# Pitched-Brick Barrel Vaults and Biaxial Cross-Vaults in Egypt's Western Desert 

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#### Abstract

Digital survey techniques allow for capturing and representing the object's shape under investigation, managing both the level of detail and the metrological accuracy. In archaeological and architectural research, a reality-based 3D model may support the implementation of geometric and metrological studies. Indeed, the geometric interpretation of the survey data allows the comprehension of the architecture and the development of reconstructive hypotheses of parts or areas that disappeared or are inaccessible. In the same way, metrological studies can support our understanding of the design and construction techniques used on the site, as well as the technical background of the local builders, and thus highlight cultural and technical influences. This method was used to study the Late Roman Fortified Settlement of Umm al-Dabadib, located in the Kharga Oasis (Egypt's Western Desert), and to broaden the knowledge related to its historical-cultural and architectural evolution. At the centre of the site stands a well-preserved Fort, around which barrel vaults cover mudbrick constructions. Most structures appear to be domestic units spread over 2-3 levels and storages. The research focuses on the analysis of the two different types of vaulted systems identified within the Fortified Settlement: pitched-brick barrel vaults and biaxial cross-vaults. The work aims at understanding and representing these specific constructions. In particular, the discussion shows how geometric/semantic modeling was fundamental to understanding the structure of these peculiar vaulted systems..


Keywords
digital survey; reality-based 3D modelling; digital reconstruction; pitched-brick barrel vaults; biaxial cross-vaults.


## Introduction

An accurate and reliable architectural survey represents the conditio sine qua non to ensure the development of geometric and metrological analyses. Digital survey techniques, imageo range-based, make it possible to capture and represent the shape of the object under investigation, allowing the management of both the level of detail and the metrological accuracy. The reality-based 3D model in the output represents a valuable support to develop geometric and metrological studies in the archaeological and architectural research [Camagni et al. 2019; Capone, Lanzara 2021; Barba et al. 2019; Santagati et al. 2019].
Indeed, the process of the geometric interpretation of the survey data allows not only the comprehension of the architecture that is being studied, but also the development of reconstructive hypotheses of the parts or areas that disappeared or are inaccessible [Pietroni and Ferdani 202 I ]. In the same way, metrological studies can support our understanding of the design and construction techniques used in the site under investigation and the technical background of local builders, highlighting any cultural influences [Messina et al. 2022; Spallone et al. 2019].
This methodological approach was used to study the Late Roman Fortified Settlement of Umm al-Dabadib, located on the borders of the Kharga Oasis in Egypt's Western Desert, and to broaden the knowledge related to its historical-cultural and architectural evolution. At the centre of the site rises a well-preserved Fort, around which mud brick constructions roofed by barrel vaults cover a more or less rectangular area (of about $9000 \mathrm{~m}^{2}$ ). Most of the structures appear to be domestic units spread over 2-3 levels and storages; to the east of the Fort, there are the remains of an early Christian church (fig. I).

## Research Aims

The site was initially recognized at the beginning of the twentieth Century [Ball 1900; Beadnell 1909]. In 1998, it was explored [Rossi 2000], and in 200I-2003, it was investigated in depth and surveyed for the first time [Rossi and Ikram 2018].
The photogrammetric surveys of the entire area of the Fortified Settlement based on a topographic network were carried out in the 2014-2015 seasons [Fassi et al. 2015]. The


Fig. 1. Position of the Fortified Settlement of Umm al-Dabadib; to the right, the Digital Elevation Model of the whole site, and below its southern and below its southern orthoimage. Graphi
elaboration by the elaboration
authors.

Fig. 2. Dimensions of the single brick and 3-head orientation of the masonry. Graphic elaboration by the authors.

palm = reformed paim
foot $=$ Ptolemaic/Egyptian foot

SIZE OF THE SINGLE BRICK


3-HEAD BRICK ORIENTATION
photogrammetric project has nearly 5,000 photos to cover the entire area of the settlement. The desert remained closed for security reasons from 2016 to 2022 when it was possible to go back and conduct more extensive photogrammetric acquisition campaigns for some of the architectural complexes. The digital surveys played a crucial role in the interdisciplinary investigations conducted within the ERC project LIFE (Living In a Fringe Environment), aimed at understanding functions, activities, and strategies of the late Roman settlements on the desert frontier.
The interpretation of the digital data [Fiorillo, Rossi, 20I7] and the subsequent metrological studies of the central Fort proved that the structure was designed and constructed using the ancient Egyptian cubit as its primary unit of measurement [Rossi and Fiorillo, 2018]. The ancient Egyptian cubit is about 52 cm , and this dimension corresponds to the more common thickness of the walls inside the settlement. Indeed, the most common brick measures about $(7 \times 16 \times 33)$ while the thickness of the mortar between the bricks is about 2 cm (fig. 2). Therefore, masonry with a 3 -head brick orientation, which turns out to be the one used most often, measures: $(16 \times 3+2 \times 2)=52 \mathrm{~cm}$. Figure 2 shows the dimensions of the single brick, pointing out the relation between centimeters and cubits and a typical 3-head brick orientation of the masonry. Figure 3 shows the southeast corner of unit $F$, where an example of 3 -head masonry is visible.
This finding implied as a first consideration that local workers were responsible for all aspects of building construction and design. Secondly, it allowed the geometric interpretation of the archaeological remains with a new metric key, the cubit, favoring the understanding of the spaces, the techniques used, and the development of reconstructive hypotheses.
The research presented focuses precisely on the analysis of the two different types of vaulted systems identified within the fortified settlement: pitched-brick barrel vaults and biaxial cross-vaults.
The work aims to understand and represent these fascinating and unique constructions. In particular, the discussion shows how geometric/semantic modeling was fundamental to understanding the structure of these unique vaulted systems.

## Vault typologies

Sure enough, mud bricks are a poor conductor of heat and are appropriate for dry environments with significant daily temperature variations. Moreover, a vaulted ceiling is ideal due to the desert climate since it allows hot air to rise and preserves the living room cooler. Indeed, the roofing system that characterizes the settlement is barrel vaults, constructed without any supporting system (centering or formwork) because the brick courses are self-supporting. This property is especially useful in a semi-arid environment where wood is scarce. The stability of the superimposed ring layers is guaranteed because the planes of the brick courses are perpendicular to the axis of the vault but not perfectly vertical, as they are inclined to lean against the back wall [Wendland 2007]. The vault profile was probably outlined as a reference on the back wall, and then the first course was placed against it.

Subsequent courses were laid one after the other, each supporting the subsequent one. The finished vault is composed of a succession of inclined brick layers and was covered with plaster [Van Beek 1987].
'Pitched-brick (barrel) vault' is the technical term for this architectural vaulting solution [Lancaster 2009; Lancaster 2012]. Moreover, this typology is also known as the 'Nubian' vault, as it was distinctive of the architecture of the Nubian region in the 20th Century [Zabrana 2018]. However, the use of this sort of vault construction system dates back to the Old Kingdom in Egypt [Kemp 2000;Arnold 2003] and has extended to all desert and semi-arid areas over time [Granier et al. 2006].
An in-depth geometric study of the surveyed 3D digital model of the numerous well-preserved barrel vaults on the Fortified Settlement of Umm al-Dabadib allowed establishing that the generatrix profile best approximates the shape of those vaults, not that of an arch or a parabola or a catenary but that of an ellipse [Rossi, Fiorillo 2020]. This basic knowledge was a fundamental first step to approaching the reconstructive modeling of the remains of the buildings in the archaeological site.
Inside the settlement, it has been found that the barrel vaults usually cover very narrow rooms on a rectangular layout with a span ranging from a minimum of 2 cubits (just over I m ) to a maximum of 8 cubits (about 4.16 m ). The vault thickness is about 2 palms (about 17 cm ), which corresponds to the size of the head of the type of brick used. On the other hand, each inclined course has a depth of I palm (about 8 cm ), which corresponds to the height of the brick (fig. 4).
Cross-vaults could also be constructed without formwork or centering by employing the same construction system used for barrel vaults with inclined courses. This type of biaxial vault was accomplished by intersecting four pitched brick vaults that started from the four walls of the room and leaning against them [Wendland 2007]. Several examples of this roofing system have been found within the Fortified Settlement of Umm al-Dabadib (figs. 3,5) and, generally, in the other North Kharga Oasis.
In particular, from the geometric and morphological analysis of the examples studied, the cross-vault may be described as made of four parts: two major and two minor vaults, each divided into symmetrical halves. The major barrel vault spans as wide as the length of the two opposite walls on which it rests. It consists of two symmetrical portions made of inclined brick layers, leaning against the two opposite walls of the room. They have approximately the same length, and between them passes the minor vault, following an orthogonal axis orthogonal; the span of the latter occupies about I/3 of the wall on which it stands. Also, the minor, teardrop-shaped barrel vault is made of two separated halves, leaning against the two opposite back walls and resting on the central courses of the major vault. Therefore, the portion corresponding to the ideal intersection between the two vaults (the minor and the major) is unloaded and has no structural function. This characteristic implies that this quadrangular-shaped area could also be left void and, thus, possibly, be a hole allowing an efficient ventilation of the room. In this case, the minor vault would also consist of two halves. Unfortunately, no biaxial vault survived intact, so we have no example of how the intersection area between the axes of the vaults looked like. Perhaps the central hole was closed in some cases and not in others, according to the practical needs. In fact, there is one case (Unit F showed in the figures 3 and 4) in which two cross-vaults were built one on top of the other: the lower vault is ruined, but it must have been closed on top, whereas the upper one might have been endowed with a hole.
Inside the settlement, this ingenious and elegant vaulting system seems to characterize the main living room of the houses. Therefore, this type of vaulted roof was of fundamental support in identifying the domestic houses inside the archaeological site. In total, 7 biaxial vaults have been identified with certainty and correspond to 7 domestic units denominated with the letters A to G (fig. I). Another 3 biaxial vaults associated with 3 other houses seem to be very probable, even if the remains are not clearly distinguishable.
The biaxial vaults usually cover a tall living room higher than the other rooms of the domestic unit they belong. Unfortunately, the net height is never exact because sand and debris always cover the floor in a layer of variable dimensions. The height appears to average 6-7

Fig. 3. Southeast corner of unit $F$, where the biaxial vaults on two levels (the only example) and a 3-head masonry example 3-head masonry example
are visible. Photograph by the authors.

Fig. 4. A portion of the fortified settlement of Umm al-Dabadib seen from the east side, where the backside of the Unit F 2-level biaxial vault is visible. Photograph by the author.

Fig. 5. Ortho-image of the west facade of Unit A (on the top) and its reconstruction (on its reconstruction (on
the bottom). Graphic elaboration by the authors


WEST ELEVATION

cubits, thus generally exceeding 3 m . The square or rectangular shape of the room they cover ranges from a maximum of $(7+1 / 3 \times 7)$ cubits - equal to $(3.82 \times 3.64)$ meters - to a minimum of $(6 \times 5+1 / 3)$ cubits - equal to about $(3.12 \times 2.78)$ meters.
The best example, because it is more easily accessible and visible, is the cross-vault of Unit A, also called 'corner house', which occupies the northwest corner of the settlement. Only tiny parts of the two halves of the major vault are still visible, but traces of the minor vault are also evident [Fiorillo et al. 202I]. In particular, it can be seen how the inclined brick courses of the bigger vaults stop to make room for the minor vault running in the orthogonal direction. The inclined courses of the major vault and the profile of the minor one leave a peculiar teardrop shape imprinted on the wall. An honorific niche is usually placed in the centre of this elegant layout, just below the apex of the smaller vault (fig. 5).
The results of these investigations were applied for the purpose of three-dimensional reconstructive modeling, as the visible biaxial vaults are all partially collapsed. Indeed, having identified a partial profile emerging from the rubble, the formal, metric, and geometric knowledge of these roofs helped to interpret and reconstruct the internal layout of the rooms that are inaccessible or have long since collapsed.
The following main procedural steps characterize the three-dimensional reconstructive modeling of these architectural elements. I) Dimensioning of the floor layout of the biaxial vaulted living room (thickness of the walls and length of the vault) on the point cloud model. 2) Reconstruction based on the digital model of the elliptical profile of the major vault. This geometric profile represents the generating line, which determines the surface of the barrel vault with its movement along the directing line. 3) Identification of its impost plan (fig. 6). 4) Extrusion along the axis of the elliptical profile (extrude or loft, fig. 7). 5) Reconstruction based on the digital model of the profile of the minor vault. 6) Identification of its impost plane. 7) Extrusion along the axis of the minor vault profile. 8) Modeling of the connections between the vaults. 9) Completion of the model with the reconstruction of the walls.


Fig. 7. Cross-vaults 3D modeling reconstruction (on the left) and the longitudinal section on the main vault facing west (on the right). Graphic elaboration by the authors.


Cross-vaults 3D recontruction

Longitudinal section on the main vault facing west

Figure 6 shows two-dimensional technical drawings (floor plans and vertical sections) elaborated on the point cloud (points I-3 and 5-6). The measurements in meters are represented in black, and those in cubits using the red color. Indeed, figure 7 illustrates the cross-vault 3D modeling, using red color for the major vault and the blu for the minor. Moreover, the longitudinal section on the main vault facing west is shown. In the 3D geometrical model of the figure 7, the central void is visible in the portion corresponding to the intersection of the two vaults. As mentioned, it may or may not have been filled.
The metric, geometric, and structural considerations were carried out by integrating data from various sources: bibliographic references, previous studies, photographs, hand-direct drawings, and the photogrammetric survey. Above all, the reality-based digital model was essential for accurately measuring the 'shape and size' of the studied elements. The elaboration of technical drawings and 3D models are therefore based on both metric survey data and bibliographic studies, thus representing a synthesis of both. In other words, this type of research highlights the importance of combining knowledge and representation: without understanding this type of structure, it would have been impossible to represent them technically and geometrically. In the same way, the need to represent them triggered and supported their cognitive process.

## Conclusions

The research presents the metric, geometric, and constructive description of two vaulted systems that characterize the desert areas of the North Khraga oasis: the pitched-brick barrel vault and the crossed biaxial vaults. The study highlights how the conceptual process for the technical graphic representation and 3D modeling was essential to understand the structure of these widespread but somehow little-known constructions.
The expansion of the analytical approach described in this paper to a larger corpus of vaulted buildings would perhaps provide new evidence for understanding how Roman and Egyptian technological and architectural expertise merged during the Late Period in the Kharga Oasis and perhaps the entire province.

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