowledge) Learning-related requirements (interaction-related knowledge) General pedagogical knowledge General didactic knowledge Pedagogical psychological knowledge Educational sociological knowledge	*Professional knowledge* Content knowledge Pedagogical content knowledge Explanatory knowledge Knowledge about the mathematical thinking of students Knowledge about mathematical tasks Pedagogical psychological knowledge Knowledge about effective class management Knowledge about teaching methods Knowledge about performance assessment Knowledge about individual learning processes Knowledge about individual peculiarities of students Advisory knowledge Organisational knowledge
Other aspects of professional competence Beliefs Epistemological beliefs Educational beliefs Professional beliefs	*Other aspects of professional competence* Beliefs, values, aims Epistemological beliefs Beliefs about teaching and learning mathematics Motivational orientations Career selection motivation Enthusiasm Self-efficacy Self-regulation

University Study" (Kunter et al., 2013). Table 2.1 compares the conceptualised components of professional competence in studies MT21 (Tatto et al., 2008) and COACTIV (Kunter et al., 2013).

The core of the MT21 study is a standardised test of the declarative and procedural knowledge as well as of the interdisciplinary, pedagogical knowledge of pre-service teachers, which were analysed in a multi-level model against the background of an effectiveness evaluation of mathematics education in international comparison. For this purpose, representative samples were drawn in 17 participating countries, taking into account two target populations of incoming mathematics teachers, namely those from primary (up to grade 4) and secondary (up to grade 8) (Tatto et al., 2008). It was shown that Germany belongs to a group of countries where both the content knowledge (CK) and the pedagogical content knowledge (PCK) of the secondary school pre-service teachers are significantly above the international average (Blömeke & Kaiser, 2014).

The COACTIV study was technically and conceptually linked to the second PISA test in Germany. Secondary school mathematics teachers whose students were part of the sample for the PISA 2003 survey of mathematical competence were interviewed. The study design used by COACTIV and PISA provided for the inclusion of complete school classes. Beyond the survey date of 2003 which PISA set at the end of the 9th grade, teachers and students were surveyed again at the end of the 10th grade in order to generate a true longitudinal section of combined teacher-student data. Grammar school teachers performed much better in both their content knowledge (CK) and in their pedagogical content knowledge (PCK) than teachers in other forms of school. Teachers with a high level of pedagogical content knowledge use tasks with a high potential for cognitive activation and provide good support for individual learning of students (Krauss et al., 2008; Kunter & Baumert, 2013).

The concepts of professional competence developed in the COACTIV and MT21 and TEDS-M studies show great similarities. These include, among other things, the use of research findings on teacher expertise and the associated assumptions on the knowledge and skills of teachers (Draughtsman & Conklin, 2005) as well as the use of a concept of competence (Weinert, 2001) from empirical education research and an overarching model of professional competence of teachers (Depaepe & König, 2018). In particular, both concepts of professional competence look at professional knowledge, which is composed of different areas of knowledge. These are content knowledge (CK), pedagogical content knowledge (PCK) and pedagogical-psychological knowledge (PK). In addition, professional competence encompasses aspects of affective and value-oriented aspects in addition to the cognitively oriented knowledge dimensions mentioned above.

However, there are differences within the areas of knowledge. For example, the concept of Blömeke and Kaiser (2014) looks at teaching and learning process-related requirements within the field of pedagogical content knowledge, while Baumert and Kunter (2013) distinguish explanatory knowledge, knowledge about the mathematical thinking of students and knowledge about mathematical tasks. The concept of COACTIV (Baumert & Kunter, 2013) also uses the categories consulting knowledge, organizational knowledge, motivational orientations and self-regulation.

2.3 Competence Dimensions for Teaching Mathematical Modelling

In order to be able to interpret the COACTIV model for the field of mathematical modelling, it is necessary to identify requirements for teachers that arise in the preparation and implementation of mathematical modelling processes. Borromeo Ferri and Blum (2010) describe skills they consider necessary for teaching mathematical modelling (see Table 2.2). Each of these dimensions is concretely specified by three facets of knowledge and/or ability. These include, in addition to declarative and procedural knowledge (e.g. recognition of phases in the modelling process) also action skills (e.g. conduct reality-related mathematics lessons) by (pre-service) teachers.

The *theoretical dimension* provides a background necessary and important for practical work, which is based on theoretical conceptualisations and empirical studies

Table 2.2 Competence dimensions for teaching mathematical modelling (cf. Borromeo Ferri, 2018)

Dimension	Facets
Theoretical dimension	Modelling cycles Aims/Perspectives of mathematical modelling Criteria/Types of modelling tasks
Task-related dimension	Processing of modelling tasks Cognitive analysis of modelling tasks Development of modelling tasks
Teaching-related dimension	Planning of reality-related mathematics lessons Conduct reality-related mathematics lessons Interventions during modelling processes
Diagnostic dimension	Identifying phases in the processing process Identifying difficulties in the processing process Evaluation of modelling tasks

of the current modelling discussion (Borromeo Ferri, 2018). It includes knowledge about modelling cycles and their suitability for various purposes (see Sect. 1.1.2). The educational aims associated with mathematical modelling (see Sect. 1.2) as well as knowledge about the different criteria of modelling tasks (see Sect. 1.3) form the basis of any teaching.

In particular, the last facet has strong constrictions on the *task-related dimension*. This includes the knowledge and ability to handle modelling tasks in various ways. This allows the teacher to identify different approaches for the task and thus gain an idea of the variety of solutions. Besides the cognitive analysis of the modelling task difficulties are anticipated (see Sect. 1.4.2). If modelling tasks are to be used in the classroom in a targeted manner, the development of tasks is necessary. This allows the development of individual partial competencies or modelling competence in a broad sense (see Sect. 1.1.3) depending on the requirement.

The *teaching dimension* includes knowledge and ability aspects for theory-guided planning and subsequent implementation of reality-related mathematics lessons (Borromeo Ferri & Blum, 2010). As described in Sect. 1.4, this is characterised by the choice of an appropriate learning environment. While teaching, attention should be paid to the observance of the characteristics of self-directed and cooperative learning. In the case of difficulties in the modelling process, the teacher supports by adaptive intervention (see Sect. 1.4.1).

The *diagnostic dimension* focuses on the classification of students' modelling activities in the different phases of the modelling cycle and on the identification of any cognitive hurdles in the processing process (see Sect. 1.4.2). For this purpose, teachers need knowledge and skill aspects from the field of pedagogical diagnostics as well as concrete access to the recognition and documentation of progress, difficulties and errors in the modelling process of students (Borromeo Ferri, 2018). Therefore, knowledge about the different phases of the modelling cycle is essential for the effective and results-oriented execution of these activities. Finally, developing and evaluating a performance test with modelling tasks is another facet of the diagnostic dimension.

2.4 A Competence Model for Teaching Mathematical Modelling

The competence dimensions shown in Table 2.2 are used in the interpretation of the COACTIV model (see Table 2.1), in particular the professional knowledge, to derive a structural model of professional competence for teaching mathematical modelling. In order to distinguish between interdisciplinary professional competences of mathematical teachers, the structural model must be as specific as possible to the field of mathematical modelling. Certain aspects and areas of the professional competence of teachers, therefore, seem more important in the context of mathematical modelling (see Fig. 2.1).

In this way, the aspects of *beliefs* can be interpreted as part of beliefs/values/aims and *self-efficacy expectations* as part of the motivational orientations through modelling-specific aspects. On the other hand, the aspect of *self-regulation* refers to the personality characteristics of the teacher and is therefore independent of any specific concretisation. In terms of *professional knowledge*, the area-specific interpretation of *pedagogical content knowledge* can be focused, since it is assumed that the pedagogical content knowledge is the central factor in determining the cognitive activation potential of teaching (Baumert & Kunter, 2013).

The conceptualisations of the three competence aspects and areas, the beliefs and self-efficacy expectations for mathematical modelling and the modelling-specific pedagogical content knowledge are presented in the following sub-sections. Based on the COACTIV study, the pedagogical content knowledge is understood as declarative and procedural knowledge (knowledge and skills) (Baumert & Kunter, 2013), which is measurable in its competence facets about competencies in the narrow sense. The constructs were examined for empirical testing using a structural equation model (Klock et al., 2019; Wess et al., 2021).

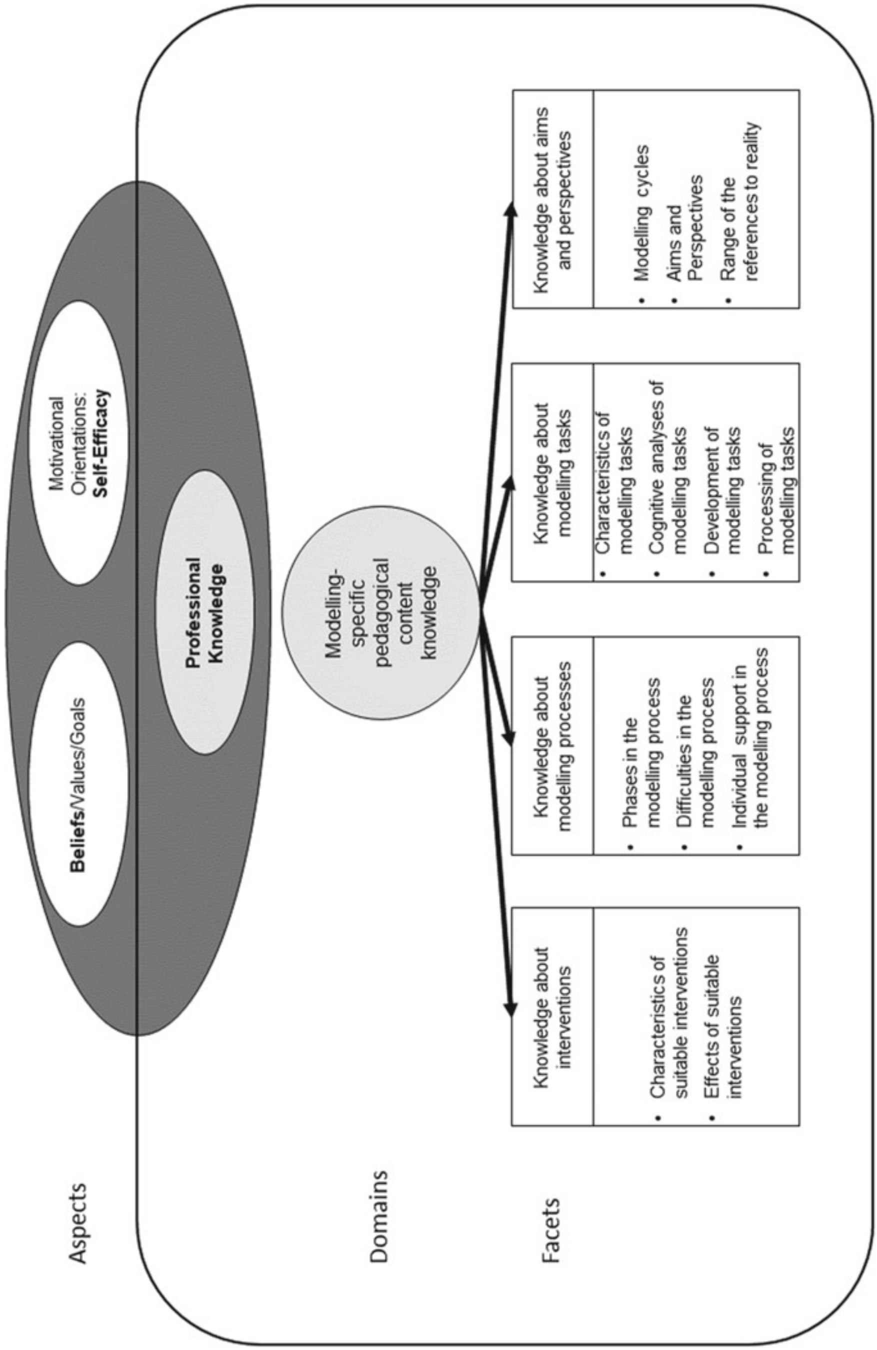


Fig. 2.1 Structure model of professional competence for teaching mathematical modelling (Wess et al., 2021)

2.4.1 Modelling-Specific Pedagogical Content Knowledge

The COACTIV model breaks down the pedagogical content knowledge into explanatory knowledge, knowledge about the mathematical thinking of students and knowledge about mathematical tasks (see Table 2.1). These three competence facets were based on the competence dimensions for teaching mathematical modelling (Borromeo Ferri, 2018; Borromeo Ferri & Blum, 2010) and further broadens the *knowledge about concepts, aims and perspectives* (see Fig. 2.1).

The explanatory knowledge facet is interpreted as knowledge about interventions using the skills "Performing *reality-related mathematics education*" and "Interventions during modelling processes" in Table 2.2. This facet of competence includes knowledge about the characteristics of adaptive interventions and the effect of different interventions on the learner's solution process. Capabilities to assess interventions in terms of adaptability and to perform interventions adequately are typical requirements for teachers in the management of mathematical modelling processes. In the field of mathematical modelling, it is specific that the interventions are characterized by a high degree of independence orientation (Smit et al., 2013; Van de Pol et al., 2010) and minimal intervention in the solution process, so that only in a few cases a direct explanation of the teacher is appropriate in the sense of explanatory knowledge. The competence facet "planning reality-related mathematics teaching" includes, among other things, the selection of appropriate social forms and methods, which is part of the scope of pedagogical-psychological knowledge. Therefore, this facet is not included in the conceptualisation.

The facet involving the knowledge about students' mathematical thinking is interpreted as a *knowledge about modelling processes by means of the competencies* "Identification of modelling phases" and "Identification of difficulties and errors" in Table 2.2. This competence facet includes the skills to diagnose modelling phases and difficulties in the modelling process and to set support goals for interventions based on these. This requires specific knowledge about the modelling process and influencing factors as well as of typical difficulties. This diagnostic component is central to the modelling-specific pedagogical content knowledge. On the one hand, diagnostic skills in the modelling process are a prerequisite for intervention-related competencies as described in the Knowledge about interventions facet. On the other hand, they are necessary for the diagnosis of demands and thus for the selection and development of cognitively activating tasks, which also have links to task-related knowledge.

The Knowledge about mathematical tasks facet is interpreted as *knowledge about modelling tasks* based on the aspects "Characteristics of modelling tasks," "Processing of modelling tasks," "Cognitive analysis of modelling tasks" and "Development of modelling tasks" in Table 2.2. This competence facet includes knowledge about different types and criteria of modelling tasks. It also includes skills for the criteria-driven development of modelling tasks, as well as their analysis and processing with regard to multiple solutions. These become even more solid when the (pre-service) teachers are given the opportunity to develop their own modelling tasks

(Borromeo Ferri & Blum, 2010). The development of mathematical tasks for certain topics and content fields is a demanding, complex and time-consuming activity. Furthermore, due to the many requirements that teachers must fulfil every day in school, there is little room for students to do modelling tasks. The comprehensive classification scheme for the categorisation and analysis of modelling tasks in accordance with Maaß (2010) in conjunction with the explanations for the task design in Czocher (2017) provide a theoretical basis for the facets mentioned here. Therefore, there is a great need for good and high-quality teaching materials, including mathematics, and in particular modelling problems (Borromeo Ferri, 2018).

The remaining competencies of the theoretical dimension "Modelling cycles" and "Goals/perspectives of modelling" in Table 2.2 are interpreted as *knowledge about concepts, aims and perspectives* in another competence facet. It consists of selected aspects of theoretical background knowledge. On the one hand, knowledge about modelling cycles and their suitability for various purposes is described, for example as a metacognitive strategy for learners or as a diagnostic tool for teachers. On the other hand, various perspectives of mathematical modelling research are presented (Kaiser & Sriraman, 2006), for example modelling as a means of learning mathematics and fulfilling other curriculum needs (Julie & Mudaly, 2007). In addition, teachers should be aware of the relevant aims of mathematical modelling in the classroom and of the different relevance of reality references for students.

In addition to specialised knowledge specific to modelling, beliefs and self-efficacy in mathematical modelling processes are also part of the professional competence to teach mathematical modelling.

2.4.2 Beliefs Regarding Mathematical Modelling

In German literature, the concepts of convictions, beliefs, ideas, notions, subjective theories, world-views and attitudes are often used in parallel without any clear distinction being made (Voss et al., 2013). In English literature, a similar blur is found, but the term "beliefs" is used predominantly (Leather et al., 2002): "The term "belief" is often used loosely and synonymously with terms such as attitude, disposition, opinion, perception, philosophy, and value" (Leder & Forgasz, 2002, p. 96).

Since the conceptual definitions overlap, uniform concrete specification is difficult (Leder & Forgasz, 2002). In this work, the concept of beliefs is therefore preferred, following the COACTIV study. These include "psychologically held understandings and assumptions about phenomena or objects of the world that are felt to be true, have both implicit and explicit aspects, and influence people's interactions with the world" (Voss et al., 2013, p. 249).

They are relatively stable cognitive structures (Voss et al., 2013) since their filtering function strengthens the perception of content that corresponds to one's own beliefs and reduces the perception of inconsistent content (Törner, 2002). According to Patrick and Pintrich (2001), change requires an intensive examination of one's beliefs and other perspectives. It is also possible to specify beliefs for specific content areas (Voss et al., 2013). Törner (2002) structures beliefs in the following three hierarchical aspects:

- *Global Beliefs*. General beliefs include beliefs about teaching and learning mathematics, the nature of mathematics, and the development of mathematical knowledge.
- *Domain-specific Beliefs*. Domain-specific beliefs include beliefs about specific mathematical sub-fields, such as analysis, stochastics or geometry. These may differ, for example, in their beliefs about the accuracy of mathematics in each field.
- *Subject-matter Beliefs*. Subject-matter beliefs are beliefs that refer to a concrete mathematical term (e.g. derivative), a mathematical object (e.g. function) or a mathematical procedure (e.g. bisection).

The first aspect is particularly attractive if beliefs for mathematical modelling are to be conceptualised, since it has a relatively high degree of generality. Woolfolk Hoy et al. (2006) distinguish *epistemological beliefs* and *beliefs about teaching and learning* mathematics with respect to the teaching–learning processes. Epistemological beliefs refer to the structure and genesis of knowledge (Buehl & Alexander, 2001). Rösken and Törner (2010) capture them via the mathematical world views that represent beliefs with respect to components of mathematics. They distinguish between the formalism aspect, the application aspect, the process aspect and the schema aspect. In terms of mathematical modelling, the application aspect is of particular interest, which relates to the meaning and utility of mathematics in the real world. Rösken and Törner (2010) summarise under this mathematical world view beliefs about the benefits of mathematics and its everyday and social significance. Due to the application orientation and the realism of modelling tasks, this aspect is suitable for the conceptualisation of *epistemological beliefs for mathematical modelling*. Thus, statements that convey a practical benefit to mathematical modelling in the world are understood as epistemological beliefs about mathematical modelling (Klock et al., 2019; Wess et al., 2021).

Beliefs about teaching and learning mathematics include beliefs about educational goals, teaching methodological preferences and classroom management. According to Kuhs and Ball (1986), three approaches can be distinguished: a learner-focused approach, a content-focused approach with a core focus on conceptual understanding, and a content-focused approach with a core focus on performance. Teachers with learner-focused beliefs see mathematical learning as an active process of knowledge construction (Voss et al., 2013). The content-oriented beliefs differ depending on whether the focus of the teacher is on promoting a conceptual understanding of the content treated or on developing the ability of the students to apply mathematical rules and procedures. This also involves differentiating the cognitive learning aims in terms of routine construction, problem solving and modelling, arguing and reasoning, and evidence. *Beliefs about modelling in mathematics education* and its goal can thus be assigned to beliefs about teaching and learning.

Both areas of beliefs about the teaching–learning processes can be viewed at a meta-level from the perspective of behaviourist and constructivist learning theories. This means that both epistemological beliefs and beliefs about teaching and learning mathematics can be *understood from a more* transmissive *or* constructivist perspective (Voss et al., 2013). Such correlations between beliefs about learning and beliefs for modelling could also be demonstrated empirically (Kuntze & Zöttl, 2008; Schwarz et al., 2008). Therefore, both positively correlated constructivist beliefs and negatively correlated transmissive beliefs contribute to the description of beliefs in mathematical modelling.

2.4.3 Self-Efficacy Expectations for Mathematical Modelling

Self-efficacy expectations are an empirically founded feature of professional competence (Kunter, 2013). The notion of the self-efficacy expectation is understood as an assessment of one's own effectiveness in certain situations. "A teacher's efficacy was a judgement of his or her capabilities to bring about desired outcomes of student engagement and learning, even among those students who may be difficult or unmotivated." (Tschannen-Moran & Woolfolk-Hoy, 2001, p. 783).

They are important for teaching and influence the performance, beliefs and motivation of students (Philippou & Pantziara, 2015). They go hand in hand with a higher quality of teaching, the use of more innovative and effective methods in teaching and a higher level of commitment of the teacher (Kunter, 2013).

Self-efficacy can be specified and concretised in relation to teacher's ideas about their own effectiveness in mathematical modelling processes. The content of the activities is determined by the facets of the modelling-specific pedagogical content knowledge. One of the main activities of the teacher during cooperative modelling processes is the diagnostics of the processing procedure. Since the diagnostic component is related to both the intervention and task-related knowledge facets (see Sect. 2.3), the self-efficacy expectations regarding the assessment of one's ability to diagnose the performance potential of students in the modelling process are operationalised.

The modelling process of the students is characterised by different activities and cognitive processes in different phases. Different diagnostic processes are necessary for the different modelling phases in which the students are currently working. This justifies the assumption that the self-efficacy of the teacher also differs according to the modelling phase. Regarding the activities of the students and the associated diagnostics, it is particularly possible to identify phases that are not specific to the modelling process and in which the activities can be traced by written materials (mathematical work), phases that are specific to the modelling process (simplification/structuring; mathematisation; interpreting; validation). The self-efficacy expectations for mathematical modelling are therefore conceptualised for the diagnosis of performance potentials for the *activities of mathematical working* and *modelling*.

2.4.4 Empirical Validation of the Structural Model

The preceding sections present the underlying structural model of professional competence for teaching mathematical modelling, which results from a specific design of the COACTIV model (see Table 2.1), taking into account theoretical and empirical insights of the current research on mathematical modelling. This naturally exploits the different research traditions that form the basis of the COACTIV model: thus, the emphasis on knowledge and skills—here in the form of modelling-specific pedagogical content knowledge—as a core of professionalism finds its connection in the highlighted work on expert research and regarding structural elements in the dimensions of Borromeo Ferri and Blum (2010). The beliefs, values and aims are also based on the statements of Rösken and Törner (2010), which were supplemented by perspectives on teaching and learning mathematics (Kuhs & Ball, 1986). Likewise, this specific interpretation shows the clear alignment of cognitive characteristics of teachers, which justify a conceptual summary of both aspects as "modelling-specific expertise." In this context too, the notion of competences as cognitive abilities and skills that can be learnt in principle is emphasised (Cochran-Smith & Draughtsman, 2005; Darling-Hammond & Bransford, 2005). Extending it to include motivational orientations—in the form of self-efficacy expectations—the present structural model follows the example of the COACTIV study and goes beyond the understanding of expertise presented by reinterpreting these aspects with regard to selected facets of teaching mathematical modelling (see Baumert & Kunter, 2013).

For the empirical review of the conceptualised structure (see Fig. 2.1), a structural equation analysis was carried out based on a data set of 156 pre-service teachers from several German universities (for a deeper consideration, see Klock et al., 2019). The fit indices CFI, RMSEA and SRMR as well as the non-significant Chi^2 test point to a global fit of the model, relying on the Hu and Bentler guide values (1998). The local fit is impaired by a minor and insignificant charge of the transmissive beliefs ($\lambda = -0.09$) and a low variance elucidation ($R^2 = 0.01$). A negative load is fully in line with expectations, as constructivist and transmissive beliefs correlate negatively due to the different theoretical perspectives of learning (Voss et al., 2013). All other scales load significantly with a medium to high significance on the respective constructs. The structure of the model can therefore be confirmed by the analysis. In addition, a significant but only weak latent connection between the beliefs about mathematical modelling and the modelling-specific pedagogical content knowledge can be demonstrated empirically ($r = 0.38$). Both these two constructs also do not correlate significantly and with little significance with the self-efficacy expectations for mathematical modelling.

Based on the inclusion of beliefs, self-efficacy and cognitive dispositions for achievement, competences are measured in a broad sense (see Sect. 2.1). The results must be compared before a relatively small and non-representative sample for the evaluation methodology. Therefore, a review of the structural model on the basis of further data was necessary in order to demonstrate any significant interaction with the expectations of self-efficacy. A sample of 349 pre-service teachers from several

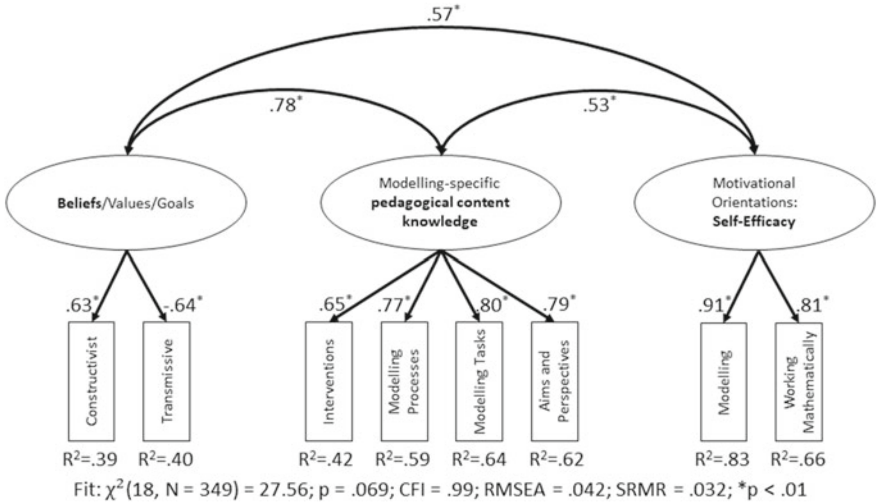


Fig. 2.2 Structural equation analysis ($N = 349$) with two belief scales (Wess et al., 2021)

German universities was used. On the one hand, the associated results demonstrate the above-mentioned interactions with self-efficacy expectations and on the other, eliminate the deficiencies in the local fit. The Chi^2 test, on the other hand, becomes significant, which can be considered problematic for the global fit (Hu & Bentler, 1998; in detail in Chap. 3). However, in view of the explanations of the beliefs on mathematical modelling (see Sect. 2.4.2), another theoretically sound model structure can be examined (see Fig. 2.2; for a deeper consideration, see Wess et al., 2021), which, instead of four belief scales, only looks at two scales located at a meta-level: a constructivist-oriented scale and a transmissive-oriented scale.

In view of the fit indices, the model specified in this manner shows a very good global fit to the current data set (Hu & Bentler, 1998). In addition, correlations of medium practical relevance between self-efficacy expectations and beliefs about mathematical modelling ($r = 0.57$) and between these and high scores can be demonstrated in the modelling-specific pedagogical content knowledge ($r = 0.53$). A significant correlation of medium to high practical significance between beliefs and specific pedagogical content knowledge can also be identified ($r = 0.78$). In addition, all scales have significant loads of high significance.

Overall, the above results show that the professional competence to teach mathematical modelling can be structurally validated in the three areas of competence. The significant latent correlations between beliefs and self-efficacy expectations for mathematical modelling, as well as between these and the modelling-specific pedagogical content knowledge, point to interdependencies between constructs, which points to an affiliation with an overarching construct—the professional competence to teach mathematical modelling.

In the context of quality development in teacher education, the theoretical fundamentals outlined serve to operationalise professional competence for teaching mathematical modelling. The development of the associated test, which also served as a survey tool for the above analyses, forms the centre of this book and is discussed in detail in the following chapter.

Part II
Assessment of Professional Competence for Teaching Mathematical Modelling

Chapter 3
Test Instrument

Key didactic aspects of mathematical modelling (see Sects. 1.1, 1.2 and 1.3), in particular the possibilities of characterising and acquiring modelling competence (see Sects. 1.1.3 and 1.4), were explained in previous sections. Against this background, the important role that the teacher plays in the development of competences among students was highlighted and the necessary professional competence was demonstrated (see Sect. 2.2). In this context, different perspectives and conceptualisations were considered, taking into account two large scale national and international studies on empirical teacher competences (see Sect. 2.2.2). The subsequent presentations focused on the competence model of the COACTIV study, which together with the competence dimensions according to Borromeo Ferri and Blum (2010; see Sect. 2.3) formed the basis for the interpretation of a *structural model of modelling-specific professional competence* (Klock et al., 2019; Wess et al., 2021; see Sect. 2.4).

This theory of underlying forms of knowledge and structures is the prerequisite for the acquisition of specific knowledge and ability and corresponding affective-motivational aspects of teachers. However, until then there had hardly been any preliminary work for a theory-based approach to the collection of modelling-specific competence of teachers in which an instrument was developed with the help of theory (Borromeo Ferri & Blum, 2010; Maaß & Gurlitt, 2011). In parallel to the structural model of professional competence for teaching mathematical modelling, as described in Sect. 2.4, a related test instrument (see Sect. 3.5) was developed, whose construction is described below.

3.1 Test Development

The operationalisation of the structural model was based on the principle of rational and effective test construction (Bühner, 2011; Burisch, 1984; Downing and Haladyna, 2011). In this process, an instrument—in German (Klock and Wess, 2018)—was

R. Wess et al., *Measuring Professional Competence for the Teaching of Mathematical Modelling*, International Perspectives on the Teaching and Learning of Mathematical Modelling, https://doi.org/10.1007/978-3-030-78071-5_3

developed to capture aspects of professional competence for teaching mathematical modelling. The content of the item construction was based on the individual components of the structure model (see Sect. 2.4). For the first version of the instrument, four scales were designed for self-reported previous experiences (15 items), four scales for beliefs (32 items), two scales for self-efficacy expectations (24 items) and four scales for modelling-specific pedagogical content knowledge (103 items). Two of the latter scales are recorded using task examples with case-based text vignettes to query the underlying facets in different non-mathematical contexts and learning situations. All items used in the test are formulated in a closed form, whereby combined true-false, multiple-choice, and mapping tasks to capture modelling-specific pedagogical content knowledge are used in addition to Likert scales to capture the self-reported prior experiences, beliefs, and self-efficacy expectations.

The items in this first test version were checked in a small sample ($N = 8$) as part of a qualitative pre-pilot with a subsequent discussion. The aim was to revise incomprehensible or inaccurate items. Subsequently, the test was first checked and quantitatively evaluated with a small sample size ($N = 66$), where critical items were excluded using statistical key parameters and didactic considerations. As a result, the number of items in the various scales decreased to 15 items for the self-reported previous experiences, 16 items for beliefs, 24 items for self-efficacy expectations and 71 items for modelling-specific pedagogical content knowledge.

Finally, the instrument was comprehensively piloted using a random sample of pre-service teachers from several German universities ($N = 156$) in various events (Klock and Wess, 2018). Excerpts of the related confirmatory factor analysis of the underlying competence model as well as a larger sample replication study ($N = 349$) confirming the design of the model in the forms were presented in Sect. 2.4.4. For a deeper consideration, please refer to the contributions of Klock et al. (2019) and Wess et al. (2021). In addition, a classification of the results of the replication study in relation to general standards in the form of quality criteria for quantitative test instruments can be found in Part 4 of this book.

The executed test duration including instructions is approximately 70 min, while the maximum processing time of the test is 60 min. It is conducted as a single test in groups. The test books are filled in anonymously. A personal code is generated at the beginning of each test book to link different test sheets of individual participants in the course of studies with multiple measurement points.

In order to provide an insight into the test book for the collection of professional competence for teaching mathematical modelling, the test items are described below and explained in detail using individual examples. All the contents of the instrument have been designed with reference to item formulation guidelines (Bühner, 2011; Impara & Foster, 2011). The complete instrument with all tasks can be found in the attached test book (see Sect. 3.5). The corresponding solutions can be found in the Appendix.

3.2 Operationalisation of Test Items: First Test Part

The test begins with the generation of the personal code for the anonymised assignment of the subjects (e.g. as part of pre-post examinations) and some brief questions on general information (gender, age, school-leaving examination grade, last mathematics grade, second subject and semester). This is followed by the items for self-reported previous experiences, beliefs and self-efficacy expectations for mathematical modelling in a first part of the test, before the focus is placed on the individual facets of the modelling-specific pedagogical content knowledge (see Sect. 3.3). The former is recorded using a five-point Likert scale with the expressions "strongly disagree" (rated at 1), "disagree" (rated at 2), "neutral" (rated at 3), "agree" (rated at 4), and "strongly agree" (rated at 5). Five points were chosen to provide participants with a medium expression and not to force them to choose a position on the statement. Five points also make it possible to express one's own degree of approval in a differentiated manner by means of a verbal gradation (Bühner, 2011; Reckase, 2000).

3.2.1 Self-reported Previous Experiences in Mathematical Modelling

Short scales were used to control the self-reported previous experiences in mathematical modelling, wherein three or six items are used to capture self-assessments of different areas of experience. For this purpose, an item pool was developed and it led to the formation of four scales after exploratory factor analysis. Items 2.2, 2.4, 2.6, 2.8, 2.10 and 2.13 form the "Teaching and preparing for mathematical modelling" scale, items 2.1, 2.5 and 2.11 form the "Treatment of mathematical modelling" scale, items 2.7, 2.12 and 2.15 form the "Modelling tasks" scale and items 2.3, 2.9 and 2.14 form the "Modelling in the classroom" scale. All items are positively formulated, so there is no need to reposition. Sample items are specified in Table 3.1.

The first scale, "Treatment of mathematical modelling," measures the extent to which mathematical modelling has generally played a role in the pre-service teachers' secondary school courses. No focus is placed on certain aspects, so that both scientific and didactic events are included in the assessment.

The second scale, "Teaching and preparing for mathematical modelling," measures the degree to which the pre-service teachers have been, and feel prepared, for teaching mathematical modelling. This scale includes both items to assess the extent to which the development of modelling competence among students has played a role in the courses that have been completed so far and items to assess their own competence in this area.

The "Modelling tasks" scale records whether modelling tasks have already been processed in another event. It gives an impression of how students have gained

Table 3.1 Sample items for self-reported previous experience in mathematical modelling

Scale	Number	Item example
Treatment of mathematical modelling	3	"Mathematical modelling has already played a role in a course I attended"
Teaching and preparing for mathematical modelling	6	"I feel well prepared to teach mathematical modelling through my previous training"
Modelling tasks	3	"I have been working on modelling examples myself during my teacher education studies"
Modelling in the classroom	3	"I have already done mathematical modelling with students"

experience with modelling tasks and acquired modelling competence. In this scale, both scientific and didactic events can contribute to the value of a scale.

A final scale, "Modelling in the classroom," records whether students already have experience of modelling. The focus is not only on school education and practical experience in the context of internships, but also extracurricular activities, such as experience in tutoring, are recorded by the scale.

3.2.2 Beliefs in Mathematical Modelling

In conceptualising the beliefs for mathematical modelling, as described in Sect. 2.4.2, epistemological beliefs and beliefs about teaching and learning mathematics are distinguished (Woolfolk Hoy et al., 2006). In addition, beliefs about learning (Voss et al., 2013), that is constructivist and transmissive beliefs, contributes to the operationalisation of mathematical modelling beliefs. Therefore, the beliefs about mathematical modelling are operationalised with the help of two scales, a *constructivist* and a *transmissive* scale, whereby the scale of constructivist-oriented beliefs can consist of three theoretical sub-scales.

Epistemological beliefs generally refer to the structure and genesis of knowledge (Buehl & Alexander, 2001) and have been operationalised in terms of mathematics teaching on the formalism aspect, the application aspect, the process aspect and the schema aspect (see Sect. 2.4.2). Items of the application aspect are suitable for the operationalisation of epistemological beliefs regarding mathematical modelling due to their application reference. The items record the extent to which mathematical modelling is considered an everyday or social benefit (see Table 3.2). This eventually led to the scale "Beliefs for the application of mathematical modelling," which consists of items 3.2, 3.5, 3.12 and 3.15. Item 3.5 is negatively worded and must be reverse-scored for scaling.

Beliefs for teaching and learning mathematics are operationalised in terms of mathematical modelling regarding educational aims of teachers (see Sect. 2.4.2). It

Table 3.2 Sample items on beliefs about mathematical modelling

Scale		Number	Item example
Constructivist-oriented beliefs	Beliefs on the use of mathematical modelling	4	"Many aspects of mathematical modelling have a practical use or a direct application reference"
	Beliefs about mathematical modelling in the classroom	4	"Mathematical modelling should be a part of mathematics education"
	Constructivist beliefs	4	"Students learn mathematics best by discovering ways to solve problems themselves"
Transmissive-oriented beliefs	Transmissive beliefs	4	"Effective teachers demonstrate the right way and methods to solve an application problem"

records the participants' agreement to statements that grant mathematical modelling a legitimate place in mathematics education and consider it important to promote modelling competence (see Table 3.2). Items 3.1, 3.3, 3.9 and 3.13 form the "Mathematical modelling in the classroom" scale.

Existing items from Staub und Stern (2002) were used to record the beliefs about learning in their constructivist and transmissive forms, as they were also used in the COACTIV study (Voss et al., 2013). Items on constructivist beliefs represent perspectives that students should discover their own ways of solving tasks, work independently and discuss their ideas for solutions (see Table 3.2). Items for transmissive beliefs represent the view that teachers should teach detailed procedures and provide schematics even for application tasks. These kinds of beliefs have been included, as constructivist views have a connection with positive beliefs about modelling and transmissive views tend to be accompanied by negative beliefs about modelling (Kuntze & Zöttl, 2008; Schwarz et al., 2008). Both types of beliefs cannot be understood as two opposing poles. Rather, they are adjoining negatively correlated constructs, such as those empirically proven by Voss et al. (2013). For this reason, two scales were formed. The "Constructivist beliefs" scale is based on items 3.3, 3.8, 3.11 and 3.14, whereas the "Transmissive beliefs" scale is based on items 3.6, 3.7, 3.10 and 3.16.

3.2.3 *Self-efficacy Expectations for Mathematical Modelling*

The self-efficacy expectations for mathematical modelling are conceptualised in Sect. 2.4.3 on the basis of ideas about the own effectiveness in the diagnosis of

Table 3.3 Sample items for self-efficacy expectations for mathematical modelling

Scale	Number	Item example
Self-efficacy expectations for mathematical modelling	13	"It is easy for me to recognise the different abilities of the students using their translation of mathematical results into reality"
Self-efficacy expectations for working mathematically	8	"It is easy for me to recognise the different abilities of the students using the mathematical formulae and symbols they used in the modelling process"

performance potentials in mathematical modelling processes. For operationalisation, items were developed that require self-assessment of one's own effectiveness and recognising the skills of students in the phases of the modelling process (see Sect. 1.1.2) using written results. Positive and negatively worded items have been created for each phase.

In factor analysis, two one-dimensional scales emerged. Sample items are shown in Table 3.3. Items 4.1, 4.4, 4.9, 4.10, 4.13, 4.14, 4.15, 4.16, 4.17, 4.18, 4.21, 4.22 and 4.23 for the modelling phases such as simplifying/structuring, mathematising, interpreting and validating form the scale of "Self-efficacy expectations for mathematical modelling," since these are specific phases with specific diagnostic requirements for the modelling process (see Sect. 2.4.3). Items 4.5, 4.7, 4.8, 4.11, 4.12, 4.19, 4.20 and 4.24 for diagnosing written results or mathematical work form the "Self-efficacy expectations for mathematical work" scale, since they are empirically distinct from the items for the other phases of the modelling process. Items 4.13 to 4.24 must be reverse-scored before scaling due to their negative formulation. Not all items in the test book were included in the scale. Items 4.2, 4.3 and 4.6 could not be assigned by factor analysis.

3.3 Operationalisation of Test Items: Second Test Part

In the second part of the test, the facets of the modelling-specific pedagogical content knowledge are collected using combined-true-false, multiple-choice, and mapping tasks (see Sect. 2.4.1). This approach serves as an economic and adequate test of the above facets by reducing the processing, evaluation and solution time and at the same time reducing the probability of guess (Bühner, 2011; Ebel & Frisbie, 1991; Impara & Foster, 2011). Alternatively, the probability of the guess could be considered as an additional parameter in the evaluation using a 3PL model. Due to the lack of model validation tests and the lack of specific objectivity of the 3PL model, this approach was abandoned.

Compared to closed answer formats, open or semi-open response formats would have required encoding of each individual answer, which would have led to low evaluation objectivity. Another test instrument under development to assess competences for teaching mathematical modelling also uses closed item formats. In this context, Borromeo Ferri (2019) also describes the above-mentioned problems that open item formats pose. The difficulty of the dichotomous evaluation was met in all knowledge scales by a normative composition based on a multi-stage expert survey as well as a theoretical foundation based on current results of didactic research on mathematical modelling.

3.3.1 Knowledge about Modelling Tasks

Combined-true-false items are used to record the sub-facet characteristics, development, and processing of modelling tasks, among other things, to gather knowledge about modelling tasks. For this purpose, three items are combined and evaluated together. The task is rated as correct if all three items have been answered correctly. The corresponding scales consist of items 5.1.[1-4], 5.2.[1-4] and 5.3.[1-4].

The example shown (see Fig. 3.1) is about the basic characteristics of modelling tasks. Modelling tasks can thus be overdetermined as well as underdetermined. An example of a specific task is the *Fire-brigade Task* (Blum, 2011), which requires only some of the information specified to be used for the solution. Likewise, the reverse is possible, where the task does not contain all the information needed to resolve it. An example of such an underdetermined task is the *Lighthouse Task* (Borromeo Ferri, 2010), where the missing information (such as the Earth's radius) must be determined using everyday knowledge, estimates or research. The first two statements would therefore be "true," while the last statement is considered to be "false," since the openness of a task is an essential feature of (good) modelling tasks (see Sect. 1.3).

Another subfacet of knowledge about modelling tasks, the analysis and classification of modelling tasks regarding an appropriate catalogue of criteria, can be measured only in conjunction with specific requirement situations which allow the

5.1	**Characteristics of modelling tasks**		
5.1.1.	Modelling tasks...	True	False
... can be underdetermined.		☐	☐
... can be overdetermined.		☐	☐
... are as closed as possible.		☐	☐

Fig. 3.1 Combined multiple-choice sample item regarding knowledge about modelling tasks

Please place the tasks "Container" (1), "Filling Up" (2), "Safe Victory" (3) and "Milk Carton" (4) in the order with regard to the following criteria for modelling tasks. Note the numbers corresponding to the tasks in the table.

8.1	Low openness					High openness

Fig. 3.2 Sample item for assigning and/or reorganising regarding knowledge about modelling tasks

classification of reality-related tasks. This is achieved by assigning or reorganising tasks that relate to the modelling tasks used to gather knowledge about modelling processes and knowledge about interventions (see Sect. 3.3.3). This item format is used for an economic review of knowledge structures, cause-effect relationships or abstraction skills, with a low probability of guess (Bühner, 2011). Specifically, items 8.1, 8.2, 8.3, 8.4 and 8.5 set out the task of analysing four of the aforementioned modelling tasks with regard to theoretically sound criteria for modelling tasks (see Sect. 1.3.3) and ranking them accordingly. Thus, the assignment is considered correct if one of the two options set out based on the multi-level expert survey has been implemented. The example (see Fig. 3.2) deals with the criterion of openness, in which the modelling tasks presented are to be entered into the columns of the table with increasing intensity from left to right. The ranking sequences "(3)(2)(4)(1)" and "(3)(2)(1)(4)" are rated as correct and are therefore coded as 1, while the remaining 22 options are rated as incorrect and coded as 0.

3.3.2 Knowledge about Concepts, Aims and Perspectives

Knowledge about concepts, aims and perspectives of mathematical modelling is gathered only using multiple-choice items. These measure selected aspects of theoretical background knowledge such as knowledge about modelling cycles and their suitability for different purposes or different perspectives of research on mathematical modelling (Kaiser & Sriraman, 2006).

The example in Fig. 3.3 covers the sub-facet of modelling cycles. The task is considered as correct when the first alternative answer has been ticked, which explicitly aims at the designs of Leiss et al. (2010) for the modelling cycle from a cognitive psychological perspective. Since the fourth statement takes a contrary position and the situation model, contrary to the third answer option, is cognitively formed by the individual (Kaiser et al., 2015), these, as well as the second statement, which is realised, for example in Schupp's cycle (1989), represent distractors of the item under consideration.

6.1.1.	Cognitive modelling distinguishes between the situation model and the real model.	☐
	A direct transition from the real situation to the mathematical model is not possible in modelling cycles.	☐
	The situation model is formed independently of the individual.	☐
	A distinction between the situation model and the real model is not conceivable in modelling cycles.	☐

Fig. 3.3 Sample item for knowledge about concepts, aims and perspectives

3.3.3 Knowledge about Modelling Processes and Knowledge about Interventions

Knowledge about modelling processes and knowledge about interventions can only be measured in conjunction with presented requirements that allow diagnosis and evaluation of intervention in a situational context. There are two main ways to provide such a situational context: (1) The situation is illustrated by a video vignette or (2) The situation is described by text vignettes. Since the analysis of video vignettes is perceived as more burdensome and the more cognitively demanding medium because of the parallelism of the actions they represent (Syring et al., 2015), the cognitively less burdensome text vignettes are used here to illustrate the requirements. In this context, an upstream general scenario (see Fig. 3.4) creates a teaching context that provides general information on the social form, on the experience of students with modelling tasks and their level of performance, on the processing time and on previous interventions, in addition to the specific requirements.

The requirements are presented by modelling tasks and related text vignettes, so-called task vignettes, which illustrate a discussion of students in a small group when they are working on the task. There are six task vignettes in total. Five of the six contexts of the modelling tasks were taken from the literature. The tasks are as follows:

- *Traffic Jam* (Maaß and Gurlitt, 2011),
- *Safe Victory* (Blum et al., 2010),
- *Milk Carton* (Böer, 2018),

You are a teacher at a secondary school and your students at the specified grade level work on the tasks in a small project in **groups of 3**. The students have **already gained experience** with modelling tasks in advance. The situations presented arise in the **first half of the processing time**. The students in consideration have an **average level of performance** for the respective grade level. You observe the students during extracts of the conversations. You have **not yet interfered in the learning process.**

Fig. 3.4 General scenario for the requirement situations

- *Filling Up* (Blum, 2011),
- *Container* (Greefrath & Vorhölter, 2016).

All tasks were slightly modified by replacing or explaining unclear terms, to use them in test situations. However, the heart of the tasks remained the same. The modelling tasks can be assigned to different class levels, which are specified with the task at hand. The test instrument includes tasks for the sixth, eighth, ninth, tenth, and twelfth class. The modelling tasks used in the test instrument have a rather low complexity. The use of more complex modelling tasks would lead to the test score not primarily determined by knowledge about modelling processes and knowledge about interventions, but also essentially the modelling competence of participants. Therefore, participants must be able to easily penetrate the modelling tasks.

The *Traffic Jam Task* (Maaß & Gurlitt, 2011) is used below to illustrate the test items (see Fig. 3.5). For each modelling task, a student discussion was formulated that describes a typical difficulty in the modelling process.

Knowledge about modelling processes

Knowledge about modelling processes is characterised, inter alia, by the identification of the modelling phase and the identification of the difficulty (see Sect. 2.4.1). As a consequence of identifying a difficulty, the teacher should reach a judgement in order to meet the requirement of adequate diagnostics (Heinrichs & Kaiser, 2018). This judgement is operationalised through the formulation of a support goal. As these three steps provide a diagnostic basis, they also operationalize important facets of teacher modelling-specific diagnostic competence of teachers (cf. Borromeo Ferri and Blum, 2010).

7.1 Traffic Jam (9th class)

Traffic jams often occur at the beginning of summer holidays. Christina is stuck in a 20 km traffic jam for 6 hours. It is very warm and she is extremely thirsty. There is a rumour that a small truck is supposed to supply the people with water, but she has not yet received anything. How long will it take for the truck to supply all people with water?

STUDENT 1:	We really need to know how many vehicles are stuck in the traffic jam.
STUDENT 2:	Huh? Right!
STUDENT 1:	How do we calculate how long it takes? A lot of things are missing from the task!
STUDENT 3:	Yeah, we don't know how long it takes for every vehicle.
STUDENT 2:	It is a dumb task.
STUDENT 1:	We can divide the 20 km by the 6 hours, then we know how fast it would take.
STUDENT 3:	Exactly! We do not have any more information.

Fig. 3.5 Task vignette for traffic jam (cf. Maaß & Gurlitt, 2011)

7.1.1.	To which phase of the solution process can the group of students **mainly** be assigned to? Please check **one** box.
Understanding	☐
Simplifying	☐
Mathematising	☐
Interpreting	☐

Fig. 3.6 Sample item to identify the modelling phase

The facets shown are recorded using three items per task vignette. Since six task vignettes are included in the test instrument, the “Knowledge about modelling processes” scale consists of 18 items. It consists of items 7.[1-6].1, 7.[1-6].2 and 7.[1-6].7. The scale consists of multiple-choice items with four answer options.

The first item type (see Fig. 3.6) asks for the identification of the modelling phase in which the learners in the shown conversation (see Fig. 3.5) are actually located. This is because students do not necessarily have to be in the same modelling phase when working together on modelling tasks. Students can simultaneously address several aspects that can be assigned to different modelling phases.

In the sample task vignette (see Fig. 3.5), statements can be clearly associated with the Simplify/Structure phase. By saying, “We really need to know how many vehicles are in the traffic jam,” and “Yes, we don’t know how long it takes for every vehicle,” it becomes clear that the students identify relevant and irrelevant aspects and thus make structuring. The two statements, "How are we going to calculate how long it takes?” and “We can divide the 20 km by the 6 hours, then we know how fast it would take.” also point to approaches of mathematisation. Since the statements on the previous phase predominate and even student 2 reacts to the impulses of his classmates, the Simplify/Structure phase can be primarily assigned for the work in a small group. At this point, the answer option “Mathematise” is a distractor that leads to high item difficulty.

After the identification of the modelling phase, participants must diagnose a potential difficulty (see Fig. 3.7). Since the Simplify/Structure modelling phase has been identified, typical difficulties in the formation of a real model can be considered. The only difficulty that can be attributed here is “problems in making assumptions” (see Sect. 1.4.2). The statement "We do not have any more information" shows that the students are not used to making assumptions. Instead, they compute based on the given data, which is an inappropriate approach to solving the problem.

7.1.2.	Diagnose students' difficulty working on the task **in this situation**. Please check **one** box.
The students...	
... have problems in making assumptions.	☐
... draw a false conclusion from their mathematical result.	☐
... have problems in understanding the context.	☐
... use an inappropriate mathematical model.	☐

Fig. 3.7 Sample item to identify the difficulty

7.1.7.	Please indicate which support goal you would like set for the group after **this situation**. Please check **one** box.
Independent acquisition and evaluation of information.	☐
Critical questioning of results in the modelling process.	☐
Independent construction of mental models for given problem situations.	☐
Secure translation of simplified real situations into mathematical models.	☐

Fig. 3.8 Sample item to determine the support goal

A final step is to set a support goal for the small group (see Fig. 3.8). The lack of ability or willingness of students to make assumptions suggests the support goal of "Independent acquisition and evaluation of information." To avoid sequential effects, the support goal items were placed after the intervention items. This should avoid simply ticking the matching support goal item after answering the diagnosis item. This approach was described by students in the framework of the qualitative pre-pilot (see Sect. 3.1). The chosen placement requires that the participants first go through and then answer the intervention items. After that, they think again about the support goal item, so that premature answering of the items is avoided.

Knowledge about interventions

Knowledge about interventions is operationalised using four items per task vignette (see Fig. 3.9). Since six task vignettes are included in the test instrument, the scale consists of 24 items. It consists of items 7.[1-6].3, 7.[1-6].4, 7.[1-6].5 and 7.[1-6].6 respectively. The items consist of statements that represent potential interventions in the situation illustrated by the task vignette. Participants are encouraged to identify potentially appropriate interventions for the independence-oriented development of

Please check which of the following **interventions** are suitable for independence-oriented development of modelling skills in this situation. Please place **a** marker for **each** intervention.		suitable	unsuitable	do not know
7.1.3	"First, estimate how long a car is."	☐	☐	☐
7.1.4	"First, consider only part of the problem, e.g. how many cars are actually stuck in the traffic jam."	☐	☐	☐
7.1.5	"Right, now calculate that value."	☐	☐	☐
7.1.6	"Think about how you can determine the missing data."	☐	☐	☐

Fig. 3.9 Sample item for knowledge about interventions

modelling competence. The scale consists of true-false items with the two answer options "suitable" and "unsuitable," one of which is rated as correct.

Since the probability of guess is 50% for two response options, the additional "do not know" response option was given. If students do not know the solution, they will be given an alternative answer and the number of correct solutions through guessing is reduced.

A decision as to whether an intervention is appropriate or unsuitable will be taken based on the criteria of adaptive intervention (see Sect. 1.4.1). The first statement, "First, estimate how long a car is" is considered as not adaptive. Although it has a content-methodical fit to the difficulty of the learner (make assumptions), it is not minimal because it strongly interferes with the content of the solution process. It is also not independence-oriented since the intervention is highly specific.

The second intervention, "First consider only part of the problem, e.g. how many cars are actually struck in traffic" can be evaluated as potentially adaptive. It is adapted in terms of content and methodology, since it addresses the difficulty of the students. It is minimal because it does not add any additional information to the solution process, and it is independence-oriented since it is less directive. Although the request makes a proposal for further work, it does not specify how the number of vehicles needs to be determined. For example, the proposal to divide the problem into sub-problems is an example of a potentially adaptive strategic intervention. The other two statements are evaluated using the same methodology.

After the operationalisation of the test, instrument has been presented, Sect. 3.5 shows the entire test book before describing the fully tested quality of the test instrument on the basis of various main and secondary criteria in Sect. 3.4. First, information on how to conduct the test is described in Sect. 3.5.

3.4 Information for Conducting the Test

The test can be conducted as a single test. It is used by pre-service teachers for secondary schools (general school, high school) with the subject of mathematics. The test includes a total of 126 items. Of these, 15 items are self-reported previous experiences, 16 items are beliefs, 24 items are self-efficacy expectations, and 71 items are professional knowledge about teaching mathematical modelling. The executed test duration including instructions is approximately 70 min. The maximum processing time is 60 min. Each test person requires a test book and a pen. No aids are allowed. The test begins with the instructions by the test instructor. It must be ensured that all participants have a pen and the test book in front of them, so that the processing can start after the instructions. These are provided on the first page of the test book. No questions are answered during the processing time.

3.5 Test Book

Create your personal code according to the following schema

First letter of your mother's first name. (For example: Anna → A)	→	
First letter of your father's first name. (For example: Tom → T)	→	
Last letter of your father's first name. (For example: Tom → M)	→	
First letter of your birthplace. (For example: Berlin → B)	→	
Last character of the day of your birthday. (For example: 07 May → 7)	→	
First letter of your mother's maiden name. (For example: Myers → M)	→	
Last letter of your first name. (For example: Jon → N)	→	

1. General Information

1.1 Gender:

☐ Female

☐ Male

☐ ____________________________

1.2 Age: ______

1.3 School-leaving examination grade: ______

1.4 Last grade in mathematics: ______

1.5 Second subject: _______________________________

1.6 Course semester: ______

2. Previous Experiences

To what extent do you agree with the following statements? Please check **one** box in each line.		strongly disagree	disagree	neutral	agree	strongly agree
2.1	Mathematical modelling was previously dealt with in a lecture/exercise/seminar.	☐	☐	☐	☐	☐
2.2	I was prepared for teaching modelling.	☐	☐	☐	☐	☐
2.3	I have already done mathematical modelling with students.	☐	☐	☐	☐	☐
2.4	I feel well prepared to teach mathematical modelling through my previous training.	☐	☐	☐	☐	☐
2.5	Mathematical modelling has already been addressed in my courses.	☐	☐	☐	☐	☐
2.6	I have already been imparted with the knowledge to teach modelling.	☐	☐	☐	☐	☐
2.7	I have resolved modelling tasks myself during my teacher education studies.	☐	☐	☐	☐	☐
2.8	As part of my previous teacher education studies, I have been able to build solid foundations for teaching mathematical modelling.	☐	☐	☐	☐	☐
2.9	Mathematical modelling played a role in my internships at school.	☐	☐	☐	☐	☐
2.10	In my education, I have already acquired knowledge to teach modelling.	☐	☐	☐	☐	☐
2.11	Mathematical modelling has already played a role in a course I attended.	☐	☐	☐	☐	☐
2.12	In my previous education, I also had to do mathematical modelling in the processing of tasks.	☐	☐	☐	☐	☐
2.13	If I have to design lessons for "mathematical modelling", I can draw from what I have learnt.	☐	☐	☐	☐	☐
2.14	I have already gained teaching experience in mathematical modelling.	☐	☐	☐	☐	☐
2.15	I have been working on modelling examples myself during my teacher education studies.	☐	☐	☐	☐	☐

3. Beliefs

To what extent do you agree with the following statements? Please check **one** box in each line.		strongly disagree	disagree	neutral	agree	strongly agree
3.1	Mathematical modelling should be a part of mathematics education.	☐	☐	☐	☐	☐
3.2	Results of mathematical modelling have a general, fundamental benefit for society.	☐	☐	☐	☐	☐
3.3	Students should be given the opportunity to do mathematical modelling in mathematics education.	☐	☐	☐	☐	☐
3.4	Students learn mathematics best by discovering ways to solve problems themselves.	☐	☐	☐	☐	☐
3.5	Mathematical modelling is a futile game, an engagement with objects with no concrete relation to reality.	☐	☐	☐	☐	☐
3.6	Effective teachers demonstrate the right way and methods to solve an application problem.	☐	☐	☐	☐	☐
3.7	Students should usually be required to solve tasks in the way they were taught in class.	☐	☐	☐	☐	☐
3.8	You should allow students to come up with their own ways of solving application problems before the teacher shows how to solve them.	☐	☐	☐	☐	☐
3.9	Among other competences, the competence for mathematical modelling should be taught in the classroom.	☐	☐	☐	☐	☐
3.10	Students should often have the opportunity to follow their teacher's model solutions.	☐	☐	☐	☐	☐
3.11	Mathematics should be taught at school in such a way that students can discover connections on their own.	☐	☐	☐	☐	☐
3.12	Many aspects of mathematical modelling have a practical use or a direct application reference.	☐	☐	☐	☐	☐
3.13	Mathematical modelling should be a specific component in mathematics education.	☐	☐	☐	☐	☐
3.14	It helps students to understand mathematics when they are asked to discuss their own solution ideas.	☐	☐	☐	☐	☐
3.15	In mathematical modelling, you work on tasks that have practical value.	☐	☐	☐	☐	☐
3.16	Teachers should provide detailed procedures for solving application problems.	☐	☐	☐	☐	☐

4. Self-efficacy

To what extent do you agree with the following statements? Please check **one** box in each line.		strongly disagree	disagree	neutral	agree	strongly agree
It is **easy** for me to recognise the different abilities of the students using ...						
4.1	... their translation of mathematical results into reality.	☐	☐	☐	☐	☐
4.2	... their written solutions when modelling.	☐	☐	☐	☐	☐
4.3	... the recording of the plausibility test of their solution.	☐	☐	☐	☐	☐
4.4	... the adequate assessment of the relationship between the mathematical result and reality that they produce.	☐	☐	☐	☐	☐
4.5	... the recording of the solutions they create for modelling tasks.	☐	☐	☐	☐	☐
4.6	... the mathematical models they chose when modelling.	☐	☐	☐	☐	☐
4.7	... the recording of their mathematical results in the modelling process.	☐	☐	☐	☐	☐
4.8	... the mathematical formulae and symbols they used in the modelling process.	☐	☐	☐	☐	☐
4.9	... the determination of their improvement of the established models presented during modelling.	☐	☐	☐	☐	☐
4.10	... the assumptions they made when modelling.	☐	☐	☐	☐	☐
4.11	... the students' solutions when modelling.	☐	☐	☐	☐	☐
4.12	... their handling of the mathematical symbols and operators used in modelling.	☐	☐	☐	☐	☐

	To what extent do you agree with the following statements? Please check **one** box in each line.	strongly disagree	disagree	neutral	agree	strongly agree
It is **difficult** for me to recognise the different abilities of the students using ...						
4.13	... their translation of mathematical results into reality.	☐	☐	☐	☐	☐
4.14	... the assumptions they have made in the modelling process.	☐	☐	☐	☐	☐
4.15	... the plausibility checks of their solution.	☐	☐	☐	☐	☐
4.16	... the recording of the real-world restructuring they have undertaken in modelling.	☐	☐	☐	☐	☐
4.17	... the mathematical models they chose when modelling.	☐	☐	☐	☐	☐
4.18	... the recording of the relationship between mathematical result and reality that they produce when modelling.	☐	☐	☐	☐	☐
4.19	... their handling of the mathematical formulae and symbols used in modelling.	☐	☐	☐	☐	☐
4.20	... their written solution when modelling.	☐	☐	☐	☐	☐
4.21	... their evaluation of the established models in the modelling process.	☐	☐	☐	☐	☐
4.22	... the simplifications and structures they have undertaken in the modelling process.	☐	☐	☐	☐	☐
4.23	... their mathematisations of a real situation.	☐	☐	☐	☐	☐
4.24	... the recording of their mathematical results in modelling.	☐	☐	☐	☐	☐

5. Knowledge about Modelling Tasks

5.1	**Characteristics of modelling tasks**		
5.1.1.	Modelling tasks...	True	False
... can be underdetermined.		☐	☐
... can be overdetermined.		☐	☐
... are as closed as possible.		☐	☐
5.1.2.	Modelling tasks...	True	False
... can also be Fermi tasks.		☐	☐
... are always also “dressed-up” word problems.		☐	☐
... can also be context-related word problems.		☐	☐
5.1.3.	Modelling tasks...	True	False
... are as close to real life as possible.		☐	☐
... are as authentic as possible.		☐	☐
... are as relevant as possible to the students.		☐	☐
5.1.4.	Modelling tasks...	True	False
... have clear solutions.		☐	☐
... are self-differentiating.		☐	☐
... may contain irrelevant information.		☐	☐

5.2	**Development of modelling tasks**		
5.2.1.	Good modelling tasks...	True	False
... require metacognitive processes of the students.		☐	☐
... require the translation of mathematics into reality.		☐	☐
... require the translation of reality into mathematics.		☐	☐
5.2.2.	Good modelling tasks...	True	False
... are always developed from inner-mathematical content.		☐	☐
... require non-mathematical knowledge.		☐	☐
... are problem-based.		☐	☐

5.2.3	Good modelling tasks...	True	False
... enable independent work.		☐	☐
... are suitable for individual work.		☐	☐
... always require cooperative learning forms.		☐	☐
5.2.4	Good modelling tasks...	True	False
... can be built on the basis of real problem situations.		☐	☐
... always require many sub-competencies of mathematical Modelling.		☐	☐
... illustrate the mathematical rigour.		☐	☐

5.3	**Processing of modelling tasks**		
5.3.1	Modelling tasks...	True	False
... are particularly suitable for use in heterogeneous learning groups.		☐	☐
... are not suitable for every grade level at secondary level.		☐	☐
... are suitable for individual development of high-performing students.		☐	☐
5.3.2	Modelling tasks...	True	False
... should encourage the practice of solution schemes.		☐	☐
... are used to record real world phenomena.		☐	☐
... can develop sub-competencies individually.		☐	☐
5.3.3	Modelling tasks...	True	False
... are only suitable for project teaching.		☐	☐
... are only suitable for regular education.		☐	☐
... are only suitable as a complement to the curriculum content.		☐	☐
5.3.4	Modelling tasks...	True	False
... are always cognitively more demanding than inner-mathematical problems.		☐	☐
... become less difficult with their degree of openness.		☐	☐
... always require passing through the complete modelling cycle.		☐	☐

6. Knowledge about Concepts, Aims and Perspectives

Please check the appropriate box (only **one** per task).

6.1	**Modelling cycles**	
6.1.1.	Cognitive modelling distinguishes between the situation model and the real model.	☐
	A direct transition from the real situation to the mathematical model is not possible in modelling cycles.	☐
	The situation model is formed independently of the individual.	☐
	A distinction between the situation model and the real model is not conceivable in modelling cycles.	☐
6.1.2	Students solve modelling tasks circularly following the modelling cycle.	☐
	Circular schemas cannot illustrate modelling processes.	☐
	Models of modelling describe the actual solution approaches of students when working on modelling tasks.	☐
	Cyclical representations of modelling distinguish between different stages or phases.	☐
6.1.3	Working mathematically is not a sub-competency of mathematical modelling.	☐
	Mathematisation is not characterised by introducing mathematically idealised objects.	☐
	Validation includes a real-world verification of models.	☐
	The use of everyday knowledge does not characterise a step in modelling.	☐
6.1.4	Mathematisation refers to all the translation processes between reality and mathematics.	☐
	In modelling, interpreting is the process of checking the solutions obtained.	☐
	The separation of important and unimportant information is not a description of the sub-competency"Simplifying".	☐
	The mental representation of the problem situation is not a description of the sub-competency "Simplifying".	☐

6.2	**Aims and perspectives of modelling**	

6.2.1	Inner-mathematical applications do not develop modelling competence.	☐
	Modelling competence cannot be developed by addressing metacognitive handling strategies.	☐
	Modelling competence must be developed in selected school levels.	☐
	Modelling competence can be developed with real problem solving tasks.	☐

6.2.2	Mathematical modelling focuses on the process of solving real world problems.	☐
	Mathematical modelling does not focus on the study of relationships between mathematics and reality.	☐
	Modelling focuses on the translation process of the mathematical language into the real-world language.	☐
	Modelling cannot be considered as the processing of non-mathematical questions by embedding them in inner-mathematical contexts.	☐

6.2.3	Structuring and development of learning processes is not a subject of pedagogical modelling.	☐
	Contextual modelling does not involve mental abstraction.	☐
	The solution of real and authentic problems is not the main focus of pedagogical modelling.	☐
	Applied modelling focuses on developing mathematical thought processes by using models as mental images.	☐

6.2.4	Using models as mental images is not a perspective of cognitive modelling.	☐
	Modelling pursues goals such as theory development on a theoretical level.	☐
	The socio-critical level of modelling does not focus on the critical understanding of the environment.	☐
	Contextual modelling historically refers to pragmatic approaches to modelling.	☐

6.3	**Range of references to reality**	

6.3.1	Proximity to life means that a task of students is already considered as significant.	☐
	Student relevance means that a task is related to the future life of the students.	☐
	Life relevance means that a task will become relevant for students only in future situations.	☐
	Proximity to life, life relevance and student relevance can be used synonymously.	☐

6.3.2	Open tasks have a strong correlation to reality.	☐
	Open tasks can be classified by initial states, transformations, and target states.	☐
	The authenticity of tasks is always to be seen in context and in isolation from the mathematics used.	☐
	The authenticity of tasks always requires that there is a real situation in the original.	☐

6.3.3	Word problems have no real reality reference.	☐
	"Dressed-up" word problem are characterised by a real reality reference.	☐
	Word problems only aim at practicing the computing skills.	☐
	"Dressed-up" word problem focus on environmental development using mathematics.	☐

6.3.4	Context-related word problems have a mathematical focus.	☐
	Tasks that are dressed-up in (complex) situations are called context-related word problems.	☐
	Context-related word problems are used for environmental development with the help of mathematics.	☐
	Context-related word problem s have no real reality reference.	☐

7. Knowledge about Modelling Processes and Interventions

Tasks and associated text vignettes that describe student conversations while performing modelling tasks are illustrated below. The tasks and text vignettes are used to diagnose, define support goals and derive appropriate interventions in these situations. The situations are characterised by the following **framework conditions**:

You are a teacher at a secondary school and your students at the specified grade level work on the tasks in a small project in **groups of 3**. The students have **already gained experience** with modelling tasks in advance. The situations presented arise in the **first half of the processing time**. The students in consideration have an **average level of performance** for the respective grade level. You observe the students during extracts of the conversations. You have **notyet interfered in the learning process.**

7.1 Traffic Jam (9th Grade)

Traffic jams often occur at the beginning of the summer holidays. Christina is stuck in a 20 km traffic jam for 6 hours. It is very warm and she is extremely thirsty. There is a rumour that a small truck is supposed to supply the people with water, but she has not yet received anything. How long will it take for the truck to supply all people with water?

STUDENT 1: We really need to know how many vehicles are stuck in the traffic jam.
STUDENT 2: Huh? Right!
STUDENT 1: How do we calculate how long it takes? A lot of things are missing from the task!
STUDENT 3: Yeah, we don't know how long it takes for every vehicle.
STUDENT 2: It is a dumb task.
STUDENT 1: We can divide the 20 km by the 6 hours, then we know how fast it would take.
STUDENT 3: Exactly! We do not have any more information.

7.1.1.	To which phase of the solution process can the group of students **mainly** be assigned to? Please check **one** box.
Understanding	☐
Simplifying	☐
Mathematising	☐
Interpreting	☐

7.1.2.	Diagnose students' difficulty working on the task **in this situation**. Please check **one** box.
The students...	
... have problems in making assumptions.	☐
... draw a false conclusion from their mathematical result.	☐
... have problems in understanding the context.	☐
... use an inappropriate mathematical model.	☐

Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **this situation**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.1.3	"First, estimate how long a car is."	☐	☐	☐
7.1.4	"First, consider only part of the problem, e.g. how many cars are actually stuck in the traffic jam."	☐	☐	☐
7.1.5	"Right, now calculate that value."	☐	☐	☐
7.1.6	"Think about how you can determine the missing data."	☐	☐	☐

7.1.7.	Please indicate which support goal you would like set for the group after **this situation**. Please check **one** box.
Independent acquisition and evaluation of information.	☐
Critical questioning of results in the modelling process.	☐
Independent construction of mental models for given problem situations.	☐
Secure translation of simplified real situations into mathematical models.	☐

7.2 Stockpile Material (6th Grade)

From both sides of the national road L1081, a route is being constructed to bring the illustrated cone dumps to the open-cast mine that is 5.5 km away. The 8.2 million m^3 of stockpile material will be transported across the L1081. The entire fleet of transporters will then transport the stockpile material 16 h a day. 12 months are planned for this transport work. To ensure transport performance, the fleet will be expanded by 10 dump trucks, each with a payload of 96 tons.

Develop a model calculation for the transport of stockpile material if 1 m^3 of the waste has a mass of approximately 2 tons and the transport has to be completed within one year.

STUDENT 1: We need to know how many dump trucks they need.
STUDENT 2: And we have to estimate how long they take to drive there.
STUDENT 1: And how long to unload.... and load.
STUDENT 3: But, if there are multiple trucks, they cannot always be loaded directly.
STUDENT 2: Yeah, lots of things to consider. They do not work for 16 hours either, they have breaks, smoking breaks and such.
STUDENT 3: How do we get all this into one formula?
STUDENT 1: Boah, [leans back] no idea. It is way too hard.

7.2.1	To which phase of the solution process can the group of students **mainly** be assigned to? Please check **one** box.
Understanding	☐
Simplifying	☐
Mathematising	☐
Validating	☐

7.2.2	Diagnose students' difficulty working on the task **<u>in this situation</u>**. Please check **one** box.
The students...	
... have problems in making assumptions.	☐
... draw a false conclusion from their mathematical result.	☐
... have problems in understanding the context.	☐
... use an inappropriate mathematical model.	☐

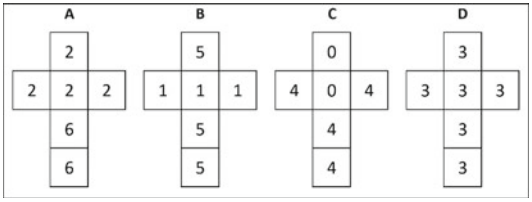
Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **<u>this situation</u>**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.2.3	"How can you deal with the missing information?"	☐	☐	☐
7.2.4	"Now assign variables to the quantities you have identified."	☐	☐	☐
7.2.5	"Divide the problem first and do not try to solve everything at once."	☐	☐	☐
7.2.6	"First of all, identify the most important information."	☐	☐	☐

7.2.7	Please indicate which support goal you would like set for the group after **<u>this situation</u>**. Please check **one** box.
Reduce the complexity of real situations independently.	☐
Secure translation of simplified real situations into mathematical models.	☐
Correct execution of mathematical operations and algorithms.	☐
Secure reference of mathematical results to a given problem situation.	☐

7.3 Safe Victory (12th Grade)

These four dice are described by their nets.

Two players choose a dice one after the other. After that, everybody throws the dice once. Whoever has the higher score wins. Develop a strategy with which the winning probability of the second player is the highest.



[Student 1 previously calculated the expected values for each cube.

$E(A) = \frac{10}{3}$, $E(B) = 3$, $E(C) = \frac{8}{3}$, $E(D) = 3$]

STUDENT 1: Is this possible?
STUDENT 2: Yeah, if I take C, you have to take A, because it is the highest.
STUDENT 3: And if I take A, you can choose between B and D, because they are the same. Makes sense, right?
STUDENT 1: Exactly.

7.3.1	To which phase of the solution process can the group of students **<u>mainly</u>** be assigned to? Please check **one** box.
Mathematising	☐
Working Mathematically	☐
Interpreting	☐
Validating	☐

7.3.2	Diagnose students' difficulty working on the task **<u>in this situation</u>**. Please check **one** box.
The students...	
... use an inappropriate mathematical model.	☐
... have problems in understanding the context.	☐
... have problems in making assumptions.	☐
... perform the calculation incorrectly.	☐

Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **this situation**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.3.3	"Check your strategy with another example."	☐	☐	☐
7.3.4	"Here you have to calculate the probabilities, not the expected values."	☐	☐	☐
7.3.5	"You have to approach the problem differently, the expected value will not get you anywhere."	☐	☐	☐
7.3.6	"Consider whether your result now delivers a correct strategy."	☐	☐	☐

7.3.7	Please indicate which support goal you would like set for the group after **this situation**. Please check **one** box.	
Reduce the complexity of real situations independently.		☐
Secure translation of simplified real situations into mathematical models.		☐
Correct execution of mathematical operations and algorithms.		☐
Critical questioning of results in the modelling process.		☐

7.4 Filling Up (10th Grade)

Mr. Stein lives in Trier, 20 km from the Luxembourg border. He drives his VW Golf to refuel in Luxembourg, where there is a fuel station just across the border. One litre of petrol costs only €1.05 here as compared to €1.20 in Trier.

Is the ride worth it for Mr. Stein?

STUDENT 1: [Has previously carried out the following calculation:

$$\mathrm{x} \cdot 0.15 \frac{€}{1} = 2 \cdot 20\,\mathrm{km} \cdot \frac{81}{100\,\mathrm{km}} \cdot 1.05 \frac{€}{1} \Rightarrow x \approx 22.41]$$

STUDENT 2: Strange, do you only have to fill up so little to make it worthwhile? But that is a very little. I would not have thought so. My father still takes canisters with him when he goes refuelling.

STUDENT 3: How much fuel goes into a car?

STUDENT 1: 50 litres, maybe?

STUDENT 3: Yes, that would be realistic. Then he would not even need to take even one canister.

7.4.1	To which phase of the solution process can the group of students **<u>mainly</u>** be assigned to? Please check **one** box.
Understanding	☐
Working Mathematically	☐
Interpreting	☐
Validating	☐

7.4.2	Diagnose students' difficulty working on the task **<u>in this situation</u>**. Please check **one** box.
The students...	
... do not adequately verify their solution for plausibility.	☐
... draw a false conclusion from their mathematical result.	☐
... have problems in understanding the context.	☐
... perform the calculation incorrectly.	☐

Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **<u>this situation</u>**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.4.3	"Check whether you have taken everything into account."	☐	☐	☐
7.4.4	"What about the wear and tear on the car?"	☐	☐	☐
7.4.5	"How accurate is your model now?"	☐	☐	☐
7.4.6	"Your calculation is still too inaccurate, you have to include several variables."	☐	☐	☐

7.4.7	Please indicate which support goal you would like set for the group after **<u>this situation</u>**. Please check **one** box.
Correct execution of mathematical operations and algorithms.	☐
Secure translation of simplified real situations into mathematical models.	☐
Critical questioning of results in the modelling process.	☐
Secure reference of mathematical results to a given problem situation.	☐

7.5 Milk Carton (12th Grade)

Not only for financial reasons, but also from an environmental point of view, it makes sense to consider what packaging should look like, so that least possible material is used. The picture shows a commercial milk carton. What should the milk carton look like so that the least possible material is used?

[The students have prepared the following calculation **in advance**:

$$V = 1\,\mathrm{l} = \mathrm{a}\cdot\mathrm{b}\cdot\mathrm{c} \Leftrightarrow \mathrm{a} = \frac{1\,\mathrm{l}}{b\cdot c}$$

$$O = 2ab + 2bc + 2ac = \frac{2\,\mathrm{l}}{c} + 2bc + \frac{2\,\mathrm{l}}{b}].$$

STUDENT 1: That is not possible now.
STUDENT 2: Why not, just derivate and then set zero.
STUDENT 1: Yeah, of what, b or c?
STUDENT 3: Mh, just go after b.
STUDENT 1: [calculates: $O' = 2c - \frac{2\,l}{b^2} = 0$] And now? I still have the b and the c.

7.5.1	To which phase of the solution process can the group of students **mainly** be assigned to? Please check **one** box.
Understanding	☐
Mathematising	☐
Working Mathematically	☐
Interpreting	☐

7.5.2	Diagnose students' difficulty working on the task **in this situation**. Please check **one** box.
The students...	
... have problems in making assumptions.	☐
... draw a false conclusion from their mathematical result.	☐
... use a completely inappropriate mathematical model.	☐
... make a computation error.	☐

Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **this situation**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.5.3	"First, consider a special case for the real problem."	☐	☐	☐
7.5.4	"Yeah, now just solve up to b."	☐	☐	☐
7.5.5	"Set a value for two variables first and then calculate the third side."	☐	☐	☐
7.5.6	"Where do you see a problem solving this equation?"	☐	☐	☐

7.5.7	Please indicate which support goal you would like set for the group after **this situation**. Please check **one** box.
Reduce the complexity of real situations independently.	☐
Critical questioning of results in the modelling process.	☐
Secure translation of simplified real situations into mathematical models.	☐
Correct execution of mathematical operations and algorithms.	☐

7.6 Container (8th Grade)

Containers are used on many construction sites to store construction goods or to collect construction waste. These containers have a special shape, which is intended to simplify loading and unloading. How much sand is in the container shown?

STUDENT 1: There is exactly 7,160,000 cubic metres of sand in there. Is that true?

STUDENT 2: I guess you were right, you calculated that with calculator.

STUDENT 1: Clearly. Then that is fine.

STUDENT 3: It is certainly right. I can present that.

7.6.1	To which phase of the solution process can the group of students **<u>mainly</u>** be assigned to? Please check **one** box.
Mathematising	☐
Working Mathematically	☐
Interpreting	☐
Validating	☐

7.6.2	Diagnose students' difficulty working on the task **<u>in this situation</u>**. Please check **one** box.
The students...	
... have problems in making assumptions.	☐
... do not adequately verify their solution for plausibility.	☐
... draw a false conclusion from their mathematical result.	☐
... use an inappropriate mathematical model.	☐

Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **<u>this situation</u>**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.6.3	"You probably made a mistake with the units somewhere."	☐	☐	☐
7.6.4	"Show me how big a cubic metre is."	☐	☐	☐
7.6.5	"Check the magnitude of your result."	☐	☐	☐
7.6.6	"How can you check the result of the calculator?"	☐	☐	☐

7.6.7	Please indicate which support goal you would like set for the group after **<u>this situation</u>**. Please check **one** box.	
Secure translation of simplified real situations into mathematical models.		☐
Independent construction of mental models for given problem situations.		☐
Critical questioning of results in the modelling process.		☐
Correct execution of mathematical operations and algorithms.		☐

8. Knowledge about Modelling Tasks

Please place the tasks “Container” (1), “Filling Up” (2), “Safe Victory” (3) and “Milk Carton” (4) in order with regard to the following criteria for modelling tasks. Note the numbers corresponding to the tasks in the table on the next page.

(1) Container

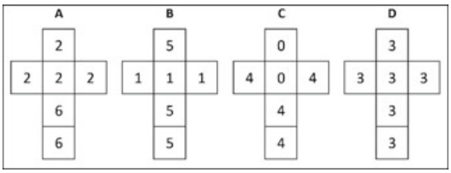
Containers are used on many construction sites to store construction goods or to collect construction waste. These containers have a special shape, which is intended to simplify loading and unloading. How much sand is in the container shown?

(2) Filling Up

Mr. Stein lives in Trier, 20 km from the Luxembourg border. He drives his VW Golf to refuel in Luxembourg, where there is a fuel station just across the border. One litre of petrol costs only €1.05 here as compared to €1.20 in Trier.

Is the ride worth it for Mr. Stein?

(3) Safe Victory



These four dice are described by their nets.

Two players choose a dice one after the other. After that, everybody throws the dice once. Whoever has the higher score wins. Develop a strategy with which the winning probability of the second player is the highest.

(4) Milk Carton

Every day tons of packaging waste is generated in Germany. Not only for financial reasons, but also from an environmental point of view, it makes sense to consider what packaging should look like, so that least possible material is used. The picture shows a commercial milk carton. What should the milk carton look like so that least possible material is used?

8.1	Low openness					High openness
8.2	Low relevance for students					High relevance for students
8.3	Low reality relation					High reality relation
8.4	Low authenticity					High authenticity
8.5	Few modelling sub-competencies					Many modelling sub-competencies

Chapter 4
Test Quality

General standards in the form of quality criteria can be used in order to assess the quality of an instrument and/or to construct a high-quality test. Three main indicators, the so-called "core quality criteria," have emerged: objectivity, reliability and validity (e.g. Bühner, 2011; Ebel & Frisbie, 1991; Linn, 2011; Miller, Linn & Grolund, 2009; Rost, 2004). These criteria must not be considered separately, but there is a logical relationship between them: objectivity is a prerequisite for reliable measurement and reliable measurement is a prerequisite for the validity of the instrument. These primary and selected secondary quality criteria (fairness and usability) are examined in more detail in the following using a data set of 349 pre-service teachers for secondary education at several German universities.

4.1 Objectivity

Objectivity is understood, in a narrower sense, as the degree of independence of the test results from the test instructor (Miller et al., 2009), while, in a broader sense, it is the degree of independence of the test results from any influences outside the participants is meant (Rost, 2004). Furthermore, in the context of the current phase of testing, a distinction is made between *implementation objectivity*, *evaluation objectivity* and *interpretation objectivity* (Bühner, 2011). In order to ensure the objectivity of implementation, the conditions under which the test is performed and the instructions provided must be as standardised as possible; in other words, the performance of a test must not vary between different examinations. One way to do this is to minimise the interaction of the test instructor with the participants. Implementation objectivity was ensured in this study by examining all participants under comparable conditions and using a pre-established standardised written introduction and printed instructions. The sole test processing was ensured by supervision and no further aid was given or additional aids were allowed. In addition, sampling checks

R. Wess et al., *Measuring Professional Competence for the Teaching of Mathematical Modelling*, International Perspectives on the Teaching and Learning of Mathematical Modelling, https://doi.org/10.1007/978-3-030-78071-5_4

on the implementation of the instructions did not reveal any deviations from the requirements.

In the context of the analysis of the test results, which are usually supported by software, the term evaluation objectivity refers to the independence of the test evaluation from the person (Good, 1973) and/or the program used for this purpose. It turns out that closed task formats, where participants have to choose between predefined answer alternatives, are the least prone to interference, although they cannot be completely excluded. On the other hand, when open task formats are used, the participant can respond with his or her own free formulation, the analysis of which depends—to a certain extent—on the subjective impressions of the coders (Miller et al., 2009). The evaluation objectivity was ensured by an automated evaluation of the tests, which was made after the encoding of the answers. The manual was created on the basis of six expert ratings from the German-speaking modelling community. In the process, critical items were discussed until consensus for evaluation was reached. In addition, part of the test logs were double-coded and checked for input errors.

Interpretation objectivity means the independence of the interpretation of the test results from the analysing person (VandenBos, 2015) and is ensured in this study by the fact that each participant can be assigned numerical values for their respective skill expressions on a fixed scale as part of the test. In addition, the effects can be interpreted on the basis of internationally accepted standards (Cohen, 1988).

4.2 Reliability

Reliability is the measurement accuracy or reliability of a test. For example, a measurement is reliable precisely when it accurately captures the personality or behavioural trait that is being measured, that is without error in measurement (Miller et al., 2009). Mathematically, the degree of reliability is determined by a so-called reliability coefficient, which describes the ratio of the variance of the true measured values to the variance of the observed and thus error-prone measured values (Bühner, 2011). In analogy to objectivity, in practice, there are also different ways of describing the reliability of a measurement and the specified variance ratio (Ebel & Frisbie, 1991). Since the variance of true values is unknown, it follows that the reliability of a test can be only estimated from the responses of the participants. This is done by means of methods that estimate reliability under certain conditions by means of a correlation between two comparisons, whereby the split-half reliability, the parallel forms reliability and the test–retest reliability are the most common methods for estimating the reliability of measurement (Bühner, 2011). For this purpose, in the first case, the tests are divided into two equivalent test halves, the results are determined separately for each test part and participant, and then both sub-test results are correlated. In the second case, the results of two strictly comparable tests that collect the same construct are correlated, whereas in the third case the results of the test (assuming that the characteristic to be captured has not changed itself) are correlated with a repeat measurement (Ebel & Frisbie, 1991). Another method for estimating reliability is internal consistency, which is essentially a generalisation of split-half reliability, where each item is considered as a separate test part (Bühner, 2011). The standard for the numerical realisation of this method is the Cronbach coefficient α (1951) developed and named after Cronbach, which sets the sum of the variances of the individual items in relation to the total variance of the test. Accordingly: the greater the number of items and the stronger the positive correlation between the items, the higher is the internal consistency (Bühner, 2011).

The concept of reliability described has been defined in the framework of classical test theory and is applied there by default (Miller et al., 2009). In contrast, reliability in the probabilistic test theory or the item response theory (for a deeper look, see, e.g. van der Linden & Hambleton, 1997), which is the basis for the analysis of the facets of the modelling-specific pedagogical content knowledge, is rarely observed, despite the extremely favourable calculation conditions. The required variance percentages in the Rasch model can be directly estimated: The variance of latent variables (i.e. the true measurement value) is estimated as a model parameter in the course of the *Marginal Maximum Likelihood Estimation* (MMLE), while the variance of the observed values corresponds to the variance of the estimated personal parameters and the error variance of the measured values can also be calculated from the standard estimation errors of the ability expressions. However, the latter two variances are based on the choice of the estimation procedure according to which, in addition to the *Unconditional Maximum Likelihood Estimation* (UMLE; often also called *Joint Maximum Likelihood Estimation*—JMLE) resulting and due to their overestimated variance inappropriate personal parameters also calculate the more suitable *Expected* a Posteriori (EAP) and *Weighted Likelihood Estimation* (WLE) estimator. The resulting reliability (EAP or WLE reliability), of which the EAP reliability is comparable to the reliability measure from classical test theory as specified by Cronbachs α (Rust, 2004), therefore represents adequate options to determine the measurement accuracy of a test within the scope of the item response theory. In view of the facets of the modelling-specific pedagogical content knowledge, reliability values are determined which tell exactly how the corresponding personal parameters (EAP or WLE estimators) can be measured in the second test part. The corresponding dichotomic items were scaled using simple Rasch models and the scales were thus checked for their sufficiency. Using the eRm package (Mair & Hatzinger, 2007) of Software R, the item difficulty parameters were estimated based on the solution rates of items and personal skill parameters based on the performance of the people interviewed. Various scale parameters have been calculated to evaluate scalability (see Table 4.1). In the course of the model validity review, items 6.1.3 and 6.2.3 for knowledge about concepts, aims and perspectives as well as items 7.1.4, 7.2.5, 7.2.6, 7.6.4 and 7.6.5 for knowledge about interventions were excluded due to insufficient discrimination and therefore not included in the scale.

In general, the reliability coefficients between 0.50 and 0.70 can be considered adequate for group comparisons (Ebel & Frisbie, 1991) as well as coefficients that are not less than 0.70 as the characteristic values of good test instruments (Bühner, 2011). All EAP reliability values are above 0.70 and are therefore acceptable. All the Andersen tests to assess the model fit are insignificant and therefore indicate a fit

Table 4.1 Dichotomous Rasch models for knowledge scales

Facet	Number	EAP reliability	Andersen test	Pt.-Bis.-Corr.
Modelling tasks	17	0.81	0.086	>0.30
Concepts, aims and perspectives	10	0.70	0.058	>0.30
Interventions	19	0.71	0.061	>0.30
Modelling processes	18	0.74	0.072	>0.30

Table 4.2 Reliabilities for the (sub-)scales of self-reported prior experiences, beliefs and self-efficacy expectations for mathematical modelling

Scale	Sub-scale	α
Self-reported previous experiences for mathematical modelling	Handling of mathematical modelling	0.89
	Teaching and preparation for mathematical modelling	0.88
	Modelling tasks	0.87
	Modelling in the classroom	0.82
Beliefs in mathematical modelling	Constructivist-oriented beliefs	0.83
	Transmissive-oriented beliefs	0.65
Self-efficacy expectations for mathematical modelling	Self-efficacy expectations for mathematical modelling	0.88
	Self-efficacy expectations for mathematical work	0.84

of the one-dimensional Rasch models. Furthermore, all point-biseral correlations of the remaining items are greater than 0.30 and therefore also of acceptable quality.

In addition, for the scale of self-reported prior experience, beliefs and self-efficacy expectations for mathematical modelling, the first part of the test calculated relics according to the classical approach (see Table 4.2).

Except for those of transmissive oriented beliefs (0.65), all the reliability values of the scales considered are above 0.80 and must therefore be described as good.

4.3 Validity

While the reliability describes the trustworthiness or measurement accuracy of the test, validity describes the extent to which the test measures what it should measure (Miller et al., 2009). A test is considered to be completely valid if its results allow accurate and immediate conclusions to be drawn about the individual characteristics of the participants' abilities or behaviour to be captured (Ebel and Frisbie, 1991). There are also three concepts as far as validity is concerned: the *content validity*, *criterion validity* and *construct validity* which is explained below (Bühner, 2011).

For content validity, it is fundamental whether the test instrument as a whole, but also whether each of its individual items represents the characteristic to be captured sufficiently well. This is not checked by numerical parameters, but rather by didactic and logical considerations (Ebel & Frisbie, 1991). Accordingly, the content validity in the present study was ensured by a rational and effective design of the test tasks (see Sect. 3.1), by a theory-based operationalisation that was closely aligned with the definitions of the aspects, areas and facets of professional competence to teach mathematical modelling. In addition, the tasks developed in this way were discussed extensively with several experts from the German-speaking modelling community to determine whether the constructs considered were adequately covered.

The criterion validity refers to the validation of a test based on the association with an external manifest criterion that should correlate with the characteristic to be recorded (Bühner, 2011). Depending on the time at which this criterion is available (before, simultaneously, later), there is a distinction between retrospective validity, consistency validity and predictive validity. In the first case, therefore, the relationship of the test result to a criterion of interest that was already known, in the second case, the relation of the measured values with a criterion that was collected simultaneously and in the third case the prediction of a future characteristic is in the foreground. These correlations can therefore be used to provide a numerical variable to represent the criterion validity (Kane, 2011). In order to consider the criterion validity in the field of professional competence for teaching mathematical modelling, the annex of the study makes it possible to rely primarily on retrospective validity. However, as there is almost no knowledge about which criteria generally correlate with modelling-specific professional competence, the main focus is on the links between the modelling-specific pedagogical content knowledge facets and the school-leaving examination grade, following the results of the COACTIV study (Krauss et al., 2008) (see Table 4.3) whereas other aspects are considered in the subsequent explanations of construct validity.

It turns out that the school-leaving examination grade is not indicative of the facets of modelling-specific pedagogical content knowledge considered, with negative correlations indicating a positive correlation due to the German grade scale. These results replicate the correlations with the pedagogical content knowledge found by Krauss et al. (2008).

In the context of construct validity, the question is examined whether the instrument also captures the theoretical construct that needs to be captured. In this respect, many authors increasingly summarise the construct validity as a general concept

Table 4.3 Correlations between facets of pedagogical content knowledge and school-leaving examination grade

	Modelling tasks	Concepts, aims and perspectives	Interventions	Modelling processes
School-leaving examination grade	−0.101*	−0.106*	−0.065	−0.095*

*The correlation is significant at the level of 0.05 (2-sided).

encompassing all aspects of validity, while, in a narrower sense, only the *convergent*, *discriminatory* and *factorial* validities are included in the aspects of construct validity (Bühner, 2011). Instead of naming individual external manifests, as with criterion validity, one formulates diverse hypotheses about the structure and contexts of the construct and the related relationships to manifest, but also latent variables. These hypotheses can therefore relate, on the one hand, to which other construct-related variables the test to be validated is closely related (convergent validity) and, on the other hand, to which non-structural variables it is not or only very little related (divergent validity) (Ebel & Frisbie, 1991). In addition, factorial validity often involves checking the established test model before the test construction (or even structural model) with the help of confirmatory factor analyses and other model matching procedures, which on the one hand examines the defined mapping of individual test pieces to specific design areas and facets and on the other hand tests the assumption of uncorrelated measurement errors (Bühner, 2011). The possibility of verifying convergent validity is limited, since inaccessible comparative tests have not allowed the use of instruments other than those described in Sect. 3. Thus, correlations between the aspects of professional competence to teach mathematical modelling considered are calculated, also based on the results of the COACTIV study (Krauss et al., 2013), which may give indications of convergent validity (see Tables 4.4 and 4.5).

Table 4.4 Correlations of facets of modelling-specific pedagogical content knowledge

	MSPCK	Modelling tasks	Concepts, aims and perspectives	Interventions	Modelling processes
MSPCK					
Modelling tasks	0.67**				
Concepts, aims and perspectives	0.60**	0.60**			
Interventions	0.72**	0.55**	0.56**		
Modelling processes	0.69**	0.41**	0.48**	0.53**	

**The correlation is significant at the level of 0.01 (2-sided)

Table 4.5 Correlations of aspects of modelling-specific professional competence

Beliefs and self-efficacy	MSPCK
Constructivist-oriented beliefs	0.25**
Transmissive-oriented beliefs	−0.22**
Self-efficacy expectations for mathematical modelling	0.19**
Self-efficacy expectations for mathematical work	0.12*

*The correlation is significant at the level of 0.05 (2-sided)
**The correlation is significant at the level of 0.01 (2-sided).

Significant correlations are consistently shown, which, in almost all cases, are comparable with the COACTIV results in terms of their significance and thus support the validity of the test designed.

In view of the discriminatory validity, it would have been desirable to use additional tests which only measure similar constructs to ensure that they are not measured, in other words, that the correlation between the present and the other test results is minimised. But this was not possible for economic reasons. Instead, comprehensive efforts have been made to verify the factorial validity of the constructs under consideration. As described in Sect. 2.4.4, the model of professional competence for teaching mathematical modelling was largely confirmed by structural equation and/or confirmatory factor analyses (see Klock et al., 2019; Wess et al., 2021). In the context of structural analyses, various Rasch models for professional knowledge have also been and will be determined to teach mathematical modelling and tested using modelling tests as well as compared with each other to ensure factorial validity (for a deeper look, see Greefrath, Siller, Klock and Wess, submitted).

In addition to the main quality criteria discussed, scalability is sometimes mentioned as another criterion. This is considered to be fulfilled if the test value formation follows a valid clearing rule, that is there is a sufficient statistic (Bühner, 2011), which is ensured by the use of fitting Rasch models (see Sects. 2.4.4 and 4.2). In addition, other so-called secondary quality criteria such as fairness and usability can be listed (e.g. Bühner, 2011; Ebel & Frisbie, 1991; Miller et al., 2009), which are briefly explained below.

4.4 Secondary Quality Criteria

One of the most well-known secondary commodity criteria is the fairness of a test. The quality criterion of fairness is met precisely when the results of a test do not systematically discriminate against groups of participants on the basis of external characteristics (e.g. ethnic, socio-cultural or gender-specific) (Zieky, 2011). For example, when designing test tasks, care was taken to ensure that the formulations were made in a language that was appropriate for the gender. In addition, the tests used to record the professional competence to teach mathematical modelling were checked for differential item functioning in the course of two dissertation projects in order to check whether certain sample groups are significantly disadvantaged by individual items, that is a systematic item bias (see Klock, 2020; Wess, 2020).

The usability criterion is met by a test when it uses relatively little financial and time resources (Miller et al., 2009) in terms of the diagnostic knowledge gained. For this purpose, it is important to keep the implementation time as short as possible and to minimise the material requirements as well as to make the test instrument easy to handle and to realise it as a group test as far as possible (Bühner, 2011). This means that the instrument used can be described as economically viable, since it has a relatively short implementation time (approximately 60 min), consumes little material, is easy to use and can be implemented as a group test.

In addition to the associated quality criteria presented, various authors cite further criteria for the test quality. This includes, among other things, acceptability, usefulness and tamper-proof characteristics. For more detailed explanations, please refer to the relevant literature (e.g. Bühner, 2011; Downing & Haladyna, 2011; Miller et al., 2009).

Part III
Discussion and Outlook

Chapter 5
Discussion

The results presented in the preceding sections were primarily aimed at developing a test on professional competence of pre-service teachers specifically for teaching mathematical modelling. In a first step, the theoretical foundations were laid and, based on this, the structure of professional competence used for teaching mathematical modelling was explained. Subsequently, the relevant constructs were operationalised and empirically checked.

The results of the analyses presented regarding the test quality are summarised below and are classified in the current state of research to discuss possible explanations for the observed results on this basis. In order to further define the scope of the results, key limits of the test instrument in relation to the present study will be addressed. The book concludes with an outlook, in which possible implications are derived from this study for didactic research as well as for university teacher education.

The chosen approach to operationalise and empirically describe the structures of professional competence for teaching mathematical modelling addresses the question formulated by Blum (1995) about appropriate structural conceptualisations and empirical underpinnings for essential skills for teaching in application-related contexts. For example, central constructs, in particular the area-specific competence, were first defined and then measured using a test instrument and recorded in sufficient quality. On this basis, a model validity check showed that the data in the field of modelling-specific pedagogical content knowledge can be described by one-dimensional Rasch models—after excluding some critical test tasks (see Sect. 4.2).

The fact that aspects of professional competence and in particular pedagogical content knowledge can be empirically captured as a facet of professional knowledge was already demonstrated in the COACTIV (Kunter et al., 2013) and TEDS-M (Blömeke et al., 2014) studies. The present study now uses a different, focused perspective, for example, because no facets of pedagogical or psychological knowledge have been measured. For example, the explanatory notes on the individual

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modelling-specific pedagogical content knowledge facets reveal additional constrictions that have not been taken into account in the models in question (see also Sect. 2.4).

Nevertheless, the results of the present work conclude on the results of the structural analysis of Klock et al. (2019) and Wess et al. (2021), which showed that the dimensioning envisaged by Borromeo Ferri and Blum (2010), despite the theoretical constraints presented, is empirically distinct and can therefore be assumed from one-dimensional knowledge facets. The one-dimensionality of the constructs under consideration is therefore in line with the theoretical, conceptual and substantive dimensions of the skills and abilities necessary to promote modelling competence among students and, in addition, with the reported homogeneity of subject-related knowledge facets from the aforementioned large-scale studies (Blömeke et al., 2014; Krauss et al., 2013).

Furthermore, beliefs/values/aims and motivational orientations in the form of self-efficacy expectations for mathematical modelling could be adequately captured and significant interactions between these constructs could be demonstrated. In this sense, the professional competence to teach mathematical modelling can be considered a complex construct (see Sect. 2.1). In the modelling-specific design, beliefs can also be understood from a stronger transmissive or stronger constructivist perspective (cf. Voss et al., 2013). The established correlations are therefore in line with the findings of Schwarz et al. (2008) and Kuntze and Zöttl (2008): both positively correlated constructivist beliefs and negatively correlated transmissive beliefs contribute to the description of beliefs in mathematical modelling.

Before identifying further uses of the test presented here for didactic research and for university teacher training, some limitations of this study will be addressed in order to define the scope of the results presented. While the objectivity of the test can be considered very good due to the type of item used and the reliability on the basis of the studies can be considered acceptable to good, the review of the criterion validity in the field of professional competence for teaching mathematical modelling was primarily based on retrospective validity. Based on the results of the COACTIV study (Krauss et al., 2008), the focus was primarily on the school-leaving examination grade as a criterion that was not indicative of the specific knowledge facets (see Sect. 4.3). These results replicate the correlations found by Krauss et al. (2008) and thus support the criterion validity of the instrument used. In addition, the possibility of verifying the convergent validity in the present work is limited, since inaccessible comparative tests have not allowed the use of instruments other than those described in Sect. 3.5. Correlations between the competencies considered and with the beliefs and self-efficacy expectations for mathematical modelling were thus calculated, also in line with the results of the COACTIV study (Krauss et al., 2013) (see Sect. 4.3). Significant correlations between the examined aspects were shown, which, in almost all cases, are comparable with the COACTIV results in terms of their expression and significance and thus contribute towards the convergent validity of the test designed. Only the strengths of constructivist beliefs were slightly lower than in the reference study. However, it can be assumed that no stronger correlations could develop due to ceiling effects in this area. These ceiling effects also suggest that the degree of differentiation between beliefs (and, where appropriate, self-efficacy expectations) and mathematical modelling should be adapted for the following studies, for example by using a seven-step Likert scale instead of a five-step one. In addition to the convergent, factorial validity is another form of construct validity. In order to ensure this, various models of professional competence for teaching mathematical modelling were identified in the framework of the structural analyses carried out and were examined in replication studies (see Sect. 2.4.4).

The use of valid Rasch models also ensured the existence of a sufficient statistic (Bühner, 2011), which provides the basis for a valid transfer rule for test value formation and thus fulfils the quality criterion of the scaling. In the course of the application review, however, both in the facet of knowledge about concepts, aims and perspectives as well as in the facet of knowledge about interventions, some critical items emerged, which must be discussed further.

In the light of the model structures confirmed by the modelling tests and item characteristics, it was also possible to determine the reliability values that indicate exactly how the personal parameters (EAP or WLE estimators) could be measured (see Sect. 4.2). In this respect, it should be noted that, in all aspects of professional competence in teaching mathematical modelling, values have been achieved that is sufficient for group comparisons and, in some cases, as indicators of a good measuring instrument (Bühner, 2011; Ebel & Frisbie, 1991).

In view of the evaluation methodology, the probabilistic test theory used to scale the raw data can be considered to be of decisive importance. The chosen methodological approach is certainly not the easiest way to calculate the measurement accuracy and to verify the dimensionality of tests. However, it offers decisive advantages while at the same time reducing certain deficits in relation to the dependency on items (cf. van der Linden & Hambleton, 1997). Various methods for estimating the ability parameters were also discussed, ultimately looking at Weighted Likelihood Estimation. Although it leads to measurable error-related measurements, these are the best point estimates of the person's abilities (Rost, 2004). These provided the basis for the analyses aimed at answering the question of the quality of the test instrument under consideration.

Chapter 6
Outlook

The presented test instrument opens up a wide range of approaches for further didactic research and valuable implications for university teacher education; it may not be considered in isolation from the findings of the above discussion or from the limits set out.

Teaching mathematical modelling is a cognitively demanding activity for (pre-service) teachers (Blum, 2015), which is why quality development in teacher education requires a detailed examination of professional competence to teach mathematical modelling. In order to analyse this competence, theoretical models (see Sect. 2.4) are required, which describe the requirements for teachers in detail, as well as measuring instruments (see Sect. 3.5) that are equally suitable for the purpose of adequately measuring the skills and abilities required. As shown in the course of this book, these could be used profitably for conceptualisation such as operationalisation of specific professional competence.

The measuring instrument covers many substantial components of modelling-specific professional competence and has been extensively examined—given the recorded constructs—in order to meet established test quality criteria (see Sect. 4). The recognition that the conceptualised domain-specific competence can be empirically recorded and the corresponding knowledge facets can be described in a rapid and homogeneous manner thus indicates an added value for further didactic research on the teaching of mathematical modelling, since for example a wide range of university courses and concepts can be evaluated in a more targeted manner and thus be assessed in a more differentiated way. Competence developments can also be analysed in more detail in order to obtain an informed basis for modelling the possible levels.

The extent to which there are correlations between individual aspects of professional competence for teaching mathematical modelling could be demonstrated with regard to the cognitive-oriented as well as the affective-motivational components. In subsequent research projects, it would be desirable to combine the aspects of professional competence for teaching mathematical modelling with other specific

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competencies. For example, Klock (2020) focuses on the correlations between intervention competency and diagnostic competency, while Wess (2020) looks at the links between the task and diagnostic competency to teach mathematical modelling. On the other hand, examinations of further correlations, for example between specific task and intervention competency or between these and other constructs, are still pending.

However, in the light of the above, it should also be noted that these relate on the one hand to pre-service teachers and on the other to individual universities in Germany. Accordingly, the results obtained primarily represent site-specific empirical confirmations of the structures and correlations shown. Further work with the aim of a possible adaptation of the conceptualised structural model as well as of the test instrument used to fit and use practicing teachers on the one hand and in international contexts on the other is therefore still to be done.

Finally, in the context of the COACTIV study, which serves as the basis for the conceptual considerations of the structural model as defined in Sect. 2.4, it would be of particular interest to gain insights into other facets of modelling-specific professional knowledge. In this context, the combination of this instrument and that of the test developed by Haines et al. (2001) to capture modelling-specific content knowledge as a profitable way to conduct future analyses. However, such a combination requires either the compilation of extensive test books or the conception of a balanced rotation design. As both instruments ensure the rapid homogeneity of the designs considered, the latter option, in particular, shows itself to be an economic way of recording a broader competence structure.

In general, it is appropriate for the following research projects to use these preparatory work as a starting point to further investigate the genesis, structure and relevance of professional competence of (pre-service) teachers in the field of mathematical modelling. For example, it would be particularly desirable to demonstrate to what extent the modelling-specific content knowledge or the modelling-specific pedagogical content knowledge as well as other affective-motivational components of professional competence to teach mathematical modelling of practitioners are predictively valid for the quality of teaching and the learning progress of their students. To answer a more global design of this question, COACTIV used the longitudinal cross-sectional component of PISA in Germany (Bruckmaier et al., 2018). On the other hand, for a local, modelling-specific design, it is advisable to use proven and valid modelling competence tests, such as those developed by Zöttl et al. (2010), Kaiser and Brand (2015) or Hankeln et al. (2019).

Overall, the effectiveness of (more) developed elements, structures and teaching formats in the context of teacher education must always be measured in terms of the developed teacher competences. On the basis of the findings presented, it also seems desirable to consider further process-related competences such as problem solving or reasoning in the context of teaching–learning laboratories, thereby contributing to a holistic, practice-related mathematics teacher education.

Test Book—Correct Answers

Create your personal code according to the following schema

First letter of your mother's first name. (For example: Anna → A)	→	
First letter of your father's first name. (For example: Tom → T)	→	
Last letter of your father's first name. (For example: Tom → M)	→	
First letter of your birthplace. (For example: Berlin → B)	→	
Last character of the day of your birthday. (For example: 07 May → 7)	→	
First letter of your mother's maiden name. (For example: Myers → M)	→	
Last letter of your first name. (For example: Jon → N)	→	

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1. General Information

1.1 Gender:

- ☐ Female
- ☐ Male
- ☐ ______________________________

1.2 Age: _______

1.3 School-leaving examination grade: _______

1.4 Last grade in mathematics: _______

1.5 Second subject: _________________________________

1.6 Course semester: _______

2. Previous Experiences

To what extent do you agree with the following statements? Please check **one** box in each line.		strongly disagree	disagree	neutral	agree	strongly agree
2.1	The mathematical modelling was previously dealt with in a lecture/exercise/seminar.	☐	☐	☐	☐	☐
2.2	I was prepared for teaching modelling.	☐	☐	☐	☐	☐
2.3	I have already done mathematical modelling with students.	☐	☐	☐	☐	☐
2.4	I feel well prepared to teach mathematical modelling through my previous training.	☐	☐	☐	☐	☐
2.5	Mathematical modelling has already been addressed in my courses.	☐	☐	☐	☐	☐
2.6	I have already been imparted with the knowledge to teach modelling.	☐	☐	☐	☐	☐
2.7	I have resolved modelling tasks myself during my teacher education studies.	☐	☐	☐	☐	☐
2.8	As part of my previous teacher education studies, I have been able to build solid foundations for teaching mathematical modelling.	☐	☐	☐	☐	☐
2.9	Mathematical modelling played a role in my internships at school.	☐	☐	☐	☐	☐
2.10	In my education, I have already acquired knowledge to teach modelling.	☐	☐	☐	☐	☐
2.11	Mathematical modelling has already played a role in a course I attended.	☐	☐	☐	☐	☐
2.12	In my previous education, I also had to do mathematical modelling in the processing of tasks.	☐	☐	☐	☐	☐
2.13	If I have to design lessons for “mathematical modelling”, I can draw from what I have learnt.	☐	☐	☐	☐	☐
2.14	I have already gained teaching experience in mathematical modelling.	☐	☐	☐	☐	☐
2.15	I have been working on modelling examples myself during my teacher education studies.	☐	☐	☐	☐	☐

3. Beliefs

To what extent do you agree with the following statements? Please check **one** box in each line.		strongly disagree	disagree	neutral	agree	strongly agree
3.1	Mathematical modelling should be a part of mathematics education.	☐	☐	☐	☐	☐
3.2	Results of mathematical modelling have a general, fundamental benefit for society.	☐	☐	☐	☐	☐
3.3	Students should be given the opportunity to do mathematical modelling in mathematics education.	☐	☐	☐	☐	☐
3.4	Students learn mathematics best by discovering ways to solve problems themselves.	☐	☐	☐	☐	☐
3.5	Mathematical modelling is a futile game, an engagement with objects with no concrete relation to reality.	☐	☐	☐	☐	☐
3.6	Effective teachers demonstrate the right way and methods to solve an application problem.	☐	☐	☐	☐	☐
3.7	Students should usually be required to solve tasks in the way they were taught in class.	☐	☐	☐	☐	☐
3.8	You should allow students to come up with their own ways of solving application problems before the teacher shows how to solve them.	☐	☐	☐	☐	☐
3.9	Among other competences, the competence for mathematical modelling should be taught in the classroom.	☐	☐	☐	☐	☐
3.10	Students should often have the opportunity to follow their teacher's model solutions.	☐	☐	☐	☐	☐
3.11	Mathematics should be taught at school in such a way that students can discover connections on their own.	☐	☐	☐	☐	☐
3.12	Many aspects of mathematical modelling have a practical use or a direct application reference.	☐	☐	☐	☐	☐
3.13	Mathematical modelling should be a specific component in mathematics education.	☐	☐	☐	☐	☐
3.14	It helps students to understand mathematics when they are asked to discuss their own solution ideas.	☐	☐	☐	☐	☐
3.15	In mathematical modelling, you work on tasks that have practical value.	☐	☐	☐	☐	☐
3.16	Teachers should provide detailed procedures for solving application problems.	☐	☐	☐	☐	☐

4. Self-efficacy

To what extent do you agree with the following statements? Please check **one** box in each line.		strongly disagree	disagree	neutral	agree	strongly agree
It is **easy** for me to recognise the different abilities of the students using ...						
4.1	... their translation of mathematical results into reality.	☐	☐	☐	☐	☐
4.2	... their written solutions when modelling.	☐	☐	☐	☐	☐
4.3	... the recording of the plausibility test of their solution.	☐	☐	☐	☐	☐
4.4	... the adequate assessment of the relationship between the mathematical result and reality that they produce.	☐	☐	☐	☐	☐
4.5	... the recording of the solutions they create for modelling tasks.	☐	☐	☐	☐	☐
4.6	... the mathematical models they chose when modelling.	☐	☐	☐	☐	☐
4.7	... the recording of their mathematical results in the modelling process.	☐	☐	☐	☐	☐
4.8	... the mathematical formulae and symbols they used in the modelling process.	☐	☐	☐	☐	☐
4.9	... the determination of their improvement of the established models presented during modelling.	☐	☐	☐	☐	☐
4.10	... the assumptions they made when modelling.	☐	☐	☐	☐	☐
4.11	... the students' solutions when modelling.	☐	☐	☐	☐	☐
4.12	... their handling of the mathematical symbols and operators used in modelling.	☐	☐	☐	☐	☐

To what extent do you agree with the following statements? Please check **one** box in each line.		strongly disagree	disagree	neutral	agree	strongly agree
It is **difficult** for me to recognise the different abilities of the students using ...						
4.13	... their translation of mathematical results into reality.	☐	☐	☐	☐	☐
4.14	... the assumptions they have made in the modelling process.	☐	☐	☐	☐	☐
4.15	... the plausibility checks of their solution.	☐	☐	☐	☐	☐
4.16	... the recording of the real-world restructuring they have undertaken in modelling.	☐	☐	☐	☐	☐
4.17	... the mathematical models they chose when modelling.	☐	☐	☐	☐	☐
4.18	... the recording of the relationship between mathematical result and reality that they produce when modelling.	☐	☐	☐	☐	☐
4.19	... their handling of the mathematical formulae and symbols used in modelling.	☐	☐	☐	☐	☐
4.20	... their written solution when modelling.	☐	☐	☐	☐	☐
4.21	... their evaluation of the established models in the modelling process.	☐	☐	☐	☐	☐
4.22	... the simplifications and structures they have undertaken in the modelling process.	☐	☐	☐	☐	☐
4.23	... their mathematisations of a real situation.	☐	☐	☐	☐	☐
4.24	... the recording of their mathematical results in modelling.	☐	☐	☐	☐	☐

5. Knowledge about Modelling Tasks

5.1	**Characteristics of modelling tasks**		
5.1.1.	Modelling tasks...	True	False
... can be underdetermined.		☒	☐
... can be overdetermined.		☒	☐
... are as closed as possible.		☐	☒
5.1.2.	Modelling tasks...	True	False
... can also be Fermi tasks.		☒	☐
... are always also “dressed-up” word problems.		☐	☒
... can also be context-related word problems.		☒	☐
5.1.3.	Modelling tasks...	True	False
... are as close to real life as possible.		☒	☐
... are as authentic as possible.		☒	☐
... are as relevant as possible to the students.		☒	☐
5.1.4.	Modelling tasks...	True	False
... have clear solutions.		☐	☒
... are self-differentiating.		☒	☐
... may contain irrelevant information.		☒	☐

5.2	**Development of modelling tasks**		
5.2.1.	Good modelling tasks...	True	False
... require metacognitive processes of the students.		☒	☐
... require the translation of mathematics into reality.		☒	☐
... require the translation of reality into mathematics.		☒	☐
5.2.2.	Good modelling tasks...	True	False
... are always developed from inner-mathematical content.		☐	☒
... require non-mathematical knowledge.		☒	☐
... are problem-based.		☒	☐

5.2.3 Good modelling tasks...	True	False
... enable independent work.	☒	☐
... are suitable for individual work.	☒	☐
... always require cooperative learning forms.	☐	☒
5.2.4 Good modelling tasks...	True	False
... can be built on the basis of real problem situations.	☒	☐
... always require many sub-competencies of mathematical Modelling.	☒	☐
... illustrate the mathematical rigour.	☐	☒

5.3 Processing of modelling tasks		
5.3.1 Modelling tasks...	True	False
... are particularly suitable for use in heterogeneous learning groups.	☒	☐
... are not suitable for every grade level at secondary level.	☐	☒
... are suitable for individual development of high-performing students.	☒	☐
5.3.2 Modelling tasks...	True	False
... should encourage the practice of solution schemes.	☐	☒
... are used to record real world phenomena.	☒	☐
... can develop sub-competencies individually.	☒	☐
5.3.3 Modelling tasks...	True	False
... are only suitable for project teaching.	☐	☒
... are only suitable for regular education.	☐	☒
... are only suitable as a complement to the curriculum content.	☐	☒
5.3.4 Modelling tasks...	True	False
... are always cognitively more demanding than inner-mathematical problems.	☐	☒
... become less difficult with their degree of openness.	☐	☒
... always require passing through the complete modelling cycle.	☐	☒

6. Knowledge about Concepts, Aims and Perspectives

Please check the appropriate box (only **one** per task).

6.1	**Modelling cycles**

6.1.1.	Cognitive modelling distinguishes between the situation model and the real model.	☒
	A direct transition from the real situation to the mathematical model is not possible in modelling cycles.	☐
	The situation model is formed independently of the individual.	☐
	A distinction between the situation model and the real model is not conceivable in modelling cycles.	☐

6.1.2	Students solve modelling tasks circularly after the modelling cycle.	☐
	Circular schemas cannot illustrate modelling processes.	☐
	Models of modelling describe the actual solution approaches of students when working on modelling tasks.	☐
	Cyclical representations of modelling distinguish between different stages or phases.	☒

6.1.3	Working mathematically is not a sub-competency of mathematical modelling.	☐
	Mathematisation is not characterised by introducing mathematically idealised objects.	☐
	Validation includes a real-world verification of models.	☒
	The use of everyday knowledge does not characterise a step in modelling.	☐

6.1.4	Mathematisation refers to all the translation processes between reality and mathematics.	☐
	In modelling, interpreting is the process of checking the solutions obtained.	☐
	The separation of important and unimportant information is not a description of the sub-competency"Simplifying".	☐
	The mental representation of the problem situation is not a description of the sub-competency "Simplifying".	☒

6.2	**Aims and perspectives of modelling**	

6.2.1	Inner-mathematical applications do not develop modelling competence.	☐
	Modelling competence cannot be developed by addressing metacognitive handling strategies.	☐
	Modelling competence must be developed in selected school levels.	☐
	Modelling competence can be developed with real problem solving tasks.	☒

6.2.2	Mathematical modelling focuses on the process of solving real world problems.	☒
	Mathematical modelling does not focus on the study of relationships between mathematics and reality.	☐
	Modelling focuses on the translation process of the mathematical language into the real-world language.	☐
	Modelling cannot be considered as the processing of non-mathematical questions by embedding them in inner-mathematical contexts.	☐

6.2.3	Structuring and development of learning processes is not a subject of pedagogical modelling.	☐
	Contextual modelling does not involve mental abstraction.	☐
	The solution of real and authentic problems is not the main focus of pedagogical modelling.	☒
	Applied modelling focuses on developing mathematical thought processes by using models as mental images.	☐

6.2.4	Using models as mental images is not a perspective of cognitive modelling.	☐
	Modelling pursues goals such as theory development on a theoretical level.	☒
	The socio-critical level of modelling does not focus on the critical understanding of the environment.	☐
	Contextual modelling historically refers to pragmatic approaches to modelling.	☐

6.3	**Range of references to reality**	

6.3.1	Proximity to life means that a task of students is already considered as significant.	☐
	Student relevance means that a task is related to the future life of the students.	☐
	Life relevance means that a task will become relevant for students only in future situations.	☒
	Proximity to life, life relevance and student relevance can be used synonymously.	☐

6.3.2	Open tasks have a strong correlation to reality.	☐
	Open tasks can be classified by initial states, transformations, and target states.	☒
	The authenticity of tasks is always to be seen in context and in isolation from the mathematics used.	☐
	The authenticity of tasks always requires that there is a real situation in the original.	☐

6.3.3	Word problems have no real reality reference.	☒
	"Dressed-up" word problem are characterised by a real reality reference.	☐
	Word problems only aim at practicing the computing skills.	☐
	"Dressed-up" word problem focus on environmental development using mathematics.	☐

6.3.4	Context-related word problems have a mathematical focus.	☐
	Tasks that are dressed-up in (complex) situations are called context-related word problems.	☐
	Context-related word problems are used for environmental development with the help of mathematics.	☒
	Context-related word problems have no real reality reference.	☐

7. Knowledge about Modelling Processes and Interventions

Tasks and associated text vignettes that describe student conversations while performing modelling tasks are illustrated below. The tasks and text vignettes are used to diagnose, define support goals and derive appropriate interventions in these situations. The situations are characterised by the following **framework conditions**:

You are a teacher at a secondary school and your students at the specified grade level work on the tasks in a small project in **groups of 3**. The students have **already gained experience** with modelling tasks in advance. The situations presented arise in the **first half of the processing time**. The students in consideration have an **average level of performance** for the respective grade level. You observe the students during extracts of the conversations. You have **not yet interfered in the learning process.**

7.1 Traffic Jam (9th Grade)

Traffic jams often occur at the beginning of summer holidays. Christina is stuck in a 20 km traffic jam for 6 h. It is very warm and she is extremely thirsty. There is a rumour that a small truck is supposed to supply the people with water, but she has not yet received anything. How long will it take for the truck to supply all people with water?

STUDENT 1: We really need to know how many vehicles are stuck in the traffic jam.

STUDENT 2: Huh? Right!

STUDENT 1: How do we calculate how long it takes? A lot of things are missing from the task!

STUDENT 3: Yeah, we don't know how long it takes for every vehicle.

STUDENT 2: It is a dumb task.

STUDENT 1: We can divide the 20 km by the 6 h, then we know how fast it would take.

STUDENT 3: Exactly! We do not have any more information.

7.1.1.	To which phase of the solution process can the group of students **mainly** be assigned to? Please check **one** box.
Understanding	☐
Simplifying	☒
Mathematising	☐
Interpreting	☐

7.1.2.	Diagnose students' difficulty working on the task **in this situation**. Please check **one** box.
The students...	
... have problems in making assumptions.	☒
... draw a false conclusion from their mathematical result.	☐
... have problems in understanding the context.	☐
... use an inappropriate mathematical model.	☐

Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **this situation**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.1.3	"First, estimate how long a car is."	☐	☒	☐
7.1.4	"First, consider only part of the problem, e.g. how many cars are actually stuck in the traffic jam."	☒	☐	☐
7.1.5	"Right, now calculate that value."	☐	☒	☐
7.1.6	"Think about how you can determine the missing data."	☒	☐	☐

7.1.7.	Please indicate which support goal you would like set for the group after **this situation**. Please check **one** box.
Independent acquisition and evaluation of information.	☒
Critical questioning of results in the modelling process.	☐
Independent construction of mental models for given problem situations.	☐
Secure translation of simplified real situations into mathematical models.	☐

7.2 Stockpile Material (6th Grade)

From both sides of the national road L1081, a route is being constructed to bring the illustrated cone dumps to the open-cast mine that is 5.5 km away. The 8.2 million m^3 of stockpile material will be transported across the L1081. The entire fleet of transporters will then transport the stockpile material 16 h a day. 12 months are planned for this transport work. To ensure transport performance, the fleet will be expanded by 10 dump trucks, each with a payload of 96 tons.

Develop a model calculation for the transport of stockpile material if 1 m^3 of the waste has a mass of approximately 2 tons and the transport has to be completed within one year.

STUDENT 1: We need to know how many dump trucks they need.

STUDENT 2: And we have to estimate how long they take to drive there.

STUDENT 1: And how long to unload…. and load.

STUDENT 3: But, if there are multiple trucks, they cannot always be loaded directly.

STUDENT 2: Yeah, lots of things to consider. They do not work for 16 hours either, they have breaks, smoking breaks and such.

STUDENT 3: How do we get all this into one formula?

STUDENT 1: Boah, [leans back] no idea. It is way too hard.

7.2.1	To which phase of the solution process can the group of students **mainly** be assigned to? Please check **one** box.
Understanding	☐
Simplifying	☒
Mathematising	☐
Validating	☐

7.2.2	Diagnose students' difficulty working on the task **in this situation**. Please check **one** box.
The students...	
... have problems in making assumptions.	☒
... draw a false conclusion from their mathematical result.	☐
... have problems in understanding the context.	☐
... use an inappropriate mathematical model.	☐

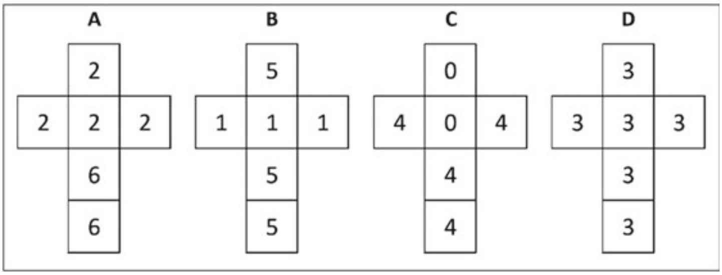
Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **this situation**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.2.3	"How can you deal with the missing information?"	☒	☐	☐
7.2.4	"Now assign variables to the quantities you have identified."	☐	☒	☐
7.2.5	"Divide the problem first and do not try to solve everything at once."	☒	☐	☐
7.2.6	"First of all, identify the most important information."	☒	☐	☐

7.2.7	Please indicate which support goal you would like set for the group after **this situation**. Please check **one** box.
Reduce the complexity of real situations independently.	☒
Secure translation of simplified real situations into mathematical models.	☐
Correct execution of mathematical operations and algorithms.	☐
Secure reference of mathematical results to a given problem situation.	☐

7.3 Safe Victory (12th Grade)

These four dice are described by their nets.

Two players choose a dice one after the other. After that, everybody throws the dice once. Whoever has the higher score wins. Develop a strategy with which the winning probability of the second player is the highest.



[Student 1 previously calculated the expected values for each cube.
$E(A) = \frac{10}{3}$, $E(B) = 3$, $E(C) = \frac{8}{3}$, $E(D) = 3$]

STUDENT 1: Is this possible?

STUDENT 2: Yeah, if I take C, you have to take A, because it is the highest.

STUDENT 3: And if I take A, you can choose between B and D, because they are the same. Makes sense, right?

STUDENT 1: Exactly.

7.3.1	To which phase of the solution process can the group of students **mainly** be assigned to? Please check **one** box.
Mathematising	☐
Working Mathematically	☐
Interpreting	☒
Validating	☐

7.3.2	Diagnose students' difficulty working on the task **in this situation**. Please check **one** box.
The students...	
... use an inappropriate mathematical model.	☒
... have problems in understanding the context.	☐
... have problems in making assumptions.	☐
... perform the calculation incorrectly.	☐

Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **this situation**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.3.3	"Check your strategy with another example."	☒	☐	☐
7.3.4	"Here you have to calculate the probabilities, not the expected values."	☐	☒	☐
7.3.5	"You have to approach the problem differently, the expected value will not get you anywhere."	☐	☒	☐
7.3.6	"Consider whether your result now delivers a correct strategy."	☒	☐	☐

7.3.7	Please indicate which support goal you would like set for the group after **this situation**. Please check **one** box.
Reduce the complexity of real situations independently.	☐
Secure translation of simplified real situations into mathematical models.	☒
Correct execution of mathematical operations and algorithms.	☐
Critical questioning of results in the modelling process.	☐

7.4 Filling up (10th Grade)

Mr. Stein lives in Trier, 20 km from the Luxembourg border. He drives his VW Golf to refuel in Luxembourg, where there is a fuel station just across the border. One litre of petrol costs only €1.05 here as compared to €1.20 in Trier.

Is the ride worth it for Mr. Stein?

STUDENT 1: [Has previously carried out the following calculation:

$$\mathrm{x} \cdot 0.15\frac{€}{1} = 2 \cdot 20\,\mathrm{km} \cdot \frac{81}{100\,\mathrm{km}} \cdot 1.05\frac{€}{1} \Rightarrow x \approx 22.41]$$

STUDENT 2: Strange, do you only have to fill up so little to make it worthwhile? But that is a very little. I would not have thought so. My father still takes canisters with him when he goes refuelling.

STUDENT 3: How much fuel goes into a car?

STUDENT 1: 50 L, maybe?

STUDENT 3: Yes, that would be realistic. Then he would not even need to take even one canister.

7.4.1	To which phase of the solution process can the group of students **mainly** be assigned to? Please check **one** box.
Understanding	☐
Working Mathematically	☐
Interpreting	☐
Validating	☒

7.4.2	Diagnose students' difficulty working on the task **in this situation**. Please check **one** box.
The students...	
... do not adequately verify their solution for plausibility.	☒
... draw a false conclusion from their mathematical result.	☐
... have problems in understanding the context.	☐
... perform the calculation incorrectly.	☐

Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **this situation**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.4.3	"Check whether you have taken everything into account."	☒	☐	☐
7.4.4	"What about the wear and tear on the car?"	☐	☒	☐
7.4.5	"How accurate is your model now?"	☒	☐	☐
7.4.6	"Your calculation is still too inaccurate, you have to include several variables."	☐	☒	☐

7.4.7	Please indicate which support goal you would like set for the group after **this situation**. Please check **one** box.
Correct execution of mathematical operations and algorithms.	☐
Secure translation of simplified real situations into mathematical models.	☐
Critical questioning of results in the modelling process.	☒
Secure reference of mathematical results to a given problem situation.	☐

7.5 Milk Carton (12th Grade)

Not only for financial reasons, but also from an environmental point of view, it makes sense to consider what packaging should look like, so that the least possible material is used. The picture shows a commercial milk carton. What should the milk carton look like so that the least possible material is used?

[The students have prepared the following calculation **in advance**:

$$V = 1\,\mathrm{l} = \mathrm{a} \cdot \mathrm{b} \cdot \mathrm{c} \Leftrightarrow \mathrm{a} = \frac{1\,\mathrm{l}}{b \cdot c}$$

$$O = 2ab + 2bc + 2ac = \frac{2\,\mathrm{l}}{c} + 2bc + \frac{2\,\mathrm{l}}{b}].$$

STUDENT 1: That is not possible now.

STUDENT 2: Why not, just derivate and then set zero.

STUDENT 1: Yeah, of what, b or c?

STUDENT 3: Mh, just go after b.

STUDENT 1: $\left[\text{calculates:} O' = 2c - \frac{2\,\mathrm{l}}{b^2} = 0\right]$ And now? I still have the b and the c.

7.5.1	To which phase of the solution process can the group of students **mainly** be assigned to? Please check **one** box.
Understanding	☐
Mathematising	☐
Working Mathematically	☒
Interpreting	☐

7.5.2	Diagnose students' difficulty working on the task **in this situation**. Please check **one** box.
The students...	
... have problems in making assumptions.	☒
... draw a false conclusion from their mathematical result.	☐
... use a completely inappropriate mathematical model.	☐
... make a computation error.	☐

Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **this situation**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.5.3	"First, consider a special case for the real problem."	☒	☐	☐
7.5.4	"Yeah, now just solve up to b."	☐	☒	☐
7.5.5	"Set a value for two variables first and then calculate the third side."	☐	☒	☐
7.5.6	"Where do you see a problem solving this equation?"	☒	☐	☐

7.5.7	Please indicate which support goal you would like set for the group after **this situation**. Please check **one** box.
Reduce the complexity of real situations independently.	☒
Critical questioning of results in the modelling process.	☐
Secure translation of simplified real situations into mathematical models.	☐
Correct execution of mathematical operations and algorithms.	☐

7.6 Container (8th Grade)

Containers are used on many construction sites to store construction goods or to collect construction waste. These containers have a special shape, which is intended to simplify loading and unloading. How much sand is in the container shown?

STUDENT 1: There are exactly 7,160,000 cubic metres of sand in there. Is that true?

STUDENT 2: I guess you were right, you calculated that with calculator.

STUDENT 1: Clearly. Then that is fine.

STUDENT 3: It is certainly right. I can present that.

7.6.1	To which phase of the solution process can the group of students **mainly** be assigned to? Please check **one** box.
Mathematising	☐
Working Mathematically	☐
Interpreting	☐
Validating	☒

7.6.2	Diagnose students' difficulty working on the task **in this situation**. Please check **one** box.
The students...	
... have problems in making assumptions.	☐
... do not adequately verify their solution for plausibility.	☒
... draw a false conclusion from their mathematical result.	☐
... use an inappropriate mathematical model.	☐

Please indicate which of the following interventions are suitable for an independence preserving support of modelling competence in **<u>this situation</u>**. Please check **one** box in each line.		suitable	unsuitable	do not know
7.6.3	"You probably made a mistake with the units somewhere."	☐	☒	☐
7.6.4	"Show me how big a cubic metre is."	☒	☐	☐
7.6.5	"Check the magnitude of your result."	☒	☐	☐
7.6.6	"How can you check the result of the calculator?"	☒	☐	☐

7.6.7	Please indicate which support goal you would like set for the group after **<u>this situation</u>**. Please check **one** box.
Secure translation of simplified real situations into mathematical models.	☐
Independent construction of mental models for given problem situations.	☐
Critical questioning of results in the modelling process.	☒
Correct execution of mathematical operations and algorithms.	☐

8. Knowledge about Modelling Tasks

Please place the tasks “Container” (1), “Filling Up” (2), “Safe Victory” (3) and “Milk Carton” (4) in order with regard to the following criteria for modelling tasks. Note the numbers corresponding to the tasks in the table on the next page.

(1) Container

Containers are used on many construction sites to store construction goods or to collect construction waste. These containers have a special shape, which is intended to simplify loading and unloading. How much sand is in the container shown?

(2) Filling Up

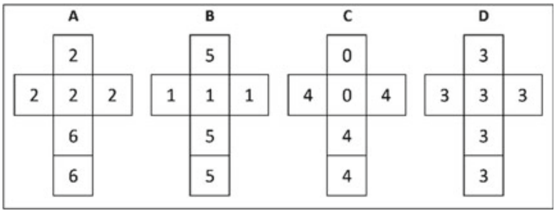
Mr. Stein lives in Trier, 20 km from the Luxembourg border. He drives his VW Golf to refuel in Luxembourg, where there is a fuel station just across the border. One litre of petrol costs only €1.05 here as compared to €1.20 in Trier.

Is the ride worth it for Mr. Stein?

(3) Safe Victory

These four dice are described by their nets.

Two players choose a dice one after the other. After that, everybody throws the dice once. Whoever has the higher score wins. Develop a strategy with which the winning probability of the second player is the highest.



(4) Milk Carton

Every day tons of packaging waste is generated in Germany. Not only for financial reasons, but also from an environmental point of view, it makes sense to consider what packaging should look like, so that least possible material is used. The picture shows a commercial milk carton. What should the milk carton look like so that least possible material is used?

8.1	Low openness	3	2	4	1	High openness
		3	2	1	4	
8.2	Low relevance for students	1	3	4	2	High relevance for students
		1	4	3	2	
8.3	Low reality relation	3	4	1	2	High reality relation
		3	1	4	2	
8.4	Low authenticity	3	1	4	2	High authenticity
		3	1	2	4	
8.5	Few modelling sub-competencies	3	4	1	2	Many modelling sub-competencies
		3	1	4	2	

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