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Mark Fossett

New Methods for Measuring and Analyzing Segregation

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ISSN 1389-6784 ISSN 2215-1990 (electronic)
The Springer Series on Demographic Methods and Population Analysis
ISBN 978-3-319-41302-0 ISBN 978-3-319-41304-4 (eBook)
DOI 10.1007/978-3-319-41304-4

Library of Congress Control Number: 2017930612

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Abstract

In this monograph, I place indices used to measure the uneven distribution dimension of residential segregation in a new framework; I cast them as simple differences of group means on individual-level residential outcomes scored from area racial composition. The "difference-of-group-means" framework places all popular indices in a common measurement framework in which index scores are additively determined by individual residential attainments. This yields new and appealing options regarding substantive interpretations of the scores of segregation indices. It also brings important methodological benefits by creating the new possibility of joining the investigation of aggregate segregation and the investigation of individual-level residential attainments together in a single analysis. Specifically, segregation index scores now can be equated with the effect of group membership (e.g., race) on individual residential attainments, and thus variation in segregation over time and across cities can be equated to the ways that the effect of group membership varies over time and with city characteristics in multilevel models of individual residential attainments. Framing segregation indices in the difference-of-group-means framework has several other desirable consequences for segregation analysis. It creates opportunities to investigate segregation in new ways by permitting researchers to assess the impact of group membership on residential outcomes in the context of multivariate attainment models that if desired can include controls for other individual characteristics (e.g., language, education, income). Relatedly, it suggests a new basis on which to evaluate and compare segregation indices – whether the individual-level residential outcomes they register and reflect are relevant for theories of residential dynamics and/or are relevant for concerns about racial differences in socioeconomic attainments and life chances. Finally, the difference-of-group-means framework paves the way for developing refined versions of indices that are free of potentially problematic upward bias intrinsic to standard formulations of these indices. Significantly, adopting the new framework outlined here does not require breaking with previous conceptions of segregation; results of empirical analyses of segregation using traditional computing formulas can be exactly replicated within this framework even as several new options for measurement and analysis become available.

Preface

In this monograph, I review findings and observations I have accumulated while grappling with issues in segregation measurement over the past decade. My explorations in this area were motivated by three concerns. The first was that, while it is obvious to all concerned that residential segregation can potentially have important consequences for group differences in residential outcomes, the literature on segregation measurement does not provide formulations of segregation indices that make it clear exactly what implications index scores have for group differences in residential outcomes. In this regard, the measurement and analysis of segregation is on a different conceptual footing from standard approaches to measuring and analyzing intergroup disparity and inequality on other socioeconomic and stratification outcomes such as education, occupation, and income. Researchers investigating disparities in these other areas routinely assess inequality and disparities based on comparisons of group means on individual-level outcomes. Consequently, the connections between scores of measures of aggregate inequality have clear and direct implications for group differences in the attainments of individuals. In contrast, the literature on segregation measurement has not established how segregation index scores are connected to group differences on residential outcomes for individuals. This is surprising and unfortunate because the substantive relevance of segregation indices ultimately rests on the presumption that their scores carry important implications for group differences on individual residential outcomes and yet these implications have remained obscure. I address this concern here by introducing new formulations of popular segregation indices that place them in an overarching "difference-of-group-means" framework that clarifies exactly how segregation index scores are connected to group differences in individual-level residential outcomes.

The second concern motivating me was that the literature on segregation measurement and analysis did not provide a straightforward means for directly linking quantitative findings from studies of micro-level processes of residential attainment to findings for segregation index scores at the aggregate level (e.g., city-level segregation scores). As a result, the research literature has been divided into two important but largely disconnected traditions. One is a tradition of macro-level studies that use aggregate-level index scores for cities to investigate how segregation varies across cities and over time; the other is a tradition of micro-level studies that examine how various individual-level residential attainments are related to social and economic characteristics of individuals and households such as income, education, nativity, English language ability, family type, and other related individual-level variables. The current state of the literature leaves researchers in both traditions in the frustrating situation of being unable to directly connect segregation index scores at the aggregate-level to the individual-level outcomes that are examined and modeled in micro-level residential attainment analyses. I address this concern here by drawing on the difference-of-group-means measurement framework to develop methods for linking index scores to individual-level residential attainment processes. In this new approach, segregation index scores now can be interpreted as the effect of group membership (e.g., race) on segregation-determining residential outcomes in an individual-level attainment model. The level of segregation in a city thus can now be assessed by estimating the effect of group membership on individual-level residential attainment in bivariate attainment model. More importantly, the model can be extended to a multivariate specification to properly take account of the role that nonracial characteristics (e.g., income) may play in shaping the level of segregation in a city. And the model can be further extended to multilevel specifications to take account of how city-level factors impact segregation net of the role of nonracial individual characteristics. Significantly, past findings of aggregate-level analyses can be exactly replicated and subsumed under this approach while giving researchers many new options for analysis.

The third concern motivating me was that, under the current state of segregation measurement, many interesting and important research questions cannot be addressed because segregation index scores exhibit problematic behavior under a wide range of commonly occurring conditions. In particular, all indices of uneven distribution are subject to inherent positive bias that can render their scores untrustworthy and potentially misleading in a variety of situations – for example, when segregation is measured at small spatial scales (e.g., at the block level) or when the groups involved in the segregation comparison are small and/or are imbalanced in size. This presents severe obstacles to many interesting and important lines of inquiry in segregation research. For example, it precludes quantitative study of segregation for newly arriving immigrant and migrant groups because, by definition, the groups initially are small in both absolute size and relative size in comparison to established population groups. Similarly, it precludes study of segregation among narrowly defined subgroups with a population (e.g., foreign- and native-born Latinos, high-income Whites and Blacks, etc.) because one or both subgroups often are small in absolute and/or relative size. Additionally, it potentially frustrates investigation of segregation dynamics using agent simulation models because studies in this tradition routinely examine segregation at small spatial scales.

The impact of these concerns on segregation research is substantial, pervasive, and hard to overstate. It has led researchers to routinely adopt two "defensive" practices. One practice is to use various ad hoc guidelines to "screen" cases to avoid measuring segregation in situations where index scores cannot be trusted. The other practice is to differentially weight cases to minimize the undesirable impact of bias on index scores even after cases have been "screened" to eliminate those where index scores are most problematic. The first practice prevents researchers from undertaking many studies that otherwise would be conducted and thus sharply restricts the scope of segregation studies. In addition, it draws on ad hoc guidelines that at best are crude and at worst have uncertain effectiveness. The second practice of differentially weighting cases is predicated on the implicit recognition that the first practice of screening cases cannot adequately deal with the problem of bias. Unfortunately, differential weighting of cases is itself inadequate. First and foremost, it leaves index scores untrustworthy on a case-by-case basis and so one cannot discuss and compare cases – otherwise weighting would be unnecessary. Second, while the strategy permits researchers to avoid "draconian" screening of cases and thus larger nominal sample sizes, differential weighting in the end amounts to assessing segregation patterns and trends based on the small subset of cases that get large weights.

I address this unsatisfying state of affairs by developing and introducing refined versions of popular segregation indices that provide trustworthy measurements of segregation over a much broader range of situations than standard measures. I demonstrate that the resulting unbiased measures have attractive properties and provide researchers the previously unavailable option of dealing with index bias directly at the point of measurement on a case-by-case basis.

As I worked to address the three concerns just mentioned, I increasingly took interest in a fourth concern – the question of whether different segregation indices yielded similar or different results and, if different, under what conditions and why. Conventional wisdom in the segregation measurement literature has been that the most widely used measures of uneven distribution tend to give similar results. But I found discrepancies between indices were common when I measured segregation over broader samples of cases and group comparisons. At first I thought the large discrepancies between scores of different indices might be a by-product of the problem of index bias. After all, using broader samples tends to include cases that are more susceptible to being adversely affected by the problem of bias, and previous methodological studies had reported that indices vary in susceptibility to scores being inflated by index bias. But on investigating the issue further, I found that the role of bias was only a minor part of the story as discrepancies between scores for different indices persisted even when using refined versions of the indices that were free of the influence of bias.

The difference-of-group-means framework provided a useful perspective for exploring this issue and led me to recognize that the discrepant scores I observed reflected an aspect of uneven distribution that is not generally widely appreciated, namely, index sensitivity, or lack thereof, to whether displacement from even distribution is concentrated or dispersed. My goal in exploring this issue was different in nature from my goals in addressing the first three concerns I noted. In this case, I was not seeking to make progress toward solving technical problems in measuring and analyzing segregation. Instead, my goal was to clarify the nature of the differences between indices to better account for why different indices sometimes yield different results. In the end, I concluded the issue could be framed succinctly in terms of index sensitivity to whether group displacement from even distribution is concentrated and dispersed. At any given nontrivial level of group displacement from even distribution, groups can be concentrated in a way that produces homogeneous areas for both groups, or groups can be dispersed in a way that minimizes homogeneous areas. Indices vary in their sensitivity to this aspect of uneven distribution. For example, the widely used index of dissimilarity (D) takes the same value regardless of whether displacement is concentrated or dispersed, while the separation index (S) takes higher values when displacement is concentrated and takes low values when displacement is widely dispersed.

I am hardly the first to recognize the technical basis for this potential difference between indices. But I believe my discussion and review of these issues makes useful new contributions to the literature on segregation measurement. First off, the analyses I report here document that important discrepancies between different index scores are much more common than previous methodological studies have suggested. Second, the difference-of-means framework for measuring segregation I introduce here provides a new basis for understanding exactly how different indices can yield discrepant index scores. Finally, I offer analytic exercises and empirical case studies to further clarify the basis of differences between indices and dispel certain misconceptions regarding of these issues.

My hope is that this monograph will contribute to a better understanding of the issues examined here and also will provide useful practical strategies for measuring and analyzing segregation. Looking back on the decade of work reflected here, I can see with hindsight that the core issues are closely interconnected. Establishing how segregation index scores related to group differences in residential outcomes was a necessary step for developing methods for conducting micro-level analysis of individual-level residential attainments that could directly account for overall segregation in a city at the aggregate level. Discovering that the residential attainments in question were rooted in a simple construct – the pairwise group proportions in the area of residence – then paved the way for a further discovery, namely, that troublesome problem of index bias could be eliminated by making surprisingly simple refinements in the calculation of pairwise group proportions. Thinking more carefully about the individual-level residential outcomes that are registered by different indices led to a better understanding of the differences between concentrated and dispersed displacement from even distribution.

The interconnections among the issues are clearer in hindsight. If I had recognized them from the start, I would have avoided muddling around for so long. I offer my findings and observations on these and related matters here in hopes that others will find them useful. I apologize in advance for the many limitations of this study but also suggest that it occasionally offers original insights and new options for segregation measurement and analysis that I hope can help other researchers move the study of segregation forward.

Many organizations and many people have provided support and encouragement that helped make my work possible. Over the past decade, I was fortunate to receive funding support for projects that helped me develop findings and observations included in this monograph. They included National Institutes of Health research grants R43HD038199 and R44HD038199 “Simulating Residential Segregation Dynamics: Phases I & II”; a proposal development grant from the Mexican American and Latino Research Center at Texas A&M University, College Station; and National Science Foundation research grant SES 1024390 “New Methods for Segregation Research.” Of course the funding agencies are not responsible for and do not necessarily endorse the findings and conclusions I offer. Finally, I also acknowledge a faculty development leave from Texas A&M University that was crucial for completing the first full draft of the monograph. I also thank the College of Liberal Arts and the Open Access to Knowledge (OAK) program at Texas A&M University which have provided generous funding to help publish this monograph as an open access work.

I have always received encouragement and support from my colleagues and good friends in the Sociology Department at Texas A&M University. In particular, I must mention Jane Sell, Dudley Poston, and Rogelio Saenz to offer special thanks for their support over this period of extended effort. I thank Wenquan “Charles” Zhang for engaging me in many stimulating and productive discussions on the issues addressed in this study and for collaborating with me as co-investigator on the aforementioned NSF project and associated empirical analyses that draw on the new measures and methods introduced in this work. Warner Henson III, currently a doctoral student in the Sociology Department at Stanford University, deserves special acknowledgment for helping me establish the difference-of-means formulation of the Theil index while an undergraduate major in sociology here at Texas A&M University. I also must thank Amber Fox Crowell, at the time a doctoral student in sociology at Texas A&M University and now an assistant professor at California State University-Fresno, who served as research assistant on research projects in which the measures and methods introduced here were developed and refined. Her insights, questions, and suggestions have been extremely helpful. She has a deep grasp of the potential value the measures and methods can bring to empirical analyses and has applied them with great success in her dissertation project and her projects as a postdoctoral researcher. I also offer a note of appreciation to the many students who have attended informal workshops on segregation measurement and analysis on a regular basis over the past several years including, in rough chronological order, Warren Waren, Lindsay Howden, Amber Fox Crowell, Gabriel Amaro, Bianca Manago, Jennifer Davis, Jessica Barron, Nicole Jones, Bo Hee Yoon, Brittany Rico, Melissa Sanchez, Chiying Huang, Xuanren Wang, Katelyn Polk, Bridget Clark, Danielle Deng, Nathanael Rosenheim, Cassidy Castiglione, and Xinyuan Zou. Their questions, puzzlements, and suggestions helped me develop better ways of explaining and thinking about some of the material presented here. As ever throughout my career, I benefit from the voice of my dissertation supervisor, mentor, and friend, Omer Galle, which always present in the back of my mind encouraging me and challenging me.

I also must acknowledge the amazing support Ms. Evelien Bakker and Ms. Bernadette Deelen-Mans with Springer have given during the process of bringing this monograph to completion. Apparently, it is impossible to exhaust their patience

and goodwill. They have been encouraging, helpful, and pleasant at every step in the process. For that I am truly grateful.

I close by thanking my wife, Betsy, and our children Kate, Tyler, and Lane for their support and patience. They have brought, and continue to bring, great joy to my life. They have kept me grounded through the ups and downs of the too-long gestation period for this monograph. I love them more than they know and appreciate that they put up with me and indulge my passion for the study of social inequality and segregation.

College Station, TX, USA Mark Fossett
December 2015
July 2017

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Chapter 1
Introduction and Goals

The literature on residential segregation is one of the oldest empirical research traditions in sociology and has long been a core topic in the study of social stratification and inequality as well as in the study of the demography of spatial population distribution. This literature is guided by the fundamental assumption that group differences in neighborhood residential outcomes are closely associated with social position and life chances. Accordingly, indices measuring segregation, especially the dimension of uneven distribution, are viewed as important summary indicators of overall group standing and scores for segregation indices have been a mainstay of research documenting levels, patterns, and trends in the residential segregation of racial and ethnic groups. Given the extensive attention social scientists have directed to the study of residential segregation, one might assume that the relationship between residential segregation and group differences in neighborhood residential outcomes is well understood. Surprisingly, this is not the case. The issue has received little attention in the literature on segregation measurement. Consequently, researchers are not able to offer precise conclusions about group differences in residential outcomes based on scores for popular and widely used indices of uneven distribution.

In this monograph I address this deficiency in the literature by outlining a new approach to measuring uneven distribution. My goal is not to replace familiar, widely-used indices with new ones. Instead, I wish to place popular indices in a new alternative framework that clarifies the implications they carry for group differences in individual-level residential outcomes. My motivation for doing this rests on two convictions. One is that understanding how segregation is related to individual residential outcomes is desirable for its own sake and brings valuable new options for interpreting segregation index scores and understanding differences between them. The other is that casting segregation indices in terms of group differences in individual residential outcomes brings benefits for segregation measurement and analysis including, as two primary examples, the ability to directly link segregation at the aggregate or macro level to micro-level processes of residential attainment and the

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_1

ability to develop versions of the indices that are free of the troublesome problem of inherent upward bias.

Moving from generalities to specifics, my goal in this monograph is to set forth the "difference of means" framework, a new framework for segregation measurement wherein popular indices of uneven distribution are cast as simple differences of group means on residential outcomes that register group contact and exposure based on area racial composition. In accomplishing this goal I establish that all widely used segregation indices including the Gini Index (G), the Delta or Dissimilarity Index (D), the Hutchens Square Root Index (R) – an index with close similarities to the Atkinson Index (A), the Theil Entropy Index (H), and the Separation Index (S) – also known as the variance ratio and a variety of other names, can be expressed as a difference of group means on individual- or household-level residential outcomes (y) that are scored on the basis of index-specific scaling of group contact based on area group proportions.

The indices just listed are all well-known and all have been reviewed in detail in many previous methodological studies (e.g., Duncan and Duncan 1955; Zoloth 1976; James and Taeuber 1985; Stearns and Logan 1986; White 1986; Massey and Denton 1988; Hutchens 2001, 2004; Reardon and Firebaugh 2002). The contribution I seek to make is to clarify a characteristic of these indices that currently is not well understood; namely, the particular way each one relates to and ultimately quantitatively registers group differences in neighborhood residential outcomes. The sociological relevance of segregation index scores rests on the presumption that they carry important implications for group differences in social position and life chances that are associated with area of residence. Segregation researchers and consumers of segregation research thus generally assume that variation in segregation index scores tends to correlate with variation in a broad range of group disparities associated with neighborhood residential outcomes.

It is definitely plausible to assume that summary index scores may serve as proxies for valuable, but usually unavailable information about residentially-based group inequality and disparity. But it is important to recognize that, in the final analysis, the calculations that yield segregation index scores revolve around a simple and very particular aspect of neighborhood residential outcomes – "pairwise" group proportions.[1] This residential outcome can be understood in multiple ways from the point of view of individuals and households. For example, it can be understood as registering levels of contact or exposure based on co-residence with members of the two groups in the comparison. Alternatively, it can be understood as registering exposure to deviations or departures from the racial composition of the city as a whole. One of my goals is to clarify how different indices register group differences on individual residential outcomes relating to area racial mix and group proportions.

[1] The expression "pairwise" ethnic mix or group proportion signifies that the calculations involved use only the population counts for the two groups in the comparison. It can be contrasted with "overall" ethnic mix or group proportion based on the full population including groups *not* in the segregation comparison. Significantly, the presence and distributions of groups other than the two in the comparison has no bearing on scores for indices of uneven distribution.

In doing so I hope to help researchers better understand what indices specifically measure in this regard when they are interpreting index scores and evaluating their relevance as proxies for group position.

Indices of uneven distribution provide quantitative summaries of how groups are differentially are distributed across neighborhoods that vary on "pairwise" racial mix. This obviously has direct implications for individual residential outcomes relating to racial mix and indices can be cast in two ways that reflect this fact. One option is to cast indices as simple, overall population averages on individual residential outcomes scored on the basis of area racial mix. I review this option briefly, but I give it limited attention because it not especially novel and it is not useful for my main goals. The second option is to cast indices of uneven distribution as group differences of means on segregation-relevant neighborhood residential outcomes scored from pairwise racial mix. This approach is the primary focus of my attention because it resonates with substantive interests that motivate much of the research on segregation – namely, concerns about group disadvantage and inequality rooted in differential residential distribution. Additionally, the difference of means approach brings several practical advantages for segregation measurement and analysis.

I offer the difference of means framework for computing indices of uneven distribution in hopes that it will be a useful alternative to prevailing approaches to computing index scores. However, I stress from the outset that I intend this new framework to be an enhancement of and supplement to traditional approaches to segregation measurement, not a wholesale replacement. The difference of means framework does not yield different values for index scores. Instead, it yields identical index scores but draws on new, mathematically equivalent index formulations to gain new understandings of segregation and new options for measurement, interpretation, and analysis. In current practice indices of uneven distribution are formulated and interpreted in ways that focus attention on aggregate-level patterns for spatial units (i.e., areas or neighborhoods). The formulas used generally feature calculations that register the extent to which the racial mix of areas (neighborhoods) within a city depart from the racial composition of the city as a whole. These widely used formulas are tried and true and they are useful and convenient for many purposes. That said, it also is important to recognize what the most widely used computing formulas neglect and obscure. Traditional approaches to measuring uneven distribution do not clarify the how segregation is connected to group differences in neighborhood residential outcomes for individuals. It is obvious that neighborhood departures from city racial composition necessarily carry implications for group differences in residential outcomes. But the specific nature of these implications is not well understood because it is not revealed in prevailing approaches to formulating, computing, and interpreting segregation indices.

The "difference of means" framework for calculating and interpreting popular segregation indices I introduce here addresses this gap in the literature on segregation measurement. The framework highlights something that currently is not widely appreciated – that differences between indices can be understood as arising from a single factor, the particular way each index registers segregation-relevant residential outcomes for individuals as scored from area racial composition. On reflection this probably should not be surprising. All indices are calculated from the same underlying distribution of residential outcomes on pairwise racial proportions. Consequently, index scores obtained from group differences of means on residential outcomes can differ only by registering these very specific residential outcomes in different ways. These cross-index differences in "scoring" area racial mix provide a new basis for comparing and evaluating indices of uneven distribution.

The difference of means formulation of indices of uneven distribution brings additional practical benefits beyond clarifying how index scores are related to group differences in residential outcomes. One example is that the approach makes it possible to join the study of aggregate segregation with the study of individual-level residential attainment in a seamless way. This becomes possible because segregation index scores now can be viewed as arising from the simple additive aggregation of segregation-relevant, neighborhood residential outcomes for individuals. As a result, segregation index scores can be equated with the effect of race in micro-level regression models predicting the residential attainments of individuals and households that additively determine segregation at the aggregate-level.[2] These micro-level attainment models can be extended to include multiple individual and household characteristics as predictors in the attainment equation. This then enables researchers to assess segregation – now equated to the effect of race on residential attainments – in multivariate specifications that control for non-racial factors (e.g., income, nativity, language ability, etc.) that also may affect the residential attainments that ultimately determine segregation. The new ability to model the individual-level residential attainments that directly and additively give rise to segregation makes it possible to undertake quantitative standardization and decomposition analyses to assess the extent to which group differences on factors other than race contribute to overall segregation based on their impact on residential outcomes that determine aggregate segregation. Finally, city-specific, individual-level models of residential attainments can be extended to multi-level specifications that can be used to investigate variation in segregation over time and across cities in new ways that previously were not feasible.

The kinds of analysis options just described have been available and used on a routine basis for decades in the broader literature investigating racial differences in most domains of socioeconomic attainment (e.g., education, income, occupation, etc.). Until now, however, they have been not been available in segregation research. The reason for this is that segregation, in contrast to racial inequality in other socioeconomic attainments such as education, occupation, and income, has not been explicitly formulated in terms of group differences on individual attainments. Placing indices of uneven distribution in the difference of means framework thus puts segregation analysis on similar conceptual footing with research traditions that analyze other aspects of racial socioeconomic disparity and inequality.

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[2] Specifically, the segregation index score is equal to the value of the unstandardized regression coefficient for race (coded as 0 or 1) in an individual-level regression predicting residential outcomes.

The difference of means formulation of indices of uneven distribution brings other benefits as well. One conceptual benefit is to introduce a new basis for evaluating and choosing among familiar indices; namely, whether and to what degree the individual-level residential outcomes registered by a given index are relevant for theories of segregation dynamics and racial socioeconomic stratification. Another practical benefit is that the approach makes it easy to implement spatial versions of popular segregation indices.

Last but not least, the difference of means formulation of segregation indices provides a basis for gaining a better understanding the source of index bias – a well-known and vexing problem that can make scores of standard versions of indices of uneven distribution untrustworthy and potentially misleading. This new understanding then makes it possible to develop unbiased versions of popular indices based on implementing surprisingly simple refinements to index formulas that eliminate this problematic behavior of index scores.

In the chapters that follow I introduce the difference of means formulations of widely used segregation indices and provide more detailed reviews of the new options for measurement, interpretation, and analysis just mentioned. In Chaps. 2, 3, 4, and 5 I introduce the difference of means framework and explore differences between indices as revealed through the lens of this framework. I begin in Chap. 2 by noting that scores of popular indices of uneven distribution can be obtained using a variety of mathematically equivalent formulas and I briefly review selected formulas to highlight how they support different insights about segregation measurement. I conclude the chapter by introducing the difference of means formulas that are used throughout this monograph. In Chap. 3 I provide a general overview of the difference of means framework. I then expand on this in Chap. 4 by offering a more detailed discussion of how individual measures of uneven distribution can be cast as difference of group means on residential outcomes scored from area racial proportions. In Chap. 5 I note a useful insight about uneven distribution that emerges from the difference of means framework; namely, that differences between indices can be seen as arising from a single source – how each index registers individual residential outcomes scored from area group proportions.

In Chaps. 6, 7, and 8 I review the logical and empirical differences among popular measures of uneven distribution and offer suggestions regarding how to understand and interpret these differences. In Chap. 6 I document that, in contrast to findings reported in some previous methodological studies, popular indices of even distribution can and often do yield highly discrepant scores. The analyses I present here establish that the findings of earlier methodological studies – which reported that popular indices tended to be highly correlated in empirical application – are a byproduct of focusing primarily on White-Minority segregation in a small subset of large metropolitan areas where the minority group is a substantial presence in terms of relative group size and where group residential distributions are characterized by a particular pattern of "prototypical" segregation. This is a pattern of uneven distribution in which group displacement from parity involves a high level of group separation and area racial polarization because *both groups* are disproportionately concentrated in homogeneous areas. I refer to uneven distribution with this pattern of "concentrated displacement" as "prototypical segregation" because this signature pattern – in which all popular measures of uneven distribution take high scores – is always present in crafted examples used to illustrate high segregation in didactic discussions of segregation measurement. Similarly, it also is invariably present in empirical cases used to illustrate high levels of segregation. So it easy to understand that many would not be aware that popular indices can take substantially discrepant scores.

The empirical analyses I present in Chap. 6 document that uneven distribution does not always take the form of prototypical segregation. To the contrary, the analyses instead reveal that broader samples of cities include a large number of cases with a sharply contrasting pattern of "dispersed displacement" wherein uneven distribution involves extensive group displacement from parity but does not involve group separation and area racial polarization. In these situations, index scores can be highly discrepant. Specifically, indices that are sensitive to differential displacement – such as the gini index (G) and the dissimilarity index (D) which Duncan and Duncan (1955) aptly also termed the displacement index – will take high scores while the Theil index (H) and the separation index (S) – which Stearns and Logan (1986) note is sensitive to residential separation and area racial polarization – will take low scores.

In Chap. 7 I review the distinction between concentrated and dispersed displacement in more detail. The chapter makes two important points. One is that the sociological implications of uneven distribution involving "prototypical segregation" and D-S concordance are fundamentally different from the sociological implications of uneven distribution with dispersed displacement and substantial D-S divergence. Simply put, a high level of group separation is obviously substantively compelling and necessarily entails a high level of displacement. But the reverse is not true. Thus, high levels of displacement do not always entail high levels of group separation and this should be noted when it occurs because the literature on segregation measurement provides no clear basis for viewing differential displacement without group separation as sociologically important. The second point I make in this chapter is that the largely unrecognized but empirically common outcome of dispersed displacement is not an artifact of relative group size or deficiencies in indices that are more sensitive to group separation than displacement. To make this point I introduce and exercise simple analytic models to show that when non-trivial displacement from even distribution is present, it can be concentrated or it can be dispersed. Concentrated displacement produces "prototypical segregation" wherein the score of S will approach or even equal the score for D indicating that displacement involves group separation and area polarization. In the case of dispersed displacement, D will be equally high but S will be low signaling that group separation and area polarization are minimal, sometimes to the point of being negligible. I review the principles of transfers and exchanges from segregation measurement theory to establish that D-S discrepancies of this sort arise because D is flawed and does not conform to these accepted principles of segregation measurement.

Chapter 8 supplements the analytic results by discussing the sociological dynamics that are likely to influence whether non-trivial displacement takes the form of "prototypical segregation" or the substantively less compelling pattern of dispersed displacement. It also reviews case studies of empirical examples of high-D-high S combinations that in communities where the minority group is small in relative size. The discussion here drives home two important points. One is that scores for D and S can be congruent or discrepant in any setting where displacement from uneven distribution is non-trivial. The other is that sociological dynamics, not artifacts of index construction, determine whether in fact D and S are congruent or discrepant in a given community.

In Chap. 9 I show how the difference of means framework creates new options for research by joining micro- and macro-level analysis of segregation. At the simplest level, casting segregation as a difference of group means on residential outcomes leads to the new insight that segregation index scores are exactly mathematically equivalent to the effect of race in bivariate regression analyses predicting segregation-determining residential outcomes for individuals. I then argue that this insight opens the door to the new possibility of using multivariate regression analyses to quantitatively assess how segregation arises from two sources. The first source is group differences on distributions of social and economic characteristics that are salient in residential attainment processes. The second source is group differences in the efficacy of how inputs to residential attainment processes translate into segregation-determining residential outcomes. In this framework, segregation can be analyzed in greater detail and sophistication by using standardization and decomposition analysis in combination with multivariate regression analysis of attainments, methods that are routinely applied to the study of racial inequality in education, occupation, income, health, and other important stratification outcomes. This is a major advance as research on segregation has lagged behind research on group disparities in other domains where aggregate-level outcomes on group disparities have long been routinely analyzed as outgrowths of micro-level attainment processes.

In Chap. 10 I show how the regression analysis of individual-level residential attainments can subsume comparative analysis of cross-city variation in segregation and create new possibilities for investigating the factors contributing to variation in segregation across cities and over time. The new approach involves extending city-specific analysis of segregation using bivariate and multivariate models of individual-level residential attainment to multi-level specifications that reveal how the process determining segregation varies across cities and over time. I first note that findings from aggregate-level analyses of cross-city variation in segregation can be exactly reproduced using multi-level specifications of segregation-attainment models. I then outline how this specification opens the door for improving the ability of researchers take accurately assess the role that non-racial characteristics such as income may play in shaping cross-city variation in segregation.

Previous research has often tried to assess the impact of group differences on income and other individual-level characteristics by the method of aggregate-level regression analysis. I note that this approach is prone to yield flawed results because it runs afoul of the "aggregate fallacy." The problem is hidden from view and less obvious when segregation is viewed only as a macro-level outcome. It becomes clear and more readily evident when segregation is analyzed within the difference of means framework where the outcome of segregation at the aggregate-level is exactly determined by individual-level attainment processes. I demonstrate the importance of the problem by showing that results of analyses that assess the impact of group income differences on segregation using aggregate-level regressions are contradicted by multi-level regression analyses that avoid the aggregate fallacy and properly take account of the effects of income at the individual-level.

In Chaps. 11, 12, and 13 I review topics that benefit from insights and perspectives gained from drawing on the difference of means framework for analyzing segregation. In Chap. 11 I note that the difference of means framework makes it easy for researchers to implement spatial versions of popular segregation indices if they desire to do so. The reason for this is simple; the residential attainments for individuals that determine segregation scores can be computed using mutually-exclusive bounded areas, or using overlapping, spatially-defined areas. The former yields a traditional aspatial index score. The latter yields a "spatial" index score that is affected by how neighborhoods that vary in racial composition are distributed in space.

In Chaps. 12 and 13 I argue that the difference of means framework leads to new perspectives regarding what aspects of residential segregation researchers will view as most compelling on substantive grounds. In Chap. 12 I argue that group separation is a more compelling substantive concern than mere displacement from even distribution. I frame the issue as follows. It is non-controversial to assert that group separation area racial polarization is substantively important because it is a logical prerequisite for group disparity and inequality on neighborhood-based residential outcomes. In contrast, there is no established basis for arguing that displacement from even distribution is substantively important when it does not involve group separation and area racial polarization. The only candidate is the "volume of movement" interpretation of D in which a high value of D does indicate that a large fraction of one group must move to bring about exact even distribution. But it is rendered irrelevant in situations where movement to exact even distribution has no impact on group separation and area racial polarization.

In Chap. 13 I consider how being sensitive to different aspects of uneven distribution makes different indices more or less relevant for theories of segregation. I note that measures rooted in the segregation curve – G and D – are sensitive to rank-order differences on the residential outcome of area racial proportion but are relatively insensitive to the quantitative magnitude of the differences involved. In contrast, the separation index is sensitive to the quantitative magnitude of the differences because it registers the residential outcome of area racial composition in its natural metric. Segregation dynamics such as "tipping," resulting from group differentials in entries and exits to areas, and discrimination to exclude groups from areas are thought to be triggered by area group proportions. In contrast, theories of segregation dynamics rarely direct attention to rank order position on area racial composition over and above its association with area racial composition itself.

In Chaps. 14, 15, and 16 I give attention to the problem of index bias. All indices of uneven distribution have the undesirable property that their scores are subject to inherent upward bias that can be non-negligible and varies in magnitude across individual cases. I draw on the difference of means framework to first identify the source of index bias and then identify a solution for obtaining unbiased versions of all popular indices of uneven distribution. I use formal analytic models and empirical exercises to demonstrate that the unbiased versions of G, D, R, H, and S behave as desired in analytic exercises and in empirical applications. Significantly, the difference of means framework is crucial because it provides the vantage point needed to identify both the source of the problem and its solution both of which turn out to be surprisingly simple and intuitive. In this new formulation index scores are calculated as differences of group means on *individual* residential outcomes scored from area racial proportion. The source of bias can be traced to how area racial proportion is assessed from the perspective of individuals. In the "standard" (biased) formulation, the individual in question is *included* in the area counts used to calculate area racial proportion. The value of this residential outcome for the individual thus reflects a combination of two things: the individual's own contribution to area racial mix and the racial mix of neighbors – the other individuals in the area. Under random assignment the racial mix of neighbors is a random draw and every individual, regardless of group membership has the same expected distribution of outcomes on racial mix of neighbors. In contrast, an individual's own contribution to area racial mix is fixed and, importantly, differs systematically with group membership. This is the source of index bias. Once seen from this vantage point, the problem of index bias can then be eliminated by assessing area racial proportion for individuals based on *neighbors* instead of *area population*.

The solution to index bias I offer in this monograph is attractive for many reasons. To begin, when working within the difference of means framework for calculating indices of uneven distribution, the solution is simple and intuitive, even "obvious." Second, the unbiased measures do not require radical changes in research practices. Researchers can continue to use the same measures they have used for decades. But now they can use refined versions of these measures that will yield scores that are free of bias at the level of individual cases in situations where previously researchers could not trust index scores and in other situations the scores of the refined versions will be essentially identical to scores obtained using standard computing formulas. In sum, the new versions will exactly replicate research findings obtained using standard index versions when measurement is non-problematic and will yield superior results when standard calculations cannot be trusted.

In Chap. 17 I offer final comments on the contributions of the monograph overall and reiterate my hope is that the new options for measurement and analysis I introduce here will enable researchers to investigate residential segregation in more detail and depth than has previously been possible. Significantly, the benefits gained from using the new options of measurement and analysis are "cost free"; there are no penalties or sacrifices associated with adopting them. Researchers do not have to put aside familiar measures and replace them with unfamiliar ones. The difference of means framework for measuring segregation permits researchers to exactly replicate results of past studies while at the same time giving them new options for refined measurement, expanded analysis, and attractive substantive interpretations. Thus, researchers can maintain continuity with previous studies of aggregate segregation while simultaneously having the option of taking advantage of opportunities to analyze segregation in new ways to gain a deeper, more detailed understanding of segregation patterns.

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Chapter 2
Alternative Formulas for Selected Indices

The values of all popular indices of uneven distribution can be obtained using a variety of mathematically equivalent computing formulas. For a given index some formulas are more familiar and widely used than others, but no single formula can be declared sacred or best for all purposes. The many alternatives can be confusing to those who are new to segregation research. But their availability benefits researchers by providing a variety of options from which to choose to best serve the needs of a particular study. The relevant considerations can include factors such as efficiency of computation, ease of explaining the index to broad audiences, relevance for establishing appealing substantive interpretations, capacity for enabling practical tasks such as decomposition analysis or the calculation of spatial versions of index scores, and utility for pinpointing technical issues in segregation measurement. Researchers may choose a particular formula specifically to serve the needs of a given study. Or they may use a formula based on familiarity and habit. But in one crucial sense the choice is unimportant as all valid formulas can be used interchangeably without affecting the results of individual index scores, research findings, and substantive conclusions.

To specialists well-versed in the literature on segregation measurement these are not surprising observations. Nevertheless, I raise the point because many researchers and most consumers of segregation research understand the quantitative underpinnings of segregation index scores based primarily on a handful of popular computing formulas. This is not a problem in itself. But problems can arise when lack of familiarity with mathematically equivalent alternatives makes individuals resistant to insights and interpretations that can be gained by drawing on alternative formulations of a particular index. This leads me to suggest that, while some formulas for popular indices of uneven distribution are better known and more widely used, it can be useful to consider other, less well known alternatives. In this chapter I discuss three classes of formulas. The formulas in the first group, which includes some well-known formulas that are very widely used in empirical research, focus attention on outcomes for areas and provide little insight into the relationship

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_2

$$G = 100 \cdot (\Sigma X_{i\text{-}1} Y_i - \Sigma X_i Y_{i\text{-}1}) \text{ (Duncan and Duncan 1955)}$$

$$D = 100 \cdot ½ \, \Sigma \, | (n_{1i}/N_1) - (n_{2i}/N_2) | \text{ (Duncan and Duncan 1955)}$$

$$R = 100 \cdot \left(1.0 - \Sigma \sqrt{(n_{1i}/N_1) \cdot (n_{2i}/N_2)}\right) \text{ (Hutchens 2001:23)}$$

Fig. 2.1 Examples of selected area-based computing formulas for indices of uneven distribution (Notes: N_1 and N_2 denote city-wide population counts for the two groups in the comparison; $T = N_1 + N_2$; i denotes area; n_{1i} and n_{2i} denote the area counts for the two groups in the segregation comparison; and X_i and Y_i denote the cumulative proportions of groups 1 and 2, respectively, over areas ranked from low to high on p_i obtained from $n_{1i}/(n_{1i}+n_{2i})$. A summary of notation used is given in Appendices)

between residential segregation and residential outcomes for individuals. The formulas in the second group establish that indices of uneven distribution are connected to the residential outcomes of individuals, but they not provide a basis for gaining insight into how residential outcomes differ across groups. The formulas in the third group go one step further and establish that indices of uneven distribution can be cast in ways that reveal how segregation is specifically connected to group differences on individual-levels residential outcomes associated with neighborhood racial composition.

Many, perhaps most, readers will have given little thought to how indices of uneven distribution are linked to individual residential outcomes. This would not be surprising as this aspect of indices of uneven distribution has not been emphasized in the literature on segregation measurement. It also is not obvious from inspecting the most widely used computing formulas for popular indices. Alternative formulas that do highlight the property tend not to be well known in addition to being infrequently used. In view of this, I use this chapter to briefly introduce formulas that highlight individual residential outcomes and contrast them with standard computing formulas. To streamline presentation, I offer minimal commentary here on the derivations of the new formulas that are introduced in this chapter. For those who are interested, I provide derivations and more detailed discussion of related technical issues as Appendices. In Chaps. 3, 4, and 5 in the body of the monograph I provide general discussions of the new formulas introduced here and then review their benefits for segregation measurement and analysis throughout the remainder of the study.

I begin by introducing computing formulas for three indices of uneven distribution that have very close relations to the segregation curve; namely, the gini index (G), the dissimilarity or delta index (D), and the Hutchens square root index (R). The formulas are given in Fig. 2.1. The formulas for G and D are likely to be familiar to many readers as they are widely used in segregation studies. In no small part this is because these formulas were introduced in Duncan and Duncan (1955), a landmark methodological study that served as the definitive guide to segregation measurement for three decades. In addition, they have continued to remain popular because they are convenient computing formulas that are relatively easy to implement in empirical analyses. The formula for R was introduced more recently (Hutchens 2001) but I include it with the formulas for D and G because all three measures have close relations to the segregation curve and, as I document later in Chap. 6, all three are highly correlated in empirical applications. G and D are better known to sociologists. But R has technical properties that make it an attractive index to consider if one is committed to using a measure with close relations to the segregation curve.

The point I make about these three formulas is that they focus attention on outcomes for areas, not outcomes for individuals. The formulas adopt this orientation in part because it is efficient for computing index scores from area tabulations – a fact of non-trivial practical import in the early era of segregation research when Duncan and Duncan's study first appeared. In addition, these formulas fit comfortably with approaches to thinking about segregation that have an aggregate-level focus and frame the assessment of even distribution from the point of view of whether or not the racial composition of *areas* or neighborhoods matches the racial composition of the city as a whole. I note, however, that something important is left mysterious and obscure in these formulas. It is the residential outcomes that the individuals residing in these areas experience and how these outcomes may or may not vary systematically for the two groups in the segregation comparison.

The formulas for G and D given here are probably the two most widely applied computing formulas for measuring residential segregation. They also are likely to be the first two computing formulas students of segregation research learn. The fact that these formulas provide little to no basis for drawing insights about how segregation is connected to residential outcomes for individuals speaks volumes about the state of the literature on segregation measurement.

Figure 2.2 provides alternative formulas for G, D, and R and adds in similar formulas for two additional indexes, the Theil entropy index (H) and the separation index (S) (also known as eta squared [η^2] and the variance ratio). With the exception of the formula for R, these computing formulas also are likely to be familiar to many readers because they have been featured in many important methodological studies (e.g., Duncan and Duncan 1955; Zoloth 1976; James and Taeuber 1985; White 1986; Massey and Denton 1988). They, or close variations on them, are widely used in segregation studies. In no small part this is because they are convenient computing formulas that are relatively easy to implement in empirical analyses.

The formulas Fig. 2.2 have a key feature in common. Each formula incorporates the term "t_i" in the core calculations leading to the index value. This term represents the combined population of the two groups in the comparison residing in the i'th area in the city. The calculations involving this term are cumulated over all areas and at some point are divided by "T," the combined city-wide total populations of the two groups. Based on this construction, the index score can be understood as an average value for a quantitative result assessed for all individuals in the segregation comparison.

The point I want to make about these formulas is that the quantitative result computed for individuals can be viewed as an individual-level residential outcome or

$$G = 100 \cdot (1/2T^2PQ) \cdot \Sigma\, t_i\, t_j \cdot \left| p_i - p_j \right| \quad \text{(James and Taeuber 1985:5)}$$

$$D = 100 \cdot (1/2TPQ) \cdot \Sigma\, t_i \cdot |p_i - P| \quad \text{(James and Taeuber 1985:6)}$$

$$R = 100 \cdot \left[1 - (1/T) \cdot \Sigma\, t_i \cdot \sqrt{p_i q_i / PQ}\right] \quad \text{(Appendix F, this monograph)}$$

$$H = 100 \cdot \Sigma\, t_i \cdot [(E - E_i)/ET] \quad \text{(Massey and Denton 1988:285)}$$

$$S = 100 \cdot (1.0 - [(\Sigma\, t_i \cdot p_i q_i)/TPQ]) \quad \text{(Zoloth 1976:282) or}$$
$$100 \cdot (1/TPQ) \cdot \Sigma\, t_i (p_i - P)^2 \quad \text{(James and Taeuber 1985:6)}$$

Fig. 2.2 Examples of area-based computing formulas for indices of uneven distribution that implicitly feature overall averages on individual-level residential outcomes (Notes: N_1 and N_2 denote city-wide population counts for the two groups in the comparison; $T = N_1 + N_2$; $P = N_1/T$; $Q = N_2/T$; i denotes area; n_1 and n_2 denote the area counts for the two groups in the segregation comparison; $t = n_1 + n_2$; $p_i = n_{1i}/t_i$; $q_i = n_{2i}/t_i$; X_i and Y_i denote the cumulative proportions of groups 1 and 2, respectively, over areas ranked from low to high on p_i; and E denotes entropy for the city overall given by $E = P \cdot Log_2(1/P) + Q \cdot Log_2(1/Q)$ and E_i denotes entropy for area i given by $E_i = p_i \cdot Log_2(1/p_i) + q_i \cdot Log_2(1/q_i)$. A summary of notation is given in the Appendices)

residential attainment. I emphasize this point with the formulas listed in Fig. 2.3. These are alternative, mathematically equivalent versions of the formulas given in Fig. 2.2. The only difference is that the formulas have been rearranged to highlight and clarify how each index can be understood as an overall average of residential outcome scores (y) for individuals. A more detailed discussion of these formulas are given in the Appendices. Here I limit my comments to noting that the residential outcome terms (y) can be characterized as registering the degree to which the racial composition in the area the individual resides in departs from the racial composition of the city. In the case of G, D, H, and the first formula for S, the calculation of the departure score involves a city-specific constant that "scales" results so the final index score will fall in the range 0–1.

These formulations show that, if one chooses to do so, all popular measures of uneven distribution can be expressed in terms of individual residential outcomes. While this option has been available for most measures for many decades, mathematical expressions of this form have not been as widely used and discussed as the standard computing formulas. One reason for this is that formulating indices of uneven distribution as overall population averages on residential outcomes does not provide any significant practical advantages. Another reason is that these formulations do not support substantive interpretations that are viewed as useful and compelling for the study of segregation. Most studies that measure uneven distribution are motivated by the assumption that it ultimately carries important implications for group differences in residential distributions and residential outcomes. Casting uneven distribution as an overall average for residential outcomes, while a viable mathematical option, does not speak directly to a substantive interest focused on group differences in residential distributions and residential outcomes. Nevertheless, these formulations are relevant for my purposes because they make it clear that all

Index	Averaging Scores for y Over All Individuals	Scores Assigned to Individuals Based on Scaling Function $y_k = f(p_i)$
G =	$100 \cdot (1/T) \cdot \Sigma y_k$	$y_k = \Sigma \left\| p_k - p_m \right\| / 2TPQ$
D =	$100 \cdot (1/T) \cdot \Sigma y_k$	$y_k = \|p_i - P\|/2PQ$
R =	$100 \cdot [1 - (1/T) \cdot \Sigma y_k]$	$y_k = \sqrt{p_i q_i / PQ}$
H =	$100 \cdot (1/T) \cdot \Sigma y_k$	$y_k = (E - E_i)/E$
S =	$100 \cdot (1/T) \cdot \Sigma y_k$	$y_k = (p_i - P)^2/PQ$ or, alternatively,
	$100 \cdot [1 - (1/T) \cdot \Sigma y_k]$	$y_k = p_i q_i / PQ$

Fig. 2.3 Formulas explicitly casting values of indices of uneven distribution as overall population averages on individual residential outcomes (y) (Notes: k and m index individual households; p_i denotes the pair-wise area proportion for the reference group in the i'th area; p_k denotes the value of p_i for the k'th household and p_m denotes the value of p_i for the m'th individual; See notes to Figs. 2.1 and 2.2 for other terms)

indices of uneven distribution have definite relations to residential outcomes for individuals.

Thinking about this led me to raise two questions that are central to this study. They are "Can indices of uneven distribution be formulated in a way that provides direct insights regarding group differences in residential outcomes?" and, if so, "How specifically do indices of uneven distribution register group differences on neighborhood residential outcomes?" The formulas presented in Fig. 2.4 address these questions. The formulas given here cast popular indices of uneven distribution as differences of means on individual residential outcomes (y) that are scored on the basis of the pairwise group proportion (p) for the area of residence. These expressions are new to this monograph and have not been presented previously in the literature on segregation measurement.

These formulas play a crucial role in this study; they constitute the mathematical basis for what I term the "difference of means" framework for segregation measurement. Accordingly, I review these formulas in more detail in Chap. 3 and I also provide additional technical discussions and derivations as Appendices. I conclude this short chapter with a few additional comments. This chapter establishes the point that all popular indices of uneven distribution can be given in a variety of mathematically equivalent formulations. Some are convenient for computing; some support attractive substantive interpretations; and some reveal how segregation is connected to residential outcomes for individuals and how these may differ across groups. All can be used to obtain correct values for index scores and thus they all are interchangeable for that narrow purpose. The new formulas introduced in Fig. 2.4 definitely can be used for this purpose. But that is not their main claim to fame. Their value to segregation research is that they provide unique advantages for segregation

	Difference of Group Means on y	Residential Outcome Scores (y) Assigned to Individuals Based on $y_i = f(p_i)$
G =	$100 \cdot 2(\bar{Y}_1 - \bar{Y}_2)$	$y_i = f(p_i) =$ relative rank (quantile scoring) on p_i
D =	$100 \cdot (\bar{Y}_1 - \bar{Y}_2)$	$y_i = f(p_i) = 0$ if $p_i < P$, 1 if $p_i \geq P$
	Alternatively, compute D as a simplified version of G based on collapsing area values for p_i into a two-category rank scheme consisting of areas where $p_i < P$ and areas where $p_i \geq P$.	
A =	No direct difference of group means solution is available but $A = 2R - R^2$ for the "symmetric" version of A (i.e., A when $\alpha = \beta = 0.5$).	
R =	$100 \cdot (\bar{Y}_1 - \bar{Y}_2)$	$y_i = Q + \left(1 - \sqrt{p_i q_i / PQ}\right) / (p_i/P - q_i/Q)$.
H =	$100 \cdot (\bar{Y}_1 - \bar{Y}_2)$	$y_i = Q + [(E - e_i)/E] \;/\; (p_i/P - q_i/Q)$.
S =	$100 \cdot (\bar{Y}_1 - \bar{Y}_2)$	$y_i = p_i$

Fig. 2.4 Formulas casting values indices of uneven distribution as differences of group means ($\bar{Y}_1 - \bar{Y}_2$) on individual residential outcomes (y) (Notes:· $\bar{Y}_1$ and $\bar{Y}_2$ are group averages given by $\bar{Y}_1 = (1/N_1)\Sigma y_i$ and $\bar{Y}_2 = (1/N_2)\Sigma y_i$ with i denoting individuals in the relevant group p_i denotes the pairwise area proportion for the reference group (p_i) in the area where the i'th individual resides and y_i is the residential outcome score generated by the index-specific scoring function $f(p_i)$. See notes to Figs. 2.1 and 2.2 for other terms)

measurement and new options for segregation analysis. They do so by placing all popular indices of uneven distribution in a common framework wherein all indices are given as group differences of means on individual residential outcomes (y) that are scored from the pairwise racial composition (p) of the area in which the individual resides. This framework provides a new basis for understanding, interpreting, and comparing familiar indices. It also opens the door to innovations in segregation measurement and analysis. I explore these possibilities in more detail in the remaining chapters of this monograph starting next with an overview to the "difference of means" framework.

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Chapter 3
Overview of the "Difference of Means" Framework

The previous chapter notes that popular indices of uneven distribution can be expressed in a variety of mathematically equivalent ways. The discussion there (and in Appendices) reviews a variety of formulas presented previously in the literature. It also introduces a set of new formulas that cast indices of uneven distribution as group differences of means on individual residential outcomes. I argue that the group difference of means formulation is an important new approach that brings many advantages and possibilities to segregation measurement and analysis. To make the case for this view I now provide a more detailed discussion comparing standard computing formulas with the difference of means formulas.

3.1 Index Formulas: The Current State of Affairs

As I noted briefly in the previous chapter, popular measures of segregation such as the widely used dissimilarity or delta index (D) traditionally have been formulated and interpreted from a perspective that focuses attention on outcomes for areas rather than outcomes for individuals. For example, the following formula from Duncan and Duncan (1955: 211) highlights area differences in relative group presence – specifically, the area's share (s) of the group's city-wide population – for the two groups in the comparison. This formula is widely used to compute D because it is computationally efficient and easy to implement. In addition, the focus on variation in area outcomes is seen as an appealing basis for understanding and assessing the extent to which two groups are distributed unevenly across the residential areas of a city.

$$D = 100 \cdot ½ \Sigma \left| \left(n_{1i} / N_1 \right) - \left(n_{2i} / N_2 \right) \right|$$

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_3

$$D = 100 \cdot ½ \Sigma \left| s_{1i} - s_{2i} \right|$$

This aggregate-level approach is not unique to the era in which Duncan and Duncan were writing or to the dissimilarity index. More than four decades later Hutchens introduced a new measure of uneven distribution termed the square root index (R) and drew on a similar formulation to clarify how R assesses the extent to which two groups are distributed unevenly across the residential areas of a city (Hutchens 2001: 23).

$$R = 100 \cdot \left(1.0 - \Sigma \sqrt{\left(n_{1i} / N_1\right) \cdot \left(n_{2i} / N_2\right)}\right)$$

$$R = 100 \cdot \left(1.0 - \Sigma \sqrt{s_{1i} \cdot s_{2i}}\right)$$

In the formula for D, uneven distribution is assessed as 0 only when the "area share scores" (s) for the two groups in the comparison are exactly equal in all areas of the city. The same is true for the formula for R.[1]

Summary measures of uneven distribution formulated in this way have been and remain valuable tools for aggregate-level description. But the focus on outcomes for areas rather than individuals and groups imposes a significant limitation that Duncan and Duncan (1955) noted over 50 years ago. The limitation is that area-oriented formulations of D and other indices provide little basis for gaining insight into how underlying micro-level social processes of residential attainment give rise to the area patterns that determine the level of residential segregation for the city. Accordingly, Duncan and Duncan stated "In none of the literature on segregation indices is there a suggestion about how to use them to study the *process* of segregation or change in the segregation pattern" (1955: 223; emphasis in original). The process of course plays out at the level of individuals and households, not for areas. Indeed, the areas often are defined as statistical units with no intrinsic sociological qualities relevant for segregation process; they are merely useful constructs for assessing group differences in residential distribution. So formulas that focus attention on outcomes for areas are at a level of abstraction removed from "where the action is" in segregation dynamics.

Duncan and Duncan additionally noted it would be desirable, but was not then possible, to incorporate controls for the role of individual-level factors (e.g., labor force status, occupation, income, etc.) beyond race when seeking to understand and explain the level of segregation in a city. Unfortunately, efforts to achieve this goal were frustrated then and are currently frustrated now by thinking about segregation solely from the point of view of the area-oriented computing formulas given above. The formulas are framed in terms of outcomes for areas, not in terms of individual

[1] This may be less obvious for R because it is a relatively recent addition to the literature. When share scores (s) for groups are equal in an area, the square root of the product of the two share scores will equal the value of the individual share scores. The resulting terms will sum to 1 when group share scores are equal in all areas and the index score will be 0.

residential outcomes. So it is no surprise that it is not easy to use them to gain insights into how index scores arise from an underlying micro-level process where potentially many factors play a role in shaping the residential outcomes individuals attain.

When segregation when conceptualized and analyzed from the point of view of outcomes for areas, it is very difficult to take account of the role of even a single social or economic characteristic beyond race and it is completely infeasible to take account of the role of several social and economic characteristics at the same time. Past efforts to achieve the goal of controlling for the role of non-racial characteristics have been limited to computing index scores using group subsamples that are matched on one or more relevant social characteristics (e.g., income). This approach is untenable in practical application because analysis quickly comes to be based on very small subgroup counts if one measures non-racial characteristics in fine-grained ways and/or if one tries to control for more than one or two non-racial characteristics at the same time. Accordingly, the approach is used infrequently in the empirical literature. When it is used, implementations are crude and unsatisfying and the resulting index scores are likely to be problematic on technical grounds. The implementations are crude because fine-grained distinctions quickly lead to small subgroup counts. Consequently, "matching" on non-racial characteristics can at most involve one or two characteristics and an interval variable such as income must be grouped into very broad categories. Yet even with these compromises, subgroup counts wind up being much smaller than overall counts and this then leads to technical problems relating to index bias, a concern I discuss in detail in Chaps. 14, 15, and 16.

In short, it is a disappointing state of affairs. In the six decades that have passed since Duncan and Duncan raised these important and fundamental concerns, the problems they identified have yet to be adequately addressed. Researchers continue to formulate indices of uneven distribution from area-oriented perspectives that leave the connections between index scores and individual-level residential attainments, and the related micro-level processes that shape them, unspecified and poorly understood. As a consequence, research on residential segregation has become increasingly out of step with the broader literatures investigating racial and ethnic inequality and disparity in socioeconomic outcomes such as education, occupation, and income. Studies of racial and ethnic differences in other socioeconomic outcomes have for many decades routinely drawn on micro-level models of individual attainment to gain insights into how many different factors may contribute to the creation of aggregate-level (i.e., national- and community-level) group disparities. In contrast, the literature on segregation has had to limit its focus to assessing aggregate-level segregation leaving the implications for and connections to group differences in individual residential outcomes uncertain and unexamined.

To be fair, a vibrant and important literature focusing on individual-level residential attainment has emerged in recent decades (e.g., Alba and Logan 1993; Logan and Alba 1993; Logan et al. 1996; Alba et al. 1999; South and Crowder 1997, 1998). But it has developed as a separate literature that is only loosely connected with research investigating segregation at the aggregate-level. The reason for this is that the dependent variables in analyses of individual residential attainment do not correspond to terms that figure directly in the calculation of segregation index scores. Accordingly, studies of individual residential attainments to date do not, and logically cannot, provide direct insights into the values of D or other aggregate-level summary indices of uneven distribution. Conversely, studies of aggregate-level segregation cannot directly provide insights into the parameters of individual-level residential attainment processes.

The current state of affairs is unfortunate and unsatisfactory. Interest in segregation generally rests on an implicit assumption that segregation has important associations with group differences on neighborhood residential outcomes that are relevant for socioeconomic attainment and inequality in life chances. Individuals and households strive to attain these residential outcomes either for their own sake (e.g., as markers of social position) or because they are closely correlated with factors that impact life chances (e.g., exposure to crime, social problems, schools, services, neighborhood amenities, etc.). In view of this, it is clearly desirable to gain a better understanding of how different segregation indices relate to group differences on individual-level residential outcomes. Surprisingly, the methodological literature on segregation measurement is nearly silent on this issue. Segregation measurement theory gives attention to many properties and qualities of aggregate-level indices but it has not taken up the question of how different indices relate to individual-level residential outcomes or carry different implications for group differences on residential outcomes.

3.2 The Difference of Means Formulation – The General Approach

I address this gap in the measurement literature by casting popular measures of uneven distribution as differences of group means on segregation-relevant individual residential outcomes. Specifically, I place familiar segregation indices in a common "difference of means" framework in which the index score "S" is given as

$$S = Y_1 - Y_2$$

where:

S is the score of the relevant segregation index (i.e., G, D, R, H, or S),
Y_1 is the mean on y for individuals in Group 1 based on either $(1/N_1)\cdot\Sigma n_{1i}y_i$ when computed for area data or $(1/N_1)\cdot\Sigma y_{1j}$ when computed for individual data,
Y_2 is the mean on y for individuals in Group 2 based on either $(1/N_2)\cdot\Sigma n_{2i}y_i$ when computed for area data or $(1/N_2)\cdot\Sigma y_{2j}$ when computed for individual data,
n_{1i} and n_{2i} are the counts of Groups 1 and 2, respectively, in the i'th area,
p_i is the pairwise area proportion for Group 1 in the i'th area based on $p_i = n_{1i}/(n_{1i} + n_{2i})$,

y_i is the residential outcome score (y) for the i'th area scored as a function of the pairwise area group proportion $y_i = f(p_i)$,
y_{1k} indicates the residential outcome (y) for the k'th individual in Group 1 (set equal to the residential outcome score for the area in which the individual resides), and
y_{2k} indicates the residential outcome (y) for the k'th individual in Group 2 (set equal to the residential outcome score for the area in which the individual resides).

I hold that formulating segregation indices in this way is useful for both conceptual and practical reasons. First, it provides a new interpretation for aggregate segregation indices; they now can be understood as registering simple group differences on residential outcomes (y) scored based on area group proportion (p) which has an easy, straightforward interpretation as (pairwise) contact with or exposure to Group 1 (i.e., the reference group) based on co-residence. Simple co-residence, of course, does not necessarily imply harmonious social interaction. But it does indicate common fate regarding many neighborhood outcomes and many shared residential experiences. On this basis, it is a potentially important and meaningful social indicator.

Second, this new approach to computing index values places different indices in a uniform, common computing framework that highlights differences between measures on a single, specific point of comparison – the manner in which each index registers neighborhood residential outcomes (y) based on area group proportion (p). Since area group proportion can be understood as contact or exposure based on co-residence with Group 1, all of the indices can be interpreted as group differences in average "scaled contact" with Group 1. Differences between indices ultimately trace to differences in the specific way that residential outcomes (y) are quantitatively scored based on area group proportion (p). Consequently, differences between indices can be seen as arising solely from differences in the index-specific form of the scaling function $y = f(p)$. This provides a new basis for evaluating segregation indices; they can be compared on the substantive relevance of how each index registers residential outcomes (y) based on contact and exposure with Group 1 as embodied by area group proportion (p).

Third, the segregation-relevant residential outcomes (y) used to compute the segregation index score can directly serve as dependent variables in individual-level residential attainment analyses. Thus, in the difference of means formulation, the segregation index score can be equated to the effect of group membership (e.g., coded 0 or 1) in an individual-level residential attainment analysis for the city. This carries minimal practical value for specific task of estimating index scores because the scores can be readily obtained by simpler methods. But it is important because it expands options for understanding and analyzing segregation. It unifies the study of aggregate segregation with the study of residential attainment in a single framework. In doing so it opens the door to a host of new options for segregation analysis including, for example, the ability to easily take account of the role that factors other than group membership (e.g., income) may play in determining segregation and the ability to use multi-level models of residential attainment to study cross-area and cross-time variation in segregation.

3.3 Additional Preliminary Remarks on Implementation

The key to implementing the new approach is to identify for each index a scoring system for neighborhood outcomes (y) that will yield the segregation index score as a difference of group means ($Y_1 - Y_2$). I have identified relevant scoring systems for five indices that are widely used to measure the unevenness dimension: the gini index (G), the delta or dissimilarity index (D), the separation index (S) (also known as the variance ratio index [V]), the Theil entropy index (H), and the Hutchens square root index (R), a measure that is closely associated with the "symmetric" implementation of the Atkinson index (A).[2,3]

For all of these indices, the residential outcome (y) is scored as a function of "pairwise" group proportion (p) for the area the individual resides in. Indexing areas by "i", p_i is given as

$$p_i = n_{1i} / \left(n_{1i} + n_{2i}\right)$$

where p_i is the Group 1 proportion in the combined population of Group 1 and Group 2 in the i'th area. In this formulation the scoring system for each index rests on a "scaling" function $y = f(p)$ that maps area group proportion scores (p) on to index-specific residential outcome scores (y). I discuss the index-specific scaling functions $y = f(p)$ in the chapters that follow and in Appendices.

Before continuing, I comment briefly to note two technical points. One is that the designation of which group serves as "Group 1" is arbitrary. One group must be so designated. But the result for the index score will be the same regardless of which of the two groups is chosen as the reference. White (1986) termed this index property as "symmetry." When one group is understood as a majority group and the other as a minority group, it has been conventional in previous research to designate the majority group as Group 1. This is not required, but it is convenient because it facilitates interpreting segregation as reflecting the extent to which the minority group has less contact with the majority group than would occur under even distribution. This has generally been viewed as useful based on the assumption that areas of majority group residence are advantaged and thus disparity and disadvantage in residential outcomes follows when contact with the majority falls below parity. But it is only a custom, not a logical requirement. If the roles of the two groups are

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[2] The separation index (S) is known by many names including the revised index of isolation (Bell 1954), the correlation ratio and eta squared (η^2) (Duncan and Duncan 1955), r or r_{ij} (Coleman et al. 1975, 1982), variance ratio (V) (James and Taeuber 1985), and segregation index (S) (Zoloth 1976).

[3] I note below that A is an exact nonlinear function of R. To date I have not identified the relevant scoring system for Atkinson's Index (A). A is rarely used in empirical studies and has been criticized on conceptual grounds for being asymmetric such that A for White-Black segregation may differ from A for Black-White segregation (e.g., White 1986). But it has received attention in the segregation measurement literature. So this remains a task for future research.

reversed, the contact interpretation will be reversed. But all substantive implications of the patterns of group differences in contact will remain intact and unchanged.

The second technical point I mention is that p is computed using only counts for the two groups in the segregation comparison. Thus, p is not Group 1's proportion among the *total* population of the area; it is Group 1's proportion among the combined count of the two groups in the segregation analysis. To emphasize this point, I sometimes term p as a "pairwise" group proportion. However, as this is the primary way I use p in this monograph, I often drop the "pairwise" modifier in the interest of economy of expression. Note that this "pairwise" construction is not at all controversial in segregation measurement; relevant terms in all of the standard formulas for measures of uneven distribution reviewed earlier are based on pairwise implementations of group proportions for areas (i.e., p and q) and for the city as a whole (i.e., P and Q).

The general outline of the approach is now set. The next task is to review how the difference of means framework can be implemented with the most popular and widely used segregation indices.

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Chapter 4
Difference of Means Formulations for Selected Indices

In this chapter I review the implementation of the difference of means framework for calculating indices of uneven distribution for five indices: the delta or dissimilarity index (D), the gini index (G), the separation index (S), the Theil entropy index (H), and the Hutchens square root index (R). For each index I introduce the relevant scoring systems for residential outcomes (y) that makes it possible to obtain the index scores as simple differences of group means on individual residential outcomes (y). To facilitate discussion, I replace the abstract terms "Group 1" and "Group 2" with the more concrete example of Whites and Blacks which has been investigated in hundreds of empirical analyses of uneven distribution in U.S. cities, urban areas, and metropolitan areas. As I move from index to index, I provide comments on the nature of the scaling function that maps scores for contact and exposure based on pairwise area proportion White (p) onto index-specific residential outcome scores (y). In addition, I sometimes offer commentary on the index. Note, however, that I do not provide a comprehensive review of the five indices because this task has been addressed previously in the existing literature and does not need to be repeated here.

4.1 Scoring Residential Outcomes (y) for the Delta or Dissimilarity Index (D)

I begin with the delta or dissimilarity index (D) because it is by far the most widely used index of uneven distribution. I review two scoring schemes for the function $y = f(p)$ for the delta index (D). One is based on interpreting D as a crude variant of the gini index (G). I discuss this below after I have introduced and reviewed the scoring scheme for G. First, however, I review a scoring scheme for D that is especially simple, easy to explain, and attractive on substantive grounds. In this scheme D is obtained as a difference of group means (Y_1–Y_2) based on assigning residential

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_4

outcomes (y) for individuals a value of either 0 or 1 based on whether area proportion White (p) for their area of residence equals or exceeds proportion White for the city (P).[1] Thus, the relevant scaling function $y = f(p)$ for D is a monotonic, binary step function where $y = 1$ when $p \geq P$ and 0 otherwise (i.e., when $p < P$), where $p = n_1 / (n_1 + n_2)$ and $P = N_1 / (N_1 + N_2)$ per expressions introduced earlier with counts for Whites being used for Group 1 (the reference group) and counts for Blacks being used for Group 2 (the comparison group).

I review the underlying formal basis for this scoring of residential outcomes in Appendices. The material also provides detailed discussions establishing the formal basis for scoring function $y = f(p)$ for all indices considered in the body of this paper. The discussions are mostly dry and tedious. But I encourage interested readers to review the discussions to verify the basis for the scoring functions and to gain additional insights into the underlying nature of different indices.

The scoring of residential outcomes as either 0 or 1 based on whether area proportion White (p) equals or exceeds the city mean (P) supports a simple, straightforward substantive interpretation of D in terms of group differences in "exposure" and "contact". Specifically, D can be understood as the White-Black difference in the proportion in each group that experiences "parity" in (pairwise) contact with Whites. Parity here is equated to attaining at least the level of (pairwise) proportion White seen for the city overall. Noting this substantive interpretation for D introduces a theme that will recur throughout this chapter. It is that:

> *D and all other popular indices of uneven distribution can be interpreted as measures of simple group differences on residential outcomes of scaled group "exposure" or "contact" for individuals.*

The contact interpretation of D is simple and easy to grasp. In light of this, it is surprising that it is so infrequently discussed in the broader literature. Instead, it is much more common for the substantive interpretation of D to be framed in terms of the extensiveness of group displacement from even distribution based on "volume of group movement". In this interpretation D indicates "the minimum proportion of one group that would have to move to a new area to bring about even distribution."[2] This interpretation of D is useful for some purposes and it is often seen as an interpretation that is easy to convey to broad audiences. For example, it is relevant for policy analysis assessing consequences of segregation in terms of the "disruption" in residential patterns (or school attendance patterns) that would result if policies

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[1] The same result is obtained if y is set to 1 when contact exceeds parity (i.e., $p > P$). This is because assigning 0 or 1 in cases where $p = P$ will shift the mean for both groups by the same amount and thus have not impact on the index score.

[2] This interpretation rests on the assumption that only one group relocates (Zelder 1977). Minimum "volume of movement" requirements can be quite different if members of both groups exchange residential locations.

promoting integration were implemented.[3] But the group difference of means interpretation of D also is useful, very easy to compute, and very easy to convey to broad audiences. So these are not decisive factors for the neglect of this straightforward contact interpretation of D.

Regardless of what factor(s) account for it, the lack of attention given to the difference of mean contact interpretation for D has an unwelcome consequence. It has led researchers to be less familiar with an important property of D; namely, that D is inherently insensitive to, and conveys very little information about, the quantitative magnitude of group differences on residential outcomes. For example, a high value of D means that a higher proportion of Whites than Blacks live in areas that attain parity with the city-wide level of proportion White. But the high value does not, *and it inherently cannot,* signal whether the kinds of areas that Whites and Blacks typically live in are relatively similar on proportion White or fundamentally different. Relatedly, while the value of D signals the minimum proportion of one group that will need to move to eliminate uneven distribution, it provides little insight into the changes in neighborhood outcomes that would result for the two groups in the comparison.

> *The value of D does not indicate whether movement to bring about even distribution will lead to substantively important changes in the residential outcomes for the individuals in either group. Specifically, it does not indicate whether movement will bring about sociologically meaningful changes in neighborhood racial composition.*

This is not a trivial concern. Accordingly I give it extended attention at several other points in this monograph as well as here. The reason it is not trivial can be put in simple terms. It is logically possible for D to take high values when Whites and Blacks live in areas that are fundamentally similar on area proportion White. In this circumstance, residential redistribution leading to integration would indeed require a high proportion of one group to move, but the movement will not lead to important changes in their residential outcomes or in their comparison on these outcomes with the other group. This important possibility appears not to be widely recognized and appreciated by segregation researchers. It is safe to say it is almost never recognized by broader consumers of segregation research.

In my experience non-specialists and researchers alike overwhelmingly interpret D in a way that is oblivious to this quality and as a result leads them to be prone to make mistaken inferences about the nature of segregation. Specifically, researchers as well as non-specialists are prone to assume that high values of D *necessarily* indicate that most members of both of the groups in question live apart from each other in areas where their group predominates and as a result elimination of uneven distribution will lead to important changes in racial mix and associated residential outcomes for at least one group. This, of course, is *sometimes* the case. But it is important to recognize that *it is not necessarily the case.* Furthermore, this latter outcome is not an esoteric or unusual hypothetical possibility that can be safely

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[3] For this concern, the replacement index might be the better choice as it assesses the minimum of overall population movement required to bring about even distribution (Farley and Taeuber 1974).

ignored. To the contrary, as I show in empirical analyses I review in Chap. 6, instances where high values of D occur but both groups on average live in neighborhoods that are fundamentally similar on neighborhood outcomes can be found with surprising frequency when one systematically examines group differences in residential distribution in detail.

It may be helpful to make the issue more concrete by considering an example where inattention to this issue can lead to an incomplete and potentially misleading understanding of segregation patterns. A relevant case is the comparison of White-Black segregation and White-Asian segregation. Studies generally find that, on average, D for White-Black segregation is higher than D for White-Asian segregation. I find a similar result based on analysis of block-level data for core-based statistical areas (CBSAs) in 1990, 2000, and 2010 with the median value of D being 71.8 for White-Black segregation and 62.8 for White-Asian segregation.[4] The difference of 9.0 points is relatively modest and suggests that, while White-Asian segregation is appreciably lower than White-Black segregation, White-Asian segregation still should be seen as fairly high.

With values of D being so high for both comparisons, one might assume that Black and Asian residential outcomes would be relatively similar and that most members of both groups would tend to reside in areas where their group predominates and not with Whites. But this is not the case at all. Blacks consistently reside in areas where Blacks predominate; across the CBSAs in the full data set the median for Black (pairwise) contact with Whites is 43.1 % and the median for Black (pairwise) contact with Blacks is 56.9 %. In contrast, Asians rarely reside in areas where Asians predominate; across CBSAs the median for Asian (pairwise) contact with Whites is 83.4 % and the median for Asian (pairwise) contact with Asians is 16.6 %. These results indicate that White-Black segregation is quantitatively fundamentally different from White-Asian segregation even when they have similar values on D. The results for D do not suggest this, but results for the separation index (S) – an alternative measure of uneven distribution I discuss in more detail below – do signal that the group comparisons are very different. In the same analysis I found the median value for the separation index was 48.3 for White-Black segregation and 13.8 for White-Asian segregation. The difference in the two segregation comparisons is much more dramatic using S; the typical level of S for White-Black segregation is three times the typical level of S for White-Asian segregation. This example highlights that D is insensitive to an important aspect of segregation; namely, group residential separation and neighborhood racial polarization. I review this issue in more detail in the next chapter.

[4] The full data set of segregation scores is discussed in more detail in the next chapter of this monograph. CBSAs areas are included in the analysis if the size of the smaller group is 2,500 or higher.

4.2 Scoring Residential Outcomes (y) for the Gini Index (G)

For the gini index (G), the relevant function $y = f(p)$ for scoring residential outcomes (y) so G can be obtained from the difference of group means (Y_1–Y_2) is based on relative rank position – that is, quantile or percentile standing – on area group proportion (p). Specifically, y = percentile scores based on p. This makes y an ever-rising, monotonic, nonlinear function of p. The percentile scores can be obtained as follows. Rank areas from low to high on p. Assign the first area (i.e., $i = 1$) the percentile score $y = 100 \cdot (½ t_i) / T$. Assign the remaining areas (i.e., areas $i = 2, 3, 4 \ldots I$) percentile scores based on $y = 100 \cdot (\Sigma t_{i-1} + ½ t_i) / T$. Under this scoring system, G/2 can be obtained from $Y_1 - Y_2$. Alternatively, G can be obtained from $2 \cdot (Y_1 - Y_2)$. I review the basis for these expressions in Appendices.

The fact that G can be obtained from percentile scoring of area group proportions results because G, when applied using its formulation as a segregation index, is a measure of ordinal or "rank order" inequality between groups. I have previously described this property of G in Fossett and Siebert (1997) and earlier, albeit less directly, in Fossett and South (1983). In Fossett and Siebert (1997: Appendix A) I note that G is equivalent to familiar indices of ordinal inequality and ordinal association. Specifically, G is mathematically equivalent to Lieberson's (1976) index of net difference (ND), a measure of ordinal inequality between groups and G also is equivalent to Somers' (1962) d_{yx}, an index of ordinal association. Based on this Fossett and Siebert (1997) show that G can be given as

$$G = 100 \cdot \Sigma\Sigma y_{ij} \cdot (n_{1i} / N_1) \cdot (n_{2j} / N_2)$$

or, more compactly,

$$G = 100 \cdot \Sigma\Sigma y_{ij} \cdot (s_{1i} \cdot s_{2j})$$

where i and j index areas ranked for low to high on area proportion White (p) and y_{ij} is scored −1 if $i < j$, 0 if $i = j$, and 1 if $i > j$ and s_{1i} and s_{2j} are group share scores given by $s_{1i} = (n_{1i} / N_1)$ $s_{2j} = (n_{2j} / N_2)$. This formula reveals that *G registers only rank position on p and does not register the size of the quantitative differences involved*.

The difference of means formulation of G supports and clarifies the interpretation of G as a measure of group difference on scaled "exposure" and "contact". In the case of White-Black segregation, G is the White-Black difference in average relative rank position on contact with Whites (p).[5] Alternatively, G is the White-Black difference in exposure to area percentile rank on proportion White. G's equivalence to the index of net difference supports a related interpretation. G indicates the difference between two probabilities for rank order comparisons of individual

[5] More carefully, G is twice the difference. Or, equivalently, G is obtained by expressing the observed difference as a percentage of its maximum possible value (which is 0.50).

Whites and Blacks on area proportion White. The first, P(A), is the probability that a randomly selected White will live in an area where proportion White is higher than that for a randomly selected Black. The second, P(B), is the probability that a randomly selected White will live in an area where proportion White is lower than that for a randomly selected Black.[6] The value of G is given by P(A) − P(B) where $P(A) = \Sigma X_{i-1} Y_i$ and $P(B) = \Sigma X_i Y_{i-1}$ with X and Y denoting cumulative group proportions over areas ranked from low to high on area proportion White. This expands to $100 \cdot (\Sigma X_{i-1} Y_i - \Sigma X_i Y_{i-1})$, the formula for G from Duncan and Duncan (1955) given in Fig. 2.1. The main value of the net difference interpretation originally introduced by Lieberson (1976) is to drive home the point that G is a measure of inter-group rank order inequality on the residential outcome of area proportion White (p).

It is useful to briefly contrast G with D. Unlike D, G satisfies the principle of transfers and on this basis is technically superior to D.[7] But G is similar to D in being unable to give a reliable signal about group separation and residential polarization. The reason for this is that G can take high values when the two groups in the comparison have similar distributions on area group composition. This is possible because G registers rank-order differences on area proportion White and will register such differences equally regardless of whether the quantitative differences on area proportion White are small or large. As a result, when on sees a high value on G, it is impossible to know whether the underlying pattern of segregation involves extensive group separation and extreme neighborhood racial polarization such as that observed for White-Black segregation in Chicago or involves a more benign pattern with minimal group separation and fundamentally similar neighborhood fate.

4.3 The Delta or Dissimilarity Index (D) as a Crude Version of G

The index of dissimilarity (D) can be understood as a special case of the gini index (G). Specifically, D is equivalent to G when areas are ranked using a two-category scheme based on whether $p_i < P$ or $p_i \geq P$, rather than being ranked on the full range of scores on p_i as would be the case with G. Thus, D is a version of G computed when areas are grouped into two categories based on whether or not they are at or above average on proportion White. Accordingly, D can be obtained using the formula for G after ranking areas on the basis of a two-value recoding of p_i as either 1, when $p_i \geq P$, or 0 otherwise. These recoded values of p_i are then used to score y in terms of relative rank position on area group proportion as described above for G.

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[6] It is also possible for Whites and Blacks to tie when compared on area proportion White. But this probability need not be computed as it does not directly determine the index score.

[7] The principle of transfers is discussed in James and Taeuber (1985) and Reardon and Firebaugh (2002).

Accordingly, the value of D/2 is given by $Y_1 - Y_2$, or alternatively, D is given by $2 \cdot (Y_1 - Y_2)$. If one were graphing the segregation curves associated with D and G for a given comparison, G would produce a conventional segregation curve the data for D would produce a segregation curve in the form of a triangle. The two curves would share three points. The two end points of the curve at (0,0) and (1,1) and one point along the curve at (X,Y) where the values of X and Y are equal to the proportion of Blacks and Whites, respectively, living in areas where $p < P$.

This provides insight into why D and G are highly correlated and why scores for D never exceed scores for G (i.e., $D \le G$). Both measures register White-Black differences in relative rank position on p_i. However, G registers *all* rank differences on p_i while D registers only rank differences where group comparisons on p_i are on opposite sides of P. This accounts for the difference between D and G in how they respond to population transfers or exchanges. G will register any transfer or exchange that affects at least one household's rank position on area proportion White (p). D will register a transfer and exchange only if it causes the value of p for at least one household to shift from $p < P$ to $p \ge P$ or vice versa.

4.4 Scoring Residential Outcomes (y) for the Separation Index (S)

I use the term Separation Index (S) to refer to a measure that has been known by many names over the decades. A partial list of past names includes: the correlation ratio and eta squared (η^2) (Duncan and Duncan 1955; Stearns and Logan 1986; Iceland et al. 2002), r or r_{ij} (Coleman et al. 1975, 1982), the variance ratio (V) (James and Taeuber 1985), and segregation index (S) (Coleman et al. 1966; Zoloth 1976).[8] I term this measure the separation index because a high value on this index gives a clear and reliable signal that the two groups in the comparison are residentially separated and generally do not reside in the same areas.[9] That is, it indicates whether the two groups live apart from each other due to being concentrated in areas that are racially polarized in a pattern of "prototypical" segregation wherein, in the example of White-Black segregation, Whites live in predominantly White areas and Blacks live in predominantly Black areas. I clarify the basis for this claim in more detail shortly.

For the separation index (S), the relevant function $y = f(p)$ for scoring residential outcomes (y) so S can be obtained from ($Y_1 - Y_2$) is quite simple; it is the

[8] Additionally, S is a special case of Bell's (1954) revised index of isolation for the situation in which the population has only two groups.

[9] As used here, the term separation does not imply that the groups live in areas that are far apart in distance. It implies only that they are residentially separated into distinctly different areas. These can be far apart but they also can be adjoining as standard implementations of all measures of uneven distribution are "aspatial" in that the arrangements of units in space does not affect index values.

identify function $y = p$. I review the formal basis for this scoring of residential outcomes for S in Appendices.[10] The scaling function used for S is distinct from those that are used for other popular indices of uneven distribution. It maps the contact score (p) directly onto residential outcome scores (y) based on a one-to-one linear relationship. In contrast, the scaling functions for all other indices map the contact score (p) onto residential outcome scores (y) based on some form of positive, monotonic, nonlinear relationship.

The separation index supports a clear and appealing interpretation based on pairwise "exposure" and "contact". In the case of White-Black segregation, S is the White-Black difference on average contact with Whites (p). From this vantage point, it becomes clear why it is appropriate to refer to this measure as the "separation index". The White-Black difference in contact with Whites can be large *only* if Whites live separately from – that is, apart from, not with – Blacks in neighborhoods that are predominantly White *and* Blacks live separately from Whites in areas that are predominantly Black. To clarify, in most applications indices of uneven distribution are implemented as "aspatial" measures. In this application, the notion of separation implies only that the groups live in different areas. It does not imply that the different areas are necessarily spatially distant from each other. This would be the case when segregation involves large-scale clustering. But the index score would be the same if Whites and Blacks lived separately from each other in different areas forming a checker board pattern.

The separation index also could be aptly termed the "contact difference" index, but that is a bit cumbersome. Alternatively, it could be named the "concentration" index following Stearns and Logan (1986), but Massey and Denton (1988) popularized the term "concentration" in association with another distinct dimension of segregation. So I adopt the term "separation index" (S) which emphasizes that the measure is sensitive to whether groups live apart from each other and are separated into different areas that differ fundamentally on group composition.

The notion of group separation is closely connected with the notion of area or neighborhood racial polarization discussed by Stearns and Logan (1986).[11] As they used it, polarization is high when the areas in which the two groups live fall primarily into two types. In the case of White-Black segregation that would be either predominantly White or predominantly Black with few areas in between. Their usage of the term polarization directs attention to a neighborhood outcome. But polarization of neighborhood racial composition has obvious implications for group differences on residential outcomes for individuals. When areas are racially polarized, individuals in both groups primarily live in neighborhoods where members of their

[10] I derived this relationship independently. But I later discovered that the relationship had been reported, based on a different derivation, in a paper by Becker et al. (1978) that unfortunately is not widely known or referenced.

[11] Stearns and Logan also used the term "concentration" to describe this aspect of uneven distribution. It is an appealing term, but I use "polarization" instead because Stearns and Logan use it as a synonym for concentration and because the influential methodological study by Massey and Denton (1988) used the term "concentration" to refer to a different aspect of segregation (relating to concentration in physical space).

group predominate. In the example of White-Black segregation, Whites live primarily in White neighborhoods and Blacks live primarily in Black neighborhoods. This resonates with the idea that groups live separate and apart from each other, a necessary, but not sufficient, precondition for experiencing disparities on neighborhood residential outcomes other than racial composition per se (e.g., crime, social disorder, inferior amenities, poor schools, and poor government services, etc.). Given this close similarity of group separation and neighborhood polarization, it would not be unreasonable to call the separation index the "polarization" index. But I reserve that term for an alternative measure which I will introduce and discuss shortly.

As noted earlier, I endorse Stearns and Logan's view that the separation index (S) taps an aspect of uneven distribution that is sociologically important and is not consistently captured by other measures. In particular, the presence or absence of group separation is not captured well by the more widely used delta or dissimilarity index (D). It is interesting then to note that D is used much more widely than S. To be sure, the separation index has been used in segregation studies for many decades – for example, it was given close attention in Duncan and Duncan's (1955) landmark article on segregation indices.[12] Moreover, it consistently receives high marks in technical reviews of indices (e.g., Zoloth 1976; White 1986; Reardon and Firebaugh 2002). Additionally it has been shown to be far less susceptible than D to the vexing problem of index bias (Winship 1977).[13] Nonetheless, S is not used nearly as widely as D in empirical studies and its attractive qualities appear not to be widely appreciated. What could explain this?

At least three factors appear to be relevant. One is that the measure has never been consistently used under the same name and interpreted in a consistent way. This alone is likely to lead many people to underestimate both the frequency of its usage and the extent to which different researchers have endorsed its value for assessing segregation.

Another factor is that much of the usage of the index has involved terminology and interpretations that do not highlight what I view as the separation index's strongest feature for substantive interpretation. For example, the measure has been used most widely under the names "variance ratio", "correlation ratio", and "eta squared" in the literature. These names are not technically incorrect or inappropriate. But they also do not call attention to the measure's most attractive characteristic – its ability to signal when group residential distributions are polarized such that the two groups live separately from each other with members of *both* groups living primarily in areas where their group predominates. Instead, the names used in the past attention toward substantive interpretations relating to the strength of the individual-level, statistical association between the binary variable of race (coded

[12] Duncan and Duncan referred to it as eta squared and the correlation ratio. The measure also is discussed in Bell (1954), but the application there is to overall isolation instead of pairwise group comparisons.

[13] In Chap. 14 I amplify Winship's early finding by showing that among all popular indices of uneven distribution S is least susceptible to distortion by index bias while G and D are the most susceptible.

0–1) and the categorical variable of area of residence. These "statistical" interpretations are mathematically defensible, but they do not resonate with broad audiences and researchers perhaps because their substantive relevance for group differences in residential outcomes is neither obvious nor easy to convey.[14]

The difference of group means formulation of the separation index can potentially address these two points and enhance the attractiveness of S to researchers and broader audiences. The computation of S under the difference of means formulation is simple and easy to implement. In addition, this formulation of S has an appealing substantive interpretation that is easy to convey to both broad and technical audiences; it signals that uneven distribution involves groups residing separately from each other with *both* groups being disproportionately concentrated in racially polarized neighborhoods such that the two groups experience fundamentally different residential outcomes on area racial composition. Importantly, D, the most widely used index of uneven distribution does not provide a reliable signal for whether or not this pattern of segregation is present.

4.5 A Side Comment on the Separation Index (S) and Uneven Distribution

A third factor that may help explain why the separation index has not been used more widely requires a longer discussion. It is that S is occasionally viewed as a measure of group isolation and exposure rather than a measure of uneven distribution. At one level I view the controversy as minor because most technical reviews correctly characterize the separation index as a measure of uneven distribution (e.g., Zoloth 1976; James and Taeuber 1985; White 1986; Reardon and Firebaugh 2002). But there are contrasting descriptions of S in the literature so the issue warrants a brief side discussion.

Massey and Denton (1988) categorize S (which they refer to as V and eta squared) as an "exposure" measure rather than a measure of uneven distribution. One reason they offer for doing so is that, unlike D and G, S does not have a definite relationship to the segregation curve. This concern should be set aside for two reasons. The first reason is that Massey and Denton themselves do not apply this criterion in a consistent way. For example, they classify the Theil entropy index (H) as an index of uneven distribution but, like S, H also does not have a definite relationship with the segregation curve.

The second reason to set aside this concern is that many authoritative reviews of measures of uneven distribution disregard the segregation curve when evaluating indices (e.g., Zoloth 1976; Stearns and Logan 1986; Reardon and Firebaugh 2002).

[14] S is equal to the eta squared (η^2) statistic from an individual-level analysis of variance predicting the mean of the binary variable of race (0–1) by area of residence (e.g., the categorical variable of tract). Relatedly, S is equal to the square of the individual-level correlation between race (coded 0–1) and p (computed for area of residence).

Some important statements explicitly and forcefully dismiss the relevance of the segregation curve altogether (White 1986; Coleman et al. 1982). I endorse these views. I recognize that the segregation curve is a visually appealing and is potentially a useful graphical tool for depicting group differences in residential distribution. But it does it not embody “the” definitive definition of uneven distribution and it also has clear limitations and deficiencies. For example, the segregation curve does not, and it logically cannot, signal whether the two groups in the comparison live apart from each other in areas that differ in substantively important ways on area racial composition. Accordingly, most authoritative methodological reviews classify both H and S as valid measures of uneven distribution often noting features of these indices that make them attractive for many purposes (e.g., Zoloth 1976; James and Taeuber 1985; White 1986; Reardon and Firebaugh 2002).

Another possible basis for Massey and Denton’s characterization of S as an exposure index is that, under certain circumstances, particular computing formula for S contain terms that are similar to terms found in formulas for exposure indexes. For example, many have noted that S has similarities to Bell’s (1954) revised index of isolation which involves terms that have exposure interpretations (e.g., Duncan and Duncan 1955; Becker, McPartland, and Thomas 1978; Iceland et al. 2002; James and Taeuber 1985: footnote 4; Stearns and Logan 1986). In the final analysis, however, it is clear that there are fundamental logical differences between S and exposure and isolation indices.

The first important logical difference is the population comparison involved. Exposure and isolation indices are calculated by comparing group counts to counts for the *full* population, not just the two groups in the segregation comparison. In contrast, S, like other indices of uneven distribution, is calculated from “pairwise” counts; that is, it is calculated using only the counts for the two groups in the comparison and is unaffected by the counts for other groups. The distinction can be crucially important in empirical applications because scores and substantive implications of “overall” and “pairwise” isolation can be and often are quite different.

The second important logical difference that distinguishes S from pairwise isolation indices is that the pairwise isolation term incorporated in some computing formulas for S is modified by a “normalizing” calculation. This calculation is crucially important to the issue at hand because it can and often does radically change its value. Equally importantly, the normalizing calculation also fundamentally changes the substantive interpretation of S. Specifically, *S does not register the level of pair-wise isolation*. It registers something distinctly different; *S registers the relative extent to which pair-wise isolation exceeds its expected value*. This is fundamentally different from pair-wise isolation itself. The normalizing calculation in this particular formulation of S has a crucially important consequence; it eliminates the mathematical correspondence between isolation and group composition. Thus, while group composition has important implications for the value of pair-wise isolation scores, it has no necessary or mathematically inherent implication for S. As a result, S can take any value over its logical range of 0 to 1 under any arrangement on group composition for the city. This is not the case for measures of isolation. They must take high values when the group in question is large in relative terms.

In sum, S is fundamentally distinct from standard indices of isolation and exposure. Isolation terms found in some formulas for computing S are based on pairwise counts, not overall counts, and they are subject to a normalizing transformation that radically changes their value and eliminates any mathematical correspondence between city ethnic composition and the value of S. Consequently, one cannot reliably infer the value of either overall or pairwise isolation from knowledge of the score of S or vice versa.

Given the confusion in the literature on this issue, it may be useful to consider the hypothetical example of a population with three groups – Whites, Blacks, and Latinos. Then assume that Whites live apart from Blacks and Latinos but that Blacks and Latinos live together. S will register the pattern of uneven distribution as high for both White-Black and White-Latino segregation and low for Black-Latino segregation. Importantly, this result will be the same regardless of the city racial composition. In contrast, both overall isolation and also Bell's revised index of isolation will vary depending on city racial mix. The revised index of isolation for Blacks will be higher in a city where Whites outnumber Latinos (e.g., Detroit and Cleveland) and it will be low in a city where Latinos outnumber Whites (e.g., El Paso or San Antonio). This issue is not narrowly academic; it can have important practical consequences for index scores and substantive conclusions. This takes on increasingly relevance in recent decades as the growth of the Latino and Asian populations has resulted in more complex racial demography in many cities.

Finally, I close this discussion by stressing that S is not unusual among measures of uneven distribution in having linkages and interpretations relating to exposure and contact. To the contrary, one of the valuable insights gained from the difference of means formulations of indices of uneven distribution set forth in this monograph is that a*ll popular indices of uneven distribution have direct and definite linkages to pairwise contact and exposure*. Thus, for the example of White-Black segregation, both S and D can be obtained as simple group differences on exposure to Whites. In the case of S exposure is assessed directly by area proportion White (p). The only difference for D is that exposure is rescaled to either 0 or 1 depending on whether p equals or exceeds P. Thus, the key differences between indices of uneven distribution are found not in *whether* the indices register contact and exposure – *all popular indices do this*. The key differences are found in *how* the specific indices scale contact and exposure differently based on the way segregation-relevant residential outcomes (y) are scored from area group proportion (p).

4.6 Scoring Residential Outcomes (y) for the Theil Index (H)

For the Theil index (H), the relevant function $y = f(p)$ for scoring of residential outcomes (y) is a continuous function of p in the manner of S, but the form of the function is more complex. Specifically, the function $y = f(p)$ is the following continuous, ever-rising, nonlinear expression

$$y = Q + \left[(E - e_i)/E\right] / (p_i / P - q_i / Q)$$

where e_i is the entropy score for area i and E is the entropy score for the city as a whole. These are given by the calculations $e_i = p_i \cdot \ln(1/p_i) + q_i \cdot \ln(1/q_i)$ and $E = P \cdot \ln(1/P) + Q \cdot \ln(1/Q)$. I owe special thanks to Warner Henson, III for helping me identify the form of this function.[15] I review the formal basis for this scoring of residential outcomes in Appendices.

Theil and Finizza (1971) and Theil (1972) argue that information theory provides an attractive conceptual grounding for using entropy calculations to assess segregation. But most researchers who use H adopt it on a more narrow and practical basis. In particular, H is often used because it is mathematically tractable in ways that facilitate decomposition analysis.[16] The substantive relevance of area (e_i) and city-level (E) entropy scores are seen narrowly as quantifying two-group racial diversity with the expression $(E - e_i)/E$ thus registering uneven distribution as departure of area racial diversity from that which would occur under even distribution given the racial mix of the city population.

I show below that the nonlinear relationship between y and p is visually simpler and more intuitively appealing than the mathematical expression introduced above might suggest. In its essence, the function maps p into y based on an ever-rising, backwards "S-curve". The undulations of the S-curve and its symmetry, or lack thereof, vary with the relative sizes of the two groups in the comparison.[17] When the groups are identical in size, the undulations in the S-curve are moderate and the resulting curve is symmetrical. In this situation results for H and S tend to track each other very closely. When the two groups are unequal in size, the undulations in the S-curve for y–p relationship for H are asymmetrical and larger in amplitude and the

[15] At the time, Mr. Henson was an undergraduate research assistant at Texas A&M University. At the time of this writing, he is a sociology doctoral student at Stanford University.

[16] Reardon and Firebaugh (2002) emphasize this property in arguing that H is attractive for investigating multi-group segregation.

[17] The graph for $y = f(p)$ for G also tends to form a forward-leaning "S-curve. However, it is not A smooth curve; it is a series of small step functions that typically take the general form an S-curve.

resulting curve departs from linearity in greater degree. In this situation results for H and S may differ.

Under this system for scoring segregation-relevant residential outcomes (y), H can be obtained from ($Y_1 - Y_2$) and thus fits in the framework for measuring segregation set forth in this paper. As in the previous examples considered, the difference of means formulation of H shows that it can be interpreted in terms of scaled contact and exposure. In the case of White-Black segregation, H is the White-Black difference in average contact with Whites (p) scored on the basis of the nonlinear function described above. When P and Q are balanced (i.e., $P = Q = 50$), the function is a symmetrical backwards "S". As a result, the measure responds less to differences in p in the middle of its range (i.e., 25–75) and more to differences in the lower and higher ranges of p. When P and Q are imbalanced, one must study the y−p relationship to understand specifically how H responds differentially to contact over different ranges of p. I discuss this in more detail below.

4.7 Scoring Residential Outcomes (y) for the Hutchens Square Root Index (R)

At this point, only one measure of residential segregation that receives regular attention in methodological studies of indices of uneven distribution has yet to be considered. This is Atkinson's index (A). While rarely used in empirical studies, it nevertheless has been discussed in several methodological studies of segregation indices. For example, James and Taeuber (1985) praise A for involving a user-specified parameter (δ) which they argue can be used to "tune" the index to be sensitive to particular regions of the segregation curve. Massey and Denton (1988) also comment that this is a potentially interesting quality of A. In contrast, White (1986) and Hutchens (2001, 2004) view this characteristic of A as undesirable. Indeed, they characterize it as a fundamental flaw. They point out that A is "asymmetric" when δ is set to any value other than 0.5 and argue that the property of asymmetry introduces conceptual complications most would view as impractical if not fatal altogether for general use of A in segregation research. For example, the property of asymmetry implies that White-Black segregation can be logically and quantitatively different from Black-White segregation. No one has endorsed this as a desirable quality of segregation indices. I follow White and Hutchens in endorsing the principle of symmetry for segregation indices and therefore limit my consideration of A to only its symmetric implementation $A_{(0.5)}$ – the special case where δ is set to 0.5. Hereafter, my references to A are to this version so I drop the subscript.

I have not been able to discover a way to express the value of A as a simple difference of means on scores of y based on area group proportion scores (p). However, I have found a difference of means solution for an index that is a close conceptual and mathematical surrogate for A. The index I refer to is the Hutchens (2001, 2004) square root index (R). This index has gained currency in the study of occupational sex segregation, but has not yet gained wide usage in studies of residential segregation. Since it may be unfamiliar to some readers, I briefly note three equivalent formulas for R.

$$R = 100 \cdot \left(1 - \Sigma\sqrt{\left(w_i / W\right) \cdot \left(b_i / B\right)}\right) = 100 \cdot \left(1 - \Sigma\sqrt{s_{Wi} \cdot s_{Bi}}\right)$$

$$R = 100 \cdot \left[1 - \Sigma\left(t_i / T\right)\sqrt{\left(p_i / P\right) \cdot \left(q_i / Q\right)}\right]$$

$$R = 100 \cdot \left[1 - \Sigma\left(t_i / T\right)\sqrt{p_i q_i / PQ}\right]$$

The similarity to Atkinson's A can be seen by comparing the last formula with the following expression for A which obtains when the tuning parameter δ is set to 0.5.

$$A_{(0.5)} = 100 \cdot \left[1 - \left\{\Sigma\left(t_i / T\right) \cdot \sqrt{p_i q_i}\right\}^2 / PQ\right].$$

The close relationship of R and A also can be seen in the fact that the two map onto each other based on the following exact nonlinear relationships

$$A = 2 \cdot R - R^2 \text{ and}$$

$$R = 1 - \sqrt{1 - A}.$$

Values of R are numerically lower than values of A. But since the relationship of their scores is exact and continuous, the two indices yield identical rank-orderings of segregation comparisons. Hutchens (2001, 2004) argues that R is an attractive measure of segregation in its own right. I include R in the discussion here on that basis as well as because it is a close surrogate for A. Additionally, values of R have a very strong relationship with values of D in empirical studies and R fares much better than D in technical reviews.[18]

For Hutchens' square root index (R), the relevant scoring of residential outcomes (y) is a continuous, ever-rising, nonlinear function of p. Specifically, the function $y = f(p)$ is

$$y_i = Q + \left(1 - \sqrt{p_i q_i / PQ}\right) / \left(p_i / P - q_i / Q\right)$$

where p_i, q_i, P, and Q are as introduced earlier. Under this system for scoring residential outcomes (y), R can be obtained from ($Y_1 - Y_2$) and thus fits in the

[18] In the data sets I examine for this study, the square root of R consistently correlates with D at 0.99 or higher.

framework for measuring segregation set forth in this paper. I establish the formal basis for this scoring of residential outcomes for R in Appendices.

As with the other indices, this supports an interpretation of R in terms of scaled contact and exposure. In the case of White-Black segregation, R is the White-Black difference in average contact with Whites (p) scored on the basis of the nonlinear function shown above. Like H, the function produces a continuous, ever-rising, nonlinear curve that forms a backwards "S". Also like H, the undulations in the nonlinear curve vary with the relative sizes of the groups in the comparison. When the groups are identical in size, the undulations in the S-curve are modest and symmetrical and the resulting curve is relatively close to linear. When the two groups are unequal in size, the undulations in the S-curve are asymmetrical and larger in amplitude and the resulting curve departs from linearity in greater degree. One must study the particular y−p relationship in each case to understand how R registers p over different ranges of p.

Hutchens (2001, 2004) argues R is an attractive index in part because it orders aggregate segregation scores in a manner consistent with the principle of segregation curve dominance advocated by James and Taeuber (1985). The Atkinson index (A), the Gini index (G), and the dissimilarity index (D) all also satisfy this principle. Accordingly, scores for R tend to correlate closely with scores for G and D and especially with scores for A.[19] As I noted earlier, however, the principle of segregation curve dominance is controversial and only a few methodological reviews endorse it.[20] One reason for this mentioned earlier is that defining segregation in relation to the segregation curve eliminates two popular indices– the Theil entropy index (H) and the Separation Index (S) – that both have attractive features to recommend them and that both fare well in technical reviews.

In essence, the principle of segregation curve dominance requires that indices place segregation comparisons involving non-crossing segregation curves in the same order as would result from segregation curve analysis.[21] Some methodological reviews explicitly reject the principle. Most reviews are less direct but, while not explicitly taking a position on the issue, they implicitly reject the principle by giving favorable evaluations of H and S which do not have the property.[22] I view the principle of segregation curve dominance as undesirable because it assigns priority to segregation indices that are *necessarily* insensitive to group residential separation

[19] Analyses of White-Minority segregation for core-based statistical areas reported later in this monograph document close, mildly nonlinear relationships among G, $A_{(0.5)}$, and R.

[20] Most controversially, it assigns logical primacy to the segregation curve – a graphical and geometric representation of group differences in cumulative rank distribution on area group proportions – without a compelling conceptual-theoretical basis for doing so.

[21] The principle does not specify how an index should rank segregation comparisons when segregation curves cross.

[22] Coleman et al. (1982) explicitly reject the principle. White (1986) also questions it value. Reardon and Firebaugh (2002), Zoloth (1976), Stearns and Logan (1986), and others, ignore the principle but praise measures such as Theil's entropy index (H) and the separation index (S) giving no concern to the fact that these indices do not conform to the principle of segregation curve dominance.

and neighborhood polarization. I emphasize the word "necessarily" because segregation curves register *rank order differences* between groups on area group proportion (p) without regard to whether group differences on p are large or small in magnitude. As a result, segregation curves can signal high levels of uneven distribution when group residential separation and neighborhood polarization are low. I view this with great concern and accordingly review the issue in more detail in the next chapter.

For now I conclude this discussion by arguing that it is important for researchers to at least have the option of focusing on uneven distribution that involves group separation and neighborhood polarization. In my view segregation that separates groups into residing apart from each other in different neighborhoods that differ fundamentally on racial composition is substantively compelling. Separation conceived in this way is a logical prerequisite for group disparities on neighborhood residential outcomes such as quality of schools, exposure to crime and social problems, availability and quality of services, etc. In contrast, uneven distribution that does not involve group separation and polarization does not necessarily create the logical potential for group differences on these kinds of stratification-related neighborhood outcomes.

To summarize, in this chapter I reviewed how five indices of uneven distribution – G, D, R, H, and S – all can be specified as differences of group means on residential outcomes (y) scored from area group proportion (p). These five indices represent the most popular, widely used, and carefully studied indices of uneven distribution in the literature on segregation measurement. Consequently, I conclude that all popular indices of uneven distribution have ready interpretations as measures of group differences in contact and exposure. All of the indices indicate that groups experience the maximum possible average difference on contact outcomes when uneven distribution is complete and groups live completely apart. Similarly, all of the indices indicate that groups experience identical contact outcomes under conditions of even distribution. From this vantage point, the substantive differences between the indices ultimately trace to one thing; the differences among them in how they register group differences on individual residential contact outcomes (y) in the intermediate ranges. The scaling function $y = f(p)$ for each index provides insight into this. Accordingly, I review how this function varies across indices in the next chapter.

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Chapter 5
Index Differences in Registering Area Group Proportions

My goal in this chapter is to help interested readers become more familiar with the residential outcomes for individuals and households that additively determine the scores of different indices of uneven distribution. To do so, I review the residential outcome scores that underlie segregation comparisons in the difference of means formulation looking in detailed at the segregation comparisons of Whites with Blacks, Latinos, and Asians in Houston, Texas in 2000. The data for these comparisons are taken from block group tabulations for families obtained from Summary File 3 of the 2000 census.[1] Table 5.1 presents the basic demographic information for the four groups and the three segregation comparisons considered here. The results for "overall" percentages document that Whites (non-Hispanic) are the largest group at 52.7 % overall, followed by Latinos (34.8 %), Blacks (16.5 %), and Asians (4.8 %). The results also document that the pairwise percentages for any group comparison are always higher than overall percentages for the obvious reason that groups outside the comparison are excluded from the denominator in the calculations.

Table 5.2 lists the values of G, D, R, H, and S obtained using standard computing formulas given in James and Taeuber (1985) for D, G, S and H, and a comparable formula for R adapted from Hutchens (2001) (reviewed in Appendices).

$$D = 100 \cdot \Sigma\, t_i \left| p_i - P \right| / 2TPQ$$

$$G = 100 \cdot \Sigma\Sigma\, t_i\, t_j \left| p_i - p_j \right| / 2T^2PQ$$

$$S = 100 \cdot \Sigma\, t_i \left(p_i - P \right)^2 / TPQ$$

[1] Specifically, I draw on Table 160 (A-I) which tabulates families by race, poverty status, family structure, and presence of related children under age 18 for block groups. In this tabulation White is Non-Hispanic White, Black and Asian counts include Hispanics, and Latinos are of any race. The information in the tabulations pertaining to social characteristics of poverty and family status are not used here, but are used in analyses presented in Chap. 9.

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_5

Table 5.1 Group counts and overall and pairwise group percentages for Houston, Texas, 2000

Group	N of families	Percentage among all families	White-Black pairwise percentage	White-Latino pairwise percentage	White-Asian pairwise percentage
White	627,613	52.7	76.2	68.0	91.8
Black	195,928	16.4	23.8	–	–
Latino	294,931	24.8	–	32.0	–
Asian	55,746	4.7	–	–	8.2
Total	1,191,102	100.0	100.0	100.0	100.0

Source: US Census 2000, Summary File 3

Table 5.2 Scores for White-Minority segregation indices obtained using standard computing formulas, Houston Texas, 2000

Group comparison	G	D	R	H	S
	Computed using standard formulas				
White-Black segregation	87.07	70.97	47.02	53.59	57.39
White-Latino segregation	74.19	58.37	28.11	35.46	40.96
White-Asian segregation	76.28	58.22	34.96	31.31	23.88

Source: US Census 2000, Summary File 3

$$\mathrm{H} = 100 \cdot \Sigma \mathrm{t_i} \left(\mathrm{E} - \mathrm{e_i} \right) / \mathrm{ET}$$

$$\mathrm{R} = 100 \cdot \left(1 - \Sigma \mathrm{t_i} \cdot \sqrt{p_i q_i \;/\; PQ} \;/\; \mathrm{T} \right)$$

Terms are defined as noted earlier (and also summarized in Appendices). In this particular analysis, the five segregation indices – G, D, R, H, and S – yield generally similar overall patterns of aggregate segregation between Whites and the three non-White groups. For example, all five indices show that substantial segregation is evident in each comparison. Similarly, all five indices show that White-Black segregation is the highest of the three segregation comparisons examined. There is one notable finding regarding how the different measures portray patterns of aggregate segregation. D, G, and R indicate that White-Latino segregation and White-Asian segregation are roughly similar. H and S indicate that White-Asian segregation is substantially lower than White-Latino segregation.

5.1 Segregation as Group Differences in Individual Residential Attainments

I next present results that demonstrate how the scores of the aggregate segregation indices can be obtained from simple differences of group means on residential attainments. Table 5.3 lists the values of D, G, S, H, and R calculated using the difference of means formulations introduced in this monograph. The three panels in

Table 5.3 Details for obtaining scores for White-Minority segregation from difference of group means on residential outcomes, Houston, Texas, 2000

Residential outcome scored from index-specific scaling function y = f(p)	Mean for Whites	Mean for Minority	White-Minority difference
White-Black segregation			
y scored for G/2 (×100)[a]	60.36	16.82	G = 87.08
y scored for G (×200)	120.72	33.65	G = 87.07
y scored for D/2 (×100)[a]	58.44	22.96	D = 70.96
y scored for D (×100)	87.73	16.75	D = 70.98
y scored for R (×100)	72.98	25.96	R = 47.02
y scored for H (×100)	82.57	28.98	H = 53.59
y scored for S (×100)	89.86	32.48	S = 57.38
White-Latino segregation			
y scored for G/2 (×100)[a]	61.86	24.76	G = 74.20
y scored for G (×200)	123.72	49.53	G = 74.19
y scored for D/2 (×100)[a]	59.33	30.14	D = 58.38
y scored for D (×100)	81.49	23.12	D = 58.37
y scored for R (×100)	63.37	35.26	R = 28.11
y scored for H (×100)	72.95	37.50	H = 35.45
y scored for S (×100)	81.12	40.17	S = 40.95
White-Asian segregation			
y scored for G/2 (×100)[a]	53.11	14.97	G = 76.28
y scored for G (×200)	106.22	29.94	G = 76.28
y scored for D/2 (×100)[a]	52.38	23.27	D = 58.22
y scored for D (×100)	75.15	16.93	D = 58.22
y scored for R (×100)	70.70	35.74	R = 34.96
y scored for H (×100)	83.47	52.16	H = 31.31
y scored for S (×100)	93.79	69.91	S = 23.88

Source: US Census 2000, Summary File 3

[a]For these scorings of y for G and D, the values of G and D are given by $2\cdot(Y_1 - Y_2)$

the table report results separately for the White-Black, White-Latino and White-Asian segregation comparisons, respectively. The first step in generating these results is to calculate the residential outcomes scores (y) at the block group level. I obtain these by applying the relevant index-specific scaling function $y = f\left(p\right)$ to the value of pairwise proportion White (p) at the block group level. The second step is to calculate the group-specific means for scaled contact with Whites (y). The resulting values are reported in Table 5.3. The last step is to calculate the difference of the group-specific means which also are reported in Table 5.3.

The results are straightforward. The values of the differences of means equal the values of the index scores reported in Table 5.2. Any apparent differences reflect only rounding error and would disappear if the results were reported to greater precision. Of course, the index scores reported in Table 5.3 are redundant with the results already presented and do not themselves provide any new insights into segregation patterns. But presenting the detailed results documents that the difference-of-means formulas yield the same results as the conventional formulas.

I noted in the previous chapter that the segregation-relevant residential outcomes (y) that determine the group means (Y_1 and Y_2) are index-specific scores for scaled pairwise contact with Whites. The exact scoring of residential outcomes (y) varies from index to index, but values of y always are a positive, monotonic function of pairwise proportion White (p) for the household's or individual's area of residence. The minority group's average pairwise contact with Whites cannot exceed that observed for Whites and it can reach parity only under the condition of exact even distribution. When there is departure from uneven distribution, mean contact with Whites for Whites will diverge from mean contact with Whites for the minority group. The average magnitude of the difference will be reflected in the difference of means ($Y_1 - Y_2$) which will yield the index score. Given this, it is instructive to consider how the different index-specific residential outcome scores compare to each other.

Figure 5.1 plots the values of residential attainment scores (y) by values of pairwise proportion White (p) for G, D, R, H, and S for the three White-Minority segre-

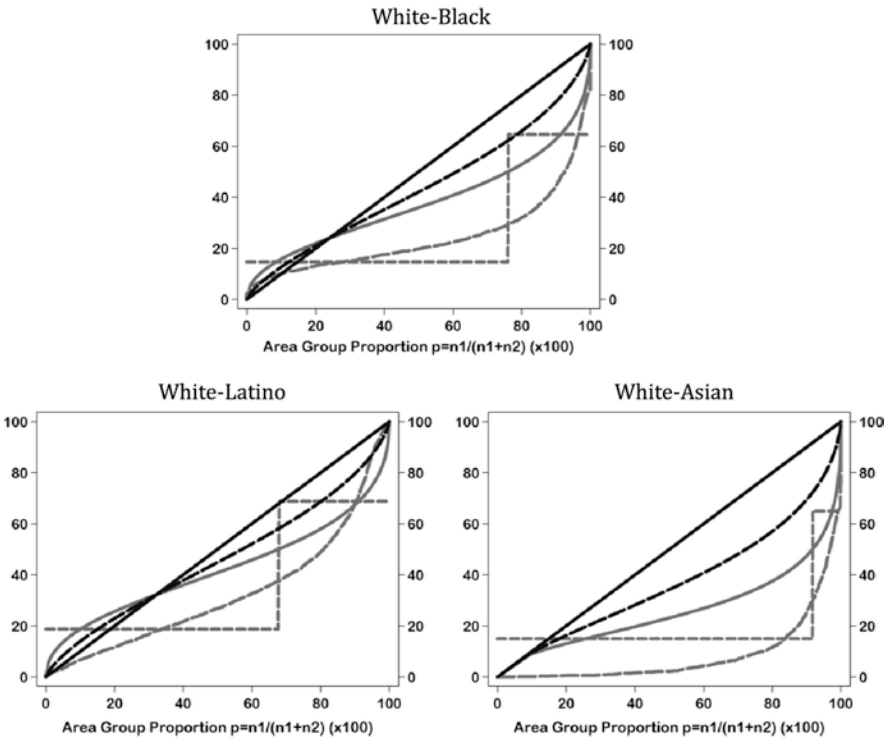

Fig. 5.1 Scoring residential outcomes (y) from pairwise proportion White (p) to compute G, D, R, H, and S as a difference of means. Legend for index-specific curves: y scored for G/2 – *gray, long dashes*; y scored for D/2 – *gray, short dashes*; y scored for R – *gray, solid line*; y scored for H – *dark, long dashes*; y scored for S – *dark, solid line*

gations comparison presented in Table 5.3. These plots provide a basis for gaining insight into how each segregation index registers residential contact outcomes (y) based on pairwise area racial mix (p). I begin with the scores for the separation index (S) because they are the easiest to describe. The scaling function $y = f(p)$ for S maps y directly to the values of p producing a diagonal line rising from (0,0) to (100,100) in all three graphs. As a result, it is very easy to interpret the relationship between y and contact with Whites (p); a one-point change in contact with Whites translates into a one-point change in y. Thus, the graph for the White-Black comparison indicates that a Black family that moves from a 20 % White area to a 70 % White area would experience an increase of 50 points on scaled contact with Whites. The graph for the White-Latino comparison shows that the same would be true for a Latino family moving from a 20 % White area to a 70 % White area and the graph for the White-Asian comparison shows that the same would be true for an Asian family moving from a 20 % White area to a 70 % White area. This similarity of change in y by change in p is not observed for the other indices because their scaling functions are nonlinear.

The scaling function $y = f(p)$ for the Theil index (H) converts values of p to values of y that fall on a smooth, ever-rising, backwards "S-curve". In these graphs the departure from nonlinearity is not dramatic, especially in comparison to what will be seen for some other indices. Accordingly, the values of residential attainment scores (y) relevant for H tend to be relatively close to residential attainment scores (y) relevant for S. This provides a new insight to why scores for the separation index (S) tend to correlate more closely with the scores of the Theil Index (H) than with the scores of other indices. Looking across the three segregation comparisons one can see that the nonlinearity is most pronounced in the White-Asian comparison and least pronounced in the White-Latino comparison. This is because nonlinearity in the y-p relationship for residential attainment scores (y) relevant for H will be less pronounced when the two groups in the comparison are more equal in size and more pronounced when one group is substantially larger than the other. As a result, residential outcomes scores (y) for H and S tend to track each other more closely when the two groups in the comparison are comparable in size and less closely when the groups are unequal in size.

The nonlinearity in the y-p relationship for H just described has another implication. It means that a change of a fixed amount in contact with Whites (p) will translate in different amounts of change in y for H depending on two factors; the initial starting value of p and relative size of the two groups. Thus, inspection of the three graphs in Fig. 5.1 indicates that a family that moves from an area that is 20 % White area to an area that is 70 % White area would experience an increase of 35.9 points on scaled contact with Whites (y) in the White-Black comparison, 36.9 points in the White-Latino comparison, and 31.8 points in the White-Asian comparison. The change in scaled contact for the White-Asian comparison is smallest because the White-Asian group size comparison is the most imbalanced. This leads to greater nonlinearity in the y-p relationship and smaller changes in y when moving from 20 to 70 on p. In contrast, the White-Latino group size comparison is the most balanced of the three and leads to milder nonlinearity in the y-p relationship and larger changes in y as p moves from 20 to 70.

In each group comparison the changes in y as p moves from 20 to 70 are smaller than the 50 point increase in y observed for S for the same group comparisons. This is because the y-p relationship is linear for S and nonlinear for H. The nonlinearity in the y-p relationship for H creates a large region in the middle portion of the range of p where the slope of the curve is less than 1.0 and thus changes in y are smaller than changes in p.[2] In addition, the degree to which changes in y are smaller than changes in p varies across the three segregation comparisons because the nonlinearity in the y-p relationship varies; specifically, the departure from linearity is more pronounced when the two groups in the comparison are more unequal in size and thus changes in y over the middle range of p are smaller in these group comparisons.

The function $y = f(p)$ for the Hutchens index (R) also generates values of y that fall on a smooth, ever-rising, backwards "S-curve". The curve is similar in form to the curve seen for the Theil index (H). But the nonlinearity in the curve for R is noticeably more pronounced. Accordingly, the patterns for the scoring of y for R are similar to those just noted for H, but "amplified". For example, as with H, changes of a fixed amount in contact with Whites (p) translate into different impacts on y depending on the initial starting value of p and relative size of the two groups. Thus, the graphs in Fig. 5.1 indicate that a family that moves from an area that is 20 % White area to an area that is 70 % White area would experience an increase of 24.2 points on scaled contact with Whites (y) in the White-Black comparison, 25.7 points in the White-Latino comparison and 18.3 points in the White-Asian comparison. The changes in y are even smaller than the changes in y noted for H because the departure from linearity in the y-p relationship for R is greater. This "flattens" the y-p curve over the middle range of p even more and causes changes in y to be smaller than changes in p. As seen with H, the changes in y vary across the different segregation comparisons; they are larger when groups are more equal in size and smaller when groups are more unequal in size.

The function $y = f(p)$ for the gini index (G/2) also produces an ever-rising, backwards "S-curve". However, in contrast to the functions for H and R, this curve is irregular rather than smooth. This is because G tracks percentile scores for p and these depend not on the specific value of contact with Whites (p) itself, but instead on how values of p translate into rank position on contact with Whites. In the case of White-Black segregation, for example, this is determined by the number of Whites and Blacks living in areas where p higher and the number of Whites and Blacks living in areas where p is lower. The nonlinearity of the function for G/2 is more pronounced than that seen for the functions for H and R and this produces larger departures from the diagonal line for S. As a result, it is reasonable to say that scoring y as the percentile transformation of p is as the most "dramatic" rescaling of contact of those considered here. Thus, the graphs in Fig. 5.1 indicate that a family

[2] For example, from inspection of the figure for the White-Black comparison changes in y are smaller than changes in p for the portion of the curve between approximately 15 and 85 on p.

that moves from an area that is 20 % White area to an area that is 70 % White area would experience an increase of 13.2 points on scaled contact with Whites (y) in the White-Black comparison, 27.7 points in the White-Latino comparison, and 6.4 points in the White-Asian comparison. In each case, the changes in y are even smaller than the changes in y seen for H and R because the pronounced nonlinearity in the y-p relationship for G "flattens" the y-p curve over the middle range of p quite dramatically causing changes in y to be much smaller than changes in p. As observed previously for H and R, the changes in y vary across the different segregation comparisons with changes being larger when groups are more similar in size and smaller when are more unequal in size. Thus, the change in y for the White-Latino comparison, where the two groups are more similar in size, is more than four times larger than the change in y for the White-Asian comparison where the two groups are more unequal in size.

In contrast to S, H, R, and G, the scoring of y for the index of dissimilarity (D) is not ever-rising as p increases. Instead, it follows a simple, two-value, monotonic step function. The scoring of y for D/2 shown in the graphs draws on the formulation of D as a version of G computed from a two-category ranking scheme with areas where $p \geq P$ being in the higher ranking category and all other areas being in the lower ranked category. For example, in the White-Black comparison, y is scored 14.6 when $p < P$ and 64.6 when $p \geq P$.[3] The scoring of y for D could alternatively be shown as a step function where values of y are either at 0 or 100 depending on whether p is above P or not. But I present the D/2 formulation here to facilitate the comparison of D with G.

The step function for D/2 produces a rescaling of contact that responds to changes in p only when p crosses from being below P to equaling or exceed it. As the graph in Fig. 5.1 indicates, this does not occur when a family moves from an area that is 20 % White area to an area that is 70 % White area in the White-Black comparison. So a family making this move would experience no change in scaled contact with Whites (y); y is 14.6 when p is 20 and y remains at this value when p is 70. The same is true in the White-Asian comparison. In contrast, the change in y for a family making a comparable move in the White-Latino comparison would be 50.0 points (the maximum possible change under the D/2 formulation).

These results highlight two things about D. They highlight that D responds to changes in p only when p crosses a specific value and otherwise D is insensitive to changes in p. The examples also highlight that the value of p that D responds to differs from one segregation comparison to another based on group size. Thus, when groups are equal in size, D responds to changes in p at 50 % White and when the minority group is smaller in size, D responds to changes in p at increasingly higher levels. Thus, the 50 point change in y occurs when p crosses from below to above

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[3] The value of 14.6 indicates that the areas in the lower ranking category contain 29.2 % of the households in the analysis and thus have an average percentile score of 14.6. The value of 64.6 is based on the average percentile score for the 70.8 % of households that are in the higher ranking category; that is, $29.2 + 70.8 / 2$.

68.0 in the White-Latino comparison, from below to above 76.2 in the White-Black comparison, and from below to above 91.8 in the White-Asian comparison.

5.2 Implications for Sensitivity to Separation and Polarization

The patterns just reviewed provide an intuitive basis for comparing indices of uneven distribution and placing them on a continuum. One end of the continuum is anchored by the separation index (S). The y-p relationship for S is linear. So it registers group differences in pairwise contact (p) in its original metric. This is well-suited for measuring group separation and neighborhood polarization. If the group means on p differ by a large amount, it follows that groups live apart from each other with members of each group living in neighborhoods where their group predominates. If the group means on p are similar, it follows that the groups live together, not apart, and thus share similar neighborhood outcomes on pairwise racial mix (p).

The other end of the continuum is anchored by the gini index (G). The y-p relationship for G is profoundly nonlinear. This is because it does not register group differences in pairwise contact (p) in its original metric. Instead, the scoring function instead converts the level of actual contact into a score for rank order position via the percentile transformation. This is well-suited for measuring ordinal differences in group contact. But it is ill-suited for measuring group separation and neighborhood polarization. Accordingly, if group means on percentile scores (y) based on pairwise group contact (p) differ by a large amount in White-Minority comparisons, one can safely conclude that Whites consistently live in neighborhoods that rank higher on proportion White than do minorities. But, one cannot conclude that the minority group lives apart from Whites in neighborhoods where the minority group predominates. This is because percentile scores logically cannot provide reliable signals about underlying quantitative differences. As a result, percentile scoring of pairwise group contact cannot provide a reliable basis for assessing group residential separation and neighborhood polarization.

This is not an esoteric point. I will present empirical analyses in the next chapter that demonstrate that high scores on G can and do occur when group residential separation and neighborhood polarization is low, and in some cases even trivial. Ultimately, researchers should decide for themselves if they view this quality of G as desirable, undesirable, or irrelevant. But to decide, they first must become aware that G has this quality. In the main they are not aware and this is understandable because the issue receives little attention in methodological discussions in the literature. As a consequence, no one has set forth a well-articulated rationale for prioritizing group differences in rank order position on contact over the group differences in quantitative "raw score" standing on contact.

The remaining three indices of uneven distribution considered here – the index of dissimilarity (D), the Hutchens square root index (R), and the Theil entropy index

(H) – fall in intermediate positions on the continuum between the gini index (G) and the separation index (S). Not surprisingly, D is closest to G. R and H fall in between with R closer to D and H close to S. The basis for this ordering is suggested by the y-p relationships for the indices depicted in the graphs in Fig. 5.1. G is at the opposite end of the continuum from S because its y-p relationship is most profoundly nonlinear – resulting due to the fact that the percentile scoring of y from p often produces scores for y that depart dramatically from the original value of p. The dissimilarity index (D) is closest to the gini index (G) because D can be understood as a crude version of G based on a two-category ranking scheme. This is indicated visually by the fact that the step-function "curve" for the y–p relationship for D overlays the "finer-grained" steps in the y-p curve for G seen in the figures.

The Hutchens square root index (R) falls near the index of dissimilarity (D) based on the fact that the y-p curve for R is closer to linear than the y-p curve for G but is more nonlinear than the y-p curve for the Theil entropy index (H). Perhaps this should not be surprising since Hutchens (2001) notes that R has the quality of ranking segregation comparisons in accord with the principle of segregation curve dominance. Since the segregation curve is a graphical depiction of rank order differences, it makes sense that R is more sensitive to group differences in rank order standing on group contact than to group differences in quantitative standing on contact.

The y-p relationship for the Theil entropy index (H) displays only mild departure from linearity and thus produces curves that align more closely with the linear y-p curve for the separation index (S). On this basis, one can infer that H is more sensitive to group residential separation and neighborhood polarization than every other index except S.

Each of the indices of uneven distribution considered here – G, D, R, H, and S – have been endorsed in methodological studies.[4] And each has been adopted by researchers who have seen the index as having qualities that are attractive for the purposes of the studies they were undertaking. The discussion here provides one additional basis for choosing among indices – sensitivity to group differences in rank order standing on group contact or sensitivity to group differences in contact measured in its "natural" metric. This can also be cast in terms of sensitivity to group residential separation and neighborhood polarization because this follows differences in actual contact, not differences in rank order position on contact.

If one is interested in identifying "prototypical segregation" as seen in traditional exemplars such as White-Black segregation in Chicago and White-Latino segregation in Los Angeles, the separation index (S) is a logical choice and the Theil entropy index (H) would be the next best choice. The basis for choosing S is this.

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[4] Of these, D is viewed as most problematic on technical grounds, but it usually receives a "conditional pass" because its technical deficiencies often do not have important practical consequences. For example, in empirical studies it typically correlates very closely with G, its close and technically superior cousin.

High values on S always signal a high level of group residential separation and neighborhood polarization of the kind featured in didactic discussions of examples of pronounced segregation.

This is not strictly the case for high values on H. But the relatively close relationship of y scored for H with y scored for S (i.e., with p), dictates that high scores on H are very likely, albeit not necessarily guaranteed, to involve a high level of group separation and neighborhood polarization. In contrast, the other three indices – R, D, and G – are not reliable in signaling the presence of prototypical segregation that involves group separation and neighborhood polarization.

If one is interested in identifying segregation assessed strictly on rank-order standing on group contact (p) as registered by the segregation curve, S and H are not good choices. The gini index (G) and the Hutchens square root index (R) would be the superior choices on technical grounds and the dissimilarity index (D) would be an attractive choice based on past usage, ease of computation and interpretation, and related practical considerations.

A few simple questions can help frame the issues researchers confront when they choose to give priority to one index over others. One is "Do the theories and substantive concerns motivating analysis of segregation lead one to naturally focus on prototypical segregation which involves substantial area racial polarization and clear group differences in quantitative levels of contact or do they lead one to instead focus on group differences in rank order standing on contact?" If the substantive focus is on rank order standing, one should be able to explain why high scores of 76.3 and 58.2 on G and D, respectively, for the White-Asian comparison are sociologically important in light of the low score of 23.9 on S. The low score on S, as well as the component group means on contact with Whites that determine it, document that White-Asian segregation in Houston is not "prototypical" segregation. White-Asian segregation does not involve substantial group separation and neighborhood polarization; Asians are more than twice as likely to live with Whites (mean pairwise contact is 69.9 %) as with Asians (mean pairwise contact is 31.1 %).

In contrast, G and D for White-Latino segregation – at 74.2 and 58.4, respectively – take values comparable to those observed for White-Asian segregation, but White-Latino segregation is more in keeping with prototypical segregation. In contrast to Asians, Latinos are much less likely to live with Whites; Latino pairwise contact with Whites is only 40.2 % while Latino pairwise contact with Latinos is 59.8 %. As a result, the score of 40.1 on S for White-Latino segregation indicates that group separation and neighborhood polarization is nearly twice as high in the White-Latino comparison as in the White-Asian comparison. Similarly, G, D, and S are 87.1, 71.0, and 57.4, respectively, for the White-Black comparison. The values of G and D are only 10.8 and 13.4 points higher, respectively, than the values observed for the White-Asian comparison. But the value of S is some 33.5 points higher and is more than double the value of S for the White-Asian comparison. The component terms of S for the White-Black comparison indicate clearly that this is "prototypical" segregation involving substantial group separation and neighborhood racial polarization. Consistent with this, *both* Whites and Blacks live apart in neighborhoods where their group predominates. White pairwise contact with Whites is 89.9 % and Black pairwise contact with Blacks is 68.5 %. The level of same group contact for Blacks is more than double the level of 31.1 % seen for Asians. In sum, G and D suggest that all three segregation comparisons are fairly similar. S suggests White-Asian segregation is distinctively different from White-Latino and especially White-Black segregation.

Figure 5.1 clarifies why G and D yield high scores for White-Asian segregation when S does not. It is because G and D assign great importance to group differences on p that have minimal impact on S because they are quantitatively small. S takes a relatively low value of 23.8 because Asian pairwise contact with Whites, while not reaching the level of 93.8 % seen for Whites, is nevertheless quite high at 69.9 %. To calculate G, values of p are converted to percentile scores and the group difference is then doubled.[5] While the group means for p do not necessarily map exactly to the group means for percentile scores (because the percentile transformation is nonlinear), it instructive to note that the values of 93.8 and 69.9 for p translate to percentile score values of 36.3 and 6.8, respectively. Taking twice the difference to obtain the implications for G yields the value of 59.0. Thus, the initial modest difference on p that produces a value of 23.8 points for S translates to an implied difference of 59.0 points for G. This is actually less than the observed value of G of 76.3 which means that the exaggeration of group differences on p is consistently larger than this particular calculation suggests.

Applying this same exercise to the group difference of medians also is "instructive." The group medians on p are 97.5 for Whites and 76.7 for Asians. This yields a group difference at the medians of 20.8 (which is close to the difference in group means of 23.8). These values of p translate to 53.9 and 9.7, respectively, when converted to percentile scores. When this difference is doubled to obtain the implications for G specified as a difference of group medians, the result is 88.4. So the original quantitative difference in "typical" residential outcome of 20.8 when p is measured in its original metric grows to more than four times that size when p is rescaled by the percentile transformation curve shown in Fig. 5.1.

A similar pattern is observed when values of p are converted from their original metric to the 0 or 100 scoring scheme used for D. The values of 69.9 and 76.7, which represent the mean and median, respectively, for Asians on p become 0.0. In contrast, the values of 93.8 and 97.5, which represent the mean and median, respectively, for Whites on p become 100.0. Thus, the original group differences at these points of comparison – 23.8 points at the group means and 20.8 points at the group medians, expand to the maximum possible difference of 100.0.

The point to take away is simple, but important. The rescaling of p from its original metric, which determines S, to the scaled contact scores for y that determine G and D serves to exaggerate small quantitative differences on p. Accordingly, values of G and D are usually larger and are never smaller than values of S.[6] Furthermore,

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[5] This is because the maximum possible group difference on percentile scores is 50.

[6] Additionally, since D is a crude version of G based on a three-point segregation curve instead of the full segregation curve, G is almost always higher and is never lower than D.

the degree to which the rescaling exaggerates quantitative differences on p is greater when groups are unequal in size as seen in the White-Asian comparison. Accordingly, the G-S and D-S discrepancies can be especially large in such comparisons.

This raises the question, "Why is it appropriate to score y in a way that dramatically amplifies group differences in contact with Whites as observed in this example?" Relatedly, "In what way is the exaggerated difference of 59.1 points on y scored for G and 100.0 points for y scored for D more sociological meaningful than the smaller difference of 23.8 points for y scored for S?" Perhaps compelling answers to these questions can be given. For now, however, the measurement literature does not provide a ready answer and I am skeptical that a compelling answer can be advanced. Regardless, it will remain the case that in these segregation comparisons examining S and its component terms reveals important information that would be missed if one looked only at G and D. Specifically, S documents that White-Asian segregation does not involve group residential separation and neighborhood polarization whereas White-Latino segregation and especially White-Black segregation do. The practical implication is straightforward; one cannot safely assume that high values of G and D indicate a prototypical pattern of segregation. One must also examine S to draw a safe conclusion on this issue.

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Chapter 6
Empirical Relationships Among Indices

In this chapter I present analyses that document various aspects of the empirical relationships among the segregation indices examined in this study. I document both situations where the indices consistently agree and also situations where they often disagree. I then offer observations on what may be learned from considering these two situations. In addition, I use portions of the chapter to review several practical issues researchers may want to consider when using the indices in empirical studies.

I start by reviewing results from a large, comprehensive data base of index scores for White-Minority segregation comparisons. More specifically, the data base contains segregation scores for White-Black, White-Latino, and White-Asian comparisons for 960 core-based statistical areas (CBSAs). CBSAs are constructed from counties. I applied the 2010 definitions to data from 1990 to 2000 to obtain index scores using constant area boundaries at these three points in time. The full data set includes index scores computed using data for three different spatial units – census blocks, census block groups, and census tracts. I focus primarily on the scores computed using block-level data because block groups and census tracts are too large to use for assessing segregation in smaller CBSAs.

Massey and Denton (1988:299) note that multiple options for areal units can be conceptually defensible. Citing prior research by Duncan and Duncan (1955b) and Taeuber and Taeuber (1965) as well as drawing on their own experiences, Massey and Denton also note that, while index scores consistently run higher when segregation is calculated using smaller areal units, block-based and tract-based index scores tended to correlate closely in the studies they considered. This suggests that findings regarding patterns in cross-city variation in segregation and trends over time in segregation will tend to be similar whether using scores computed from tracts, block groups, or blocks. However, an important qualification must be noted on this point. It is that these findings are based on studies using a relatively small number ($N \approx 60$) of large metropolitan areas and the findings do not hold in broader data sets. Thus, I obtain similar findings as reported in these earlier studies when I restrict the

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_6

analysis here to include only the largest metropolitan areas. However, I find that the choice of spatial unit is much more consequential when I use the full data set which includes hundreds of smaller metropolitan CBSAs and micropolitan CBSAs.

The reason choice of spatial unit matters more in broader samples is simple; tracts are too large to reveal segregation patterns in smaller CBSAs. Indeed, the number of tracts in micropolitan CBSAs is often very small – sometimes falling to single digits. As a result, tracts are not viable units for assessing segregation in smaller communities; tracts consistently yield low scores when closer inspection of residential patterns reveals that segregation is clear and pronounced. In contrast, census blocks can reliably detect segregation patterns in all CBSAs regardless of size. The difference between index scores based on tracts and index scores based on blocks is consistently much larger in small- and medium-sized CBSAs. Accordingly, I use scores based on block data in analyses involving the full range of metropolitan and micropolitan CBSAs. When I use scores based on tract or block group data I restrict analysis to include only large metropolitan CBSAs.

Index scores for my full CBSA analysis data set are based on block-level group population counts obtained from Summary File 1 in 2000 and 2010 and from the PL-94 (voter redistricting) File for 1990. The data for Whites, Blacks, and Asians do not include Latinos and the data for Latinos include persons of all races. The analyses reported here are based on 4,319 White-Minority comparisons for CBSAs where both groups in the segregation comparison have overall population counts of at least 1,500. In all there are 1,718 White-Black comparisons, 1,754 White-Latino comparisons, and 847 White-Asian comparisons.

Table 6.1 provides descriptive statistics summarizing the distributions of index scores for G, D, R, H, and S obtained for each of the three White-Minority comparisons. Several patterns stand out in the results. One is that scores for G and D consistently run higher than scores for R, H, and S. This is evident when comparing values at the mean and also at the five quantile values examined. A related pattern is that scores for R, H, and S are relatively similar at the median and above (i.e., at P_{50}, P_{75}, and P_{90}), but scores for H and especially S are noticeably lower below the median (i.e., at P_{25} and especially at P_{10}). The analyses reported in the previous chapter provide a basis for understanding both of these patterns. S typically generates smaller group differences on contact with Whites because S registers the original untransformed pairwise contact scores (p). In contrast, G, D, R, and H subject the original or "raw" contact scores (p) to a nonlinear rescaling that consistently serves to exaggerate group differences in contact with Whites when the original "raw-score" contact differences are small (i.e., when average values of p are relatively high for both groups) and S is likely to take a low value. As noted in the previous chapter, the nonlinearity in the y-p scaling function is more dramatic for G and D. This causes their scores tend to consistently run somewhat higher than the other indices. One practical implication of these findings is that one should keep these inherent "scale" differences in index values in mind when making comparisons across different indices. For example, as a rule of thumb, I suggest the three- and four-category schemes for characterizing levels of segregation in broad categories in Fig. 6.1.

Table 6.1 Descriptive statistics for indices of uneven distribution for White-Minority comparisons for CBSAs for 1990, 2000, and 2010

	N of cases	Mean	SD	IQR	IDR	P_{10}	P_{25}	P_{50}	P_{75}	P_{90}
Gini Index (G)										
White-Black	1,718	86.8	6.9	8.4	16.8	77.7	83.3	88.1	91.7	94.5
White-Latino	1,754	76.1	9.0	12.2	23.7	63.2	70.6	77.3	82.7	87.0
White-Asian	847	79.8	7.5	10.8	19.6	69.1	74.8	80.9	85.6	88.6
Dissimilarity Index (D)										
White-Black	1,718	71.5	8.6	10.9	21.8	60.3	66.2	72.1	77.2	82.1
White-Latino	1,754	59.3	9.3	12.8	24.6	46.6	53.1	59.7	65.8	71.3
White-Asian	847	64.1	8.8	12.6	23.9	52.0	57.8	64.6	70.4	75.9
Hutchens Square Root Index (R)										
White-Black	1,718	51.5	10.9	14.5	28.2	37.5	44.1	51.8	58.7	65.8
White-Latino	1,754	36.6	10.6	15.4	28.0	22.6	28.6	36.5	44.1	50.5
White-Asian	847	42.5	10.3	14.7	27.8	28.0	35.3	43.0	50.0	55.8
Theil Entropy Index (H)										
White-Black	1,718	48.9	12.9	18.0	35.0	30.3	39.9	49.4	58.0	65.3
White-Latino	1,754	32.6	9.1	12.7	22.9	21.1	26.0	32.2	38.7	44.0
White-Asian	847	30.7	6.8	8.4	16.5	22.6	25.9	30.4	34.4	39.1
Separation Index (S)										
White-Black	1,718	43.5	18.3	28.1	49.7	16.1	29.4	46.4	57.5	65.8
White-Latino	1,754	25.6	12.3	19.2	33.2	9.6	15.4	24.8	34.6	42.8
White-Asian	847	15.7	8.4	9.9	20.4	7.7	9.7	13.2	19.6	28.1

Source: Index scores are calculated use block-level data from U.S. Census summary files. Comparisons are excluded if the minority group total population is under 1,500. SD is standard deviation, IQR is interquartile range, IDR is interdecile range, and P_{10}-P_{90} are selected percentiles

Level	G	D	R	H	S
Three Broad Categories					
High	80-100	65-100	50-100	50-100	50-100
Medium	50-79	35-64	20-49	20-49	20-49
Low	0-49	0-34	0-19	0-19	0-19
Four Broad Categories					
Very High	85-100	70-100	60-100	60-100	60-100
High	65-84	50-69	35-59	35-59	35-59
Medium	50-64	30-49	15-34	15-34	15-34
Low	0-64	0-29	0-14	0-14	0-14

Fig. 6.1 Suggested schemas for placing index scores within broad groupings for levels of segregation

Regarding group comparisons, all five indices suggest that White-Black segregation is consistently higher than both White-Latino segregation and White-Asian segregation. Index scores are higher for the White-Black comparison at the mean and at every quantile listed in the table. Interestingly, the absolute and relative differences in how scores vary across group comparison are smallest for G, which has the highest scores on average, and they are largest for S, which generally takes much lower scores. The magnitude of the differences across group comparisons for D, R, and H fall in between the larger differences seen for S and the smaller differences seen for G. When comparing median values, the maximum difference across group comparisons is 10.8 points for G, 12.4 points for D, 15.3 points for R, 19.0 points for H, and 33.2 points for S.[1]

The indices tell a less consistent story regarding the comparison of White-Latino segregation and White-Asian segregation. G indicates the two are roughly similar but with White-Asian segregation being slightly higher. D and R clearly indicate that White-Asian segregation is higher. H indicates the two comparisons are similar but with White-Latino segregation being slightly higher. In contrast, S indicates that White-Latino segregation is considerably higher than White-Asian segregation. At both the mean and the median, S for the White-Latino comparison is higher by at least 10 points than S for the White-Asian comparison and the mean and median for S for the White-Latino comparison is at least double the level of S for the White-Asian comparison.

Close inspection of the underlying distributions of residential outcomes reveals general patterns similar to those seen in the example for Houston, Texas discussed earlier. Specifically, S is higher for the White-Black comparison because the White-Black segregation routinely involves high levels of group separation and neighborhood polarization and S is lower for the White-Asian comparison because White-Asian segregation almost never involves even moderate levels of group separation and neighborhood polarization. White-Latino segregation stands in between; it routinely involves moderate levels of group separation and polarization and occasionally involves high levels. The level of White pairwise contact with Whites across CBSAs is very high in both of these White-Minority comparisons; for example, at the median it is 94.4 % for White-Black comparisons, 94.5 % for White-Latino comparisons, and 97.9 % for White-Asian comparisons. Thus, the difference in S across the different White-Minority comparisons arises primarily due to differences in the levels of pairwise contact Blacks, Latinos, and Asians have with Whites. For Blacks the median for pairwise contact with Whites across CBSAs is 46.6 %, for Latinos it is 68.0 %, and for Asians it is 84.4 %. The "flip" side of these values – that is, average pairwise same-group contact for the minority group – tells a similar story. It averages 15.6 % for Asians, 33.0 % for Latinos, and 53.4 % for Blacks.

Taken together, these results reveal that residential separation from Whites is low for Asians, moderate for Latinos, and high for Blacks. Recall that, for separation and polarization to be high, *both* groups in the comparison must reside in neighborhoods where their group predominates (i.e., when both have high levels of pairwise

[1] Comparisons at the means of the distributions yield similar patterns.

same-group contact). This is why, in sharp contrast to overall or pairwise isolation, separation and polarization are independent of city racial composition. Under even distribution, an imbalanced racial mix for the city will cause one group to experience a high level of same-group contact but it also will cause the smaller group to experience a low level of same group-contact. So, regardless of city ethnic composition, segregating forces must be operating for *both* groups to have high-levels of same-group contact. The results just reviewed indicate that Whites consistently have high-levels of (pairwise) same-group contact. This is not simply due to city racial composition. If it was merely a function of racial composition, Blacks, Latinos, and Asians also would experience high levels of contact with Whites when same-group contact is high for Whites. But the reality is that same-group contact for *both* groups is above the level expected under even distribution.

Other indicators (not reported in the table) further confirm that White-Black segregation routinely involves substantial group residential separation and neighborhood polarization while White-Asian segregation almost never does and the pattern for White-Latino segregation falls in between. One such indicator is whether at least half of the population in *both* groups in the comparison lives in a neighborhood where their group constitutes at least 60 % of the population. This outcome can never occur under even distribution under any city racial composition. So when it is observed, it is a clear sign that segregation dynamics have produced group separation and neighborhood polarization. This result is seen in 44.5 % of White-Black comparisons, 11.8 % of White-Latino comparisons, and only 1.5 % of White-Asian comparisons. Thus, clear separation and polarization is rare for White-Asian segregation and uncommon for White-Latino segregation but common for White-Black segregation.

6.1 When Do Indices Agree? When Can They Disagree?

Table 6.2 presents simple and squared correlations among the scores of the indices for White-Minority segregation comparisons for CBSAs in 1990, 2000, and 2010 previously reported in Table 6.1. Squared correlations are reported above the diagonal and are in bold typeface. Simple linear correlations are reported below the diagonal. As noted earlier the full analysis data set includes a total of 4,319 White-Minority segregation comparisons where the minority population was 1,500 or more. Due to this large sample size all of the correlations reported in the table are statistically significant at conventional levels and so statistical significance is not specifically noted in the table. As a last preliminary comment, note that the table includes correlations for scores for the symmetric version of the Atkinson index ($A_{[0.5]}$) as an added point of comparison.[2]

[2] This is primarily to document that A is an exact function of H, which is less well known to sociologists.

Table 6.2 Relationships among indices of uneven distribution for White-Minority segregation comparisons in CBSAs in 1990, 2000, and 2010[a]

	All cases (N = 4,319)					
	G	A	D	R	H	S
G – Gini Index	1.0000	***0.9671***	***0.9679***	***0.9355***	***0.7982***	***0.3031***
A – Atkinson Index ($A_{[0.5]}$)	0.9834	1.0000	***0.9673***	***0.9793***	***0.6277***	***0.2170***
D – Dissimilarity Index	0.9838	0.9835	1.0000	***0.9692***	***0.6838***	***0.2709***
R – Hutchens Index	0.9672	0.9896	0.9845	1.0000	***0.6739***	***0.2520***
H – Theil Index	0.8934	0.7923	0.8269	0.8209	1.0000	***0.8181***
S – Separation Index	0.5505	0.4658	0.5205	0.5020	0.9045	1.0000

[a]Squared correlations are reported above the diagonal (in *bold*, *italic*). Index scores are computed using block-level data from U.S. Census Summary File 1 and PL-94. Cases are for White-Black, White-Latino, and White-Asian segregation comparisons excluding CBSAs where the total minority population is under 1,500

The results in the table document several interesting findings. One is that scores for indices that are related to the segregation curve – namely, G, A, D, and R – correlate very closely.[3] The associations among G, A, and D are particularly high. The lowest simple linear correlation among them is 0.984 and the lowest squared correlation is 0.967. Correlations of R with A and D also are very high. The correlation of R with G appears to be lower with a squared correlation of 0.936 but closer inspection reveals that G and R have a very close relationship that is mildly nonlinear. This is not surprising as R has an exact nonlinear relationship with A, specifically $A = (2R - R^2)$, which in turn has a close linear relationship with G.

Figure 6.2 provides graphical depictions of the associations among indices reported in Table 6.2. The scatterplots make it clear that relationships among these four indices – G, A, D, and R – are exceedingly close, even closer than the high correlations suggest if one takes account of the mild nonlinearities in several of the relationships. Indeed, in any pair combination, the multiple squared correlations for predicting the values of any one index based on the value of one of the other indices plus either its square or its square root (depending on the index combination) exceeds 0.969 in all cases. These close associations reflect the fact that the G, A, D, and R all assess segregation outcomes consistent with the principle of segregation curve dominance. As noted earlier, this means that all of these indices are geared to registering group differences in rank order standing on pairwise contact with Whites (p).

The results reported in Table 6.2 also document a second important finding; the correlations involving H and S are lower than the correlations observed among G,

[3] Specifically, G, A, D, and R satisfy the principle of "segregation curve dominance" which means that when comparing two cases the index will indicate that segregation is lower for a case if its segregation curve is somewhere above and nowhere below the segregation curve for the other case.

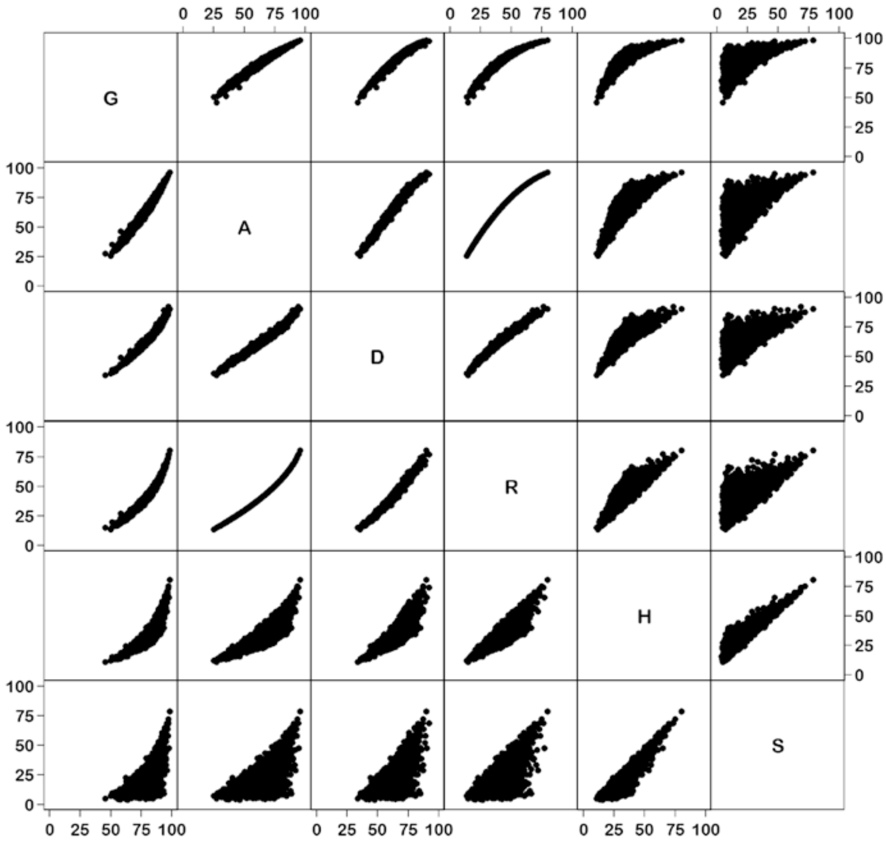

Fig. 6.2 Scatterplots depicting relationships among indices of uneven distribution for White-Minority segregation comparisons in CBSAs in 1990, 2000, and 2010 – full analysis sample (Note: scores are for White-Black, White-Latino, and White-Asian segregation comparisons computed using block-level data from U.S. Census summary files)

A, D, and R. Unlike G, A, D, and R, H and S are not related to the segregation curve. It is perhaps not surprising then that H and S are more strongly associated with each other (squared correlation of 0.818) than with the other indices. The H-S scatterplot in Fig. 6.2 documents that the correspondence between H and S is close at high values but is weaker when one of the indices takes a lower value. This accounts for why the correlation between H and S is not as high as those seen among G, D, A, and R. Generally, but not always, scores for H run higher than scores for S. This tendency is more pronounced when S is in the low-to-moderate range (e.g., below 40). The squared correlations of H with G, A, D, and R are not as high as the squared correlation of H with S; but they are moderately strong and run from a low of 0.628 to a high of 0.798. The squared correlations of S with G, A, D, and R are much lower across the board. They run from a low of 0.217 to a high of only 0.303.

Figure 6.3 presents selected scatterplots from Fig. 6.2 to highlight particular results. It shows that the correspondence of H with G, D, and R is relatively close at high and low values of H, but it is looser in the mid-ranges of H. In the case of the relationship of R with H, values of R rarely fall more than a few points below values of H; less than ten percent of cases are lower by more than five points and none are lower by 10 points. However, in the low-to-middle ranges of H (i.e., 25–50), the values of R often are substantially higher than the values of H with R exceeding H by more than 10 points in over a quarter of cases. In the case of the relationship of D with H, values of D always are well above values of H and again it is evident that the D-H discrepancies are largest in the low-to-middle range of H (i.e., 25–50). A similar pattern is seen in the relationship of G with H. Values of G always are well above values of H and the G-H discrepancies tend to be largest in the lower middle range of H (i.e., 20–40).

S has a close correspondence with G, D, and R only when values of S are high-to-very high. When values of S are not high, the relationships between S and these three indices are weak and inconsistent. The reason for this is that values of G, D, and R can and frequently do vary over wide ranges when S is at low-to-moderate values. To be sure, G, D, and R can and sometimes do agree with S and take low-to-moderate values when S takes low-to-moderate values. But G, D, and R also can and often do take high values when the value of S is low.

It is instructive to consider the comparison of S with D. Scores of D are never lower than scores of S, but the amount by which D exceeds S can and does vary dramatically across comparisons. For example, when S is in the range of 15–25, the interdecile range for the difference between D and S is 27.5 points with more than ten percent of scores for D falling below 47 and more than 10 % exceeding 73. Similarly, when S is in the range of 35–45, the interdecile range for the D-S difference is 22.6 with over ten percent of scores for D below 56 and more than 10 % above 78. The patterns for S compared with G are similar. Scores for G are never below D and thus run considerably higher than scores for S. But the amount by which G exceeds S varies greatly. For example, when S is in the range of 15–25, the interdecile range for the difference between G and S is 25.3 points with more than 10 % of scores for G falling below 64 and more than 10 % falling above 88. Similarly, when S is in the range of 35–45, the interdecile range for the G-S difference is 19.0 points with more than 10 % of scores of G falling below 73 and more than 10 % exceeding 91.

The pattern for S compared with R is similar to those just described for D and G but with one difference; scores for R occasionally are lower than scores for S. This is not typical and, when it occurs, R is lower than S only by a small amount. The more important finding is that the values of R, like values of D and G, can vary greatly at a given level of S. For example, when S is in the range of 15–25, the interdecile range for the R-S difference is 31.3 points with over 10 % of scores for R below 23 and more than 10 % above 53. The same variability in scores for R is seen when S is in the range of 35–45. In this situation, the interdecile range for the R-S difference is 29.9 points and it is not uncommon to observe scores of R ranging at or below 29 to at or above 59.

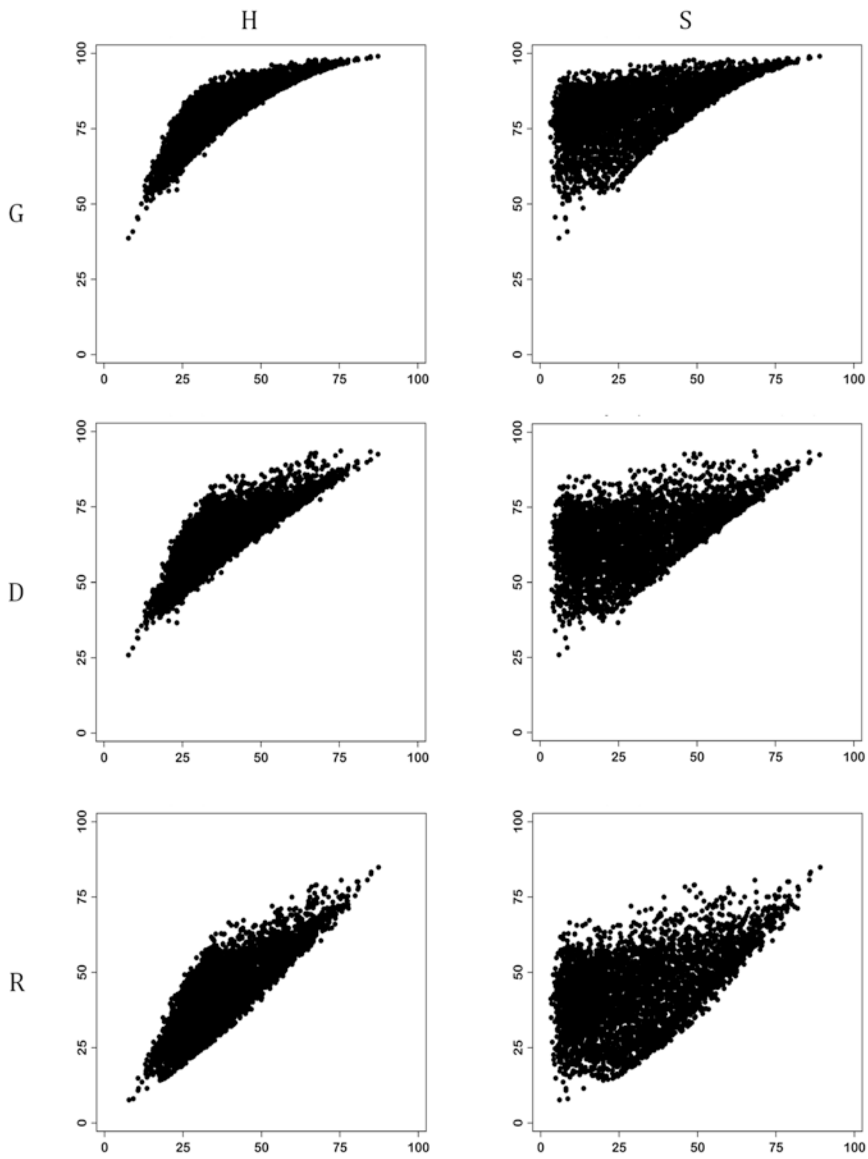

Fig. 6.3 Scatterplots depicting relationships of H and S with G, D, and R for White-Minority segregation comparisons in CBSAs in 1990, 2000, and 2010 – full analysis sample (Notes: scores are for White-Black, White-Latino, and White-Asian segregation comparisons computed using block-level data from U.S. Census summary files)

Summing up, indices that are closely associated with the segregation curve – namely, G, A, D, and R – correlate at high levels with each other, but less so with H and much less so with S, two measures not linked to the segregation curve. These findings depart dramatically from previous findings of high correlations among all indices of uneven distribution. For example, Duncan and Duncan's (1955a) landmark methodological study reported that D, G, and S were correlated at high levels and suggested the correlations were so high that there was little practical benefit to gain from considering measures beyond D which had advantages in ease of calculation and interpretation. More recently, the valuable and influential methodological study by Massey and Denton (1988) similarly reported very high levels of correlation among G, $A_{(0.50)}$, D, H, and S with the lowest correlation among the indices being 0.89 (for the correlation between G and S).

Why are these correlations reported in these previous studies so high when correlations of G, A, D, and R with H and S reported here are moderate-to-weak? The answer traces to basic differences in research design across the studies. Specifically, the difference in findings traces to difference in the samples of cities considered and to differences in the spatial units used when computing segregation scores. Regarding the differences in the samples of cities, the studies by Duncan and Duncan (1955a) and Massey and Denton (1988) both were based on 60 cities consisting primarily of the largest metropolitan areas in the country. Duncan and Duncan examined cities for which tract data had been tabulated in the 1940 census and the sample was primarily, but not exclusively, comprised of the largest metropolitan areas in the country. Massey and Denton developed their analysis sample by first taking the 50 largest metropolitan areas and then including an additional 10 metropolitan areas with large Latino populations. Regarding spatial units, both studies used tract-level data when computing segregation scores. While this is a common practice, it is not well suited for assessing segregation for smaller groups or for assessing segregation in smaller communities. These two aspects of the samples used in the landmark studies by Duncan and Duncan (1955a) and Massey and Denton (1988) tend to minimize differences between measures that emerge in the much broader sample used here. To be clear, the results reported in these earlier studies are not incorrect. But the results reported in these studies do not generalize beyond large metropolitan areas.

I provide evidence to support this conclusion with several analyses. To begin I replicated the analysis reported in Table 6.2 using a subset sample of 58 CBSAs that corresponds as closely as possible to the cities used in Massey and Denton's (1988) study.[4] I found that the correlations among indices obtained using this subsample were consistently higher, often by substantial amounts and were never significantly lower in comparison to correlations using the broader sample. For example, the correlation of D and S using scores computed from block data was 0.5205 in the broader sample and 0.6433 in the Massey and Denton subsample. I then examined

[4] Two cases in the Massey and Denton sample are not included in the subset of cases examined here. In the 2010 CBSA definitions used here their areas of Paterson-Clifton and Jersey City are assigned to the New York-White Plains-Wayne CBSA Division.

Table 6.3 Relationships among indices of uneven distribution by size of combined group populations for White-Minority segregation comparisons in CBSAs in 1990, 2000, and 2010[a]

CBSAs by size of combined group populations (in 1,000 s)							
	All	< 100	100–249	250–499	500–999	1,000–1,999	2,000 or More
Correlation of Dissimilarity Index (D) and Separation Index (S)							
Tracts	0.7678	0.6821	0.7042	0.8108	0.8739	0.9154	0.9355
Block groups	0.7862	0.7467	0.7337	0.8043	0.8634	0.9111	0.9306
Blocks	0.5205	0.6581	0.4099	0.3123	0.4649	0.6979	0.7190
Squared Correlation of Dissimilarity Index (D) and Separation Index (S)							
Tracts	0.5895	0.4652	0.4959	0.6574	0.7637	0.8380	0.8752
Block groups	0.6181	0.5576	0.5383	0.6469	0.7470	0.8301	0.8660
Blocks	0.2709	0.4331	0.1680	0.0975	0.2161	0.4871	0.5170
N of cases	4,319	1,689	1,183	631	392	277	147

[a]Index scores are computed using data from U.S. Census Summary File 1 and PL-94. Cases are for White-Black, White-Latino, and White-Asian segregation comparisons excluding CBSAs where the total minority population is under 1,500

correlations using index scores computed from tract-level data instead of block-level data. The correlations among indices increased by substantial amounts and closely matched the correlations reported in Massey and Denton (1988). For example, the correlation of scores for D and S based on tract-level data in the subsample of cases corresponding to the Massey and Denton sample was 0.9248 and replicates the value of 0.92 reported in Massey and Denton.

These analyses establish that the associations among segregation indices are markedly lower when study designs draw on a broader sample of cities and assess segregation using block data instead of tract data. For example, when computing scores using tract data the squared correlation between D and S is 0.8552 (r = 0.9248) for the Massey and Denton subsample of CBSAs. It drops to 0.5895 (r = 0.7678) when using the broader sample. Both values are much higher than the squared correlation of 0.2709 (r = 0.5205) observed for the broader sample of CBSAs using scores computed from block-level data.

Table 6.3 explores the issue in more detail by reporting the correlation and squared correlation of D and S using subsets of segregation comparisons grouped by the size of the populations in the segregation comparisons (a close correlate of city population size). Correlations are reported separately for index scores based on tract, block group, and block data. Several patterns are clear.

- Correlations are consistently stronger for scores computed using tract data and weaker for scores computed using block data.
- Correlations are stronger for comparisons for CBSAs with populations of 500,000 and even stronger for CBSAs with populations of 1,000,000 or more. This pattern holds generally for scores computed using tract, block group, and block data.

These results support the general conclusion that correlations between indices are consistently weaker when using broader, more heterogeneous samples of cities and when using index scores computed for blocks instead of tracts. As a final check, I replicated these results using alternative versions of index scores that corrected for index bias (discussed in Chaps. 14 and 15), a potential concern when using index scores computed from block-level data. The relevant results were fundamentally similar and strengthen the conclusion I offer here.

I now answer the questions posed in the heading for this section of the chapter, "When do different indices agree?" and "When can they disagree?" The previous discussion provides a preliminary answer. Indices are more likely to agree in studies that focus on large metropolitan areas and compute index scores using tract-level data. Conversely, indices are more likely to disagree in studies that use broader samples and/or compute index scores with block-level data. But why is this so? Two findings provide clues. One is that cities in the Massey and Denton sample have higher levels of relative minority presence and the other is that correlations among indices are consistently higher when the relative size of the minority population is larger. Among the CBSAs segregation comparisons that meet the criterion of having at least 1,500 in population for the minority group, relative minority presence is consistently higher in the subset of CBSAs in the Massey and Denton subsample and this is true for all three White-Minority comparisons considered.

This is consequential because correlations among indices are higher when pairwise minority group proportions are moderate-to-high.[5] Evidence for this is presented in Fig. 6.4 and in Table 6.4. Table 6.4 is organized in three panels. The top panel gives correlations among index scores computed from block-level data for the subset of White-Minority segregation comparisons where the two groups in the comparison are similar in relative size; specifically, these are the subset of 510 segregation comparisons where the pairwise proportion for the smaller group in the comparison is in the range of 0.30–0.50. The key finding documented here is simple and compelling; the correlations among all of the indices are extremely high. The weakest relationship observed is between G and R with a simple linear correlation of 0.9697 and a squared correlation of 0.9403. Figure 6.4 presents the scatterplots for these same relationships. It documents that the relationships are even stronger than the simple linear correlations suggest as the lower correlations involve relationships that are very close but mildly nonlinear. When the nonlinearities are taken into account, all relationships are near exact. For example, the G-R combination has the lowest squared linear correlation (0.9403) but regressing G on R and the square root of R yields a multiple R-square statistic of 0.9859.

The middle panel of Table 6.4 presents results for White-Minority segregation comparisons where the pairwise proportion for the smaller group in the comparison is the range of 0.10–0.30. The key finding documented here is that, while the correlations are generally lower, they all remain very high. Thus, the lowest squared

[5] More carefully, correlations are higher when the two groups are similar in size; that is, when P and Q are equal. The distinction is relevant in segregation comparisons where Whites are the smaller group; for example, White-Latino segregation in San Antonio and El Paso.

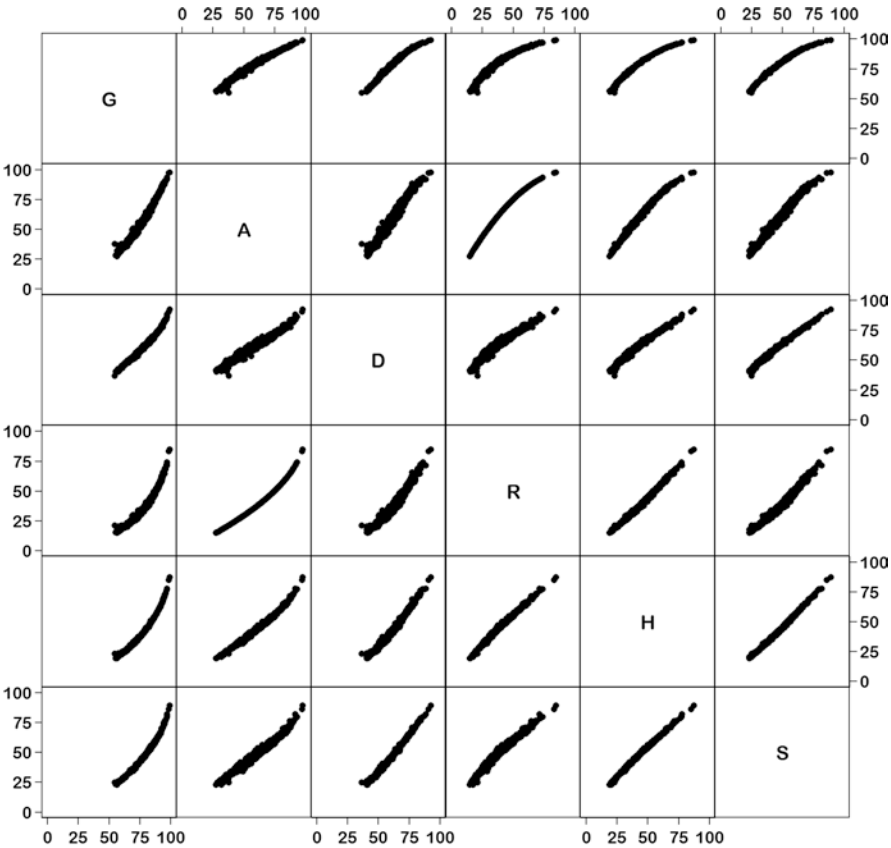

Fig. 6.4 Scatterplots depicting relationships among indices of uneven distribution for White-Minority comparisons in 1990, 2000, and 2010 – subset of CBSAs with Minority Proportion ≥ 0.30 (Notes: scores are for White-Black, White-Latino, and White-Asian segregation comparisons computed using block-level data from U.S. Census summary files)

correlation is 0.8660 for the D-S relationship and nine of the fifteen correlations exceed 0.95.

The bottom panel of Table 6.4 reports correlations among index scores for the subset of White-Minority segregation comparisons where the minority group is small in relative size. Specifically, it reports correlations for cases where the pair-wise proportion for the smaller group in the comparison is under 0.10. Two findings warrant mention. First, the correlations among G, D, A, and R – the four measures related to the segregation curve – remain high; the lowest squared correlation is 0.9432 for the G-R combination. Second, and more importantly, the squared correlations involving H and S – the two measures not related to the segregation curve – drop off considerably, especially correlations involving S. The squared correlation of 0.8370 between H and S is fairly high. But squared correlations of H with G, D, A, and R fall in a substantially lower range of 0.7056–0.7543 for H and the

Table 6.4 Relationships among indices of uneven distribution by group proportions for White-Minority segregation comparisons in CBSAs in 1990, 2000, and 2010[a]

	G	A	D	R	H	S
CBSAs where the smaller pairwise group proportion ≥ 0.30 (N = 510)						
G – Gini Index	1.0000	***0.9799***	***0.9795***	***0.9403***	***0.9618***	***0.9690***
A – Atkinson Index ($A_{[0.5]}$)	0.9899	1.0000	***0.9690***	***0.9799***	***0.9825***	***0.9754***
D – Dissimilarity Index	0.9897	0.9844	1.0000	***0.9641***	***0.9843***	***0.9898***
R – Hutchens Index	0.9697	0.9899	0.9819	1.0000	***0.9932***	***0.9789***
H – Theil Index	0.9807	0.9912	0.9921	0.9966	1.0000	***0.9958***
S – Separation Index	0.9844	0.9876	0.9949	0.9894	0.9979	1.0000
CBSAs where the smaller pairwise group proportion ≥ 0.10 and < 0.30 (N = 1,163)						
G – Gini Index	1.0000	***0.9750***	***0.9751***	***0.9339***	***0.9339***	***0.8699***
A – Atkinson Index ($A_{[0.5]}$)	0.9874	1.0000	***0.9748***	***0.9805***	***0.9520***	***0.8660***
D – Dissimilarity Index	0.9875	0.9873	1.0000	***0.9663***	***0.9580***	***0.8857***
R – Hutchens Index	0.9664	0.9902	0.9830	1.0000	***0.9714***	***0.8874***
H – Theil Index	0.9664	0.9757	0.9788	0.9856	1.0000	***0.9694***
S – Separation Index	0.9327	0.9306	0.9411	0.9420	0.9846	1.0000
CBSAs where the smaller pairwise group proportion < 0.10 (N = 2,646)						
G – Gini Index	1.0000	***0.9761***	***0.9628***	***0.9432***	***0.7515***	***0.3756***
A – Atkinson Index ($A_{[0.5]}$)	0.9880	1.0000	***0.9791***	***0.9801***	***0.7056***	***0.3132***
D – Dissimilarity Index	0.9812	0.9895	1.0000	***0.9797***	***0.7271***	***0.3325***
R – Hutchens Index	0.9712	0.9900	0.9898	1.0000	***0.7543***	***0.3581***
H – Theil Index	0.8669	0.8400	0.8527	0.8685	1.0000	***0.8370***
S – Separation Index	0.6129	0.5596	0.5766	0.5984	0.9149	1.0000

[a]Squared correlations are reported above the diagonal (in bold, italic). Index scores are computed using block-level data from U.S. Census Summary File 1 and PL-94. Cases are for White-Black, White-Latino, and White-Asian segregation comparisons excluding CBSAs where the minority population is under 1,500

squared correlations of S with these measures fall in a much lower range of 0.3132–0.3756.

I highlight the most important points of the above discussion as follows.

- Scores for all popular segregation indices consistently agree and correlate closely with one another when the two groups in the comparison are similar in size.
- Scores for popular segregation indices that are closely related to the segregation curve – G, D, A, and R –consistently agree and correlate closely with one another regardless of relative group size.
- Scores for popular segregation indices not related to the segregation curve – H and S – correlate closely with each other even when relative group size is imbalanced (i.e., when the pairwise proportion for the smaller group is under 0.10).
- Scores for H and S correlate closely with scores for G, D, A, and R when relative group size is relatively balanced (i.e., when the pairwise proportion for the smaller group is ≥ 0.10). But the correlations fall off substantially, especially

those involving S, when the pairwise proportion for the minority group is low (i.e., below 0.10).

6.2 Why Does Relative Group Size Matter?

The difference of means framework provides a basis for gaining insight into these findings. In this framework segregation index scores are obtained as differences of group means on segregation-relevant residential outcomes (y) that are scored from area proportion White (p) via index-specific scaling functions $y = f(p)$. It is obvious that scores for different indices will correlate more closely when the index-specific scaling functions $y = f(p)$ for the indices involved are similar. Conversely, correlations among scores will be lower when the scaling functions involved differ. The graphs in Fig. 5.1 introduced earlier documented how the scaling functions vary across indices. In the case of S, the scaling function is linear. The scaling functions for the other indices are nonlinear with nonlinearity being more pronounced for some indices than for others. Specifically, the graphs in Fig. 5.1 documented that the nonlinearity is least pronounced for H and progressively more pronounced for R, D, and G. This helps explain why scores for G, D, and R consistently correlate closely. It also helps explain why scores for S correlate more closely with scores for H than with scores for G, D, and R.

The scaling function for S is invariant across variation in relative group size; y is always a simple, one-to-one linear function of p. Significantly, the scaling functions for all of the other indices vary systematically with relative group size. Specifically, the "amplitude" of the nonlinearity in the scoring function is most pronounced when relative group size is highly imbalanced and it is least pronounced when relative group size is equal (i.e., 50/50). Figures 6.5 and 6.6 document this for the Theil index (H) and the Hutchens square root index (R) by plotting the scaling function $y = f(p)$ with values of relative group size set variously at 0.01, 0.05, 0.20, 0.50, 0.80, 0.95, and 0.99. The variation in nonlinearity is particularly easy to summarize for these two functions because they are smooth and continuous. Nonlinearities in the scaling functions for G and D behave in a similar manner, but are more complicated visually because the functions involve monotonic but irregular step functions.

Figures 6.5 and 6.6 show that in all four cases the nonlinear functions departure from linearity is mildest when groups in the segregation comparison are similar in size and it grows increasingly more pronounced as groups become more unequal in size. Since the scaling function for S is always linear, this explains why scores for S correlate more closely with scores for the other indices when groups are equal in size and less closely, sometimes markedly so, when the two groups in the comparison are unequal in size. In general, the difference between any two index-specific scaling functions is least pronounced when groups are equal in size and it grows

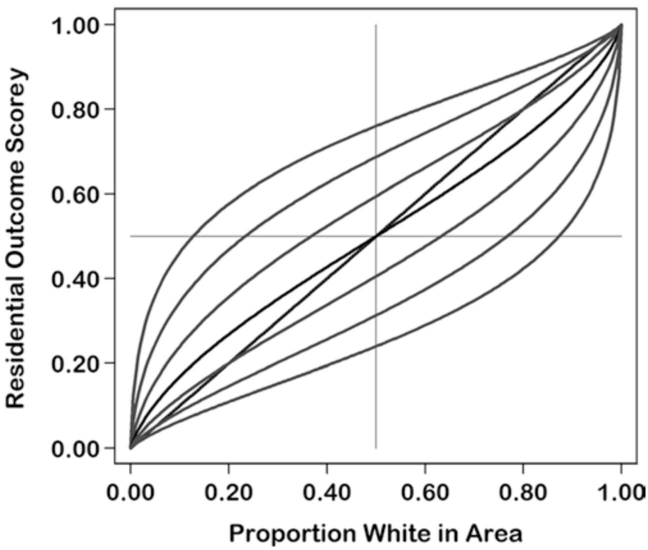

Fig. 6.5 Scoring y = f(p) for computing Theil's H as a difference of group means on scaled contact (Curves reflect y = f(p) for Theil's H based on y = Q + [(E-e)/E] / (p/P − q/Q) for selected values of proportion White in the city (P). Moving from the top curve to the bottom curve, the selected values for P are: 0.01, 0.05, 0.20, 0.50, 0.80, 0.95, and 0.99, respectively. The diagonal line reflects y = f(p) for S)

larger as groups become more unequal in size. This accounts for why index scores generally correlate more closely when groups are equal in size and correlate less closely when groups are unequal in size.

The potential discrepancies between scores for different indices follow a very clear pattern. At one end of the spectrum there are indices like G and D which register residential outcome scores (y) based on scaling functions that involve more pronounced nonlinearities (as seen in Fig. 5.1). On the other end of the spectrum are indices like H and S which register residential outcome scores (y) based on scaling functions that involve only mild nonlinearity (H) or simple linear scaling (S). Under all conditions scores for G and D consistently run higher than scores for H and S. But there are big differences in how this plays out depending on the group size comparison. When group size is relatively balanced (e.g., pairwise proportion for the smaller group is 0.15 or higher), scores for G and D will run higher than scores for H and S and will fall in a narrow range of variation at any particular level of H or S. In contrast, when group size is imbalanced (e.g., pairwise proportion for the smaller group under 0.10), scores for G and D will run higher than scores for H and S but they may fall in a sizeable range of variation at any particular level of H or S.

This is documented in Fig. 6.7 which plots values of D against values of H and S for three sets of cases.[6] The first panel of the figure depicts the D-H and D-S rela-

[6] Results for G and R are not shown, but are similar. I highlight results for D because it is used more often in empirical studies.

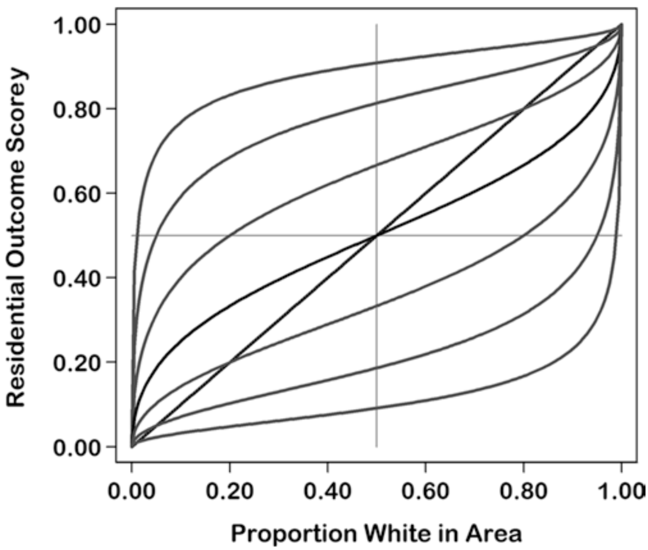

Fig. 6.6 Scoring y = f(p) for computing Hutchens' R as a difference of group means on scaled contact. Curves reflect y = f(p) for Hutchens' R based on $y = Q + \left(1 - \sqrt{pq / PQ}\right) / \left(p / P - q / Q\right)$ for selected values of proportion White in the city (P). Moving from the top curve to the bottom curve, the selected values of P are: 0.01, 0.05, 0.20, 0.50, 0.80, 0.95, and 0.99, respectively. The diagonal line reflects y = f(p) for S

tionships for all CBSAs. The second panel depicts the same relationships for the subset of CBSAs where the pairwise proportion for the smaller group is 0.15 or higher. The third panel depicts the relationships for the subset of CBSAs where the pairwise proportion for the smaller group is below 0.10. Note that I exclude CBSAs with the very lowest values (i.e., values below 0.02) on pairwise group proportion so it will be clear that the pattern observed in this panel is not determined by extreme cases. The scatterplots in the second panel document that, when the groups in the segregation comparison are relatively similar in size, D varies in a narrow range at any particular level of H or S. The scatterplots in the third panel document that, when the groups in the segregation comparison are somewhat unequal in size, D varies in a much larger range at any specific level of H and S. On the low end, the variation in D extends down to the levels seen in the second panel of the figure. On the high end the variation in D is considerable and often ranges 25–35 points above scores on the low end. The first panel combines the CBSAs in the second and third panels and also includes CBSAs where the smaller group meets the group size requirement of 1,500 in population but has a pairwise proportion of less than 0.02. This amplifies the pattern seen in the third panel by extending the range of variation on both the high and low ends at any given level of H and S.

Figure 6.7 documents that popular indices of uneven distribution can and often do yield highly discrepant results. When this happens, a specific substantive interpretation applies. The pattern of segregation in these situations involves extensive

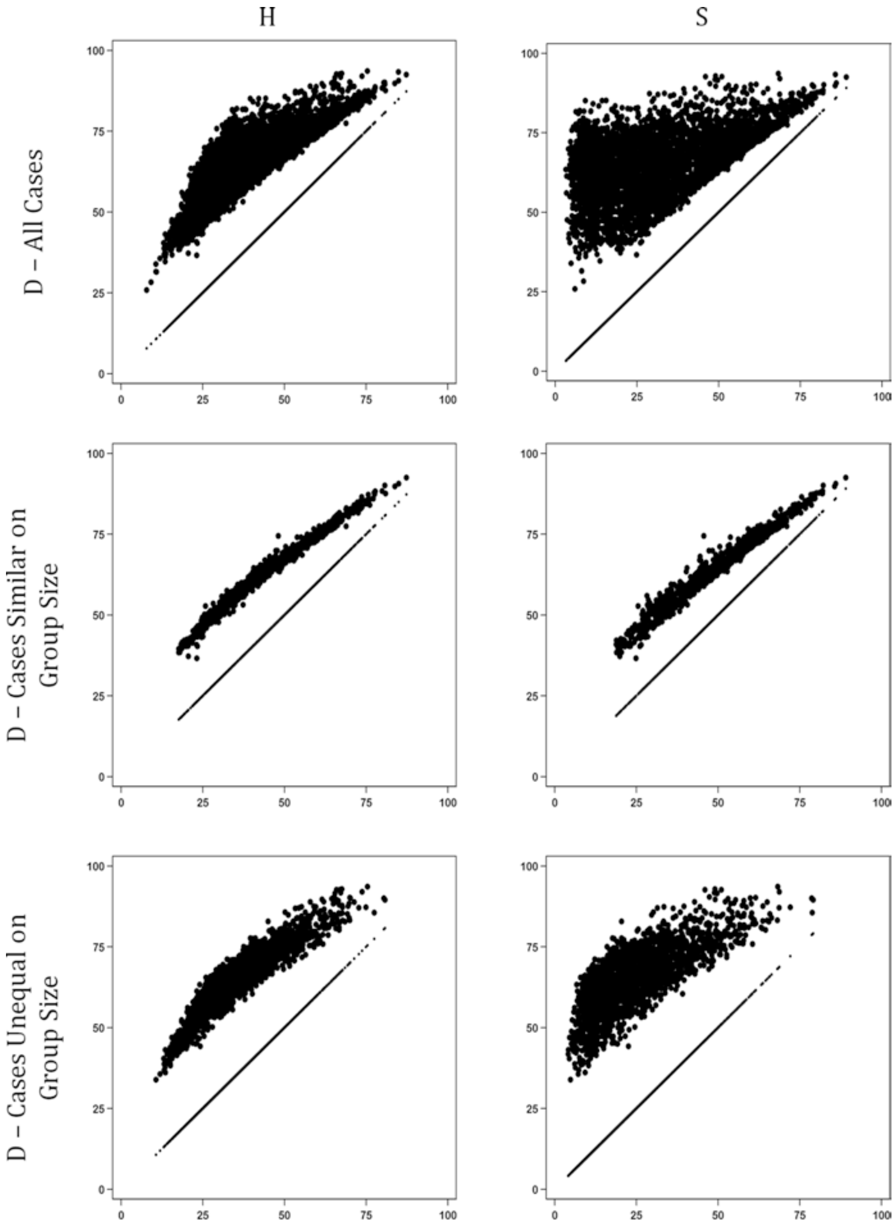

Fig. 6.7 Scatterplots depicting relationships of D with H and S for White-Minority comparisons for CBSAs in 1990, 2000, & 2010 (Notes: scores are for White-Black, White-Latino, and White-Asian segregation comparisons computed using block-level data from U.S. Census summary files)

group differences in displacement from parity but does not involve high levels of group residential separation and neighborhood polarization. The combination comes about because indices such as G, D, and R can respond with high scores when displacement from parity involves group differences in pairwise contact that are quantitatively small. Indices that register group separation and neighborhood polarization take low values in these situations because the two groups are living together, not apart, with most minority individuals living with Whites and few residing in predominantly minority residential areas (e.g., ghettos and barrios). It is important to be aware of this possibility for many reasons not the least of which being that it affects the potential policy implications of eliminating uneven distribution. When group separation and area polarization are absent, majority-minority differences in residential outcomes will change little when uneven distribution is eliminated. When separation and polarization are present, the residential outcomes experienced by minority individuals can potentially change dramatically when uneven distribution is eliminated. I believe this is an important aspect of the correspondence, or lack of it, between different indices. Accordingly, I review the issue in more detail in Chaps. 7 and 8.

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Chapter 7
Distinctions Between Displacement and Separation

The previous chapter documents that the separation index (S) can reveal the presence of important aspects of residential segregation that cannot be reliably established by examining the more widely used dissimilarity index (D). Specifically, S reliably indicates whether groups are separated and live apart from each other in different areas of the city and experience substantially different residential outcomes – at minimum with respect to area racial composition and potentially also on other neighborhood outcomes that co-vary with area racial composition. High values on S thus signal that groups are residentially separated and reside apart from each other in areas that are polarized on racial composition. The same cannot be said for D. To the contrary, D can and sometimes does take high values when two groups are not residentially separated and in fact live together in the same neighborhoods and experience quantitatively similar residential outcomes on area racial composition. Thus, high values on D cannot and do not reliably signal the presence of group residential separation and neighborhood racial polarization.

I view the issue of whether groups are separated and live apart in different neighborhoods or live together in the same areas and share neighborhood outcomes as fundamental to segregation research. The following two quotes from Massey and Denton's (1988) landmark methodological study are consistent with this view. Speaking of segregation in broad terms they state "At a general level, residential segregation is *the degree to which two or more groups live separately from one another, in different parts of the urban environment*." (1988:282, emphasis added). Speaking more specifically of the dimension of uneven distribution they state "Evenness is minimized and segregation maximized when no minority and majority members share a common area of residence" (1988:284).

These statements resonate with prevailing substantive intuitions about residential segregation. Researchers and broader audiences alike presume that high scores on segregation signal that the groups in the comparison live apart from each other in different neighborhoods and thus do not share common fate based on area of

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_7

residence. The separation index (S) provides a reliable signal on this count. The dissimilarity index (D) does not. D does not because it measures something different from whether groups live together or apart. Specifically, D provides a reliable signal regarding whether groups differ in their extent of being displaced from parity. Significantly, however, *group differences in being displaced from parity and group residential separation are not the same things and they do not necessarily correlate closely empirically*. Displacement and separation often do take high values together. But, importantly, group difference in displacement from parity can be high when group residential separation is low.

In this chapter I seek to clarify the differences between separation (S) and displacement (D) in more detail. I begin by noting that the issue has become more important in recent decades because conceptual distinctions between separation and displacement have come to take on greater practical significance in empirical analyses. The main reason for this is that the scope of segregation studies has expanded and the racial demography of US urban areas in the United States has become more complex. As a result, researchers are now frequently investigating segregation in situations where large differences between scores on separation and displacement are more common than was the case in an earlier era of segregation research.

I frame the substantive issues involved by introducing two terms. The first is "prototypical segregation" which is associated with a pattern of "concentrated displacement". The second is the opposite condition of "dispersed displacement" a pattern of segregation that is empirically common but largely unrecognized in the measurement literature.

In the pattern of "prototypical segregation" displacement from even distribution concentrates the populations of the two groups into homogeneous areas that differ by quantitatively large amounts on area racial composition. When such a pattern of "concentrated displacement" is present, group residential separation and area racial polarization as indicated by S will approach the maximum levels possible at a given level of displacement from parity as indicated by D. In the logical extreme where displacement is concentrated to the maximum possible extent, the value of S will equal the value of D. The pattern of "dispersed displacement" is at the opposite end of the spectrum. Under this pattern levels of group residential separation and area racial polarization are far below the maximum levels possible for a given level of displacement. In sum, under "prototypical segregation" involving concentrated displacement values of D and S correspond closely. Under dispersed displacement, values of D and S diverge by large amounts.

I next explore these issues in two extended technical discussions that clarify the basis for D-S congruence and divergence. In the first discussion I contrast how D and S respond differently to residential exchanges that promote integration and/or segregation and I describe how this can lead to D and S taking either similar or discrepant values. In the second discussion I introduce simple analytic models that reveal more precisely how displacement (D) and separation (S) can vary independently to produce residential patterns ranging from "prototypical segregation" to "dispersed displacement" at any given combination of displacement (D) and overall city racial composition (P). I then "exercise" the models to produce graphical results that reveal the nature and range of potential combinations of displacement (D) and group separation (S) by level of city racial composition.

I close the chapter by considering the question of whether displacement (D) and separation (S) should be seen as distinctly different dimensions of segregation. My discussion gives attention to three alternative views. One is the position suggested by Stearns and Logan (1986) which holds that group separation and area racial polarization should be seen as a distinctive dimension of segregation to be considered along with uneven distribution and exposure. Another view takes the position that group separation and area racial polarization can be seen as an important aspect of uneven distribution that may or may not be present when group distributions are displaced from even distribution. I also consider and dismiss a mistaken third view, sometimes suggested in the literature, that group separation and area polarization reflects exposure.

In the end I endorse a practical compromise. In my view it ultimately is not crucial whether one classifies group separation and area racial polarization as a distinct dimension of segregation or is a particular aspect of uneven distribution. What is crucial is for researchers to recognize that separation, displacement, and exposure all provide useful information and all three can and do vary independently in empirical analyses. This knowledge will help researchers choose measures that best serve their research interests. My view is that this will lead researchers to pay closer attention to group separation and area polarization as measured by S because S provides a reliable signal about the presence or absence of "prototypical segregation" which researchers and broad audiences alike find more interesting and compelling than "dispersed displacement".

7.1 The Increasing Practical Importance of the Distinction Between Displacement and Separation

Stearns and Logan (1986) argued that the distinction between D and S is important noting that the measures "are responsive to different aspects of changes in racial residential patterns" and can "lead to divergent, sometimes contradictory, results" (1986:125–126). To support their view they noted the example of Logan and Schneider (1984) who found that D and S gave different results regarding trends in White-Black segregation in suburban areas with S showing increasing segregation while D indicated declining segregation. Studies by Schnare (1980) and Smith (1991) also reported finding different patterns and trends in residential segregation when using D and S.

Coleman et al. (1982:177–179) had previously argued that D and S differ in ability to provide a reliable signal of when group have important differences in residential outcomes and noted that D can take high values when the two groups in the comparison have fundamentally similar distributions on residential outcomes. Zoloth (1976) made similar points in an earlier methodological study. Unfortunately, the findings and observations reported in these studies have had minimal impact on prevailing practices in segregation research. Empirical studies overwhelmingly use D over alternative measures and typically do not report whether findings are similar or different depending on whether alternative indices such as S are used. This suggests that researchers generally are not aware of two points. The first is that D and S can take highly discrepant scores and can move in different directions. The second is that whether scores for D and S align or diverge it has important implications about fundamental aspects of the nature of segregation.

Prevailing practices may have been more understandable and less consequential in an earlier era of segregation research. For many decades empirical studies focused primarily on White-Black segregation in large metropolitan areas where Black populations were substantial in size and typically were concentrated in large ghettos. The empirical analyses in the Chap. 6 showed that discrepancies between displacement (D) and separation (S) tend to be less dramatic when analysis is restricted to this particular subset of segregation comparisons. So, while D and S are not exactly interchangeable in these situations, displacement typically is highly concentrated. As a result the values of D and S tend to correlate closely and index choice may be less likely to lead to important practical differences in findings.

Times have changed. The racial and ethnic composition of cities in the United States has undergone dramatic demographic transformation. Additionally, the scope of segregation studies has expanded to consider segregation across a wider range of group comparisons and a wider range of community settings. *In these new circumstances of segregation research, researchers cannot safely assume that index choice does not matter*. To the contrary, nowadays the logical differences between displacement (D) and separation (S) routinely take on greater practical importance. Over the last four decades the Latino and Asian populations have grown rapidly and diffused from traditional settlement areas to wider distribution nationally. Consequently, segregation studies now examine a broader range of group comparisons beyond the earlier narrow focus on White-Black segregation and routinely give attention to White-Latino and White-Asian segregation. Additionally, the focus of research has expanded from beyond considering just large metropolitan areas where minority presence often is sizeable. Empirical studies now increasingly consider a broader range of communities including communities where minority population presence is relatively small. This is reflected, for example, in studies that examine White-Latino and White-Asian segregation in "new destination" communities where Latino and Asian populations are newly arrived and growing rapidly. Additionally, segregation studies nowadays investigate segregation over an increasingly wide range of settings including not only the largest metropolitan areas but also smaller metropolitan areas, micropolitan areas, noncore counties, and small towns.

All of these trends make the topic of this chapter more relevant to current and future segregation studies. The empirical analyses of White-Minority segregation across CBSAs reviewed Chap. 6 document that the correlation of D and S is weaker when examining White-Latino and especially White-Asian segregation, weaker when examining segregation in smaller communities, and weaker in communities where minorities are smaller in relative size. As a result, we should expect discrepancies between scores for D and S to be increasingly common and larger in size and for the discrepancies to carry increasing substantive importance. Accordingly, it is useful to understand the substantive issues that are relevant when D and S align and when D and S diverge. To serve this goal I now explore the notion of "prototypical segregation" and the contrast between "concentrated" and "dispersed" displacement.

7.2 Prototypical Segregation and Concentrated Versus Dispersed Displacement

I use the term "displacement" to refer to group differences in distribution across neighborhoods that are above or below "parity." Taking Whites as the reference group in an analysis of White-Black segregation, displacement is high when a large share or proportion of White population resides in "above-parity" areas (i.e., where $p_i < P$) and a similarly large share or proportion of the Black population resides in "below-parity" areas. Alternatively, displacement is high when Whites and Blacks differ on the proportion residing in "above-parity" areas or on the proportion residing in "below-parity" areas. These are all slightly different ways of describing the same arrangement and all result in the same values on displacement as measured by D.

Significantly, the notion of displacement from parity does not specify anything further about group residential distributions beyond the narrow confines of what was just stated. Displacement varies in extensiveness – the degree to which it involves large differences in group portions. But extensiveness of displacement does not carry specific implications for the quantitative magnitude of the differences in area racial composition between above-parity neighborhoods and below-parity neighborhoods. To the contrary, the magnitude of the differences involved can vary dramatically at a given level of displacement. The notion of displacement is captured well by the dissimilarity index (D) as its value directly registers majority-minority differences in proportions residing in "parity" or "above-parity" areas.[1] This quality of D was recognized by Duncan and Duncan (1955) who referred to D as the "displacement index."

I use the terms "group separation" and "neighborhood polarization" to refer to residential distributions that are characterized by groups living apart from each other such that members of both of the groups in the comparison are disproportionately located in areas where their own group predominates. Significantly, *displacement does not necessarily involve group separation*. Thus, D is not a valid proxy for group separation. Whether or not displacement involves separation depends on additional consideration; namely, whether displacement is "concentrated" or

[1] Alternatively, the value of D can be obtained from the Black-White difference in group proportions residing in "below- parity" areas.

"dispersed." Under concentrated displacement both groups reside apart from each other in racially homogeneous areas that differ markedly on racial composition. Under dispersed displacement, the groups reside together in areas that differ modestly on racial composition.

To clarify, at a given level of displacement, separation is maximized when displacement is *concentrated* in a way that maximizes same-group contact *for both groups*.[2] Conversely, group separation is minimized when displacement is *dispersed* in a way that produces a low level of same-group contact for at least one of the two groups. The notion of separation just outlined is captured well by S which registers the majority-minority difference in (pairwise) contact with the majority group.

7.2.1 Prototypical Segregation

I use the term "prototypical segregation" to refer to a residential pattern where group separation approaches the maximum that can occur at a given level of displacement. I characterize this pattern as prototypical because, without exception so far as I have been able to find, this is the pattern of segregation depicted when examples of high levels of segregation are introduced and reviewed in didactic discussions of residential segregation. For example, it is the kind of segregation pattern seen in didactic illustrations and discussions provided by Taeuber (1964), Taeuber and Taeuber (1965), Jaret (1995), and Iceland et al. (2000). It also is the kind of segregation pattern seen in familiar examples of high levels of segregation as observed for White-Black segregation in cities such as Chicago, Detroit, Cleveland, and Milwaukee and as observed for White-Latino segregation in Los Angeles. What is common in these situations of prototypical White-Minority segregation is this: White households are living in above-parity neighborhoods that are predominantly White in racial composition and, similarly, minority households are living apart from Whites in below-parity neighborhoods that are predominantly minority in racial composition. Accordingly, non-parity areas are "polarized" into areas that differ greatly on racial composition with Whites being concentrated in predominantly White areas and minorities being concentrated in predominantly minority areas typically forming enclaves, barrios, and ghettos.

Values of D and S correspond closely when the condition of prototypical segregation hold because displacement from parity is concentrated rather than dispersed. When prototypical segregation is pronounced, values of both D and S are high; displacement from parity is extensive for both groups and the populations residing in non-parity areas are concentrated into areas that are ethnically homogeneous. Because the two groups live apart in neighborhoods that are fundamentally different in terms of racial composition, residential redistribution that substantially reduces

[2] I place emphasis on "for both groups" because this distinguishes separation from simple same-group contact and isolation. Isolation is intrinsically affected by city racial composition and separation is not.

or eliminates displacement from even distribution also will bring about correspondingly large quantitative changes in neighborhood racial composition. This will in turn carry the potential to also bring about large changes in group differences on neighborhood outcomes that are correlated with area racial composition (e.g., social problems, amenities, services, etc.).

My strong sense is that broad audiences, most academics, and even many segregation researchers generally assume that the residential patterns associated with "prototypical" segregation will be present when scores on widely used segregation indices such as the dissimilarity index (D) are high. This assumption is mistaken. In fairness, however, it is easy to understand why this mistaken view is so widely held. Standard examples and didactic discussions encourage the assumption and little in the standard methodological literature cautions otherwise. That is,

> *Methodological discussions that present examples illustrating how residential segregation is captured by the segregation curve and the dissimilarity index (D) rarely, if ever – feature residential distributions with low group separation (S) resulting from dispersed displacement. Instead, they feature residential distributions with high levels of group separation resulting from concentrated displacement.*

As a result, the prevailing understanding of segregation measurement rests on a widely shared but incorrect assumption that high scores on popular segregation indices always signal the condition of prototypical segregation involving concentrated displacement and group residential separation. This is not the case. In particular, high values of the dissimilarity index (D), the most widely used segregation index, do not and intrinsically cannot provide a reliable signal about the presence of prototypical segregation.[3] In contrast, high values of the separation index (S) provide a certain indication that a high level of prototypical segregation is present.

The outcome of high displacement but with low separation – that is, high D and low S – occurs when residential distributions are characterized by "dispersed displacement." In the pattern of dispersed displacement, individuals residing in non-parity areas are not concentrated in areas where their group predominates. Instead, the residential distribution for at least one of the groups – usually the smaller of the two groups, which in White-Minority comparisons in US cities is typically, but not always, the non-White minority group – is dispersed widely and thinly across non-parity areas such that most members of the group live in "mixed" areas where their group is not the predominant presence. Indeed, it can be the case that few members of the group live in areas where their group is a majority presence and instead most members of the group live in areas where the other group in the comparison is the predominant group. As a result, under dispersed displacement the two groups in the comparison live together in areas with similar racial composition, not apart from each other in areas where racial composition is polarized.

[3] The same can be said for any index that ranks segregation comparisons consistent with the principle of segregation curve dominance. In addition to the dissimilarity index (D), this includes the gini index (G), the symmetric version of the Atkinson index ($A_{0.5}$), and the Hutchens square root index (R).

The contrasting notions of prototypical segregation and dispersed displacement can be clarified by comparing two logically possible but fundamentally different outcomes that can occur at a given level of displacement. One outcome is that all group members not living in parity areas reside in perfectly segregated, homogeneous areas. For example, in the case of White-Black segregation, Whites and Blacks not living in parity areas would reside in all-White and all-Black areas, respectively. I term this "maximally concentrated displacement." The other outcome is that all group members not residing in parity areas reside in areas that come as close to matching parity as is demographically feasible. I term this "maximally dispersed displacement."

Importantly, the values of D and S vary dramatically across these two logical possibilities. The value of D will necessarily be the same in both cases. In contrast, the value of S will vary across these two cases, potentially by a very large amount. For the level of displacement in question, the value of S will take its highest possible value, in which case it will equal the value of D, under maximally concentrated displacement. S will take its lowest possible value under maximally dispersed displacement. This leads to a broad rule of thumb for characterizing segregation patterns. At a given level of displacement, "prototypical segregation" holds when the value of S is relatively close to its highest possible value and "dispersed displacement" holds when the value of S is relatively close to its lowest possible value.

Under "prototypical segregation," D-S combinations are characterized by close agreement; their scores roughly correspond at low-low, medium-medium, high-high, and so forth. Under "dispersed displacement," D-S combinations are characterized by disagreement, sometimes very dramatic disagreement, with scores for D being much higher than scores for S. Figure 7.1 places combinations of D and S in four general categories based on a two-by-two classification of high and low outcomes on the dissimilarity index (D) and the separation index (S). The purpose of this simplified presentation is to focus attention of the fundamental differences between the logically possible combinations.

To begin I note that the D-S combination in the upper-left cell of the figure cannot occur. As I show below, displacement as measured by D sets the upper limit for group separation as measured by S. Accordingly, high values of group separation (S) always are accompanied by values of displacement (D) of equal or greater size. Consequently, a low-D, high-S combination is not logically possible. The lower-left cell (A) is labeled "Low Prototypical Segregation." It involves a low-level of group displacement from even distribution (D) and a corresponding low level of group separation (S). The upper-right cell (C) is labeled "High Prototypical Segregation." It involves a high level of group displacement from even distribution (D) and a corresponding high level of group separation (S). The lower-right cell (B) is labeled "Displacement without Separation." It involves a high level of displacement from even distribution (D) but with levels of group separation substantially below what is possible (S). Since this pattern involves dispersed rather than concentrated displacement, the alternative label of "Dispersed Displacement" also is appropriate.

Recall from discussion in earlier chapters that the dissimilarity index (D) can be characterized as summary index of group inequality in rank-order position on area

Value of S	Value of D: Low	Value of D: High
High	This outcome cannot occur	(C) High Prototypical Segregation (Concentrated Displacement) Displacement from even distribution is extensive and it is concentrated so it involves maximal group separation and area racial polarization. The group difference in percentage attaining parity on area group proportion (p) is large *and* the group difference of means on (p) is large. Example generating process – implement as many "segregation-promoting" exchanges as possible without changing D·
Low	(A) Low Prototypical Segregation Displacement from even distribution is low and group separation and residential polarization also are low. The group difference in percentage attaining parity on area group proportion (p ≥ P) is small *and* the group difference of means on (p) is small. Example generating process – quota allocation or random distribution.	(B) Displacement without Separation (Dispersed Displacement) Displacement from even distribution is extensive, but it is dispersed and involves minimal group separation and area racial polarization. The group difference in percentage attaining parity on area group proportion (p) is large, but the group difference of means on (p) is small. Example generating process – implement as many integration-promoting exchanges as possible changing D.

Fig. 7.1 Possible combinations of high and low values on displacement (D) and separation (S)

racial composition. Specifically, in the case of White-Minority segregation, D, like the gini index (G), reflects rank-order inequality on area proportion White (p).[4] Similarly, the separation index (S) can be characterized as a summary index of group inequality on the original or "raw" quantitative scores on area racial composition (p). With this in mind, the four cells in Fig. 7.1 can be described in the

[4] Thus, the value of G can be given as twice the value of the group difference in mean percentile scores on area group proportion (p). The value of D can be given in the same way based on collapsing scoring of area group proportion into two categories of "above parity" ($p > P$) or not.

following terms. The lower-left cell (A) "Low Prototypical Segregation" and the upper-right cell (C) "High Prototypical Segregation" both involve situations where group distributions on area proportion White (p) produce similar high levels of inequality in rank-order position (D) and quantitative difference (S). The lower-right cell (B) "Displacement without Separation" involves a high level of group inequality on rank order position on area proportion White (p) but a low level of group inequality on quantitative differences on area proportion White (p). The combination indicates that Whites are consistently ranked above Blacks on area proportion White – as indicated by the high value of D, but the quantitative differences involved are small and thus result in the low value of S. Thus, the rank-order differences on area proportion White do not translate into group separation because the two groups have similar distributions on area racial composition (p) and thus the two populations are living together, not apart from each other.

7.3 Clarifying the Logical Potential for D-S Concordance and Discordance – Analysis of Exchanges

Scores for D and S can diverge because they assess group differences in residential distribution in fundamentally different ways. D measures group differences on area proportion White (p) in a crude way; it assesses the group difference in relative distribution ***between*** two kinds of areas; those that are "above-parity" on area proportion White (p) and those that are "below-parity."[5] In contrast, S measures group difference in area proportion White (p) based on quantitative differences over the full distribution of values for area proportion White (p). Thus, where S registers all quantitative information about group differences on area proportion White (p), D instead collapses this information into a dichotomous rank-order scoring of "above P" or not. Thus, at any value of D, the value of S can vary by a considerable amount because, unlike D, S registers group differences in distribution on area proportion White (p) both within and across "non-parity" areas.

Methodological studies establish that the potential for scores of D and S to diverge traces to two technical differences between D and S. The first is a well-known technical deficiency with D. It is that D does not register all integration-promoting exchanges of White and Black households between two areas (Reardon and Firebaugh 2002).[6] The value of D changes only for a partial subset of integration-promoting exchanges – those that cause at least one of the two areas involved in the exchange to move from being above the value of proportion White for the city (P) to at or below P when the exchange is completed, or, alternatively, to move from being below P to at or above P. When integration-promoting exchanges involve

[5] The same quantitative result is obtained if the distinction is "at-or-above-parity" and "below-parity."

[6] The nature of integration-promoting and segregation-promoting exchange is discussed in more detail in a separate section below.

households from areas on the same side of the cut point (P) before and after the exchange, the value of D does not change. In contrast, S behaves as accepted principles of segregation measurement require; the value of S goes down when any integration-promoting exchange occurs and the value of S goes up when any segregation-promoting exchange occurs (Reardon and Firebaugh 2002).

This provides the initial basis for understanding how the value of S can move independently of the value of D. It is that, at any value of D, integration-promoting exchanges that involve areas on the same side of overall proportion White (P) before and after the exchange will cause the value of S to go down while the value of D remains fixed. Similarly, segregation-promoting exchanges that involve areas where proportion White (p) is on the same side of overall proportion White (P) before and after the exchange will cause the value of S to go up while the value of D remains fixed. Under accepted principles of segregation measurement the changes in values of S that take place while D is remaining constant are desirable; they occur because S is registering changes in uneven distribution within non-parity areas. In contrast, the non-responsiveness of D is undesirable; it occurs because D is insensitive to changes in uneven distribution that are taking place within non-parity areas.

There is a second basis for why the value of S can move independently of the value of D. It is that, even in cases where D does register the impact of an integration-promoting exchange, D has a "flat" or "uniform" response regardless of the impact of the exchange on group separation as it relates to the magnitude of the changes in area racial composition. In contrast, S responds differentially depending on the impact the exchange has on group separation by responding more strongly when the two areas involved in the exchange are more "polarized" based on being further apart on area proportion White (p). That is to say, all else equal, for any exchange producing a change in D, the impact on the value of D will be the same regardless of the magnitude of the difference on area proportion White (p) between the two areas in the exchange but the impact on the value of S will be larger when the difference is larger rather than smaller. This conforms to the substantively appealing property that exchanging White and Black households across all-White and all-Black areas reduces segregation more than exchanging White and Black households across areas that are nearly identical on area proportion White (p). The former exchange reduces group separation to a greater degree than the latter exchange because it has a larger impact on reducing area racial polarization and White-Black differences in distribution on area proportion White (p).

I review the formal basis for this conclusion in the next two sections. I motivate the discussion by trying to briefly give an intuitive sense of why the issue is important. At a given level of displacement from even distribution as measured by D households not residing in parity areas can be maximally segregated or minimally segregated under the exchange criterion. Under maximal segregation, all possible segregation-promoting exchanges that do not change the value D are implemented. The value of S will equal the value of D and White and Black households residing in non-parity areas will be separated into maximally polarized, homogeneous areas. Under minimal segregation, all possible integration-promoting exchanges that do not change the value of D are implemented. The value of S will be very low in comparison to the value of D because White and Black households residing in non-parity areas will live together in areas that are relatively similar on racial composition. The difference between the two extremes is unquestionably sociologically meaningful. So it is important to understand how D and S differ in their ability to reveal these two fundamentally different residential patterns.

7.3.1 *Overview of D-S Differences in Responding to Integration-Promoting Exchanges*

In this section I review how D and S respond to exchanges of White and Black households across areas. To begin, I note that uneven distribution emerges when two areas with the same racial composition – in the White-Black comparison, the same area proportion White (p) – exchange a White and Black household. The area receiving the White household and losing the Black household now has a higher proportion White and the area losing the White household and receiving the Black household now has a lower proportion White. Reversing the exchange restores even distribution. Accordingly, an "integration-promoting exchange" is one in which the White household in the exchange moves from an area where proportion White (p_i) is higher to an area where proportion White (p_j) is lower (i.e., $p_i > p_j$) *and* the Black household in the exchange moves from an area where proportion White (p_j) is lower to an area where proportion White (p_i) is higher (Reardon and Firebaugh 2002:38). Conversely, a "segregation-promoting exchange" is one in which the White household in the exchange moves from an area where proportion White (p_i) is lower to an area where proportion White (p_j) is higher (i.e., $p_i < p_j$) *and* the Black household in the exchange moves from an area where proportion White (p_j) is lower.

In the theory of segregation measurement, the "exchange" criterion holds that indices should register all integration-promoting and segregation-promoting exchanges by decreasing or increasing in value, respectively, when the exchange is completed (Reardon and Firebaugh 2002). The separation index (S) meets this criterion. The dissimilarity index (D) does not.

I note that it is reasonable to term segregation-promoting exchanges as "polarizing" and "concentrating" and it is similarly appropriate to term integration-promoting exchanges as "depolarizing," "deconcentrating", and "dispersing." A segregation-promoting exchange is "polarizing" because it moves the two areas involved in the exchange further apart on area proportion White since $|p_i - p_j|$ is larger after the exchange is completed. At the same time, the exchange is "concentrating" because pairwise same-group contact goes up for both Whites and Blacks in the affected areas. Since the residential distribution of Whites and Blacks in other areas is unchanged, the result of the exchange is greater overall area polarization, greater overall group concentration, greater overall group separation, and a higher value of S.

An integration-promoting exchange is "depolarizing" because it moves the two areas involved in the exchange closer together on area proportion White since $|p_i - p_j|$ is smaller after the exchange is completed. At the same time, the exchange is "deconcentrating" because pairwise same-group contact goes down for both Whites and Blacks in the affected areas. Again, since the residential distribution of Whites and Blacks in other areas is unchanged, the exchange reduces overall area polarization, reduces overall group concentration, reduces overall group separation, and lowers the value of S.

Based on this, it is clear that the underlying logic of the separation index (S) resonates well with the exchange criterion. In contrast, the underlying logic of the dissimilarity index (D) is often at odds with the criterion. D registers integration-promoting exchanges only in the circumstance that the racial composition of the two areas involved in the exchange are on opposite sides of P, proportion White for the city overall. Integrating-promoting exchanges that involve areas with racial compositions on the same side of P have no impact on D.

In addition to meeting the minimum requirements for satisfying the exchange criterion, the separation index (S) has additional properties that in my opinion are desirable for assessing whether groups live apart or together. I list them as follows.[7]

- All else equal, an integration-promoting exchange produces a larger reduction in S when the two areas involved in the exchange are more polarized.

  I term this the "polarization" property with polarization or dispersion being based on the initial size of $|p_i - p_j|$. Substantively, this is appealing because, assuming area size is constant, exchanges between more polarized areas reduces same group contact for larger fractions of the affected population.

  No surprisingly, D does not have this property.

- All else equal, an integration-promoting exchange produces a larger reduction in S when the two areas involved in the exchange are closer to one of the polarization boundaries of all-White or all-Black. That is, the reduction is larger when the minimum of the two values $|p_i - 1|$ and $|p_j - 0|$ is closer to 0.0.

  The substantive appeal of this characteristic is similar to that for the "polarization" property. Here again exchanges that involve areas that are nearer to the homogeneous "poles" of 0 and 1 reduce same group-contact for larger fractions of the affected population.

  D does not have this property.

- The "polarization" property holds throughout the full range of area proportion White (p). Thus, in contrast to D, integration-promoting exchanges have desirable impacts on reducing S regardless of whether the two areas involved in the

[7] I establish these properties by drawing on previous methodological discussions (e.g., Zoloth 1976; James and Taeuber 1985; Reardon and Firebaugh 2002) and by simulation analyses that systematically exercise the possible "event-space" of exchanges between areas in a model city.

exchange have racial composition on opposite sides of P – the racial composition of the city overall – or on the same side of P.

This is substantively attractive because it is nonsensical to limit the principle of exchanges to apply to exchanges on opposite sides of P (i.e., where $p_i > P > p_j$). It is possible to achieve integration by making only exchanges of this nature. But substantial integration also can be achieved with exchanges on the same side of P (i.e., where $p_i > p_j > P$ or $P > p_i > p_j$.

There is no substantive basis for ignoring the impact of integration-promoting exchanges involving areas with racial compositions on the same side of P.

7.3.2 Examples of D-S Differences in Responding to Integration-Promoting Exchanges

To illustrate selected points from the preceding discussion, I compare four integration-promoting exchanges for a hypothetical city that is populated by only White and Black households and has an overall proportion White of 0.50. For simplicity, I assume all areas are the same size and are populated with 100 households. Under these assumptions, relative impact of an exchange on S is strictly determined by the impact the exchange has on the White-Black difference in segregation-relevant average contact with Whites (p) for the 200 households residing in the two areas involved in the exchange.[8] For the purposes of this discussion I will designate this difference with the Greek letter lambda (λ) and express it in percentage form (instead of as proportions) for ease of presentation and discussion.

Figure 7.2 presents results for two pairs of hypothetical exchanges. The first panel summarizes results for a pair of integration-promoting exchanges that involve areas on opposite sides of P, one above parity and the other below parity. The second panel summarizes results for a pair of integration-promoting exchanges that involve two areas that are not above parity. I begin by discussing the pair of exchanges in the first panel. The first exchange shown involves two areas that are highly polarized on racial composition. The first area (Area 1) is an all-White area with 100 White and 0 Black households. The second area (Area 2) is all-Black area with 0 White and 100 Black households. The integration-promoting exchange moves a White household from Area 1 (higher p) to Area 2 (lower p) and a Black household from Area 2 (lower p) to Area 1 (higher p). Following the exchange, Area 1 has 99 White households and 1 Black household and Area 2 has 1 White household and 99 Black households.

The integration-promoting exchange could be imagined as two "pioneering" residential moves. For example, the exchange could involve the moves of a "pioneering" Black household and a "gentrifying" White household. The pioneering Black household leaves a predominantly Black neighborhood and moves to a pre-

[8] The racial composition of all other areas remains unchanged. So the any change in S derives solely from the impact of changes in the areas involved in the exchange.

Two Examples of Exchanges Involving Areas on Opposite Sides of P

	First Exchange		Second Exchange	
Area Population Distributions	White N	Black N	White N	Black N
Area 1 – Before Exchange	100	0	51	49
Area 2 – Before Exchange	0	100	49	51
Area 1 – After Exchange	99	1	50	50
Area 2 – After Exchange	1	99	50	50
Impact on Index	S	D	S	D
Initial White Mean (y·100)	100.00	100.00	50.02	51.00
Initial Black Mean (y·100)	0.00	0.00	49.98	49.00
λ Before Exchange (x100)	100.00	100.00	0.04	2.00
Final White Mean (y·100)	98.02	99.00	50.00	0.00
Final Black Mean (y·100)	1.98	1.00	50.00	0.00
λ After Exchange (x100)	96.04	98.00	0.00	0.00
λ Change (x100)	−3.96	−2.00	−0.04	−2.00

Two Examples of Exchanges Involving Below-Parity Areas

	Third Exchange		Fourth Exchange	
Area Population Distributions	White N	Black N	White N	Black N
Area 1 – Before Exchange	49	51	26	74
Area 2 – Before Exchange	1	99	24	76
Area 1 – After Exchange	48	52	25	75
Area 2 – After Exchange	2	98	25	75
Impact on Index	S	D	S	D
Initial White Mean (y·100)	48.04	0.00	25.04	0.00
Initial Black Mean (y·100)	17.32	0.00	24.99	0.00
λ Before Exchange	30.72	0.00	0.05	0.00
Final White Mean (y·100)	46.16	0.00	25.00	0.00
Final Black Mean (y·100)	17.95	0.00	25.00	0.00
λ After Exchange (x100)	28.21	0.00	0.00	0.00
λ Change (x100)	−2.51	0.00	−0.05	0.00

Fig. 7.2 Impacts of selected integration-promoting exchanges on the value of the separation index (S) and the dissimilarity index (D)

dominantly White neighborhood. The "gentrifying" White household leaves a predominantly White neighborhood and moves to a predominantly Black neighborhood.

In the difference of means framework, the impact of the exchange on an index score can be assessed by considering how segregation-relevant residential outcomes (y) change for the 200 households in the affected neighborhoods. For S, y is simply area proportion White $(y = p)$ so average contact with Whites is initially 100.0 points for Whites and 0.0 points for Blacks. This yields a value of λ – the White-Black average difference for the population in the two areas – of 100.0 points. After the exchange, average contact with Whites falls to 98.02 points for Whites in the two areas and rises to 1.98 points for Blacks, producing a value of λ of 96.04 points. Thus, the exchange causes the average White-Black contact difference for the subset of affected households – quantified as λ – to fall by 3.96 points.

The second integration-promoting exchange involves White and Black households residing in two areas that differ only slightly on racial composition. Before the exchange the first area (Area 1) has 51 White and 49 Black households and the second area (Area 2) has 49 White and 51 Black households. After the exchange Area 1 and Area 2 both change to 50 White and 50 Black households thus bringing about integration. For the subset of households in the affected households, average contact with Whites is initially 50.02 points for Whites and 49.98 points for Blacks, producing a White-Black difference (λ) of 0.04 points. After the exchange, average contact with Whites falls to 50.0 points for Whites and rises to 50.00 points for Blacks, producing a White-Black difference (λ) of 0.00 points. Thus, the exchange causes the average White-Black contact difference for the subset of affected households (λ) to fall, but only by 0.04 points.

The larger reduction in λ for S in the first exchange compared to the second exchange −3.96 points versus −0.04 points – highlights a property of S discussed above and noted previously by Zoloth (1976), James and Taeuber (1985), and Reardon and Firebaugh (2002). The property is that S responds more strongly to integration-promoting exchanges between areas that are more polarized in terms of area racial composition (i.e., exchanges when $| p_i - p_j |$ is larger). The reduction in the first exchange is larger by 3.92 points than the reduction in the second exchange and in relative terms is 99 times larger.

I view this as sensible and desirable. In substantive terms the first exchange has a larger impact on reducing group separation because it does more to "deconcentrate" the group distributions across the two areas because it brings together White and Black households from areas that initially were at opposite extremes on area racial composition.

The first exchange reduces White's contact with Whites by a larger amount – 1.98 points compared to 0.02 points – while simultaneously increasing Black's contact with Whites by a larger amount – 1.98 points compared to 0.02 points. As a result, the first exchange reduces the White-Black difference in contact with Whites by a larger amount. In contrast, the second exchange has a small impact on reducing group separation because it brings people together from areas that initially were minimally different on area racial composition. Accordingly, the exchange has less impact on group separation as measured by S because the affected White and Black households were already living together.

The relative impact of these integration-producing exchanges on D can be assessed by calculating lambda (λ) in the same manner as just performed for S. The only difference is that segregation-relevant contact with Whites (y) is scored differently for D than for S. Specifically, contact with Whites (y) is scored 1 for "above parity" and 0 otherwise. For D, the relative impact of the exchanges on the value of D is the same for both of the exchanges. Specifically, the White-Black difference in average (scaled) contact with Whites for the affected households (λ) is reduced by two points under both scenarios. The reason for this is that in both cases a single White household changes from being scored 1 for "above parity" to being scored 0 for "not above parity." Similarly in both cases only a single Black household changes from being scored 0 for "not above parity" to being scored 1 for "above parity". The initial average on contact with Whites as measured by D is 100.0 for Whites and 0.0 for Blacks, producing an average White-Black difference of 100.0 for the population in the affected neighborhoods. After the exchange, the average on contact with Whites as measured by D is 99.0 for Whites and 1.0 for Blacks resulting in a difference of 98.0. The exchange thus reduces the value of λ by 2.0 points.

The second exchange also produces a reduction in the value of λ of 2.0 points. In this case, average contact with Whites as measured by D is initially 51.0 for Whites and 49.0 for Blacks, producing an average White-Black difference of 2.0 for the population in the affected neighborhoods. After the exchange, the average on contact with Whites as measured by D is 0.0 for Whites and 0.0 for Blacks resulting in a difference of 0.0 since now no one in either group lives in an "above-parity" area. The exchange thus reduces the value of λ by 2.0 points, a reduction identical to the amount in first exchange.

The "flat" or "fixed" response of the relative impact of λ on D can be seen as appropriate for the goal of assessing "displacement" conceived narrowly in terms of population fractions moving from one side of parity to the other. These fractions are the same for both exchange scenarios, so λ is the same for both scenarios. The fact that the two exchanges in question have fundamentally different effects on group separation and area polarization is not relevant to the narrow conception of displacement embodied in D.

The contrast of the flat response for D for these two exchanges and the variable response for S highlights how displacement and separation are distinct and can vary independently. This point is further established by considering the pair of integration-promoting exchanges summarized in the second panel of Fig. 7.2. The most important difference between this pair of exchanges and the pair summarized in the top panel is that both of the areas in the bottom panel are "below-parity" on area proportion White.

The third exchange depicted involves one area (Area 1) with 49 White and 51 Black households and a second area (Area 2) with 1 White and 99 Black households. Both are "below-parity" areas. The integration-promoting exchange involved moves a White household from Area 1 (higher p) to Area 2 (lower p) and a Black household from Area 2 (lower p) to Area 1 (higher p). Following the exchange Area 1 changes to 48 White and 52 Black households and Area 2 changes to 2 White and 98 Black households.

This exchange involves two areas that differ substantially on racial composition with values on area proportion White of 49.0 and 1.0, respectively. The impact on S can be assessed as before by examining the value of λ – the White-Black difference on segregation-relevant contact with Whites for the population in the affected areas. Initially, average contact with Whites as measured for S is 48.04 for Whites and 17.32 for Blacks yielding a value of λ of 30.72. After the exchange, average contact with Whites as measured for S is 46.16 for Whites and 17.95 for Blacks yielding a value of λ of 2.21. Thus, under this exchange scenario the White-Black contact difference (λ) for the subset of affected households is reduced by 2.51 points.

The fourth exchange involves two areas that together have the same number of White and Black households as in the two areas in the third exchange; 50 White and 150 Black households, respectively. The initial distribution is less polarized than in the previous example. One area (Area 1) begins with 26 White and 74 Black households and a second area (Area 2) that begins with 24 White and 76 Black households. The integration-promoting exchange involves moving one White household from the area of higher p to the area with lower p and moving one Black moving from the area of lower p to the area of higher p. Following the exchange, Area 1 and Area 2 both change to having 25 White and 75 Black households. As with the third exchange, this exchange involves two areas that are "below-parity". Here, however, the two areas initially are very similar on racial composition with area proportion White at 0.26 and 0.24, respectively. As a result, the White-Black contact difference (λ) for the affected households changes by a very small amount. Initially, average contact with Whites as measured by S is 25.04 for Whites and 24.99 for Blacks yielding a value of λ of 0.05. After the exchange, average contact with Whites as measured by S is 25.00 for Whites and 25.00 for Blacks yielding a value of λ of 0.0. Thus, the exchange reduces the White-Black contact difference (λ) for the subset of affected households by 0.05 points.

There are two key findings. One is that both exchanges produce reductions in S whereas we will soon see that neither exchange produces a reduction in D. Another key finding is that the impact on reducing S is much larger in the third exchange than in the fourth exchange. The reduction in the third exchange is 2.46 points larger than in the fourth exchange and in relative terms is almost 50 times larger. Again, considered in relation to the goal of assessing whether groups live together or apart, it is substantively sensible that the third exchange has a bigger relative impact on S than the fourth exchange. As previously seen in the first exchange, the third exchange does more to "deconcentrate" the group distributions. The third exchange reduces White's contact with Whites by a larger amount than the fourth exchange – 1.88 points compared to 0.04 points, respectively – while simultaneously increasing Black's contact with Whites by a larger amount – 0.63 points compared to 0.01 points, respectively.

In substantive terms, the third exchange could be imagined to reflect a "middle-stage" integrating sequence where a pioneering Black household leaves a predominantly Black neighborhood and moves to diverse (50/50) neighborhood and a gentrifying White household leaves a 50/50 area and moves to a predominantly Black area. In contrast, the fourth exchange is a small-impact integrating exchange. Like the second exchange reviewed earlier, the two areas involved are near-identical in terms of racial composition before the exchange so on balance the households affected by the exchange experience minimal changes in neighborhood outcomes and very small reductions in pairwise same-group contact.

The response of D in the third and fourth exchanges in the second panel is easy to summarize. D does not change in either case because all households in both groups reside in areas that are "below-parity" both before and after the exchanges. So again, D has a flat response of no change while S registers a decline in both exchanges. The response by S varies from the response by D in two ways. First, S responds to both integrating moves while D does not. Second, S responds more strongly to the third exchange which clearly reduces group separation and area racial polarization by a larger amount.

One implication from the comparison of how S and D are affected by these two exchanges is readily obvious. It is that the value of D can remain fixed while the value of S can run higher or lower depending on whether integrating moves involving areas that are not above parity reduce polarization or whether segregating moves increase polarization. I discuss this more carefully in the next section.

7.3.3 Implications of Analysis of Example Exchanges

A couple of important implications follow from these examples of how D and S respond to exchanges. I start first with integration-promoting exchanges where both areas involved in the exchange are on opposite sides of P. In these exchanges D has a "flat" response to all integration-promoting exchanges; its value declines by the same amount in all cases. In contrast, S will respond more strongly when the exchange is between more polarized areas (and therefore more distant from parity) and S will respond less strongly when the exchange is between areas of similar racial composition (and therefore closer to parity). This leads to the following conclusion.

> *Values of D and S can be similar or they can diverge depending on whether displacement from uneven distribution arises from segregation-promoting exchanges that produce maximally polarized areas (higher S, closer to D) or minimally polarized areas (lower S, further from D) on opposite sides of P.*

The second important implication concerns integration-promoting exchanges where both areas involved in the exchange are on the same side of P. D again has a "flat" response. It does not change. In contrast, S will always respond and S will respond more strongly when the exchange is between more polarized areas and S will respond less strongly when the exchange is between areas of similar racial composition. This leads to the following conclusions.

> *Values of D and S can be similar or they can diverge depending on whether displacement from uneven distribution arises from segregation-promoting exchanges that produce maximally polarized areas (higher S, closer to D) or minimally polarized areas (lower S, further from D) on the same side of P.*

Stated another way, S will take higher values when the population residing in non-parity areas is concentrated to form racially polarized areas and S will take lower values when the population residing in non-parity areas is dispersed widely to form areas that are similar on racial composition instead of being polarized.

The practical consequence for D-S comparisons is this. At a given level of displacement as measured by the dissimilarity index (D), the value of the separation index (S) can vary independently and by substantial amounts depending on whether group distributions both *between* "above-parity" areas and "other" areas and *within* "non-parity" areas tend toward maximum area racial polarization or minimum area racial polarization. The former concentrates both groups in homogeneous areas and maximizes same-group contact and group separation. The latter disperses both groups across less homogeneous areas and minimizes same-group contact and group separation.

Ultimately, as I show below, this leads to the following conclusion about the relationship between D and S. At a given level of displacement (D), the value of the separation index (S) can vary substantially depending on whether group distributions within "non-parity" areas tend toward concentration or dispersion. When concentration within non-parity areas is at its maximum, the value of S will equal the value of D. But when concentration is at its minimum – that is, when groups are maximally dispersed across non-parity areas, the value of S will be lower, sometimes much lower, than the value of D.

Intuitively, one can get to these two alternative outcomes via simple steps as follows. At a given level of displacement, implement as many segregation-promoting exchanges as possible *within* non-parity areas. If such exchanges can be made, group residential distributions will shift toward the pattern of "prototypical segregation" and the value of S will increase. The value of D will not change so the D-S disparity will decrease. Ultimately, the value of S will rise until it reaches the value of D and D-S disparity will be zero.

Alternatively, implement as many integration-promoting exchanges as possible *within* non-parity areas. If such exchanges can be made, group residential distributions will shift toward the pattern of "dispersed displacement" and the value of S will decrease. The value of D will not change so D-S disparity will increase. Ultimately, S will fall until it reaches its minimum possible level and the D-S disparity reaches its maximum. At the conclusion of the process, S will take a value substantially below the value of D.

7.4 Clarifying the Potential for D-S Concordance and Discordance – Analytic Models

I further clarify the potential for both D-S agreement and disagreement by reviewing a series of analytic exercises that illustrate how group separation and area polarization (S) can vary independently from the level of displacement as measured by dissimilarity (D) while holding city-level racial composition (P) constant. To keep the exercises simple and easier to follow, I limit the hypothetical city to only three kinds of neighborhoods designated as Areas 1, 2, and 3 with the following characteristics.

Area_1 is "Above parity" (i.e., disproportionately White with $p_1 > P$ and $q_1 < Q$)
Area_2 is at "Parity" (i.e., exactly average on proportion White with $p_2 = P$ and $q_2 = Q$)
Area_3 is "Below parity" (i.e., disproportionately Black with $p_3 < P$ and $q_3 > Q$)

The model can be extended to allow for more variation in area racial composition, but this provides no benefit for present purposes.

I first note that, at a given level of displacement from even distribution as registered by D, S will take its maximum value of $S = D$ when the population residing in non-parity areas is maximally concentrated. This occurs when non-parity areas are either all-White or all-Black and thus are perfectly "polarized" as either 1.0 or 0.0 on area proportion White (p_i). This result can be produced by a "Maximum Concentration" or "Maximum S" algorithm involving three steps as follows.

1. Set the share of Whites in Area_1 to D (i.e., $s_{W1} = w_1 / W = D$). Proportion White in the area will be 1.0 (i.e., $p_1 = 1.0$).
2. Set the share of Blacks in Area_3 to D (i.e., $s_{B3} = b_3 / B = D$). Proportion White will be 0.0 (i.e., $p_3 = 0.0$).
3. Place remaining Whites and Blacks in Area_2. Area share scores for Whites and Blacks will be $s_{W2} = s_{B2} = (1 - D)$ and proportion White for the area will be at parity (i.e., $p_2 = P$).

The resulting group distributions will produce a distinctive "four-point" segregation curve that Duncan and Duncan (1955: Figure 5) termed a "William's model" segregation curve. In this distribution, S takes its maximum possible value (S_{Max}) under the prevailing level of displacement from even distribution with $S_{Max} = D$.

This establishes that logical upper bound on separation (S) is the level of displacement (D). In addition, since D can vary independently of racial composition (P) and S can always match D, this result also establishes that group separation (S) can vary independently of city racial composition (P). This finding lays to rest any claim that the value of S is inherently dependent on city racial composition. S can match D when displacement is concentrated. Whether displacement is concentrated or not depends on sociological dynamics governing population distribution, not the inherent nature of S.

The next issue to take up is whether D and S can vary independently. This is relatively easy to establish as S will take a lower value than D when groups residing in non-parity areas are dispersed rather than concentrated.[9] When groups residing in non-parity areas are concentrated, higher values of S result and in the situation of complete concentration S reaches a maximum value of D. When groups residing in non-parity areas are dispersed widely, values of S will be substantially lower than values of D. When groups are exactly equal in size, a relatively uncommon but logically possible situation, the value of S can fall to at least D^2. In cases where groups are unequal in size, values of S can fall well below D^2 and in some circumstances S can potentially fall to very low values.[10]

A variety of algorithms will produce patterns of dispersed displacement from even distribution that yield low values of S while maintaining a specified value of D. In a more detailed discussion of this issue (Fossett 2015) I review a progression of algorithms. For present purposes, I introduce an algorithm that produces the lowest levels of S I have been able to obtain under the three-area scenario under discussion. This "Minimum S" (S_{Min}) algorithm actually uses just two areas, one area that is "above parity" and one that is "below parity".

The algorithm to obtain S_{Min} involves two variations which I term here Model A1 and Model A2. Each version will produce the lower value of S over some ranges of city racial composition (P) as follows.

$$
\begin{aligned}
&\text{if } (P>0.5)\ S_{MinA1} = S_{Min} \le S_{Min\,A2}\\
&\text{if } (P=0.5)\ S_{Min\,A1} = S_{Min} = S_{Min\,A2}\\
&\text{if } (P<0.5)\ S_{Min\,A1} \ge S_{Min} = S_{Min\,A2}
\end{aligned}
$$

Accordingly, one can obtain the value of S_{Min} by assigning the value of S generated by Model A1 when $P \ge 0.5$ and the value of S generated by Model A2 when $P \le 0.5$.

Both versions of the algorithm proceed to an intermediate step with one homogeneous area and one mixed area. The A1 version of the algorithm begins as follows.

A1 Step 1. For $Area_1$, set the group share for Whites (s_{W1}) to D and the group share for Blacks (s_{B1}) to 0.0.

A1 Step 2. For $Area_3$, set the group share of Whites (s_{W3}) to $1-D$ and the group share for Blacks (s_{B3}) to 1.0.

This produces an "above-parity" area ($Area_1$) that is all-White and a "below-parity" area ($Area_3$) that is mixed White and Black.

Similarly, the A2 version of the algorithm begins as follows.

---

[9] The exceptions are when D is close to boundary values of 0 and 1.0. Under these conditions, scores for all popular measures of uneven distribution will agree.

[10] I offer this conclusion based on exercising the models discussed here over the full "event space" of possible combinations of D and P. In all instances where $0 < D < 1$, I obtained values of S below the value of D^2.

A2 Step 1. For $Area_1$, set the group share for Whites (s_{W1}) to 1.0 and the group share for Blacks (s_{B1}) to $1-D$.

A2 Step 2. For $Area_3$, set the group share for Whites (s_{W3}) to 0.0 and the group share for Blacks (s_{B3}) to D.

This produces an "above-parity" area ($Area_1$) that is mixed White and Black and a "below-parity" area ($Area_3$) is all-Black.

For most logically possible combinations of D and P the value of S can be reduced even further by transferring an optimal amount (X) of equal shares of Whites and Blacks from the "mixed" area to the homogeneous area to reduce concentration (increase dispersion). For Model A1 these transfers move equal group shares (X) of Whites and Blacks from $Area_3$, which is mixed, to $Area_1$, which is all-White. For Model A2, these transfers move equal group shares (X) of Whites and Blacks from $Area_1$, which is mixed, to $Area_3$, which is all-Black.

The value of D is unaffected when equal group shares are transferred from one area to another. But the transfers can have substantial impacts on the value of S. A wide range of alternative group transfer share values are logically possible subject to the restriction that the transfers cannot produce area group share values below 0.0 or above 1.0. The task is to find the optimal value (X) that will reduce the value of S to S_{Min}, the lowest possible value under the three-area model under consideration. One strategy is to conduct a numerical search over the feasible values of X. I developed algorithms that implemented this approach and used them to establish benchmarks for what is possible. Using this approach I found I could obtain the same result for S_{Min} regardless of whether starting from the residential distributions created at the intermediate steps of Model A1 or Model A2.

With additional exploration I discovered that the same residential distributions and resulting value of S_{Min} can be obtained using a direct analytic solution. This solution involves modifying the transfer of equal group shares so it brings the share of total population (i.e., Whites and Blacks combined) in "above-parity" and "below-parity" areas as close to 0.5 as possible. This is accomplished as follows. First, identify the range of logically possible share transfer values (X) that will maintain the value of D. These will range from a minimum of 0.0 to a maximum of (1-D). Next calculate the value of $|s_{T3}-0.5|$, the unsigned difference between the total population share in $Area_3$ and 0.5. Tentatively adopt this as the share amount (X) to be transferred. If the value of X is larger than the maximum feasible value $(1-D)$, set the group transfer share value (X) to $(1-D)$. In other words, set X to the minimum of $|s_{T3}-0.5|$ and $(1-D)$. Next implement the transfer of the identified group share amounts (X) from the mixed area to the homogeneous area.

Thus, when $P \geq 0.5$, use the A1 algorithm with these additional steps.

A1 Step 3. Set the optimal share (X) of Whites and Blacks to transfer from the mixed area ($Area_3$) to the all-White area ($Area_1$) as the minimum of $|s_{T3}-0.5|$ and $(1-D)$.

A1 Step 4. Implement the transfer, thus increasing s_{W1} to $D+X$ and s_{B1} to X and reducing s_{W3} to $1-D-X$ and s_{B3} to $1-X$.

When $P \leq 0.5$, use the A2 algorithm with these additional steps.

A2 Step 3. Set the optimal share (X) of Whites and Blacks to transfer from the mixed area ($Area_1$) to the all-Black area ($Area_3$) as the minimum of $|s_{T3} - 0.5|$ and $(1 - D)$.

A2 Step 4. Implement the transfer, thus reducing s_{W1} to $1 - X$ and s_{B1} to $1 - D - X$ and increasing s_{W3} to X and s_{B3} to $D + X$.

7.4.1 Examples of Calculating Values of S_{Min} Given Values of D and P

Figure 7.3 provides a summary listing of formulas for calculating terms relating to group residential distributions under the "Maximum S" and "Minimum S" analytic models just introduced. I establish the basis for the formulas in a more detailed review of analytic models for group separation (Fossett 2015). The formulas in Fig. 7.3 establish how, in the context of the three-area analytic model considered here, algorithms for dispersed and concentrated displacement will generate group residential distributions producing lower and higher values of group separation (S) under a given combination of fixed values for displacement (D) and city racial composition (P). As best I have been able to determine, the formulas in Fig. 7.3 establish the logically possible range for S under a given combination of D and P by yielding the minimum possible value for S (S_{Min}) under dispersed displacement and the maximum possible value for S (S_{Max}) under concentrated displacement.

In this section I review examples to illustrate how values of S_{Min} and S_{Max} can be calculated for a given combination of displacement (D) and city racial composition (P). The value of S under the "Maximum S" algorithm can be obtained by using the formulas in Fig. 7.3 to first establish the values of relevant component terms – area group share distributions (s_{Wi} and s_{Bi}) and area group proportions (p_i) – used in computing formulas for S and then carry through the calculations to obtain S.

Consideration of the two general computing formulas for S given below (as well as earlier) reveals that the "parity area" in the three-area analytic model under consideration can be ignored because calculations for this area yield values of zero (0) and have no impact on the value of S.

$$S = \Sigma s_{Ti} (p_i - P)^2 / PQ, \text{ and}$$

$$S = \Sigma s_{Wi} \cdot p_i - \Sigma s_{Bi} \cdot p_i .$$

The value of S thus results from the calculations for the "above parity" and "below parity" areas and can be given as either

$$S = (s_{W1} \cdot p_1 + s_{W3} \cdot p_3) - (s_{B1} \cdot p_1 + s_{B3} \cdot p_3), \quad \text{or}$$

	Area 1 Above Parity ($p_i > P$)	Area 2 Parity ($p_i = P$)	Area 3 Below Parity ($p_i < P$)
	S_{Max}, S = D under Maximum Concentration Model		
White Share (s_{Wi})	D	1−D	---
Black Share (s_{Bi})	---	1−D	D
Total Share (s_{Ti})	PD	1−D	QD
Prop. White (p_i)	1	P	0
	$S_{Min\ A1}$, S under Dispersed Displacement Model A1		
White Share (s_{Wi})	D+X	---	1−D−X
Black Share (s_{Bi})	X	---	1−X
Total Share (s_{Ti})	PD+X	---	1−PD−X
p_i	P(D+X)/(PD+X)	---	P(1-D-X)/(1-PD-X)
	$S_{Min\ A2}$, S under Dispersed Displacement Model A2		
White Share (s_{Wi})	1−X	---	X
Black Share (s_{Bi})	1−D−X	---	D+X
Total Share (s_{Ti})	P+Q(1−D)−X	---	QD+X
Prop. White (p_i)	P(1-X)/(1-QD-X)	---	PX/(QD+X)

Fig. 7.3 Summary of formulas for group residential distributions by level of dissimilarity (D) and racial composition (P) under selected algorithms for producing concentrated and dispersed displacement from even distribution (Notes: Per discussion in text, $X = \min(|s_{T3} - 0.5|, (1 - D))$ where S_{T3} is $(1 - PD)$ under Model A1 and QD under Model A2, respectively)

$$S = (s_{W1} - s_{B1})p_1 + (s_{W3} - s_{B3})p_3.$$

Taking the example combination of displacement as measured by the dissimilarity index (D) set to 60 and pairwise city proportion White (P) set to 0.90, the resulting value of S under the "Maximum S" Model can be obtained by first establishing the values of relevant component terms and then carrying through the computations. The relevant component terms for the non-parity areas can be obtained as follows.

$$s_{W1} = D = 0.60$$

$$s_{W3} = 0.0$$

$$s_{B1} = 0.0$$

$$s_{B3} = D = 0.60$$

$$\left(s_{W1} - s_{B1}\right) = D - 0.0 = D = 0.60$$

$$\left(s_{W3} - s_{B3}\right) = 0.0 - D = -D = -0.60$$

$$p_1 = 1.0$$

$$p_3 = 0.0$$

The following calculations now demonstrate that $S_{Max} = D.$

$$\begin{aligned} S &= \left(s_{W1} \cdot p_1 + s_{W3} \cdot p_3\right) - \left(s_{B1} \cdot p_1 + s_{B3} \cdot p_3\right) \\ &= \left(0.60 \cdot 1.0 + 0.0 \cdot 0.0\right) - \left(0.0 \cdot 1.0 + 0.60 \cdot 0.0\right) = 0.60 \end{aligned}$$

$$S = \left(s_{W1} - s_{B1}\right)p_1 + \left(s_{W3} - s_{B3}\right)p_3 = 0.60 \cdot 1.0 + -0.60 \cdot 0.0 = 0.60$$

This expression reveals something interesting and important. It is this.

> *The value of P is not directly involved in the formulas for the component terms. This indicates that the value of S_{Max} is unaffected by city racial composition. Accordingly, under concentrated displacement, S can equal D for any city racial composition.*

The calculations for "Minimum S" (S_{Min}) under dispersed displacement are more involved. Model A1 applies when city racial composition (P) is ≥ 0.50 and thus would be the relevant model for most White-Minority comparisons in US cities. Model A1 also is relevant for the example just considered where D is 60 and pairwise city proportion White (P) is 0.90. The value of S under Model A1 can be obtained by first establishing the values of relevant component terms and then carrying through subsequent calculations. The relevant component terms can be obtained as follows.

$$\begin{aligned} X &= \min\left(\left|\left(1 - PD\right) - 0.5\right|, \left(1 - D\right)\right) = \min\left(\left|\left(1 - 0.90 \cdot 0.60\right) - 0.5\right|, \left(1 - 0.60\right)\right) \\ &= \min\left(\left|\left(1 - 0.54\right) - 0.5\right|, 0.40\right) = \min\left(\left|0.46 - 0.5\right|, 0.40\right) = \min\left(0.04, 0.40\right) \\ &= 0.04 \end{aligned}$$

$$s_{W1} = D + X = 0.60 + 0.04 = 0.64$$

$$s_{W3} = \left(1 - D - X\right) = 1 - 0.60 - 0.04 = 0.36$$

$$s_{B1} = X = 0.04$$

$$s_{B3} = 1 - X = 1 - 0.04 = 0.96$$

$$\left(s_{W1} - s_{B1}\right) = \left(D + X\right) - X = D = 0.60$$

$$\left(s_{W3} - s_{B3}\right) = \left(1 - D - X\right) - \left(1 - X\right) = -D = -0.60$$

$$\begin{aligned} p_1 &= P(D+X)/(PD+X) = 0.90(0.60+0.04)/(0.90\cdot 0.60+0.04) \\ &= 0.576/0.58 = 0.9930 \end{aligned}$$

$$\begin{aligned} p_3 &= P(1-D-X)/(1-PD-X) = 0.90(1-0.60-0.04)/(1-0.90\cdot 0.60-0.04) \\ &= (0.90\cdot 0.36)/(1-0.58) = 0.324/0.42 = 0.7714 \end{aligned}$$

Note that ($s_{W1} - s_{B1}$) resolves to D and ($s_{W3} - s_{B3}$) resolves to –D. As a result, the expression

$$S = (s_{W1} - s_{B1})p_1 + (s_{W3} - s_{B3})p_3$$

can be restated in the following convenient computing formula.

$$S = D(p_1 - p_3)$$

The following calculations illustrate that any of the three expressions can be used to obtain $S_{Min} = 0.1330$ under Model A1.

$$\begin{aligned} S &= (s_{W1}\cdot p_1 + s_{W3}\cdot p_3) - (s_{B1}\cdot p_1 + s_{B3}\cdot p_3) \\ &= (0.64\cdot 0.9930 + 0.36\cdot 0.7714) - (0.04\cdot 0.9930 + 0.96\cdot 0.7714) \\ &= (0.6355 + 0.2777) - (0.0397 + 0.7405) = 0.9132 - 0.7802 = 0.1330 \end{aligned}$$

$$\begin{aligned} S &= (s_{W1} - s_{B1})p_1 + (s_{W3} - s_{B3})p_3 = (0.64-0.04)0.9930 + (0.36-0.96)0.7714 \\ &= 0.60\cdot 0.9930 - 0.60\cdot 0.7714 = 0.5958 - 0.4628 = 0.1330 \end{aligned}$$

$$S = D\cdot(p_1 - p_3) = 0.60\cdot(0.9930 - 0.7714) = 0.60\cdot 0.2216 = 0.1330$$

Model A2 applies when the city racial composition (P) is ≤ 0.50. Typically this model is not relevant for most White-Minority comparisons in US cities. But it is occasionally relevant, perhaps most often for White-Latino comparisons in the border region of the southwestern United States. As with Model A1, the value of S can be obtained from any of the following three equivalent expressions.

$$S = (s_{W1}\cdot p_1 + s_{W3}\cdot p_3) - (s_{B1}\cdot p_1 + s_{B3}\cdot p_3)$$

$$S = (s_{W1} - s_{B1})p_1 + (s_{W3} - s_{B3})p_3$$

$$S = D(p_1 - p_3)$$

Thus, for example, if D is 60 and pairwise city proportion White (P) is 0.30 (similar to the value of P for many White-Latino comparison in Texas border region cities) the resulting value of S under Model A2 can be obtained by first establishing the values of relevant component terms and then carrying through computations. The relevant component terms are as follows.

$$X = \min\left(|\left(1 - QD\right) - 0.5|, \left(1 - D\right)\right)$$
$$= \min\left(|\left(1 - 0.70 \cdot 0.60\right) - 0.5|, (1 - 0.60)\right)$$
$$= \min\left(|\left(1 - 0.42\right) - 0.5|, 0.40\right) = \min\left(|0.58 - 0.5|, 0.40\right) = \min\left(0.08, 0.40\right)$$
$$= 0.08$$

$$s_{W1} = 1 - X = 1 - 0.08 = 0.92$$

$$s_{W3} = X = 0.08$$

$$s_{B1} = 1 - D - X = 1 - 0.60 - 0.08 = 0.32$$

$$s_{B3} = D + X = 0.60 + 0.08 = 0.68$$

$$\left(s_{W1} - s_{B1}\right) = \left(1 - X\right) - \left(1 - D - X\right) = \left(1 - 1\right) + D + \left(X - X\right) = D = 0.60$$

$$\left(s_{W3} - s_{B3}\right) = X - \left(D + X\right) = -D = -0.60$$

$$p_1 = P\left(1 - X\right) / \left(1 - QD - X\right)$$
$$= 0.30\left(1 - 0.08\right) / \left(1 - 0.70 \cdot 0.60 - 0.08\right) = 0.30 \cdot 0.92 / \left(1 - 0.42 - 0.08\right)$$
$$= 0.276 / 0.50 = 0.552$$

$$p_3 = PX / \left(QD + X\right) = 0.30 \cdot 0.08 / \left(0.70 \cdot 0.60 + 0.08\right) = 0.024 / 0.50 = 0.048$$

The following calculations illustrate that $S_{Min} = 0.3024$ under Model A2 can be obtained using any one of the following three expressions.

$$S = \left(s_{W1} \cdot p_1 + s_{W3} \cdot p_3\right) - \left(s_{B1} \cdot p_1 + s_{B3} \cdot p_3\right)$$
$$= \left(0.92 \cdot 0.552 + 0.08 \cdot 0.048\right) - \left(0.32 \cdot 0.552 + 0.68 \cdot 0.048\right)$$
$$= \left(0.5078 + 0.0038\right) - \left(0.1766 + 0.0326\right) = 0.5116 - 0.2092) = 0.3024$$

$$S = \left(s_{W1} - s_{B1}\right) p_1 + \left(s_{W3} - s_{B3}\right) p_3$$
$$= \left(0.92 - 0.32\right) 0.552 + \left(0.08 - 0.68\right) 0.048$$
$$= \left(0.60\right) 0.552 + \left(-0.60\right) 0.048$$
$$= 0.3312 - 0.0288 = 0.3024$$

$$S = D \cdot \left(p_1 - p_3\right)$$
$$= 0.60 \cdot \left(0.552 - 0.0480\right) = 0.60 \cdot 0.5040 = 0.3024$$

7.4.2 Examining D, S_{Max}, and S_{Min} over Varying Combinations of D and P

The models for obtaining maximum and minimum values of the separation index (S) just reviewed provide a basis for establishing the potential for D and S to vary across varying combinations of the level of displacement from even distribution as measured by the dissimilarity index (D) and the racial composition of the city (P). I used these models to compute values of S_{Max} and S_{Min} over possible combinations of D ranging from 0 to 100 with P ranging from 1 to 99. Results from these calculations are depicted graphically in Figs. 7.4 and 7.5 which depict the upper and lower bounds of the relationship between D and S at selected values for city racial composition (P). Figure 7.4 depicts the relationship by plotting values of S_{Max} and S_{Min} against values of D. Figure 7.5 depicts the relationship by plotting values of D against values of S_{Min}.

I comment first on the diagonal line on Fig. 7.4. This results from plotting values of S_{Max} against the value of D over all values of D and all values of P. The diagonal documents that S will equal D at any combination of values for D and P when displacement from parity involves concentration of both groups in racially polarized areas wherein Whites in non-parity areas live apart from Blacks in areas that are

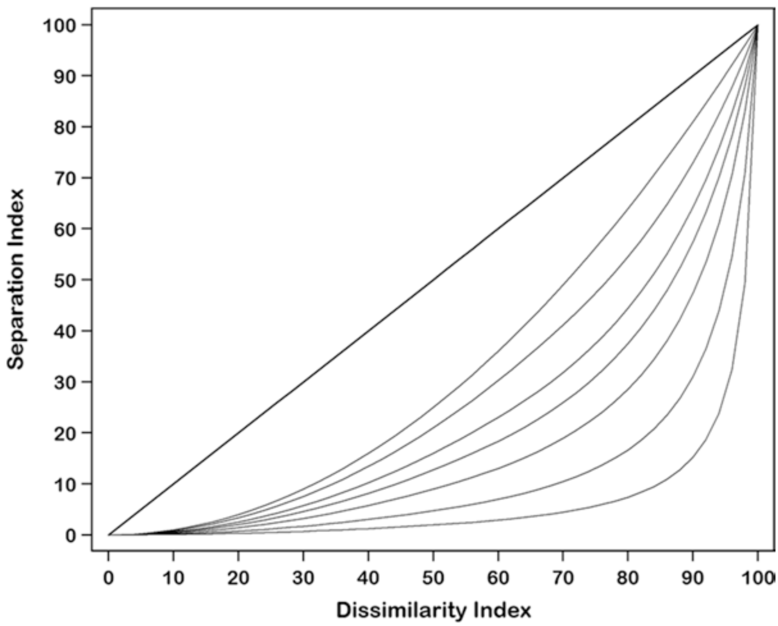

Fig. 7.4 Maximum and minimum values of the separation index (S) by values of the dissimilarity index (D) for selected values of city percent White (P) under a three-area analytic model (Notes: Maximum and minimum values of S under three-area analytic exercise. See text for disscussion of analytic model. Curves are plotted for values of percent White (P) of 50, 70, 80, 90, 95. and 98)

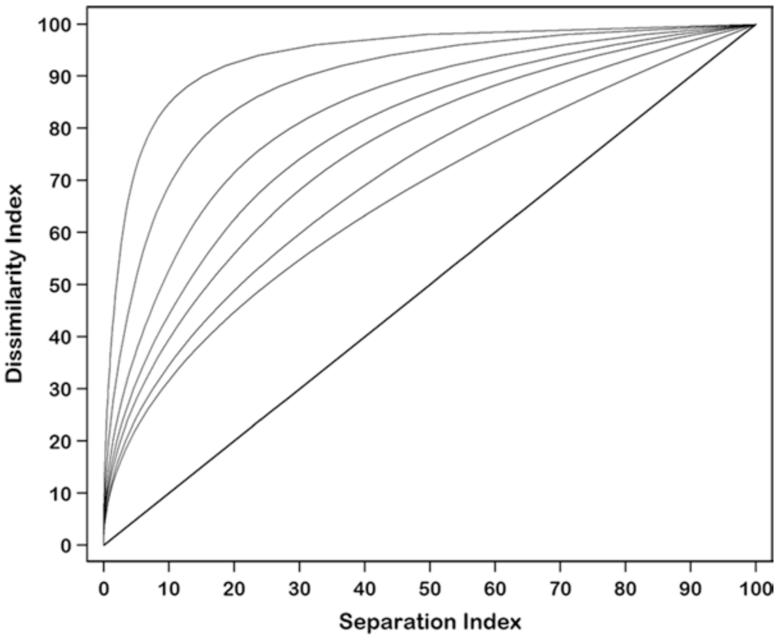

Fig. 7.5 Maximum and minimum values of the dissimilarity index (D) by values of the separation index (S) for selected values of city percent White (P) under a three-area analytic model (Notes: Maximum and minimum values of S under three-area analytic exercise. See text for discussion of analytic model. Curves are plotted for values of percent White (P) of 50, 70, 80, 90, 95, and 98)

all-White and Blacks in non-parity areas live apart from Whites in areas that are all-Black. The diagonal in the figure thus serves as a reference line indicating the maximum degree to which groups can be residentially separated at a given level of displacement from even distribution.

The graph in Fig. 7.4 also plots the values of S_{Min} against the value of D over values of D ranging from 0 to 100 and at selected values of P ranging from 2 to 50. Note that it is not necessary to plot the same relationships for values of P above 50 they are identical to the relationships already shown for values of $1-P$ already shown. Thus, for example, the curve obtained when $P = 98$ is identical to the curve obtained when $P = 2$. Importantly, all of the curves fall below the diagonal and thus visually depict the fact that S can take a lower value than D at any combination of values for D and P when group displacement from even distribution is dispersed in a way that maximizes group residential mixing instead of being concentrated in a way that maximizes group residential separation. The set of curves also makes it clear that the maximum possible difference between D and S is conditioned by city racial (P). This is visually indicated by the fact that different curves result for each value of P.

The maximum possible size of the D-S difference is smallest when the two groups in the comparison are equal in size (i.e., $P = Q = 0.5$). Intuitively, this is because the maximum departure of S from D occurs when one group is dispersed widely across areas where it is over-represented, thus resulting in small departures of p_i from P in these areas. This is demographically more feasible when one group is small in comparison to the other and it is less feasible when groups are equal in size. Elsewhere I establish that the $D - S_{Min}$ relationship when groups are equal in size is $S = D^2$ (Fossett 2015). This relationship is reflected in the curve that is closest to the diagonal. This curve documents that the absolute and relative magnitude of the possible D-S difference can be substantial even when it is at its minimum. The D-S difference when groups are equal in size reaches a maximum of 25 points when D is 50 and it is 20 points or more when D is in the range 28–72. In relative terms, the value of S can be up to 20 % lower than the value of D when D is 80; up to 30 % lower when D is 70; up to 40 % lower when D is 60; up to 50 % lower when D is 50; and so on.

The $D - S_{Min}$ curves plotted at selected values of P depart further from the diagonal as the racial composition of the city becomes progressively more imbalanced. Since most White-Minority segregation comparisons in empirical studies involve groups that differ greatly in size, these curves are highly relevant. They document that potential D-S differences can be very large in both absolute and relative terms under combinations of D and P that are common in "real world" settings. When P is 85, the $D - S_{Min}$ difference exceeds 25 when D is in the range of 30–93 and it exceeds 40 when D is in the range of 56–83. In relative terms, the value of S can be up to 50 % lower than the value D when $D \leq 82$ and 70 % lower or more when $D \leq 58$. The potential D-S differences are even more dramatic when P is 95 or higher. For example, when P is 95, the $D - S_{Min}$ difference exceeds 25 when D is in the range of 28–98 and it exceeds 40 when D is in the range of 44–96. In relative terms, the value of S can be up to 50 % lower than the value D when $D \leq 94$ and 70 % lower or more when $D \leq 84$.

Importantly, group size differentials of this magnitude are common in empirical studies of segregation in US cities. For example, they are typical of White-Asian comparisons in most cities and they are typical of White-Latino comparisons in the "new destination" communities of the Midwest, South, and Northeast. The potential for D-S differences to be very large in these situations is clearly revealed in Fig. 7.4. The patterns seen here provide compelling evidence that the prevailing practice of examining only D in empirical studies of segregation should be reconsidered. The curves in the figure document that the level of group separation and area racial polarization as measured by S can vary widely across cities that are identical in terms of group displacement from even distribution (D) and relative group size (P).

Figure 7.5 makes the same point but from the vantage point of the separation index (S) instead of the dissimilarity index (D). Here the diagonal depicts the values of D plotted by S when displacement from even distribution is maximally concentrated (S_{Max}). The curves in the figure depict the values of D plotted by S when displacement from even distribution is maximally dispersed. The implication of these curves is straightforward. If one is interested in group separation as measured by S, D is an unreliable indicator because D can take very high values when groups are not residentially separated. This occurs when group displacement from even distribution is extensive but the group populations are dispersed across non-parity areas in a way that minimizes group concentration and maximizes group mixing and co-residence.

7.4.3 *Implications of Findings from Analytic Models for S_{Max} and S_{Min}*

The preceding discussion establishes that scores for D and S can differ depending on three factors. The first is whether displacement of groups from even distribution is present and is substantial. All else equal, the potential for D and S to differ is greatest when D is high (e.g., at or above 60) but less than its maximum of 100.[11] The second factor is whether the group displacement from even distribution in question is concentrated or dispersed. When displacement is maximally concentrated, $S = S_{Max} = D$; when displacement is maximally dispersed (minimally concentrated), $S = S_{Min} \leq D^2$. The third factor is the relative sizes of the groups in the segregation comparison. All else equal, the maximum possible difference between D and S is larger when groups are unequal in size. Accordingly, the logical possibility for a large $D - S$ difference is greatest under the following conditions: (1) displacement from even distribution is extensive (i.e., D is high), (2) displacement is maximally dispersed, and (3) the groups are highly unequal in relative size (e.g., $|P - Q| > 90$). Analysis of empirical segregation patterns presented in Chap. 8 will document examples of such situations and establish that large D-S discrepancies are not just logically possible, they can and do occur with some regularity in empirical studies.

7.5 Is Separation a Distinct Dimension of Segregation?

I conclude this chapter by considering the issue of whether group separation and area racial polarization as measured by S should be viewed as a distinct dimension of segregation. Stearns and Logan (1986) argued that D and S tap different aspects of group differences in residential distribution and noted that D and S can differ both in overall value and in direction of change. On this basis they argued that S is a distinct dimension of segregation and should be routinely examined in empirical studies. The core of their position is that, unlike D, S registers whether or not groups live apart due to *both* groups being concentrated in homogeneous areas, a residential pattern of compelling substantive interest to researchers.

[11] When displacement reaches its maximum possible level, $S = D = 100$ and the D-S difference is necessarily zero. Similarly, if there is no displacement from even distribution, $S = D = 0$ and the D-S difference is zero.

The view Stearns and Logan advocate runs counter to most methodological studies which view S as one among many alternative measures of uneven distribution including the gini index (G), the dissimilarity index (D), the Theil entropy index (H), and the Atkinson index (A) represented here by the closely related Hutchens square root index (R) (Zoloth 1976; James and Taeuber 1985; White 1986; Reardon and Firebaugh 2002).[12] These various alternative indices all differ from each other in at least the narrow sense that they can yield different numerical scores when applied to the same residential distributions. So the question arises, when does one measure become different enough from the alternatives that it should be considered a distinctive dimension of segregation?

One basis for grouping indices together is similarity of computing formulas – the operational implementations of the conceptions of segregation embodied in the indices. On this basis one can argue that the separation index (S) is a measure of even distribution based on the close similarity of one of its computing formulas with a computing formula for the index of dissimilarity (D).

$$\mathrm{D} = 100 \cdot \left(1/2\mathrm{TPQ}\right) \cdot \Sigma \mathrm{t_i} \mid \mathrm{p_i} - \mathrm{P} \mid$$

$$\mathrm{S} = 100 \cdot \left(1/\mathrm{TPQ}\right) \cdot \Sigma \mathrm{t_i} \left(\mathrm{p_i} - \mathrm{P}\right)^2$$

The view can also be supported by noting the close similarity of the following computing formulas for the separation index (S) and the Hutchens square root index (R) which empirically is closely related to D as well as to Atkinson's A.[13]

$$\mathrm{R} = 100 \cdot \left[1 - \left(1/\mathrm{T}\right) \cdot \Sigma \sqrt{p_i q_i / PQ}\right]$$

$$\mathrm{S} = 100 \cdot \left[1 - \left(1/\mathrm{T}\right) \cdot \Sigma \mathrm{p_i q_i} / \mathrm{PQ}\right]$$

Similarity of computing formulas for measures of uneven distribution also can be summarized in another, more abstract way. S is like G, D, R, and H, in that all of these indices can be described in the following way. The value of each of these indices registers the population weighted average of quantitative scoring of the deviations of area pairwise racial composition (p_i) from the pairwise racial composition of the city (P) overall, normalized to the range 0–1 where 0 indicates no

[12] I make two qualifications. First, technically, Massey and Denton (1988) classified S as an exposure measure, but they noted others classify it as a measure of uneven distribution. Second, James and Taeuber (1985), Massey and Denton (1988), and White (1986) include the Atkinson index (A) as a measure of uneven distribution. But I instead list the Hutchens square root index (R) which is a superior and closely related substitute for the Atkinson index.

[13] D and R both rank segregation comparisons in accord with the principle of segregation dominance. Using the data set for analyses reported in this chapter the simple linear correlation of D and R is extremely high (0.962) and the correlation is even higher when allowing for nonlinearity (the correlation of D with the square root of R is 0.984).

deviations and 1 indicates that deviations have reached the maximum possible result.[14]

Finally, S fares well when it is reviewed on non-controversial technical criteria suggested for measures of uneven distribution. Ironically, it fares much better than D, the most widely used measure of uneven distribution (Reardon and Firebaugh 2002).

From the points just reviewed there is a clear case for grouping S with other measures of uneven distribution. But there is room for further discussion on both conceptual and practical grounds. On conceptual grounds, the theory of segregation measurement can be described as "incomplete." This means that the generally accepted criteria for evaluating measures of uneven distribution are compatible with a variety of measures each of which embodies a unique, albeit implicit, conception of uneven distribution. For now, however, the ambiguity of the situation is not likely to be eliminated. Some criteria for measuring even distribution such as the exchange principle discussed earlier in this chapter, have been endorsed widely (e.g., James and Taeuber 1985; White 1986; Reardon and Firebaugh 2002). But other criteria that would reduce ambiguity in measurement have been offered but not widely accepted.

In particular, the criterion of "composition invariance" offered by James and Taeuber (1985) is seen as controversial so too is Taeuber and James' (1982) criticism of the separation index (termed V in their discussion) based on related concerns. This principle has the practical consequence of requiring indices to order segregation comparisons in agreement with the principle of "segregation curve dominance."[15] Two widely used indices – the separation index (S) and the Theil entropy index (H) – do not satisfy this criterion. However, the criterion itself is controversial. Some have explicitly and forcefully rejected it (e.g., Coleman et al. 1982; White 1986). Others note the criterion has been suggested but do not endorse it (e.g., Reardon and Firebaugh 2002). The "revealed consensus" in the empirical literature has been that researchers ignore the criteria and use H and S when they find these indices to be useful for meeting the needs of a their study.[16]

So where do things stand? If one accepts the principles of "composition invariance" and "segregation curve dominance" as integral and essential to the measurement of uneven distribution, the separation index (S) and also the Theil index (H) cannot be considered measures of uneven distribution. Under this circumstance,

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[14] Thus, the index scores are normalized to the range 0-1 by dividing the average deviation scores by the maximum value the average can take under complete segregation.

[15] Even if this principle is accepted, segregation measurement theory is still technically incomplete because the principle is silent on how segregation comparisons should be ranked when segregation curves cross, as they sometimes do. This is less important on practical grounds as indices that satisfy the principle of segregation curve dominance tend to correlate at very high levels.

[16] Subordinating measurement principles to researcher needs is typical, not uncommon, as the most widely used index, D, does not satisfy the non-controversial principles of transfers and exchanges.

Stearns and Logan (1986) would then be correct in arguing that S taps a distinct dimension of segregation.[17]

Personally, I am comfortable with this position. It would reduce ambiguity in the current relatively flexible notion of uneven distribution by distinguishing between indices that measure displacement and indices that measure separation. Displacement would be compatible with the geometric interpretation of the gini index (G) in relation to the segregation curve and the closely related vertical distance and volume of movement interpretations of the dissimilarity index (D). Displacement also would be compatible with notions of group difference on rank-order position on area racial composition. G would then stand as an attractive index of displacement as it satisfies the principle of exchanges and responds to all directional changes in rank-order differences between groups and thus supports interpretation as the "net difference" in group rank order advantage noted by Lieberson (1976). D would then stand as a crude version of G that may be useful due to its simplicity and ease of calculation even though it does not satisfy the principle of changes and responds only to directional changes in rank-order distribution above and-below P.

Two other measures – the symmetric version of Atkinson index (A) and the Hutchens square root index (R) – also could be categorized as measures of displacement. So far as I am aware, they do not offer the specific geometric interpretation of displacement that is available for G and D. But they are like G and D in satisfying the criterion of segregation curve dominance and their values correlate very closely with values of D and G in empirical analyses. Hutchens (2004) makes the case that R has attractive options for certain kinds of analysis based on being "additively decomposable" where G and D are not.

Separation registers differences in group distribution that are not registered by displacement as measured by G, D, and R. Separation assesses group differences in *quantitative* position, instead of rank-order position, on area racial composition. A formal distinction can be made between displacement and separation by adopting a "polarization" criterion to supplement the "exchange criterion." The current exchange criterion is minimal; it requires only that an index register an integration-promoting exchange. A polarization criterion supplement would additionally require the following.

All else equal, exchanges involving more polarized areas and resulting in larger average reductions in same-group contact should have greater impact on an index than exchanges involving less polarized areas and resulting in smaller average reductions in same-group contact.

Specifically, the principle would require the impact of the exchange on the index score to increase as the value of $|p_i - p_j|$ increases. Thus, in the example exchanges discussed earlier in this chapter, S will respond more strongly to an exchange

[17] The issue is more complicated than my statement suggests. The dissimilarity index (D) does not satisfy the principle of transfers – a principle that does enjoy consensus support – yet methodological reviews typically characterize D as a valid measure of uneven distribution on the grounds that the practical consequences of violating the principle of transfers are not sufficient to justify disallowing the measure.

between highly polarized areas such as an exchange between one area with 100 Whites and 0 Blacks and another area with 0 Whites and 100 Blacks and less strongly to an exchange between minimally polarized areas such as an exchange between one area with 51 Whites and 49 Blacks and another area with 49 Whites and 51 Blacks. In contrast, as demonstrated in examples reviewed earlier and established more carefully elsewhere (e.g., James and Taeuber 1985; Reardon and Firebaugh 2002), D will treat these exchanges as identical in impact.

G responds in a more complicated way that ultimately is similar in nature to D. G will potentially treat these exchanges differently, but not based on the quantitative magnitude of the level polarization; that is, not in proportion to the value of $|p_i - p_j|$. Instead, since G assesses group differences in rank order position, it will treat these exchanges differently when they differ in terms of the share of the combined group populations residing in areas with values on racial composition (p) that fall in between the values on racial composition for the two areas involved in the exchange. Specifically, G would be reduced by a larger amount when the exchange causes the moving White and Black households to cross over a larger "intermediate" population; that is a larger share of the combined group populations residing in "intermediate" areas where area proportion White (p) is larger than that for the area receiving the White household (p_j) and smaller than that for the area sending the White household (p_i). This property of G has little practical consequence for overcoming insensitivity to polarization because, if the quantitative difference between the two areas (i.e., $|p_i - p_j|$) is small, polarization is small and G, like D, can respond strongly to group differences in distribution across areas that are similar in terms of area racial composition.

One potential benefit of adopting a strong conceptual distinction between displacement and separation is that it would reduce ambiguity in segregation measurement. It would make something clear both to researchers and also to consumers of segregation research. Namely, it would clarify that

> *Segregation indices that rank segregation comparisons in terms of the segregation curve are poor choices for measuring group residential separation and area racial polarization.*

Similarly, it would signal that

> *Segregation indices that measure group residential separation and area racial polarization are poor choices for measuring group displacement from even distribution.*

It appears, however, that prevailing practices in empirical research place greater priority on practical concerns such as flexibility and ease of use rather than conformity to technical measurement criteria. When approaching segregation guided by these priorities, which some might view as appropriate since key aspects of segregation measurement theory are unresolved, one could argue that D and S both measure uneven distribution construed broadly. However, even when one adopts this view, it is important to recognize and acknowledge the following.

> *D and S are sufficiently different in behavior that the choice between them has potentially important consequences for empirical findings that should not be overlooked.*

Once this point is acknowledged, the responsibility falls to researchers to first determine whether index choice matters for the findings obtained in any given empirical analysis and, when it does, to then report this and note the implications it may carry.

All indices of uneven distribution register group differences in residential distributions differently. But some differences are negligible on practical grounds while others are potentially more important. In the case of D and S, the differences are especially likely to have important practical consequences for findings in empirical studies because they are at opposite ends of a continuum in how indices respond to group difference in distribution on area group proportion (p). Specifically, as a crude form of G, D is sensitive to rank order differences without regard to the quantitative magnitude of the differences involved while S is sensitive to quantitative differences that are large in size and is only weakly responsive to rank order differences that involve small quantitative differences.[18] Understanding this difference helps clarify the nature of segregation patterns when D and S yield different results. Because D is sensitive to group differences in rank position on area group proportion (p), D can take high values even when the group differences on p are small in quantitative magnitude but are extensive. In contrast, S takes high values only when group differences on area group proportion (p) are quantitatively large, and will take low values when group differences in rank position on p are extensive but the quantitative differences involved are small.

Whether one sees this practical difference between D and S as justifying the conclusion that they measure distinctly different dimensions of segregation is a matter of judgment. I take the position that, at the very least, it is important to note that the two measures are similar in measuring group differences on area group proportions (p_i) and give researchers the option of assigning priority to rank-order differences or to quantitative differences. The choice between the two options is important because rank-order differences can be high even when groups live together in areas that differ by small amounts on area racial composition and quantitative differences can only be high when groups live apart in areas that differ substantially on area racial composition.

Once this point is "on the table", the choice between indices becomes sharply defined. If one adopts the separation index (S) one is choosing to focus on quantitative differences between group residential outcomes on racial composition and the question of whether groups live together or apart. When S takes high values, it also necessarily implies the presence of substantial differences in rank order position on racial composition as values of D cannot fall below values of S. This clearly fits well with prevailing, albeit usually implicit, notions regarding what I term "prototypical" segregation wherein rank-order and quantitative differences track each other closely. The contrasting possibility is when group differences in rank-order position on area racial composition (p_i) are widespread, but they are small in magnitude resulting in a high-D, low-S outcome. This possibility of this outcome is not widely recognized.

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[18] As noted earlier, G registers group differences in rank position on area group proportion (p) regardless of the quantitative magnitude of the differences involved. D is a crude version of G and behaves in a similar way.

Perhaps because of this, compelling arguments for why one would prioritize this result over assessments of prototypical segregation have not been articulated in the literature.

My own sense of the matter is that researchers should always examine S because it registers an aspect of residential distributions that is sociologically compelling and clearly relevant for the concerns that motivate researchers to assess uneven distribution in the first place. For example, Taeuber, a leading segregation researcher whose efforts popularized the use of the dissimilarity index, motivated one of his influential studies of White-Black differences in residential distribution by stating that "[r]esidential segregation of whites and nonwhites effects their *separation* in schools, hospitals, libraries, parks, stores, and other institutions" (1964:42; emphasis added).

The distinction between separation and "mere" displacement is important because residential separation is a logical prerequisite for groups to have fundamentally different neighborhood outcomes and life chances based on area of residence. To the extent that residential outcomes and life chances are liked to area of residence, groups will tend to have similar residential outcomes and life chances when the two populations live together.[19] All else equal, populations that reside together share the same physical and built environment whether despoiled and blighted or scenic and well kept; they likewise share the same neighborhood amenities such as roads, sidewalks, air and water quality; they have the same neighbors; they share the same neighborhood institutions, businesses, and public services; they have the same public schools; they have the same exposure to noise, crime and social problems; and so on.

Alternatively, as Stearns and Logan pointed out, polarization of neighborhoods into White and minority areas makes minority households concentrated in minority areas vulnerable to discriminatory practices such as formal and informal redlining for loans and insurance coverage for homes and businesses that can undermine property values and inhibit private and public investment. Similarly, area racial polarization puts minority areas and minority households at risk of disadvantage in neighborhood outcomes resulting from differential siting of less desirable public institutions such as prisons, half-way houses, low-income housing developments, waste management facilities, etc., and similarly at risk for inequality in quantity and/or quality of schools, parks, libraries, government services, roads and other public infrastructure, and so on (Stearns and Logan 1986:127–128).

[19] Of course, residential outcomes and life chances can differ substantially for groups that live together when stratification processes are tied to group-membership independently of area of residence. The Jim Crow South is an example where groups could live together at the neighborhood level but have fundamentally different life chances based on group membership. For example, in the extreme, Whites and Blacks living together in the same neighborhood – and sometimes even on the same block and residential property – went to different schools and used different public amenities such as water fountains, restrooms, and swimming pools. Even in this circumstance, however, many public goods aspects of neighborhoods – such as desirable amenities, roads, exposure to natural and man-made hazards, etc. – are shared equally when groups reside together.

In the ideal, research motivated by the kind of concerns just noted would assess group disparities on the relevant neighborhood characteristics directly. But, unfortunately, the requisite data are not available in comprehensive form. The next best option is to determine whether group residential separation creates the logical potential for disparities to be pronounced. The separation index (S) is directly relevant for this concern. Measures of displacement, D in particular, are not reliable substitutes.

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Chapter 8
Further Comments on Differences Between Displacement and Separation

In Chap. 6 I documented that displacement (D) and separation (S) routinely diverge by large amounts in some empirical analyses. Then in Chap. 7 I provided technical discussions to clarify how D and S can vary independently. I also stressed that the combination of high-D, low-S – which occurs when displacement from uneven distribution is dispersed rather than concentrated – has important sociological implications and I advised researchers to check for this pattern and guard against incorrectly assuming that high levels of displacement (D) are accompanied by high levels of group separation (S). In this chapter I try to encourage researchers to follow this advice by discussing three topics relevant to measuring separation and understanding how it may diverge from displacement.

I begin by revisiting the empirical relationship of D and S originally discussed in Chap. 6 and reviewing it in more detail in light of the material presented in Chap. 7. I then review plausible scenarios for how displacement can come to be dispersed in a way that creates large D-S differences. Discussions of this topic are not common in the literature. I address this gap to help researchers become more comfortable with giving attention to the contrast between dispersed and concentrated displacement from uneven distribution. I next focus on a practical issue researchers should bear in mind when seeking to measure and compare displacement and separation. I then conclude the chapter by noting an alternative option for measuring group separation and area racial polarization some researchers may find useful because it is easy to compute and explain and also tends to correlate closely with the separation index.

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_8

8.1 Revisiting the Empirical Relationships of Displacement (D) and Separation (S)

I now examine empirical differences between D and S in more detail by revisiting the data on White-Minority residential segregation in core based statistical areas (CBSAs) for 1990, 2000, and 2010 originally discussed in Chap. 6. My goal in this discussion is to discuss D-S differences in light of perspective gained from the material presented in Chap. 7. Figure 8.1 plots scores for the separation index (S) by scores of the dissimilarity index (D) for CBSAs in 1990, 2000, and 2010. The plot includes 4,319 White-Minority segregation comparisons screened on having at least 1,500 persons in both groups in the comparison. The diagonal reference line plots D against itself. The figure shows that in empirical application values of S are consistently lower than values of D. Logically, it is possible for the values of D and S to be equal in any comparison. But this occurs only when all group displacement from even distribution is concentrated in all-White or all-minority areas. It is readily apparent from the figure that even an approximation of this outcome is an uncommon occurrence for the cases in this data set. Figure 8.2 reverses the point of view for the relationship and plots scores for the dissimilarity index (D) by scores of the

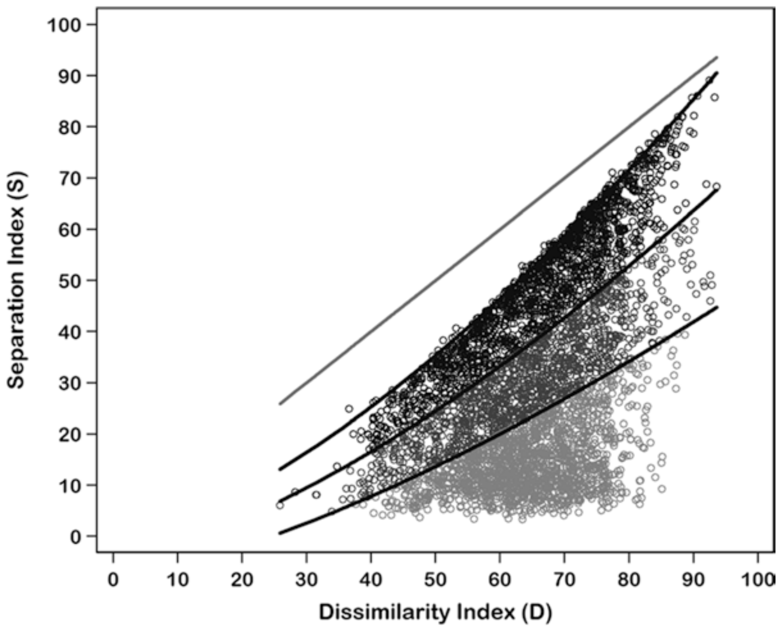

Fig. 8.1 Separation index (S) by dissimilarity index (D) for White-Minority segregation comparisons computed using block-level data for CBSAs in 1990, 2000, and 2010 (Reference lines: Diagonal for D by D and reference curves for 100 %, 75 %, and 50 % of $D^{3/2}$. 4,319 cases for White-Black, White-Latino, and White-Asian segregation comparisons with at least 1,500 persons in both groups)

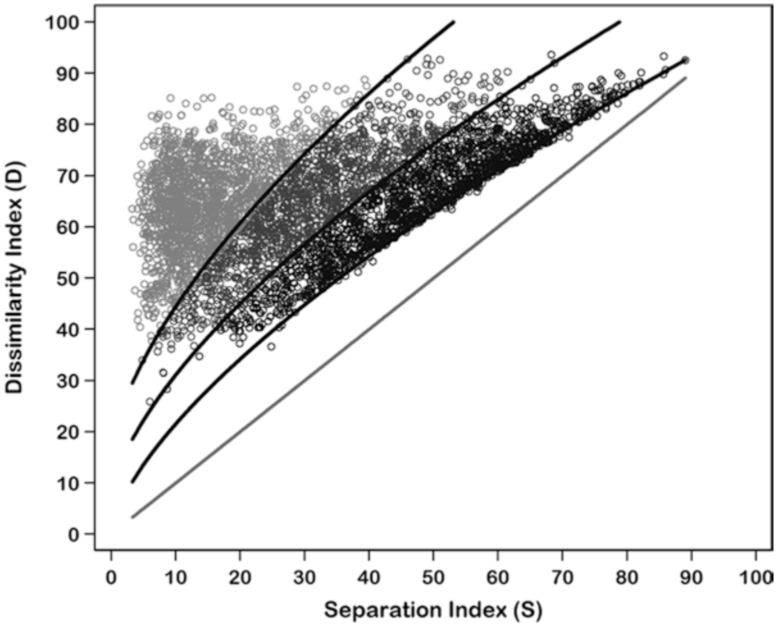

Fig. 8.2 Dissimilarity Index (D) by separation Index (S) for White-Minority segregation comparisons computed using block-Level data for CBSAs in 1990, 2000, and 2010 (Reference lines: Diagonal for S by S and reference curves for 100 %, 75 %, and 50 % of $S^{2/3}$. 4,319 cases for White-Black, White-Latino, and White-Asian segregation comparisons with at least 1,500 persons in both groups)

separation index (S). Here the diagonal reference line plots S against itself. Unsurprisingly, the figure shows that values of D in this data set are consistently higher than values of S. The main benefit of this figure is to highlight how values of D can be misleading if one's goal in measuring segregation is to identify prototypical segregation involving group residential separation.

The curved reference lines near the diagonal in each figure serve to highlight a "stylized fact" for D-S correspondence. It is the empirical regularity that, while it is logically possible for S to take a value equal to D in any comparison, values of S rarely exceed $D^{3/2}$ in empirical analyses. Similarly, values of D rarely fall below $S^{2/3}$. In view of this empirical relationship, I characterize cities that fall along the interior boundary of the empirical D-S relationship depicted in Figs. 8.1 and 8.2 as cities where segregation follows a "prototypical" pattern. By this I mean that group displacement from even distribution registered by D is substantially concentrated and produces group residential separation registered by S.

More specifically, I characterize segregation as clearly "prototypical" when scores for D and S track each other in parallel based on the mild nonlinear relationships of $D \approx S^{2/3}$ and $S \approx D^{3/2}$. Thus, for example, to characterize a city as having

General Categories	D Range	S Range
Very High / Pronounced	75-100	65-100
High / Substantial	60-74	45-64
Medium / Moderate	45-59	30-44
Low / Limited	20-44	10-29
Very Low / Minimal	0-19	0-9

Fig. 8.3 Guidelines for identifying prototypical segregation based on concordant scores for dissimilarity (D) and separation (S) when displacement from even distribution is substantially concentrated

a prototypical pattern of segregation I would expect S to be near or above 65 when D is 75; or, conversely, I would expect D to be near or below 75 when S is about 65. The reference lines in the two figures reflect how values of D and S will correspond when "prototypical" segregation varies from low to medium to and high. For convenience and consistent use of terms for describing the levels of segregation when displacement and separation are concordant, I offer guidelines in Fig. 8.3 for broad categories of prototypical segregation where dissimilarity (D) and separation (S) are concordant. When D and S align as they do in these broad categories, it is reasonable to describe displacement from even distribution as being substantially concentrated such that groups are living apart, rather than together, roughly in keeping with the degree possible at the observed level of displacement from even distribution.

In Fig. 8.4 I offer a more detailed set of guidelines for judging when D and S do not correspond as one would expect when displacement from even distribution is concentrated in the manner that produces a pattern of "prototypical segregation." The first two columns list values of D and S that are "clearly concordant" meaning that the D-S combinations listed involve values of the separation index (S) that are in the higher range of what is possible given the level of displacement from even distribution indicated by the dissimilarity index (D). The quantitative guideline I apply for "clear concordance" of D and S is for the value of S to be equal to or higher than 95 % of $D^{3/2}$. The third column lists values of S that lead me to characterize the D-S comparison as "Discordant" meaning that, instead of being substantially concentrated, displacement from even distribution is substantially dispersed and consequently produces a level of group separation that is well below that expected under prototypical segregation. The quantitative guideline I apply is that S is at or below 75 % of $D^{3/2}$. The fourth column lists values of S that lead me to characterize the D-S comparison as "Very Discordant" meaning that displacement from even distribution is highly dispersed and produces a level of group separation that is very low in comparison to that expected under prototypical segregation. The quantitative guideline I apply is that S is at or below 50 % of $D^{3/2}$.

D Value	S Value is Concordant (Displacement is Substantially Concentrated)	S Value is Discordant (Displacement is Dispersed)	S Value is Highly Discordant (Displacement is Highly Dispersed)
D = 90	S ≥ 81	S ≤ 60	S ≤ 35
D = 80	S ≥ 68	S ≤ 50	S ≤ 28
D = 70	S ≥ 56	S ≤ 40	S ≤ 21
D = 60	S ≥ 44	S ≤ 31	S ≤ 15
D = 50	S ≥ 34	S ≤ 23	S ≤ 10
D = 40	S ≥ 24	S ≤ 15	S ≤ 5
D = 30	S ≥ 16	S ≤ 8	---
D = 20	S ≥ 9	S ≤ 3	---

Fig. 8.4 Guidelines for assessing concordance-discordance of dissimilarity (D) and separation (S)[a] ([a]Concordant (displacement is substantially concentrated) with $S \geq 95\,\%$ of $D^{3/2}$; Discordant (displacement is dispersed) with $S \leq 75\,\%$ of $D^{3/2}$; and highly discordant (displacement is highly dispersed) with $S \leq 50\,\%$ of $D^{3/2}$)

Figures 8.1 and 8.2 include reference lines that correspond to the quantitative guidelines just outlined. The figures thus document that many White-Minority comparisons in these cities do have scores on D and S that place the cities in question comfortably within the category of having "prototypical segregation" wherein displacement from even distribution is accompanied by a correspondingly level of group separation and area racial polarization. At the same time, however, the figures also make it clear that a great many White-Minority comparisons in these cities have D-S combinations that are either discordant or very discordant indicating that segregation does not follow the "prototypical" pattern that researchers and broad audiences assume is typical.

In individual cases of a particular White-Minority comparison in a given city, $D-S$ discrepancies can be discussed and evaluated in several ways including the following.

- Comparing the simple $D-S$ difference
- Expressing S as a percentage of D (i.e., $100 \cdot S / D$)
- Expressing S as a percentage of $D^{3/2}$ (i.e., $100 \cdot S / D^{3/2}$)

If the simple $D-S$ difference is small, the situation involves concentrated displacement from even distribution that produces group separation at near the maximum level possible given the extent of group displacement. When the D-S difference is large, it is clear that the situation involves "dispersed displacement" that wherein group separation and neighborhood racial polarization are well below what is possible given the level of displacement. That is, while the groups are differ substantially in proportions residing in below-parity areas, they nevertheless tend to live together in neighborhoods that vary in a relatively narrow range on racial mix (p) and are not residentially separated into racially homogeneous neighborhoods.

The relative comparison of D and S should be considered when the simple D-S difference is non-negligible, but not extreme. Expressing S as a percentage of D indicates the relative extent to which displacement from even distribution is concentrated. If the value reaches 100, it indicates that group displacement is maximally concentrated in a way that produces non-parity neighborhoods that are racially homogeneous (all same-group) or nearly so.

The relative comparison of S and $D^{3/2}$ provides another reference point for assessing whether D and S are discordant. Values at 80 % and above indicate that the values of D and S align in reasonable correspondence to what is expected when segregation follows a prototypical pattern at a levels characterized as low, medium, high, etc. as suggested above. This means that, at a given level of group displacement from even distribution (D), the degree of group residential separation (S) is in line with standard expectations. If the value drops below 75 %, it signals a D-S discrepancy wherein at least one group's displacement from even distribution is dispersed rather than concentrated. Values that fall below 50 % indicate that at least one group's displacement from even distribution is highly dispersed and thus it not appropriate, and may even be substantially misleading, to characterize the two groups as living apart from each other.

When focusing on individual cases in detail, these guidelines for "quick comparisons" can be supplemented with detailed comparisons of group distributions on area racial composition. Elsewhere I provide a more extended review of graphical and quantitative analyses highlighting selected cases of White-Minority segregation that illustrate a variety of outcomes for D-S comparisons ranging from concordance (prototypical segregation) to very discordant (displacement without separation) in Fossett (2015).

8.2 Scenarios for How D and S Discrepancies Can Arise

Segregation researchers rarely comment on whether displacement measured by D involves group separation and neighborhood polarization measured by S. This is understandable because the issue is rarely discussed in either empirical studies or in the literature on segregation measurement. Accordingly, some might wonder if it is easy for D and S to differ in dramatic ways. In Chap. 6, I reviewed data showing that this is indeed the case empirically when the scope of segregation analysis is broad (i.e., expands beyond large metropolitan areas) and when samples include cities where minority populations are small in relative size.

Given the lack of discussion of dispersed displacement and D-S divergence, it is understandable that consumers of segregation research and researchers themselves may wonder "How can such patterns come about?" In this section I review some scenarios for how high-D, low-S situations can arise. My goal is to help readers gain a more intuitive understanding of how displacement can come to be extensive without also producing the high levels of group separation needed to create the pattern of prototypical segregation.

To begin, imagine a city with 100 neighborhoods each of which has 1000 residents. Additionally assume the city population is 98 % White and 2 % Black with 98,000 White residents and 2000 Black residents. Under exact even distribution all 100 neighborhoods will have 980 White residents and 20 Black residents. This, of course, would be a pattern of "no segregation" and the values of D and S will both be zero (0.0).

Now consider two alternative scenarios for how the same population could be rearranged to create a high level of uneven distribution. The first scenario produces a pattern of "prototypical segregation" – displacement from even distribution with substantial group separation and area racial polarization. It involves taking 49 of the 100 neighborhoods and exchanging the Black residents in these neighborhoods with White residents in one of the remaining 51 neighborhoods. This will leave 49 "above-parity" neighborhoods with 1000 Whites and no Blacks, 50 "parity" neighborhoods with 980 Whites and 20 Blacks, and 1 "below-parity" neighborhood with no Whites and 1000 Blacks. The resulting value of D will be 50 and the value of S also will be 50. The combination of $S = D$ signals a residential pattern of uneven distribution with the maximum polarization possible at this level of displacement.

Note that the pattern is logically easy to create even though the Black population is small.[1] I will review empirical examples along these lines in a couple of case studies considered below. The key feature of the situation is that the Black residents displaced into "below-parity" areas are concentrated in a small number of homogeneous areas – a single area in this hypothetical case – creating the pattern associated with prototypical segregation.

The second scenario I consider produces uneven distribution in the form "displacement without separation" or "dispersed displacement". In this situation a larger fraction of the Black population lives in "below-parity" areas where Whites are under-represented (and Blacks are over-represented) but at the same time there is minimal group separation and no neighborhood polarization. This scenario involves taking 50 of the 100 initial neighborhoods and exchanging the Black residents in these neighborhoods with White residents in the other 50 neighborhoods. In this case, however, the exchanges are implemented so no single neighborhood gains more than two new Black residents or loses more than two White residents. Implementing these exchanges will leave 50 "above-parity" neighborhoods with

[1] All that is required is that the size of the minority population exceeds the size of the typical neighborhood. In this example, the size of the Black population (2000) is twice the size of the typical neighborhood (1000).

1000 Whites and no Blacks, and 50 "below-parity" neighborhoods with 960 Whites and 40 Blacks.

In contrast to the first scenario, Black households displaced into "below-parity" areas are dispersed "thinly" across areas that are overwhelmingly White in terms of racial composition. As a result, displacement is extensive and affects half of the Black population but it does not produce group separation because it does not concentrate displaced Black households in areas that are predominantly Black. The resulting value of D for this scenario will be 51 and the value of S will be 2. Note that D is high under this scenario and in fact is slightly higher than in the first scenario that produced prototypical segregation. In contrast, S is much lower and indicates extremely low group separation. The resulting combination of high-D, low-S indicates uneven distribution with extensive displacement but minimal group separation and residential polarization.

Both scenarios of population residential distribution are simple and feasible demographically. However, if one assumes that Blacks are a minority population with little influence in the city's political system, the sociological implications may vary markedly across the two scenarios. In the first scenario, half of Blacks reside in an all-Black ghetto or enclave. One can imagine that this makes them vulnerable to disadvantages in neighborhood conditions as neglect of the "Black" neighborhood by city administrators would have no adverse impacts on Whites. In the second scenario, all Blacks reside in neighborhoods that are 96 % White. While these areas are overwhelmingly White, they are technically "below parity" and contain a large share of the Black population. Accordingly, the residential patterns involved are fundamentally different from that produced in the first scenario. Black separation from Whites and area racial polarization are essentially absent. As a result, one can imagine that Blacks are less vulnerable to disadvantages in neighborhood conditions because city administrators are unlikely to neglect "below-parity" neighborhoods where Blacks are "over-represented" because this would have adverse impacts on many more Whites than Blacks. Additionally, for neighborhood outcomes that are truly shared, Whites and Blacks would share a common fate and even if Black interests were not served well, they would be "protected" from harm when Whites interests are satisfied.

"Fair enough" someone might say. But can one imagine "real world" sociological processes that would produce the two very different patterns of segregation associated with these two scenarios? Again the answer is yes. One example of a potentially plausible historical scenario is the case of White-Black segregation in northern cities before and after the Great Migration. Lieberson's (1980, 1981) analyses of Black residential patterns 1890–1930 suggests that the relative numbers for Blacks in northern cities at the beginning of the time period were low and he speculates that due to the modest levels of Black presence Whites may not have perceived Blacks as a major threat to White residential areas. Accounts of the time suggest that, while Whites were hardly welcoming to Blacks, they did not yet engage widely in virulent anti-Black violence and other severe forms of discrimination that later would become widespread. The pre-Great Migration setting thus afforded opportunity for wider dispersal of the Black population which Lieberson reports is indicated by low average scores for Black isolation in a set of 17 "leading non-southern cities" for which data are available. Lieberson's analysis indicates that Blacks initially resided disproportionately in "below parity areas" with moderate to high displacement but they did not at this time experience the high levels of concentration and isolation in ghettos that would later come to characterize Northern and Midwestern urban areas.[2]

Lieberson then notes that the Black population grew rapidly in relative size in these cities as the Great Migration progressed in subsequent decades. White concerns about residential encroachment by Blacks increased and acts of anti-Black violence and both legal and informal housing discrimination against Blacks become more dramatic and more frequent. Increasingly, Blacks were driven from White residential areas and concentrated in predominantly Black areas that over time became large ghettos. With this, displacement as measured by D increased over this period. That is not surprising. What Lieberson points out as more intriguing is that Black isolation also increased at an even faster pace. More specifically and importantly for this discussion, Black isolation in these cities increased at a pace well beyond that which would result from Black population growth alone. This is consistent with Blacks being increasingly disproportionately concentrated in predominantly Black areas. By 1930 large ghettos were emerging across northern cities generally and familiar patterns of "prototypical segregation" came into being where previously they were not the norm.

The account Lieberson builds by combing quantitative analysis of data on residential distributions with historical information from the time period lays out a process of Black displacement from even distribution changing over time from being moderate and somewhat dispersed to being both more substantial and much more concentrated. This account is plausible and intriguing. But it also is quantitatively less than definitive because the analysis of residential patterns of the era is hampered by absence of data for small areas. Lieberson necessarily made use of data for larger areas such as "wards" in combination with historical accounts of relative dispersal of the Black population transitioning to concentration in ghettos.

In light of this I give attention to some other examples that are quantitatively more definitive but less well known. The examples involve Latino migration to "new destination" communities in recent decades. Detailed analysis of block-level data over the period 1990–2010 shows that high-D, low-S patterns of dispersed displacement for White-Latino segregation are common in new destination communities and in many cases transition over time into high-D, high-S patterns of prototypical segregation (Fossett, Crowell, Saenz, and Zhang 2015).

Several qualitative studies of Latino settlement in new destination communities including as examples a study of Garden City, Kansas (Broadway 1990), a study of

[2] Lieberson does not report values of the separation index. However, in the context of a near-binary White-Black city composition, overall isolation is a close proxy for pair-wise isolation. When it is low in comparison to its logical maximum of 1.0, as Lieberson reports, it implies that S also is low.

Marshalltown, Iowa (Grey and Woodrick 2006), and a study of Durham, North Carolina (Flippen and Parrado 2012) provide a basis for suggesting a plausible "composite" scenario of possible social dynamics underlying the quantitative patterns.[3] In this composite scenario Latino individuals and families initially migrate in small numbers drawn by economic opportunities. Since it is a new Latino destination with minimal prior Latino presence, White-Latino ethnic relations are inchoate and not yet well-formed. Demographically, there are no pre-exiting barrios or Latino residential areas for Latino immigrants and migrants to settle in. The qualitative accounts noted above suggest that early arriving Latino families do not initially encounter strong, widespread discrimination in housing, possibly due to their small numbers and their novelty in the absence of established White-Latino relations. As a result, early-arriving Latino settlers tended to locate idiosyncratically following available affordable housing vacancies distributed across many neighborhoods. These early arriving Latino families and households did tend to live in "below-parity" areas. But, as confirmed by quantitative analysis of block-level data, they typically lived in areas that were predominantly White, often overwhelmingly White. Few Latinos at this time lived in predominantly Latino neighborhoods.

This pattern produces a "classic" high-D, low-S index score combination associated with the segregation pattern of high displacement without group separation and area racial polarization. Quantitatively, it is a fundamentally at odds with an alternative and sociologically plausible scenario in which early arriving Latinos are concentrated in rapidly forming barrio and enclave neighborhoods due to multiple causes including as two examples housing discrimination based on linguistic and cultural differences and dynamics ethnic congregation based on mutual-support and ethnically structured flows of information regarding housing opportunities.

The key point to bear in mind that empirical studies that rely solely on D cannot differentiate between the two alternative scenarios. But the D-S comparison makes it possible to use data to sort the story out more carefully and the observed high-D, low-S outcomes are more consistent with the "dispersed displacement" scenario.

Many new destinations continue to attract Latino migrants and experience steady, sometimes rapid, Latino population growth. As the Latino population grows, the White population often begins to take greater notice and becomes less tolerant of the presence of Latinos. Anti-immigrant and nativist sentiment increases and discrimination against Latinos in housing increases and constrains residential opportunities for Latino families and households. As Latino neighborhoods emerge, they may be attractive locations for settlement for later arriving Latino migrants, especially those with limited English language skills. Such options were not available initially, of course, because the Latino presence was too limited.

These complementary dynamics of increasing discrimination and immigrant congregation dynamics can serve to concentrate larger shares of the Latino population in predominantly Latino areas forming enclaves or barrios. As this transition

[3] Special thanks to Cassidy Castiglione, an undergraduate research assistant who helped identify these case-studies during her participation in an National Science Foundation Research Experiences for Undergraduates Summer Institute at Texas A&M University in summer 2015.

occurs, the pattern of segregation also undergoes a transition wherein S rises faster than D. Indeed, the value of D itself may remain relatively stable or may even fall. The reason for this is that displacement – that is, White-Latino differences in proportion residing in "above-parity" areas was already high. But the pattern of displacement is changing from being dispersed to being concentrated. Over the span of a few decades, the high-D, low-S pattern of dispersed displacement for Latinos may then shift to a high-D, high-S combination of "prototypical segregation." The data reviewed in Fossett, Crowell, Saenz, and Zhang (2015) indicate that the quantitative trend just described can be seen across many Latino new destinations over the period 1990–2010.

These are just two examples of how possible, and I argue plausible, scenarios for social dynamics and trends could potentially produce White-Minority uneven distribution in the form of both "dispersed displacement without separation" and "concentrated displacement" resulting in "prototypical segregation". Accordingly, sociologists should be mindful of the possibilities and should consider systematically examining segregation indices that can reveal the presence of these distinctive residential patterns. The easiest option for doing so is to examine both D and S and note when instances of D-S concordance and discordance are found.

8.3 A Practical Issue When Comparing D and S – Size of Spatial Units

Values of S and D can and sometimes do disagree. When the differences are large, the discrepancy will always be in a particular direction; D will be high and S will be low. This outcome is rich with sociological implications but its occurrence is rarely discussed. The example introduced earlier in which I contrasted median scores for White-Black segregation with White-Asian segregation illustrated this point. D was high for both group comparisons with scores of 72.1 and 64.6, respectively. In contrast, S for White-Black segregation (46.4) was more than three times higher than S for White-Asian segregation (13.2). This result suggests something potentially important about the difference between White-Black segregation and White-Asian segregation. It is that consistently high levels of displacement from even distribution are evident in both comparisons, but group separation and residential polarization are present only in White-Black segregation. Uneven distribution for White-Asian segregation does not involve group separation and residential polarization. Instead, Asian displacement from parity on area proportion White (p) involves dispersed displacement with quantitatively small departures from parity. Consequently, Asians live alongside Whites and experience similar residential outcomes. Blacks experience similar extensiveness of displacement from parity on area proportion White (p), but the departures from parity are much larger quantitatively and as a result Blacks do not live alongside Whites and do not experience similar residential outcomes with regard to area proportion White (and presumably also with area characteristics that are correlated with area proportion White). Based on this, it is reasonable to conclude that the potential for differences in life chances and other consequences to arise from segregation are much greater for Blacks than from Asians even though typical values on D are relatively close.

This example along with the examples discussed in the preceding section of this chapter make a compelling case for the value of comparing S with D. However, I now caution that, before researchers finalize conclusions based on comparing D and S, they should take to review certain aspects of study design to make sure that the conclusions offered will be sound. The aspect of research design to review is the comparison of group size and the population size of the spatial units used to assess segregation. This aspect of research design is potentially important for both S and for D. But its consequences can be different for D and S and in some conditions can exaggerate D-S differences.

It is of course well known that using larger spatial units will result in lower segregation scores for any index of uneven distribution. Conventional wisdom is that this is not generally a major concern so long as it is reasonable to assume that the effect is approximately constant across cases. In that situation, researchers will know that overall levels of segregation will be lower, but at the same time they can expect that comparisons across cities or for a given city over time will still reveal fundamental variations in patterns and trends over time.

Unfortunately, it is not always reasonable to assume that the impact of areal unit choice is approximately constant across measures or across individual cases. One potentially serious problem can arise when spatial units used to measure segregation are large in relation to overall group size.[4] It is that segregation index scores will be misleadingly biased down when smaller homogeneous regions are "hidden" within larger heterogeneous areas. The problem affects both D and S but not to the same degree. The previous chapter noted that S is sensitive to large differences in area racial composition that reflect area racial polarization and group residential separation. But measurement of polarized differences is susceptible to being diminished when smaller homogeneous areas occur within larger units. This leads to lower values on D as well as S. But in this case, D is protected by its crudity as, whether due to true social dynamics or due to limitations of research design, reductions in area polarization only impact D when the associated changes cause one area to cross from one side of overall city racial composition (P) to the other. In essence, using areal units that are "too large" imposes an artifactual "ceiling" on scores for group separation and neighborhood polarization by pulling area-specific values on racial composition (p) toward the grand mean (P).

The problem of underestimating segregation will be worse under at least two conditions. The first is when segregation is manifest at a relatively low spatial scale – for example, at the block level – *and* segregation also follows a pattern of small-scale "checkering" instead of large-scale clustering. In this situation the aggregation of smaller homogeneous units within larger heterogeneous units can

[4] The key issue is absolute group size, not relative size. However, the two often go hand in hand and so the issue often will be salient when relative group size is small.

reduce values of both D and S dramatically. Fortunately, the practical consequence is usually modest because segregation patterns in US cities are characterized more by large-scale clustering than by small-scale checkering.

The second condition is when segregation patterns include homogeneous regions that are smaller than the areal units used in the study design. The practical consequence of this problem is greater when groups are small in size. Even when area polarization is substantial and homogeneous areas for a group are clustered, the value of S cannot reach its maximum value if the overall size of the smaller group does not comfortably exceed the population size of the areal units used to assess segregation. As noted above, the impact will be potentially important for both D and S, but more so for S. As a result, using large spatial units when investigating segregation involving small groups can distort comparisons of D and S making D-S differences appear larger than would be the case if a better research design was used.

In light of this, researchers should give the issue careful thought when making decisions about research design. Happily, the problem is easy to understand and, once appreciated, major problems are easy to avoid. The solution is to confirm that the spatial units used to assess segregation have the logical capacity to capture group separation and residential polarization for the groups in the comparison.

Brief discussion of a hypothetical example can illustrate the key issues. Assume a hypothetical city with 4 equal size census tracts each containing 4000 people. Also assume that each tract is subdivided into 4 equal size block groups (for a total of 16 block groups) each containing 1000 people. Next assume that the city has two groups, one with 15,000 people and one with 1000 people, and then assume that everyone in the smaller group resides in a single block group. Finally, assume that each block group is divided into 10 equal size blocks each containing 100 people.

In this example, S and D will both register perfect segregation ($D = S = 100.0$) if their values are computed using block data or block group data. However, if they are computed using tract data their values will be 80.0 for D and 20.0 for S. This contrast illustrates two points. The first is that both displacement (D) and separation (S) can be measured without error if the spatial unit used in the research is "right sized" as it is in this example when using blocks and block groups.

The second point is that when the spatial unit used is too large – meaning specifically that the population of the smaller group is too small to fill multiple areas, as is the case when using tracts in this example – the value of all indices of uneven distribution will be underestimated. Furthermore, while both D and S will be underestimated based on this problem with research design, the impact will tend to be more dramatic for S for reasons give above. This in turn can distort the comparison of D and S. In the worst case scenario, it would produce an incorrect impression that a high D, low S situation of "dispersed displacement" or "displacement without separation" prevails when a better research design would reveal a high-D, high-S combination indicating a pattern of "prototypical segregation".

A simple practice can guard against the problem; avoid using spatial units that are too large to reveal group separation and neighborhood polarization involving small groups. A practical rule of thumb is that typical population size for spatial units should be one-third to one-fifth the total size of the smaller group. Alternatively, group size should be 3–5 times larger than typical area population size. When this condition is met, it will be possible to detect group separation and neighborhood polarization when it is present. However, if the spatial units are too large – that is, if their typical population size approaches or is larger than the size of the smaller group, it will be impossible to fully "see" group separation and residential polarization when it is present.

8.3.1 A Case Study of White-Black Segregation Cullman County Alabama

I now review a real world example that illustrates both the problem and the solution. The case is White-Black segregation in Cullman county Alabama, which constitutes the Cullman, Alabama core-based statistical area (CBSA). In 2000 the county population included 73,940 Whites and 726 Blacks with Blacks comprising less than 1 % of the population. A *New York Times* article (Dawidoff 2010) reports that Black residents of the area describe the county as having a racist history including vigorous KKK activities and a hostile attitude toward Blacks in the Jim Crow era and beyond as exemplified by the fact of "sundown town" signs being posted in Cullman, the largest urban center of the county, well into the 1970s.[5] Historically, this caused Blacks to be excluded from the city of Cullman and the demographic legacy remains evident in contemporary residential distributions for the county. As of 2000, a majority of the Black population residing in the county lived in or near the small city of Colony, an outlying hamlet traditionally known as a "safe haven" for Blacks located in the hilly countryside to the south of Cullman, which was originally settled by former slaves who received land during Reconstruction following the Civil War (Kaetz 2013; Dawidoff 2010).

The social history of the county explains why Blacks are few in number in the local population and it provides a basis for expecting that the small Black population present would be residentially separated from Whites. This is in fact the case. But it is crucial to use "right sized" spatial units to "see" this pattern in a quantitative analysis of White-Black segregation. Group separation and residential polarization is readily evident in analysis using data for census blocks ($S = 62.6$). But it is less evident in analysis using data for census block groups ($S = 21.0$) and it is not evident at all using data for census tracts ($S = 5.8$). In comparison, values of D do not differ so dramatically by type of spatial unit. The progression for D is 94.2 for blocks, 82.6 for block groups, and 73.8 for tracts. Values for both S and D are lower when using tracts instead of blocks. But the difference between block- and tract-based scores for D is modest in comparison to the same difference observed for

[5] Loewen (2005) study of "Sundown" towns discusses Cullman and many other cases and notes that sundown signs proclaimed messages such as "Nigger Don't Let the Sun Go Down on You in This Town" and were common place in Alabama and many other states of the South and Midwest.

S. The progression in D-S difference is from 31.6 for blocks, 61.6 for block groups, and 68.0 for tracts. Recalling guidelines for D-S comparison offered in earlier chapters, the comparison based on block data indicates high-D, high-S and "prototypical segregation" based on a pattern for concentrated displacement from even distribution. In contrast, the comparison based on tract data suggests high-D, low-S consistent with a pattern of "dispersed displacement" or "displacement without separation".

The explanation for these results is simple; the typical population sizes of census tracts and even census block groups are too large to detect White-Black residential separation in a situation where the Black population is small. The typical tract in Cullman County has a population of approximately 4,000 so, even if all Blacks in the county lived in a single tract, they would live in a predominantly White tract. In contrast, the typical block in Cullman County has approximately 24–28 people (similar to block data for other communities around the country) and thus block-level analysis has the logical capacity to easily detect White-Black separation and residential polarization if it is present. And it definitely is. Out of 2,449 populated blocks in Cullman County in 2000, a subset of twelve (12) blocks that were at least 75 % Black (and with at least 10 residents) contained over 370 Blacks, over half of the Black population in the county. GIS-based mapping of population distribution for the Cullman CBSA (not reviewed here) reveals that these 12 blocks are located in a cluster of contiguous blocks in and around the hamlet of Colony. The high value of the separation index ($S = 62.6$) computed from block data registers this pattern of group separation and residential polarization clearly and unambiguously. Its interpretation is simple, straightforward, and sociologically meaningful. Whites and Blacks in Cullman County are residentially separated from each other and members of both groups primarily live in racially polarized neighborhoods where their own group predominates.

The lesson from this case is that tracts can be too large to detect White-Black residential separation even when the size of the Black population exceeds the size of the typical tract. This problem can occur under at least two conditions. One is when segregation involves "checkering" occurring at a spatial scale smaller than the tract. Checkering could occur for example when multiple small predominantly Black neighborhoods arise in different parts of the city. Extreme clustering would occur when predominantly black neighborhoods are contiguous and form a single Black ghetto. Analysis using block level data will detect segregation in both cases. Analysis using tract data will detect segregation only in the second case.

A second condition can further complicate the situation. It is when tract boundaries do not coincide with the perimeters of clusters of homogenous subareas (e.g., blocks). Census guidelines call for tract boundaries to follow social homogeneity in population distribution when feasible. But even at time of original "founding" boundary alignment may not be perfect because other competing concerns (e.g., tract population size, features of the natural and built environment, political boundaries, etc.) also must be taken into account. Even when boundaries initially delimit homogeneous populations, this can change over decades based on dynamics of neighborhood change and population redistribution. Analysis using block level data will be minimally affected by this problem because of their small spatial and population size. Analysis using tract level data can be affected in non-negligible degree, especially when minority population size is small.

8.3.2 *A Case Study of White-Minority Segregation in Palacios TX*

Palacios Texas is a small city found in the southwest corner of Matagorda County which constitutes the Bay City Texas CBSA. The case of Palacios is interesting because it is characterized by a segregation pattern not seen frequently in empirical studies – a high-D, high-S combination for White-Asian segregation in a community with a relatively small Asian population. Before proceeding, I first pause to make the case that it is reasonable to examine the city of Palacios separately from the rest of the Bay City CBSA. Palacios is a small spatially isolated coastal community located on Matagorda Bay some 28 miles away from the larger, inland community of Bay City. Significantly, Palacios and the nearby region is home to approximately 16 % of the total population in the CBSA but about 79 % of the CBSA's Asian population.[6] The counts by group for Palacios are 2,895 Latinos, 2,236 Whites, 706 Asians, and 239 Blacks.

The D-S combinations for all White-Minority segregation comparisons in Palacios follow patterns of "prototypical segregation." The White-Black segregation comparison involves a high-D, high-S combination (D = 79.6, S = 50.1) and White-Latino comparison involves a medium-D, medium-S combination (D = 54.9, S = 39.9). These are not particularly unusual for the region. What is unusual is that in Palacios White-Asian segregation also is characterized by a high-D, high-S combination (D = 75.3, S = 64.2) that is rarely seen for White-Asian comparisons.

Close review of the residential pattern by GIS analysis and also with an in-person, on-site visit confirms what the quantitative analysis suggests; namely, White-Asian segregation in Palacios follows a prototypical pattern of extensive displacement from even distribution that is highly concentrated resulting in a high level of group separation and neighborhood racial polarization. GIS analysis confirmed by on-site review of contemporary residential patterns combined with review of historical materials reveals that the Asian population in Palacios has for at least three decades been concentrated in a small set of six adjoining blocks that are home to a thriving Vietnamese community that came into existence in 1976–1983 as a result of a refugee settlement program.[7]

This example provides further evidence that segregation patterns can span a wide range of logically possible outcomes in terms of D-S combinations and that valu-

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[6] Population counts and other analyses reported below are based on the census tract in Matagorda County Texas that contains all block groups overlapping with or adjoining Palacios.

[7] This discussion draws on an article "A Shrimp Tale" by Robert Draper in the October 1996 issue of *Texas Monthly* magazine which recounts the history of Vietnamese settlement in Palacios and its reception by and impact on the local community.

able insights can be gained by examining both displacement (D) and separation (S). In the case of Palacios TX, the unusual high-D, high-S combination for White-Asian segregation prompted a closer inspection. This in turn revealed an interesting community history with social dynamics that serve to produce and perpetuate a pattern of White-Asian segregation that is quite different from that seen in most communities. In particular, White-Asian is highest of all White-Minority comparisons and much higher than the White-Latino comparison and closer qualitative review confirms that the quantitative finding of high-D, high-S identifies a city with a unique history of ethnic relations and residential segregation.

This example also provides further evidence that the concern that values of the separation index (S) will necessarily be low when groups are small is clearly unfounded. The comparisons of D and S for Palacios, Texas show that these indices can reveal much about segregation of small groups in small communities so long as the research design uses spatial units that are appropriate for the research setting. In this case that requires using block data. When using block data interesting and varied patterns of segregation are revealed by contrasting values of D and S across White-Minority comparisons. GIS analysis of group residential distributions and in-person, on-site inspection of the residential patterns confirms the patterns suggested by the D-S contrasts.

Indeed, the unusually high level of group separation in the White-Asian comparison is both obvious and quite striking when one is "on the ground" in Palacios. But due to the small size of the group populations involved, all of these patterns would be missed if segregation were assessed using tract-level data or even block group-level data. A single tract includes all of Palacios and also the surrounding area so tract-level analysis is infeasible. The tract containing Palacios is comprised of six block groups so block-group analysis is technically possible. But it would be highly misleading. In 2000 the tract containing Palacios had 237 populated blocks. A small cluster of six (6) contiguous blocks located on the northern side of the city forms a Vietnamese enclave easily identified by GIS analysis and on-site inspection. The six blocks contained over half (50.7 %) of the Asian population in the Palacios area and had a population of 41 (10.3 %) non-Asians and 358 (89.7 %) Asians. The enclave cannot be identified using block group data because it is located in a block group where the other blocks (not in the enclave) have a population of 888 (98.1 %) non-Asians and 17 (1.9 %) Asians. Accordingly, computing D and S using block group data yields values of 26.7 for D and 6.3 for S and equally low values for the other White-Black and White-Latino comparisons as well.

8.3.3 Reiterating the Importance of Using "Right-Sized" Spatial Units

The takeaway point from these two quantitative case studies is that it is important to use "right-sized" spatial units when assessing residential segregation and particularly when using S to assess group separation and residential polarization for groups that are small in size. The good news is that S will reliably detect residential separation between two groups so long as the spatial units used in the research design are appropriate for the analysis. In the cases just examined, block data readily revealed patterns of segregation even when some of the groups in the segregation comparisons were very small in overall population size.

Block data were once widely used in segregation analysis including most notably the landmark study by Taeuber and Taeuber (1965) and dozens of studies that used and supplemented these measures (e.g., Schnore and Evenson 1966; Roof 1972; Roof and Van Valey 1972; Sorenson et al. 1975). But in recent decades, with only occasional exceptions such as Lichter and colleagues (2010) and Allen and Turner (2012), segregation studies have relied primarily on tract-level data. The examples reviewed above highlight how the practice of using larger spatial units such as tracts and even block groups can limit the potential scope of segregation studies by creating problems for assessing residential separation between groups when one group is small. This sometimes is mistakenly viewed as a problem inherent in the indices themselves. Indeed, some have raised concerns that the separation index will "necessarily" yield low values when segregation involves small groups. The examples just reviewed show this view is mistaken on two counts. First, to the extent that there is a problem, it is not limited to the separation index; it applies to all popular indices of uneven distribution. Second, the problem is not inherent in the indices; the problem is with basic features of research design in failing to use spatial units appropriate for obtaining valid assessments of segregation.

The analyses just reviewed demonstrate that both D and S can yield misleading low values when computed using tract-level and block group-level data but will correctly signal the presence of substantial segregation when computed using block-level data. This suggests that studies should use block-level data to guard against the problem. But as noted above this practice has become uncommon. The prevailing use of tract-level data is partly due to the fact that census tabulations for tracts provide more detailed breakdowns of population groups. But another important factor is that methodological studies have noted problems that can arise when measuring segregation using small spatial units. Taeuber and Taeuber's thorough discussion of issues in segregation measurement (1965: Appendix 1) noted one reason. It is that it can be difficult or even impossible to achieve even distribution with small areas and small groups because populations are distributed in indivisible, whole number "clumps" associated with individuals, families, and households, not fractional parts, and this makes it difficult to exactly reproduce city-wide racial proportions in small areas. Winship (1977) pointed out a second reason that has been seen as more important. It is that indices measuring uneven distribution are inherently susceptible to undesirable, non-negligible upward bias when segregation is assessed using small spatial units.

The potential undesirable impact of both factors is more consequential for D than for S. But it is an important concern and, accordingly, I review it at length in analyses I present in Chaps. 14, 15 and 16. I save the details of that discussion for later. For now, I note that the new methods introduced in this monograph make it possible to identify the underlying basis for the problem of index bias and introduce new versions of popular indices that eliminate undesirable impact of bias while retaining

Type of Area	Factor for Ratio of Group Population Count to Area Population Size: 3	5	10
Blocks @ 30 persons	90	150	300
Block Groups @ 1,250 persons	4,750	6,250	12,500
Tracts @ 4,000 persons	12,000	20,000	40,000

Fig. 8.5 General guidelines for group population thresholds needed to assess displacement and group separation and area racial polarization

other desirable index properties such as familiar substantive interpretations. Based on this, I have no reservations in advising researchers to use data for smaller spatial units when investigating segregation involving small groups. Concerns about index bias when using block-level data can be readily addressed using methods outlined in this monograph.[8]

8.3.4 More Practical Guidance for Using S

The discussion to this point raises the concern that all aspects of segregation in general and group separation and residential polarization in particular may not always be assessed accurately in studies that investigate segregation involving small groups using tract data. Earlier I suggested a "rule of thumb" that the size of the smaller group in the analysis should be 3–5 times the size of the areal units used to assess segregation. This informal guideline provides a basis for diagnosing the situation and considering alternative options for research design. I summarize the implications of this guideline for studies using blocks, block groups, and tracts in Fig. 8.5. Note that the guidelines do not focus on relative size per se. That is appropriate because for this issue relative size is not the true source of the problem. The guidelines instead focus instead on group population counts and indicate that to be "safe" the population size of both groups in the comparison should be at least 3–5 times the typical population size for the areal unit used. In addition, I have added an even more conservative factor of 10 to 1 and then have listed the associated group size "thresholds" for being able to "safely" analysis of displacement from even distribution and group separation and residential polarization when using data for blocks, block groups, and tracts:

[8] The results for the examples of block-level analysis discussed in this chapter are for "standard" versions of D and S, not the "unbiased" versions I introduce in Chap. 15. In these particular cases, the issue of bias does not distort the findings presented. So I use standard versions of D and S to avoid introducing unnecessary complication to the discussion here.

These calculations make it clear that fairly large city group counts are needed to reliably assess displacement from uneven distribution and group separation and area racial polarization with tract-level data. The "safe" threshold ranges from 12,000 to 40,000 depending on whether one chooses a liberal (3:1) or conservative (10:1) ratio of group population size to typical area population size. Studies using census tract data often include cases where the size of the smaller group in the comparison falls below these thresholds, especially the conservative threshold. This raises questions as to whether assessments of displacement from even distribution and group separation and area racial polarization have been equally reliable across all cases in past studies using tract-level data. The basis for concern is not as great when segregation is measured using data for block groups because the thresholds for group size requirements are lower. The basis for concern is smaller still when segregation is assessed using block data because the thresholds for group size requirements are very small. This indicates that using block data is the safe way to go on this aspect of research design.

8.4 A Simple Index of Polarization

I conclude this chapter with a brief discussion of an alternative option for measuring group separation and area racial polarization. I offer the alternative because I recognize that D is popular in part because it is easy to compute and explain. In my opinion, S also is attractive on these counts and compares favorably with D, especially when both indices are presented in the difference of means formulation. But I also recognize that it others may it useful to have an alternative measure of separation when even greater simplicity is a priority. I suggest such a measure here terming it the "Polarization" index.

The index is constructed as follows. First, for both groups, calculate the percentage in each group that resides in areas where their group predominates based on a user-chosen "threshold" or "cut-point" such as 65 % same-group presence (POL_{65}). To illustrate using Cullman County, I first calculate the percentage of Whites that reside in areas that are 65 % White and I then calculate the percentage Blacks that reside in areas that are 65 % Black. The results show that 99.9 % of Whites lived in predominantly White areas and 61.8 % of Blacks lived in predominantly Black areas. The value of the polarization index is set to the lower of these two values and so POL_{65} is 61.8.

The logic for this measure is as follows. If the residential distributions of the two groups are polarized, the percentage residing in predominantly same-group neighborhoods must be high for *both* groups. If the distributions are not polarized, the number will be low for at least one of the two groups. So taking the minimum of the two values can serve as a simple "polarization" index. In addition to being easy to compute, the score of 61.8 for White-Black polarization (at 65 %) in Cullman County is easy to interpret; it indicates that at least 61.8 % of both groups reside in neighborhoods where their group predominates (at a level of 65 % or higher).

The main benefit of this measure is that it may be useful for helping broad audiences gain an immediate intuitive grasp of group separation and neighborhood polarization. I have conducted detailed analyses (not reported here) of the behavior of this simple index of separation and polarization and I find that the measure can be highly serviceable. It ranks cases in a consistent way over different user-choices for the threshold for same-group presence and its values typically track the separation index (S) fairly closely. For example, when using threshold levels for group predominance over the range of 55–75 %, the index values for Cullman fall between 53.3 and 62.5 and thus are roughly comparable to the value of S at 61.8.[9]

Consistency between S and POL also is seen when considering a broader range of cases. For the large, multi-year CBSA data set introduced earlier in Chap. 6, the correlations among "cut point" polarization indices using thresholds set at 5 point increments over the range 55–80 % ranged from 0.93 and 0.94. Of course, while these correlations are very high, they are not perfect. That is to be expected because S registers separation and polarization across the full spectrum of area racial composition, not just in relation to a single threshold value. The trade-off then is between precision of measurement (S) and easy of discussion and presentation ("cut point" polarization indices).

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[9] Setting the threshold at 5 point increments ranging from 55 to 80 % yields polarization scores of 62.5, 62.5, 61.8, 54.7, 53.3, and 53.3.

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Chapter 9
Unifying Micro-level and Macro-level Analyses of Segregation

Casting segregation indices in the difference of means framework provides a valuable option previously not available to researchers. It enables them to seamlessly connect macro-level segregation – as measured by the index score for a city – to micro-level processes of residential attainment. At the simplest level the value of any index placed in the difference of means framework can be obtained by performing an individual-level attainment analysis that predicts index-relevant residential outcomes (y, scored from area group proportion p) for individuals with a dummy variable (0,1) for racial group membership. The regression coefficient for race will exactly equal the index score obtained by standard computing formulas. This introduces a new interpretation of segregation index scores; their values reflect the effect of race on the attainment of residential outcomes that determine the segregation index score for the city.

Establishing the equivalence of between macro-level measures of segregation and the effect of race on residential attainments in a bivariate individual-level regression model paves the way for at least three important new options for segregation analysis. The first is to give researchers the ability to extend and elaborate bivariate models to investigate segregation in more detail using multivariate analyses. These models make it possible for researchers to address fundamental questions that previously could not be directly investigated. For example, researchers can assess whether or not the impact of race on segregation-determining residential outcomes seen in the bivariate analysis continues to persist when controls are introduced for other relevant individual- and household-level social characteristics (e.g., age, education, income, marital status, household composition, nativity, etc.) that may exert independent influence on residential outcomes.

A second new option for segregation analysis is to give researchers the opportunity to quantitatively dissect the underpinnings of segregation in more detail than has previously been possible. Specifically, researchers can use familiar tools of standardization and decomposition analysis to assess how the index score for a city is

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_9

quantitatively linked to group differences in the resources each group brings to the residential attainment process and to group differences in the parameters of the attainment process where resources (inputs) are converted to residential outcomes. Thus, one can develop improved answers to questions such as “Does segregation arise primarily because groups differ on income and other resources that affect residential contact with the reference group?” Or, “Does segregation arise primarily because groups differ with respect to their ability to convert income and other resources into residential contact with the reference group?” Or, “Do both factors play an important role in creating segregation?” Questions of this sort have been raised for many decades. But answers have been unsatisfactory because the available options for addressing the question have been crude and difficult to implement. The difference of means formulation provides new and superior options for developing answers to these long-standing questions.

A third new option for segregation analysis is for researchers to investigate cross-area and over-time variation in segregation in more detail using multi-level specifications of bivariate and multivariate segregation attainment models. Segregation attainment models are individual-level attainment models that predict the residential outcomes that exactly determine the level of segregation in a city. Multi-level specifications of the basic bivariate segregation attainment model enable researchers to investigate ecological variation in segregation by assessing how segregation – equated in this approach to the effect of race on segregation-determining residential outcomes – varies over time and across different cities depending on the time period and characteristics of the metropolitan area such as its size, rate of growth, industrial and occupational structure, unemployment rate, military presence, etc.

Multi-level specifications of individual-level, multivariate segregation attainment models make it possible to investigate these patterns in more detail and sophistication than ever before. Importantly, these models provide a superior approach for taking account of the role of non-racial social characteristics in shaping variation in segregation over time and across areas. Researchers routinely hypothesize that group differences on income, nativity, and other social characteristics may play a role in explaining cross area variation in segregation. Currently these hypotheses are assessed with aggregate-level models in which measures such as group income ratios, or percent foreign born for Latinos are used to predict segregation index scores for cities. The difference of means framework and the associated new option of analyzing segregation via attainment models make it clear that this long-standing practice is fundamentally flawed and should be discontinued.

Current practice carries risks of erroneous inference associated with the so-called “ecological fallacy” – the fallacy of using aggregate indicators to assess or control for the effects of variables that operate at the micro level. Researchers have relied on the aggregate-level approach to address these important questions because until now they did not have better options for analysis. Multi-level implementations of multivariate segregation attainment models now allow researchers to properly take account of variables that affect segregation-determining outcomes at the micro level (e.g., income, nativity, English language ability, etc.) when investigating cross-area and cross-time variation in segregation.

The difference of means framework makes these three new options for segregation analysis possible. I discuss the first two in more detail in the remainder of this chapter. I provide a detailed discussion of the third option in Chap. 10.

9.1 New Ways to Work with Detailed Summary File Tabulations

To begin I illustrate how the difference of means formulation makes it possible for researchers to investigate segregation in new ways by revisiting and expanding on the analysis of White-Minority segregation in Houston, Texas reported earlier in Chap. 5 (Tables 5.1, 5.2 and 5.3). The summary file tabulations underpinning these analyses provide more than just simple counts of families by race for census block groups. The tabulations also provide counts of families by poverty status, family type, and presence of related children separately by race.[1] The analysis of segregation reported in Tables 5.1, 5.2 and 5.3 was simple and conventional. It assessed segregation in terms for race differences in residential outcomes without consideration for the role of the other social and economic characteristics available in the tabulation. There was no need to do so because index scores for the *overall* level of segregation between groups can be calculated using just group counts by race over areas. Accordingly, the scores reported in Tables 5.2 and 5.3 were obtained by collapsing the original detailed tabulations to obtain just the marginals for race.

The difference of means formulation of segregation indices makes it possible to draw on the detailed information in the full tabulation to gain a deeper understanding of how overall segregation is related to group differences in distribution across poverty status and family type. It has always been recognized, at least implicitly, that segregation arises out of group differences in distribution on individual residential attainments. And it also is widely recognized that residential attainments may vary, not only by race, but also with social characteristics such as age, gender, education, income, family status, and so on. Accordingly, researchers extending back at least to Duncan and Duncan (1955) have always wished for the option to take account of the possible role of social characteristics other than race when investigating racial segregation. They have been frustrated in this goal, however, because until now the macro-level outcome of segregation could not be directly linked to individual-level residential outcomes in a way that would allow researchers to undertake the kinds of quantitative analyses needed to explore the issues with greater detail and sophistication.

[1] Specifically, I draw on Tabulations P160 A-I of Census Summary File 3 of the 2000 Census.

Table 9.1 Descriptive statistics for poverty status and distribution of poverty status by family type for Whites, Black, Latinos, and Asians in Houston, Texas, 2000

Variable	Whites	Blacks	Latinos	Asians
Distribution of families by social characteristics				
Percent families not in poverty	95.8	80.9	80.2	90.4
Percent families in poverty	4.2	19.2	19.8	9.6
Percent families married couple	84.0	51.2	74.1	84.9
Percent families with children	47.3	62.0	69.2	59.3
Detailed distribution of families by poverty status and family type				
Families not in poverty by family type				
Married couple, no children	43.2	19.1	16.0	28.6
Married couple, children	38.7	27.8	46.0	49.5
Female headed, no children	3.9	7.9	2.9	3.4
Female headed, children	6.1	19.9	7.4	3.7
Other family type	4.0	6.2	7.9	5.3
Families in poverty by family type				
Married couple, no children	1.1	1.6	1.6	2.2
Married couple, children	1.1	2.7	10.6	4.6
Female headed, no children	0.3	1.7	0.5	0.5
Female headed, children	1.4	11.6	5.2	1.6
Other family type	0.4	1.6	1.9	0.7
	100.2	100.0	100.0	100.1
Sample N	627,613	195,928	294,931	55,746

Source: US Census 2000, Summary File 3

The difference of means framework provides a solution to this problem. Casting segregation index scores as a group difference of means on residential outcomes for individuals opens the door for researchers to apply a standard toolkit of methods that are currently used to investigate race differences on education, income, poverty status, and other socioeconomic outcomes. Specifically, researchers now can analyze segregation by combining individual-level attainment analysis with demographic techniques of standardization and components analysis to better assess the roles that race and other social characteristics play in determining segregation.

9.2 Some Preliminaries

Tables 9.1 and 9.2 present the relevant descriptive data for the case of Houston, Texas. Table 9.1 documents that Whites, Blacks, Latinos, and Asians differ in their distribution across categories of family type and poverty status. Table 9.2 documents how averages on the residential outcomes (y) that determine the separation index (S) vary across families grouped by family type, poverty status, and race. Table 9.3 similarly documents how averages on the residential outcomes (y) that

Table 9.2 Means on pairwise contact with Whites (y) scored for the separation index (S) by poverty status and family type for White-Minority comparisons, Houston, Texas, 2000

	Whites		Minority group	
Family type	Non-poverty	Poverty	Non-poverty	Poverty
White-Black comparison				
Married couple, no children	89.9	87.4	34.2	19.7
Married couple, children	91.3	86.0	41.3	29.4
Female headed, no children	86.1	87.3	24.9	19.7
Female headed, children	87.7	83.7	31.1	23.0
Other family type	87.2	83.9	30.6	20.1
All families	89.9		32.5	
Value of separation index (S)	57.4			
White-Latino comparison				
Married couple, no children	81.2	72.5	46.9	30.7
Married couple, children	83.9	74.0	42.6	30.2
Female headed, no children	73.3	70.7	38.4	31.3
Female headed, children	77.6	71.9	41.6	31.6
Other family type	75.7	66.9	36.2	28.2
All families	81.1		40.2	
Value of separation index (S)	40.9			
White-Asian comparison				
Married couple, no children	93.9	94.5	71.2	61.8
Married couple, children	93.9	94.0	71.7	62.3
Female headed, no children	93.2	94.6	65.6	63.6
Female headed, children	92.7	94.5	70.4	58.2
Other family type	93.3	92.9	64.8	59.4
All families	93.8		69.9	
Value of separation index (S)	23.9			

Source: US Census 2000, Summary File 3

determine the dissimilarity index (D) vary across families grouped by family type, poverty status and race. In the difference of means framework the patterns in these three tables carry clear and direct implications for segregation. The overall segregation index score for the group comparison is determined by the group difference of means on residential outcomes (y) and the mean for each racial group is in turn determined by the weighted average of the subgroup means for that racial group.

From that vantage point the data presented in Tables 9.2 and 9.3 can be understood as providing a simple "ANOVA-style" micro-level attainment analysis of residential segregation as measured by the separation index (S) and the dissimilarity index (D), respectively. The essence of the analysis is that individual families are cross-classified by the "independent variables" of race, family type, and poverty status and means on the "dependent variable" of scaled contact with Whites (y) are reported for the subgroups that are broken out in the cross tabulation. The overall group means reported in Table 9.2 in the rows labeled "All Families" reflect the

Table 9.3 Means on scaled pairwise contact with Whites (y) scored for the dissimilarity index (D) by poverty status and family type for White-Minority comparisons, Houston, Texas, 2000

	Whites		Minority group	
Family type	Non-poverty	Poverty	Non-poverty	Poverty
White-Black comparison				
Married couple, no children	87.5	82.2	20.2	8.8
Married couple, children	90.6	80.6	24.2	14.5
Female headed, no children	80.8	83.4	9.9	7.1
Female headed, children	84.6	77.8	13.5	8.8
Other family type	82.5	74.9	14.5	7.1
All families	87.7		16.8	
Dissimilarity index (D)	70.9			
White-Latino comparison				
Married couple, no children	81.5	67.6	32.9	13.3
Married couple, children	86.2	68.9	24.9	12.5
Female headed, no children	68.4	69.1	22.3	17.2
Female headed, children	76.6	65.8	23.6	13.5
Other family type	72.5	56.9	18.5	11.1
All families	81.5		23.1	
Dissimilarity index (D)	58.4			
White-Asian comparison				
Married couple, no children	75.3	78.8	19.2	13.0
Married couple, children	75.6	78.8	17.1	13.7
Female headed, no children	73.8	84.7	16.4	26.9
Female headed, children	70.7	78.6	15.7	12.1
Other family type	73.7	77.6	10.3	8.7
All families	75.2		16.9	
Dissimilarity index (D)	58.3			

Source: US Census 2000, Summary File 3

weighted sum of the subgroup means by family type and poverty status based on the relative frequencies reported in Table 9.1. The difference between the two "overall" group means yields the index score for the comparison. Thus, the score for the separation index (S) for the White-Black comparison is 57.4 based on the difference between Whites having mean (pairwise) contact with Whites of 89.9 compared to 32.5 for Blacks. Similarly, the score for the dissimilarity index (D) for the White-Black comparison is 70.9 based on the difference between Whites having a mean of 87.7 on (scaled pairwise) contact with Whites compared to a mean of 16.8 for Blacks.

It is not standard practice to analyze overall segregation index scores as arising from group differences in the distribution of individual families across subgroups with different average levels on residential outcomes (y) of scaled contact with Whites. In light of this I briefly review how the analysis presented in Tables 9.1, 9.2, and 9.3 can be performed using census summary tables. To begin, the data contained in the block group-level census summary tabulation must be reconstituted as a micro-level data set for families. The first step is to recognize that the count for each "interior" cell in the full summary file tabulation represents a set of micro-level "cases" – families in this example – that have a particular configuration of social characteristics. The poverty status by family type summary file tabulation in question has eighteen (18) interior cells (note that tabulation marginals are excluded). The tabulation is repeated for all four racial groups yielding 72 separate "cases" (i.e., cells) for each block group. The final data set thus has one "record" for each interior cell in the summary file tabulation; that is a total of 72 separate records for each of the block groups in Houston. Each record has a unique combination for the characteristics of race, family type, and poverty status. The cell frequency indicates how many families with this unique combination of characteristics are found in each block group in the metropolitan area.

Next a set of variables is coded for each of the records. The first variable is area of residence (i.e., the block group code). The second is "nfamilies" which is set to the value of the cell frequency for this case (i.e., the count of families in that cell of the tabulation). This will later be used as the frequency weight for the record when performing statistical calculations.[2] Next a series of additional variables are coded to represent the social characteristics of each family – namely, their race, family type, poverty status, etc. – in the table. Each characteristic is coded as a separate variable and assigned values as appropriate for the needs of the analysis. Each record in the resulting data set represents a set of families that reside in a particular block group and hold a specific combination of social characteristics.

The variables that register social characteristics will serve as "independent" variables in micro-level residential attainment analyses. They may be coded a variety of equivalent ways. I created dummy (0,1) variables for race to select records for Whites, Blacks, Latinos, or Asians as relevant. I also created a dummy variable for "poverty" and I similarly created a set of dummy variables to represent the five categories of family type. Finally, I also created additional dummy variables to capture the possible interaction of poverty status and family type. Viewed from the perspective of analysis of variance (ANOVA) the set of dummy variables includes all combinations needed to estimate a "saturated" ANOVA model which includes all main effects and all possible interactions.

The next step is to prepare a separate block group data set. The cases in this data set are block groups. The first variable for the case is the block group code which will be used for merging with the first micro-level data set. In addition, a set of variables are coded for the total counts of families by race; specifically, separate variables for the count of White, Black, Latino, and Asian families. Next compute a set of variables

[2] Alternatively, one could create an individual-level data base by generating the relevant number of individual records for the families represented in each cell of the cross tabulation and assigning relevant codes for the social characteristics of the families as appropriate based. Of course, it is mathematically equivalent and computationally more efficient to use cells as cases and weight by cell frequency when performing analyses. However, the alternative approach can be used when statistical software cannot apply frequency weights for cases.

with the values of pairwise proportion White (p) for each of the three possible White-minority comparisons. These provide the basis for computing variables that score residential outcomes (y) from area (pairwise) proportion White (p) as relevant for different segregation indices. For example, in the case of the separation index (S), the relevant residential outcome (y) is the value of p. In the case of the dissimilarity index (D) the relevant residential outcome is the value of either 1 or 0 depending on whether area proportion White (p) is greater than proportion White for the city (P) or not. The resulting block group-level data set will then contain variables that will serve as dependent variables in micro-level segregation attainment analyses.

The final analysis data set is created by merging the second data with the first data set based on the common block group code. The resulting data set can then be used to perform micro-level statistical analyses to analyze residential segregation.

I followed the procedures just described to prepare a data set I used to perform the analyses establishing how means on the residential outcome of scaled contact with Whites (y) varies across subgroups and groups as reported in Tables 9.2 and 9.3. The results in these tables were obtained by via tabulation routines that calculate means on the relevant dependent variables (y) across the categories of a cross classification table based on micro-level variables measuring the social characteristics of race, family type, and poverty status. In the analysis the records in the family-level data set were weighted by the variable "nfamilies" which has the number of families that have the specific combination of social characteristics and reside in the block group in question. The same family-level data set can be used to perform micro-level statistical analyses such as analysis of variance (ANOVA) and multiple regression analysis predicting the dependent variable of individual residential attainments using the independent variables of race and other social characteristics.,[3,4] I report regression results obtained in this way later in the chapter.

9.3 Substantive Findings

I now discuss the analysis results in more detail. Table 9.2 shows that in all three White-Minority comparisons scaled (pairwise) contact with Whites varies across categories of poverty status and family type as well as by race. Group means on this residential outcome determine the value of the separation index (S). Two clear patterns warrant mention even on cursory inspection of the table. The first is that minority contact with Whites is consistently lower for poverty families compared with non-poverty families. The second is that, within non-poverty families, married

[3] Weighting cases by the cell counts from the summary file tabulation makes this an individual-level regression because the cell count registers the number of families that reside in the block group in question and have the exact combination of race, family type, and poverty status coded for the case.

[4] These can be termed "saturated" models because they include all possible effects of poverty status and family type (including all interactions).

couple families have higher levels of contact with Whites. Table 9.1 also documents that overall and within categories of family type minority families are consistently more likely to be in poverty than are White families but with Asians being substantially less disadvantaged than Blacks and Latinos. Table 9.1 also shows that the overall percentage of families that are married couples and non-poverty is much higher for Whites (81.9 %) and Asians (78.1 %) than for Blacks (46.9 %) and Latinos (62.0 %). The combination of these two patterns suggests it is plausible to hypothesize that group differences on poverty and family composition may play a role in making White-Black and White-Latino segregation more pronounced than White-Asian segregation.

Closer inspection of the patterns in Table 9.2 lends additional credibility to this conjecture. In the White-Black comparison pairwise contact with Whites (p) varies within a narrow interval of 7.6 points for White families ranging from a low of 83.7 % for female-headed families with children and in poverty to a high of 91.3 % for non-poverty married couples with children. For Black families contact with Whites is generally much lower than that observed for Whites in every category of family type. This suggests that race is a crucial factor in shaping the value of S (the group difference of means on p). However, it also is the case that Black contact with Whites varies by 21.6 points over categories of poverty status and family type for Blacks. The lowest level of 19.7 % is seen for married couple families without children and in poverty and this level also is seen for female-headed families without children and in poverty. The highest level of 41.3 % is seen for non-poverty married couples with children. The contrast is dramatic; the level of contact seen for the latter group is 21.6 points higher and more than double the level see for the first two groups. This suggests that, in addition to the important role of race alone, group differences in family type and poverty status also might impact the value of S for the White-Black comparison.

Similar patterns are evident in the results for the White-Latino comparison and the White-Asian comparison. In the White-Latino comparison Latino contact with Whites (p) is lower than that observed for Whites for every combination of family type and poverty status suggesting a clear "across the board" race effect. But it also is clear that contact with Whites varies across categories of family type and poverty status; by 18.7 points for Latinos and by 13.2 points for Whites. Combining this information with the knowledge that Latinos are disproportionately concentrated in categories of family type and poverty status that experience lower levels of contact with Whites suggests that group differences in distribution by poverty and family type may impact the level of White-Latino segregation.

In the White-Asian comparison Asian contact with Whites (p) is lower than that observed for Whites across all categories of family type and poverty status again suggesting an across the board" race effect but Asian contact with Whites varies much more (by 13.5 points) across categories of family type and poverty status than is observed for Whites (only 1.9 points) thus lending plausibility to the hypothesis that group differences in distribution by poverty and family type may impact the level of White-Asian segregation.

In sum, the patterns documented in Tables 9.1 and 9.2 lend plausibility to the hypothesis that group differences in social characteristics might play a non-trivial role independent of race in contributing to overall segregation. Without going into the same level of detail, I note that similar conclusions can be drawn based on reviewing the data on residential outcomes that determine the value of the dissimilarity index (D) presented in Table 9.3. The key finding is that the subgroup means that determine D vary across poverty and family type within race. This raises the possibility that group differences in distribution across these social categories may be a factor contributing to segregation as measured by D.

9.4 Opportunities to Perform Standardization and Components Analysis

The micro-level data set used to prepare Tables 9.1, 9.2, and 9.3 also can be used to apply the workhorse demographic techniques of standardization and components analysis (e.g., Kitagawa 1955; Winsborough and Dickinson 1971; Althauser and Wigler 1972; Iams and Thornton 1975; Jones and Kelley 1984) to gain insights into what factors give rise to segregation. The technique of standardization involves adopting a "standard" relative frequency distribution for poverty status and family type and using it, not the "observed" distributions given in Table 9.1, to weight the group-specific means on residential outcomes over poverty status and family type to calculate "expected" group means on residential outcomes. The resulting "standardized" group means can be interpreted as the group averages on segregation-relevant residential outcomes (y) that would result if both groups had the same "standard" distribution" on social characteristics while continuing to experience their "observed" residential outcomes documented in Table 9.2. The difference between the two group means in the standardized comparison can be interpreted as the level of segregation that remains when group differences in distribution by family type and poverty status have been "taken into account" by statistically setting them to be equal.

Table 9.4 reports results of standardization analyses of the type just outlined. In conducting this analysis I adopted the observed distribution of *all* families (both White and minority group combined) over the categories of poverty status by family type as the relevant "standard" for the distribution of social characteristics. The top panel of the table reports results for the average levels on residential outcomes (y) that determine the value of the separation index (S) that would obtain for Whites and minorities if they had the same "standard" distribution for social characteristics. In the White-Black comparison the standardized mean for Whites is 89.46. This is about 0.40 points lower than the observed mean for Whites of 89.86. The standardized mean for Blacks is 35.07. This is about 2.59 points higher than the observed mean for Blacks of 32.48. The difference of the standardized group means can be interpreted as the value of the separation index (S) standardized to the condition of Whites and Blacks having identical distributions across family type and poverty

Table 9.4 Observed and standardized White-Minority segregation comparisons, Houston, Texas, 2000

	White-Black	White-Latino	White-Asian
	Separation index (S)		
Observed group means on scaled contact with Whites (y) observed			
White mean (y)	89.86	81.12	93.79
Minority mean (y)	32.48	40.17	69.91
Difference	*57.38*	*40.95*	*23.88*
Means standardized on overall distribution for family type and poverty status			
White mean (y)	89.46	80.35	93.78
Minority mean (y)	35.07	42.06	69.59
Difference	*54.38*	*38.29*	*24.19*
	Index of dissimilarity (D)		
Observed group means on scaled contact with Whites (y)			
White mean (y)	87.73	81.49	75.15
Minority mean (y)	16.75	23.12	16.93
Difference	*70.98*	*58.37*	*58.22*
Means standardized on overall distribution for family type and poverty status			
White mean (y)	87.04	80.26	75.28
Minority mean (y)	19.40	25.58	16.83
Difference	*67.64*	*54.68*	*58.45*

Source: US Census Summary File 3

status. The initial observed value of S was 57.38 points. The standardized value of S is 54.38 points. Thus, "standardizing" the comparison to a common distribution on poverty status and family type reduces the value of S by 3.00 points. This result provides a statistically sound basis for concluding that White-Black differences in the social characteristics considered here play only a small role in determining the overall level of White-Black segregation; simply put, "controlling" for group differences on social characteristics using sound methods of statistical analysis produces only a modest reduction in segregation.

This result also can be interpreted as indicating that the level of segregation as assessed by the observed value of S traces primarily to the effect of race. That is, group separation as measured by S traces to group differences in contact with Whites that arise independent of poverty status and family type. A more thorough decomposition analysis (per Kitagawa 1955; Althauser and Wigler 1972; Iams and Thornton 1975; Jones and Kelly 1984) could quantify this in a more careful way. Of course, like all standardization and decomposition exercises, thoughtful interpretations must consider the theoretical relevance of the "control" variables and the adequacy of the micro-level analysis that seeks to capture the relationship between non-racial social characteristics and segregation-relevant residential attainments.

Table 9.4 also reports results of standardization analyses for the separation index (S) for the White-Latino and White-Asian comparisons. These analyses also indicate that differences in group distribution over family type and poverty status do not play a major role in determining the overall level of segregation between the groups. In the case of the White-Latino comparison, standardizing on poverty status and family type reduces S by 2.66 points lowering it from 40.95 to 38.29. In the case of the White-Asian comparison, standardizing on poverty status and family type increases S by 0.31 points raising it from 23.88 to 24.19. This suggests that group differences in family type and poverty status serve to obscure the impact of race on overall White-Asian segregation.

The lower panel of Table 9.4 reports results of a set of parallel analyses focusing on segregation measured using the index of dissimilarity (D). To perform this parallel analysis, I made only one change; I used a new dependent variable; namely, y as scored for D (reported in Table 9.3) instead of y as scored for S (reported in Table 9.2). Recall that in this case y is now scored 1 if $p \geq P$ and 0 otherwise. The impact of standardizing the White-Minority comparison to a common distribution on poverty status and family type here is very similar to that seen for the analysis for S. In the case of the White-Black comparison, standardizing on poverty status and family type reduces D by 3.44 points from 70.98 to 67.64. For the White-Latino comparison, the standardization exercise reduces D by 3.69 points from 58.37 to 54.68. For the White-Asian comparison, standardizing on poverty status and family type increases D by 0.23 points from 58.22 to 58.45. Thus, as seen in the analysis for S, the level of segregation measured using D changes little when one uses appropriate statistical methods to take account of the possible impact of group differences in family type and poverty status.

I also performed similar standardization exercises for other segregation indices – specifically, G, R, and H. However, I do not report the details here as the basic finding is the same in all cases.[5] That is, when analyzing group differences in residential outcomes that determine segregation as measured by G, R, and H, standardizing the White-Minority comparisons to a common group distribution on poverty status and family type reduces segregation by only modest amounts.

9.5 Comparison with Previous Approaches to "Taking Account" of Non-racial Social Characteristics

The ability to conduct the standardization exercises just reviewed is a completely new option made possible by the difference of means framework for measuring segregation. Several considerations make this approach superior to current practices for assessing or controlling for the role of non-racial social and economic characteristics of individuals on segregation. First, the approach can be easily extended to directly "control for" the role of many social characteristics in a single analysis where previously this has not been feasible. Second, the approach can draw on a broader range of information and a larger number of cases than is typical in current approaches to taking account of non-racial social characteristics and as a

[5] This is not surprising as the discussions in Chaps. 5, 6, and 7 note that when D and S give similar results all popular indices of uneven distribution will give similar results.

result yields results that are more appropriate and statistically reliable. Third, the results of the approach are much less susceptible to problems of distortion resulting from index bias and ecological fallacies than are results of current practices. I now briefly comment on each of these points.

The prevailing approach for taking account of the impact that factors other than group membership may have on segregation involves calculating segregation scores for subsets of individuals from the two groups that are matched on social characteristics. In the present context, that would involve calculating as many as 10 different White-Black segregation scores, one each based on the just the families found in the 10 categories of family type and poverty status. Or, for simplicity, the analysis might be limited to calculating the index score for one carefully chosen subgroup comparison such as non-poverty, married couple families with children, the family type with the largest number of families across all four racial groups. When the obtained index scores is lower than the score for the overall segregation comparison, the result is interpreted as indicating that segregation is lower when social characteristics are “controlled” and thus supports the conclusion that the impact of group differences on social characteristics on segregation is important. Alternatively, when the scores obtained is not lower than the score for overall segregation, the result is interpreted as indicating that the impact of group differences on social characteristics on segregation is modest or unimportant.

Unfortunately, basing the analysis on segregation scores calculated for matched comparisons involving small subgroup numbers often introduces non-trivial complications and concerns. One problem is that the approach subtly changes the substantive and quantitative relevance of the analysis. Note that the standardized segregation index scores reported in Table 9.4 are based on the full group distributions over many combinations of social characteristics and thus register the full spectrum of patterns of segregation for racial comparisons between and across all combinations of the 10 categories of family type and poverty status.

Anchoring the scores on the full range of data for both groups carries statistical and substantive benefits. Using the full group makes the comparison more statistically reliable; thus, for example, the standardized group means that determine the standardized values of S and D have smaller standard errors than group means computed for narrow subgroups. Substantively, using the full group data is attractive because it assesses segregation patterns between and across all combinations of social characteristics not just for a narrowly specified comparison that could potentially be idiosyncratic. Arguably this protects against getting unusual results for a particular narrowly defined comparison. Importantly, the approach also does not exclude the cross-category comparisons which quantitatively make large contributions to determining overall segregation but are completely ignored when comparisons are restricted to only one-to-one matches on social characteristics.

Another more technical problem is that scores based on narrowly defined subgroups are prone to being distorted by index bias. The problem of index bias is well-known and potentially vexing. Accordingly I give it extended attention in Chaps. 14, 15, and 16. Concern about index bias is especially relevant when group counts in spatial units are small and group ratios are imbalanced (Winship 1977).

This problem is likely to be salient when subgroup comparisons are based on small subsets of cases that exactly match on non-racial social characteristics. For example if one matches White and Black families on poverty status and family type, the counts families in each area will drop substantially. Furthermore, the underlying problem is likely to be even worse than it appears on first consideration. The reason for this is that the census tabulations that include other social characteristics in addition to race are based on samples instead of full counts. The summary file tabulations report "estimated" full counts. In fact, the analysis rests on a much smaller number of underlying cases. In the present example using data for 2000, the data are based on an approximate 1-in-6 (16.7 %) sample. Using more recent five-year summary files from the American Community Survey, the data would be based on a 1-in-20 (5 %) sample. Analysis of segregation between "matched" subsets of cases thus is likely to rest on a small set of cases in each block group.

Another problem is that, even under the best of conditions, it is usually infeasible to extend this conventional approach to take account of more than one or two non-racial characteristics at a time. Restricting the comparison to White and minority families matched on several characteristics at once will almost always result in basing the analysis on an unacceptably small number of micro-level cases. In contrast, the standardization approach applied in this chapter draws on the full population in each group and can in principle include many more social characteristics. The "ANOVA-style" reliance on categories instead of continuous predictors in the examples considered here can run into problems when means for some subgroups are less reliable due to being based on a small number of cases. However, the problem is less troublesome than the usual approach used in the literature. Moreover, it can be mitigated by using continuous measures in place of categories and adopting refined regression modeling strategies such as using multi-level specifications (discussed in Chap. 10) to improve estimation of effects. Thus, the difference of means framework provides clear advantages when researchers wish to take account of several non-racial characteristics at once.

9.6 Aggregate-Level Controls for Micro-level Determinants of Residential Outcomes

Segregation studies sometimes "take account" of group differences on social characteristics that play a role in residential outcomes in a fundamentally different way; namely, by estimating aggregate-level regressions where measures of group disparity on a relevant social characteristic (e.g., income or poverty status) is used to predict cross-city variation in segregation index scores. This strategy raises concerns about the risk of flawed inference associated with the "ecological" or "aggregate" fallacy.

It is fair to say that this concern does not seem to be widely recognized because the practice is routine in empirical studies and apparently not subject to strong criticism.[6] Two factors may help explain why the prevailing practice is seen as non-controversial instead of seriously flawed. One is that traditional formulations of segregation indices encourage the view that the index score is an aggregate-level characteristic of cities that is not directly a product of individual-level attainment processes in way that would raise strong concerns about the undesirable consequences of the aggregate fallacy. The second is that, while studies in the location attainment tradition could potentially promote the view that segregation should be understood as arising out of micro-level residential attainment processes, they ultimately do not do so because until now micro-models could not be used to directly investigate segregation as measured by the dissimilarity index (D) and other popular aggregate-level indices.

The findings in this chapter show that analysis of segregation using popular aggregate-level measures can be joined seamlessly with analyses of micro-level residential attainment processes. The difference of means formulation of standard segregation indices makes this possible by establishing that segregation can be understood as a difference of group means on individual-level residential outcomes that in a given city are determined by a micro-level attainment process where many individual-level characteristics can impact segregation. The data and analyses presented in Tables 9.2, 9.3, and 9.4 clarify how the individual-level characteristics of race, poverty status, and family type affect residential outcomes (y) that then aggregate in a simple additive way to determine the level segregation in the city. This example establishes that the parallel with analyses of group differences other socioeconomic attainment outcomes (e.g., education, occupation, income, home ownership, etc.) is exact. This then highlights a lack of correspondence on another point; namely, the failure of segregation researchers to show appropriate concern for the aggregate fallacy in aggregate-level segregation studies.

Researchers analyzing group differences in income understand that the aggregate-level outcome of inter-group income inequality in a particular city emerges as a product of an underlying micro-level process of income attainment for that city. As a result, it is easier for these researchers to recognize that the ideal way to obtain a sound assessment of the role that non-racial social characteristics play in producing group income inequality in a city is to draw on detailed micro data for that city. It also is easier for these researchers to recognize that attempts to take account of the role of non-racial social characteristics in producing inter-group income inequality using only aggregate data carries a high risk of mistaken inference due to the aggregate fallacy. I reviewed these issues more than two decades ago in an article that outlined the nature of the problem in detail and provided an empirical demonstration of how aggregate-level analysis leads to errors of inference and mistaken conclusions about the role of group differences in social characteristics for cross-area variation in group income inequality (Fossett 1988). Researchers interested in this topic appear to have adapted and moved forward. In recent decades there has been a fundamental change in the research literature. Aggregate-level analyses of cross-

[6] For example, in my experience journal reviewers not only do not object to this practice, they often *request* that it be incorporated into the analysis.

city variation in group income inequality were common in earlier decades and they routinely included aggregate-level measures to "control" for the impact of group differences on individual-level characteristics that predict income (e.g., income).[7] Such studies are no longer accepted as most researchers now understand that one must use disaggregated data to properly investigate these issues.

A similar reckoning is looming for the literature investigating cross-city variation in residential segregation. Concern about the aggregate fallacy currently is minimal because segregation researchers are not in the habit of viewing city-level segregation scores as mapping directly onto micro-level residential outcomes. Accordingly, segregation researchers do not automatically think in terms of using micro data to take account of the role of non-racial social characteristics in shaping residential segregation. This creates a "blind spot" for the possibility that key findings from studies that investigate cross-city variation in segregation may be suspect because the studies use research designs that incorporate the aggregate fallacy.

The data and analyses presented in Tables 9.2, 9.3, and 9.4 provide examples of how the differences of means approach makes it possible to "take account of" the impact of group differences on social characteristics on segregation in a way that is superior and offers a better chance to make correct inferences in comparison to past approaches. The data in these tables cast segregation as a group difference of means on residential outcomes (y) that emerge from a micro-level attainment process where race, poverty status, and family type all play a role in influencing residential outcomes. Once segregation is conceptualized in this way, it is clear that the proper statistical approach for taking account of group differences on poverty and family type is to perform city-specific standardization analyses using relevant attainment data disaggregated at the micro level for the city in question.

The limitations of the prevailing practice are revealed by the standardization analyses reported in Table 9.4. The results from the analyses directly answer the question of whether racial segregation arises due to group differences in poverty status and family type for the city in question. In each group comparison, the answer obtained is conceptually and statistically sound. The answer developed from analyses reported in Tables 9.1, 9.2, 9.3, and 9.4 also is definitive and complete. Group differences on social characteristics do not play a significant role in accounting for the observed level of White-Black segregation. This conclusion is anchored in a direct examination of the micro-level relationships between White and Black residential attainments in Houston in 2000. It cannot be improved by examining aggregate-level data for other White-Minority comparisons in the same city or even hundreds of such comparisons across other cities.

Moreover, analysis using only aggregate-level measures can easily lead to mistaken conclusions. For example, the analyses show that White-Asian segregation is lower than White-Black segregation and they also show that White-Asian differences in poverty are smaller than White-Black differences in poverty. The logic of

[7] Examples include Becker (1971 [1957]), Bahr and Gibbs (1966), Jiobu and Marshall (1971), Roof (1972), LaGory and Magnani (1979), and Elgie (1980) among many others as the practice was routine.

aggregate-level analysis would infer from this pattern that segregation is more pronounced when group differences in poverty are large. But analysis of the relationship using relevant micro-level data establishes that the impact of group differences in poverty is minimal.

Similarly, the result for the answer to the question cannot be improved by examining aggregate-level data for White-Black segregation in other cities. For example, if one examined a large sample of metropolitan areas and found a strong positive aggregate correlation between White-Black segregation and White-Black differences in poverty or income, the conclusion about the impact of White-Black differences in poverty on White-Black segregation in Houston based on the standardization analysis in Table 9.4 is not challenged and will stand unchanged. The aggregate-level findings are "trumped" by the direct analysis of relevant micro-level data for White-Black segregation in Houston.

In reviewing the general issues in detail in an earlier study (Fossett 1988) I noted that, while it is certainly plausible that group differences on social and economic characteristics could give rise to group differences on relevant attainment outcome, aggregate-level correlation is not a sound way to assess this possible effect. The sound way to assess the impact in a given city and group comparison is by working with relevant disaggregated data to examine the relationship at the micro level. Resorting to aggregate-level controls is tempting, but there are compelling reasons to discontinue this practice. One such reason can be summarized as follows.

> *Urban-ecological theories of cross-area variation in racial stratification provide a strong basis for expecting group differences on inputs to attainment processes to be spuriously correlated with group differences on outcomes of attainment processes at the aggregate level* (Fossett 1988).

The fundamental premise of urban-ecological theories of racial stratification is that some community-level factors shape group relations "across the board." If so, a general climate of minority disadvantage, tracing for example to comprehensive Jim Crow laws or high levels of White prejudice and discrimination in socioeconomic attainment processes, can lead to both high levels of White-Black differences in poverty and White-Black segregation. However, the resulting correlation of segregation with group differences in poverty and income produced by this social dynamic can easily be spurious, not causal. Thus, for example, if discrimination in housing severely constrains the residential opportunities of non-poverty Black households reducing their segregation-relevant contact with Whites, eliminating group differences in poverty will have no impact segregation.

It is not possible to sort out whether the aggregate relationship is spurious or causal with aggregate-level data. One must ultimately examine relevant micro data to directly assess whether reducing group differences in poverty or income would in fact reduce segregation in a given city. In Chap. 10 I present empirical analyses that illustrate how to perform such analyses. These analyses document and affirm that the empirical findings and central conclusions I reported in Fossett (1988) also apply to analyses of residential segregation. Specifically, the analyses document two parallels with the earlier study. The first is that aggregate-correlations and regressions suggest that group differences in income play a major role in accounting for cross-city variation in segregation. The second is that this conclusion is shown to be incorrect when one uses micro-level data to properly take account of the impact of income differences on segregation. Based on this I caution segregation researchers to take seriously the concern that the practice of using aggregate-level regressions to assess the role of factors that operate at the micro-level is unsound and can yield misleading results.

9.7 New Interpretations of Index Scores Based on Bivariate Regression Analysis

Investigation of segregation using the technique of standardization analysis joins aggregate-level analysis with residential attainment analysis by clarifying how segregation index scores for a city arise from micro-level residential attainment processes shaped by racial and non-racial social characteristics. This point can be highlighted by noting that the data presented in Tables 9.2 and 9.3 correspond to predictions of mean residential attainments derived from individual-level models of residential attainment. More precisely, the subgroup means on residential outcomes correspond to predictions from individual-level analysis of variance (ANOVA) models or, alternatively, individual-level regression models. The tables reports means for residential attainments (y = scaled pairwise contact with Whites as relevant for S or D) for individual families grouped by category of race, poverty status, and family type. This corresponds to an individual-level ANOVA or regression analysis predicting residential attainments based on three categorical independent variables: family type (five categories), poverty-status (two categories), and race (two categories). Thus, the subgroup means reported in Tables 9.2 and 9.3 correspond to predictions from a "fully saturated" model which estimates all possible additive and non-additive effects for race, poverty status, and family type on residential attainments. The standardization analyses reported in Table 9.4 implicitly rest on these attainment models. It is a natural next step to explicitly focus on the results of the attainment model to assess more specifically how the effects of the independent variables shape residential segregation.

The difference of difference of means framework yields a set of new and potentially attractive interpretations for segregation index scores. It is that the values of scores for indices such as S and D now can be described as reflecting the effect or impact of race on residential outcomes that determine segregation. This interpretation is straightforward in a bivariate model of individual-level residential attainment where race is the only predictor. When introducing the difference of means formulations, I offered computing formulas for obtaining index scores as a difference of group means. I now note that the index scores also can be obtained via an individual-level bivariate regression analysis in which a dummy variable for race (i.e., group membership) is used to predict the residential outcomes (y) that are relevant for a particular index.

Table 9.5 Bivariate segregation attainment regressions predicting residential outcomes (y) that additively determine White-Minority segregation for selected indices, Houston, Texas, 2000

Independent variable	G*	G/2*	D/2*	D	R	H	S
	White-Black comparison (N = 811,924)						
White (0,1 = White)	87.07	43.54	35.48	70.97	47.02	53.59	57.39
Constant	33.64	16.82	22.96	16.75	25.96	28.98	32.48
R Square	0.412	0.412	0.442	0.442	0.495	0.557	0.574
Implied index score	87.07	87.08	70.96	70.97	47.02	53.59	57.39
	White-Latino comparison (N = 911,060)						
White (0,1 = White)	74.19	37.09	29.19	58.37	28.11	35.46	40.96
Constant	49.53	24.76	30.14	23.12	35.26	37.50	40.17
R Square	0.359	0.359	0.317	0.317	0.385	0.404	0.410
Implied index score	74.19	74.18	58.38	58.37	28.11	35.46	40.96
	White-Asian comparison (N = 672, 968)						
White (0,1 = White)	76.28	38.14	29.11	58.22	34.96	31.31	23.88
Constant	29.94	14.97	23.27	16.93	35.74	52.16	69.91
R Square	0.131	0.131	0.122	0.122	0.135	0.198	0.239
Implied index score	76.28	76.28	58.22	58.22	34.96	31.31	23.88

Source: US Census 2000, Summary File 3

Notes: G* is given by $(Y_1–Y_2)$ when y is scored 0–200 and G/2* and D/2* are given by $2·(Y_1–Y_2)$ when y is scored 0–100. All regression coefficients are statistically significant at 0.001 or better

In the case of White-Black segregation as measured by the separation index (S), the regression would include a dummy variable for "White" coded 1 if White and 0 if Black to predict the residential outcome of pairwise contact with Whites ($y = p$). The value of the estimated regression intercept (b_0) will indicate the average contact with Whites for Blacks (i.e., the baseline group coded 0 on race). The value of the unstandardized regression coefficient for White (b_1) will indicate the extent to which the White mean for contact with Whites (i.e., the group coded 1 on race) deviates from the Black mean for contact with Whites. Accordingly, *the value of the regression coefficient also will exactly equal the value of the segregation index score;* that is, $b_1 = S$. (And, for the sake of completeness, mean contact with Whites for Whites will be given by $b_0 + b_1$.) At one level, this is not surprising as most readers will already be aware that bivariate dummy variable regression is mathematically equivalent to a difference of means comparison. But it is a new development in segregation measurement theory to interpret a segregation index score as the effect of race in a micro-level process of residential attainment. Thinking in this way opens up new avenues for exploring and interpreting segregation.

Table 9.5 reports results for a series of bivariate regressions of the type just described estimated using the micro-level data set for Houston, Texas introduced earlier in this chapter. Recall that this data set reconstitutes the block group-level summary tabulations so the information in the tabulation is organized in a data set appropriate for performing individual-level attainment analysis. Cells in the tabulation are treated as cases and are coded on independent variables – race, poverty status, and family type – to suit the needs of the analysis. Dependent variables relating to index-specific scores based on block-group race counts are assigned to block groups and then merged with the individual level data based on block group codes. The resulting data set can be used to estimate regression analyses in the conventional way with the proviso that a variable representing cell counts be used as a case-level frequency weight.[8]

The independent variable used in the bivariate regressions reported in Table 9.5 is a "dummy" (0, 1) variable for "White" coded 1 for White and 0 for minority depending on the race of the family's householder. The dependent variables are residential outcome scores (y) scaled from pairwise area proportion White (p) as appropriate for the segregation index of interest and the relevant group comparison. Table 9.5 reports results for separate regression analyses for five segregation indices – namely, G, D, R, H, and S.[9]

An important finding is evident in the results. In each regression analysis, the unstandardized regression coefficient for the dummy variable for race (here coded 1 if White and 0 otherwise) yields the value of the relevant index score (previously reported in Table 5.2). In the case of G, individual residential outcomes (y) are scored two ways; one to yield G and one to yield G/2. In the latter coding, the value of the coefficient for race must be doubled to obtain the value of G. The table also reports results for D taken as a crude version of G and thus scores residential outcomes (y) in relation to D/2. In this regression the coefficient for race must be doubled to obtain the value of D. The table also reports results based on the alternative formulation of D where residential outcomes (y) are coded as either 0 or 1.

Of course, relatively little is gained if we stop with the simple bivariate regression analysis. It merely recasts the difference of means comparison reviewed earlier in Table 5.2 in the regression (or ANOVA) framework. The most important descriptive findings to be gleaned from the analysis – namely, the group means and the group difference of means – are exactly the same as those previously reported earlier in Tables 5.2 and 5.3. So no new information is gained.

Regression analysis does potentially provide a useful framework for hypothesis testing regarding the level of segregation. But this has minimal practical value at the bivariate level of analysis as statistical significance is typically not a central concern in segregation analysis. Sample sizes and race effects both tend to be large in analyses of the overall level of segregation and thus statistical tests tend to be significant at levels far beyond conventional standards (i.e., 0.05 and 0.01). For example, the

[8] When estimation routines in statistical programs have this capability, the data set can be stored an efficient, compact form. Alternatively, one may create a separate record for each family included in the summary file tabulation. The resulting data set will be much larger. Some might find the less compact form of the data set more familiar for conducting individual level regression analysis. But regression results will be identical either way.

[9] The regressions were estimated using OLS regression. This is satisfactory for present purposes and is a convenient choice because it simplifies the presentation and discussion of results. In other situations, it may be necessary to use more technically appropriate regression procedures such as fractional logit regression (Papke and Wooldridge 1996; Wooldridge 2002) to deal with the problem that OLS assumptions are not valid when modeling bounded variables and OLS can yield predictions outside of the 0-1 bounds of segregation indices.

t-ratio for the effect coefficient of race in the bivariate regression of pairwise contact with Whites on a dummy variable for race for the White-Black comparison is over 1000 and the probability of chance deviation from 0.0 is zero out to many decimal places. In such circumstances, the usual concerns about statistical significance and technical regression assumptions fade into the background.

The more significant potential benefit of regression analysis is that it provides an opportunity to put segregation research on a new path for gaining a better, more direct understanding how segregation arises. Specifically, analyzing segregation from the difference of means framework sets segregation researchers on the path of investigating segregation using the methods and modeling strategies that status attainment researchers routinely use to investigate racial disparities and inequality on education, occupation, income, health, and other socioeconomic and life chance outcomes. These methods and modeling strategies previously have not been available to segregation researchers because the link between micro-level attainment and aggregate-level segregation (city segregation scores) was not established. The difference of means formulation of segregation indices thus allows researchers to move away from focusing simply on the calculation of descriptive index scores that summarize the state of segregation at the aggregate level. It instead allows researchers to move toward investigating segregation through the more analytically flexible method of performing multivariate analyses to assess the zero-order and net effects of race (group membership) on individual-level residential outcomes that directly determine the level of segregation.

I discuss the extension to multivariate analysis of segregation-relevant residential outcomes in more detail below. But first it is useful to point out that different indices register residential outcomes in different ways – based on index-specific functions $y = f(p)$ – and that these differences carry implications for interpreting the effect of race on residential outcomes in individual-level attainment analyses. Here it is useful to recall Fig. 5.1 which clarifies how these five segregation indices differ in registering residential outcomes (y) scored from area group proportion (p). In the case of G, D and R, y is scored as a nonlinear transformation of p that in these group comparisons tends to exaggerate group differences at high levels in p and minimize group differences over the middle ranges of p. H also involves a similar nonlinear transformation, but it is much less dramatic. In contrast, S scores y simply on the basis of p and does not subject p to a transformation. This makes the regression results for the separation index (S) especially easy to interpret and a good place to begin.

Table 9.5 reports the results for the bivariate regression $y = b_0 + b_1$ (race) relevant for investigating White-Black segregation as measured by S as $y = 32.5 + 57.4$ (race) where race is coded 1 for White and 0 for Black. In this example, the value of the regression constant (b_0) is 32.5 and reflects Blacks' average contact with Whites (Y_B). The value of the unstandardized regression coefficient for race (b_1) is 57.4. It reflects the impact that race has on average contact with Whites; namely, to raise contact with Whites by 57.4 points above the level of contact that Blacks experience. The sum of b_0 and b_1 gives the predicted value of 89.9 for Whites' average contact with Whites (Y_W). These values map exactly onto the terms reported in

Table 5.3 which showed how index scores can be obtained as differences of group means on residential outcomes. Thus, the value of S for White-Black segregation overall is 57.4 resulting because White families live in neighborhoods where pairwise percent White averages 89.9 while Black families live in neighborhoods where pairwise percent White averages 32.5.

This highlights the new interpretation available for S as indicating that race – specifically, being White instead of Black – "matters" for residential outcomes and in this case has the impact of increasing contact with Whites by 57.4 points in comparison with the reference group of Blacks. The magnitude of the effect makes it clear that race differences in residential attainment produce substantial residential separation between Whites and Blacks as Whites are predicted to reside in predominantly White areas and Blacks are predicted to live in predominantly Black areas.

It is instructive to compare the effect of race in the White-Black comparison with the effects of race in the bivariate segregation attainment analyses for the White-Latino and White-Asian comparisons. The race effect of 41.0 points in the White-Latino regression is approximately 16 points lower than that in the White-Black regression. Thus, we can conclude that race "matters less" in promoting residential separation of Whites from Latinos than it does in promoting residential separation of Whites from Blacks. However, the effect is still large and has the consequence of on average placing Whites in predominantly White areas while Latinos are in predominantly Latino areas. The race effect of 23.9 points in the White-Asian regression is approximately 34 points lower than in the White-Black regression. Based on this we can conclude that, while the effect of race is not trivial, race matters much less in promoting residential separation of Whites from Asians than it does in promoting residential separation of Whites from Blacks. One clear indication of this is that the effect of race on average leaves both Whites and Asians being predicted to reside in predominantly White areas.

The bivariate results for D suggest a somewhat different story. I focus on the results for D based on scoring residential outcomes as 0 or 1 based on whether the family attains parity on contact with Whites based on whether area proportion White equals or exceeds the level for the city as a whole (i.e., 1 if $p \geq P$, 0 otherwise). For this residential outcome, race matters a great deal in all three group comparisons. The unstandardized regression coefficients for race take high values in each analysis reaching approximately 71.0 in the White-Black analysis, 58.4 in the White-Latino analysis, and 58.2 in the White-Asian analysis. In substantive terms, we can interpret these effects as indicating that, in each comparison, race – that is being White in contrast to being minority – has a large impact on the probability of residing in an area where proportion White attains parity with city-wide proportion White.

This information is not without value. But it also is important to be aware of what is not revealed when modeling micro-level outcomes that determine the value of D. Namely, this analysis fails to provide a basis for assessing the quantitative differences in the racial composition of the neighborhoods the groups live in. If one does not bear this in mind, one can come away with an incomplete and potentially misleading impression of the nature of segregation in these three comparisons. This is particularly true in the case of the White-Latino and White-Asian analyses. The comparison on the effect of race in these analyses shows that it is essentially the same in both two regression equations. This indicates that the White advantage in the probability of attaining parity on area proportion White is the same in relation to Latinos and Asians. In addition, comparison of the regression constants indicates that, overall, Asians are less likely than Latinos to attain parity on area proportion White. The combination suggests that White-Latino segregation and White-Asian segregation are very similar.

But it is important to bear in mind that D is sensitive to group differences in attaining "parity" on neighborhood proportion White *where "parity" is assessed in relation to the citywide pairwise racial proportions*. As a result, the effect of race in the models for D does not support inferences and interpretations relating to group differences in the actual level of pairwise contact with Whites or to group differences in "fixed" outcomes such as probabilities of residing in neighborhoods that are majority (50 %) White, two-thirds (67 %) White, or predominantly (e.g., 80 %) White. For example, in the case of Houston, Texas, Latinos are a much larger group than Asians. Accordingly, the "cut point" for scoring of residential outcomes as attaining "parity" on area proportion White for the White-Latino comparison is much different – specifically, much lower – than the "cut point" for scoring of residential outcomes as attaining "parity" on area proportion White for the White-Asian comparison. Consequently, a naive interpretation of the race effect in the attainment analysis for D might suggest the conclusion that Latinos and Asians fare similarly in comparison to Whites but with Asians being less likely than Latinos to live in areas that are disproportionately White. But the analysis for S shows that Asians live in areas that on average are 69.9 % White, a full 29.7 points higher than Latinos who on average live in majority Latino areas. This suggests that the substantive value of scoring residential "disadvantage" based on "parity" is open to reconsideration. In particular, I pose the question, "What are the substantive and sociological implications of Asians experiencing near-identical disadvantage as Latinos on attaining "parity" when the two groups differ greatly in terms of their residential separation from Whites?"

9.8 Multivariate Segregation Attainment Analysis (SAA)

The bivariate regression examples just discussed are interesting and useful in their own right. They illustrate some of the benefits of directly modeling the individual-level residential outcomes that give rise to segregation index scores. Specifically, the approach enables and encourages more thoughtful and careful interpretation of race effects on residential outcomes across group comparisons and different indices. In the long run, however, the bivariate regressions are just a useful preliminary step toward investigating how the impact of race on segregation compares with the impacts of other social characteristics. This can be done by investigating micro-level analyses segregation-relevant residential outcomes using multivariate attainment models in the manner that is already universal in other literatures investigating racial disparities.

I term this new approach "segregation attainment analysis" (SAA). The justification for the label is that the effect of race in bivariate models corresponds directly to the aggregate level of segregation in the city and its effect in multivariate models yields insights into how the impact of race should be assessed and interpreted when taking account of the role of non-racial factors that also impact residential outcomes.

This can be accomplished by extending the micro-level attainment regressions to include additional independent variables beyond race. In this case, I used the tabulation of race by family type by poverty status to fashion the following independent variables: poverty status (0,1), married with spouse present (0,1), and presence of children under age 18 (0,1). Table 9.1 previously presented descriptive statistics based on this data set. It documents that the four groups in the analysis vary greatly on these variables. Non-poverty status runs from a high of 95.8 % for Whites to a low of 80.2 % for Latinos. Percent of families that are married couple families runs from a high of 84.9 % for Asians to a low of 51.2 % for Blacks. And percent of families with children under age 18 runs from a high of 69.2 % for Latinos to a low of 47.3 % for Whites. Given these group differences in distribution across social characteristics an obvious questions arise: "What role to these characteristics play in shaping residential outcomes that determining segregation?", "How does their role compare with the role of race?", and "How does the estimated effect of race change when other characteristics are controlled?"

Tables 9.6, 9.7, and 9.8 report results of bivariate and multivariate regression analyses that can be used to address these and related questions. Each table has five panels. Each of the five panels presents results from regression analyses predicting dependent variables that additively determine segregation indices. The regression analyses are estimated and reported separately by racial group for ease of discussion and presentation. For hypothesis testing and for cross-time and cross-city comparisons it may be more appropriate to estimate single-equation specifications which incorporate additive and non-additive race effects. White-Black comparisons are reported in Table 9.6, White-Latino comparisons in Table 9.7, and White-Asian comparisons in Table 9.8.

Analyses of this sort can be used to gain a richer understanding of the residential attainment process that gives rise to segregation by permitting direct examination and comparison of the separate effects of racial *and* non-racial social characteristics on residential outcomes. Table 9.6 presents results relevant for the analysis of White-Black segregation. Results are presented separately for five indices. I begin by discussing the results for the separation index (S) reported in the fifth panel in the table. The first and second columns report separate regressions for Whites and Blacks with no other independent variables included in the model. The constants in these equations of course equal the group means for scaled contact with Whites (y). In the case of the separation index (S) y is given by the pairwise proportion White (p) in the block group and difference between the two group means yields the value of the separation index. This is reported as "White Advantage (S)" which has the

Table 9.6 Group-specific attainment regressions for White-Black segregation

	Simple comparison		Comparison with controls		
Variable	Whites	Blacks	Whites	Blacks	Net impact
Residential outcome (y) scored for gini index (G)					
Non-poverty Family	–	–	5.66	8.37	–2.71
Married couple family	–	–	8.62	7.83	0.79
Children present	–	–	0.79	4.58	–3.79
Constant	120.71	33.64	107.67	20.03	87.64
White advantage (G)	87.07		87.64		81.93
Residential outcome (y) scored for index of dissimilarity (D)					
Non-poverty family	–	–	6.72	6.45	0.27
Married couple family	–	–	5.76	8.95	–3.19
Children present	–	–	2.89	3.07	–0.18
Constant	87.73	16.75	75.09	5.05	70.04
White advantage (D)	70.98		70.04		66.94
Residential outcome (y) scored for hutchens index (R)					
Non-poverty family	–	–	3.72	6.70	–3.98
Married couple family	–	–	3.67	5.29	–1.62
Children present	–	–	0.33	3.98	–3.65
Constant	72.98	25.96	67.14	15.38	60.42
White advantage (R)	47.02		60.42		42.52
Residential outcome (y) scored for theil index (H)					
Non-poverty family	–	–	3.37	8.19	–4.82
Married couple family	–	–	3.75	6.86	–3.11
Children present	–	–	0.93	4.83	–3.90
Constant	82.57	28.98	75.75	15.85	59.90
White advantage (H)	53.59		59.90		48.07
Residential outcome (y) scored for separation index (S)					
Non-poverty family	–	–	3.58	9.59	–6.01
Married couple family	–	–	3.34	8.19	–4.85
Children present	–	–	1.20	5.67	–4.47
Constant	89.86	32.48	83.06	17.01	66.05
White advantage (S)	57.38		66.05		50.72

Source: Summary File 3, Houston, Texas, 2000. Sample N: 627,613 for Whites and 195,928 for Blacks. All coefficients are statistically significant at 0.001 or better

value of 57.38. This value of S was reported previously in Tables 5.2, 5.3 and 9.5 and thus confirms the equivalence of the different approaches to assessing segregation.

The third and fourth columns report multivariate regressions separately for Whites and Blacks. Each equation has three independent variables – non-poverty status, married couple family, and presence of children – which have been coded as dummy (i.e., 0,1) variables. In this specification, the intercept of the equation can be interpreted as the expected group mean on scaled contact with Whites for families that are in poverty, are not married couple families, and do not have children residing with them.

Table 9.7 Group-specific attainment regressions for White-Latino segregation

	Simple comparison		Comparison with controls		
Variable	Whites	Latinos	Whites	Latinos	Net impact
Residential outcome (y) scored for gini index (G)					
Non-poverty family	–	–	14.94	15.33	–0.39
Married couple family	–	–	16.56	6.33	10.23
Children present	–	–	5.28	–2.70	7.98
Constant	123.72	49.53	93.01	34.41	58.60
White advantage (G)	74.19		58.60		76.42
Residential outcome (y) scored for index of dissimilarity (D)					
Non-poverty family	–	–	12.92	11.46	1.46
Married couple family	–	–	9.94	5.32	4.62
Children present	–	–	4.90	–3.76	8.66
Constant	81.49	23.12	58.46	12.58	45.88
White advantage (D)	58.37		45.88		60.62
Residential outcome (y) scored for hutchens index (R)					
Non-poverty family	–	–	5.13	7.18	–2.05
Married couple family	–	–	5.10	2.56	2.54
Children present	–	–	1.69	–0.73	2.42
Constant	63.37	35.26	53.38	28.10	25.28
White advantage (H)	28.11		25.28		28.19
Residential outcome (y) scored for theil index (H)					
Non-poverty family	–	–	6.60	9.33	–2.73
Married couple family	–	–	6.01	3.34	2.67
Children present	–	–	2.91	–1.01	3.92
Constant	72.95	37.50	60.49	28.24	32.25
White advantage (H)	35.45		32.25		36.11
Residential outcome (y) scored for separation index (S)					
Non-poverty family	–	–	7.65	11.30	–3.65
Married couple family	–	–	6.23	3.97	2.26
Children present	–	–	2.72	–1.24	3.96
Constant	80.12	40.17	67.28	29.01	38.27
White advantage (S)	40.95		38.27		40.84

Source: Summary File 3, Houston, Texas, 2000. Sample N is 627,613 for Whites and 294,931 for Latinos. All coefficients are statistically significant at 0.001 or better

The difference between the intercepts of the two equations can be interpreted as a White-Black segregation comparison that has been "standardized" to control for group differences in distributions on social characteristics. That is, the comparison reflects group differences on model predicted means on segregation-determining residential outcomes for White and Black families that are matched on social characteristics. For both Whites and Black the level of average contact with Whites for the subgroup reflected at the intercept is lower than the group's overall mean. The value for Whites is 83.06 which is 6.80 points lower than the value of 89.86 reported in the "constant only" equation for Whites. The value for Blacks is 17.01

Table 9.8 Group-specific attainment regressions for White-Asian segregation

	Simple comparison		Comparison with controls		
Variable	Whites	Asians	Whites	Asians	Net impact
Residential outcome (y) scored for gini index (G)					
Non-poverty family	–	–	−11.69	5.97	−17.66
Married couple family	–	–	0.91	4.11	−3.20
Children present	–	–	−2.67	*−0.11*	−2.56
Constant	106.22	29.94	117.91	21.12	96.79
White advantage (G)	76.28		96.79		73.37
Residential outcome (y) scored for index of dissimilarity (D)					
Non-poverty family	–	–	−4.98	3.09	−8.07
Married couple family	–	–	2.72	3.95	−1.23
Children present	–	–	*−0.17*	−1.29	1.12
Constant	75.14	16.93	77.72	11.55	66.17
White advantage (D)	58.21		66.17		57.99
Residential outcome (y) scored for hutchens index (R)					
Non-poverty family	–	–	−5.68	4.38	−10.06
Married couple family	–	–	*0.17*	2.42	−2.25
Children present	–	–	−1.57	*0.27*	−1.84
Constant	70.69	35.74	76.74	29.56	47.18
White advantage (H)	34.95		47.18		33.03
Residential outcome (y) scored for theil index (H)					
Non-poverty family	–	–	−2.91	6.71	−9.62
Married couple family	–	–	0.74	3.48	−2.74
Children present	–	–	−0.59	0.54	−1.13
Constant	83.47	52.16	85.91	42.82	43.09
White advantage (H)	31.31		43.09		29.60
Residential outcome (y) scored for separation index (S)					
Non-poverty family	–	–	−0.72	8.86	−9.58
Married couple family	–	–	0.81	4.18	−3.37
Children present	–	–	*0.00*	0.97	−0.97
Constant	93.79	69.91	93.80	57.78	36.02
White advantage (S)	23.88		36.02		22.10

Source: Summary File 3, Houston, Texas 2000. Sample N is 627,613 for Whites and 55,746 for Asians. Regression coefficients not significant at 0.01 are in gray italics

which is 15.47 points lower than the value of 32.48 reported in the "constant-only" equation for Blacks.

The White-Black difference of 66.05 at the intercept (83.06 minus 17.01) is reported in the third column of the "White Advantage" row. (As discussed below, it also is reported as a "net impact" in the fifth column.) This value can be understood as the impact of race on expected scaled contact with Whites for White and Black families that have the specific configuration of social characteristics associated with the intercept of the multivariate equation. Thus, it is the White-Black difference on average scaled contact with Whites predicted under the model for families coded zero on all three independent variables (i.e., for non-married couple families with no children present and in poverty). In the bivariate regressions the impact of race represents the level of segregation in the city because it is exactly equal to the segregation index score. In the multivariate specification the impact of race can be interpreted as the expected level of segregation between Whites and Blacks when group differences in distribution on other social characteristics is controlled.

The group-specific regression coefficients reported in columns 3 and 4 give insights into how the three social characteristics included in the regression impact the residential attainments that additively determine segregation as measured by S. The regression coefficients for this analysis indicate that all three variables – non-poverty status, married couple status, and presence of children – have positive effects on family attainments of the residential outcome of scaled contact with Whites. This pattern is generally consistent across the multivariate regression analyses reported for all three White-Minority comparisons and for all five segregation indices considered. The effect of non-poverty status is positive and statistically significant in all equations. The effect of married couple status is positive and statistically significant in almost all equations. The effect of presence of children is less consistent. In the analyses for White-Black segregation it is positive and statistically significant in all of the equations but is small in size for Whites in analyses for some measures of segregation. In the analyses for White-Latino segregation the effect is positive for Whites and negative for Latinos. In the analyses for White-Asian segregation it is mixed in terms of direction but consistently small (absolute value under 1.0 in 7 of 10 possible cases).

The question of how these social characteristics impact segregation is answered by examining whether their effects ultimately reduce White-Minority differences on segregation-determining residential outcomes. For example, in the analyses for White-Black segregation moving from poverty to non-poverty status increases Black contact with Whites by 9.59 points. The comparable effect for Whites is 3.58. The "net impact" (i.e., White-Black effect difference) is –6.01 points and is reported in column five. This has direct implications for segregation. Specifically, it indicates that if one starts with White and Black families in poverty that are matched on other social characteristics and then move these families from poverty to not in poverty it would reduce segregation by 6.01 points. As a quick methodological aside, this "net impact" interpretation is based on using a linear, additive regression specification. Moving to a nonlinear and/or non-additive model for estimating effects of non-racial characteristics would require a more nuanced approach to assessing effects.[10]

In the analysis of White-Black segregation as measured by S the "net impact" (i.e., White-Black effect difference) is negative for all three social characteristics considered. Thus, in the same sense that the "net impact" indicates that moving from "poverty" to "non-poverty" reduces segregation by 6.01 points, moving from "non-married couple" to "married couple" on family type reduces segregation by

[10] Specifically, in nonlinear and/or non-additive models, the impact of a change from poverty to non-poverty would have to be assessed separately for each the initial configuration of the other social characteristics.

4.85 points and moving from "without children" to "with children" reduces segregation by 4.47 points. In the context of the linear, additive model used here, implementing all three "net impact" effects simultaneously would reduce the expected "White Advantage" in contact with Whites by 15.33 points; it would move the "White Advantage" from 66.05 at the intercept – that is, for the White-Black comparison standardized to non-married couple families without children and in poverty – to 50.72 for the White-Black comparison standardized to non-poverty, married couple families with children. This is reported in column five of the "White Advantage" row in the results.

These results help clarify how the impacts of racial group membership and non-racial social characteristics on segregation can be investigated in a more careful and nuanced way. The "net impact" reported in column five provides insight into the proximate impact of group differences on social characteristics on segregation. The regression coefficients reported in columns three and four clarify how the "net impact" comes about. Including the group-specific regression constants in the discussion provides a basis for comparing how the additive and non-additive effects of race compare with the effects of other factors in shaping segregation.

In the multivariate framework a wide range of logical possibilities can be imagined. At one extreme all block groups could have identical values on pairwise proportion White. In this possible but unlikely scenario of exactly zero race segregation the regression coefficients for non-racial social characteristics would be zero in both group equations and the intercepts of both equations would be identical. Another possibility is that race segregation is present and is due only to simple additive effects of race. In one scenario for this pattern, the regression coefficients for non-racial social characteristics are zero in both group equations but the intercept is higher in the equation for Whites and lower in the equation for Blacks. In a more complex scenario, the group equations differ at the intercept as just described and the regression coefficients for other social characteristics are not zero but are identical for both groups and both groups have identical distributions on the social characteristics.

A more plausible scenario is that race segregation is present and is produced by a complex combination of contributing factors including the following: additive race effects (i.e., differences at the intercepts of the attainment equation), non-additive race effects (i.e., race differences in the effects of non-racial characteristics), race differences in distribution on non-racial social characteristics, and the "interaction" of the last two factors. The results in Table 9.6 provide evidence that additive race effects are the most important factor contributing to segregation. The "White Advantage" of 66.05 reported in column three is one estimate of the quantitative contribution. This value can be described as the impact of race on segregation-determining residential outcomes for non-married couple families without children who are in poverty. That is, it is the value that would be estimated for the effect of being White (coded 1 if White and 0 if Black) if the regression analyses reported in columns 3 and 4 were replicated in a single equation specification using the combined samples.

The value of the intercept enters into all predictions and in this model specification the intercept corresponds to a set of families with a specific profile on social characteristics. So it is fair to describe the observed race difference at the intercept as applying “across the board” since reflects the expected level of segregation when social characteristics are fixed. Of course, the specific value of the intercept can vary depending on how variables are coded. So it is reasonable to ask whether the value of 66.05 is a fair or representative choice among all of the possible estimates of expected segregation for White-Black comparisons matched on social characteristics. The model predictions provide one answer to that question. Since all net impact calculations in column 5 are negative, 66.05 is the *maximum* race difference the attainment models will predict for White and Black families matched on all social characteristics. In contrast, the race difference of 50.72 predicted for the White-Black comparison for non-poverty, married-couple families with children present is *minimum* difference the attainment analysis will predict for White and Black families matched on all social characteristics.

This is useful information to consider. One can also apply predictions from the model to a hypothetical “standard” distribution of social characteristics to obtain expected White-Black differences on segregation-determining residential outcomes for “matched distributions.” Results for this kind of standardization analysis were reported in Table 9.4 based on adopting the combined group distributions as the “standard” for matching Whites and Blacks on social characteristics. The White-Black difference obtained under this calculation was 54.38, which necessarily falls between the minimum and maximum predicted differences of 50.72 and 66.05.

The question at hand here is how the effect of race on segregation compares to the effect of other social characteristics. A range of estimates of the impact of race are on the table. The “net impact” estimates in column 5 provide one basis for assessing the impact of other social characteristics on segregation. The separate net impact estimates range from –4.47 to –6.01 and are small compared to the race effect. The impact of non-poverty status is the largest of the three values and its magnitude is less than 12 % of the lower-bound estimate of the additive impact of race. If one combines the impacts of all social characteristics to obtain the maximum possible combined effect on reducing segregation, the result is 15.33 which is about 30 % of the lower-bound estimate of the additive impact of race. On this basis, one can argue that race is clearly the dominant factor impacting residential outcomes that determined segregation. Poverty status, family type, and presence of children do impact segregation. But their effects on segregation are small compared to the broad effect of race.

Standardization and decomposition analysis can provide additional perspective on the role non-racial social characteristics play in shaping segregation. For example, Table 9.1 reported that 80.9 % of Black families were in not in poverty compared to only 95.8 % of White families. If the non-poverty rate for Black families was increased to match the rate of observed for White families, the model indicates segregation would be reduced by 1.43 points. This is less than 3 % of the lower-bound estimate of the effect of race. From many different vantage points, the analysis consistently indicates that White-Black differences in distribution on social characteristics are not the major factor in determining segregation; the vast majority of segregation is due to expected mean differences on segregation-determining outcomes – in the case of S, pairwise contact with White – between Whites and Blacks matched on social characteristics.

Similar findings emerge in the analyses of residential outcomes relevant for determining the separation index (S) for White-Latino segregation (reported in Table 9.7) and for White-Asian segregation (reported in Table 9.8). In both cases, the net impact calculation for race based on the multivariate analysis of segregation-determining residential attainments (i.e., the value of White advantage reported in column 5) is much larger than the net impact calculations for the other social characteristics included in the analysis. The same general finding holds up across all three White-Minority segregation comparisons in analyses focusing on residential attainments relevant for G, D, R, and H. That is, the net impact of race on index-specific, segregation-determining residential outcomes is consistently much larger than the net impact estimates for the other social characteristics considered in these analyses.

These general conclusions are appropriate. But close inspection of the detailed results reveals interesting differences across White-Minority comparisons and across analyses focusing on different segregation indices. For example, in the case of White-Black segregation, the net impact calculation for non-poverty status varies across indices. Its absolute and relative magnitude is largest in the analysis focusing on the separation index (S) and is small and modest in the analyses focusing on the gini index (G) and the dissimilarity index (D). This is also true in the case of White-Latino segregation. But the pattern is different in the analysis results for White-Asian segregation. Here the net impact calculation for non-poverty status is sizeable for all indices and largest of all in the results for the gini index (G).

I conclude this section by noting that other interesting results can be discovered by making comparisons across groups. For example, the combined net impact calculations for married couple status and children present serve to *reduce* White-Black segregation across analyses for all segregation indices. A very similar pattern is also found in the results for the analyses of White-Asian segregation. But a much different pattern is seen in the results for the analyses of White-Latino segregation. The combined net impact calculations for married couple status and children present serves to increase segregation in the analyses for all segregation indices with the magnitude of the combined impact being especially large in the case of the gini index (G) and the dissimilarity index (D). These intriguing results and highlight how the new approach opens the door for pursuing more careful exploration of the social processes that produce White-Minority segregation. Future research may provide insight into why family structure appears to play a different role in White-Latino segregation in comparison with White-Black and White-Asian segregation. These and other possibilities for future analysis highlight the advantages of adopting the difference of means framework and embracing its capabilities for exploring segregation patterns in more detail.

9.9 Unifying Aggregate Segregation Studies and Studies of Individual-Level Residential Attainment

For many decades, dating back at least to the late 1960s, studies of segregation have followed one path while studies of racial and ethnic inequality and disparity on socioeconomic outcomes such as education, occupation, income, wealth, and home ownership have followed a different path. In the broader literature on racial socioeconomic inequality and disparity it is conventional to see racial disparities on socioeconomic attainment outcomes (e.g., education, income, etc.) as emerging from micro-level processes of attainment. Accordingly, research focusing on intergroup inequality and disparity on most socioeconomic outcomes draws on micro-level attainment models to understand and analyze group differences on socioeconomic attainments.

This has not been the case in the study of residential segregation. To be fair, researchers understand that at some level residential segregation arises from micro-level processes wherein individuals and groups seek, compete for, and attain (or fail to attain) particular residential outcomes. But past statements on segregation measurement have focused almost exclusively on the task of aggregate-level description. Relatively little attention has been given to developing connections between index scores for uneven distribution and residential outcomes for individuals and families that are considered in studies of residential attainment.

I noted earlier that Duncan and Duncan lamented this fact observing that the literature on segregation indices provided no "suggestion about how to use them to study the *process* of segregation" (1955:216, emphasis in original). Unfortunately, the negative assessment they offered more than five decades ago applies with equal force today. Research clarifying how micro-level attainment dynamics give rise to aggregate segregation as measured by popular indices of uneven distribution is not well-developed. In my view, the point of concern that Duncan and Duncan raised has taken on much greater importance in the five decades that have passed since their study. In general, research on racial and ethnic differences in socioeconomic outcomes has advanced considerably based on steady, cumulative improvements in our understanding of how group differences in aggregate attainments arise from micro-level attainment dynamics. But this has not been the case in the subfield of segregation research. Until now there has been little progress in developing a better understanding of how aggregate level segregation (as measured by indices of uneven distribution) is linked with individual-level residential outcomes and the micro-level processes that shape them.

Of course, there is a large and vital literature that investigates micro-level dynamics of residential attainment. Studies using individual-level data to focusing on spatial assimilation and spatial attainment first appeared in the 1980s (e.g., Massey and Mullan 1984; Massey and Denton 1985) and then with increasing frequency in the 1990s and beyond (e.g., Alba and Logan 1993; Alba et al. 1999; Bayer et al. 2004; Crowder and South 2005; Crowder et al. 2006; Logan et al. 1996; South and Crowder 1997, 1998; South et al. 2005a, b; South et al. 2008). But, as valuable as this literature has been, it has remained fundamentally disconnected from the literature investigating segregation at the aggregate level. The reason for this is simple; the literature on segregation measurement has never provided a simple, direct strategy for connecting segregation at the aggregate level (i.e., for a city) to individual residential attainments.

Casting indices of uneven distribution as group differences in means on individual residential outcomes addresses this gap in segregation studies. It establishes a simple, direct connection between individual residential outcomes and segregation index scores and in doing so creates the possibility of unifying studies of aggregate segregation and studies of residential attainment in a common overarching framework. Specifically, this approach allows researchers to *simultaneously* investigate both individual residential attainments *and* aggregate segregation in a single analysis. I noted earlier in this chapter that aggregate segregation now can be understood as the effect of group membership (coded 0–1) on the relevant residential outcome in a simple bivariate regression model of individual residential attainment.[11] But this is only a starting point for analysis, not an end point. The approach can be readily extended in a variety of ways that move the investigation of segregation beyond simply assessing aggregate-level uneven distribution.

Casting segregation as a difference of means on individual residential outcomes puts the investigation of segregation on the same methodological footing as the investigation of inter-group inequality and disparity on other important socioeconomic outcomes such as education and income. The key to this is that group disparity is conceived and modeled as emerging directly from an individual-level attainment process. This fundamental change in conceptualization opens up important new options for research. For example, it makes it possible to assess the role of social characteristics such as income using fine-grained measurement such as continuous measurement of income instead of crude category distinctions as used in current practice. Even more importantly, it makes it possible to take account of multiple social characteristics in analyses investigating group segregation; something that is difficult if not impossible to implement using standard methodological approaches to investigating segregation.

These new options become possible because multivariate modeling of individual residential outcomes provides a superior – specifically, a statistically more efficient – framework for taking account of the role of multiple social characteristics (including both race and non-racial characteristics). In this context, implications for aggregate-level segregation can be assessed using methods that are widely used in

[11] The analysis can be conducted using conventional OLS regression or analysis of variance. Statistical tests that rest on the assumption of normality and equal variances for the error term across groups may be questionable on technical grounds in some cases. But the typically large sample sizes used in segregation studies will minimize concerns about these issues. In any event, many good statistical alternatives are available. Boot-strapping or other methods may be used to perform statistical tests that do not rely on assumptions regarding normality and equal variances. Alternatively, the effect of group membership can be assessed using more technically appropriate modeling frameworks such as fractional regression (Papke and Wooldridge 1996) or beta regression (Smithson and Verkuilen 2006; Buis 2006; Buis et al. 2006).

the study of racial inequality and disparity in other socioeconomic outcomes. For example, regression standardization methods can be used to examine differences in residential outcomes for groups that are statistically matched on relevant social characteristics (i.e., other than group membership). Similarly, components analysis can be used to assess the contributions to aggregate segregation of group differences in attainment resources and group differences in ability to convert resources into attainments. These and related methods provide valuable new options for gaining a better understanding of the factors that produce segregation and new options for exploring the potential of different policies to impact aggregate segregation.

Regression-based analysis carries advantages on all these points. In general, the advantages derive from the fact that multivariate regression analysis is a more statistically efficient method with which to account for the effects of multiple social characteristics when comparing groups on average attainments on residential outcomes. Specifically, the statistical efficiencies of the regression standardization approach make it feasible to: (a) incorporate multiple non-racial social characteristics in the analyses and obtain reliable estimates of their separate effects on relevant residential attainments, (b) model the role of continuous social characteristics (e.g., income) in as much detail as the tabulations (or, as will be discussed below, micro-data) will permit, (c) perform comparisons in cities where the small relative size of the minority population makes application of previous approaches problematic, and (d) perform significance tests of the role of race (i.e., group membership) on residential outcomes with social characteristics controlled.[12]

The empirical examples reviewed here provide preliminary illustrations of how the new methods can be used to good effect. But the next section shows that the examples introduced above only hint at what is possible. The new methods used in these examples permit one to imagine new options for analysis using micro data that can go far beyond what might be accomplished using traditional approaches for incorporating non-racial social characteristics into segregation analyses.

9.10 New Possibilities for Investigating Segregation Using Restricted Data

The methods introduced in this chapter permit researchers to investigate segregation in more detail than was previously possible. But the potential benefits of the new methods are relatively modest when segregation is investigated using publicly distributed census summary file tabulations. Summary file tabulations have been the "life blood" of segregation research to date. They have sustained traditional approaches to investigating residential segregation and, at least to some degree, they also can sustain analyses of individual residential attainments of the kind just reviewed. But public summary file tabulations have major limitations. For example,

[12] As discussed in an earlier note, this is based on performing pooled regression analyses to test additive and non-additive effects of race with non-racial social characteristics controlled.

tabulations rarely include more than a few non-racial social characteristics at one time, tabulations often provide only limited detail on non-racial social characteristics, and researchers have no control over the sample universe for the tabulations.[13]

The new methods outlined here can help researchers get more out of these traditional sources of data for segregation analysis. But the potential benefits of the new methods can be realized more fully and to greater effect if one draws on a new source of data for performing segregation analysis. The new source is restricted census datasets that contain individual-level data with detailed information about both individual social characteristics and also geographic information needed to pursue analyses of the residential attainment processes that produce segregation.[14] Working with restricted access census files is difficult, time consuming, and expensive. But it also affords great opportunities. For example, it is conceivable that one could use the most recent files from the American Community Survey (ACS) or the American Housing Survey (AHS) to investigate segregation without having to rely on summary file tabulations. This is possible because the difference of group means formulation of segregation indices allows segregation scores to be estimated by the effect of race in city-specific individual-level models of residential attainment. So, if one has access to detailed micro data, one has tremendous flexibility to investigate segregation in a wide range of new ways.

Additionally, because this approach allows for more efficient multivariate analysis, it expands the possibilities for investigating segregation reliably with smaller samples.[15] This is not only relevant for using smaller samples such as are found in the ACS and AHS. It also raises the possibility of investigating segregation using *non-census surveys*.[16] This is intriguing because non-census surveys can permit investigators to expand residential attainment analyses to consider variables such as individual racial attitudes, residential preferences, and other relevant measures that are not available in census datasets whether micro-data files or summary tabulations.

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[13] For example, the attainment analyses I reported in the previous section found weak income effects based on a crude poverty/non-poverty distinction. Stronger income effects can be discerned using detailed income tabulations, but these tabulations do not include the other social characteristics in the analysis.

[14] A study by Bayer et al. (2004) takes a step in this direction by using restricted access census data to conduct refined individual-level analyses of residential contact. The framework I set forth here makes it possible to implement this kind of study to investigate uneven distribution.

[15] Significance tests and confidence intervals for the effect of race on residential attainments provides clear information about the reliability of segregation estimates obtained from analyses using smaller samples.

[16] Non-census surveys such as the Multi-City Study of Urban Inequality (MCSUI) can be used to study refined models of segregation so long as the households in the study are coded for area of residence at census geographies relevant for studying segregation (e.g., tract, block group, or block). Residential outcomes scored from census data can then be merged with the survey data to permit refined micro-level analyses of aggregated segregation.

9.11 An Example Analysis Using Restricted Microdata

A series of recently completed studies by Amber Fox Crowell provides insight into what the future of research on residential segregation is going to look like.[17] The primary focus of her research is on the factors determining White-Latino segregation. Her dissertation research (Fox 2014) presents detailed analyses investigating White-Latino in six major metropolitan areas. The analyses draw on restricted micro-data files of the 2000 decennial census and the restricted micro-data files of the 2008–2012 American Community Survey. Crowell applies the methods discussed in this work to the full potential that can be achieved with extant data. She measures residential outcomes at the level of census blocks and performs sophisticated quantitative analyses using the method of fractional regression to assess the impact of social and economic characteristics on White and Latino residential attainments. She then performs standardization and components analysis to assess the role of group differences in social and economic characteristics in explaining White-Latino residential segregation. Her studies present detailed results for analyses pertaining to segregation measured both using the separation index (S) and the dissimilarity index (D). I limit the presentation here to selected results from her analyses focusing on group separation (S) but note that the results for the dissimilarity index are similar in overall pattern.

The most striking contribution of her research is her ability to investigate how a comprehensive set of social and economic characteristics shape residential outcomes for Latino households. The list of micro-level predictors and the estimated coefficients indicating their impact on the residential attainments of Whites and Latinos in Houston, Texas in 2000 and in 2010 is presented in Table 9.9. Results for other cities are not presented to conserve space, but the results for Houston give the full flavor of the analyses Crowell is able to conduct. Her attainment equations include a wide range of relevant predictors including age, level of education, household income, military service, nativity and citizenship, year of immigration, English ability, marital/family status, and recent immigration experiences. No previous study has ever been able to take all of these factors into account simultaneously to quantitatively assess their impact on overall (city-level) residential segregation.

The results reported in Table 9.9 show that all of the micro-level variables have statistically significant effects in both the equation for Whites and the equation for Latinos. The "centered" constant reported in the table is the expected value of contact with Whites when independent variables are set at reference categories (for categorical variables) or values (for interval variables). The coefficients reported are fractional effects. These are additive effects on the logit value of the mean for contact with Whites. Positive effects are seen for education, income, and English language ability, produce greater average contact with Whites. Negative effects are

[17] The studies originate with analyses reported in Dr. Crowell's dissertation (Fox 2014) and elaborated and extended in presentations at professional meetings (Crowell and Fossett 2015; Fox and Fossett 2014a, b).

Table 9.9 Coefficients from fractional regressions predicting residential outcomes (y) determining the separation index (S) for White-Latino segregation in Houston, Texas in 2000 and 2010

	Whites		Latinos	
Variable	2000	2010	2000	2010
Degree (0–5)	0.1907[a]	0.1395[a]	0.2794[a]	0.2293[a]
Income (Ln)	0.0998[a]	0.0666[a]	0.0771[a]	0.0535[a]
Military	–0.0990[a]	–0.1103[a]	0.1088[a]	0.0828
U.S.-born citizen (ref)	–	–	–	–
Non-U.S. citizen	–0.0981[b]	–0.0873	–0.2506[a]	–0.2318[a]
Nat. U.S. citizen	–0.0991[a]	–0.1312[a]	–0.0315	–0.0280
Recent immigrant	–0.1836[a]	–0.0877	–0.1874[a]	–0.0042
English ability	0.2923[a]	0.3097[a]	0.1679[a]	0.2526[a]
Age 30–59 (ref)	–	–	–	–
Age 15–29	–0.1713[a]	–0.1673[a]	–0.1902[a]	–0.1950[a]
Age 60+	0.1579[a]	0.1386[a]	–0.0025	0.0843[a]
Married couple (ref)	–	–	–	–
Single mother	–0.2871[a]	–0.3010[a]	–0.1655[a]	–0.2784[a]
Other family	–0.3715[a]	–0.3325[a]	–0.0489[a]	–0.1283[a]
Recent mover	0.0940[a]	–0.0814[a]	0.2334[a]	0.0530[b]
Constant	–0.6652[a]	–0.6285[a]	–1.7437[a]	–1.8607[a]
Constant (centered)	1.5908[a]	1.2448[a]	0.0903[a]	–0.1099[a]

Notes: [a]denotes $p<0.01$ and [b]denotes $p<0.05$
Source: Restricted microdata files from the 2000 decennial census and the 2008–2012 American Community Survey

seen for foreign born status, non-citizen status, and recent immigration which all produce lower average contact with Whites. All of the effects are consistent with expectations from spatial assimilation theory. Group differences in the efficacy of the social and economic characteristics reflect the impact of minority status on contact with Whites. Altogether the results provide a wealth of information about the role of social and economic characteristics in shaping White and Latino residential outcomes and ultimately White-Latino segregation.

The implications of the results for White-Latino segregation in Houston are summarized in Table 9.10 which also presents results for the other cities included in Crowell's analyses. The results document that White-Latino differences in mean contact with Whites – the residential outcome that determines the value of the separation index (S) – vary across substantively relevant standardization scenarios. The scenario labeled "Latino group means & Latino rates of return" yields the predicted level of contact with Whites for Latinos in the Houston given their observed distribution on the social and economic characteristics in the attainment equations. Similarly, the scenario labeled "White group means & White rates of return" yields the predicted level of contact with Whites for Whites in the Houston given their observed distribution on the social and economic characteristics in the attainment equations. The difference between these two means yields the observed value of the

Table 9.10 Standardization analyses for White-Latino differences in residential outcomes (y) determining the separation index (S)

	Atlanta		Chicago		Houston		Los Angeles		San Diego		Seattle	
Standardization comparison	Mean	*S**	Mean	*S**	Mean	*S**	Mean	*S**	Mean	*S**	Mean	*S**
2000												
Latino group means & Latino rates of return	72.2	*23.9*	52.2	*40.4*	43.2	*42.1*	30.5	*51.7*	52.3	*34.4*	88.0	*8.4*
White group means & Latino rates of return	86.9	*9.3*	66.9	*25.7*	62.9	*22.4*	54.1	*28.1*	69.6	*17.1*	91.7	*4.7*
Latino group means & White rates of return	91.4	*4.7*	87.0	*5.6*	74.2	*11.2*	71.7	*10.5*	81.3	*5.3*	95.0	*1.4*
White group means & White r ates of return	96.1	–	92.6	–	85.3	–	82.2	–	86.6	–	96.4	–
2010												
Latino group means & Latino rates of return	63.1	*29.6*	49.5	*39.8*	36.8	*42.3*	27.4	*50.4*	47.8	*34.2*	82.5	*11.3*
White group means & Latino rates of return	80.9	*11.8*	63.1	*26.3*	54.5	*24.5*	46.4	*31.4*	61.9	*20.1*	88.7	*5.1*
Latino group means & White rates of return	85.9	*6.8*	84.2	*5.2*	67.4	*11.7*	72.5	*5.3*	76.7	*5.3*	90.8	*2.9*
White group means & White rates of return	92.7	–	89.4	–	79.1	–	77.8	–	82.0	–	93.7	–

Notes: "Mean" denotes predicted mean contact with Whites based on the standardization scenario. S* denotes the value of the separation index (S) (i.e., the White-Latino mean difference in contact with Whites) under the standardization scenario. S* under the "Latino group means & Latino rates of return is the observed value of the separation index. Source: Restricted micro-data files from the 2000 decennial census and the 2008–2012 American Community Survey separation index (S) for White-Latino segregation in Houston. That is, the value of 42.1 in 2000 reflects the difference between the mean of 85.3 for Whites and the mean of 43.2 for Latinos and is reported in the column labeled "S*" under Houston on the first row of the panel reporting results for 2000.

Scanning the values reported on this row of the table reveals that White-Latino separation varies greatly across the six cities in Crowell's analysis. The separation index (S) is highest in Los Angeles (51.7) and only slightly lower in Houston (42.1) and Chicago (40.4). It is somewhat lower in Atlanta (23.9) and very low in Seattle (8.4). Drawing on methods reviewed earlier in this chapter, Crowell performed standardization analyses to explore address the question of whether White-Latino segregation is due to group differences in resources for residential attainment or the impact of group status itself in the residential attainment process. In the interests of space group distributions on predictors are not shown but they are reported in Crowell's studies. Not surprisingly, Latinos tend to have deficits on predictors that have positive effects on contact with Whites (e.g., income) and surpluses on predictors that reduce contact with Whites (e.g., non-U.S. citizen).

The role of group differences in resources is documented in the row labeled "White group means & Latino rates of return." The values reported here indicate how Latino residential outcomes would change if Latinos had the White "profile" on social and economic characteristics. The implications for S* show that the role of group differences assessed in this manner is always positive and substantively important. Equalizing Latino inputs to residential attainment process reduces the value of S by between 34 and 61 %.

The role of minority status is documented in the row labeled "Latino group means & White rates of return" which indicates how Latino residential outcomes would change if Latinos experienced White rates of converting inputs to the attainment process into contact with Whites. The implications for S* show that the role of this factor also is always positive and substantively important. Indeed, equalizing Latino rates of return in the attainment process would reduce the value of S by between 74 and 89 %.

I close this chapter by noting that the results presented in Tables 9.9 and 9.10 provide a wealth of information warranting additional discussion. Unfortunately, a more detailed review is beyond the scope of the present discussion so I encourage interested readers to seek out Crowell's research for more in-depth discussion of her findings. The central point I stress here is this. Crowell's research shows that combining the new methods outlined in this monograph with the restricted census micro-data files opens the door to exciting new options for segregation analysis. Crowell's research provides the best example to date of how segregation can be analyzed in great detail in a single-city analysis. In the next chapter I outline how this approach can be expanded to cover a larger sample of cities and explore the impact of city-level characteristics on residential segregation via estimation of multi-level models of residential attainments.

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Chapter 10
New Options for Investigating Macro-level Variation in Segregation

The previous chapter established that the difference of means framework for measuring segregation makes it possible to investigate segregation in a single city using individual-level models of residential attainment. The discussion in this chapter reviews how this approach can be extended to investigate ecological (i.e., aggregate-level) variation in segregation across cities and over time using multi-level models of individual residential attainments. The key is that ecological variation in segregation can be investigated by assessing how the effect of race on segregation-relevant individual residential outcomes is conditioned by time and/or city characteristics. A central advantage of this approach is that it permits researchers to also include relevant non-racial social and economic characteristics in the micro-model. This allows effects of community characteristics to be estimated at the "zero order" level or "net" of controls for non-racial factors. It also can help overcome the risk of errors of inference that are likely to occur in aggregate-level analyses that attempt to control for relevant individual-level social and economic characteristics using aggregate-level indicators of group disparity on these variables.

10.1 New Specifications for Conducting Comparative and/or Trend Analyses of Segregation

Investigations of how segregation varies across metropolitan areas and over time are a staple of segregation studies. *The new methods outlined here can be used to first exactly replicate earlier studies and then to extend them in new ways*. Results from previous studies can be exactly replicated by estimating contextual and multi-level models where variation in segregation over time and across metropolitan areas is captured by assessing how the effect of race in individual-level models of residential attainment varies with time and/or the ecological characteristics of metropolitan areas. For example, consider the question of how White-Black segregation

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_10

measured by the index of dissimilarity (D) varies across cities based on city size (lnpop = the natural logarithm of total population) and relative minority size (rpb = the square root of proportion Black for the city population).

Following the typical aggregate-level approach, cities are taken as units of analysis, D is calculated separately for individual cities, and scores for D then are taken as dependent variables (y) in a city-level OLS regression analysis that includes city size (lnpop) and relative minority size (rpb) as predictors as follows.

$$\mathrm{y_i} = a_0 + a_1\left(\mathrm{lnpop}\right) + a_2\left(\mathrm{rpb}\right)$$

I estimated this equation using city-level data for the 201 metropolitan areas that had 50,000 total households and 2000 Black households in the 2000 Census.[1] I obtained the following results.

$$\mathrm{y_i} = 55.92 + 3.90\left(\mathrm{lnpop}\right) + 4.79\left(\mathrm{rpb}\right)$$

These effect values also can be obtained from an individual-level, contextual model that takes as its dependent variable the individual-level residential outcome (y) relevant for D – that is, whether the (pairwise) proportion White (p) for the area in which the individual resides is equal to or greater than the city-wide figure (P) – and then also includes as a predictor the individual characteristic of race (coded 1 if White and 0 if Black) and appropriate interactions to capture how the effect of race on residential outcomes varies by city size (lnpop) and relative minority size (rpb). The result is the following individual-level OLS regression specification.

$$\mathrm{y_i} = b_0 + b_1\left(\mathrm{race}\right) + b_2\left(\mathrm{lnpop}\right) + b_3\left(\mathrm{rpb}\right) + b_4\left(\mathrm{race{\cdot}lnpop}\right) + b_5\left(\mathrm{race{\cdot}rpb}\right)$$

I estimated this equation using individual-level-level data for the White and Black households in the same set of 201 metropolitan areas used in the aggregate analysis just reported above and I obtained the following results.[2]

$$\begin{aligned}\mathrm{y_i} &= 17.58 + 55.92\left(\mathrm{race}\right) + -1.671\left(\mathrm{lnpop}\right) + 12.75\left(\mathrm{rpb}\right)\\ &\quad + 3.90\left(\mathrm{race}\cdot\mathrm{lnpop}\right) + 4.79\left(\mathrm{race}\cdot\mathrm{rpb}\right)\end{aligned}$$

[1] The data for the analyses are obtained from the tabulation of household income by race for census block groups in Tables 15.1 (A-I) distributed in Summary File 3 of the 2000 Census.

[2] One must give attention to how cases are weighted to replicate the unstandardized regression coefficients from the city-level regression. The city-level regression implicitly gives equal weight to each group's mean for the relevant residential outcomes (y) in every city. To implement the same weighting scheme at the individual level, I first calculated each household's proportionate share of the race-specific group total for the city in which it resided. I then multiplied these share values by 2000, the minimum number of households for any group in any city, and used the resulting number as the case weight for the household. One may consider other weighting approaches at the individual level. But something along the lines of the approach just noted is required to exactly reproduce the city-level regression coefficients.

The results document that the additive and non-additive effects of race in the individual-level contextual regression correspond exactly to the coefficients in the city-level regression. Specifically, $b_1 = a_0 = 55.92$, $b_4 = a_1 = 3.90$, and $b_5 = a_2 = 4.79$.

I present the city-level and household-level regressions in Table 10.1. The table also includes regressions for the additive components that define D; namely, the percentages of Whites living in areas where proportion White exceeds the city proportion and the comparable percentage of Blacks. Inspection of the results shows that the effects of city size (lnpop) and relative minority size (rpb) on D can be traced to the differential effects these city characteristics have on the levels of residential contact White and Black households have with Whites.

The table also provides a parallel analysis for the separation index (S). As seen for the analysis for D, coefficients in the city-level regression for S map exactly onto coefficients in the individual-level contextual regression and the separate regressions and the results for S can be traced to the differential effects city characteristics have on the residential contact Whites and Blacks have with Whites.

I next extended the analysis to do something that previously has not been possible – namely, to investigate variation in segregation across cities and/or over time while simultaneously taking account of non-racial characteristics of households at the micro-level. This is possible because the summary file tabulations – namely, Table 151 (A-I) from Summary File 3 of the 2000 Census – provide the individual level data needed to perform this analysis. To accomplish the task, I re-estimated the contextual regressions reported in Table 10.1 after adding household income and the interaction of race and household income as predictors in the analysis. The results are presented in Table 10.2.

The impact of race on White-Black differences in residential outcomes and how these differences vary with city characteristics are registered in the same way as before. But here the segregation effects – that is the effect of race on residential outcomes – can be interpreted as being estimated *net of the effects of that income has on residential outcomes*. In the model specification used here, higher income is seen to lead to greater contact with Whites, a finding consistent with results reported in the literature on residential attainment. But note that *the introduction of the control for income at the individual level has little impact on the effects of city size and relative minority size on segregation*. That is, the impacts of city size and relative minority size on the coefficient for race in this analysis closely parallel the same effects observed for these variables in the city-level and individual-level contextual regressions that do not include individual income as a control variable.

I do not present city-level regressions in Table 10.2 because aggregate specifications cannot properly take account of the role of group differences in socioeconomic characteristics. I have outlined the general basis for this conclusion in an earlier paper focusing on the logically similar task of assessing the role of group differences in education in city variation in racial income inequality (Fossett 1988). The conclusions of that methodological study apply with full force to the present situation. That is, to correctly assess how group differences in socioeconomic attainments impact city variation in segregation, one must draw on data that disaggregates

Table 10.1 Regression results illustrating that effects in city-level analyses of segregation and contact can be obtained using individual-level, contextual regressions predicting group differences in contact

City-level regressions for dissimilarity index (D)	D	D contact $P_{W(p \geq P)}$	D contact $P_{B(p \geq P)}$
City-level effects on segregation (group contact difference) and group contact terms[d]			
City size (lnpop) (a_1, b_4)	3.90[a]	2.23[a]	−1.67[a]
Relative minority size (rpb) (a_2, b_5)	4.79	17.54[a]	12.75[a]
City-level intercept (a_0, b_1)	55.92[a]	73.50[a]	17.58[a]
Sample N	201	201	201
Individual-Level, Contextual Regressions for Dissimilarity Index (D)	y for D Pooled	y for D Whites	y for D Blacks
City-level effects on segregation (group contact difference) and group contact terms[d]			
City size (lnpop) (a_1, b_4)	3.90[a]	2.23[a]	−1.67[a]
Relative minority size (rpb) (a_2, b_5)	4.79[a]	17.54[a]	12.75[a]
City-level intercept (a_0, b_1)	55.92[a]	73.50[a]	17.58[a]
Additional individual-level effects			
City size (main effect lnpop, b_2)	−1.67[a]	–	–
Relative minority size (main effect rpb, b_3)	12.75[a]	–	–
Individual-level intercept (b_0)	17.58[a]	–	–
Sample N	804,000[c]	402,000[c]	402,000[c]

City-level regressions for separation index (S)	S	S Contact P^*_{WW}	S Contact P^*_{BW}
City-level effects on segregation (group contact difference) and group contact terms[d]			
City size (lnpop) (a_1, b_4)	5.83[a]	0.59[b]	−5.24[a]
Relative minority size (rpb) (a_2, b_5)	70.51[a]	−35.39[a]	−105.90[a]
City-level intercept (a_0, b_1)	11.76[a]	99.78[a]	88.02[a]
Sample N	201	201	201
Individual-level, contextual regressions for separation index (S)	y for S Pooled	y for S Whites	y for S Blacks
City-level effects on segregation (group contact difference) and group contact terms[d]			
City size (lnpop) (a_1, b_4)	5.83[a]	0.59[a]	−5.24[a]
Relative minority size (rpb) (a_2, b_5)	70.51[a]	−35.39[a]	−105.90[a]
City-level intercept (a_0, b_1)	11.76[a]	99.78[a]	88.02[a]
Additional individual-level effects			
City size (main effect lnpop, b_2)	−5.24[a]	–	–
Relative minority size (main effect rpb, b_3)	−105.90[a]	–	–
Individual-level intercept (b_0)	88.02[a]	–	–
Sample N	804,000[c]	402,000[c]	402,000[c]

(continued)

Table 10.1 (continued)

Source: Summary File 3, Census 2000

[a] $p < 0.001$

[b] $p < 0.01$

[c] Weighting of cases is described in the text

[d] In the city-level regressions for D and S, the equation is $y = a_0 + a_1(\text{lnpop}) + a_2(\text{rpb})$ and the city-level effects are a_0, a_1, and a_2. In the individual-level, contextual regressions y is scaled pairwise contact based on y = f(p) as appropriate for D and S. The equation is $y = b_0 + b_1(\text{race}) + b_2(\text{lnpop}) + b_3(\text{rpb}) + b_4(\text{race}\cdot\text{lnpop}) + b_5(\text{race}\cdot\text{rpb})$. City-level effects are captured by coefficients $b_1 = a_0$, $b_4 = a_1$, and $b_5 = a_2$. The variables lnpop and rbp are centered on values at the observed sample minimum, lnpop on 12.0, and rbp on 0.10, to make the regression intercepts substantively meaningful

residential outcomes by race and socioeconomic status as is the case for the individual-level contextual regressions in Table 10.2.

Due to the lack of viable alternative methods, past studies sometimes have instead adopted the approach of including aggregate (i.e., city-level) measures of group differences in socioeconomic status as control variables in analyses predicting segregation (e.g., Marshall and Jiobu 1975; Roof et al. 1976; Farley and Frey 1994; Massey and Denton 1987). Unfortunately, this approach is flawed. As noted earlier, it can yield misleading results because it runs afoul of the "aggregate" or "ecological" fallacy in using aggregate-level measures to take account of the role of variables whose impact should properly be assessed at the micro level. I do not provide an extended discussion of the general issues to since I have reviewed them in an earlier study (Fossett 1988). But I do highlight the practical significance of the problem by reporting analyses in Table 10.3 that replicate central findings reported in Fossett (1988) using an empirical example investigating cross-city variation in segregation. The first column in Table 10.3 reports results of conventional city-level regressions that predict D and S using city characteristics. The second column reports results of regressions that add a city-level measure of Black-White income inequality as a predictor. Many aggregate analyses of segregation have used similar model specifications motivated by the plausible conjecture that segregation between groups will be larger when their disparity on income is larger.

The results of the aggregate regression suggest that group income differences have dramatic impacts on segregation. But this finding is contradicted by the results of the contextual analyses reported in Table 10.2. It also is at odds results from the city-specific standardization exercises for Houston, Texas reported earlier in Table 9.4. These analyses controlled for socioeconomic characteristics at the individual level and the results indicated that socioeconomic characteristics were not generally important in shaping racial segregation. Specifically, these analyses indicated that

Table 10.2 Analyses illustrating how city-level analyses of segregation can be conducted using individual-level, contextual regressions that control non-racial characteristics

Regressions for Dissimilarity Index (D)	y for D	y for D
City size (lnpop) (a_1, b_4)	3.90[a]	3.97[a]
Relative minority size (rpb) (a_2, b_5)	4.79[a]	2.24[a]
City-level intercept (a_0, b_1)	55.92[a]	56.35[a]
Additional individual-level effects		
City size (main effect lnpop, b_2)	−1.67[a]	−2.39[a]
Relative minority size (main effect rpb, b_3)	12.75[a]	15.62[a]
Income (b_6)	–	3.36[a]
Race-income interaction (b_7)	–	−1.16[a]
Individual-level intercept (b_0)	17.58[a]	11.40[a]
Regressions for Separation Index (S)	**y for S**	**y for S**
Implied city-level effects on segregation & contact[c]		
City size (lnpop) (a_1, b_4)	5.83[a]	6.26[a]
Relative minority size (rpb) (a_2, b_5)	70.51[a]	68.05[a]
City-level intercept (a_0, b_1)	11.76[a]	15.40[a]
Additional individual-level effects		
City size (main effect lnpop, b_2)	−5.24[a]	−5.88[a]
Relative minority size (main effect rpb, b_3)	−105.90[a]	−103.33[a]
Income (b_6)	–	3.00[a]
Race-income interaction (b_7)	–	−2.28[a]
Individual-level intercept (b_0)	88.02[a]	82.51[a]
Sample N	804,000[b]	804,000[b]

Source: Summary File 3, Census 2000

[a]$p<0.001$

[b]Weighting of cases is described in the text

[c]In individual-level, contextual regression for D and S the specification is $y = b_0 + b_1(\text{race}) + b_2(\text{lnpop}) + b_3(\text{rpb}) + b_4(\text{race} \cdot \text{lnpop}) + b_5(\text{race} \cdot \text{rpb}) + b_6(\text{income}) + b_7(\text{income} \cdot \text{race})$. Implied city-level effects are b_1, b_4, and b_5. To make intercepts substantively meaningful, lnpop and rbp are centered on values near the low end of the observed sample distribution; specificall, lnpop is centered on 12.0 and rbp is centered on 0.1

White-Black differences in residential contact with Whites (coded to reflect how contact determines values of D and S) decrease only modestly when White-Black differences in socioeconomic characteristics are taken into account at the individual-level; that is, by drawing on micro-level data to standardize the White-Black comparison to take account of the impact of group differences in income separately in each city based on the city-specific race-income-residence relationship at the individual level for that city.

The third column of Table 10.3 presents city-level regressions that replicate another finding reported in Fossett (1988). The dependent variables for these analyses, D* and S*, are values of D and S that have been "standardized" so they represent differences in residential outcomes between Whites and Blacks with identical

Table 10.3 Analyses illustrating how city-level analyses of segregation can yield misleading results when aggregate measures are used to control for group differences on non-racial characteristics

City-level regressions for White-Black segregation – Observed Dissimilarity (D) and Standardized Dissimilarity (D*)			
	D	D	D*
Unstandardized regression coefficients			
City size (lnpop)	3.90[a]	4.23[a]	4.43[a]
Relative minority size (rpb)	4.79	−10.36[c]	−6.79
Ratio of mean incomes (B/W)	–	−40.29[a]	−29.43[a]
Standardized regression coefficients			
City size (lnpop)	0.399[a]	0.423[a]	0.427[a]
Relative minority size (rpb)	0.063	−0.136[c]	−0.086
Ratio of mean incomes (B/W)	–	−0.538[a]	−0.379[a]
Sample N	201	201	201
City-level regressions for White-Black segregation – Observed Separation Index (S) and Standardized Separation Index (S*)			
	S	S	S*
Unstandardized regression coefficients			
City size (lnpop)	5.83[a]	6.32[a]	6.24[a]
Relative minority size (rpb)	70.51[a]	48.04[a]	52.01[a]
Ratio of mean incomes (B/W)	–	−59.75[a]	−45.37[a]
Standardized regression coefficients			
City size (lnpop)	0.360[a]	0.391[a]	0.406[a]
Relative minority size (rpb)	0.571[a]	0.389[a]	0.444[a]
Ratio of mean incomes (B/W)	–	−0.494[a]	−0.396[a]
Sample N	201	201	201

Source: Summary File 3, Census 2000

[a] $p < 0.001$

[b] $p < 0.01$

[c] $p < 0.05$

income distributions. Drawing on techniques discussed earlier in Chapter 9, the standardization is accomplished by calculating D* and S* from predicted means on segregation-relevant residential outcomes for Whites and Blacks with identical levels of income based on city- and group-specific individual-level regressions of residential outcomes on income. Since D* and S* reflect White-Black differences in residential outcomes for families that have identical levels of income, city variation in D* and S* cannot be attributed to city variation in group income differences. Nevertheless, the city-level measure of racial income inequality continues to have very strong and statistically significant effects on D* and S* in the city-level regressions.

There is a ready explanation for this result. It is that the aggregate-level association of segregation and socioeconomic differences reflects much more than the narrow impact of racial income differences on racial differences in residential outcomes.

Drawing on arguments set forth in more detail in Fossett (1988) I suggest that the strong effect of racial income inequality in this equation is misleading and primarily reflects a spurious association. My interpretation is guided by the simple hypothesis that *aggregate racial inequality in all important areas of socioeconomic attainment are likely to co-vary because they all are likely to share a common cause; they vary together based on the general salience of race and minority disadvantage in socioeconomic attainment dynamics in the community stratification system*. To the extent that this is so, racial segregation and racial income inequality will be correlated at the aggregate level even when White-Black income differences play a minor role in shaping White-Black residential segregation. The regression results reported in column 3 are consistent with this hypothesis. This in turn indicates that the strong effects of racial income inequality in the regression results reported in column 2 are misleading.

The interpretation I offer regarding the effect of income inequality in aggregate-level regressions predicting segregation is at odds with the usual interpretation offered aggregate-levels studies of segregation. But it is consistent with findings from micro-level studies of the role of group income differences in shaping White-Black segregation. Studies that draw on micro-data that disaggregate residential outcomes by income and race simultaneously consistently report that White-Black income differences are not a major factor contributing to segregation between the groups. For example, analyses performed for individual cities typically report that index scores for White-Black segregation are as high when computed for households that are matched on income (or other socioeconomic characteristics) as when computed for the full populations (Farley 1977; Denton and Massey 1988; Massey and Fischer 1999). I observe the same pattern in the city-specific income standardization exercises that generated the D* and S* index scores used in the aggregate analyses reported here.

In sum, then, there is little available evidence from analysis of detailed micro-data for individual cities to indicate that White-Black income differences play a major role in producing residential separation of Whites and Blacks. The reason is simple; Whites at all income levels tend to live apart from Blacks at all income levels. Findings of this sort based on analysis of disaggregated micro-data should be seen as more compelling than findings from aggregate correlations of racial income differences and racial segregation. *Researchers seeking to properly assess the impact of group differences in non-racial characteristics (*e.g.*, income) on segregation must directly examine how residential attainments vary with those characteristics separately by race in each community using disaggregated micro-level data*. The framework for studying segregation set forth here allows researchers to investigate these questions in a methodologically sound way. It allows them to assess the role of group differences on non-racial characteristics such as income using individual-level contextual models of attainment. The alternative approach of including measures of socioeconomic inequality in city-level analyses of segregation is flawed and prone to yielding misleading results as seen in the example here. It should be abandoned.

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Chapter 11
Aspatial and Spatial Applications of Indices of Uneven Distribution

The difference of means formulations of indices of uneven distribution makes it relatively straightforward to implement segregation measurement in either conventional *aspatial* formulations or in *spatial* formulations. Aspatial versions of segregation indices are familiar because they are widely used in empirical studies. They are obtained by applying any of the computing formulas reviewed here using data for non-overlapping, bounded areas such as census tracts, block groups, or blocks. It is appropriate to designate the resulting scores as "aspatial" because the spatial arrangements of the units (e.g., blocks, block groups, tracts) have no implications for the scores obtained. Spatial formulations would differ on this key point; namely, the spatial arrangement of units can potentially impact index scores.

In truth, opportunities to compute spatial versions of indices of uneven distribution have always existed. But apparently this has not been widely appreciated. Or, more carefully, researchers have rarely taken advantage of this possible option. One simple way to implement popular indices of uneven distribution in either aspatial or spatial versions is to use computing formulas that give index scores as population averages for area-specific residential outcomes. Figures in Appendices present formulations of this type for all popular indices of uneven distribution. Here I note only two such formulations, one for D and one for S. Both take the general form $100 \cdot (1/T) \cdot \Sigma y$ where y is a residential outcome for individuals scored on the basis of their area of residence. The value of D can be obtained using $y = |p_i - P|/2PQ$ and the value of S can be obtained using $y_k = (p_i - P)^2 / PQ$.

If y and p are calculated using only the data for the block the individual resides in, the calculations will yield the usual index score which is aspatial because how individual blocks are arranged in space has no impact on index scores. However, if one chooses to do so, one can calculate y and p based on spatially defined neighborhoods. For example, one could define the neighborhood as a "first order" contiguity neighborhood based on combining data for the block the individual resides in and also the blocks that are adjacent to that block. This is the only modification that is required; all other steps in the calculations remain the same.

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_11

This neighborhood formulation makes the index "spatial" because how blocks are arranged in space will now potentially affect index scores. The key change is that an individual's neighborhood has shifted from being equated with a discrete "bounded" area that applies only to individuals in the area to a spatially-defined region that in some degree is shared with individuals in adjacent areas. I ignore the fact that the size of the neighborhood has changed because it is not a fundamental issue. It can be rendered irrelevant by defining bounded areas and spatially defined areas to be comparable in size.

Following this example, it is obvious that difference of means formulations of indices also can be implemented as either spatial or aspatial. The key terms that determine the index scores are individual residential outcomes (y) that are scored from area group proportion (p). Calculate p for bounded areas and the index is aspatial; calculate p for spatially defined areas and the index is spatial. Assessment of group means and associated segregation index scores is easy to accomplish either way and results will be spatial or aspatial depending on this choice of how area group proportions are calculated. I have drawn on these options when conducting simulation studies of segregation dynamics using the SimSeg simulation model (Fossett and Waren 2005; Fossett and Dietrich 2008; Clark and Fossett 2008; Fossett 2011a) and also in applications using block-level census data to assess segregation using neighborhoods that vary in spatial scale (Fossett 2011b).

Spatial and aspatial implementations of indices are both potentially interesting. However, my own experience in empirical analyses has been that they rarely yield different substantive findings when they are implemented at spatial scales that yield comparable neighborhood-level population counts. But it is logically possible that they might yield different findings in some circumstances. For example, one can imagine that some administrative boundaries (e.g., school district lines, city boundaries, zoning areas, etc.) and/or urban ecological barriers (e.g., highways, roads patterns, rivers, etc.) could delimit sociologically meaningful spatial domains that are sharply "bounded" based on the impact of physical barriers or administrative boundaries on social interaction. In the extreme case, racial composition in adjacent areas would not matter because social interaction and common residential fate are determined solely inside the boundaries of the spatial units used. In practice, however, boundaries for the spatial units used most often in segregation research can be somewhat arbitrary and spatially defined areas may potentially correspond more closely to sociologically meaningful neighborhoods. For example, a block located near the boundary of census tract may have more in common with the nearby blocks in an adjacent tract than with blocks on the far side of the same tract. So both approaches can be defended on conceptual grounds.

Again, there is as yet little evidence to indicate that the choice between spatial and aspatial implementations of segregation indices carries compelling practical consequences for findings regarding aggregate segregation patterns. However, I discuss the issue here because I can think of at least one practical reason for investigators to consider using spatially defined neighborhoods. It is for studying segregation involving smaller groups and segregation in smaller communities (e.g., small cities and CBSAs). I noted earlier that in conducting analyses of segregation in CBSAs I have found that census tracts and even census block groups can be too large to capture segregation patterns in smaller CBSAs. In particular, I find that tracts and block groups are not well suited for studying segregation involving smaller populations – for example, studying segregation for Latinos in areas of recent settlement. Among available census geography that leaves census blocks as the best option to use for computing standard aspatial segregation indices. However, some researchers might worry that census blocks are too small. One way to address this concern is to assess segregation using first- or second-order spatial neighborhoods based on block data. These would meet the needs of using spatial units that are small enough to capture segregation in smaller communities and for smaller groups while at the same time being potentially more appealing with regard to reflecting sociologically meaningful neighborhoods.

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Chapter 12
Relevance of Individual-Level Residential Outcomes for Describing Segregation

The new options for segregation analysis introduced here suggest a *new basis for evaluating segregation indices – the substantive relevance of the individual-level residential outcomes registered by the indices*. Three of the segregation indices considered here – G, D, and S – have been used widely in empirical analyses for more than five decades and each has been reviewed many times in methodological studies.[1] Until now little attention has been given to the substantive relevance of the individual-level residential outcomes each index registers. In this chapter I argue that it is instructive to consider how indices differ on this important point of comparison.

In their difference of means formulations G, D, and S register group differences on average residential outcomes (y) scored from (pairwise) area group proportion (p). G rescales p to register relative rank or percentile scoring. D rescales rank distinctions on p to register only a 0, 1 coding of whether or not p is above the city average (P). S does not rescale p; it registers it in its original metric. Because S registers p directly, a given value of p yields the same value of y in all cities. In contrast, G and D assign values of y based on monotonic, rank position scoring schemes that vary in functional form in complex ways across cities. In particular, the scoring of y from p is nonlinear and the magnitude of the departure from linearity varies with city racial mix (as discussed previously in Chap. 5). Consequently, identical values of area racial proportion (p) can be and often are assigned very different values on the residential outcome of scaled contact (y) in different cities.

Residential outcomes (y) registered by S – for example, area proportion White (p) in the familiar case of White-Black segregation – have clear substantive appeal. The residential outcome of group contact in its "natural" metric is directly meaningful to individuals and households both in its own right and also because area proportion White (p) tends to correlate with neighborhood characteristics that have clear relevance for life chances (e.g., crime rates, quality of schools, neighborhood services and amenities, property values, etc.). The same cannot be said for the

[1] I bring R and H into the discussion later in this chapter.

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_12

neighborhood outcomes (y) used in computing scores for G and D. G rescales values of p into ordinal-level, relative rank (percentile) scores. D collapses values of p to just two relative rank scores.

The value and sociological relevance of scoring residential outcomes (y) as G and D do is not obvious. Few if any discussions of group differences in residential outcomes explicitly prioritize ordinal position on contact with Whites over contact with Whites in its natural metric. Similarly, discussions that view area proportion Whites as relevant for the impact of area of residence on life chances rarely if ever suggest that this is best captured by coding area proportion White in terms of relative rank position or in terms of "parity." To the contrary, theories of majority group discrimination and avoidance of minority groups usually presume that exclusionary discrimination by Whites and White avoidance of minority areas is aimed at maintaining neighborhoods as predominantly-majority (e.g., 85 % White or higher) rather than simply "above parity" in comparison to proportion White in the city which of course can vary dramatically across cities. In view of this, I believe there is no compelling basis for giving "relative rank" scoring of p or "above parity" scoring of p *priority* over the natural interval metric for p.

S also is attractive because it has clear, straightforward implications that are easy to explain to general audiences. For example, if White-Black segregation as measured by S is high – say 60 – it means Blacks' average contact with Whites is 60 points lower than Whites' average contact with Whites. This yields an unambiguous signal about the consequences of segregation for individuals and groups; it indicates that, on average, Whites live in predominantly White neighborhoods and Blacks live in predominantly Black neighborhoods. This score on S also sends a signal about the extent to which Whites and Blacks can potentially experience differences on life chances based on neighborhood characteristics that correlate with area proportion White. When S is zero, Whites and Blacks will necessarily experience the same average on all residential outcomes. As S increases above zero, so too does the potential for Whites and Blacks to experience differences on other important residential outcomes (e.g., crime, poverty, schools, amenities, etc.).

The simple and clear conclusions one can draw based on knowing that S takes a high score do not necessarily hold when G and D take high scores. To the contrary, as discussed in Chaps. 7 and 8, it is possible for G and D to be very high – say 80 – and for both Whites and Blacks to live in neighborhoods that on average are similar on area proportion White (p). In these cases it could be highly misleading to assume that high scores on G and D carry consequences for group differences on neighborhood outcomes that are relevant for life chances (e.g., crime, poverty, schools, amenities, etc.) and are correlated with area proportion White. The reason for this is simple. If Whites and Blacks experience similar outcomes on area proportion White, they will, all else equal, tend to experience similar outcomes on factors that are correlated with area proportion White.

The mathematical basis for how G and D can take high values when Whites and Blacks share similar neighborhood outcomes was discussed earlier in Chap. 5. It was illustrated in the graphs in Fig. 5.1 which depict how G and D register group differences in contact with Whites (p) after p has been subjected to a dramatic nonlinear rescaling. This nonlinear rescaling of p reduces the importance of group differences on contact with Whites (p) over some ranges of p and it exaggerates the importance of group differences on p over other ranges of p. In the graph for the White-Asian comparison in Fig. 5.1, for example, group differences in p over the range of 0–80 are of minor importance while group differences on p over the range 80–100 take on great importance. D is even more extreme in this regard; it rescales values of p into values of y based on a one-step function that registers only differences on either side of P. The discussion in Chap. 7 further outlined the technical basis for how these characteristics of G and D create the possibility that they can and often will take values that differ substantially from values of S.

The key implication is that high values on G and D have uncertain implications for group differences on sociologically meaningful residential outcomes because values of D and G can be highly sensitive to small differences in area group proportion. Specifically, G and D for White-Black segregation can in principle take very high values when Whites and Blacks live in neighborhoods that, on average, are fundamentally similar on area proportion White (p) and other sociologically important neighborhood outcomes.

This may be surprising to some. If so, it is instructive to carefully consider the familiar interpretation of D as indicating the minimum proportion of one group that must move to bring about even distribution. Note that this interpretation implies nothing specific about whether the residential movement that eliminates uneven distribution as measured by D will cause either group's residential outcomes to change in sociologically important ways. In fact, the movement associated with eliminating a high value for D can and sometimes will produce small, potentially trivial, average changes in substantively meaningful residential outcomes for the members of a group.

This frames a point of clear contrast between G and D on the one hand and S on the other. *High values of S always signal that residential movement needed to bring about even distribution will produce dramatic changes in group differences in residential outcomes. This is not necessarily true for G and D*. This is a consequence of the fact that high values of G and D can occur under "prototypical segregation," which involves high levels of group separation, but also under "dispersed displacement" or "displacement without separation" as discussed in Chap. 7 (Fig. 7.1).

The potential for G and D to manifest this characteristic is not uniform across all circumstances. It varies dramatically with relative group size. The underlying technical basis for this was reviewed in Chap. 7 and the logically possible consequences for D-S differences were summarized graphically in Figs. 7.4 and 7.5. The implications for empirical analyses also were illustrated in the comparison of the function $y = f(p)$ for G/2 in the graphs in Fig. 5.1. The graph for the White-Latino comparison has the mildest nonlinearity in the scoring of y from p because it has the most balanced group ratio of 68/32. In contrast, the group ratio of 92/8 for the White-Asian comparison is much more imbalanced and the nonlinearity is much more pronounced in the graph for this comparison. The White-Black comparison is in between on both the group ratio of 76/24 and the nonlinearity of the y–p relationship.

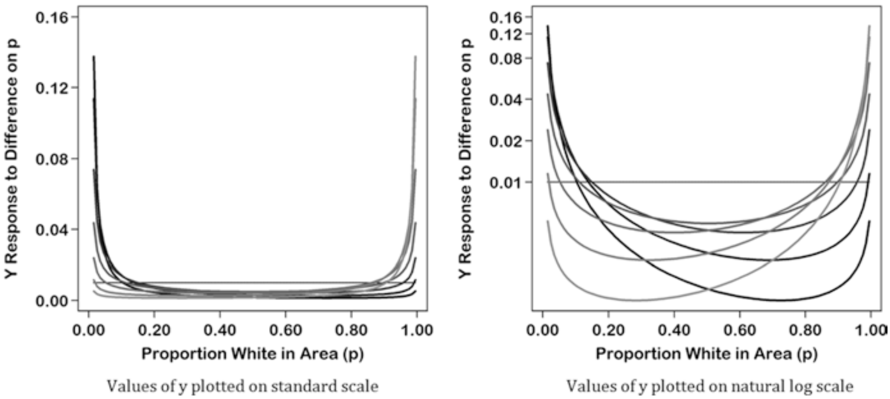

Fig. 12.1 Response of group contact (y) scored for Hutchens R by proportion White in area (p) and selected values for city proportion White (P). Curves reflect the response of Hutchens' R to a change in area proportion White (p) by level of p and selected values of proportion White for the city (P). $\mathrm{y=f(p)} = Q+\left(1-\sqrt{pq/PQ}\right)/\left(p/P-q/Q\right)$. Moving from darker curves to lighter curves, the values of P are: 0.01, 0.05, 0.20, 0.50, 0.80, 0.95, and 0.99. The horizontal line is for reference and reflects the "flat" response of the separation index (S)

Further insight into these patterns can be gained by again considering the behavior of the function $\mathrm{y=f(p)}$ for the Hutchens square root index (R) shown earlier in Fig. 6.6. The y–p relationship for R is continuous and thus lends itself more easily to mathematical and graphical analysis than the y–p relationship for G which is mathematically less tractable because it is based on a percentile transformation. In other key respects, however, R and G are quite similar: the y–p relationships for both R and G have similar nonlinear forms (i.e., both follow a backwards S curve); the nonlinearity in the y–p relationships for both R and G become more pronounced when group size is more imbalanced; and R and G are closely correlated in empirical data sets.

The graph in Fig. 6.6 plots the function $\mathrm{y=f(p)}$ for R over selected values of city racial mix (P). The graph documents that $\mathrm{y=f(p)}$ for R is always a continuously rising backwards S curve. The nonlinear nature of the y–p relationship means that R responds to differences on p in a much different way than S. S registers differences arithmetically according to p's original, "natural" metric. R responds more strongly to differences on p over ranges of p where the curve is relatively "steep" and less strongly to differences on p over ranges of p where the curve is relatively "flat". The graph also reveals that where steep and flat regions of the curve occur over the range of p is strongly conditioned by the racial mix of the city (P). When the two groups are balanced, the y–p curve for R is symmetric and differences between how R and S respond to p are modest. When group size is imbalanced, the y–p curve for R becomes asymmetric and more profoundly nonlinear. Under these conditions, the differences between how R and S respond to p can be dramatic.

This is documented further in Fig. 12.1 which depicts graphically how changes in p are registered as changes in y as scored for R. The graph on the left uses the original metric scoring of y; the graph on the right uses a natural log scale on the y axis. These two graphs make it clear that R responds more strongly to changes in p near the extremes of p and this tendency becomes more asymmetric and more dramatic when the racial composition of the city departs from balance (i.e., 50/50). This establishes the mathematical basis for how and when R (and G and D) can take high values when S is low. Regarding the "how" part of the story, *R (and G and D) can take high values when S is low by responding dramatically to very small differences on p*. Regarding the "when" part of the story, the potential for R to depart from S is greatest when the city racial mix (P) is highly imbalanced.

It is clear from these results that R must be high when S is high, but R can be either high or low when S is low. As noted earlier, this also applies with equal force to G and D. Thus, if S is high, G and D must be high, but when S is low D and G can be either high or low. This is consistent with results presented earlier in Figs. 8.1 and 8.2 which depicted graphs of plotting scores for D against scores for S (and vice versa) for White-Minority segregation comparisons for CBSAs in 1990, 2000, and 2010. It is readily evident here that when S is high, D also is high. But when S is low, values of D vary dramatically; sometimes they are low and sometimes they are high. This raises an obvious question, "When S is low and D (or G or R) is high, is there a compelling reason for assigning sociological importance to the high values of D (or G or R)?" I am not aware of a reason that is (or could be) grounded in the consequences segregation will have for sociologically important group differences in residential outcomes.

The one reason that comes to mind is grounded, not in consequences for group differences in residential outcomes, but more literally in "volume of movement" consequences of policies seeking to redress segregation. High values of D do imply that a large fraction of one group must change area of residence to bring about even distribution. That can be sociologically consequential in policy situations such as school desegregation where students are literally redistributed across schools. Historically, the consequence has been especially important for minority populations who have often disproportionately born the burden of bussing.

The sociological relevance of this volume of movement policy consequence cannot be denied. But its relevance for choosing segregation indices can be discounted for two reasons. The first is that it is "beside the point" because historically literal "volume of movement" policy implications of high values of D have almost always played out in contexts of "prototypical segregation" where values of S also are high. The driving concern behind the policy to redress segregation of course was that racial segregation adversely impacted life chances in education by creating group separation and unequal educational opportunities. The sociological import of D is fundamental and real; but it is beside the point for the issue under discussion because S captures the same concern and thus D does not identify a "life chances" implication that S misses.

The second reason is that the policy implications of a high value of D are much less likely to have practical consequences in situations where D is high and S is low. The basis for saying this is that policy concerns about reducing segregation usually are rooted in concerns about the impact of segregation on inequality in life chances.

When D is high and S is low, groups live together and experience similar neighborhood outcomes. In these situations moving across neighborhoods to achieve exact even distribution will have limited impact on group differences in neighborhood outcomes. Thus, since policies to promote integration are unlikely to be pursued solely for the purpose of achieving exact even distribution without implications for life chances, the policy implications of D's volume of movement interpretation are unlikely to come into play in practice.

So we come back to the issue of why one would focus on values of D, or its technically superior "close cousins" G and R, over values of S. To argue that high values of R, G, and D are sociologically important when S is low, one must advocate two unusual views about the sociological relevance of residential outcomes.

First, one must view differences on p as both very important over certain narrow ranges of p and also much less important over the rest of the logical range of p.

Second, one also must view it as desirable to amplify this differential evaluation of differences on p by greater amounts when a city's racial mix is imbalanced.

To the best of my knowledge, no segregation researcher has articulated a compelling basis for assessing group differences in residential outcomes in this manner. Measurement approaches of this sort are not used when group differences on other socioeconomic outcomes such as education, occupation, and income are studied. So it is not obvious why such an approach would be seen as attractive when studying group differences in residential attainments relating to area racial mix and group contact.

To be clear, I am not arguing that G, D, and R should not be used to measure uneven distribution. Researchers can be interested in uneven distribution for many different reasons. In some cases they may determine that one of these indices is the best choice to serve the needs of a particular study. As just noted, these measures might be defensible choices if one is interested in certain consequences of segregation in relation to a social policy such as bringing about school integration where D could be seen as superior to S in signaling how much potential "social disruption" will be involved in achieving segregation. This would be sociologically important regardless of whether movement to achieve integration brings about big changes in racial proportions in different schools.

At the same time, I argue against the prevailing view that G, D, and R should be seen as the best available choices or even appropriate choices for serving most research interests. Personally, I am interested in measures of uneven distribution that are well suited for signaling the consequences segregation may have for group differences in residential outcomes that are both meaningful to individuals and households and relevant for life chances associated with residential outcomes. Given this focus, I am drawn to S because, among popular indices of uneven distribution, it registers residential outcomes that have clear and compelling implications for racial differences in residential outcomes. Focusing on the example of White-Black segregation, I know that when S is high, Whites and Blacks are residentially separated and are living apart from each other in neighborhoods that differ markedly on racial mix. I also know that there is a clear structural potential for the neighborhoods that Whites and Blacks live in to differ in other respects as well (e.g., amenities, crime, poverty, exposure to social problems, etc.). Furthermore, I know that when S is high, R, G, and D also will be high and as a result knowledge of their values adds limited additional information that is relevant to my concerns.

When S is low, I know that Whites and Blacks are not residentially separated; instead, they are living together in the same neighborhoods. Because of this, I additionally, know that, all else equal, the possibility for Whites and Blacks to experience fundamentally different neighborhood outcomes on other dimensions (e.g., amenities, crime, poverty, etc.) is logically constrained because people who reside in the same neighborhoods necessarily experience the same neighborhood outcomes. If S is exactly zero, Whites and Blacks cannot on average experience different neighborhood outcomes based on race alone. As S takes higher values, the logical potential increases for Whites and Blacks to differ on residential outcomes based on race alone.

Of course S does not reflect all relevant aspects of race differences in residential outcomes by itself. Other characteristics such as income can interact with race and influence race differences in neighborhood outcomes. For example, a low-to-moderate level of S, say 15–20, could result because Whites and Blacks have substantial contact across all income strata. Alternatively, the same level of S may result due to Blacks having higher levels of contact with low income Whites that offset Blacks having lower levels of contact with high income Whites. All else equal, the second scenario will be associated with greater White-Black differences in exposure to poverty and low income. This does not change the fundamental implications of high S versus low S situations. It merely acknowledges that consequences of racial differences in residential distributions are not necessarily simple.

What can be said about White-Black neighborhood differences when R, G, and D take high values? This is much harder to pin down. When S is high, R, G, and D will be high. But the reverse is not true. S can be low when R, G, and D are high, particularly when group size is highly imbalanced. When this occurs, the high values of R, G, and D do not provide a basis on their own for offering conclusions regarding White-Black differences in residential outcomes. This monograph has established that, as a matter of arithmetic, when R, G, and D have high values when S is low it is because these indices are responding strongly to small quantitative differences on neighborhood racial mix (p) over relatively narrow ranges of p. This provides little basis for speculating about the consequences of uneven distribution for residential differences. This is made worse by the fact the "crucial" range of p varies from city-to-city depending on racial mix. For my research interests, this index behavior is not attractive.

What about Theil's H which I have not yet discussed? Like S, Theil's H usually receives favorable treatment in methodological studies of segregation indices but has been used less frequently than D in empirical studies. For purposes of this discussion, H falls between S and indices rooted in the segregation curve (G, D, and R). Figures 6.5 and 6.6 introduced earlier show that the function $y = f(p)$ for H is similar to the same function for R in several respects. The nonlinearity in the y–p relationship is similar in form, but the magnitude of the departure from nonlinearity is much less

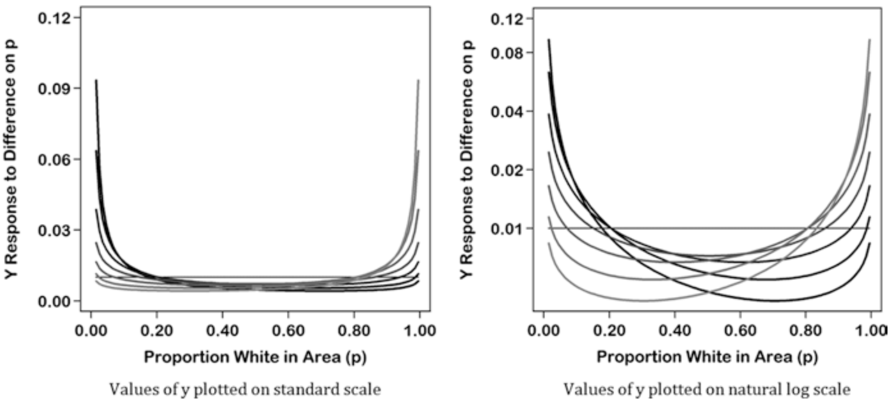

Fig. 12.2 Response of group contact (y) scored for Theil's H by proportion White in area (p) and selected values for city proportion White (P) (Curves reflect the response of Theil's H to a change in area proportion White (p) by the level of p and selected values of proportion White for the city (P). $y = f(p) = Q+[(E-e)/E]/(p/P-q/Q)$. Moving from darker curves to lighter curves, the values of P are: 0.01, 0.05, 0.20, 0.50, 0.80, 0.95, and 0.99. The horizontal line is for reference and reflects the "flat" response of the separation index (S))

dramatic and the degree to which it varies with city racial mix (P) also is less dramatic. So, in comparison with S, H has similar tendencies as R, but in milder degree. Figures 12.1 and 12.2 document that H has similar tendencies to R in terms of how changes in area racial composition (p) translate into changes in residential outcomes (y). The figures document similarity in the form of the response. One must note the values on the "Y" scale in the figures to see that the responses by H are milder than the responses by R.

What distinguishes H from R is this. H is rooted in a conception of uneven distribution that draws on the information-theoretic notion of relative deviation from expected entropy. Individuals who find this conceptual approach attractive may accordingly prefer H. But like G, D, and R, H is differentially sensitive to changes in p over relatively narrow ranges and the relevant ranges vary with city racial mix. I am not aware of a basis for prizing this quality and leave it for others to make the case.

12.1 An Example Analysis of Segregation and Exposure to Neighborhood Poverty

I conclude this chapter by presenting an empirical analysis intended to speak to the issues reviewed here in a more "concrete" way. The issue I explore is whether high scores for measures of uneven distribution carry implications for racial stratification on residential and neighborhood outcomes. To investigate this, I used block group data from Summary File 3 of the 2000 census and computed scores for the indices of uneven distribution discussed in this section – specifically, G, D, R, H, and S – for Core Based Statistical Areas (CBSAs). I computed scores for White-Minority comparisons – specifically, White-Black, White-Latino, and White-Asian – using data for non-Hispanics for Whites, Blacks, and Asians. For economy of presentation, I focus on the results for D and S, noting that index scores for G and R correlate closely with scores for D and noting that scores for H takes an intermediate position between scores for D and S.

I additionally calculated group-specific exposure to neighborhood poverty based on poverty rates for neighborhoods (calculated using data for the total population) and also group-specific exposure to neighborhood income rank (percentile standing based on the city-specific income distribution for the total population). I then calculated the White-Black, White-Latino, and White-Asian differences on exposure to neighborhood poverty and exposure to neighborhood income rank. The differences were constructed so positive scores indicated White advantage.[2] I restricted the analysis to CBSAs where the minority group in the segregation comparison had a population of 1,500 or more and where the number of block-groups was adequate for assessing segregation patterns.[3] This resulted in 1,455 CBSA-group comparisons; 571 White-Black comparisons, 605 White-Latino comparisons, and 279 White-Asian comparisons.

I then addressed the following question; "Do scores on D and S for White-Minority segregation carry similar or different implications for White-Minority differences on these residential inequality outcomes?" To a certain extent they do carry similar implications, at least in this analysis, as the scores for both D and S are positively associated with White-Minority inequality on exposure to poverty and neighborhood income rank. The White-Minority difference in exposure to neighborhood poverty (coded so higher scores indicate White advantage) is correlated with D at 0.645 ($r^2 = 0.417$) and with S at 0.715 ($r^2 = 0.512$). The White-Minority difference in exposure to neighborhood income rank (also coded so higher scores indicate White advantage) is correlated with D at 0.619 ($r^2 = 0.383$) and with S at 0.702 ($r^2 = 0.494$). These results indicate that S provides a better signal for when segregation carries implications for racial inequality in neighborhood outcomes. But in this analysis D is not awful for this purpose. One reason for this is that scores on D and S are often concordant. The story changes substantially when attention is focused on cases where D and S are discordant.

Probing more deeply into the data lends additional support to the idea that S is more attractive than D for the purpose of signaling when it is likely that segregation is associated with White-Minority inequality in residential outcomes. To do this, I coded each White-Minority segregation comparison on the consistency of D and

[2] The poverty difference is minority exposure minus White exposure. Positive scores indicate that the minority group is exposed to higher levels of neighborhood poverty than White. The income rank difference is White exposure minus minority exposure with positive scores indicating Whites are exposed to neighborhoods that rank higher on income (White advantage).

[3] The cut-off was at least 10 populated block groups. I replicated the results using cut-off values of 15 and 20 block groups. The results were the same.

S. Recall that D can take high values when S is low. Based on this, I classified outcomes on D and S into four categories. The first is a baseline category of "concordant" as occurs in prototypical segregation where displacement from even distribution is substantially polarized. The other three categories capture D exceeding S by increasingly large amounts. Holding D constant, distribution across the three categories of D-S discrepancy indicates variation in the extent to which displacement from uneven distribution is dispersed and produces lower levels of group separation and neighborhood polarization.

I then estimated the regression of the White-Minority difference on exposure to neighborhood poverty on D and the three categories of D-S discrepancy. The multiple R-square for the regression was 0.502 compared to 0.417 when using D alone. This indicates that knowing that D is discordant from S added to the ability to predict the White-Minority difference in exposure to neighborhood poverty over what could be predicted from knowledge of D alone. As expected, the pattern of the effects indicated that when D was high in relation to S, the White-Minority difference in exposure to neighborhood poverty was lower (all effects were statistically significant at $p < 0.001$). The impact of the largest D-S discrepancy category was −4.3 which is clearly large in relation to the value of 6.9 for interquartile range of 6.9 for the dependent variable.

I obtained similar results for the regression predicting the White-Minority difference on neighborhood income rank. The multiple R-square for the regression using D and the three categories of D-S discrepancy as predictors was 0.483 compared to 0.383 when using D alone. The results indicated that knowing that D was high in relation to S added to the ability to predict White-Minority difference in exposure to neighborhood income rank over what could be predicted from knowledge of D alone (all effects statistically significant at $p < 0.001$). As expected, discrepant categories had lower levels of White-Black inequality on income rank and the impact of the largest D-S discrepancy category was −4.0 which is clearly large when compared to the value of the interquartile range of 5.8 for the dependent variable.

I next estimated parallel regressions where S and categories of D-S discrepancy were used to predict White-Minority disadvantage in exposure to poverty and neighborhood income rank. The results were different and quite revealing. For the regression of the White-Minority difference on exposure to neighborhood poverty the multiple R-square for the regression was 0.529 compared to 0.512 when using S alone. This signals that knowing D was high relative to S increased the ability to predict the White-Minority difference in exposure to neighborhood poverty by only a small amount over what could be predicted from knowledge of S alone. The coefficients for the three categories of discrepancy were all statistically significant (all at $p < 0.001$) but impacts were more modest than in the parallel analysis focusing on D as the largest effect here was 1.9 which was less than half the size of the largest effect of −4.3 seen in the parallel analysis focusing on D.

I found similar results for the regression predicting the White-Minority difference on neighborhood income rank. The multiple R-square for the regression using S and the three categories of D-S discrepancy as predictors was 0.508 compared to 0.494

when using S alone. So, again, knowing that D was high relative to S increased ability to predict White-Minority difference in exposure to neighborhood income rank by only a small amount over what could be achieved from knowledge of S alone. The effect coefficients for D-S discrepancy were statistically significant (all at $p < 0.001$), but effects were small compared to the parallel analysis focusing on D as the largest effect was 1.5 compared to −4.0 in the parallel analysis focusing on D.

I draw the following conclusions based on these analyses. *In comparison with the dissimilarity index (D), the separation index (S) speaks more directly to the question of whether uneven distribution is associated with group differences in residential outcomes such as income and poverty*. This is because S registers whether or not groups live separately in neighborhoods that are polarized on racial mix. This is a logical precondition for White-Minority differences on neighborhood-level stratification outcomes such as socioeconomic standing. D can take high values when groups live together in neighborhoods with similar racial composition and the logical potential for group differences in neighborhood outcomes is limited. Accordingly, S is the stronger predictor of White-Minority differences on neighborhood-based stratification outcomes such as indicators of neighborhood socioeconomic standing. Not surprisingly, I obtained parallel findings when contrasting S with the gini index (G) and the Hutchens square root index (R). This is because these two measures correlate closely with D and can take high values when groups are not residentially separated.

In view of these results, I suggest that researchers always examine multiple indices and give particularly close attention to cases where S and D (or its close correlates) diverge. Such cases involve uneven distribution without residential separation and neighborhood polarization. These situations are likely to be fundamentally different from cases of prototypical segregation where D and S both take high values. Specifically, group inequality in neighborhood-based residential outcomes is likely to be higher under a high level of prototypical segregation (i.e., a high-D, high-S combination) and lower under a high level of "displacement without separation" (i.e., a high-D, low-S combination). Personally, I am primarily interested in those aspects of segregation that have greater potential consequences for stratification in neighborhood outcomes and associated life chances. So I pay closer attention to S when S and D disagree.

Chapter 13
Relevance of Individual-Level Residential Outcomes for Segregation Theory

The residential outcomes that give rise to segregation index scores can be assessed in terms of whether they are relevant for investigating different theories of segregation dynamics. In the final analysis, theories of segregation must reckon with the micro-level dynamics that produce the residential patterns that aggregate indices summarize. It is easy to see how the residential outcome registered by S – namely, area racial mix (p) – is relevant for theories of residential attainment dynamics. For example, Lieberson advanced the hypothesis that segregation arises in part when Whites strive to maintain high levels of same-group contact and avoid more than incidental levels of contact with minorities (Lieberson 1980, 1981: 75; Lieberson and Carter 1982). Combining this hypothesis with the assumption that Whites have greater ability to influence residential dynamics leads to straightforward predictions regarding how S will vary when city racial composition varies over time or across cities. For example, the hypothesis that discrimination by Whites serves to keep White contact with Whites from falling below fairly high levels (say 85 % or higher) leads to the prediction that S will vary as a positive, nonlinear function of proportion Black in the city.[1]

The implications for D, G, R, and H are much more complicated and indirect. I not aware of any theories that suggest Whites may specifically strive to attain or avoid particular levels on the residential outcomes that determine the values of these indices. Figure 5.1 introduced earlier shows D, G, R, and H score values of p differently across cities depending on the racial mix of the city. For present discussion, consider D when formulated as a difference of means when neighborhoods are

[1] In a city where proportion Black is very low – say 1–5 %, S can be low since Blacks can experience high levels of contact with Whites without causing problems for White's desires to have limited contact with Blacks and high contact with Whites. This changes when proportion Black increases. In order to maintain White contact with Whites (P_{WW}) at 0.90 or higher as proportion Black in the city increases, Black contact with Whites (P_{BW}) must fall. This will cause S to increase since, in the two-group case, $S = P_{WW} - P_{BW}$. The relationship will be nonlinear. Initially, S will increase rapidly as proportion Black in the city (Q) increases; then the rate of increase will decline.

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The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_13

scored as either 0 or 1 depending on whether p for the area exceeds P for city. In this formulation, a neighborhood where p is 90 would be scored 1 in a lower-P city such as Birmingham and 0 in a higher-P city such as Minneapolis. The literature on race and residential dynamics gives no basis for expecting residential outcomes to revolve around these 0-1 scores instead of the original values of p. In contrast, the literature does provide a basis for expecting p scored in its natural metric to predict residential dynamics; specifically, Lieberson hypothesizes that Whites in all cities will prefer residential outcomes of $p=95$ over $p=85$ and $p=85$ over $p=75$, and so on. Thus, one can plausibly argue that S registers White-Black differences on residential outcomes that are meaningful in residential attainment dynamics. I know of no basis for making this kind of argument for the residential outcomes registered by D, G, R, or H.

With this in mind, it is interesting to note that the results presented earlier in Table 10.1 indicate that the impact of relative minority size on segregation is much greater in the analysis of S than in the analyses for D. For example, cities that are at 4 % and 25 % Black are predicted to differ by 24.0 points on S but only 1.6 points on D.[2] Furthermore, the effects of relative minority size on patterns of residential contact relating to S are more sensible in my view. For S, both White and Black contact with Whites declines as relative minority size increases, but the rate of decline is greater for Blacks thus leading to higher levels of group separation as minority size increases. This pattern is consistent with the Lieberson hypothesis. Contact with Whites as registered by D *increases* for both Whites and Blacks as relative minority size increases. These effects do not lend themselves to ready substantive interpretation and in any event the pattern has minimal implications for city-level variation in segregation across cities.

I conclude this discussion by noting again that it is unproductive to claim that any one segregation index is best for all circumstances and purposes. Accordingly, I advocate the following position. Ideally, researchers should be able to offer a sound justification for why a particular index is an appropriate choice for the substantive question(s) they are investigating. My comments endorsing S are rooted in a particular set of research interests. I am interested in segregation as it relates to racial stratification and socioeconomic inequality and thus I assign priority to the implications segregation may have for group differences in life chances linked with residential outcomes. From this vantage point, I believe S registers outcomes that are meaningful to individuals and households and relevant to residential attainment dynamics that produce aggregate segregation. But *I do not argue that this is the only valid vantage point from which to advocate the use of a particular segregation index*. Others may offer good justifications for viewing other indices as valid and attractive either for addressing specific research questions that interest them or on various practical grounds. For example, while I have expressed reservations about G and D based on the unusual way they register group differences in area racial mix, I expect many researchers will continue to use them, especially D, in order to main-

[2] These values on relative minority size translate to 0.14 and 0.50 on the square root of minority proportion. The difference of 0.34 is multiplied by the effect coefficients of 4.79 and 70.51 in the equations for D and S, respectively. This translates into $1.63 = 0.34(4.79)$ and $23.97 = 0.34(70.51)$.

tain continuity with previous research and because they find D's aggregate-level "volume of movement" interpretation to be attractive.

I conclude with a practical observation. It is that sometimes index choice is not that important and it is easy enough to check to determine whether this is the case. Recall that the analyses reviewed in Chap. 6 provided evidence that popular indices of uneven distribution correlated at very high levels ($r^2 \geq 0.85$) when group size was not highly imbalanced (e.g., when $0.10 \leq P \leq 0.90$). It is easy to see if this welcome situation prevails; examine the correlation of scores for D and S and check to see if results differ using these two indices. When these two measures correlate closely, all popular measures correlate closely. Accordingly, if the correlation is high and the key findings do not differ by index, it is safe to conclude that index choice is not an important factor in this situation.

When D and S are not highly correlated one must give the issue of index choice more attention. To the extent possible one should provide a sound justification for choosing to use a particular index. Additionally, it would be wise to check for and acknowledge whether key empirical findings and substantive conclusions vary depending on index choice.

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Chapter 14
Index Bias and Current Practices

Standard versions of indices of uneven distribution take their minimum value of zero only under the condition of exact even distribution. Most segregation researchers and consumers of segregation studies are habituated to accepting this benchmark for social integration. On reflection, however, it is an unusual point of reference for assessing segregation. For one thing, exact even distribution usually is not logically possible because individuals, families, and households cannot be distributed in fractional parts as almost always is needed to achieve exact even distribution. The resulting departure from uneven distribution is likely to be negligible when segregation is being assessed for broad group comparison using relatively large spatial units such as census tracts. But it will be non-negligible when measuring segregation for small groups and/or when using small spatial units such as blocks.

A second reason for viewing exact even distribution as an unusual reference point is that it does not correspond to the notion that race (or more generally "group membership") is statistically unrelated to neighborhood of residence in keeping with the usual "baseline" null hypothesis adopted in studies seeking to assess quantitative group disparities on socioeconomic outcomes. To the contrary, *exact even distribution is an unexpected outcome under a model of random distribution wherein race and neighborhood are statistically independent*. Thus the occurrence of exact even distribution can signal that race is systematically associated with residence through some kind of structured social dynamic (e.g., a group quota allocation process).

As a consequence of these two factors, scores for all popular indices of uneven distribution are inherently subject to upward bias in the following sense; they have positive expected values when residential distributions of individuals and households are random and thus standard indices will signal that segregation exists even when there is no significant statistical association between group membership (e.g., race) and residential location.

Index bias is a concern for several reasons. One is that, while bias is sometimes negligible and can safely be ignored; bias can be and often is non-negligible. When

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_14

this is the case, bias can distort index scores and result in misleading assessments of the level of segregation in a particular case as well as misleading assessments of how the case in question compares with other cases including the same city at another point in time. A second reason for concern about bias is that it varies in complex ways that can make it difficult for researchers to diagnose its presence and deal with its undesirable consequences. A third reason for concern is that, because researchers are aware that bias can render index scores untrustworthy, they guard against it by foregoing many kinds of segregation studies that they would otherwise undertake if index scores could be trusted.

The current state of affairs presents difficult challenges to researchers. They want to view index bias as negligible for all cases in a given study so they can set aside concerns that assessments of segregation are untrustworthy when examining values of individual cases at a point in time, or when comparing values for a case over time, or when comparing values across cases. Unfortunately, it is not always safe to assume that scores can be trusted. In response to this situation, researchers routinely adopt multiple ad hoc strategies with the goal of avoiding and/or "dealing with" the undesirable consequences of index bias.

A few methodological studies have advocated dealing with bias directly at the point of measurement by adjusting observed scores to remove the impact of bias and obtain unbiased scores (e.g., Winship 1977; Carrington and Troske 1997; Allen et al. 2009; Mazza and Punzo 2015). To date, however, few researchers have embraced such strategies. The main reason for this appears to be that the resulting index scores are complicated to explain and interpret and the best approaches to implementing the adjustments are technically and computationally demanding.

What most researchers do instead is adopt "indirect" rather than "direct" approaches to dealing with index bias. That is, they measure segregation using "standard" (i.e., biased) versions of indices and then they adopt a variety of strategies to cope with the problem that scores may be differentially distorted by bias. Unfortunately, the strategies researchers use are a patchwork of informal, ad hoc practices. They are well-intentioned, but they are subject to criticism on multiple counts. The most important criticism is that the prevailing practices do not directly deal with index bias at the point of measurement for individual cases. Consequently, index scores for individual cases that are suspect of being distorted by bias are never "corrected" and in most studies these cases are not even identified. Consequently, index scores for individual cases affected by bias remain untrustworthy and cannot be safely used for even elementary descriptive tasks such as: assessing the level of segregation for individual cities on a case-by-case basis, making direct comparisons of segregation between any two cases, assessing differences in segregation between different group comparisons for a single city, or following a single case over time.

There is no sugar-coating the current situation. *Prevailing practices for dealing with bias do not yield trustworthy segregation index scores for individual cases*. At one level this is not surprising because the strategies researchers use to cope with index bias do not adopt the goal of obtaining trustworthy index scores for individual cases that have only a negligible amount of bias (e.g., less than 2 points). They instead employ a two-pronged strategy. They first try to screen out cases most likely to be distorted by severe levels of bias. They then try to "work around" the problem of moderate levels of bias for many of the "surviving" cases. The main strategies researchers use in pursuing this approach are informal "rule-of-thumb" practices for screening cases from the analysis and/or minimizing the undesirable consequences of cases where bias is likely to be a non-trivial concern. Common strategies for dealing with bias include the following:

- assess segregation using larger spatial units such as census tracts instead of smaller units such as blocks;
- focus on comparisons of broad group populations and avoid comparisons involving smaller subgroups within populations – for example, compare all Whites with all Blacks instead of comparing low-income Whites with low-income Blacks;
- apply a variety of ad hoc sample restrictions to exclude potentially problematic cases in the full data set from the subset of cases used for the final analyses; and
- weight cases in the analysis data set differentially in hopes of minimizing the influence that potentially problematic cases may exert on results.

These strategies and ones similar to them are widely used primarily because they are easy to implement. More rigorous alternative approaches are available but are rarely adopted due partly because they are less well known but also because they are more complex and demanding. I view the current state of affairs with concern. First, as I noted above, the practices researchers use do not improve the measurement of index scores at the level of individual cases. Second, the "protective" practices are applied inconsistently and in patchwork fashion. Third, there is little formal methodological work to show that the practices being used are in fact effective in eliminating and/or minimizing the undesirable impact of untrustworthy index scores.

Finally, and perhaps most importantly, I worry that the "cures" adopted for dealing with index bias have undesirable side effects that in some cases may be "as bad as the disease." In particular, prevailing practices restrict the scope of segregation studies and constrain research designs in nonrandom and ultimately undesirable ways. They shift study designs toward investigating a narrower set of questions that can be addressed using a smaller subset of cases and group comparisons where standard index scores are viewed as more trustworthy.

Obviously, this is not the situation researchers want. They would prefer to have trustworthy index scores for as many cases as possible and for as wide a range of group comparisons and research situations as possible. Happily, the difference of means framework I introduce in this monograph makes it possible to take a major step toward this goal. Working from within this framework I am able to develop refined versions of widely used indices of uneven distribution to correct the problem of index bias directly at the point of measurement. The new measures are attractive on several counts. First, they are not exotic or dramatically different. They are refined versions of popular indices and researchers do not have to adopt unfamiliar approaches to measuring uneven distribution. Second, the refinements that yield unbiased versions of indices involve minor adjustments in index calculations that are simple and easy to implement but yet very effective in providing robust protection against index bias over a broad range of conditions and group comparisons. Third, the technical basis for achieving unbiased index scores allows researchers to continue to invoke familiar substantive interpretations of popular indices with only subtle changes. Finally, the new measures can be used at little cost or risk. When bias in fact is negligible, as sometimes is the case, scores of unbiased versions of indices track scores of standard versions very closely and the two versions will yield essentially identical results. The scores for standard and unbiased versions of indices differ only when bias is non-negligible and scores for standard versions of indices do not yield trustworthy assessments of uneven distribution.

Based on these points I suggest that the unbiased versions of indices that I introduce in this monograph provide valuable new alternatives for research. They can be used interchangeably with standard versions of indices in any situation where standard index scores can be trusted and results will be the same. But, more importantly, the unbiased versions can be used in many additional situations where standard indices cannot be safely used. Thus, the unbiased versions of index scores I introduce here expand the potential scope of segregation studies to include group comparisons and study situations that researchers currently would avoid.

I devote the remainder of this chapter to the task of "setting the stage" for introducing the unbiased versions of popular indices. I serve this goal by first reviewing the general problem of index bias. I then review the prevailing practices researchers use to try to minimize the undesirable effects of index bias and note my concerns about these practices focusing on technical questions of their efficacy considered narrowly and also on the insidious impact of these practices on segregation research more broadly. Finally, I review options that have been previously suggested for how bias might be addressed directly at the point of measurement and consider why they have not gained wider adoption. The existence of this chapter indicates that I believe it is worthwhile to review these topics in some detail. However, I will not be surprised and will take no offense if some readers choose to skip forward to Chap. 15 where I outline the basis for formulating unbiased versions of popular indices and Chap. 16 where I review their behavior in empirical applications. I turn now to reviewing basic issues and current practices.

14.1 Overview of the Issue of Index Bias

The dissimilarity index (D) is the most widely used measure of uneven distribution so it comes as no surprise that it has received especially close scrutiny on the issue of index bias. Taeuber and Taeuber (1965) provided a thoughtful early discussion of the issue in their appendix chapter reviewing issues in segregation measurement. They noted that zero, the value of D that signals integration conceived as exact even distribution, does not obtain under random distribution and furthermore is usually logically impossible even under strategic, purposive assignment because individuals and households cannot be assigned in fractional parts (1965: 231–235).[1] Later methodological studies characterized D's positive expected value under random assignment (i.e., $\mathrm{E}\left[\mathrm{D}\right]>0$) as "bias" and raised awareness that bias in D varies in complex ways that can make scores for D problematic in many situations (e.g., Cortese et al. 1976; Winship 1977). The issue has now received regular attention for four decades and a large literature has grown with contributions from many methodological studies that have considered the nature of index bias, its practical consequences, and possible approaches for diagnosing and dealing with it (e.g., Taeuber and Taeuber 1976; Cortese et al. 1976, 1978; Blau 1977; Winship 1977, 1978; Massey 1978; Falk et al. 1978; Farley and Johnson 1985; Boisso et al. 1994; Carrington and Troske 1997; Ransom 2000; Allen et al. 2009; Mazza and Punzo 2015).

Consensus exists on many important points relating to certain technical aspects of index bias. Several key understandings trace to Winship's (1977) influential early analysis of the bias behavior of D and S. Of particular note, Winship introduced two analytic formulas for calculating the expected value of D (denoted by E[D]) under random distribution. Both formulas are based on a formal model of random distribution of households from two groups over areas of constant population size (t_i). He termed one formula "exact" because it implements detailed calculations based on the binomial probability distribution and can be applied at both small and large values of area population size. He termed the other formula an "approximation" because it draws on simpler calculations that yield satisfactory results when area population size is <u>not small</u> (i.e., when $\mathrm{t}_\mathrm{i}\geq 25$). Examining the approximation formula, $\mathrm{E}\left[\mathrm{D}\right]=1/\sqrt{2\pi t_i PQ}$, clarifies how E[D] varies over study design and demographic conditions. Specifically, it reveals that two terms –area population size (t_i) and the relative size of the reference group (P) – determine how the value of E[D] varies with city racial composition and with study design (i.e., the size of spatial units used in assessing segregation).

The first term, the area pairwise population count (t_i), has an inverse relationship with E[D]; all else equal, E[D] declines as t_i increases. This relationship can provide a rationale for why research moved from once commonly assessing segregation using small areas such as blocks to more often using larger areas such as census tracts. It also provides a rationale for avoiding group comparisons which involve small combined populations. The practice provides a measure of protection against index bias, but this comes with substantial costs. It eliminates the option of investigating segregation in smaller cities and communities where tracts are too big to capture segregation patterns. It also eliminates the option of studying segregation involving small groups and subpopulations.

[1] The latter point is not widely appreciated but deserves greater attention because individuals typically are imbedded in a family or household that sociologically cannot be viewed as divisible and in most cases will be racially homogenous. Accordingly, Winship (1977) advocates assessing segregation for households rather than individuals because individuals within households are not "statistically independent".

The second key term in Winship's approximation formula for E[D] is PQ, the product of group population proportions. The value of this term is controlled by P – the pairwise proportion of the reference group in the combined city-wide population of the two groups. P in turn determines Q, the pairwise proportion of the comparison group, based on $Q = 1 - P$, and so also determines the value of PQ. The value of PQ has an inverse relationship with E[D]; all else equal, E[D] is lower when PQ is higher. The maximum for PQ occurs when the two groups are equal in size ($P = Q = 0.5$). So bias in D (E[D]) grows larger as groups become more imbalanced in size (i.e., as P departs from 0.5). This relationship can provide a rationale for excluding cases from analysis when one group in the segregation comparison is small in relative size. Again, the practice provides protection against index bias, but it comes at a cost; it eliminates the option of investigating segregation in communities where groups are imbalanced in size. Thus, for example, it precludes the possibility of investigating segregation in the initial stages of a new group's entry into a residential system since group size will in most cases be highly imbalanced.

Winship assessed the impact of area population size (t_i) and city racial composition (P) on index bias (E[D]) by tabulating the values of E[D] obtained from analytic formulas over varying combinations of t_i and P. The results he reported showed that area size and city racial composition have complex, non-linear, non-additive effects on E[D]. Later studies confirm his findings with similar results obtained by analytic and simulation exercises investigating the issue of index bias (e.g., Carrington and Troske 1997; Allen et al. 2009; Mazza and Punzo 2015). I summarize the most important findings of these studies as follows.

- D is subject to bias under all conditions; that is, the expected value of D under random distribution always is greater than zero (e.g., $E[D] > 0$).

  Significantly, the positive value of E[D] truncates the range of D in empirical analyses by setting a "floor" for the minimum value below which D is unlikely to fall in the absence of exceptional circumstances (e.g., assignment of individuals and households by quota and in fractional parts).
- In many situations the index value of 0, which obtains only under exact even distribution, is not logically possible due to the integer nature of population counts and the non-independence of individuals in families and households.[2]
- The magnitude of bias for D varies inversely with the population size of areal units (t_i).

  Other things equal, E[D] grows smaller as area population size grows larger; it moves toward being negligible when area population size is very large.
- The magnitude of bias varies inversely with pairwise balance in city racial composition.

[2] By non-independence of individuals in families and households I mean that attaining exact even distribution would require that individuals living together within families and households would be separated and distributed independently across areas.

- Other things equal, E[D] grows larger as city racial composition becomes more imbalanced. More exactly, E[D] is lowest when $P = Q = 0.5$ and increases at an increasing rate as P departs further from 0.5.
- The joint impact of area population size (t_i) and city racial composition (P) on the magnitude of bias is complex. Specifically, the effects of each factor are non-additive and nonlinear such that a bias-promoting change in one factor amplifies the other factor's impact on bias.

The most important conclusion to be drawn from these studies is more general and deserves to be separated from the others.

Bias can be non-trivial in magnitude in many cases and it can vary greatly in magnitude from case to case including different cities, different group comparisons, or a given city-group comparison tracked over time.

Consequently, bias can complicate measurement and potentially lead researchers to draw incorrect conclusions about the levels and patterns of variation in uneven distribution across group comparison, across cities, and over time.

Significantly, all of the points just listed apply to all popular indices of uneven distribution except one. More specifically, the points listed above apply to the gini index (G), the Atkinson index (A), the Hutchens square root index (R), and the Theil entropy index (H). One popular measure – the separation index (S) – is an exception; index bias is less of a problem for this index than for any other widely used index of uneven distribution.

Bias for S is smaller in magnitude than for any other popular index. In addition, variation in bias for S across cases is less complicated than for any other popular index. The major reason for this is that bias for S is determined by just one factor – area population size (t_i) – with E[S] being given by the simple calculation $E[S] = 1/t_i$ (Winship 1977). Thus, in contrast to other indices, bias for S does not vary with city racial composition (P). Accordingly, analyses reported in Chap. 16 show that the separation index (S) exhibits a lower level of bias than other indices under all conditions and especially when city racial composition is imbalanced. Indeed, the levels of bias for the separation index (S) are so much lower and so much less complicated than for other indices, this alone could be a compelling reason to always consider using S in empirical analyses. That said, E[S] is never zero and bias can render scores for S problematic in some extreme circumstances. Consequently, while using S to measure uneven distribution can go a long way to protecting against the potential distorting impact of index bias, using S cannot in itself guarantee that bias does not adversely affect index scores.

14.1.1 Effective Neighborhood Size (ENS): A Further Complication

Previous methodological studies provide valuable insights about the nature of index bias. Unfortunately, however, these insights do not necessarily provide an adequate basis for diagnosing the presence of bias in empirical studies. The reason for this is that the expected values of index scores (i.e., E[•]) under random assignment are more complicated in empirical studies than in analytic studies. Three factors pose difficulties for researchers seeking to assess and deal with index bias in empirical studies.

- Neighborhood size often varies substantially across spatial units.
- The non-negligible presence of other groups not included in the segregation comparison often varies markedly across cases.
- The extent to which other groups not included in the segregation comparison co-reside with the two groups in the comparison often varies across cases.

Each of these three factors complicates bias because they affect the value of t_i which, as noted above, plays a central role in determining the expected values of indices under random assignment (i.e., E[•]). In empirical studies area population size (t_i) can be highly variable and this makes its impact on E[•] more difficult to establish. As a rule of thumb, t_i varies in predictable ways across the kind of areal units used in measuring segregation. For example, t_i is lower when using census blocks compared to census tracts. So, all else equal, one can safely expect bias will be a greater concern for blocks than for tracts. But there is a further complication in empirical studies; the population size of the areal unit used (e.g., tracts) can vary considerably across units.

The exact impact of variation in area size (t_i) on bias can be complicated to assess for a given case. But it is easy to grasp that it can be important because empirical distributions of population counts for areas often span a wide range and tend to be skewed right with unusual outliers. Variation in area population size occurs for many reasons including: differences between areas with high-density apartment buildings vs. areas with low-density, single-family detached housing; the presence of non-institutional group quarters such as work camps, college dorms, and military barracks, convents, etc.; and the presence of institutional group quarters such as prisons, facilities for the elderly and disabled, and other institutions. As a result, it can be inappropriate to use a single value of area population size (t_i) when estimating E[D] by analytic formulas. As an alternative, one could extend the formulas for E[D] to take account of variation in area size. Another alternative is to adopt computation-intensive methods such as estimating the sampling distribution of E[D] under random distribution using city- and comparison-specific bootstrap simulations as advocated by Carrington and Troske (1997) and Allen et al. (2009).

Unfortunately, all of these options introduce complexity and substantial computational burdens and so are unlikely to be widely adopted by researchers.[3]

The next complication arises when other groups not in the segregation comparison are present in the city population. To see this, first note that, strictly speaking, it is not area population size per se that is relevant to index bias; it is the "pairwise" population count in the area. In view of this I introduce the term "effective neighborhood size" (ENS) to refer to the value of the combined population counts for the two groups in the comparison in the areal unit. The value of effective neighborhood size (ENS) sometimes corresponds to the value of area population size, but ENS is conceptually distinct and can depart from overall area population size. Indeed, ENS can take dramatically different values from overall area population size when the combined relative size of other groups not in the segregation comparison is large.[4]

Effective neighborhood size (ENS) equals area population size (t) only when the city population consists of just the two groups in the segregation comparison. This situation is often assumed in methodological studies to simplify analysis, but the assumption is untenable in empirical studies where the presence of other groups in the population can cause the value of effective neighborhood size (ENS) to depart dramatically from overall area population size. Under random distribution for *all* groups ENS will be smaller than area population size and estimates of index bias based on overall area population size will be too low. This can cause commonly used "rules-of-thumb" for protecting against bias to fail. For example, researchers may use census tracts as the spatial units for assessing segregation in hopes that bias will be negligible because tract populations are large. But ENS can still be low even when using census tracts if the two groups in the segregation comparison are both small. For example, this might occur when investigating the segregation of Asian subgroups (e.g., the Chinese and Korean subpopulations) or when investigating segregation across income subgroups (e.g., Whites and Blacks in the top quintile or decile of the distribution of household income).

In simple situations one could replace the value of area population size with a smaller value of ENS by multiplying average area population size by the proportionate representation of the two groups in the comparison in the total population.[5] Unfortunately, this is inadequate because the value of effective neighborhood size (ENS) is affected by another complicating factor; namely, the extent to which the other groups in the city population co-reside with the two groups in the segregation comparison. If the other groups co-reside extensively with the two groups in the comparison (as would be the case under random distribution of all groups), ENS will be smaller than area population size (t_i) and approach its minimum possible value. All else equal, index bias would then be higher. But, if the other groups in the

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[3] Analytic techniques advocated by Mazza and Punto (2015) may help reduce the computational burden. But the task will still likely be too complex for wide adoption in empirical studies.

[4] The impact of this factor has not previously been carefully studied. I provide analyses in the next chapter that show how it can have important impacts on E[D] and how these impacts can make previous strategies for dealing with index bias (e.g., Winship 1977) less effective.

[5] For the moment I set aside the factor of variation in area size.

population are completely segregated from the two groups in the comparison, the two groups of interest will be the only groups present in the areas where they reside. In this situation the value of ENS then will take its maximum possible value and match area population size. All else equal, index bias would then be lower. The "correct" value of ENS in empirical analyses will typically fall somewhere between these minimum and maximum values depending on whether the other groups in the city population are weakly or strongly segregated from one or both of the groups in the segregation comparison. Since multi-group distributions vary widely in real cities, this issue carries complex implications for index bias and greatly complicates the assessments of index bias across group comparisons and cities.

In sum, the distinction between overall area size and effective neighborhood size (ENS) and the other complications noted above can have important practical implications for assessing index bias. When ENS is known with precision, analytic formulas for calculating expected values of bias (e.g., E[D]) can potentially provide a reasonable guide to identifying when bias is negligible or problematic. When one or more of the complications noted in the above discussion are present, the same formulas can yield incorrect expected values of bias. Previous methodological studies have not recognized this problem. As a result, strategies for dealing with bias that rely on estimating expected values of index scores under random distribution (E[•]) can perform poorly in empirical studies.

14.1.2 The Practical Relevance of Variation in Effective Neighborhood Size

In the face of these complications, one option is to estimate values of E[•] by bootstrap simulation methods (per Carrington and Troske 1997; Allen et al. 2009, and Mazza and Punzo 2015). In principle, applying these methods with observed residential distributions can yield superior results of E[•] because the estimates do not depend on simplifying assumptions about the value of effective neighborhood size (ENS).

I explored using this option by examining expected values the dissimilarity index (D) for block-level segregation between Whites and Blacks for CBSAs in 2000. For this analysis I computed values of E[D] by three methods. First I computed two values of E[D] using Winship's (1977) "approximation" and "exact" formulas. To establish the value of ENS to use in the formulas, I calculated the median value of ENS over blocks in the CBSA that had nonzero counts for the combined White and Black population. I additionally computed values of E[D] based on bootstrap simulations that do not make simplifying assumptions about ENS.[6]

[6] Specifically, I estimated E[D] by distributing White and Black individuals randomly over areas where Whites and Blacks were found in the observed residential distribution for the city and calculating D from the resulting random distribution. I repeated the exercise 1000 times for each CBSA and took the average of the obtained values of D as the estimate of E[D] for the CBSA.

I found that the values of E[D] based on the three methods were highly correlated ($r^2 \geq 0.95$). But, importantly, they were not exact substitutes for one another and the differences often had important consequences. First, while values of E[D] correlated across methods, the average values of E[D] varied by method. Values of E[D] based on Winship's approximation formula were much higher than those based on the exact formula (consistent with results presented in Winship (1977)). Second, values of E[D] based on bootstrap simulation methods were lower than values obtained using analytic formulas. Specifically, estimates of E[D] based on the Winship's exact formula were on average 40 % higher than estimates from bootstrap simulations. This indicates that assessment of bias using analytic formulas will be too high and adjustments of index scores using estimates of bias based on analytic formulas would tend to significantly "over-correct" and yield estimates of unbiased segregation that are too low.

I conducted similar exercises for other popular indices of uneven distribution; specifically, G, R, H, and S. The results for these indices were similar to what I just described for D. Estimates of bias obtained by analytic formula were higher than estimates based on bootstrap simulation methods. The key point for present concerns is that the magnitude of estimates of E[•] varies by method. This indicates that estimating expected values of index scores under random distribution is not a simple task in empirical studies. For now it appears that the most accurate alternative is to use the computationally demanding method of bootstrapping (per Carrington and Troske (1997) and Allen et al. (2009)) to obtain estimates of expected values (E[•]) of measures of uneven distribution. The estimates are superior because they do not rely on strong assumptions (i.e., that areas are all the same size and that effective neighborhood size is constant across areas) but instead directly incorporate the observed variation in ENS across areas. Unfortunately, the practical burdens associated with this approach will deter most researchers from adopting the methods.

14.1.3 Random Distribution Is a Valid, Useful, and Conceptually Desirable Reference Point

The literature on segregation measurement includes many statements noting that random distribution can serve as a valid and desirable reference point for assessing segregation (e.g., Jahn et al. 1947; Reiner 1972; Zelder 1972; Cortese et al. 1976; Winship 1977; Blau 1977; Boisso et al. 1994; Carrington and Troske 1995; Carrington and Troske 1997; Ransom 2000; Allen et al. 2009; Mazza and Punzo 2015). For example, Cortese, Falk, and Cohen offer the succinct argument that it is "natural" to "construct an index which takes a value of zero when the distribution is random" (1976: 631). The unbiased measures suggested by Winship (1977), Carrington and Troske 1995, Carrington and Troske (1997), Allen et al. (2009), and Mazza and Punzo (2015) all have this property. The measures I introduce in Chap. 15 also have this property.

One obvious benefit is that when indices have this property the value of zero can then serve as the reference point for evaluating whether the index value obtained indicates that race or other group membership plays a role in segregation over and above the consequences of chance. Using indices with this quality would bring segregation research into conformity with long-standing convention in the study of group disparities in socioeconomic outcomes. Inequality research in all domains except the study of residential segregation evaluate group disparities on socioeconomic outcomes (e.g., education, occupational status, income, etc.) based on comparisons of group means that take expected values of zero when group membership (i.e., race) has no statistical association with the stratification outcome in question.

No significant objection has been or can be raised against the goal of seeking "unbiased" segregation indices with these properties. Taeuber and Taeuber (1976) and Winship (1977) have correctly noted that segregation resulting from random factors can be substantively meaningful in its own right. But this of course does not undercut the desirability of having unbiased indices whose scores provide a trustworthy signal that segregation departs from levels expected under random distribution. Winship argues that measures possessing this quality are especially desirable when interest is focused on the *causes* of segregation rather than its *consequences* (1977: 1065). Moreover, even when one is interested in the consequences of segregation, it can be valuable to know whether the segregation involved reflects systematic social dynamics, stochastic variation in residential distributions, or artifactual components of index values.

14.2 Prevailing Practices for Avoiding Complications Associated with Index Bias

I noted at the beginning of this chapter that most segregation researchers are aware of the problem of index bias and based on concern about this potential problem they routinely adopt strategies to minimize its undesirable consequences. This represents a practical compromise between the ideal of assessing and dealing with bias directly at the point of measurement – which until now has not been possible – and foregoing segregation research altogether. Researchers thus face the dilemma that segregation is an important social phenomenon that warrants sustained investigation but methodological studies establish that bias can distort segregation index scores and have adverse impacts on results and findings. Because direct solutions to this problem have not been available, researchers have adopted two general approaches for coping with concerns about index bias. One is to identify and avoid using especially problematic cases. The other is to differentially weight cases to try to minimize the impact of problematic cases.

Surprisingly, researchers almost never use direct methods of assessing bias to identify potentially problematic cases. This is difficult to understand and raises the question of why researchers use inferior proxy approaches instead of more rigorous methods. Computation intensive bootstrap methods – which arguably yield the best estimates of E[•] – are relatively new and arguably are too demanding for general use. But analytic methods for assessing E[•] set forth in Winship (1977) have rigorous foundations and are easy to implement. It would seem that these methods provide an obvious and compelling option for identifying segregation comparisons that are most likely to be distorted by bias. Nevertheless, researchers instead rely on informal "rules of thumb" to screen cases. These informal methods tend to be crude and imprecise in comparison to available analytic methods for directly evaluating E[•]. Common examples include the following practices.

- Restrict segregation studies to comparisons involving broad population groups; avoid comparisons involving small populations or subgroups within broader populations.
- Assess segregation using larger spatial units such as census tracts; avoid smaller spatial units such as census blocks or census block groups.
- Restrict segregation studies to only comparisons where group ratios are relatively balanced and avoid comparisons where group ratios are highly unbalanced.
- Assess segregation using full count (100 %) data; avoid sample data.
- Weight cases differentially – discounting cases presumed to be distorted by bias – when performing statistical analyses assessing variation in segregation over time or across groupings of cases and when performing regression analyses investigating cross-area variation in segregation.

The practices just listed are not necessarily all implemented in every study and the individual practices are not always implemented in exactly the same way. But almost all empirical studies adopt some combination of multiple practices similar to the ones listed above. The best justification one can offer for these "rule-of-thumb" practices for dealing with index bias is that, while they are not necessarily optimal, they are easy to implement and may be useful.

14.2.1 Unwelcome Consequences of Prevailing Practices

Researchers adopt the practices just described with the best of intentions and the practices probably do provide a measure of protection from situations where undesirable consequences of index bias are especially great. My concern is that segregation studies rely too heavily and uncritically on these informal practices. One basis for my concern can be expressed in the simple question, "Is there compelling evidence to indicate that the practices are effective in accomplishing the intended goal of eliminating undesirable impacts of index bias?" Unfortunately, the answer is "no, not really." The practices are appropriately characterized as rough-and-ready "rules-of-thumb" whose efficacy has not been established by rigorous methodological studies.

I comment on these issues further in the next section to explain the points more carefully. But I should note here that I see these issues as secondary because it is easy to imagine substituting better practices. The more serious concern is that *even if these prevailing practices for dealing with the problems associated with index bias are refined to work as well as possible they still have the undesirable consequence of restricting the scope of segregation studies*. This issue is insidious because it is less obviously "visible." But its impact on segregation research is substantial and far reaching.

Importantly, this undesirable consequence is not reduced when one adopts more rigorous practices for diagnosing situations where index bias is likely to be problematic. The practices researchers adopt to avoid problems associated with index bias make it impossible to conduct many studies that researchers would otherwise undertake if index bias were not a concern. The following is a list of research topics that are of clear scientific interest but currently are "off limits" because prevailing practices for dealing with index bias will preclude analyses that could address questions relating to these topics.

- studying segregation at finer levels of neighborhood resolution such as using small spatial units such as census blocks,
- studying segregation in smaller metropolitan areas and non-metropolitan areas (because segregation in these areas can only be captured well using smaller spatial units such as blocks),
- studying segregation involving populations that are small in absolute size such as Asian and Latino subgroups (e.g., Vietnamese or Salvadoran) or "first settler" and early arriving" Latino and Asian populations in new destination communities,
- studying segregation between population subgroups based on social characteristics such as education, income, family/household type, or other similar characteristics, especially considered in combination, and
- studying segregation involving groups that differ substantially in relative size.

As the situation currently stands, these and many other kinds of studies are precluded due to researchers' concerns that index scores obtained for the comparisons involved cannot be trusted. The undesirable consequence of this is that the research literature is severely skewed toward examining a narrow subset of segregation comparisons that survive a gauntlet of restrictions placed on group comparisons, analysis samples, and study design (e.g., size of spatial unit). Accordingly, most empirical studies of segregation in the contemporary literature focus on tract-level segregation for large metropolitan areas and on group comparisons involving minority populations that are large in terms of both absolute and relative group size. Of course these cases are important and sociologically interesting in their own right. But researchers should not lose sight of the fact that this is a narrow subset of cases and is not representative of the full range of situations and group comparisons that research would consider if study designs were not narrowly restricted to reduce concerns about index bias.

This raises the concern that our understanding of segregation patterns is based on a particular subset of cases and comparisons chosen for practical, not theoretical and substantive, reasons. Equally importantly, it raises the related concern that researchers cannot undertake studies of segregation in many situations that have potentially important value for understanding segregation dynamics. For example, it is of obvious scientific interest to study the trajectory of segregation over time for new immigrant populations. But this currently is not possible because prevailing restrictions on study designs preclude the possibility of assessing segregation in the early stages of this process when the group is small in both absolute and relative size.

In some areas of inquiry the impact of concerns about index bias on the scope of segregation studies is pervasive and near-total. One example of this is the near total disappearance from the literature of studies that assess segregation at smaller spatial scales. Analysis of segregation based on block-level data once was common (Taeuber 1964; Taeuber and Taeuber 1965; Sorenson et al. 1975; Schnore and Evenson 1966; Farley and Taeuber 1968, 1974; Roof and Van Valey 1972; Van Valey and Roof 1976). Nowadays it is rare.

This change in the literature is not based on theoretical or substantive concerns. To the contrary, assessing segregation at small spatial scales has obvious substantive value because it can potentially detect segregation that might otherwise be missed. Accordingly, block-level analysis is better suited for studying the emergence of segregation patterns for newly arriving migrant or immigrant populations because patterns of segregation during their initial settlement would not be evident if segregation is measured using larger units such as census tracts.[7] Similarly, block data are relevant for nonmetropolitan areas and non-core counties where census tracts are too large to sustain meaningful segregation analysis. But contemporary empirical studies rarely investigate segregation using block data. It is not because segregation in these settings just mentioned is substantively unimportant or scientifically uninteresting. Instead, it is because *segregation study designs have "retreated" to supposedly safer ground to avoid the complications of index bias* that arise when measuring segregation based on small areas. The unfortunate byproduct of this is that it has inhibited the investigation of segregation in smaller cities and communities.

Another closely related example is that empirical segregation studies systematically avoid examining segregation in metropolitan areas where one of the populations in the analysis is a relatively small proportion of the population or is small in absolute population size. For example, Farley and Frey's (1994) influential study of trends in segregation from Whites for Blacks, Latinos, and Asians restricted its analysis to metropolitan areas where the minority group in the comparison either reached 20,000 in overall population or represented 3 % or more of the city population. As a result, out of 318 total metropolitan areas, their analysis included only 232 areas for

[7] Lichter et al. (2010) used block-level data to study White-Latino segregation in new destinations. They offered compelling arguments for why block level data was necessary. But they did not address the problem of index bias.

White-Black segregation, only 153 areas for White-Latino segregation, and only 66 areas for White-Asian segregation.

The metropolitan areas excluded from comparison were those for which the minority group was small in relative and/or absolute size. Many of the excluded cases have non-negligible populations for the groups in question ideally would be included in studies investigating how segregation varies with basic factors such as size of city, relative group size, and trends in absolute and relative group size. However, since prevailing practices exclude cases over key ranges of these variables, many interesting research questions cannot be addressed.

Similar consequences are seen in studies of segregation among subgroups within various minority populations. For example, in a study of segregation patterns for five Asian-origin groups (Chinese, Japanese, Korean, Vietnamese, and Asian Indian), Massey and Denton (1992) restricted their analysis to metropolitan areas where the size of the Asian-origin group in question was 5,000 or higher. This limited the scope of their analysis to no more than 11 metropolitan areas for any single group. In addition, they reported segregation scores only for group comparisons where both groups in the segregation comparison had 5,000 persons and this eliminated 20–30 % of possible comparisons involving other Asian-origin groups. They explicitly justified these restrictions in terms of concerns about index bias stating "Since the index of dissimilarity is inflated by random variation when group sizes get small (Massey 1978), we only compute indices when the group size in the SMSA exceeds 5,000" (Massey and Denton 1992: 171). Massey and Denton are clear that they did not adopt these restrictions on study design based on theoretical interest or other substantive concern but rather adopted the restrictions solely as a means of guarding against adverse consequences of index bias.

A final example I note is the impact on research examining racial segregation between racial groups after they have been secondarily grouped on socioeconomic status or other social characteristics relevant for group differences in residential distributions. Empirical investigations of this type routinely limit their analyses to a handful of very large cities. Furthermore, to proceed with analysis in this small subsample of cities they collapse the detailed data on socioeconomic characteristics (e.g., income) into a small number of broad groupings (e.g., 3–5 categories). Again, these restrictions in study design are adopted primarily to avoid complications associated with index bias. Evidence of this is found in the following statements from two important studies investigating racial-ethnic segregation across socioeconomic standing.

> Since the number of minority members is small in some socioeconomic categories, particularly those at the upper end of the socioeconomic spectrum, we focus attention on three sets of 20 SMSAs that have the largest numbers of blacks, Hispanics, and Asians … Focusing on the top 20 SMSAs for each group maximizes the number of minority members within each socioeconomic category and increases the stability of the segregation indices. (Denton and Massey 1988: 799–800)

> Since dissimilarity indices become unreliable and difficult to interpret when the number of minority members is very small (Massey 1978), we only compute figures for those metropolitan areas where the minority population reached 5,000. Massey and Fischer (1999: 318)

The several examples reviewed above illustrate that empirical studies of segregation routinely adopt restrictions on study designs to avoid situations where index bias can complicate assessments of the level of segregation and its variation across cases. In the absence of better alternatives for dealing with index bias, these practices can perhaps be seen as necessary precautions. Nevertheless, it is important to recognize that the practices have many unwelcome consequences and it would be more desirable to have unbiased versions of indices of uneven distribution so the current restrictions on the scope of segregation studies can relaxed.

14.2.2 Efficacy of Prevailing Practices: Screening Cases on Minority Population Size

In the ideal, the practices researchers adopt to minimize complications associated with index bias would have clear rationales and be established as effective by rigorous methodological studies. One approach would be to identify potentially problematic cases by using either analytic formulas (Winship 1977) or bootstrap methods (e.g., Carrington and Troske 1997; Allen et al. 2009; Mazza and Punzo 2015). For example, one might require that expected values of E[D] be below some value deemed "acceptable" – say 3–5 points. But empirical studies of segregation do not screen cases this way nor do they report the levels and ranges of E[D] for the cases in the analysis sample.

Instead, empirical studies rely on informal practices such as screening cases based on "thresholds" on absolute and relative group size. The potential concern is that this is an imprecise way to screen problem cases. I explored the issue empirically using a data set with observations on White-Minority segregation for CBSAs in 1990, 2000, and 2010. I screened cases requiring that each case have at least 2,500 persons in both groups in the decade of observation and with the smaller group in the comparison comprising at least 3 % of the combined group total.[8] Screening criteria similar to these are routine in empirical studies. Their application here yielded an analysis data set with 3,570 cases.

This result itself deserves comment. Relaxing the case selection criteria to require cases to have only 500 persons and for the smaller group in the comparison to comprise only at least one-half of one percent of the combined group total would yield 6,655 cases. The additional 3,085 cases would be highly relevant for assessing how segregation compares in smaller communities and communities where one

[8] The data set included White-Black, White-Latino, and White-Asian comparisons.

group in the comparison is small in relative size. This could apply, for example, to establishing "baselines" for White-Latino segregation in micropolitan areas and non-core counties of the Midwest and South that emerged as new destination communities experiencing Latino population growth during the period 1980–2000. Current practices do not permit these cases to be considered. The unbiased indices I introduce in Chap. 15 make it possible for researchers to focus on these communities using spatial units as small as blocks (instead of tracts) if they wish to do so.

For each segregation comparison I calculated the value of D and estimates of bias based on values of E[D] obtained using both Winship's analytic formulas and also by bootstrap methods. The question I address is whether the restrictions on the study design and analysis sample yield an analysis data set where concern about bias is negligible. The main conclusions are the same whether using either set of estimates of E[D] so I report results for E[D] computed by formula because few researchers are likely to compute bootstrap estimates in empirical studies. I first consider results when segregation is assessed using tract-level data, the most conservative choice for minimizing potential bias. Here the mean for E[D] was 7.36. Equally and perhaps more importantly, its values displayed considerable variation across cases with an inter-decile range of 8.86 with 10 % of cases at or below 3.74 and 10 % of cases at or above 12.60. So the first takeaway point is that the screening criteria did not reduce the typical potential for bias to negligible levels. A second takeaway point is that screening cases did not yield an analysis data set where the potential for bias is uniform across cases. This is not surprisingly because relative group size is an important determinant of E[D] and it varies widely across cities even after screening out cases where percent minority is below 3 %.

Another finding is that the level of underlying potential for bias in D varies across group comparisons. The mean for E[D] is 6.10 for White-Black segregation, 7.02 for White-Latino segregation, and 10.86 for White-Asian segregation. The cross-group variation traces to the fact that, on average, the relative size of the minority population is smaller for the comparisons involving Latinos and even more so for comparisons involving Asians. This raises concerns that bias might distort cross-group comparisons on segregation. The means on D are 48.48 for the White-Black comparisons, 35.13 for the White-Latino comparisons, and 39.21 for the White Asian comparisons. It is interesting to observe that the difference of 3.84 between the White-Asian and White-Latino averages for E[D] is almost as large as the difference of 4.08 between the White-Asian and White-Latino averages for D.

The important point here is that the conventional approach to screening cases does not do away with nagging concerns about the potential role of bias. Furthermore, these results only get worse when segregation is measured using data at lower levels of geography such as for block-groups and blocks. For example, when calculated using block-level data, the means for E[D] are 21.98 for White-Black segregation, 35.13 for White-Latino segregation, and 39.21 for White-Asian segregation. The results for E[D] also varied considerably across areas and across group comparisons as observed for E[D] computed using tract-level data.

14.2.3 Efficacy of Prevailing Practices: Weighting Cases by Minority Population Size

Researchers often are aware of concerns that index bias can distort results even after applying sample restrictions aimed at excluding the most problematic cases. In many studies researchers address this concern by weighting cases by minority population size for the city when performing statistical analyses such as computing summary statistics (e.g., means) for groups of cases or estimating regression equations. Unfortunately, the efficacy of this strategy is not rigorously established.

The practice is sometimes described as being an appropriate way to deal with "unreliable" cases but this rationale is open to question. Cases with biased index scores are not "unreliable" in the usual statistical sense of that term. To the contrary, biased index scores are highly reliable in the sense of yielding consistent results under given study conditions. The problem is not that the scores are inconsistent; the problem is that they are consistently high; that is, they are reliable but still untrustworthy because they are biased upward.

Weighting cases by minority population size does not "correct" the higher and potentially misleading index scores that may result from bias for some cases. So what does the practice accomplish? One clear consequence is to strongly skew analysis results in the direction of reflecting segregation patterns found in cities that have large minority populations. In most studies this means that a relatively small subset of cases will receive larger weights and have a disproportionate influence on results of statistical analyses. In contrast, a larger number of remaining cities will receive smaller weights and have modest-to-negligible influence on results. This amounts to reducing the "nominal" sample size for the macro units (usually cities) as results will be similar those obtained when excluding cases with small minority populations.

Minority population size is at best only a crude proxy for bias potential (i.e., E[D]). Accordingly, screening and weighting on this item can introduce at least two kinds of distortions to results. Holding relative group size constant, many smaller cities will be discounted or excluded from the analysis altogether when more careful diagnostic analysis would show that their index scores are as trustworthy as those for larger cities (because bias is intrinsically related to relative group size, not to absolute size). The practical result is that weighting cases to protect against bias will tend to be "hit and miss" in effectiveness but the practice will definitely skew results to more closely reflect segregation patterns for cities with large minority populations.

The main point is that current approach of guarding against undesirable consequences of bias based on using informal proxy criteria is open to question. Moreover, even if problematic cases were identified more carefully (e.g., using bootstrap methods to estimate E[D]), an important underlying problem would remain; current practices do not correct flawed scores so the cases can be trusted and used in the analysis. Instead, the cases that are impacted by index bias are excluded or discounted and analysis results thus reflect segregation patterns observed for a small subset of cases that are not adversely impacted by bias. This is hardly an ideal study design. These cases, while important in their own right, are not necessarily representative. So one is left hoping, but not knowing, that "true" segregation patterns in the large fraction of cases that are excluded or discounted do not differ from the segregation patterns in the smaller subset of cases that dominate the analysis results.

14.2.4 An Aside on Weighting Cases by Minority Population Size

Statistical theory provides a different and potentially defensible rationale for case weighting when performing statistical analyses of variation in segregation across cities and communities. It is that the dependent variable (i.e., the index score) exhibits differential variability across cities. The relevant statistical issue is heteroskedasticity – a violation of the ordinary least squares (OLS) regression assumption that error variance is constant across cases. This issue is distinct and separate from index bias. Index bias is systematic with regard to the direction of its impact on index scores; biased cases have consistently inflated values for index scores. In contrast, heteroskedasticity does not involve bias; it involves greater volatility in scores around the model-predicted average and the volatility reflects scores that are below the predicted average as well as scores that are above the predicted average. When heteroskedasticity is present, estimates of means and regression coefficients are unbiased but significance tests in OLS regression may be questioned because the assumptions underlying the tests are not met.

One strategy for dealing with heteroskedasticity in aggregate-level regressions is to perform weighted least squares (WLS) regression using case weights (w) that are proportional to the inverse of each case's expected error variance (Hanushek and Jackson 1977). Statistical theory indicates the appropriate weight (w) would be the reciprocal of the expected error variance of D. This can be calculated directly.[9] But some might view absolute size of the minority population as a potentially acceptable proxy and defend weighting cases by population size on this count.

This would perhaps be justified if variation in index scores was greater when minority population size is small. But empirical analysis suggests this is not the case. This is due to two reasons, one simple and one complex. I explored the issue by examining the empirical associations among three variables – the score for D, predisposition for bias measured by E[D], and minority population size – using the data set and measures introduced and described in the previous section. The simple

[9] D is the White-Black difference of proportions ($p_1 - p_2$) residing in areas where proportion White for the area equals or exceeds that for the city as a whole. The expected variance (σ^2) of a difference of proportions is obtained by squaring the standard error of the difference of proportions – $\sigma\left(p_1 - p_2\right)$ – given by $\sqrt{p_1 q_1 / n_1 + p_2 q_2 / n_2}$ or, alternatively, $\sqrt{pq\left(1/n_1 + 1/n_2\right)}$ if using the pooled calculation of $p = \left(p_{1.} n_1 + p_{2.} n_2\right) / \left(n_1 + n_2\right)$.

part of the story is that values of D do not display heteroskedasticity in relation to minority population size. More specifically, dispersion in the values of D around the mean is relatively constant across minority population size so there is no obvious empirical basis for weighting cases by minority population size to compensate for heteroskedasticity.

The complex part of the story is that predisposition for bias (i.e., E[D]) is moderately and inversely correlated with minority population size.[10] This might lead one to expect that dispersion in residuals would be larger when minority population size is small. Instead, however, the dispersion in residuals for D is lower, not higher, when E[D] is high. This is because index bias raises the "floor" for D since bias precludes low scores. This then truncates the range of variation in D in comparison to the range of variation in D when E[D] is low.

Since the argument for weighting cases by minority population size to deal with the statistical issue of heteroskedasticity is weak, it is appropriate to ask whether the practice is warranted on any basis. The best one can say in defense of the practice is that it may tend to reduce the influence of cases that on average have higher levels of bias (i.e., higher values on E[D]). But this purpose could be better served by establishing weights based on direct assessments of bias. However, even if case weights were well-calibrated to reflect bias, the practice of down-weighting cases proportional to bias is a weakly justified ad hoc procedure. It does not "repair" or "correct" inflated index values for individual cases. Misleading cases remain misleading. What the practice does accomplish is to minimize the influence of potentially misleading scores when they are averaged in with other scores that are viewed as less misleading.

If the rationale for case-weighting is not particularly strong, is it at least benign? This question is hard to answer. One thing is clear; weighting by minority population size skews results toward patterns of segregation observed in cities with large minority populations. This is definitely a non-representative subset of cities disproportionately including large cities and medium-sized cities where percent minority is higher. Whether this influences findings in undesirable ways or not is unclear and may depend on the question being addressed. If one is investigating patterns and variation in segregation *for all cities* – that is, to understand how segregation varies across cities based on urban-ecological factors (e.g., population size, racial composition, population growth, etc.) equal weighting of all cases is more appropriate. Weighting cases by minority population shifts the focus away from outcomes for all cities and toward outcomes for minority individuals residing in cities with large minority populations. Skewing results in this way may be tolerable for some research questions. But it would be best for researchers who use these practices to acknowledge the issue and reflect on how findings might be affected.

[10] This is because absolute minority size has a moderate association with relative group size which is intrinsically related to index bias for D and all other indices of uneven distribution except S.

14.2.5 Summing Up Comments on Prevailing Practices

In this section I have argued that the research designs of empirical studies of residential segregation are shaped in important ways by researchers' concerns about the possible undesirable consequences of index bias. Motivated by these concerns, and with the best of intentions, segregation researchers routinely adopt a variety of informal practices such as restricting analysis samples to exclude cases where they suspect bias may render index scores untrustworthy and differentially weighting remaining cases when conducting statistical analyses. The goal is to minimize the potentially undesirable impacts of bias on index scores for cases that are included in the analysis sample.

I raised concerns that the efficacy of this patchwork of informal practices is open to question on various counts not the least of which being that bias is "flagged" by crude proxies instead of by using best available direct approaches for diagnosing the potential for bias. In the final analysis, I argued that the greater concern is that, even if these prevailing practices for dealing with index bias are refined and improved, they would continue to have an important but largely unappreciated undesirable consequence. This is that the practices narrow the scope of segregation studies in two important ways. First, they restrict empirical analysis to a subset of potentially non-representative cases and group comparisons where index scores are presumed to be less problematic. Second, they eliminate the possibility of investigating many important research questions that involve situations where standard indices are viewed as prone to non-negligible bias.

Based on this I argue that the most desirable strategy all around is to deal with bias at the point of measurement and obtain index scores that are not distorted by index bias. Having unbiased index scores would make it possible to use individual cases "as is". It would eliminate the need to screen and exclude cases due to concerns about bias. It would eliminate the need to use weighting procedures to minimize the influence of cases with biased scores on results of statistical analyses. The attractiveness of this kind of solution has not been overlooked. But past efforts to deal directly with index bias at the point of measurement have not gained acceptance. I review the reasons for this in the next section.

14.3 Limitations of Previous Approaches for Dealing Directly with Index Bias

The potential benefits of dealing directly with the index bias at the point of measurement have not gone unrecognized and a variety of suggestions for developing unbiased versions of segregation indices have been offered over the decades. To this point, however, none of these suggestions has gained wide acceptance in empirical research. The kind of approach proposed most often is to adjust scores of standard versions of index scores downward to eliminate the impact of upward bias associated with their expected values under a baseline model of random distribution (e.g., Cortese et al. 1976; Winship 1977; Farley and Johnson 1985; Carrington and Troske 1997; Allen et al. 2009; Mazza and Punzo 2015). For example, Winship (1977) and Carrington and Troske (1997) have proposed a relatively simple "norming" adjustment that has intuitive appeal.[11] They propose calculating "unbiased" or "bias adjusted" scores for D, designated here as D*, based on the following calculation.

$$D^* = \left(D - E[D]\right) / \left(1 - E[D]\right)$$

The justification for the calculation is that the value obtained indicates the degree to which observed departure from uneven distribution (D) exceeds the departure expected under a baseline model of random distribution (i.e., E[D]). In principle this adjustment can be applied to any index of uneven distribution for which the expected value under random distribution (E[•]) can be estimated.

Unfortunately, conceptual and practical issues have worked against wide adoption of this procedure. Regarding conceptual issues, the interpretation of D* is more technical and abstract than the interpretation of the conventional version of D. For example, negative values are possible and, while this is a valid result under the procedure, it is unsettling to many researchers. This negates one of the appealing aspects of D; namely, the ease with which its interpretation can be conveyed to broad audiences as well as professional audiences. Regarding practical issues, the method requires estimating E[D] as part of the analysis. In principle this can be accomplished using either analytic formulas or bootstrap simulation methods. But so far these options have not been embraced by segregation researchers due at least in part to the technical and computational burdens associated with estimating E[D].

Prospects for adoption of this approach in the future are poor. One reason for this is the formula-based methods for estimating E[D] that are most easily implemented can perform poorly when the full population of the city includes groups other than the two groups in the segregation comparison. Unfortunately, this condition is common in many research situations. It undercuts the potential value of using simple formula-based approaches to estimating E[D] because estimated values tend to be too high and in turn can lead values of D* to be too low because the adjustment to remove the impact of index bias is too aggressive. Until now this problem has gone unnoticed in the literature. In principle, the problem can be overcome by drawing on refined versions of formula-based estimates of E[D] or using estimates based on bootstrap simulation methods, but complexity and increased computational burden associated with these superior approaches to estimating E[D] makes it unlikely researchers will adopt these options.

[11] Allen and colleagues (2009) also suggest a similar strategy.

14.4 Summary

In this chapter I pointed out that empirical studies of residential segregation are strongly influenced by concerns about index bias. These concerns are reflected in the study designs researchers adopt and in the methods of statistical analyses researchers use. One important consequence of this is that researchers carefully avoid studying segregation in situations where they suspect bias will render scores of standard versions of indices of uneven distribution untrustworthy. Accordingly, they avoid studying group comparisons involving small groups; they avoid studying group comparisons where groups are imbalanced in size; they avoid measuring segregation using smaller spatial units such as census blocks; and they avoid examining segregation in smaller communities. Even after adopting these restrictions on study design, researchers continue to have concerns that bias makes some index scores untrustworthy. Analysis reviewed in the chapter shows their concern is well justified. Motivated by these concerns researchers routinely weight cases differentially based on minority population size when performing statistical analyses on the assumption that this will minimize the impact cases with scores inflated by bias will have on results. In a very real sense this has the practical effect of reducing the sample size even further and skewing it toward a non-random subset of cases. Taken collectively, these several practices limit the scope of segregation studies so attention is focused disproportionately on patterns of segregation for large metropolitan areas with minority populations that are large in absolute and relative terms. And even among this subset of cases, results of statistical analyses disproportionately reflect segregation patterns for cities with larger minority populations.

The adoption of these practices is well intentioned. But the current state of affairs is far from ideal. As things currently stand, even after restricting study designs to avoid problematic cases, researchers remain less than confident about scores for the individual cases in their studies and routinely weight cases differentially when performing statistical analysis to minimize the impact of index bias. This concern complicates elementary tasks in segregation analysis such as being confident about the index score for a given case, or comparing scores for two cases, or following the score for a single case over time. More importantly, concern about index bias leads researchers away from investigating segregation in a wide range of situations that would be theoretically relevant and sociologically interesting if index scores could be trusted.

The better alternative is to deal with problem of index bias directly at the point of measurement. Previous suggestions for accomplishing this task have involved applying after the fact adjustments to standard versions of index scores. These "bias adjusted" indices have never gained wide usage. In part this is because they have involved complex and often computationally demanding procedures. In addition many researchers find the resulting measures to be unfamiliar and therefore more difficult to interpret and explain to nontechnical audiences. Finally, researchers simply have not yet been convinced that the approach of applying corrective adjustments to standard index scores yields robust and effective results over the wide range of situations encountered in "real world" empirical studies.

In the next chapter I introduce a new solution for moving beyond the current unsatisfactory situation. By drawing on the difference of means formulation of indices of uneven distribution, I identify new insights about the nature of index bias that make it possible to address index bias at the point of measurement. The insight is that, when segregation is cast as a group difference on average levels of scaled group contact, bias can be traced to a relatively simple source; namely, how group contact with the reference group is impacted by self-contact which inherently differs for the reference group and the comparison group. Eliminating self-contact from index calculations by assessing group contact based on "neighbors" instead of "area population" eliminates this inherent source of bias in index scores. Chapter 15 reviews the basis for establishing unbiased versions of popular indices. Chapter 16 reviews the performance of the "unbiased" versions of popular indices to establish that, as desired, they have expected values of zero under random assignment. It also makes the case that the new measures allow researchers to use familiar indices with greater confidence and dispense with most of the ad hoc practices that currently restrict the scope of segregation studies.

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Chapter 15
New Options for Understanding and Dealing with Index Bias

In this chapter I introduce a new approach for addressing the problem of index bias at the point of measurement. Specifically, I introduce new formulations of popular indices of uneven distribution that are free of bias and take expected values of zero when individuals and households are randomly assigned to residential locations. I accomplish this task by drawing on the difference of means formulations of segregation indices introduced in earlier chapters to first identify and then eliminate the root source of bias in standard versions of popular indices of uneven distribution. The crucial insight from the difference of means formulation is that the values for all popular indices of uneven distribution can be seen as resting on person-specific scores for pairwise group contact (p). Close consideration reveals that the source of index bias is found in these group contact scores. Happily, a surprisingly simple refinement in the calculation of these scores eliminates index bias.

I review the root problem and its solution in more detail in the body of this chapter but offer a brief preview the essence of the problem and the solution here. To begin, recall that the difference of means framework establishes that all popular indices of uneven distribution can be formulated in terms of group differences in scaled residential exposure or contact. More specifically, the score for a particular index of uneven distribution can be obtained by calculating the difference of group means on individual residential outcomes (y) scored using an index-specific scaling function $y = f(p)$. The input to the scaling function, "p", is the individual's level of pairwise contact with the reference group in the comparison. The value of p is calculated from the area population counts for the two groups in the segregation comparison based on $p_i = n_{1i} / (n_{1i} + n_{2i})$. This approach to calculating the value of p introduces inherent upward bias in group differences on scores for p and also group differences on scores of y.

The source of bias is simple; the count terms (i.e., n_{1i} and n_{2i}) used in the calculation of group contact (p_i) include the individual in question. The score for contact thus combines two components of contact – *contact with self* and *contact with neighbors*. For any individual the component of contact that derives from contact

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_15

with neighbors can vary widely; it can range from no (0 %) contact with the reference group to only (100 %) contact with the reference group. In principle, this component of contact can be random for any individual regardless of group membership. Thus, under random assignment the expected value of this component of contact will be the same for every individual regardless of group membership and expected group differences will be zero (0). In contrast, the component of contact that derives from self-contact cannot be randomly assigned; it is fixed and invariant for each individual. Contact with self distorts group comparisons on contact because this component of contact inherently differs by race. Specifically, self-contact makes the assessed value of contact (p) *intrinsically higher* for members of the reference group and *intrinsically lower* for members of the comparison group. This is the source of bias in indices of uneven distribution.

This can be understood intuitively by considering the situation where residential assignments are random. The expected representation of the reference group among neighbors will obviously be same for all individuals and for both groups. But when self-contact is added in, the distribution of values on p necessarily shifts up for members of the reference group and necessarily shifts down for members of the comparison group. Index scores are computed from the difference of groups means on scaled contact (y) scored from simple pairwise contact (p). Since all of the index-specific scaling functions (i.e., $y = f(p)$) score y as a positive, monotonic function of p, the expected distribution of y will necessarily be higher for the reference group than for the comparison group. As a result, standard versions of indices of uneven distribution are biased upward; that is, their expected values under random assignment (E[•]) are positive.

I eliminate index bias in indices of uneven distribution by making a simple refinement to the contact calculation for individuals; *I assess contact using counts for neighbors instead of area population.* For purposes of discussion, I designate the revised version of contact as p′. This modification removes the fixed contribution of self-contact from the calculation of group contact scores for individuals. Intuitively, the expected representation of the reference group among neighbors is the same for all individuals under random assignment regardless of group membership. As a result, the expected distribution of values on contact with neighbors (p′) is the same for both groups. It follows necessarily that the same is true for the expected distribution of scaled contact (y′) scored from p′. Accordingly, the expected value of the group difference of means on scaled contact (y′) also is zero under random assignment. Thus, indices of uneven distribution calculated in this way are unbiased. Below I develop this conclusion more carefully. In Chap. 16 I report results of empirical analyses demonstrating that indices of uneven distribution computed using this relatively simple refinement take an expected value of zero under random assignment.

15.1 The Source of the Initial Insight

I should give credit where credit is due and note that a study by Laurie and Jaggi (2003) set me on the path to discovering a general strategy for developing unbiased versions of all popular indices of uneven distribution. Laurie and Jaggi used a Schelling-style agent simulation model to produce model-generated residential patterns in a virtual city.[1] As is common in agent models they assessed segregation at very small spatial scales. For purposes of the discussion here I consider the example of a city with simple housing grid that is divided into small "blocks" based on 3×3 square sections that contain 9 households.[2] Ordinarily, segregation assessed at this fine-grained spatial resolution would be subject to extremely high levels of index bias. For example, in a city with an 80/20 White-Black group ratio the value of E[D] would be 37.9 and the value of E[S] would be 11.1. Laurie and Jaggi (2003) measured segregation using an index of their own construction which they termed the "ensemble averaged, von Neumann segregation coefficient." They designated their measure as "S" but I term it "LJ" here to credit them and also to avoid confusion with using S to designate the separation index. Lauri and Jaggi claimed their index had an expected value of zero under random distribution; that is $E[LJ] = 0$. Initially I was skeptical of the claim. But I examined the behavior of their index in detail and discovered the claim was valid; Laurie and Jaggi's LJ index was indeed "unbiased." That is, over repeated trials of randomly generated residential distributions the distribution of values for scores on the LJ index will have a mean of zero.

Intrigued by this property and its potential benefits for measuring segregation in agent-models, I examined the formula for their index more closely to see how it related to more well-known indices of uneven distribution (Fossett 2007). I found the formula yielded the average over all individuals of a "scaled" score on same-group contact. For each individual the scaled score is obtained by first taking the difference between the observed proportion same-group among the individual's neighbors from the expected proportion based on the group's representation in the population and then expressing this result as a proportion of the maximum possible deviation under complete segregation. Putting this in notation more familiar to demographers and sociologists, scores for White households (agents) were given by $(p_i - P)/(1 - P)$ where P is proportion White in the population of agents and p_i is

[1] Laurie and Jaggi (2003) is one of many recent studies using Schelling-style agent simulation models – computer-implemented elaborations of the influential agent model of segregation dynamics first introduced in Schelling (1971).

[2] Laurie and Jaggi actually used a smaller, spatially delimited "von Neumann" or "rook's" neighborhood which consists of the 4 neighboring households who share sides with a focal household in a housing grid. I use the 3×3 "bounded" neighborhood to correspond better with practices in sociological segregation studies. All findings I note in this discussion also apply to spatially delimited neighborhoods of any spatial scale. But I defer detailed discussion of this topic for another time.

proportion White for the individual's neighbors.[3] Similarly, scores for Black households (agents) were given by $(q_i - Q)/(1-Q)$ where Q is proportion Black in the population of agents and q_i is proportion Black for the individual's neighbors. The sum of these scores is then divided by T, the total number of households (agents), to obtain the overall average. The resulting expression (dropping subscripts for convenience of presentation) is

$$LJ = (1/T)\cdot\left[\Sigma(p-P)/(1-P)+\Sigma(q-Q)/(1-Q)\right].$$

Interestingly, I found the separate averages for Whites and Blacks calculated as shown below also gave the same result. That is,

$$LJ = (1/W)\cdot\Sigma(p-P)/(1-P) = (1/B)\cdot\Sigma(q-Q)/(1-Q).$$

These expressions can be restated as follows

$$LJ = (\Sigma p/W-P)/(1-P) = (\Sigma q/B-Q)/(1-Q).$$

This expression reveals a close correspondence between LJ and Bell's (1954) revised index of isolation (I_R). Bell's I_R expresses a group's average for same-group contact as a proportion of its possible logical range. For Whites and Blacks, respectively, I_R would be given as

$$I_R = (P_{WW} - P)/(1-P), \text{and}$$

$$I_R = (P_{BB} - Q)/(1-Q)$$

where: $P_{WW} = (1/W)\cdot\Sigma(w_i \cdot p_i)$; $P_{BB} = (1/B)\cdot\Sigma(b_i \cdot q_i)$; $P = (W/T)$; $Q = (B/T)$; W, B, and T are the city totals for the White, Black, and Total populations, respectively; w_i, b_i, and t_i, are the counts for White, Black and Total population in area i; and p_i and q_i are area proportion White and Black, respectively, based on w_i/t_i and b_i/t_i.

The contact expressions P_{WW} and P_{BB} can be restated as $\Sigma(w_i \cdot p_i)/W$ and $\Sigma(b_i \cdot q_i)/B$, respectively. If the calculations are expressed from the point of view of individuals, as in Lauri and Jaggi, they can be given as $\Sigma p/W$ and $\Sigma q/B$. Thus, I_R for Whites and Blacks will take the same form given above for LJ. Thus,

$$I_R = (\Sigma p/W - P)/(1-P), \text{ and}$$

[3] To clarify terms in this discussion, city-level terms are given as follows: W and B are totals for Whites and Blacks, respectively, $T = W + B$, $P = W/T$, and $Q = B/T$. For each individual, w and b are the number of White and Black *neighbors* in the relevant neighborhood, $t = w + b$, and p and q are proportion White and Black, respectively, based on $p = w/t$ and $q = b/t$.

$$I_R = (\Sigma q / B - Q)/(1 - Q)$$

As shown here the two measures – LJ and I_R – appear to be equivalent, but there is an important difference between them that causes I_R and LJ to exhibit fundamentally different behavior. The difference in behavior is that Bell's I_R will manifest positive bias (i.e., $E[I_R] > 0$) while Laurie and Jaggi's LJ will be unbiased (i.e., $E[LJ] = 0$). The difference in behavior traces to one crucial difference between the calculations for the two indices. It is the difference in how the values of p and q are calculated for LJ and I_R. For Bell's I_R the calculation of contact terms follows the standard methodological practice in sociological segregation studies; the contact terms p and q are calculated using count terms for the full *area population*. Significantly, this calculation *includes* the focal household in the count terms that appear in the numerator and the denominator of the contact calculations. In contrast, for Laurie and Jaggi's LJ the calculation of contact terms p and q is based on a different procedure; it uses count terms for the focal household's *neighbors.* Thus, the approach Laurie and Jaggi use *excludes* the focal household from the count terms used in the calculations. To clarify, the contact scores used in calculating I_R and LJ differ as follows.

For I_R, $p = w / t$ and $q = b / t$.

For LJ, $p' = (w - 1)/(t - 1)$ and $q' = (b - 1)/(t - 1)$.

I use the prime symbol to differentiate contact based on neighbors from contact based on area population.

Closely comparing the design and behavior of the two measures led me to draw several conclusions. One is that, when focusing on a two group comparison, the LJ index can be described as an unbiased version of I_R. Another is that the only difference between the standard (biased) and unbiased versions of I_R is how contact is calculated. Specifically, self-contact is eliminated in the unbiased LJ version and this is accomplished by the simple exercise of excluding the focal household from the count terms that appear in the numerator and denominator of the contact calculations. This revealed that bias in I_R traces to a single source – the impact of incorporating self-contact into the calculation of group contact scores for individuals. It also revealed that bias could be eliminated by following Laurie and Jaggi's example and making the simple adjustment of computing group contact for individuals based on count terms for *neighbors* instead of count terms for *area population.* When this adjustment is implemented, values of I_R take an average value of zero when calculated over repeated trials for random residential distributions.

15.2 Building on the Initial Insight

Based on these intriguing findings, I focused on the question of whether this measurement strategy could be adapted in a general way for application with measures of uneven distribution. I focused first on the separation index (S) as a natural first choice because it is equivalent to Bell's revised index of isolation (I_R) in the special case where the city population consists of only two groups (James and Taeuber 1985; Stearns and Logan 1986; White 1986).[4] In light of this it is straightforward to describe Laurie and Jaggi's LJ index as an unbiased version of the separation index (S). Thus, Laurie and Jaggi deserve credit for establishing the core strategy for developing an unbiased version of S.

Initially I was frustrated in applying this insight to other indices of uneven distribution. The crucial insight of the strategy is to eliminate bias by eliminating the impact of self-contact from group contact calculations. But the best known computing formulas for indices of uneven distribution do not provide an obvious opportunity for acting on this insight because they do not yield index scores as group differences in average contact outcomes for individuals. As one example, James and Taeuber (1985: 6) give the following widely used computing formula for calculating the value of separation index

$$S = 1/NPQ \cdot \Sigma t_i \left(p_i - P\right)^2 .$$

This formula is efficient for computing values of S. But it does not give the value of S as a group difference in average contact scores for individuals. Moreover, I found that implementing the p_i adjustment used by Laurie and Jaggi in this formula did not yield an unbiased version of S with the desirable properties of the version established by Laurie and Jaggi.

I then struck on a second key insight. It is that eliminating bias from index scores first requires that the index be formulated as a difference of means on residential outcomes scored from pairwise contact. This isolates the impact of group differences in self-contact separately by group so its role can be eliminated. This prompted me to search for a formulation of the separation index that (a) would highlight the role of average group contact outcomes for individuals and (b) could be used as a template for deriving similar formulations for other popular indices of uneven distribution.

Appendices outline a derivation I that achieved this goal by expressing the separation index (S) as a group difference of means on contact with the reference group in the comparison.[5] I review a generic formulation in the additional material but give the result here using the example of White-Black segregation with Whites being

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[4] That is, one can describe the separation index (S) as a special case of Bell's Revised Index of Isolation (I_R) computed using only pairwise population counts.

[5] Later I found a similar derivation had been reported much earlier in a little known methodological paper by Becker et al. (1978).

designated as the reference group. Thus, S is the White-Black difference in average contact with Whites based on

$$S = P_{WW} - P_{BW}$$

where $P_{WW} = (1/W) \cdot \Sigma(w_i \cdot p_i)$, and $P_{BW} = (1/B) \cdot \Sigma(b_i \cdot p_i)$, with "W" and "B" designating total population for the reference group (Whites) and the comparison group (Blacks), respectively, w_i and b_i indicating area counts for the two groups, and $p_i = w_i / (w_i + b_i)$ indicating pairwise contact with the reference group for individuals residing in area "i".

Refining the contact calculations to eliminate the role of self-contact, leads to the unbiased version of S given as

$$S' = P'_{WW} - P'_{BW}$$

where P'_{WW} and P'_{BW} are contact expressions based on counts for neighbors instead of area population. They are obtained as follows. $P_{WW} = (1/W) \cdot \Sigma(w_i \cdot p'_i)$, and $P_{BW} = (1/B) \cdot \Sigma(b_i \cdot p'_i)$, with p_i' being calculated from $(w_i - 1)/(w_i + b_i - 1)$ for Whites and from $(w_i - 0)/(w_i + b_i - 1)$ for Blacks.

15.3 A More Detailed Exposition of Bias in the Separation Index

I now review the issue of index bias for the separation index (S) in more detail. I continue with the example of White-Black segregation and for simplicity consider a situation where the city in question is not small, consists of only Whites and Blacks, and is divided into areas of constant size in terms of area population (t).[6] I start with the question of "What can be expected when households are distributed randomly across housing units in all areas of the city?" For any household, White or Black, the expected contact with Whites is assessed using counts for *neighbors.* Normally I designate this as p′ but for the current discussion I also sometimes designate it as p_N using the subscript "N" to indicate "computed for neighbors." The expected value of this calculation is essentially equal to proportion White in the city (i.e., $E[P'_{WW}] = E[P'_{BW}] = P = W/(W+B)$).[7]

Intuitively, this is easy to understand. When a household's neighbors are obtained by a random draw from a large city population, the expected proportion Whites for the neighbors will be the city proportion White ($E[p'] = P$). Note that the expected

[6] The assumption that the city is not small assures that an individual household has a negligible impact on the city-wide group proportion for the reference group (P).

[7] For ease of presentation, I ignore the impact of the focal household's contribution to P for the city as a whole. In most empirical applications, the impact is negligible.

result is essentially the same for all households whether White or Black. More exactly, there is a very slight difference in expected value associated with the contribution the focal household makes to the combined total of Whites and Blacks and how this varies with the race of the focal household. In a very small city the P and Q results might differ slightly by race if one calculated P′ as $(W-1)/(W+B-1)$ for Whites and $(W-0)/(W+B-1)$ for Blacks. In larger cities this potential difference becomes negligible and I ignore it here for convenience of exposition.

The results for expected contact in local areas can be quite different when contact with Whites (p) is assessed using counts for *area population* instead of counts for *neighbors*. Expected contact with Whites (E[p]) will now reflect the weighted average of two contributions. The first contribution is the household's contact with White neighbors (p_N). As noted in the previous paragraph, this reflects a random draw of Whites and Blacks and its expected value is equal to P for both Whites and Blacks. The second contribution is the household's self-contact with Whites (p_S). The value of self-contact will be 1 for White households and 0 for Black households so the contribution of self-contact to contact with Whites in the area population (p) varies systematically by race. The relative contribution of the two components of contact depends on the value of area (pairwise) population size (t). Contact with Whites based on area population can be given by

$$p = p_N \cdot (t-1)/t + p_S \cdot (1/t) \cdot$$

where t is area population and $t-1$ is the number of neighbors a household has.

Under random distribution, the expected value of the term $p_N \cdot (t-1)/t$ is the same for every household in the city. But the term $p_S \cdot (1/t)$ is systematically different for Whites and Blacks. Specifically, p_S is 0/t for Blacks and 1/t for Whites. This causes the expected value of the White-Black difference in mean contact with Whites to differ by 1/t.

To further clarify, I examine expected contact separately by race. A White household's expected number of White neighbors under random assignment is given by the household's number of neighbors $(t-1)$ multiplied by expected contact with Whites for neighbors (p_N) which as noted above is $E\left[P'_{WW}\right] = P = W/(W+B)$. Unsurprisingly, the White household's expected self-contact with Whites in the area population (p_S) is 1. As a result the expectation for White contact with Whites in the standard contact formulation based on area population (i.e., $E[P_{WW}]$) can be given as follows.

$$E\left[P_{WW}\right] = E\left[P'_{WW}\right] \cdot \left((t-1)/t\right) + 1.0 \cdot (1/t)$$

A Black household's expected number of White neighbors under random assignment is the same as that expected for a White household. It is given by the household's number of neighbors $(t-1)$ multiplied by expected contact with Whites for neighbors (p_N) which as noted above is $E\left[P'_{BW}\right] = P = W/(W+B)$. Unsurprisingly, the Black household's expected self-contact with Whites in the area population (p_S)

is 0. As a result the expected value for Black contact with Whites in the standard contact formulation based on area population (i.e., $E[P_{BW}]$) can be given as follows.

$$E\left[P_{BW}\right]=E\left[P'_{BW}\right]\cdot\left(\left(t-1\right)/t\right)+0.0\cdot\left(1/t\right)$$

Because $E\left[P'_{WW}\right]=E\left[P'_{BW}\right]=P$*, it is now becomes clear that upward bias in the separation index (S) traces solely to role of self-contact in the group contact calculations for the standard formula for the index.*

In the difference of means formulation $S=P_{WW}-P_{BW}$ (given here in pairwise P* contact notation) and the expected value of S is given by the expected value of its components. That is, $E\left[S\right]=E\left[P_{WW}\right]-E\left[P_{BW}\right]$. This can be evaluated as follows.

$$E\left[S\right]=E\left[P_{WW}\right]-E\left[P_{BW}\right]$$

$$E\left[S\right]=\left[\left(\left(t-1\right)/t\right)\cdot E\left[P'_{WW}\right]+\left(1/t\right)\cdot 1.0\right]-\left[\left(\left(t-1\right)/t\right)\cdot E\left[P'_{BW}\right]+\left(1/t\right)\cdot 0.0\right]$$

$$E\left[S\right]=[\left(\left(t-1\right)/t\right)\cdot E\left[P'_{WW}\right]-\left[\left(\left(t-1\right)/t\right)\cdot E\left[P'_{BW}\right]+\left[\left(1/t\right)\cdot 1\right]-\left(1/t\right)\cdot 0\right]$$

$$E\left[S\right]=\left[\left(\left(t-1\right)/t\right)\cdot P_{W}-\left(\left(t-1\right)/t\right)\cdot P_{W}\right]+\left[\left(1/t\right)\cdot 1\right]-\left(1/t\right)\cdot 0]$$

$$E\left[S\right]=\left(1/t\right)\cdot 1-\left(1/t\right)\cdot 0$$

$$E\left[S\right]=1/t$$

Note that this result is identical to the expected value for S previously established and reported by Winship (1977: 1064).

Now consider the expected value for the separation index when contact for individuals is assessed using counts for neighbors instead of counts for area population.[8]

$$E\left[S'\right]=E\left[P'_{WW}\right]-E\left[P'_{BW}\right]$$

$$E\left[S'\right]=P-P$$

$$E\left[S'\right]=0$$

[8] Again, this assumes city size is sufficiently large that an individual household's contribution to P is negligible.

This establishes that an unbiased version of the separation index (i.e., S′ with $E\left[S'\right]=0$) can be obtained by eliminating the role of self-contact when assessing each individual's contact with the reference group.

15.4 Situating This Result and Its Implications in the Difference of Means Framework

I now recast the results for S just presented in the notation of the more general difference of means framework. In that framework the standard formula for S is

$$\mathrm{S} = 100\cdot\left(\bar{Y}_1-\bar{Y}_2\right)=\left(1/\mathrm{W}\right)\cdot\Sigma\left(\mathrm{w_i}\cdot\mathrm{y_i}\right)-\left(1/\mathrm{B}\right)\cdot\Sigma\left(\mathrm{b_i}\cdot\mathrm{y_i}\right).$$

When computing S by this formula, values of y_i are set according to the index-specific scaling function $y=f(p)$. In the case of S, the scaling function is the identity function and thus $y_i=p_i$. Accordingly, the contact formula for S White-Black segregation given in the preceding section

$$\mathrm{S} = \mathrm{P_{WW}}-\mathrm{P_{BW}} = \left(1/\mathrm{W}\right)\cdot\Sigma\left(\mathrm{w_i}\cdot\mathrm{p_i}\right)-\left(1/\mathrm{B}\right)\cdot\Sigma\left(\mathrm{b_i}\cdot\mathrm{p_i}\right)$$

can be converted into the difference of means formula for S by simply substituting y_i for p_i.

I introduce the unbiased version of the separation index (S′) first for two reasons. One is that, as mentioned earlier, it was the first index for which I was able to establish an unbiased version. The second is that the nature of bias for S is especially straightforward and easy to explain. But S is not a special case among indices of uneven distribution. The core strategy of revising the formula to remove the contribution of self-contact can be applied to any index of uneven distribution that can be placed in the difference of means framework.

In standard index calculations group contact is assessed using area population counts and thus reflects the weighted average of two components. The first component registers contact with *neighbors*. This expected value of this component of contact is the same for all individuals and groups in the comparison and so does not contribute to index bias. The second component registers self-contact which is fixed for every individual and differs systematically by group. This introduces bias by systematically inflating contact scores for members of the reference group and reducing contact scores for members of the comparison group. Eliminating the second component from contact calculations yields unbiased group means on contact scores and this results in an unbiased index score.

To summarize, the following two important conclusions apply to all popular indices of uneven distribution – including G, D, A, R, and H – that can be place in the difference of means framework.

- bias in standard index formulations traces to calculating group contact (p_i) for households based on area population counts, and
- unbiased versions of the index can be obtained by calculating group contact based on counts for neighbors.

I now briefly review how these conclusions generalize and apply to other popular and widely used indices of uneven distribution.

15.4.1 Expected Distributions of p' and y' Under Random Assignment

When households are randomly assigned to areas, the expected distribution of raw contact scores calculated using counts for *neighbors* (hereafter designated p_i') will be the same for both Whites and Blacks. As a result, expected values for group means on scaled exposure (y_i') scored based on *any* index-specific scaling of "raw" contact among neighbors (p_i') will be the same for both Whites and Blacks (i.e., $E\left[Y_W'\right] = E\left[Y_B'\right]$).

This can be established as follows. The expected distribution of values for raw contact with the reference group (p_i') calculated using counts for neighbors will be given by the binomial probability distribution for a given number of neighbors. This expected distribution will be the same regardless of whether the focal household for this set of neighbors is White or Black. Thus, the expected distribution of p_i' will be the same for Whites and Blacks. Values of contact with the reference group (p_i') determine residential outcome scores (y_i'). So the expected distribution of contact scores (p_i') directly determines the expected distribution of residential outcomes scores (y_i'). This also will be the same for Whites and Blacks. The expected distribution of residential outcomes (y_i') determines the expected mean on scaled contact (Y') and this also will be the same for Whites and Blacks. Because the expected means on scaled contact are the same for Whites and Blacks (i.e., $Y_W' = Y_B'$), the expected group difference of means (i.e., $Y_W' = Y_B'$), difference under random assignment is zero. This leads to the following general conclusion.

> *Scores for indices computed as a difference of means in scaled contact with the reference group calculated for neighbors (instead of area population) will be unbiased. That is, the expected value of index scores under random assignment will be zero (0.0).*

15.5 Reviewing a Simple Example in Detail

It is instructive to review a simple example in some detail to show how expected group means on residential outcomes (y) differ depending on whether an individual's contact with the reference group (p) is assessed using counts for neighbors or counts for area population. For purposes of illustration I consider the example of a

Table 15.1 Calculations to obtain values of D and S for White-Black segregation from differences of group means on residential outcomes (y) based on contact with Whites for area population and among neighbors under random distribution

Count of Whites	Whites p (×100)	Blacks p (×100)	Share of Whites	Share of Blacks	Whites y_D (×100)	Blacks y_D (×100)	Whites y_S (×100)	Blacks y_S (×100)
Among neighbors								
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1–11	–	–	–	–	–	–	–	–
12	60.00	60.00	0.04	0.04	0.00	0.00	60.00	60.00
13	65.00	65.00	0.20	0.20	0.00	0.00	65.00	65.00
14	70.00	70.00	0.89	0.89	0.00	0.00	70.00	70.00
15	75.00	75.00	3.19	3.19	0.00	0.00	75.00	75.00
16	80.00	80.00	8.98	8.98	0.00	0.00	80.00	80.00
17	85.00	85.00	19.01	19.01	0.00	0.00	85.00	85.00
18	90.00	90.00	28.52	28.52	100.00	100.00	90.00	90.00
19	95.00	95.00	27.02	27.02	100.00	100.00	95.00	95.00
20	100.00	100.00	12.16	12.16	100.00	100.00	100.00	100.00
Sum or mean			100.00	100.00	67.69	67.69	90.00	90.00
For area population								
0	N/A	0.00	0.00	0.00	N/A	0.00	N/A	0.00
1–11	–	–	–	–	–	–	–	–
12	57.14	57.14	0.01	0.04	0.00	0.00	57.14	57.14
13	61.90	61.90	0.04	0.20	0.00	0.00	61.90	61.90
14	66.67	66.67	0.20	0.89	0.00	0.00	66.67	66.67
15	71.43	71.43	0.89	3.19	0.00	0.00	71.43	71.43
16	76.19	76.19	3.19	8.98	0.00	0.00	76.19	76.19
17	80.95	80.95	8.98	19.01	0.00	0.00	80.95	80.95
18	85.71	85.71	19.01	28.52	0.00	0.00	85.71	85.71
19	90.48	90.48	28.52	27.02	100.00	100.00	90.48	90.48
20	95.24	95.24	27.02	12.16	100.00	100.00	95.24	95.24
21	100.00	N/A	12.16	N/A	100.00	N/A	100.00	N/A
Sum or mean			100.00	100.00	67.69	39.17	90.48	85.71

Notes: "N/A" indicates the combination does not occur. "–" indicates outcomes are omitted because their frequency is negligible

hypothetical city where the population consists of only Whites and Blacks, proportion White for the city (P) is equal to 0.90, and area size (t_i) is equal to 21 households.[9] Table 15.1 presents the expected distributions for contact scores (p) and index-specific residential outcomes scores (y) for the dissimilarity index (D) and the

[9] The number of households is substantially higher than would be found in typical census blocks but substantially lower than would be found in typical census block groups.

Table 15.2 Calculations to obtain values of R and H for White-Black segregation from differences of group means on residential outcomes based on contact with Whites for area population and among neighbors under random distribution

Count of Whites	Whites p (×100)	Blacks p (×100)	Share of Whites	Share of Blacks	Whites y_R (×100)	Blacks y_R (×100)	Whites y_H (×100)	Blacks y_H (×100)
Among neighbors								
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1–11	–	–	–	–	–	–	–	–
12	60.00	60.00	0.04	0.04	28.99	28.99	42.11	42.11
13	65.00	65.00	0.20	0.20	31.24	31.24	45.70	45.70
14	70.00	70.00	0.89	0.89	33.74	33.74	49.56	49.56
15	75.00	75.00	3.19	3.19	36.60	36.60	53.79	53.79
16	80.00	80.00	8.98	8.98	40.00	40.00	58.54	58.54
17	85.00	85.00	19.01	19.01	44.24	44.24	64.06	64.06
18	90.00	90.00	28.52	28.52	50.00	50.00	70.83	70.83
19	95.00	95.00	27.02	27.02	59.23	59.23	80.08	80.08
20	100.00	100.00	12.16	12.16	100.00	100.00	100.00	100.00
Sum or mean			100.00	100.00	55.96	55.96	73.69	73.69
For area population								
0	N/A	0.00	0.00	0.00	N/A	0.00	N/A	0.00
1–11	–	–	–	–	–	–	–	–
12	57.14	57.14	0.01	0.04	27.79	27.79	40.15	40.15
13	61.90	61.90	0.04	0.20	29.82	29.82	43.45	43.45
14	66.67	66.67	0.20	0.89	32.04	32.04	46.95	46.95
15	71.43	71.43	0.89	3.19	34.51	34.51	50.73	50.73
16	76.19	76.19	3.19	8.98	37.35	37.35	54.87	54.87
17	80.95	80.95	8.98	19.01	40.73	40.73	59.52	59.52
18	85.71	85.71	19.01	28.52	44.95	44.95	64.93	64.93
19	90.48	90.48	28.52	27.02	50.68	50.68	71.57	71.57
20	95.24	95.24	27.02	12.16	59.85	59.85	80.63	80.63
21	100.00	N/A	12.16	N/A	100.00	N/A	100.00	N/A
Sum or mean			100.00	100.00	56.56	46.34	74.35	66.04

Notes: “N/A” indicates the combination does not occur. “–” indicates outcomes are omitted because their frequency is negligible

separation index (S) under random residential distributions based on a binomial probability model. Table 15.2 presents similar results for the Hutchens square root index (R) and the Theil entropy-based index (H). The first panel in each table gives the results when households’ contact with Whites is assessed using counts for neighbors. The second panel in each table gives the parallel results when households’ contact with Whites is assessed using counts for area population in the standard way.

I first review the results in Table 15.1. The first column in the first panel of the table lists the possible counts for Whites among neighbors. The areas in the example have 21 total households so every household has exactly 20 neighbors, a situation that would be common when measuring segregation using block-level data. Except for the outcome of 0, which warrants separate comment, the outcomes for counts of White neighbors below 12 are omitted from the listing because their occurrence under random distribution is quantitatively negligible. The values of proportion White among neighbors (p′) is given separately for Whites and Blacks in the next two columns. Note that proportion White among neighbors is the same for both Whites and Blacks under all possible combinations. The share – that is, the proportion – of households in the group expected to experience each of the possible levels of contact under random distribution is given separately for Whites and Blacks in the next two columns. Note that group shares at every outcome are the same for both White and Black households. Scores of residential outcomes y′ scored from p′ using in computing the dissimilarity index (D) under the difference of means calculation approach are reported separately for Whites and Blacks in the next two columns. Scores of residential outcomes (y′) relevant for computing the separation index (S) are reported separately for Whites and Blacks in the last two columns. The results for the expected group means on index-specific residential outcomes are given in the bottom row of the panel. These are obtained by summing the products of group shares and residential outcomes scores (y′).

Table 15.2 continues the exercise and has the same structure as Table 15.1. The only difference is that it provides information on the residential outcomes (y′) that are used in computing the Hutchens square root index (R) and the Theil entropy index (H).

The results for the analysis in the first panels in Tables 15.1 and 15.2 are easy to summarize. For all four indices – D, S, R, and H, Whites and Blacks both experience all possible outcomes on p′ and both groups identical expected distributions across possible outcomes on number of White neighbors. Accordingly, they have identical expected values for the means on the unbiased version of the residential outcome scores (y′) that determine each segregation index score. Consequently, the expected values of D′, S′, R′, and H′ all are zero (0.0). For example, proportion White among neighbors equals the city-wide proportion (0.90) when the count of White neighbors is 18, 19, or 20. Residential outcomes (y′) relevant for calculating D are scored 1.0 in these cases and 0.0 in all other cases. Column 4 shows that 67.69 % of Whites experience this residential outcome. Column 5 shows that the same is true for Blacks. Accordingly, the expected mean for the 0–1 scoring of y scored for D is 0.6769 for both Whites and Blacks (values shown in the final row of columns 6 and 7). This result shows that Whites and Blacks are equally likely to reside in areas where their contact with White neighbors equals or exceeds the proportion White in the city as a whole. As a result, the expected value of D′ is 0.0 (i.e., $E\left[D'\right] = \left(E\left[Y'_W\right] - E\left[Y'_B\right]\right) = \left(0.6769 - 0.6769\right)$).

The group means reported in columns 8 and 9 show that Whites and Blacks also experience identical average levels of contact with Whites neighbors; specifically, on average 90.0 % of their neighbors are White, a level of contact matching the representation of Whites in the city population overall. So the expected value of S′ also is 0.0 (i.e., $D\left[S^{'}\right]=\left(E\left[Y_{W}^{'}\right]-E\left[Y_{B}^{'}\right]\right)=\left(0.9000-0.9000\right)$). Similar results are seen when residential outcomes are scored as relevant for computing the Hutchens square root index (R′) (i.e., $E\left[R^{'}\right]=0.0=\left(E\left[Y_{W}^{'}\right]-E\left[Y_{B}^{'}\right]\right)=\left(0.5596-0.5596\right)$) and the Theil entropy index (H′) (i.e., $E\left[H^{'}\right]=0.0=\left(E\left[Y_{W}^{'}\right]-E\left[Y_{B}^{'}\right]\right)=\left(0.7369-0.7369\right)$).

These results are easy to summarize. When neighbors are a random draw, Whites and Blacks have identical probability distributions for experiencing different levels of unbiased contact with White neighbors (p′). It then follows that Whites and Blacks also have identical group means on residential outcomes (y′) scored from unbiased contact with White neighbors (p′).

I now review the results in the second panel of Tables 15.1 and 15.2 where contact with Whites is computed in the standard way based on counts for area population. The results here play out much differently. The key change producing the differences is that counts in the numerator and denominator of the calculation of proportion White (p) now include the focal household. Accordingly, the value for a household's contact with Whites (p) based on area population reflect a weighted average of the household's contact with Whites for neighbors (p′) and the household's self-contact with Whites designated here by p_S which is $1=\left(1/1\right)$ for White households and $0=\left(0/1\right)$ for Black households. The relevant expression is

$$p\ =\ p^{'}\cdot\left(20/21\right)+p_{S}\cdot\left(1/21\right)$$

The distribution of values for contact with Whites among neighbors (p′) remains the same as before. This means that all changes in contact with Whites in the lower panel trace to the impact of self-contact with Whites (p_S) which is systematically different for Whites and Blacks.

To see the implications it is useful to consider how the results change for a household with 18 White neighbors, the case that in this example has important implications for the expected value of the dissimilarity index. For both White and Black households who have 18 White neighbors the value of contact with Whites among neighbors (p′) is 0.90 and results in a value of $y^{'}=1$ when residential outcomes (y′) are scores as relevant for the dissimilarity index (D). The results change when contact with Whites is based on area population (p). For a White household the value of contact with Whites based on area population (p) is given by

$$\begin{aligned}p&=p^{'}\cdot\left(20/21\right)+p_{S}\cdot\left(1/21\right)\\&=\cdot\left(18/20\right)\cdot\left(20/21\right)+\left(1/1\right)\cdot\left(1/21\right)\cdot\\&=\left(0.90\cdot0.9524\right)+0.0476\\&=0.8571+0.0476\\&=0.9048.\end{aligned}$$

For a Black household the value of p is given by

$$\begin{aligned} p &= p' \cdot (20/21) + p_S \cdot (1/21) \\ &= (18/20) \cdot (20/21) + (0/1) \cdot (1/21) \cdot \\ &= (0.9524 \cdot 0.90) + 0.0 \\ &= 0.8571 + 0.0 \\ &= 0.8571. \end{aligned}$$

The White and Black households have identical contact with Whites among neighbors and accordingly in the upper panel are scored identically on the residential outcome ($y' = 1$) relevant for computing D′. But in the lower panel the residential outcome (y) relevant for computing D is scored 1 for the White household – based on $0.9048 \geq 0.90$ – and 0 for the Black household – based on $0.8571 < 0.90$.

The expected proportion of households that have 18 White neighbors is 0.2852 for both Whites and Blacks. The difference in how these households are scored on scaled contact with Whites in the upper and lower panels contributes to determining the level of bias in D. Whites are scored the same in both the upper and lower panels; $y' = y = 1$. But Blacks are scored differently in the upper and lower panels; $y' = 1$ in the upper panel and $y = 0$ in the lower panel. This difference reduces the expected Black mean on scaled contact with Whites from $E\left[Y_B'\right] = 0.6769$ based on neighbors in the upper panel to $E\left[Y_B\right] = 0.3917$ based on area population in the lower panel. In contrast, the expected White mean on scaled contact with Whites is the same – $E\left[Y_W'\right] = E\left[Y_W\right] = 0.6769$ – under both calculations. Thus, the expected value of D changes from 0.0 when contact with Whites (p′) is based on neighbors ($E\left[D'\right] = E(Y_W'] - E(Y_B'] = 0.6769 - 0.6769 = 0.0$) to 0.2852 when contact with Whites (p) is based on area population ($E\left[D\right] = E(Y_W] - E(Y_B] = 0.6769 - 0.3917 = 0.2852$).

Scaling to 100 in keeping with convention, the "bias" in the standard version of the index of dissimilarity (D) under random distribution is 28.52. The parallel calculations for the separation index (S) ($E\left[S\right] = E(Y_W] - E(Y_B] = 0.9048 - 0.8571 = 0.0477$) indicate that bias in the standard version is 4.77. The interested reader can confirm that these values for E[D] and E[S] are equal to values of E[D] and E[S] obtained using analytic formulas given in Winship (1977).

This example reveals in detail how bias enters into the picture and distorts scores for standard versions of indices of uneven distribution. The example also documents how the simple refinement of assessing group contact based on neighbors instead of area population eliminates index bias for all indices of uneven distribution that can be placed in the differences of means formulation. The basis for this welcome result is easy to summarize. When self-contact is eliminated from that calculation, the two groups in the comparison will have identical expected distributions for the number of neighbors from the reference group and the number of neighbors from the comparison group. It then follows that expected group means on residential outcomes (y′) scored of the distribution of unbiased contact values (p′) will be identical for both groups.

15.5.1 Additional Reflections on Results Presented in Tables 15.1 and 15.2

The analysis presented in Tables 15.1 and 15.2 clarifies how index bias originates in the role of self-contact. The results provide an intuitive basis for understanding why bias is greater when effective area size (ENS) is small. It is because self-contact will have a bigger impact on assessments of an individual's contact with the reference group when area counts are small as they are in this example. If the same exercise were repeated with area population size set to 5,001 instead of 21, the resulting magnitude of index bias would be much smaller. Alternatively, if the exercise were repeated with area counts of 9 (equivalent to a "Queen's" neighborhood of eight adjacent neighbors plus the focal household), the magnitude of index bias would be even larger.

Reflecting on the difference between unbiased contact (p′) and standard contact (p) also yields additional insight into why the expected level of bias varies from index to index. The role of self-contact in standard calculations of contact is to shift the distribution of values of p up for the reference group and down for the comparison group. These shifts in p are then translated into impacts on scaled contact (y) based on the index-specific scaling function $y = f(p)$. I established earlier that the scaling functions for G, D, R, and H are nonlinear. The nonlinearity has implications for bias. Specifically, *bias at the level of group differences on raw contact (p) will translate into larger group differences in scaled contact (y) when the scaling function is nonlinear and the magnitude of bias is greater when the scaling function is more strongly nonlinear*. This provides a succinct explanation for why levels of bias are higher for G and D compared to R and H and why the level of bias is lowest for S. The scaling function $y = f(p)$ for S is linear; so bias impacting the value of p is carried forward unchanged. The scaling functions for G and D depart from linearity the most; so bias impacting p is "amplified" to a greater degree when values of y are assigned for these indices. The scaling functions for R and H involve milder nonlinearity; so, while bias impacting p also is amplified when values of y are assigned, the resulting distortion is not as dramatic.

Finally, this also provides an explanation for why bias in S does not vary with group size, but bias in the other measures, and especially in G and D, does vary with group size. The reason is that the nonlinear scaling functions for G and D measures become more strongly nonlinear when groups are unequal in size. This means that the role of nonlinearity in exaggerating group differences in y scored from p is magnified for these measures when groups are more imbalanced in size.

15.6 Summary

This chapter reviews how the difference of means formulation of indices of uneven distribution leads to new insights about the nature of index bias and makes it possible to address index bias at the point of measurement. The insight is that, when segregation is cast as group differences in means on scaled group contact, bias can be traced to a relatively simple source; namely, the role of self-contact which inherently and unsurprisingly differs by race. Eliminating self-contact from index calculations by assessing group contact based on neighbors instead of area population eliminates this inherent source of bias in index scores. The chapter shows that resulting "unbiased" versions of unbiased indices are attractive for many reasons. They are attractive on formal grounds because analysis based on binomial probability models shows that they have expected values of zero under random assignment. They are attractive because the index refinements are easy to explain; for any individual group contact can be a random draw when computed using neighbors but it is always inherently biased when computed using area population that includes the individual. Finally, the unbiased versions of indices introduced here are attractive because they allow researchers to use familiar indices and apply familiar substantive interpretations as well as new interpretations.

The next chapter presents evidence on another aspect of the unbiased versions of indices of uneven distribution introduced here; their behavior over varying circumstances of study design. It uses simulation methodology to generated residential distributions over a wide range of circumstances and shows that the unbiased versions of popular indices introduced in this chapter behave as desired in circumstances where bias renders scores for standard versions of the indices untrustworthy and potentially misleading. It also shows that unbiased indices are attractive because they near-exactly replicate the behavior of standard versions of indices in situations where bias is negligible and they yield clearly superior assessments of segregation in situations where the impact of bias on standard versions of indices is non-negligible.

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Chapter 16
Comparing Behavior of Unbiased and Standard Versions of Popular Indices

In the previous chapter I outlined the rationale for unbiased versions of indices of uneven distribution. Additionally, I presented results from analysis of expected group residential distributions under a binomial probability model to establish that the unbiased versions of popular indices have expected values of zero when residential distributions are random. In this chapter I report analyses of the behavior of standard and unbiased versions of indices of uneven distribution to document two things: the potential undesirable impact of bias on the scores of standard versions of indices and the attractive behavior of the scores of unbiased versions of the same indices.[1]

To document index behavior I conducted a series of simulation experiments to systematically "exercise" standard and unbiased versions of popular indices under a wide range of demographic contexts and neighborhood definitions. I performed the analyses using residential distributions generated by SimSeg, a computational model that simulates residential segregation dynamics. The SimSeg program has been described in more detail elsewhere (e.g., Fossett and Waren 2005; Fossett 2006, 2011a, b; Fossett and Dietrich 2009; Clark and Fossett 2008). Examining results generated by the SimSeg program is useful for the purposes of this chapter for two reasons. First, the program implements routines that calculate both standard and unbiased versions of G, D, R, H, and S. Second, the program can systematically generate residential distributions over a wide range of study designs that can reveal how the behavior of standard and unbiased versions of indices differ under varying circumstances.

Using SimSeg I designed and executed simulation experiments that implemented a two-group city in which segregation is assessed using bounded neighborhoods of uniform size. The two groups in the simulation are of course "virtual", but for convenience of discussion and consistency with examples discussed in earlier chapters

[1] The analyses I report in this chapter elaborate and extend analyses I conducted for an earlier study on this topic (Fossett and Zhang 2011)

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_16

I refer to them as "White" and "Black". I varied the conditions of the experiments to exercise index behavior by varying the racial mix of the city randomly from 2 to 98 % White separately in each experiment. I then ran 2,500 experiments separately for each of eight neighborhood sizes based on a square housing grid for the bounded area ranging from 3 to 10 houses on a side. The resulting neighborhood sizes were 9, 16, 25, 36, 49, 64, 81, 100, and 225. The simulation experiments conducted using these varying settings for neighborhood size and city racial composition were relatively simple.[2] The program first created the relevant virtual neighborhoods and housing units within them. Next it created the virtual population of households according to the racial demography setting. It then distributed households distributed randomly across housing units. Then, it calculated and recorded a battery of segregation index scores including scores for standard versions of all popular measures of uneven distribution G, D, A, R, H, and S and unbiased versions for G, D, R, H, and S.[3]

Tables 16.1 and 16.2 report the means and standard deviations for scores for standard versions of indices of uneven distribution under random distribution at the initialization of the city landscape over varying conditions of effective neighborhood size (ENS) and percent White for the city (P). For economy of presentation, results are given only for ENS settings of 9, 16, 25, 49, 100, and 225. Inspection of Table 16.1 shows that the level and pattern of index scores varies systematically by index and over different combinations of settings for ENS and P. Inspection of the results presented in Table 16.2 shows that index scores vary in a relatively narrow range around the mean for index scores under any particular combination of settings for ENS and P and the results also show that the degree of dispersion in index scores is generally similar in magnitude across different indices.

Figure 16.1 provides visual documentation of the patterns of index behavior summarized in Tables 16.1 and 16.2. The figure provides separate graphs for each index considered; namely, G, D, A, R, H, and S. Each graph plots the values of the relevant index score calculated from the random residential distribution at the beginning of the simulation experiment (i.e., cycle 0) against percent White in the city population (P). The graphs plot index scores for the simulations where effective neighborhood size (ENS) is set to value of 9, 16, 25, 49, and 100. In addition, each graph also plots a black line tracing the expected index score (e.g., E[D]) based on calculations using a binomial model (per Winship 1977). To reduce visual clutter and facilitate clarity of patterns, the graphs do not depict results for ENS settings of

[2] Other details of the simulations are uniform across all simulations and have no impact on results. For example, neighborhoods are arranged to form an approximately circular form for the overall city. The dimensions of the city were calibrated to yield between 6400 and 8500 virtual households depending on number of households per neighborhood and the number of neighborhoods in the simulated city. The resulting virtual cities are similar in form to those described in Fossett (2006, 2011a, b). I conducted additional simulations using larger cities with more neighborhoods and more virtual households. All relevant index behavior was fundamentally similar. So I used smaller cities to keep the computational burdens for generating the analysis data sets at reasonable levels.

[3] Results for an unbiased version of Atkinson's A are not shown because I have not been able to place this index in the difference of means framework. Hutchens R is a closely related measure.

Table 16.1 Means for standard versions of popular indices of uneven distribution computed for random residential distributions under varying combinations of relative group size (P) and neighborhood size

		Neighborhood size					
Index	P[a]	9	16	25	49	100	225
Gini (G)	≤5	81.4	70.5	60.1	45.2	33.0	22.0
	11–15	56.5	43.7	35.1	25.1	17.6	11.7
	36–50	40.2	30.1	24.0	17.1	12.0	8.0
Dissimilarity (D)	≤5	77.7	62.8	48.0	33.4	24.0	15.6
	11–15	40.8	31.6	25.2	17.9	12.5	8.3
	36–50	28.7	21.4	17.1	12.2	8.5	5.7
Atkinson[b] (A)	≤5	78.4	64.4	49.8	28.4	12.9	4.5
	11–15	41.9	23.3	13.0	5.5	2.6	1.1
	36–50	14.5	7.5	4.7	2.4	1.1	0.5
Hutchens (R)	≤5	54.0	40.8	29.6	15.6	6.7	2.3
	11–15	23.8	12.4	6.7	2.8	1.3	0.5
	36–50	7.6	3.8	2.4	1.2	0.6	0.3
Theil (H)	≤5	34.8	24.1	16.9	9.0	4.4	1.8
	11–15	18.8	10.5	6.4	3.1	1.5	0.6
	36–50	9.9	5.3	3.3	1.7	0.8	0.4
Separation (S)	≤5	12.3	6.9	4.4	2.2	1.1	0.5
	11–15	12.3	7.0	4.4	2.3	1.1	0.5
	36–50	12.3	6.9	4.4	2.3	1.1	0.5

[a]Here P denotes the city-wide group percentage for the smaller group in the comparison
[b]Atkinson index (A) is computed with δ set at 0.5, the value at which A is "symmetric"

36, 64, 81, and 225. However, Table 16.2 documents that the results for these settings are consistent with the results shown in the figure. For example, the means for index scores when ENS is set at 36, 64, and 81 fall between the scores for ENS settings immediately above and below the ENS setting in question.

The results presented in Fig. 16.1 document several clear patterns. First, all of the indices take values above zero in each and every simulation trial reflected in the 12,500 data points plotted in the figure. The gray points for individual simulation trials indicate that index scores calculated from the random residential distributions at initialization in individual simulation trials vary in relatively narrow ranges around their expected values based on binomial theory. The Black lines show that the expected values of the indices based on analytic calculations vary systematically with effective neighborhood size and percent White in the city population. As noted earlier, the nature of the systematic variation in index scores is simple in its main features. For all indices, scores for both the expected values under random assignment and the observed random segregation at initialization in the simulations are systematically higher when effective neighborhood size (ENS) is lower. Thus, the highest curve is for the set of simulations that use the lowest value of ENS (in this case 9) and the curves move systematically lower as ENS moves to successively higher values. Also, for all indices *except the separation index (S)*, both the expected

Table 16.2 Standard deviations for standard versions of popular indices of uneven distribution computed for random residential distributions under varying combinations of relative group size (P) and neighborhood size

		Neighborhood size					
Index	P[a]	9	16	25	49	100	225
Gini (G)	≤5	4.9	6.6	7.2	7.4	5.7	3.9
	11–15	2.4	2.4	2.4	2.1	1.4	1.0
	36–50	1.2	1.2	1.2	1.2	0.9	0.6
Dissimilarity (D)	≤5	6.4	9.6	10.3	5.9	4.2	2.8
	11–15	1.8	2.2	1.9	1.5	1.1	0.8
	36–50	0.9	0.9	0.9	0.9	0.6	0.4
Atkinson[b] (A)	≤5	6.1	8.9	10.4	10.7	6.3	2.0
	11–15	3.8	3.3	2.5	1.1	0.4	0.2
	36–50	1.2	0.6	0.5	0.3	0.2	0.1
Hutchens (R)	≤5	6.7	7.6	7.5	6.5	3.4	1.0
	11–15	2.5	1.9	1.3	0.6	0.2	0.1
	36–50	0.6	0.3	0.2	0.2	0.1	0.1
Theil (H)	≤5	3.9	3.9	3.5	2.7	1.5	0.6
	11–15	1.4	1.1	0.9	0.5	0.2	0.1
	36–50	0.6	0.4	0.3	0.2	0.1	0.1
Separation (S)	≤5	0.7	0.5	0.4	0.3	0.2	0.1
	11–15	0.6	0.5	0.4	0.3	0.2	0.1
	36–50	0.7	0.5	0.4	0.3	0.2	0.1

[a]P is the city-wide, pairwise group percentage for the smaller group in the comparison
[b]The Atkinson index (A) is computed with "tuning" value δ set at 0.5, the value at which A is symmetric

values and the observed random outcomes are systematically higher when proportion White for the city (P) departs from balance at 0.50 and the expected values and observed outcomes take especially high values when P falls below 0.10 or rises above 0.90.

The existing methodological literature has documented similar patterns of random variation for D many times before and also occasionally for G. But reports on patterns of variation for expected values of A, R, H, and S under random assignment are rare if they exist at all. To my knowledge, the results presented here are the first to systematically compare the bias behavior of all popular indices of uneven distribution.

Comparing the figures for each index reveals several noteworthy differences in their behavior under random assignment. One obvious pattern is that indices vary considerably in the magnitude of bias under random assignment. The highest expected values under random assignment are observed for G followed closely by D and then A. The lowest scores under random assignment are for S. H and R have the next lowest scores for expected values. The "takeaway" point here is that D, the most popular and widely used index of uneven distribution has higher expected values under random assignment than all other indices except G.

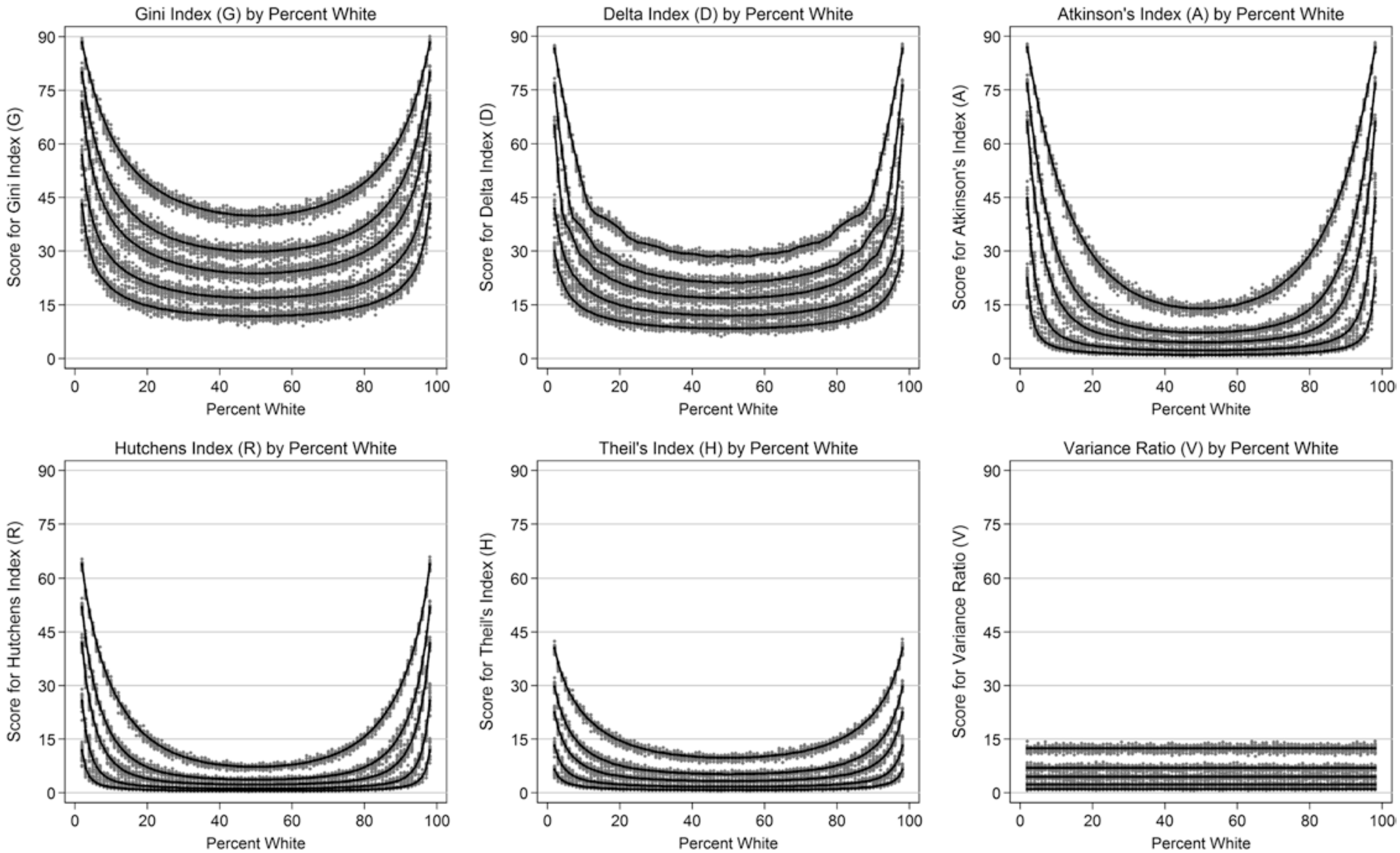

Fig. 16.1 Scores for "Standard" versions of indices of uneven distribution under random assignment by city percent White and neighborhood size (Note: Points plotted in *light gray* are values calculated from random residential distributions. Points plotted in *black* are expected values (e.g., E[D]). Values for effective neighborhood size (ENS) are 9, 16, 25, 49, and 100)

Another clear pattern is that expected values under random assignment are lower for every index when effective neighborhood size (ENS) is larger. One additional finding is that, for all indices except S, index bias is highest, often alarmingly so, when group size is imbalanced. These findings provide at least some justification for two crude rules-of-thumb for research designs used in many segregation studies. One practice is that most studies in recent decades examine segregation scores calculated using data for spatial units with larger population counts (e.g., use tracts over blocks). This tends to promote, but does not guarantee, higher levels of effective neighborhood size which, all else equal, serves to reduce bias. Another practice is that studies often avoid analysis of comparisons involving groups that are small in relative population size. All else equal, this tends to exclude comparisons where bias is likely to be larger.

Another common practice in the empirical literature is to avoid analysis of comparisons that involve groups that are small in absolute population size. The results presented here provide no support for this practice. Analytic formulas for bias (e.g., Winship 1977) identify a clear role of neighborhood size and relative group size but they do not identify a role for absolute group size. Empirically, absolute size may be correlated with relative group size but only relative size has a consequence for index bias. So if one is screening cases on relative group size there is no justification for additional screening on absolute size, at least not for the purpose of avoiding problematic bias.[4]

Similarly, there is no support in these results for the practice of "dealing with bias" by weighting cases in aggregate-level analyses by the size of the minority population. Absolute group size has no bearing on bias. Accordingly, weighting cases on minority size serves only to skew results toward findings for cities with larger minority populations.

Figure 16.1 also reveals a few findings that are not currently widely appreciated. One is that *for most indices, and especially for G and D, effective neighborhood size (ENS) and group ratio (GR) interact such that index bias is especially high when ENS is low and GR is highly imbalanced*. This has an important practical implication. It indicates that the standard rules-of-thumb commonly used in restricting analysis samples in empirical studies are crude and are not necessarily reliable for their intended purpose of identifying cases prone to high levels of bias. The standard rules of thumb are crude first because they applied using a "rough-and-ready" cut points when bias behavior varies continuously across ENS and GR and second because the rules are applied in a simple additive way and do not take account of the important interaction between ENS and GR that is so clear in these results. As a result, the prevailing practices can easily exclude cases where bias may low enough to be viewed as negligible (e.g., $E[\bullet] < 2-3$); particularly when using R, H, and

[4] In Chap. 8 I noted that screening on absolute group size may be relevant for other reasons. For example, if group size is insufficient to "fill" 3–5 of the spatial units used in measuring segregation, scores for indices that measure residential polarization will likely be biased downward because the spatial units being used may be too large to capture concentrated displacement for the group in question.

S. Conversely, they can sometimes include cases where bias is high and problematic; particularly when using G, D, and A.

In sum, not only do current practices for dealing with bias greatly restrict the scope of segregation studies, they also are likely to be less reliable and effective for their intended purpose than researchers may realize. If researchers apply these practices in future research, they should revise them to take account of the findings reported here.

16.1 Documenting the Attractive Behavior of Unbiased Versions of Indices of Uneven Distribution

I now review the behavior of the new *unbiased* versions of popular indices of uneven distribution under random distribution at the initialization of the city landscape. Tables 16.3 and 16.4 report the means and standard deviations, respectively, for the sampling distributions of scores for the unbiased versions of the indices over the simulations conducted over varying conditions of effective neighborhood size (ENS) and percent White in the city (P). Figure 16.2 documents these patterns visually with separate graphs for G′, D′, R′, H′, and S′.[5] As with Fig. 16.1, each graph plots the values of the relevant index score at the beginning of the simulation experiment (i.e., cycle 0) against percent White in the city population. Also as before the individual graphs plot observed segregation outcomes from simulations in which effective neighborhood size (ENS) is variously set to 9, 16, 25, 49, and 100. I should note two important differences from Fig. 16.1. One is that the expected values of the unbiased indices (e.g., E[D′]) all are zero under calculations using an “exact” binomial model (per Winship 1977). So the resulting plotted “curve” for the expected values for all of the indices is a horizontal straight line centered on zero on the vertical (y) axis of the figure. The other is that the vertical range of the “y” axis of the figures is covers a much smaller range of scores than in Fig. 16.1. This aids in making visual inspection of patterns in Fig. 16.2. But it is important to take account of the difference when making visual comparisons with Fig. 16.1. The range of variation is much smaller in Fig. 16.2 but this is not visually obvious.

The graphs in Fig. 16.2 show that the unbiased index scores based on the 12,500 random residential distributions vary in an approximately bell-shaped distribution around zero and thus take both negative and positive values. The vertical dispersion of unbiased index scores around the expected value of zero gives intuitive insight into the expected sampling distribution of the scores for the unbiased versions of the different indices. The dispersion depicts the range and pattern of index scores that occur when there is no statistical association between race and residential location; that is when residential distributions are random. Intuitively, this provides a basis

[5] There is no graph for Atkinson’s A′ because I have not been able to place it in the difference of means framework. Hutchens R is a closely related index with an available difference of means formulation needed to develop and unbiased version of the index.

Table 16.3 Means for unbiased versions of popular indices of uneven distribution computed for random residential distributions under varying combinations of relative group size (P) and neighborhood size

		Neighborhood size					
Index	P[a]	9	16	25	49	100	225
Gini (G)	≤5	–0.1	–0.5	–0.4	–0.7	–1.0	–1.5
	11–15	–0.2	0.2	–0.5	–0.6	–0.7	–1.1
	36–50	–0.2	–0.4	–0.5	–0.8	–0.8	–1.0
Dissimilarity (D)	≤5	–0.0	–0.4	–0.4	–0.8	–0.2	–0.6
	11–15	–0.0	0.1	–0.1	0.3	–0.1	0.1
	36–50	–0.2	–0.2	–0.1	–0.1	–0.1	0.1
Hutchens (R)	≤5	–0.1	–0.2	–0.1	–0.1	–0.1	–0.1
	11–15	–0.1	0.2	–0.0	0.0	0.0	–0.0
	36–50	–0.0	–0.0	–0.0	–0.1	–0.0	0.0
Theil (H)	≤5	–0.0	–0.1	–0.1	–0.1	–0.1	–0.1
	11–15	–0.0	0.1	–0.0	0.0	0.0	–0.0
	36–50	–0.0	–0.0	–0.0	–0.0	–0.0	–0.0
Separation (S)	≤5	–0.0	–0.0	–0.0	–0.0	–0.0	–0.0
	11–15	–0.0	0.0	–0.0	–0.0	0.0	–0.0
	36–50	–0.0	–0.0	–0.0	–0.0	–0.0	–0.0

[a]P is the city-wide, pairwise group percentage for the smaller group in the comparison

Table 16.4 Standard deviations for unbiased versions of popular indices of uneven distribution computed for random residential distributions under varying combinations of relative group size (P) and neighborhood size

		Neighborhood size					
Index	P[a]	9	16	25	49	100	225
Gini (G)	≤5	4.9	5.3	5.8	6.7	5.2	3.2
	11–15	3.3	3.5	3.6	3.6	2.5	1.9
	36–50	2.4	2.4	2.4	2.4	1.8	1.2
Dissimilarity (D)	≤5	4.8	5.4	5.8	6.9	6.1	4.3
	11–15	3.3	3.6	3.9	4.6	3.5	3.3
	36–50	2.5	2.7	2.9	3.7	3.0	2.6
Hutchens (R)	≤5	3.4	3.3	3.2	3.0	1.6	0.5
	11–15	1.7	1.3	0.9	0.5	0.2	0.1
	36–50	0.5	0.3	0.2	0.2	0.1	0.0
Theil (H)	≤5	2.2	1.9	1.7	1.4	0.8	0.3
	11–15	1.2	0.9	0.7	0.4	0.2	0.1
	36–50	0.6	0.4	0.3	0.2	0.1	0.1
Separation (S)	≤5	0.8	0.5	0.4	0.3	0.2	0.1
	11–15	0.7	0.6	0.5	0.3	0.2	0.1
	36–50	0.8	0.5	0.4	0.3	0.2	0.1

[a]P is the city-wide, pairwise group percentage for the smaller group in the comparison

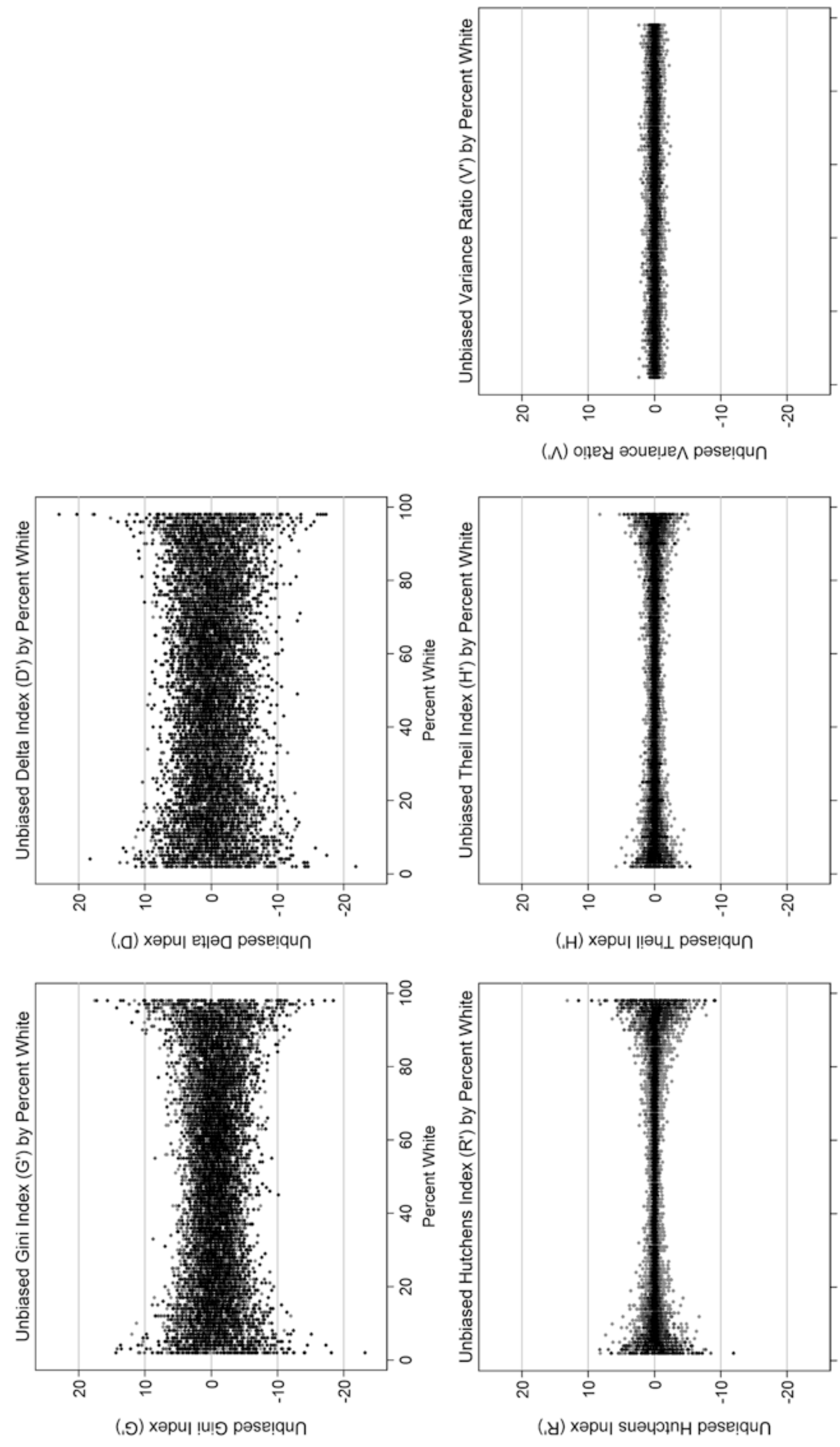

Fig. 16.2 Scores for unbiased versions of indices of uneven distribution under random assignment by percent White and neighborhood size (Note: Values for effective neighborhood size (ENS) are 9, 16, 25, 49, and 100. Cases for higher values of ENS are plotted in darker shades)

for evaluating observed scores for residential segregation. Observed scores that fall within the middle portion of the sampling distribution can easily occur by chance. But chance is a less plausible explanation for observed scores that fall in the low probability tails of the sampling distribution. Accordingly, scores in these regions are likely to reflect the impact of structured social processes that promote either greater or lesser segregation than would occur based on chance.

Because the expected value for an unbiased index under the null hypothesis of no association between race and residential location is zero and the sampling distribution is bell-shaped, one half of the values in the sampling distribution of an unbiased index will be negative. Some segregation researchers may not be initially comfortable with seeing negative scores for unbiased indices. But negative scores have a straightforward interpretation on both narrow statistical grounds and also on substantive grounds. On statistical grounds *negative scores indicate that scores for the standard version of the index take values that are lower than would be expected under random assignment*. Under the null hypothesis, negative values that fall in the middle region (e.g., in the middle 95 % region) of the sampling distribution for unbiased index scores can be set aside in the usual way; they can be attributed to chance and the observed departure from the expected value of zero can be viewed as not statistically significant. In contrast, negative scores that fall in the left tails of the sampling distribution can be viewed as statistically significant; they are unlikely to occur by chance and thus invite a substantive sociological explanation of how (scaled pairwise) contact with Whites among neighbors could come to be higher on average for Blacks than for Whites.

I note below that interesting sociological explanations are available. But I first pause to note that unbiased indices necessarily take negative values under exact even distribution. For example, consider the values of the standard and unbiased versions of the separation index for a city that is 90 % White and 10 % Black and has exactly 10 households per block. Under exact even distribution every block will have nine White households and one Black household. Proportion White among *neighbors* differs by race and will be 0.889 (i.e., 8/9) for every White household and 1.000 (i.e., 9/9) for every Black household. In contrast, proportion White for area population will be 0.900 (i.e., 9/10) for every White and every Black household. Accordingly, the standard version of S will be zero but the unbiased version S′ will be –0.111.

The comparison on D would be even more extreme. The value of the standard version of D would again be zero. But the value of the unbiased version D′ would be–1.000 because all White households are scored 0 on attaining parity (i.e., 0.90 or higher) on proportion White among neighbors while all Black households are scored 1.[6]

These negative values for unbiased indices under conditions of exact even distribution will be unfamiliar and perhaps also surprising to most readers, but they are

[6] The example serves to highlight a difference between D and S; namely, that, whether in standard or unbiased form, D responds much more strongly than S to quantitatively small deviations from parity on racial proportions.

fully expected and have a clear substantive interpretation. Negative values result because exact even distribution – the zero point for standard measures of uneven distribution – is a highly unexpected outcome under random distribution. The occurrence of such an unexpected residential distribution invites a sociological explanation identifying the structured social process that could bring about exact even distribution. Ready examples could include social dynamics such as quota systems in state policies governing assignments of households to housing units or institutional housing policies that structure housing assignments in dorms at colleges and universities, public housing, barracks in military bases, juvenile detention facilities, jails and prisons, orphanages, institutions for persons with disabilities, and the like. Thus, statistically significant negative values for unbiased indices are not only possible, they can and should obtain in certain empirical settings (albeit not ones that are commonly studied) where group distributions are highly structured to produce even distribution. Thus, negative scores for unbiased indices are valid and carry a clear sociological meaning.

Table 16.4 and Fig. 16.2 document patterns of dispersion in scores for unbiased indices under random distribution. The main differences across the five unbiased indices are seen in three areas. The first is the general level of volatility in the dispersion of scores around the expected value of zero. Holding simulation conditions constant, scores for G′ and D′ consistently exhibit greater variability under random assignment; scores for R′ and H′ exhibit less variability; and scores for S′ exhibit the lowest variability of all.

Another interesting pattern in the sampling distributions of the unbiased indices is how the dispersion of index scores under random distributions varies with effective neighborhood size (ENS). Table 16.4 documents that variability in the distribution of scores around zero is greater when effective neighborhood size (ENS) is small. This pattern is highlighted in visual form in Fig. 16.2 by plotting the points in successively darker shades of gray as ENS increases in size from 9–16 to 25–49 to 100 producing a concentration of the darkest points near the center of the distribution.

A third pattern in the sampling distributions of the unbiased indices is how the dispersion of index scores under random distributions varies with city racial proportion; in this case proportion White in the city (P). Here the unbiased separation index (S′) stands apart from the other indices. Other things equal, the dispersion in the scores for S′ is constant across levels of percent White in the city (P). In contrast, a much different pattern holds for G′, D′, R′, and H′; they all exhibit greater dispersion in index scores when percent White in the city (P) departs further from balance (i.e., 50). Figure 16.2 documents that the increase in the magnitude of the dispersion in index scores becomes especially pronounced when P begins to approach the bounds of 0 and 100.

I offer the following intuitive explanation for these patterns. The pattern of dispersion in values of the unbiased version of the separation index (S′) serves as a ready benchmark. Variation in dispersion is a simple function of effective neighborhood size. This is easy to understand; smaller samples of neighbors lead to greater volatility in residential outcomes. Dispersion in S′ is unaffected by relative group size because values of unbiased contact (p′) map on segregation-determining scores for residential outcomes (y′) without change. For all other indices, the scaling functions mapping scores of p′ onto scores of y′ are nonlinear. This assures that random deviations of p′ from P will be exaggerated. Furthermore, because nonlinearity in the scaling functions is stronger when group size is imbalanced, the impact will be greater when group size is more imbalanced.

Finally, it is important to note that Fig. 16.2 documents that scores for unbiased indices are distributed symmetrically around zero at all levels of effective neighborhood size (ENS) and all levels of percent White for the city (P). So, while the magnitude of dispersion for scores for unbiased indices varies across indices and over study conditions, the expected value (zero) and shape of dispersion in scores (symmetrical and bell-shaped) remain constant for all of the indices.

16.1.1 Summary of Behavior of Unbiased Indices

In sum, under random distribution, dispersion in scores of unbiased indices varies in magnitude depending on the particular index, the value of effective neighborhood size (ENS), and, with the lone exception of S′, percent White in the city (P). These patterns indicate that one must be mindful of these distinctive sampling distributions for different indices when evaluating the statistical significance of particular index scores. Exact analytic solutions for standard errors of unbiased index scores under varying circumstances have not yet been established. For exploratory analysis "t" and "Z" tests for group differences of means on scaled contact with the reference group may perhaps serve as reasonable approximations. For more definitive assessments, researchers should use bootstrapping or other similar computation-intensive approaches that require less stringent assumptions regarding the nature of error distributions.

16.2 Documenting Additional Desirable Behavior of Unbiased Indices Based on the Difference of Means Formulation

I now review the behavior of standard and unbiased versions of popular indices of uneven distribution in multi-group situations. My purpose is to show that "norming" adjustments proposed by Winship (1977) and Carrington and Troske (1997) and discussed in Chap. 14 can be problematic in these situations while the unbiased indices that I introduce here behave in desirable ways.

The essence of the problem with norming adjustments is that the expected values of indices under random assignment are more complicated in multi-group situations than previous methodological discussions have acknowledged. The logic of performing "norming" adjustments proposed previously in the literature rests on the crucial assumption that the expected value of standard indices under random distribution is invariant (is a constant) under a given combination of area size and (pairwise) group proportions. Unfortunately, this assumption is not correct. Instead, the expected value of standard indices is uncertain and can vary substantially even when area size and group proportions are known and simple in nature (e.g., all areas are constant size). The variation in index behavior traces to the presence of other groups in the population; the residential distributions for these groups can have nontrivial impacts on expected values of standard indices. This possibility ultimately undermines the potential effectiveness of previously proposed procedures for performing norming adjustments to deal with the impact of bias on the scores of standard indices.

I present results from simulation analyses conducted using the SimSeg simulation model to highlight the complex problems of bias in standard indices. The simulations all involve three groups; one large minority group, and two smaller minority groups. At the initialization of each simulation trial the households in the majority group are highly segregated from the households in the two minority groups but the households in the two minority groups are randomly distributed in relation to each other. This is depicted in the top panel in Fig. 16.3.[7] The simulation is then run for ten cycles (i.e., time periods). During each cycle, 25 % of households are chosen at random and are assigned randomly to a new residential location. Not surprisingly, systematic segregation between the majority group and the two minority groups quickly dissipates under this process of random movement resulting in majority households being randomly intermixed with minority households. This is depicted in the bottom panel in Fig. 16.3. At all times, starting at initialization and continuing to conclusion, the households in the two minority groups are randomly distributed in relation to each other.

The simulation experiments I used to generate the results for the analysis here follow the general design used in the simulations described earlier. The simulations here use the same neighborhood size (25) and the same city size and area configuration (i.e., 256 areas and 6,400 housing units). The racial composition of the city is set at 80-10-10. A total of 2,500 separate simulation experiments are run using this setting.

Index behavior is depicted in Fig. 16.4 which provides four graphs, two on the top row for the unbiased formulation of the dissimilarity index (D′) and two on the bottom row for the standard formulation of the dissimilarity index (D). The graphs in the left column depict majority-minority segregation; the graphs in the right column depict minority-minority segregation. The box plots in the top left graph show how D′ for the majority-minority comparison starts at very high levels and falls to zero as the ten cycles of random movement dissipate the initial segregation at the start of the simulation. The box plots in top right graph show that the distributions of D′ for minority-minority segregation are always centered on zero as

[7] Note that to facilitate visual inspection the example city depicted in the figure is smaller (about 1/3rd size) than the city size used in the simulations.

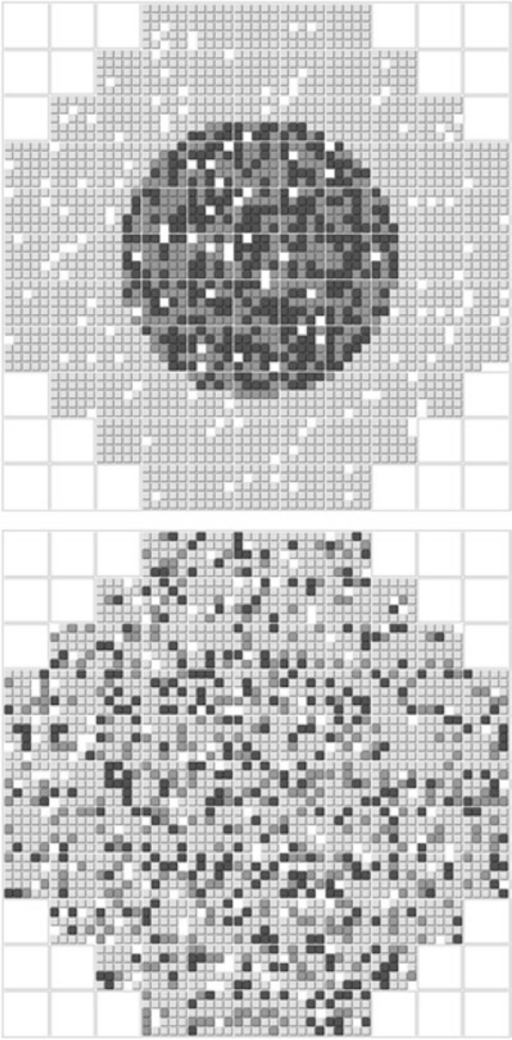

Fig. 16.3 Illustration of the transition from the initial state of minority-minority integration and high majority-minority segregation to the end state of all-way integration (Random distribution) (Note: Households from the majority group and two minority groups are depicted in shades of *gray* (light, medium, and dark gray, respectively). Vacant housing units are in *White. Grid lines* delimit areas. For easy visual review, the city here is 40% the size of the city in the simulations but faithfully depicts city shape and residential patterns)

expected since households in the two minority groups are distributed randomly in relation to each other over the entire course of the simulation.

The box plots in the bottom left graph depict the distribution of scores for the standard version of the index of dissimilarity (D) for majority-minority segregation. This shows that D is very high at the beginning of the experiment and then falls sharply as households move randomly for ten cycles. But D does not fall to zero due to the intrinsic bias in D. Thus, the final level of D essentially reflects a "bootstrap" estimate of the expected value of D (E[D]) for majority-minority segregation under random assignment. The box plots in the bottom right graph depict the distributions of scores for D for minority-minority segregation. These reflect only random residential variation over the course of simulation. The surprising finding here is that D

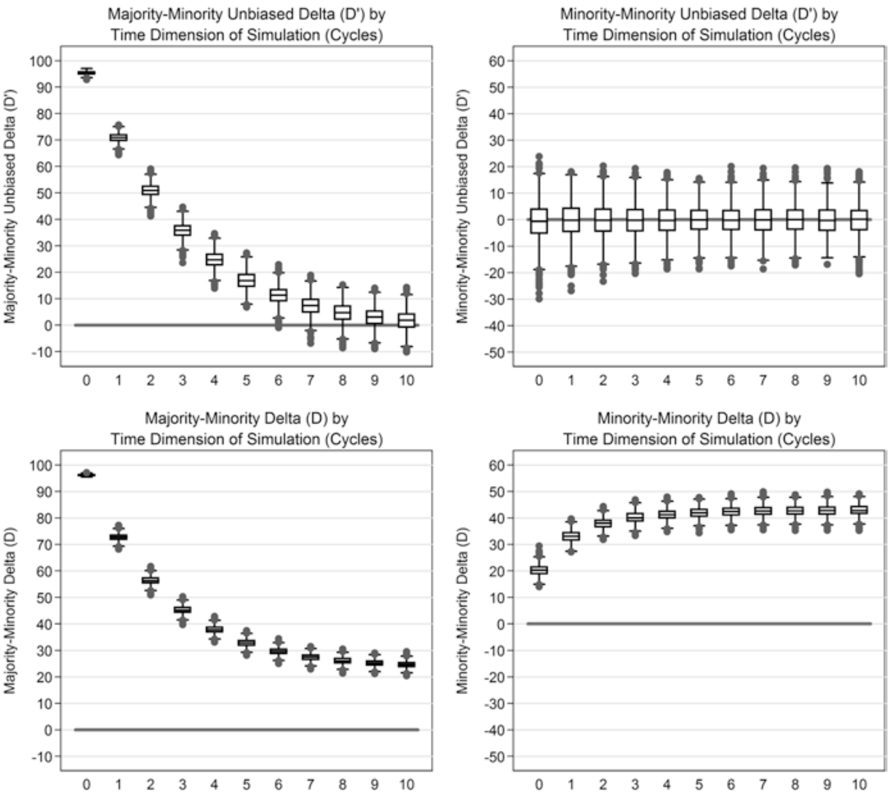

Fig. 16.4 Box plots depicting distributions of scores for unbiased and standard delta Index (D′ and D) for majority-minority segregation and minority-minority segregation over ten simulation cycles (Note: The graphs in the *top row* depict unbiased delta Index (D′) for majority-minority segregation on the *left* and minority-minority segregation on the *right*. The graphs on the *bottom row* depict values for standard delta (D) for the same comparisons. See text for details regarding the simulation designs)

increases over the course of the simulation. Why does this occur when the two minority groups are distributed randomly in relation to each other over the entire simulation? The answer traces to the complicated nature of effective neighborhood size in residential patterns for cities with three or more groups.

As illustrated in Fig. 16.4, the simulations begin with the two minority groups being highly segregated from the majority group. Under this pattern, effective neighborhood size (ENS) for the minority-minority segregation comparison is approximately 25 (i.e., the size of the neighborhoods) because households from the two minority groups live together in a small subset of the city's areas where majority households are absent. But the value of ENS for the minority-minority comparison changes over the course of the simulation. Under the final pattern of random distribution for all groups, effective neighborhood size (ENS) for minority-minority segregation falls to approximately 5 (i.e., 20 % of the neighborhood size of 25).[8] The change in ENS has important implications for the expected value of D under random assignment (i.e., E[D]) because E[D] is a negative function of effective neighborhood size. Consequently, over the course of the simulation, ENS falls from 25 to 5 and the value of E[D] for the minority-minority segregation comparison increases.

Figure 16.5 graphically summarizes results from additional analyses that replicate the analysis just reviewed using additional multiple racial demographic distributions for the virtual city. These are for group distributions of 80-15-5 and 91-6-3. The findings closely parallel those presented in Fig. 16.5. The results document two key findings. The first is that the unbiased version of D that is set forth in this study behaves in a desirable way under a wide range of conditions. The second is that standard version of D behaves in an undesirable way under these same conditions.

These findings document that previous suggestions by Winship (1977) and Carrington and Troske (1997) for dealing with index bias face a serious obstacle. They suggest adjusting observed values of D in relation to D's expected value under random distribution based on the calculation $D^* = \left(D - E\left[D\right]\right) / \left(1 - E\left[D\right]\right)$. The obstacle this approach faces is that the proposed adjustments can be effective only when the value of E[D], whether estimated by formula or by bootstrap methods, is accurate. Unfortunately, the results just reviewed show that the value of E[D] for the minority-minority segregation is not a simple constant. In the simulations under review here the two minority groups are distributed randomly in relation to each other. Accordingly, the value of D for this comparison reflects a bootstrap simulation estimate of E[D] for the minority-minority segregation comparison. The results from the simulations show that the value of E[D] is significantly impacted by an important factor that is not considered in previous discussions of potential solutions for dealing with index bias. Specifically, the value of E[D] is impacted by how the two groups in the comparison are distributed in relation to a third group – that is, the value of E[D] for the minority-minority comparison is impacted by how the two minority groups are distributed in relation to the majority group. In more general terms, the findings reviewed here indicate that E[D] for any two-group comparison is complicated in the multi-group situation and will be affected by: (a) the extent to which the two groups in the comparison are jointly segregated from other groups and (b) the relative size of other groups in the city population.

Space does not permit a detailed review of the issue, but in analyses not reported here, I have found that this finding applies to all standard indices of uneven distribution and that two broad conclusions hold in multi-group situations. One is that expected values of index scores under random assignment (i.e., E[•]) can potentially vary over wide ranges. The other is that adjustments of index scores in relation to expected values (E[•]) based on assumptions of simpler conditions can be inappropriate and perform poorly. *In the extreme the adjustments can generate assessments of segregation that are as problematic as the original unadjusted index scores*.

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[8] I say approximately because a precise discussion of effective neighborhood size would take account of the city vacancy rate (which is 6 % in these simulations).

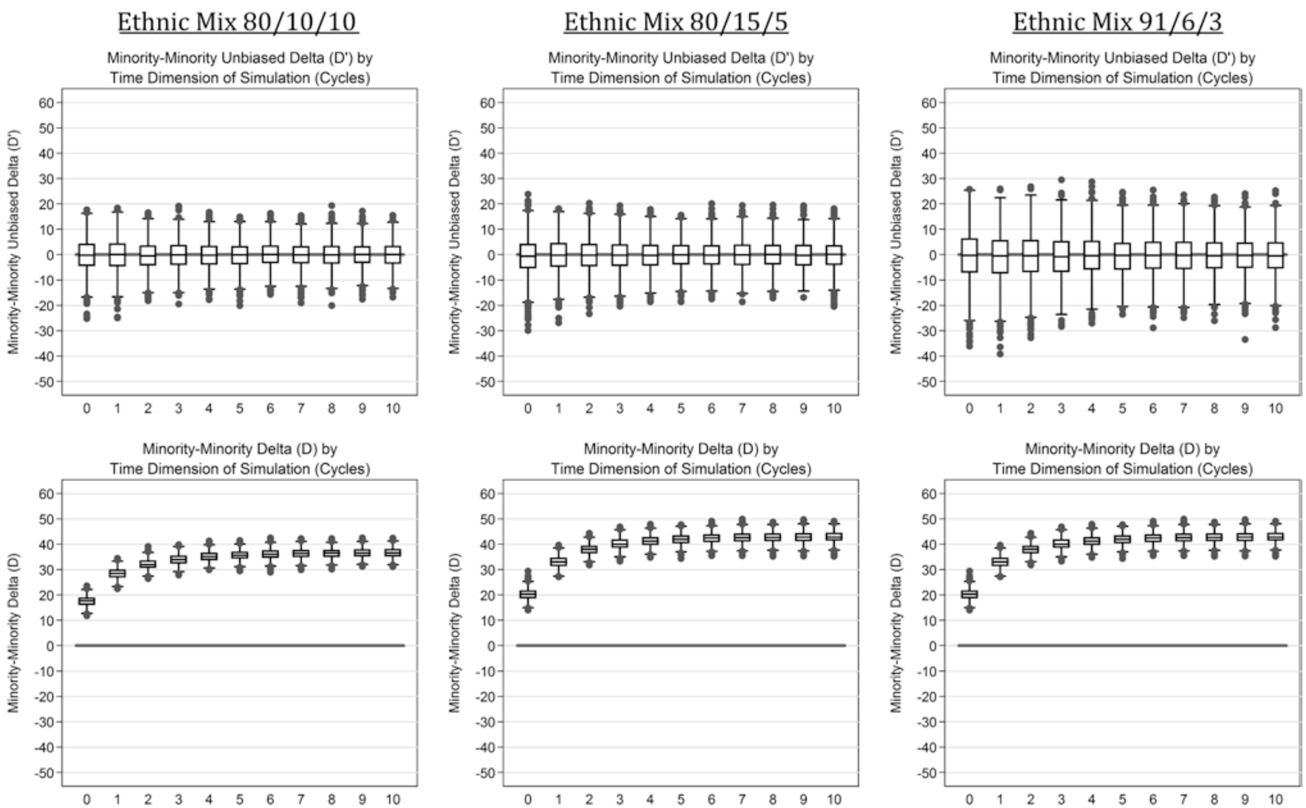

Fig. 16.5 Scores for unbiased and standard delta index (D′ and D) for minority-minority segregation over time for three combinations of ethnic mix (Note: *Top row* depicts the unbiased delta index(D′) for minority-minority segregation; *bottom row* is the standard delta index (D) for minority-minority segregation. Each simulation begins with an initial residential distribution in which the majority group is very highly segregated from two minority groups and the two minority groups are randomly distributed in relation to each other. Ten periods of random residential movement follow and the segregation pattern moves rapidly toward random distribution for *all* groups. Ethnic mix settings are 80/10/10 (column 1), 80/15/5 (column 2), and 91/6/3 (column 3). Neighborhood size is 25)

This may help explain why adjustment methods such as those proposed by Winship (1977) and Carrington and Troske (1997) are rarely used in empirical analyses. My own experience has been that the adjustment methods work quite well in methodological exercises where the underlying assumptions of the method are met (or closely approximated). However, when I apply the adjustments in the context of multi-group situations, they tend to "break down" and often yield unexpected results sometimes including results that are substantively implausible.

It is possible that the general approach of adjusting standard index scores could be "salvaged." This could be accomplished by using more sophisticated methods to develop refined estimates of expected index values under random assignment (i.e., E[•]) that take account of the complications associated with population groups not included in the segregation comparison. For example, I have found that bootstrap methods can be used to obtain serviceable situation-specific estimates of E[•]. One approach that appears to work well is to take the observed distribution across areas of the combined count of the two groups in the segregation comparison. Then perform bootstrap simulations wherein households from the two groups in the comparison are assigned randomly to areas until the observed area counts for the two groups combined are duplicated in each area. Performing a sufficiently large number of bootstrap simulations (e.g. 1,000 or more) will then establish the expected value of the index of interest under random assignment.

Alternatively, one could apply formula-based methods to obtain expected values of indices. But the formulas would have to be refined to take into account the observed distribution of effective neighborhood size across areas of the city. This makes implementing the formulas more complicated and also more computationally demanding.

Estimates of E[•] obtained in these ways are specific, not only to the nature of the multi-group residential pattern, but also to other potential complicating factors such as variation in area size. Unfortunately, most researchers are likely to view these technical refinements as exceedingly burdensome to implement. For example, in the simulation results just reviewed, the values of E[D] would have to be recalculated anew – using computation-intensive bootstrap methods or complex analytic computations – at least at the beginning of every time period of the simulation and perhaps even more frequently in the early stages of the simulations when the empirically assessed value of E[D] is changing rapidly. For this reason, reason it is unlikely that this approach will ever gain wide use.

The good news is that the unbiased indices I introduce in this monograph provide a superior alternative. The approach I propose is effective in both simple and complicated conditions, is conceptually appealing and easy to understand, and is much easier to implement in empirical analyses. The new unbiased indices I propose eliminate the source of bias at its root cause and do not rely on "after the fact" adjustments to purge unwanted consequences of index bias. Accordingly, the expected values of the unbiased indices are zero regardless of whether other groups are present in the population and, if so, regardless of the nature of the residential segregation pattern between the two groups of interest and other groups. Indeed, the only impact I have been able to discern so far is that the dispersion of the sampling distribution of the unbiased indices is affected by the presence of other groups. More specifically, while the mean for unbiased indices is always approximately zero, the standard error of the mean varies inversely with ENS as basic sampling theory would lead one to expect. But this pattern holds for the expected distributions of scores of both standard and unbiased versions of indices of uneven distribution and so does not diminish the advantage of using unbiased versions of indices.

16.3 Conceptual and Practical Issues and Potential Impact on Research

When should researchers use the new unbiased versions of indices of uneven distribution I have introduced here? One simple and reasonable answer is that researchers can and should use the unbiased versions of the indices in most if not all situations. Unbiased versions of index scores are not burdensome to compute; they support familiar substantive interpretations; they also expand available substantive interpretations; they eliminate concerns that index bias may distort findings; and they give researchers the option to expand research designs to consider a wider range of situations where standard versions of index scores would be untrustworthy and misleading.

Significantly, *few, perhaps no, unwelcome consequences are associated with using unbiased indices*. If standard versions of indices of uneven distribution are non-problematic, the unbiased versions indices will closely replicate their scores. This is because scores of unbiased indices differ from scores of standard indices in meaningful ways only when the scores for the standard indices are problematic. When this is happens, the scores of the standard version of the index are called into question as untrustworthy for many research purposes and the scores of the unbiased version of the index provide a more trustworthy assessment of the nature of group differences in residential distribution.

Will using the unbiased versions of familiar indices lead to major changes in research findings? I answer this question in two parts. The first part of my answer begins by noting that studies conducted in recent decades have tended to use research designs that try to guard against index bias. I have characterized the strategies used as a patchwork of practices that can be criticized for being crude and in some cases weakly justified. But in general the strategies do tend to minimize the most egregious impacts of index. As a result, findings of many, perhaps most, previous studies using standard indices are not necessarily likely to be contradicted in dramatic ways if they are *exactly replicated* but using unbiased indices. I place emphasis on the phrase "exactly replicated" to stress that this means using exactly the same set of cases. Below I note that future studies may differ from past studies by being able to use a wider range of cases and more varied group comparisons instead of being limited to using the smaller, restricted set of cases and group comparisons used in past research.

The reason why the specific findings of many past studies are not likely to change when exactly replicated using unbiased indices is straightforward. To the extent that the practices researchers have incorporated into research designs have been conservative and excluded cases that are most seriously affected by problems with index bias, replications that use unbiased versions of indices for the same cases will not be likely to yield dramatically different results. This is because the unbiased versions of indices yield scores similar to standard versions when bias is low. Substantively meaningful differences might arise for marginal cases that were not effectively screened because the ad hoc screening practices were crude and imprecise. But in many, perhaps most, studies these cases should not dominate the findings and so results will likely remain similar when the analysis is replicated using unbiased indices.

Certain kinds of past studies would be most susceptible to changes in results if "exactly" replicated using unbiased versions of indices of uneven distribution instead of standard versions. These are studies where research designs were less stringent in screening out cases where index scores are most susceptible to bias. Examples would include: studies that use block-level data instead of tract data; studies that focus on segregation for groups that are imbalanced in size, studies that focus on subgroups that are small in combined size, and studies that are based on sample data instead of full count data.

Another kind of study result that might change when replicated using unbiased measures are studies where findings differ when cases are weighted by minority population size in comparison to when cases are weighted equally. Presumably findings do often differ. Otherwise the practice of weighting cases would not be so widely used. Instead, an early study would report the finding that it makes no difference and study designs would weight cases equally. The results reviewed here show that minority group size has no intrinsic relationship to bias. So the logical justification for weighting cases by minority group size to minimize the consequences of index bias can be questioned under all circumstances. The practice would clearly be unwarranted if studies are replicated using unbiased versions of indices. I suspect this might lead to some changes in findings. The current widespread practice of weighting by minority group size skews findings toward the cases in the sample that have larger minority populations. To the extent that this subset of cases has different segregation outcomes, from the remainder of the cases, findings would change when studies are replicated using unbiased versions of indices.

A broader interpretation to the notion of study replication would lead to a different answer. "Exact" replications of past studies involves excluding many cases that can be included when using unbiased versions of indices. Similarly, "exact" replications of past studies means foregoing many group comparisons that can be examined when using unbiased versions of indices. The availability of unbiased indices frees the literature from the need to accept these past compromises in study design. With this in mind I now offer the second part of my answer.

There are at least three ways that results for empirical studies are likely to change in welcome and potentially important ways when researchers adopt unbiased indices. One is that *using unbiased index scores will give researchers much greater* *ability to discuss and compare specific cases without concern for the distorting influence of bias*. These discussions are more difficult when standard scores are used. Scores for individual cases are potentially subject to different levels of distortion by index bias. Researcher recognition of this concern motivates the widespread practice of weighting cases differentially in statistical analyses. Concern about case-to-case variation in the impact of bias on index scores complicates the interpretation of scores of individual cities and it also complicates the direct comparison of scores for any given city with the scores of any other cities. Such complications are eliminated when using unbiased scores. Scores for individual cases can be evaluated with ease. Similarly, scores for two cases and scores for the same case at two points in time can be compared without concern.

A second way results may change is that *the logic of case weighting as implemented in statistical analyses in current studies will no longer be justified when using scores for unbiased versions of indices*. The stated motivation for differentially weighting cases – that is, to minimize the distorting impacts biased cases may exert on findings – is of course negated entirely. The main implication of this is that results of statistical analyses will no longer be driven by segregation patterns for cities with large minority populations. It is unclear whether this will in fact lead to important changes in findings. But it is a distinct possibility that results of statistical analyses may differ because many cases which previously would have had little or no influence on results of statistical analyses will now carry equal weight.

The third way using unbiased indices will impact segregation studies is the most important. It is that *researchers will be free to greatly expand the scope of segregation studies*. Researchers will no longer need to limit analysis to the small subset of cities that survive sample restrictions and receive weights that give them disproportionate influence on results after prevailing practices exclude and discount potentially problematic cases to guard against index bias. Instead, future studies will be able to conduct expanded analyses that may investigate segregation in many situations that previously were not examined because conventions in restricting study designs foreclosed this possibility. Relatedly, using unbiased indices will allow researchers to consider many kinds of group comparisons that previously could not be considered. This includes, for example, comparisons involving small population groups and comparisons involving small subgroups within particular populations. In the past, such comparisons have gone unexamined because index scores are potentially subject to high levels of bias. These concerns can be set aside when unbiased versions of indices are used.

Eliminating the need to impose draconian restrictions on research designs of segregation studies can only be a good thing. It will allow researchers to expand samples and explore a broader range of research questions. The following is a brief list of research applications where the benefits of using unbiased indices are especially likely to be seen.

- studies assessing segregation at small spatial scales such as the census block and block group; or classrooms within schools; or the very small neighborhoods typically used in agent simulation analyses of segregation[9];
- studies assessing segregation when groups are imbalanced in size; for example, studies of segregation involving small population groups such as Asian and Latino populations in areas of new settlement; and
- studies assessing segregation for subgroups within broader populations which will result in small effective neighborhood size; for example, the segregation of Latino and Asian subgroups, and the segregation of high-income Whites and high-income African Americans.

I conclude by strongly encouraging researchers to take advantage of the new option to use unbiased versions of popular indices of uneven distribution. One is never worse off for examining the new unbiased versions of popular indices and there are many ways they may yield benefits. Accordingly, I argue that it will always make good sense to examine the scores of the unbiased versions of indices. As I said, one can never be worse off for doing so because findings will be unchanged if bias is not a problem and the positive confirmation on this point will provide an additional basis for placing confidence in one's findings. Moreover, there are many reasons to expect one would be better off, perhaps by a great deal, in comparison to following prevailing practices. Current "rule-of-thumb" practices that aim to minimize undesirable complications associated with index bias are crude and imprecise and can be "hit and miss" in effectiveness. Concerns on this point can be completely set aside by examining the unbiased versions of the indices even if one in the end elects to report results for standard versions of indices. However, it is likely that standard indices will be used as often as in the past because the availability of unbiased versions of indices of uneven distribution makes it possible for researchers to examine segregation in a wider range of situations than was previously possible. Once this occurs, scores for standard indices will be even less trustworthy than they currently are and researchers will increasingly need to rely on unbiased versions when attempting to answer the new questions these measures permit researchers to investigate.

[9] In fact, I began pursuing the development of unbiased measures of uneven distribution to cope with the problem of bias in measuring segregation in simulation studies. In that context, the unbiased measures allow researchers to explore a much wider range of combinations of neighborhood scale and population composition than can be considered using standard versions of segregation indices.

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Chapter 17
Final Comments

In this study I have shown that popular measures of residential segregation – the dissimilarity index (D), the gini index (G), the separation index (S), Theil's index (H), and Hutchens' index (R), a measure closely related to Atkinson's index (A) – can be cast as group differences of means on residential outcomes (y) scored from area group proportions (p). This approach yields identical results as previous approaches to calculating index scores, so nothing is lost when adopting this formulation – all past research findings can be reproduced and replicated. Importantly, however, many benefits accrue from adopting the new approach to assessing segregation.

One is that the approach serves to "demystify" aggregate segregation by revealing its direct connections to residential outcomes for individuals. Segregation studies generally have focused on describing aggregate distributions at the city level and have given little attention to the implications index scores have for the residential outcomes of the individuals in the groups being compared. This is very different from the approach that guides studies of group disparities in education, occupation, income, wealth, and other socioeconomic attainments. In these analyses, both the relevant attainment outcome and the attainment process that shapes its distribution are usually clearly in focus. Obviously, the literature on residential segregation rests on an implicit presumption that aggregate segregation arises from micro-level attainment dynamics that have consequences for the residential attainments of the individuals and households in the groups being compared. But, as Duncan and Duncan (1955) noted half a century ago, methodological approaches to measuring aggregate segregation have not pursued index formulations that can facilitate exploring these issues. The formulations I present here address this need by establishing that segregation indices reflect group differences on residential outcomes relating to group contact with differences between indices arising from differences in how they scale group contact.

Another benefit of the approach I have outlined here is that it creates the possibility of seamlessly joining the study of aggregate segregation with the study of

M. Fossett, *New Methods for Measuring and Analyzing Segregation*,
The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4_17

residential attainments. Previously, the two were necessarily separate. Now it is possible to directly investigate how residential attainment dynamics give rise to uneven distribution by framing aggregate segregation as the effect of race on individual-level residential attainments that additively determine the city-level segregation score. This directly addresses the concern Duncan and Duncan (1955) raised that segregation indices serve to describe aggregate-level distributions but do not lend themselves to studying the underlying social processes that create these distributions. In addition, it creates new opportunities for refining segregation analysis by including non-racial characteristics in residential attainment models. This makes it possible to perform standardization and components analyses to investigate the extent to which segregation arises out of group differences in resources relevant for residential attainment and group differences in rates of converting their resources into residential attainments. It also makes it possible to use restricted access census micro files and non-census surveys to explore questions about aggregate segregation that previously could not be explored.

Joining the study of aggregate segregation with the study of micro-level residential attainments also creates new options for investigating variation in segregation over time and across different metropolitan areas. If desired, city-level segregation can now be assessed by estimating the effect of race in city-specific individual-level models of residential attainment. Then the effect of race can itself be modeled as varying over time or with the ecological characteristics of metropolitan areas using multi-level models. The city-specific micro models can optionally include other social characteristics which may also influence residential outcomes. If not included, the effect of race in the model registers how aggregate segregation at the city-level varies with time and urban context. If included, the effect of race registers the level of and variation in racial segregation assessed net of controls for other characteristics.

The approach to assessing segregation I have outlined here establishes a new basis for discussing, comparing, and evaluating segregation indices – namely, the substantive and theoretical relevance of the residential outcomes (y) the index registers and responds to. When evaluating indices in terms of their qualities for summarizing and describing group differences in residential outcomes, one may consider whether the residential outcomes they register are substantively compelling for individuals and households or for particular policy goals. When evaluating indices in terms of their relevance for investigating segregation dynamics, one may consider whether the residential outcomes they register are salient in residential attainment dynamics. Do indices register outcomes that individuals and groups seek and potentially compete for? That is, do individuals and groups strive for the outcomes because they value them for their own sake and/or because they are correlated with other valued residential outcomes? Are the outcomes consequential for important aspects of life chances? Are the outcomes relevant to theories of residential attainment and stratification?

In the body of this monograph I reviewed how different indices register residential outcomes (y) based on scaling area group proportions (p) in different ways. D, G, H, and R score y in complicated ways. D scores y as a two-value step function based on P – the pairwise group proportion for the city in question. G scores y as an irregular nonlinear monotone function of p based on relative rank position (i.e., the percentile transformation). H and R score y as continuous nonlinear functions of p. For all four indices, the scaling of residential outcomes varies, often quite dramatically, with the racial mix of the city. To be clear, the functional forms of $y = f(p)$ for D, G, H, and R can, and often will, vary with the groups involved in the comparison, across cities for the same group comparison, or over time for the same group comparison in a given city. In contrast, S scores y directly from p under all conditions. In this regard, S stands out as the only index for which the scoring of y based on p is the same across different group comparisons, over different points in time, and across different cities. Because of this quality, I am drawn to the one-to-one scoring of $y = p$ that S registers. It is simple and easy to understand and it is related to an aggregate segregation outcome that can easily be explained to non-technical audiences. In addition, there are good reasons to believe that the area group proportions that S registers are meaningful to individuals and households and consequentially are salient in residential dynamics. However, I recognize that discussion and debate on this issue is just beginning and I invite others to give attention to questions concerning what substantive concerns about residential dynamics and group differences in residential outcomes should guide the choice to focus on particular specifications of aggregate segregation.

Finally, I note that casting uneven distribution as a difference of group means on residential outcomes (y) based on area group proportion scores (p), provides a new vantage point for understanding the origins and nature of bias in standard versions of popular indices of uneven distribution. In addition, it opens the door for a surprisingly simple and compelling solution that allows one to eliminate bias from index scores if desired. The scores of the resulting new "unbiased" versions of indices of uneven distribution are near identical to the scores of the conventional versions in situations where the conventional scores are non-problematic and they provide attractive alternatives in situations where conventional scores cannot be used – for example in the study of White-Latino segregation in new destinations (Fossett et al. 2015).

In closing, I note that the approach to investigating segregation I have outlined here complements and extends previous traditional approaches to studying aggregate segregation. It does not put approaches and findings from past research to the curb. To the contrary, the framework I offer here is fully compatible with mainstream traditions of research focusing on aggregate segregation. Casting segregation in terms of group differences in individual residential outcomes and equating index scores with the effect of race in micro-level attainment models does not preclude pursuing traditional analysis of aggregate-level segregation; that remains as an option for those who prefer that approach. In addition, however, there now is a new set of alternatives for computing the indices that are used in such studies. More importantly, the framework I offer provides researchers new options for interpretation and analysis that I believe many will view as potentially useful. These include: new options for extending previous research investigating variation in segregation across cities and over time; new options for taking account of non-racial social characteristics when investigating racial segregation; new alternatives for assessing the substantive implications of segregation based on the consequences it has for group differences in residential outcomes; and new options for theorizing about and investigating the social attainment processes that give rise to aggregate segregation. I encourage researchers to adopt the refined measures and new options for analysis outlined here because I believe they will enable researchers to move forward in ways that will yield more trustworthy measurement of segregation and better understanding of how group differences in residential distributions arise from group differences in residential attainments resulting from the role of race in residential dynamics.

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Appendices

Appendix A: Summary of Notation and Conventions

This appendix reviews the notation and conventions for terms used in this monograph. Where appropriate it provides commentary to clarify usage by context.

Pairwise Calculations

In standard applications, indices of uneven distribution are based on pairwise population counts and group proportions. The adjective “pairwise” indicates that calculations use only population counts for the two groups in the segregation comparison. If other groups are present in the population, their counts are excluded and have no impact on index scores. Accordingly, unless indicated by direct statement or by obvious context, references here to total counts and terms based on total counts (e.g., group proportions) should be taken as being based on pairwise comparisons; that is, based on the sum of the population counts for just the two groups in the comparison.

Reference and Comparison Groups (Groups 1 and 2)

When index scores are calculated using the difference of means formulation introduced in this monograph it is necessary to designate one of the two groups in the segregation comparison as the “reference” or “focal” group. The second group is then designated the “comparison” group. The choice of which group is designated as the “reference” is arbitrary and it has no impact on the resulting index scores. The choice is necessary to organize calculations and facilitate presentation. For subscripting purposes it is convenient to designate the reference group as “Group 1” and the comparison group as “Group 2”.

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The Springer Series on Demographic Methods and Population Analysis 42,
DOI 10.1007/978-3-319-41304-4

The empirical literature on residential segregation in US urban areas overwhelmingly focuses on majority-minority segregation comparisons such as White-Black, White-Latino, and White-Asian comparisons. Based on substantive concerns regarding majority-minority inequality and assimilation, it is customary to assess residential distributions for different minority groups – e.g., Blacks, Latinos, and Asians – in relation to the residential distribution of the majority group – Whites. I follow this custom and thus designate the majority group – Whites – as the reference group.

This has no consequence for index scores or for their substantive implications. But it does structure discussion and interpretation of results to focus on implications for majority-minority inequality and residential assimilation.

City-Wide Terms for Pairwise Calculations

$N_1 =$ the city-wide population count for Group 1, the "reference" or "focal" group.
$N_2 =$ the city-wide population count for Group 2, the "comparison" group.
$T =$ the combined city-wide pairwise population count ($T = N_1 + N_2$).
$P =$ the city-wide proportion for Group 1 ($P = N_1/[N_1 + N_2]$).
$Q =$ the city-wide proportion for Group 2 ($Q = N_2/[N_1 + N_2]$; $Q = 1 - P$).

Area-Specific Terms for Pairwise Calculations

$i =$ index for the areas of the city; applied where appropriate, omitted to reduce clutter when unnecessary (e.g., when clear based on context).
$j =$ a second index for the areas of the city used in formulas where one area (denoted by i) is compared to other areas (denoted by j).
$n_1 =$ the area population count for Group 1, the reference group.
$n_2 =$ the area population count for Group 2, the comparison group.
$t =$ the combined area pairwise count ($t = n_1 + n_2$).
$p =$ the area proportion for Group 1 ($p = n_1/[n_1 + n_2]$).
$q =$ the area proportion for Group 2 ($q = n_2/[n_1 + n_2]$; $q = 1 - p$).
$s_1 =$ the area share of the city-wide Group 1 population ($s_1 = n_1/N_1$).
$s_2 =$ the area share of the city-wide Group 2 population ($s_2 = n_2/N_2$).

Terms for Individuals or Households

$k =$ an index for individuals in a group or, depending on context, in the city-wide population.

m = an index similar to k for individuals in a group or in the city-wide population. This is relevant for some formulas for the Gini Index (G) where individuals indexed by k are compared to all other individuals in the population indexed by m.

Selected Terms and Conventions Relevant for the Gini Index (G)

X_i = cumulative proportion of Group 1 based on ordering areas from low to high on p_i and then summing area group share terms ($X_i = \Sigma s_1$ over relevant areas).
Y_i = cumulative proportion of Group 2 based on ordering areas from low to high on p_i and then summing area group share terms ($Y_i = \Sigma s_2$ over relevant areas).

Selected Terms and Conventions Relevant for the Theil Entropy Index (H)

The original derivation of the Theil index is grounded in an information theory framework (Shannon 1948; Theil and Finizza 1971; Theil 1972) drawing on a notion of entropy (E) quantified as given below.

E = entropy for the city overall given by $E = P \cdot Log_2\left(1/P\right) + Q \cdot Log_2\left(1/Q\right)$.

E_i = entropy for area i given by $E_i = p_i \cdot Log_2\left(1/p_i\right) + q_i \cdot Log_2\left(1/q_i\right)$.

Note that Log_2 denotes the base 2 logarithm. Many applications use natural logarithms in place of base 2 logarithms.

Selected Terms and Conventions Relevant for the Atkinson Index (A)

Formulas for the Atkinson index (A) include two constants – α and β. Values for α are restricted to fall between 0 and 1 exclusive of end points (i.e., $0 < \alpha < 1$). β is obtained by $1 - \alpha$. The Atkinson index is symmetric when α is 0.5 and is asymmetric otherwise. When A is asymmetric it yields different index values depending on which of the two groups in the comparison is adopted as the reference group in the comparison. This leads some to view asymmetric versions of A as unacceptable for use as a general measure of segregation (White 1986). I agree with this view. Accordingly, discussion of the Atkinson index in this monograph is limited to the symmetric version where $\alpha = \beta = 0.5$. This version of the Atkinson index has close relations with the Hutchens square root index (R) which is more tractable mathematically.

Appendix B: Formulating Indices of Uneven Distribution as Overall Averages of Individual-Level Residential Outcomes

This appendix chapter reviews alternative formulations of indices of uneven distribution to clarify how aggregate segregation is related to individual residential outcomes. This is useful for at least two inter-related reasons; one substantive and one methodological. The substantive reason is that sociological interest in segregation usually rests on the assumption that it has important implications for individual life chances associated with area of residence. Based on this concern, it would be useful to better understand how indices of uneven distribution register individual residential outcomes. The methodological reason is that formulating indices of uneven distribution in terms of individual residential outcomes is a necessary step for clarifying how segregation emerges from individual-level residential attainment processes.

The view that segregation emerges from micro-level attainment processes and carries important implications for group differences in residential outcomes is hardly new or controversial. In light of this it is surprising that methodological discussions of indices of uneven distribution give little attention to this issue. For example, consider two familiar formulas for the widely used Gini Index (G) and the Delta or Dissimilarity Index (D) shown in Fig. B.1.[1] These formulas were featured five decades ago in Duncan and Duncan's (1955) landmark methodological study. These formulas and close variations on them are widely used in empirical studies in part because they are computationally efficient and are easy to implement. However, Duncan and Duncan raised the concern that "[i]n none of the literature on segregation indices is there a suggestion of how to use them to study the *process* of segregation" (1955:216, emphasis in original). The reason for this is that the formulas given in Fig. B.1 provide little basis for understanding how segregation is connected to the residential outcomes of individuals. Indeed, individual-level residential outcomes are "invisible" in these formulas.

Advances in computing technology have rendered the issue of computing efficiency mostly irrelevant. Yet it is still typical for the measurement of segregation using G, D, and other indices to be discussed in relation to convenient computing formulas. It is fine to use efficient computing formulas for the narrow purpose of obtaining index values. But researchers and broad audiences who gain their understanding of segregation based solely on these formulas will have, at best, only vague notions regarding how segregation arises from micro-level attainment processes. This problem can be addressed by considering alternative formulations of popular segregation indices that clarify how index scores are connected to individual residential outcomes.

[1] Figure B.1 also includes a similar style formula for the more recently introduced Hutchens square root index (R) (2001).

G = $100 \cdot (\Sigma X_{i-1}Y_i - \Sigma X_iY_{i-1})$ where X and Y are group proportions cumulated over areas ranked low to high on p_i (Duncan and Duncan 1955)

D = $100 \cdot \frac{1}{2} \Sigma \,|\, (n_{1i}/N_1) - (n_{2i}/N_2) \,|$ (Duncan and Duncan 1955)

R = $100 \cdot \left(1.0 - \Sigma \sqrt{(n_{1i}/N_1) \cdot (n_{2i}/N_2)}\right)$ (Hutchens 2001:23)

Fig. B.1 Area-based computing formulas for indices of uneven distribution that do not draw on individual-level residential outcomes (Note: N denotes city-wide population count, n denotes area population count, subscripts 1 and 2 denote the two groups in the segregation comparison, subscript i denotes area, X_i and Y_i denote the cumulative proportions of groups 1 and 2 over areas ranked from low to high on p_i – the group 1 (reference group) proportion in the combined group population in area i ($p_i = n_{1i}/[n_{1i} + n_{2i}]$))

Focusing Attention on Individual-Level Residential Outcomes

All widely used indices of uneven distribution can be formulated in terms of individual-level residential outcomes (y) that are scored from area group (e.g., racial) proportions (p). This can be done in two distinct ways. One is to formulate index scores as simple *overall averages* of individual-level residential outcomes (y). The other is to formulate index scores as a *difference of group means* on individual-level residential outcomes (y). Both approaches can be used to obtain "correct" index values. But that is a minor benefit as convenient formulas for obtaining correct index values are readily available. The main benefit of these formulations is that they can be used to gain insight into how different indices register and summarize individual residential outcomes. In addition, formulating indices in terms of individual attainments brings certain practical advantages which I note below.

Figure B.2 presents computing formulas that highlight how individual-level residential outcomes are registered by six popular measures of uneven distribution – the Gini Index (G), the Delta or Dissimilarity Index (D), the Atkinson Index (A), the Hutchens Square Root Index (R), the Theil Entropy index (H), and the Separation Index (S) (also known as the variance ratio [V], and eta squared [η^2]). The calculations indicated in these formulas involve first computing area-specific scores (i.e., neighborhoods) based on pairwise group proportions and then averaging these scores over individuals. More specifically, the formulas have the following features:

- the core terms in the calculations are scores computed for areas (indexed here by "i") based on calculations involving area group proportions; that is involving the values of p_i and q_i as given in Appendix A,
- the area-specific scores are summed over all *individuals* based on weighting the score for each area by the area-specific combined population count (t) for the two groups in the segregation comparison,
- the population-weighted sum of area-specific scores is then divided by the combined population of the two groups for the city (T) to obtain an overall average, and

$G = 100 \cdot (1/2T^2PQ) \cdot \Sigma \Sigma t_i \cdot t_j \, |p_i - p_j|$

$100 \cdot (1/2TPQ) \cdot \Sigma t_i \cdot [(1/T) \cdot \Sigma t_j \, |p_i - p_j|]$ (noted to clarify area-specific term)

$D = 100 \cdot (1/2TPQ) \cdot \Sigma t_i \, (|p_i - P|)$ (noted to highlight similarities with S)

$A = 100 \cdot [1 - (Q/P) \{\Sigma t_i \cdot (p_i^{\alpha} q_i^{\beta}/QT)\}^{1/\alpha}]$ where $0 < \alpha < 1$ and $\beta = 1 - \alpha$

Setting $\alpha = \beta = 0.5$ yields the "symmetric" version of A, the version most relevant for use in segregation analysis. Using this setting for α, the formula of A can be expressed in the two formulations shown below to highlight similarities with formulas for the Hutchens square root index (R) and the separation index (S).

$100 \cdot [1 - \{\Sigma (t_i/T) \sqrt{p_i q_i}\}^2/PQ]$ (noted to highlight similarities with R & S)

$100 \cdot [1 - \{(1/T) \cdot \Sigma t_i \cdot \sqrt{p_i q_i}\}^2/PQ]$ (noted to highlight similarities with R & S)

$R = 100 \cdot [1.0 - \Sigma (t_i/T) \sqrt{p_i q_i/PQ}]$ (noted to highlight similarities with A & S)

$100 \cdot [1.0 - (1/T) \Sigma t_i \sqrt{p_i q_i/PQ}]$ (noted to highlight similarities with A & S)

$H = 100 \cdot \Sigma [(E - E_i)/E] \cdot (t_i/T)$

$100 \cdot (1/T) \cdot \Sigma t_i \cdot [(E - E_i)/E]$

where E is entropy for the city overall given by $E = P \cdot \mathrm{Log}_2(1/P) + Q \cdot \mathrm{Log}_2(1/Q)$ per information theory (Shannon 1948; Theil 1972) and E_i is entropy for area i and is given by $E_i = p_i \cdot \mathrm{Log}_2(1/p_i) + q_i \cdot \mathrm{Log}_2(1/q_i)$. If desired, one can use natural logarithms as well as base 2 logarithms.

$S = 100 \cdot (1/TPQ) \cdot \Sigma t_i \, (p_i - P)^2$ (noted to highlight similarities with D)

$100 \cdot [1 - \Sigma (t_i/T)(p_i q_i/PQ)]$ (noted to highlight similarities with A & R)

$100 \cdot [1 - (1/T) \cdot \Sigma t_i \cdot (p_i q_i/PQ)]$ (noted to highlight similarities with A & R)

Fig. B.2 Area-based computing formulas for indices of uneven distribution that implicitly feature averages for individual-level residential outcomes

- any other terms present in the formula serve only to rescale the resulting overall average to the range 0–1.

Based on these features, it is appropriate to describe the resulting index value as an overall individual-level average on area-specific residential outcomes scored from area group composition (p).

Figure B.3 reorganizes the expressions in Fig. B.2 to present them in a form that explicitly casts each index in terms of an index-specific, individual-level residential outcome (y) that is averaged over all individuals in the two groups in the comparison. The formulas in this figure are not necessarily the most convenient for computing index scores. But they make it clear that aggregate segregation index scores can be understood as simple summary measures (i.e., means) for individual residential outcomes.

The individual level residential outcomes (y) identified in Fig. B.3 can be characterized as follows: the outcomes register the degree to which the group proportion for the area (p_i) departs from the group proportion for the city as a whole. The specific way in which this departure is quantified varies from one index to another and that becomes the basis for each one's unique way of registering uneven distribution.

Averaging Scores for y Over Individuals		Scores Assigned to Individuals
G =	$100 \cdot (1/T) \cdot \Sigma y_k$	$y_k = \Sigma \mid p_k - p_m \mid / 2TPQ$ where k and m index individuals, p_k denotes the pairwise area proportion for the reference group (p_i) for the k'th individual, p_m denotes area proportion for the reference group (p_i) for the m'th individual (note, this reorganizes the terms in the second formula for G in Figure B.2)
D =	$100 \cdot (1/T) \cdot \Sigma y_k$	$y_k = \lvert p_i - P \rvert / 2PQ$
A =	No comparable solution is available but the value of the "symmetric" version of A (given by setting $\alpha=\beta=0.5$) can be obtained from $2R\text{-}R^2$	
R =	$100 \cdot [1 - (1/T) \cdot \Sigma y_k]$	$y_k = \sqrt{p_i q_i / PQ}$
H =	$100 \cdot (1/T) \cdot \Sigma y_k$	$y_k = (E - E_i)/E$ with E_i and E as given in Figure B.2
S =	$100 \cdot (1/T) \cdot \Sigma y_k$ or	$y_k = (p_i - P)^2/PQ$
	$100 \cdot [1 - (1/T) \cdot \Sigma y_k]$	$y_k = p_i q_i / PQ$

Fig. B.3 Alternative formulas for uneven distribution that explicitly cast indices as overall averages of residential outcomes (y) for individuals (Note: k and m index individuals, p_k denotes the pairwise area proportion for the reference group (p_i) for the k'th individual, p_m denotes area proportion for the reference group (p_i) for the m'th individual)

But all of the indices can be understood as registering average exposure to departures from the group mix that would obtain under even distribution. If all neighborhoods have the group mix of the city as a whole, all of the values of y will be 0 and the final index value also will be 0. If members of the two groups never reside in the same areas, the values of y move to the extreme values that can apply to individuals residing in neighborhoods where p_i is 1 or 0 and the sum of y goes to the maximum value possible for the city given its group composition. The resulting sum is then rescaled to yield an index value of 1 by incorporating index-specific constant terms (e.g., 2PQ for D).

Options for Spatial Versions of Indices of Uneven Distribution

These index formulations carry at least one practical benefit; they can be used to calculate spatial segregation scores as well as aspatial segregation scores for any of the indices. That is,

Formulas that cast segregation index values as overall averages on individual-level residential outcomes can readily be adapted for computing *spatial* as well as *aspatial* versions of the segregation indices.

Aspatial versions of segregation indices are familiar and widely used in empirical studies. They are obtained by applying the computing formulas introduced here, or any of the formulas introduced earlier, using data for non-overlapping "bounded" areas such as school districts, census tracts, block groups, or blocks. In the aspatial formulation, each bounded area represents a particular neighborhood and every individual or household in the area is treated as having the residential outcome calculated for this area.

When index values are cast as overall averages of individual-level residential outcomes as in Fig. B.3, the indices also can be implemented in spatial measures. This is accomplished by computing averages for individual residential outcomes (y) that scored for "overlapping" spatially-defined neighborhoods that are specified uniquely for each individual based on the population residing within a spatially defined neighborhood. For example, the spatial formulation could be implemented using census data by taking small bounded areas such as census blocks and defining the spatial neighborhood as the population residing in the "focal" block plus the surrounding adjacent blocks. In this approach the population in any particular block will be part of uniquely-defined, spatially-delimited neighborhood.

When using these formulas, the question of whether the index is viewed as aspatial or spatial depends only on how "neighborhoods" are conceived. This can be stated in general terms as follows. Whether or not the index values obtained using these formulas are properly described as spatial or aspatial is determined by the definitions of the neighborhoods used to calculate the individual-level residential outcomes used in the relevant index calculations. If the residential outcomes are for non-overlapping bounded areas, the index values are aspatial. If the residential outcomes are for individual-specific, overlapping neighborhoods, then the index values are spatial.

Summary of Difference of Means Formulations

I now review a second way in which indices of uneven distribution can be formulated in terms of individual-level residential outcomes. This is to cast each index as a difference of group means on individual-level residential outcomes. Groups are designated as groups 1 and 2 with group 1 being taken as the reference group.[2] Each segregation index value (S) is then given as the difference of group means ($\overline{Y}_1 - \overline{Y}_2$) on individual residential outcomes (y) that are scored as a function of the pairwise proportion for group 1 in the area in which the individual resides (i.e., $\mathrm{y} = \mathrm{f}(\mathrm{p})$).

Figure B.4 gives formulas for calculating values of popular segregation indices in this manner. My intent here is only to introduce formulas that place popular indices of uneven distribution in the general "difference of group means" framework. Appendices C-F provide detailed discussions of the mathematical basis for the formulas given here. The body of the monograph provides a more general discussion of this new measurement approach and the benefits associated with adopting it.

[2] The choice of which group serves as the reference is arbitrary in the sense that the index score obtained is the same either way.

Index Formulated as a Difference of Means	Residential Outcome Scores (y) Assigned to Individuals Based on y = f(p)
$G = 100 \cdot 2(\overline{Y}_1 - \overline{Y}_2)$	$y_i = f(p_i) =$ relative rank (quantile scoring) on p_i
$D = 100 \cdot (\overline{Y}_1 - \overline{Y}_2)$	$y_i = f(p_i) = 0$ if $p_i < P$, 1 if $p_i \geq P$
Alternatively, compute D as a simplified version of G based on collapsing area values for p_i into a two-category rank scheme consisting of areas where $p_i < P$ and areas where $p_i \geq P$.	
A = No direct solution is yet found but $A = 2R - R^2$ for the "symmetric" version of A given by on setting $\alpha = \beta = 0.5$.	
$R = 100 \cdot (\overline{Y}_1 - \overline{Y}_2)$	$y_i = Q + \left(1 - \sqrt{p_i q_i / PQ}\right) / (p_i/P - q_i/Q)$
$H = 100 \cdot (\overline{Y}_1 - \overline{Y}_2)$	$y_i = Q + [(E - e_i)/E] / (p_i/P - q_i/Q).$
$S = 100 \cdot (\overline{Y}_1 - \overline{Y}_2)$	$y_i = p_i$

Fig. B.4 Formulas casting indices of uneven distribution (S) as group differences of means ($\overline{Y}_1 - \overline{Y}_2$) on individual residential outcomes (y) (Note: p_i denotes the pairwise area proportion for the reference group (p_i) in the area where individual i resides and y_i is the residential outcome score generated by the index-specific scoring function $f(p_i)$)

For the moment I note that the approach is attractive on conceptual grounds because these formulas clarify that segregation indices measure whether groups to experience similar or different averages on specific residential outcomes. Additionally, the formulas reveal that differences between indices arise from a single source; the specific nature of the scaling function $y = f(p)$ that scores residential outcomes (y) from values of area group proportion (p). Area group proportion (p) reflects simple group contact or exposure in its original or "natural" metric. The scoring function $y = f(p)$ rescales group contact and maps it onto an alternative scaling metric for residential outcomes (y) specific to the index in question. From this perspective all popular indices of uneven distribution register group differences of means on "scaled" pairwise group contact.

Appendix C: Establishing the Scaling Functions $y = f(p)$ Needed to Cast the Gini Index (G) and the Dissimilarity Index (D) as Differences of Group Means on Scaled Pairwise Contact

This is the first of several appendix chapters which establish how popular indices of uneven distribution can be placed in the "difference of group means" framework. The feature of this framework is that the values of each index are obtained as a simple difference of group means on individual residential outcomes (y) that are scored from to 0 to 1 based on area group proportion (p) computed from pairwise population counts. Taking the familiar example of White-Black segregation, area group proportion (p) can be set to proportion White of the combined White and Black population in the area; that is, $p = w/(w+b)$ where w and b are the counts of Whites and Blacks, respectively, in the area.[3] Residential outcome scores (y) are then obtained from an index-specific scaling function $y = f(p)$ that takes values of p that range from 0 to 1 and rescales them to new values that also range from 0 to 1. The segregation index score is then obtained from the difference ($Y_W - Y_B$) where Y_W and Y_B are the group means for Whites and Blacks, respectively, on residential outcomes (y).

For individuals, p registers simple pairwise "contact" or "exposure" to the reference group based on residing in a given area. In the example under consideration the reference group is Whites and p thus registers "contact with" or "exposure to" Whites. The residential outcome score (y) can be described as "scaled pairwise contact" or "scaled pairwise exposure". Accordingly, the segregation index score can be described as a difference of group means on scaled contact; in the example under consideration, it is the White-Black difference in average scaled contact with Whites.

The General Task

The key to placing a particular index of uneven distribution in the difference of means framework is to identify a scaling function $y = f(p)$ that accomplishes the goal of scoring residential outcomes (y) from area group proportions (p) such that the scores for y fall over the range 0–1 and yield the value of the index of interest as a difference of means on y for the two groups in the segregation comparison. I have identified scaling functions meeting these criteria for all popular indices of uneven distribution including: the gini index (G), the delta or dissimilarity index (D), the Hutchens square root index (R), the Theil entropy index (H) and the separation index (S). Placing these various indices in the difference of means framework gives them a common basis for interpretation and a specific basis for comparison. The common basis for interpretation is that all indices measure White-Black differences in average scaled contact with Whites. The specific basis for comparison is that the differences between index scores arise *solely* from differences in how index-specific scaling functions $y = f(p)$ map values of pairwise contact from its original or "natural" metric based on area group proportion (p) onto values of residential outcomes (y).

The main task of this appendix chapter and the ones that follow it is to establish the particular scaling function $y = f(p)$ that will yield the value of the index in question. The general way task is to start with a generic expression of the difference of means formulation.

[3]Alternatively, p can be set to area proportion Black. The choice is arbitrary as the index score is the same either way.

$$\text{Difference of Means Formula} = \left(Y_W - Y_B\right) = \left(1/W\right) \cdot \Sigma\, w_i y_i - \left(1/B\right) \cdot \Sigma\, b_i y_i$$

Then equate this formula to a standard formula for the index of interest and then manipulate the full expression to obtain a solution for y. In this appendix chapter and the ones that follow it I review steps that accomplish this task and establish a basis for an index specific scaling function $y = f(p)$ relevant for G, D, R, H, and S.

I expect that many readers will not be especially interested in the derivations of the relevant scaling functions. With this in mind, I presented only the final formulas in the main body of this monograph and in the overview discussion just provided in Appendix B. Readers who are not interested in the details of these derivations can rely on these earlier presentations and skip the remainder of this chapter and the additional appendix chapters that follow. For those who elect to slog through the technical details, I thank you in advance for your patience and forbearance. I claim only that the derivations accomplish what is needed and apologize for the fact that they are tedious and inelegant.

Introducing the Function $y = f(p)$ for the Gini Index (G)

For the Gini Index (G) the relevant scaling function $y = f(p)$ is relatively simple; it is the quantile (percentile) or *relative rank* transformation.

$$y = \text{quantile(p), or, more exactly}$$
$$y = 2 \cdot \text{quantile}(p).$$

Under this scaling approach, households are assigned values on residential outcomes (y) based on the population-weighted relative rank position of their area of residence on area group proportion (p); more specifically, the quantile score on p for individuals.

I review the quantile scaling function in more detail below. For the moment I note briefly that the scaling function $y = f(p)$ for G is a continuous, monotonic, nonlinear transformation of p that changes p from its original or “natural” metric to a new scaling metric. The nonlinear transformation produces a curve that tends to rise faster when p is low and when p is high and tends to rise more slowly when p is in the middle ranges. As a result, the scaling transformation serves to exaggerate group differences on p over portions of the lower and upper ranges of the scale of p (i.e., $p < 0.25$ and $p > 0.75$) while compressing group differences on p over middle portions of the range of p (i.e., $0.30 < p < 0.70$). Thus, the quantile transformation can and often does change small quantitative differences between Whites and Blacks on p into large differences on rank-order quantile scores. This in turn makes average White-Black differences on y larger than average White-Black differences on p. The tendency is moderate when groups are approximately equal in size. It becomes more and more pronounced when groups become increasingly unequal in size.

As formulated for the difference of group means framework, the Gini Index (G) for White-Black segregation can be given by

$$\begin{aligned} &Y_W - Y_W = G/2, \text{ or} \\ &(Y_W - Y_B)/0.5 = 2(Y_W - Y_B) = G \end{aligned} \tag{C.1}$$

for $y = \text{quantile}(p)$, or, alternatively, for $y = 2 \cdot \text{quantile}(p)$,

$$(Y_W - Y_B) = G. \tag{C.1a}$$

In this formulation residential outcomes (y) register each household's *relative rank* position on area proportion White (p), Y_W is the mean on y for White households, and Y_B is the mean on y for Black households. One way to describe the formulation is that the value of G is the observed difference of group means on quantile scores for p divided by 0.5, the maximum value possible when scoring y as quantile scores. Alternatively, if y is scored as twice the quantile score (i.e., $2 \cdot \text{quantile}(p)$), G is the simple difference of means.[4]

G Is a Measure of Rank Order Inequality on Contact

Surprisingly, methodological reviews of segregation indices rarely make, much less emphasize, the point that the Gini Index (G) assesses uneven distribution in terms of group differences in rank order standing on area group proportion scores (p). This quality of G has been noted in methodological studies that review the application of G as a measure of inter-group inequality on ordinal variables. Lieberson (1976) introduced a measure of inter-group inequality on ordinal outcomes which he termed the index of net difference (ND). He characterized ND as being "analogous" to G (1976:281). Fossett and South (1983) noted that ND and G are more than analogous; they are mathematically equivalent (this is established in expressions (C.2a) and (C.2b) below). Accordingly, ND can be characterized as an alternative computing formula for G that supports an explicit and potentially attractive substantive interpretation in terms of group difference in rank advantage.

This provides an initial basis for interpreting G for White-Black segregation as an index of relative rank difference between Whites and Blacks in their distribution on residential contact with Whites (p). Specifically, in the ND formulation, the value of G is the difference of two probabilities; (a) the probability that a randomly chosen White will have greater residential contact with Whites than will a randomly

[4] Under maximum uneven distribution all Whites live in neighborhoods that are 100 % White and all Blacks live in neighborhoods that are 100 % Black. Their respective average quantile scores on area proportion White will be $1-P/2$ for Whites and $Q/2$ for Blacks. The group (White-Black) difference of means will be $(1-P/2)-Q/2$ which resolves to $1-(P/2+Q/2)=1-(P+Q)/2=1-1/2=0.5$.

chosen Black, and (b) the probability that a randomly chosen Black will have greater residential contact with Whites than will a randomly chosen White.

Fossett and South (1983:861) note that the value of ND, and therefore G, can be obtained from the following computing formula

$$\mathrm{ND} = \mathrm{G} = \Sigma_i \Sigma_j \, x \cdot \left(w_i / W\right)\left(b_j / B\right)$$

where i and j index areas ranked on area proportion White (p), and x is scored: 1 if $(i > j)$, 0 if $(i = j)$, and −1 if $(i < j)$. This formula highlights that G responds solely to White-Black comparisons on rank order standing on area proportion White (p). Thus, it gives insight into why G is insensitive to the quantitative magnitude of group differences on p; G treats all White-Black differences on p as either 1 or −1, regardless of the difference involved is large or small.

Fossett and Siebert (1997, Appendix A) also explore the formulation of G as a measure of inter-group inequality on ranked outcomes. They showed that G is a special case of Somers' d_{yx}, a measure of ordinal (rank-order) association. Consequently, G can be interpreted as an ordinal slope coefficient that indicates the impact of race (i.e., group membership) on the rank order standing of individuals on residential contact with Whites (p). Of more direct relevance for the present discussion, Fossett and Siebert also noted that the value of G can be given as twice the difference of group means on percentile (or quantile) scores for ranked outcomes. In application to White-Black segregation this means that G registers the White-Black difference of means on quantile scores for contact with Whites (p).

Calculating G as a Difference of Means

The procedure for obtaining the value of G for White-Black segregation as a difference of means on residential outcomes (y) can be given as follows. First implement the relative rank scoring function $y = f(p)$ by ordering areas from low to high based on values of area proportion White (p_i).[5] Note that p_i is calculated using only counts for Whites and Blacks (i.e., $p_i = w_i / (w_i + b_i)$). Designate the number of households in the area ranked lowest on area proportion White (p_1) by t_1 based on $t_1 = w_1 + b_1$ where w_1 and b_1 are the counts for Whites and Blacks, respectively, in the area. Then calculate the average relative rank position (y_1) on area proportion White (p_1) for households in this area as $y_1 = (t_1/2)/T$ where T is the combined population of Whites and Blacks in the city based on $T = W + B$. The calculation reflects the fact that households in this area occupy ranks 1 through t_1 on area proportion White (p) and so they all are assigned the average for this range of relative rank positions. The number of households in the area ranked next lowest on area

[5] Areas that are identical on area proportion White (p) can be combined and treated as single areas, or they can be handled separately. There is no practical difference as the average score for y will be the same either way.

proportion White (p_2) is designated by t_2. The average relative rank position (y_2) for these households on area proportion White (p) is $\left[t_1 + \left(t_2 / 2\right)\right] / T$, the average for the relative rank position for households in the area. Continue with this procedure until all areas are scored on y.

The resulting White-Black difference of means on y is then given by

$$Y_W - Y_B = \Sigma\, w_i y_i / W - \Sigma\, b_i y_i / B.$$

This result takes a value equal to G/2.

Deriving G as a Difference of Means

The next several sections establish that the difference of means formulation of the Gini Index (G) maps exactly onto the usual computing formulas for G. Unfortunately, the discussion is long and tedious. Readers who are not interested in these details should skip forward to the section that discusses the differences of means formulation of the Dissimilarity Index (D).

Specifying Some Useful Terms and Relationships

To begin, it is helpful to introduce several terms and establish certain relationships among them. I start by introducing the following three terms:

$pt_i = t_i / T$, this term registers the i'th area's proportion (share) of the city's combined population of Whites and Blacks,
$pw_i = w_i / W$, this term registers the i'th area's proportion (share) of the city's White population, and
$pb_i = b_i / B$, this term registers the i'th area's proportion (share) of the city's Black population.

When calculating G the areas of the city are ordered from lowest to highest value on area proportion White (p). This leads to the following terms

$cpt_i = \Sigma pt_i = \Sigma t_i / T$, cumulative proportion (share) of the city's combined population of Whites and Blacks residing in areas ranked 1 through i on area proportion White (p),
$cpw_i = \Sigma pw_i = \Sigma w_i / W$, cumulative proportion (share) of the city's White population residing in areas ranked 1 through i on area proportion White (p), and
$cpb_i = \Sigma pb_i = \Sigma b_i / B$, cumulative proportion (share) of the city's Black population residing in areas ranked 1 through i on area proportion White (p).

These terms can be used to give the familiar computing formula for G introduced by Duncan and Duncan (1955: 211) as

$$G = \Sigma pw_i \cdot \Sigma pb_{i-1} - \Sigma pb_i \cdot \Sigma pw_{i-1}. \tag{C.2}$$

This can be restated with alternative notation as

$$G = \Sigma\left(cpw_i \cdot cpb_{i-1}\right) - \Sigma\left(cpb_i \cdot cpw_{i-1}\right). \tag{C.2a}$$

Recognizing that $\left(cpw_i \cdot cpb_{i-1}\right) = \left(pw_i \cdot cpb_{i-1}\right) + \left(cpw_{i-1} \cdot cpb_{i-1}\right)$, and that $\left(cpb_i \cdot cpw_{i-1}\right) = \left(pb_i \cdot cpw_{i-1}\right) + \left(cpb_{i-1} \cdot cpw_{i-1}\right)$, (C.2a) can be restated as

$$G = \Sigma\left(pw_i \cdot cpb_{i-1}\right) - \Sigma\left(pb_i \cdot cpw_{i-1}\right) \tag{C.2b}$$

Expressions (C.2), (C.2a), and (C.2b) are mathematically equivalent variations of the standard computing formula for G. Expression (C.2a) corresponds to the traditional computing formulas for G given in Duncan and Duncan (1955). Expression (C.2b) is an alternative computing formula for G which Lieberson (1976) termed ND.

A Brief Demonstration

I begin with an example that applies the terms introduce above to obtain G by the conventional formula and also demonstrates how the value of G can be obtained by the simpler approach of computing the difference of group means from percentile scores. The example case has just five areas, each one with 100 people. These are listed from high to low based on proportion White in the area. Appendix Fig. C.1 lists for basic terms for each area. These include the group count terms (t_i, w_i, b_i), proportion White for the area (p_i), the proportion of the group population residing in the area (pt_i, pw_i, pb_i), and the cumulative proportion of the group population residing in areas with area proportion White at or below p_i (cpt_i, $cptw_i$, $cptb_i$).

Appendix Fig. C.2 presents terms that are used directly to calculate the value of G. The second and third columns in the figure present the terms used to calculate the value of G via the Lieberson (1976) "net difference" variation of the formula given in Duncan and Duncan (1955) (expression (C.2b) above). The difference between the sums for the two columns (i.e., 0.903–0.027) yields the value of G as 0.876. The fourth column gives the percentile score for each area as ranked on area proportion

Area	t_i	w_i	b_i	p_i	pt_i	pw_i	pb_i	cpt_i	cpw_i	cpb_i
5	100	100	0	1.000	0.200	0.286	0.000	1.000	1.000	1.000
4	100	95	5	0.950	0.200	0.271	0.033	0.800	0.714	1.000
3	100	90	10	0.900	0.200	0.257	0.067	0.600	0.443	0.967
2	100	65	35	0.650	0.200	0.186	0.233	0.400	0.186	0.900
1	100	0	100	0.000	0.200	0.000	0.667	0.200	0.000	0.667
	500	350	150			1.000	1.000			

Fig. C.1 Example of calculating the Gini index – intermediate terms

Area	$pw_i \cdot cpb_{i-1}$	$pb_i \cdot cpw_{i-1}$	y_i	$pw_i \cdot y_i$	$pb_i \cdot y_i$
5	0.286	0.000	0.900	0.257	0.000
4	0.262	0.015	0.700	0.190	0.023
3	0.231	0.012	0.500	0.129	0.033
2	0.124	0.000	0.300	0.056	0.070
1	---	---	0.100	0.000	0.067
	0.903	0.027		0.631	0.193

Fig. C.2 Example of calculating the Gini index – final terms

White (p). This is the residential outcome (y) relevant for computing G in the difference of means framework. The fifth and sixth columns give weighted sum calculations for obtaining separate group means on y for Whites and Blacks. Twice the difference of the sums for the two columns (i.e., $2 \cdot \left(Y_W - Y_B\right) = 2 \cdot \left(0.631 - 0.193\right) = 2 \cdot 0.438$) also yields the value of G as 0.876.

Getting on with the Derivation

This example illustrates that the difference of means approach for obtaining G is simple and straight forward. The next task is to show how these formulas for G (C.2, C.2a, and C.2b) map onto the terms in the formulation of G as the White-Black difference of means $Y_W - Y_B$ on relative rank position on area proportion White (p). I apologize in advance for the fact that the derivation to follow is long and tedious. I suspect a simpler derivation can be given but I have not discovered it. What follows is one way to accomplish the task.

My first step is to introduce the term RRT_i as an alternative designation of y_i as “relative rank” standing on area proportion White (p). Thus,

$$RRT_i = y_i = \left(\Sigma pt_{i-1} + pt_i/2\right) = \left(\Sigma t_{i-1} + t_i/2\right)/T.$$

The “RR” in “RRT” refers to *relative rank* and the “T” indicates that it is calculated for the total of the combined population of White and Black households (ignoring other households). Multiplying relative rank by 100 gives a percentile score. Given these terms, the White-Black difference of means for y_i is given by

$$\begin{aligned} Y_W - Y_B &= \Sigma pw_i \cdot y_i - \Sigma pb_i \cdot y_i, \text{ or, alternatively,} \\ Y_W - Y_B &= \Sigma pw_i \cdot RRT_i - \Sigma pb_i \cdot RRT_i. \end{aligned} \tag{C.3}$$

Next I introduce two related terms – RRW_i and RRB_i. RRW_i registers average relative rank position on area proportion White (p) based on the distribution of *White households only* and is given by

$$\mathrm{RRW_i} = \left(\Sigma \mathrm{pw_{i-1}} + \mathrm{pw_i}/2\right) = \left(\Sigma \mathrm{w_{i-1}} + \mathrm{w_i}/2\right)/\mathrm{W}.$$

RRB_i registers the relative rank position on area proportion White (p) based on the distribution of *Black households only* and is given by

$$\mathrm{RRB_i} = \left(\Sigma \mathrm{pb_{i-1}} + \mathrm{pb_i}/2\right) = \left(\Sigma \mathrm{b_{i-1}} + \mathrm{b_i}/2\right)/\mathrm{B}.$$

The terms RRT_i, RRW_i, and RRB_i, are closely interrelated. Specifically, each one can be defined in terms of the other two according to the following expressions.

$$\mathrm{RRT_i} = \mathrm{P}\cdot\mathrm{RRW_i} + \mathrm{Q}\cdot\mathrm{RRB_i} \tag{C.4a}$$

$$\mathrm{RRW_i} = \left(\mathrm{RRT_i} - \mathrm{Q}\cdot\mathrm{RRB_i}\right)/\mathrm{P} \tag{C.4b}$$

$$\mathrm{RRB_i} = \left(\mathrm{RRT_i} - \mathrm{P}\cdot\mathrm{RRW_i}\right)/\mathrm{Q} \tag{C.4c}$$

The basis for expression (C.4a) can be clarified as follows

$$\begin{aligned}
\mathrm{RRT_i} &= \left(\Sigma \mathrm{pt_{i-1}} + \mathrm{pt_i}/2\right) \\
&= \left(\Sigma \mathrm{t_{i-1}} + \mathrm{t_i}/2\right)/\mathrm{T} \\
&= \left(\Sigma \mathrm{w_{i-1}} + \Sigma \mathrm{b_{i-1}} + \mathrm{w_i}/2 + \mathrm{b_i}/2\right)/\mathrm{T} \\
&= \left(\Sigma \mathrm{w_{i-1}} + \mathrm{w_i}/2\right)/\mathrm{T} + \left(\Sigma \mathrm{b_{i-1}} + \mathrm{b_i}/2\right)/\mathrm{T} \\
&= \left(\Sigma \mathrm{w_{i-1}} + \mathrm{w_i}/2\right)/\left[\mathrm{W}\cdot\left(\mathrm{T}/\mathrm{W}\right)\right] + \left(\Sigma \mathrm{b_{i-1}} + \mathrm{b_i}/2\right)/\left[\mathrm{B}\cdot\left(\mathrm{T}/\mathrm{B}\right)\right] \\
&= \left(\mathrm{W}/\mathrm{T}\right)\cdot\left(\Sigma \mathrm{w_{i-1}} + \mathrm{w_i}/2\right)/\mathrm{W} + \left(\mathrm{B}/\mathrm{T}\right)\cdot\left(\Sigma \mathrm{b_{i-1}} + \mathrm{b_i}/2\right)/\mathrm{B} \\
&= \left(\mathrm{W}/\mathrm{T}\right)\cdot\left(\Sigma \mathrm{w_{i-1}} + \mathrm{w_i}/2\right)/\mathrm{W} + \left(\mathrm{B}/\mathrm{T}\right)\cdot\left(\Sigma \mathrm{b_{i-1}} + \mathrm{b_i}/2\right)/\mathrm{B} \\
&= \mathrm{P}\cdot\left(\Sigma \mathrm{pw_{i-1}} + \mathrm{pw_i}/2\right) + \mathrm{Q}\cdot\left(\Sigma \mathrm{pb_{i-1}} + \mathrm{pb_i}/2\right) \\
&= \mathrm{P}\cdot\mathrm{RRW_i} + \mathrm{Q}\cdot\mathrm{RRB_i}\,.
\end{aligned}$$

Expressions (C.4b) and (C.4c) are simple rearrangements of (C.4a).

The relationships among RRT_i, RRW_i, and RRB_i help clarify how G relates to $Y_W - Y_B$. Expression (C.3) shows that the values of RRT_i are directly used in computing Y_W and Y_B. Expression (C.4a) establishes that RRT_i can be given in terms of RRW_i and RRB_i. These two terms can be incorporated into familiar computing expressions for G (yielding Eq. (C.5) below).

Before reviewing this in more detail I first digress to note that values of RRW_i and RRB_i define points on the segregation curve, the well-known graphical representation of uneven distribution that supports an appealing geometric interpretation of G. The segregation curve is constructed by taking areas in ascending order of area proportion White (p) and then plotting cumulative proportion White ($\mathrm{cpw_i} = \Sigma \mathrm{w_i}/\mathrm{W}$) against cumulative proportion Black ($\mathrm{cpb_i} = \Sigma \mathrm{b_i}/\mathrm{B}$). The curve is contrasted with the diagonal line that would result under conditions of exact even distribution and the value of G is given by ratio of the area between the diagonal and the curve to the total area under the diagonal. The values of RRW_i by RRB_i fall on the midpoints of the line segments that form the segregation curve.

The values of RRW_i and RRB_i can be used to directly calculate the value of G. To see this, start with the following familiar computing formula for G given by Duncan and Duncan (1955: 211)

$$G = \Sigma pw_i \cdot \Sigma pb_{i-1} - \Sigma pb_i \cdot \Sigma pw_{i-1}. \qquad \text{(C.2, restated)}$$

Then add 0 in the form of $\Sigma pw_i \cdot pb_i/2 - \Sigma pw_i \cdot pb_i/2$ to obtain

$$G = \left[\Sigma pw_i \cdot \Sigma pb_{i-1} - \Sigma pb_i \cdot \Sigma pw_{i-1}\right] + \left[\Sigma pw_i \cdot pb_i/2 - \Sigma pw_i \cdot pb_i/2\right].$$

Rearrange terms

$$G = \Sigma pw_i \cdot \left[\Sigma pb_{i-1} + pb_i/2\right] - \Sigma pb_i \cdot \left[\Sigma pw_{i-1} + pw_i/2\right].$$

Drawing on terms given earlier, substitute RRB_i for $\left[\Sigma pb_{i\ 1} + pb_i/2\right]$ and RRW_i for $\left[\Sigma pw_{i\ 1} + pw_i/2\right]$ to obtain

$$G = \Sigma\, pw_i \cdot RRB_i - \Sigma\, pb_i \cdot RRW_i. \qquad \text{(C.5)}$$

For later notational convenience, I designate $\Sigma\, pw_i \cdot RRB_i$ as G_W and $\Sigma\, pb_i \cdot RRW_i$ as G_B to get the compact expression

$$G = G_W - G_B. \qquad \text{(C.5a)}$$

Note that the terms G_W and G_B support straightforward substantive interpretations. Specifically, G_W indicates the proportion of total comparisons between White and Black households where the White household is higher on area proportion White (p) and G_B similarly indicates the proportion of comparisons where the Black household is higher.[6]

$$Y_W - Y_B = \Sigma\, pw_i \cdot RRT_i - \Sigma\, pb_i \cdot RRT_i. \qquad \text{(C.3, restated)}$$

Expression (C.5) is very similar in form to expression (C.3) (restated here for convenience). This suggests that the relationship of G to $Y_W - Y_B$ can be expressed in terms of specific relationships between the core terms in (C.3) and (C.5). This is indeed the case. The first relationship involves the terms $\Sigma\, pw_i \cdot RRB_i$ from (C.5) and $\Sigma\, pw_i \cdot RRT_i$ from (C.3). Their relationship can be given as

$$\Sigma\, pw_i \cdot RRB_i = \left(\Sigma\, pw_i \cdot RRT_i - P/2\right)/Q. \qquad \text{(C.6)}$$

[6] This corresponds closely to Lieberson's (1976) index of net difference (ND) interpretation of G. The only difference computationally is how ties are handled in the computations. In Lieberson's calculations, ties are dealt with separately. In this calculation, ties are apportioned in equal halves to each outcome. The resulting value of G (or ND) is identical.

The second relationship involves the terms $\Sigma\, pb_i \cdot RRW_i$ from (C.5) and $\Sigma\, pw_i \cdot RRT_i$. from (C.3). Their relationship can be given as

$$\Sigma\, pb_i \cdot RRW_i = \left(\Sigma\, pb_i \cdot RRT_i - Q/2\right)/P. \tag{C.7}$$

Similarly, the central terms in (C.2) for $Y_W - Y_B$ can be expressed in relation to the terms in (C.5) for G based on

$$Y_W = \Sigma\, pw_i \cdot RRT_i = Q \cdot \Sigma\, pw_i \cdot RRB_i + P/2 = Q \cdot G_W + P/2, \text{ and} \tag{C.8}$$

$$Y_B = \Sigma\, pb_i \cdot RRT_i = P \cdot \Sigma\, pb_i \cdot RRW_i + Q/2 = P \cdot G_B + Q/2. \tag{C.9}$$

Restating these using more compact notation yields

$$G_W = \left(Y_W - P/2\right)/Q. \tag{C.6a}$$

$$G_B = \left(Y_B - Q/2\right)/P. \tag{C.7a}$$

$$Y_W = Q \cdot G_W + P/2 \tag{C.8a}$$

$$Y_B = P \cdot G_B + Q/2 \tag{C.9a}$$

Establishing Expressions (C.6, C.6a) and (C.8, C.8a)

For the sake of completeness I show here how expressions (C.6, C.6a) and (C.8, C.8a) can be obtained. I begin by drawing on (C.4b) to restate the term $\Sigma\, pw_i \cdot RRB_i$ from (C.5) and then rearrange the result as follows.

$$\Sigma\, pw_i \cdot RRB_i = \Sigma\, pw_i \cdot \left[\left(RRT_i - P \cdot RRW_i\right)/Q\right]$$

$$\Sigma\, pw_i \cdot RRB_i = \Sigma\, pw_i \cdot \left[\left(RRT_i/Q\right) - \left(RRW_i \cdot P/Q\right)\right]$$

$$\Sigma\, pw_i \cdot RRB_i = \Sigma\, pw_i \cdot \left(RRT_i/Q\right) - \Sigma\, pw_i \cdot \left(RRW_i \cdot P/Q\right)$$

$$\Sigma\, pw_i \cdot RRB_i = \left(\Sigma\, pw_i \cdot RRT_i\right)/Q - \left(\Sigma\, pw_i \cdot RRW_i\right)\left(P/Q\right)$$

The value of the term $\Sigma\, pw_i \cdot RRW_i$ is 0.5 because the mean of relative rank position is necessarily $0.5 = ½$. Accordingly, the last expression can be simplified by substituting (½) for $\Sigma\, pw_i \cdot RRW_i$ to obtain (C.6) as follows

$$\Sigma\, pw_i \cdot RRB_i = \left(\Sigma\, pw_i \cdot RRT_i\right)/Q - \left(½\right)\left(P/Q\right)$$

$$\Sigma\, pw_i \cdot RRB_i = \left(\Sigma\, pw_i \cdot RRT_i - P/2\right)/Q \qquad \text{(C.6, restated)}$$

Or in more compact notation

$$G_W = \left(Y_W - P/2\right)/Q \qquad \text{(C.6a, restated)}$$

Reversing sides and rearranging terms to isolate Y_W yields

$$\left(Y_W - P/2\right)/Q = G_W = G_W$$

$$Y_W/Q - P/2Q = G_W$$

$$Y_W/Q = G_W + P/2Q$$

$$Y_W = Q \cdot \left(G_W + P/2Q\right)$$

$$Y_W = Q \cdot G_W + P/2 \qquad \text{(C.8a, restated)}$$

Expanding to less compact notation

$$\Sigma\, pw_i \cdot RRT_i = Q \cdot \Sigma\, pw_i \cdot RRB_i + P/2. \qquad \text{(C.8, restated)}$$

Establishing Expressions (C.7, C.7a) and (C.9, C.9a)

Next I show here how expressions (C.7, C.7a) and (C.9, C.9a) can be obtained. I begin by drawing on (C.4a) to restate the term $\Sigma\, pb_i \cdot RRW_i$ from (C.5) and then rearrange the result as follows.

$$\Sigma\, pb_i \cdot RRW_i = \Sigma\, pb_i \cdot \left[\left(RRT_i - Q \cdot RRB_i\right)/P\right]$$

$$\Sigma\, pb_i \cdot RRW_i = \Sigma\, pb_i \cdot \left[\left(RRT_i\ /P\right) - \left(RRB_i \cdot Q/P\right)\right]$$

$$\Sigma\, pb_i \cdot RRW_i = \Sigma\, pb_i \cdot \left(RRT_i\ /P\right) - \Sigma\, pb_i \cdot \left(RRB_i \cdot Q/P\right)$$

$$\Sigma\, pb_i \cdot RRW_i = \left(\Sigma\, pb_i \cdot RRT_i\right)/P - \left(\Sigma\, pb_i \cdot RRB_i\right)\left(Q/P\right)$$

Since $\Sigma\, pb_i \cdot RRB_i$ is $0.5 = ½$, the last expression can be simplified by substituting (½) for $\Sigma\, pb_i \cdot RRB_i$ to obtain (C.8) as follows

$$\Sigma\, pb_i \cdot RRW_i = \left(\Sigma\, pb_i \cdot RRT_i\right)/P - (½)\left(Q/P\right)$$

$$\Sigma\, pb_i \cdot RRW_i = \left(\Sigma\, pb_i \cdot RRT_i - Q/2\right)/P. \qquad \text{(C.8, restated)}$$

Or more compactly

$$G_B = (Y_B - Q/2)/P. \qquad \text{(C.8a, restated)}$$

Reversing sides and rearranging terms to isolate Y_B yields

$$Y_B/P - Q/2P = G_B$$

$$Y_B/P = G_B + Q/2P$$

$$Y_B = P \cdot (G_B + Q/2P)$$

$$Y_B = P \cdot G_B + Q/2 \qquad \text{(C.9a, restated)}$$

$$\Sigma\, pb_i \cdot RRT_i = P \cdot \Sigma\, pb_i \cdot RRW_i + Q/2. \qquad \text{(C.9, restated)}$$

Some Implications of Expressions (C.6) and (C.7)

Based on (C.6) and (C.7), G as given in (C.5) can be obtained from the core terms that define $Y_W - Y_B$ in (C.3) as follows

$$G = (\Sigma\, pw_i \cdot RRT_i - P/2)/Q - (\Sigma\, pb_i \cdot RRT_i - Q/2)/P \qquad \text{(C.10)}$$

or, in more compact notation,

$$G = (Y_W - P/2)/Q - (Y_B - Q/2)/P. \qquad \text{(C.10a)}$$

Similarly, based on (C.8) and (C.9), the term $Y_W - Y_B$ in (C.3) can be obtained from the terms that define G in (C.5) as follows

$$Y_W - Y_B = (Q \cdot \Sigma\, pw_i \cdot RRB_i + P/2) - (P \cdot \Sigma\, pb_i \cdot RRW_i + Q/2) \qquad \text{(C.11)}$$

or, in more compact notation,

$$Y_W - Y_B = (Q \cdot G_W + P/2) - (P \cdot G_B + Q/2). \qquad \text{(C.11a)}$$

These results establish that the value of the Gini Index (G) can be directly and exactly mapped onto the terms of the group difference of means ($Y_W - Y_B$) on residential outcomes (y) scored on the basis of relative rank position on area group proportion (p).

The Role of P and Q in Scaling Terms when Groups Differ in Relative Size

The results just reviewed show that, while the relationship between G and ($Y_W - Y_B$) is exact, it also is complex. Expressions (C.10, C.10a) and (C.11, C.11a) clarify how scores for G map onto scores for ($Y_W - Y_B$). In this, it is clear that the terms for relative group size – P and Q – play important roles. How can this be understood? One answer to that question is that the operations involving P and Q in these expressions rescale the core terms of G so they will map onto the core terms of $Y_W - Y_B$, and vice versa. This is necessary because the core terms in G and $Y_W - Y_B$ have different logical ranges. Accordingly, the operations involving P and Q in expression (C.10) rescale the core terms of $Y_W - Y_B$ so they will take the same value as their corresponding terms in G. Similarly, the operations involving P and Q in expression (C.11) rescale the core terms of G so they will take the same value as their corresponding terms in $Y_W - Y_B$.

The logical ranges for both G and its core terms are constant across all combinations of P and Q. The core term $\Sigma pw_i \cdot RRB_i$ (i.e., G_W) has a logical range of 0.5 based on having a minimum possible value of 0.5 under even distribution and a maximum value of 1.0 under complete segregation. The core term $\Sigma pb_i \cdot RRW_i$ (i.e., G_B) also has a logical range of 0.5 based on having a minimum possible value of 0.0 under complete segregation and a maximum value of 0.5 under even distribution. Thus, G ranges from a minimum of 0.0 under even distribution based on

$$G = \Sigma pw_i \cdot RRB_i - \Sigma pb_i \cdot RRW_i = 0.5 - 0.5 = 0.0$$

to a maximum of 1.0 under complete segregation based on

$$G = \Sigma pw_i \cdot RRB_i - \Sigma pb_i \cdot RRW_i = 1.0 - 0.0 = 1.0$$

The logical range for $Y_W - Y_B$ also is always constant but it is 0.5 not 1.0. This accounts for why G is divided by 2 in expression (C.1). Note, however, that the logical ranges for the two core terms Y_W and Y_B are not constants. In each case one boundary of their logical range is a constant but the other boundary varies with the values of P and Q. For the term $Y_W = \Sigma pw_i \cdot RRT_i$ the fixed boundary is its minimum possible value of 0.5, which occurs under even distribution. Its upper boundary (i.e., maximum possible value) is given by $Q + P/2$, which occurs under complete segregation and varies in exact value with city ethnic composition. For the term $Y_B = \Sigma pb_i \cdot RRT_i$, the fixed boundary of its logical range is 0.5, its maximum possible value which occurs under even distribution. Its lower boundary (i.e., the minimum possible value) is given by Q/2 which occurs under complete segregation and varies in exact value with city ethnic composition.

Thus, $Y_W - Y_B$ ranges from a minimum of 0.0 under even distribution based on

$$Y_W - Y_B = \Sigma pw_i \cdot RRT_i - \Sigma pb_i \cdot RRT_i = 0.5 - 0.5 = 0.0$$

to a maximum of 0.5 under complete segregation based on

$$Y_W - Y_B = \Sigma pw_i \cdot RRT_i - \Sigma pb_i \cdot RRT_i = (Q + P/2) - (Q/2)$$
$$= Q/2 + P/2 = (Q + P)/2 = 0.5.$$

In light of these points, Expression (C.10) can now be understood as follows. The values of P/2 and Q in the term ($\Sigma pw_i \cdot RRT_i - P/2$) / Q rescale the value of the core term $\Sigma pw_i \cdot RRT_i$ used in computing Y_W in $Y_W - Y_B$ to map its position in the logical range of 0.5 to ($Q + P/2$) onto the correct position in the logical range of 0.5 to 1.0 for the parallel core term $\Sigma pw_i \cdot RRB_i$ used in computing G. Similarly, the values of Q/2 and P in the term ($\Sigma pb_i \cdot RRT_i - Q/2$) / P rescale the core term $\Sigma pb_i \cdot RRT_i$ used in computing Y_B in $Y_W - Y_B$ to map its position in the logical range of Q/2 to 0.5 onto the correct position in the logical range of 0.0 to 0.5 for the parallel core term $\Sigma pb_i \cdot RRW_i$ used in computing G.

Expression (C.11) can be interpreted in a similar way. P/2 and Q in the term ($Q \cdot \Sigma pw_i \cdot RRB_i - P/2$) / Q rescale the core term $\Sigma pw_i \cdot RRB_i$ used in computing G to map its position in the logical range of 0.5 to 1.0 on to the correct position in the logical range of 0.5 to $Q + P/2$ for the core term $\Sigma pw_i \cdot RRT_i$ used in computing $Y_W - Y_B$. Similarly, Q/2 and P in the term ($P \cdot \Sigma pb_i \cdot RRW_i - Q/2$) / P rescale the core term $\Sigma pb_i \cdot RRW_i$ used in computing G to map its position in the logical range of 0.0 to 0.5 onto the correct position in the logical range of Q/2 to 0.5 for the core term $\Sigma pb_i \cdot RRT_i$ used in computing $Y_W - Y_B$.

The Special Circumstance When $P = Q$

Things are relatively simple when $P = Q$. This can be seen by rearranging terms in (C.10) to obtain the alternative expression.

$$G = (1/Q) \cdot \Sigma pw_i \cdot RRT_i - (1/P) \cdot \Sigma pb_i \cdot RRT_i + Q/2P - P/2Q \qquad \text{(C.12)}$$

When $P = Q$, this resolves to

$$G = 1/(1/2) \cdot \Sigma pw_i \cdot RRT_i - 1/(1/2) \cdot \Sigma pb_i \cdot RRT_i$$
$$+ (1/2)/[2 \cdot (1/2)] - (1/2)/[2 \cdot (1/2)]$$

$$G = 2 \cdot \Sigma pw_i \cdot RRT_i - 2 \cdot \Sigma pb_i \cdot RRT_i + (1/2) - (1/2)$$

$$G = 2 \cdot (\Sigma pw_i \cdot RRT_i - \Sigma pb_i \cdot RRT_i)$$

$$G/2 = \Sigma pw_i \cdot RRT_i - \Sigma pb_i \cdot RRT_i$$

$$G/2 = Y_W - Y_B . \qquad \text{(C.1, restated)}$$

This corresponds to expression (C.1) presented at the beginning of this section.

Similarly, rearranging terms in (C.11) leads to the following alternative expression.

$$Y_W - Y_B = Q \cdot \Sigma pw_i \cdot RRB_i - P \cdot \Sigma pb_i \cdot RRW_i + P/2 - Q/2 \qquad (C.13)$$

When $P = Q$, this resolves to

$$Y_W - Y_B = (1/2) \cdot \Sigma pw_i \cdot RRB_i - (1/2) \cdot \Sigma pb_i \cdot RRW_i + (1/2)/2 - (1/2)/2$$

$$Y_W - Y_B = (1/2) \cdot (\Sigma pw_i \cdot RRB_i - \Sigma pb_i \cdot RRW_i)$$

$$Y_w - Y_B = G/2 \qquad \text{(C.1, restated)}$$

And this also corresponds to expression (C.1).

Summary Comments on Formulating G as a Difference of Means ($Y_W - Y_B$) on Relative Rank

The relationship in expression (C.1) now can be placed in broader context as follows. The core terms that define G in expression (C.2) map directly and exactly onto the core terms that define $Y_W - Y_B$ in expression (C.3). Consequently, G can be described as registering the White-Black difference in average relative rank on area proportion White (p). Examined in the "natural" metric of relative rank scores, the difference of means $Y_W - Y_B$ has a logical range of 0.0–0.5 while the logical range of G is 0.0–1.0. Hence, expression (C.1) equates the two measures based on $Y_W - Y_B = G/2$.

The Dissimilarity Index (D) – A Special Case of the Gini Index (G)

The dissimilarity or delta index (D) is closely related to the Gini Index (G). More specifically, D can be described as a special case of G where G is computed after areal units ordered on area group proportion scores (p) are collapsed into two categories: areas where the group proportion score exceeds the city-wide group proportion (i.e., $p > P$) and areas where it does not (i.e., $p \leq P$). Based on this, D can be expressed as a difference of group means on residential outcomes (y) scored from area group proportions (p) in a manner comparable to that just outlined for G.

D and G both are intimately related to the segregation curve, a graphical device for depicting uneven distribution popularized by Duncan and Duncan (1955). An example of a standard segregation curve is shown in Fig. C.3. The curve is based on block group data for Whites and Blacks in the Houston, Texas metropolitan area in 2000 and is constructed as follows. First the areas (in this case block groups) are placed in ascending order based on proportion White (p) in the area. Then the curve

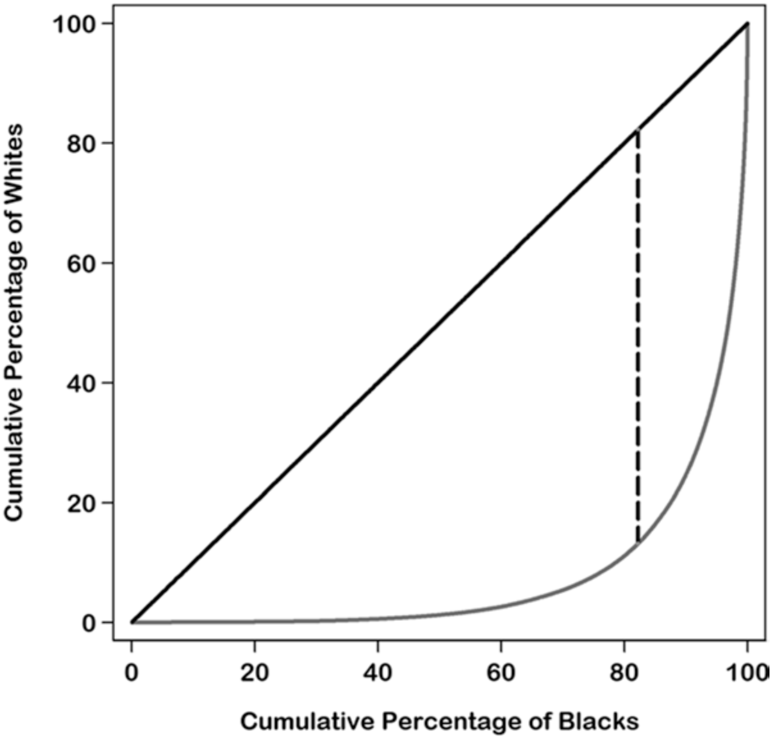

Fig. C.3 Example Segregation Curve for White-Black Comparison (Note: Units are ordered from low to high on area proportion White. Gini index is 84.7, Delta is 69.0)

is traced by drawing line segments connecting the sequence of (x,y) pairings for the cumulated proportion of the White population (on the y-axis) and the cumulated proportion of the Black population (on the x-axis) as areas are taken in ascending order on the value of p. The resulting curve is contrasted with the diagonal line between the starting point (0,1) and ending point (1,1) of the curve. The diagonal represents the segregation curve that would obtain under the condition of *exact* even distribution. The gap between the curve and the diagonal visually indicates the degree of departure from even distribution.

As is well known, G and D both have direct quantitative and geometric relations to the curve's departure from the diagonal. G registers the departure quantitatively based on the ratio of the area between the curve and the diagonal to the total area under the diagonal. In the example shown, the value of G is 84.7. D registers the degree of departure quantitatively based on the maximum vertical difference between the curve and the diagonal and in the example shown has a value of 69.0.

The geometric relationships to the segregation curve for G and D highlight an important difference between the two measures. The area interpretation of G makes it clear that its value is determined by the shape of the full curve. In contrast, the vertical line interpretation of D makes it clear that its value is determined by a single

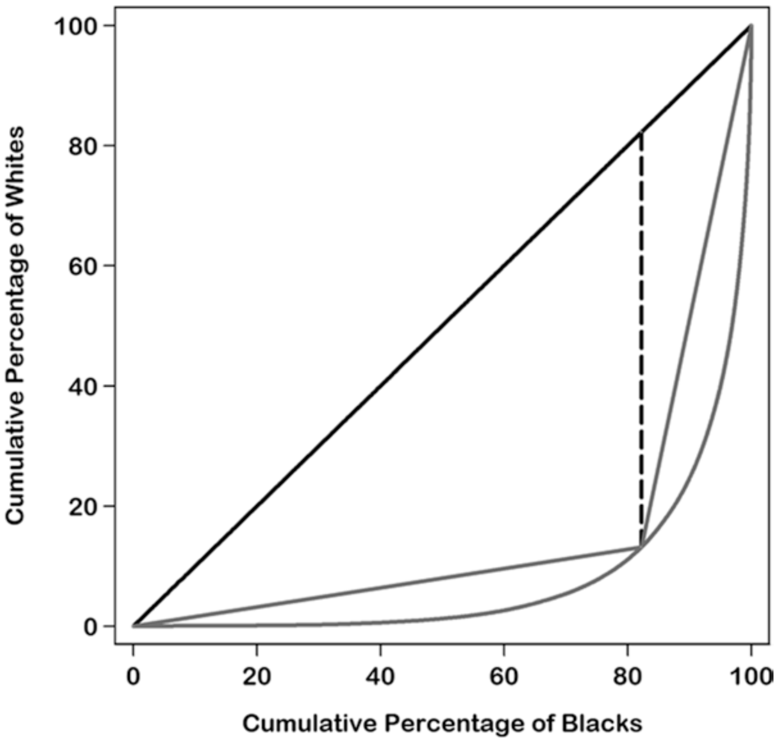

Fig. C.4 Example segregation curves for white-black comparison (Note: Units are ordered from low to high on area proportion White. Gini index is 84.7, Delta is 69.0)

point on the segregation curve. Accordingly, G responds to any residential shifts that promote more even distribution (i.e., that reduce the area between the diagonal and the curve) while D responds to such changes only if they affect the position of a particular point on the curve. The difference is highlighted in Fig. C.4. Here the segregation curve in the first graph is supplemented with a second segregation curve. This is a three point segregation curve defined by the triangle involving three points from the full segregation curve; the two end points (0,0) and (1,1) of the diagonal and the point on the full curve where the vertical distance between the curve and the diagonal is at its maximum. This last point determines the value of D so I designate it as (x_D, y_D). In the example shown it is (0.132, 0.822).[7]

[7] Becker et al. (1978) present a similar graphical analysis of D.

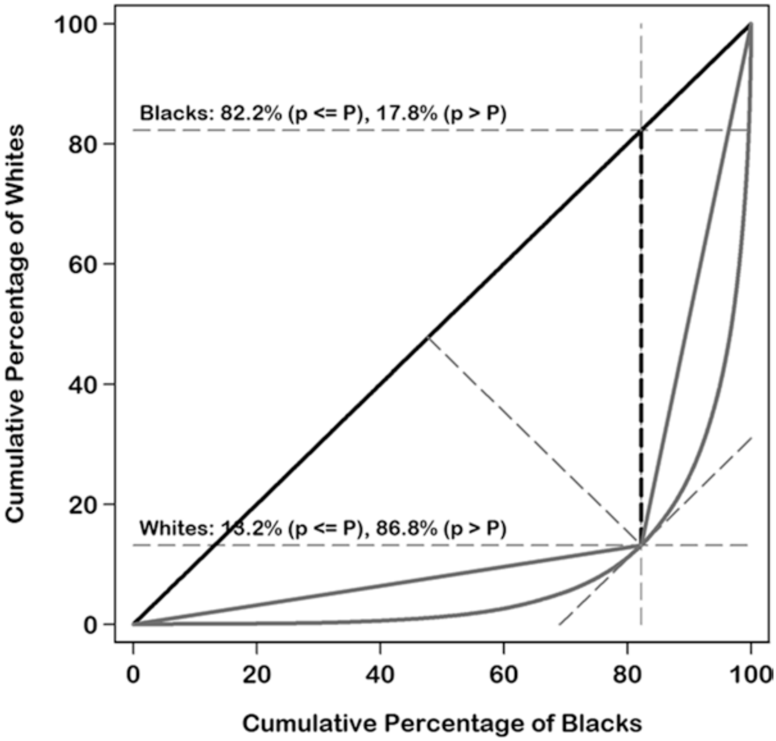

Fig. C.5 Example segregation curves for G and D with details (Note: Units are ordered from low to high on area proportion White. Gini index is 84.7, Delta is 69.0 = (86.8−17.8))

D Is G Calculated from a Special Three-Point Segregation Curve

D can be seen as a special case of G calculated for the three-point segregation curve defined by the points (0,0), (x_D, y_D), and (1,1). More specifically, D represents the minimum value of G that can obtain for a curve that has the point (x_D, y_D). This is depicted graphically in the detailed example in Appendix Fig. C.5. The relationships involved can be outlined in a general way as follows. Recall that the value of G is given by A/T where A is the area between the diagonal and the segregation curve and T is the total area under the diagonal which is ½. For the three point segregation curve associated with D, A is equal to the area of the triangle that forms the three-point segregation curve. Accordingly, $A = ½ \cdot b \cdot h$ where A is the area of the triangle, b is the length of the base of the triangle, and h is the height of the triangle. The base of the triangle is the diagonal and thus b is equal to the length of the diagonal which is $\sqrt{2}$. The height of the triangle (h) is equal to the length of the line that extends perpendicular from the diagonal and ends on the segregation curve at the point (x_D, y_D). This line is a side of a right isosceles triangle whose base has a length equal to the value of D – the maximum vertical distance from the segregation curve to the diagonal. Thus, $h = D/\sqrt{2}$.

It follows that the area (A) between the diagonal and the three point segregation curve for D is given by $A = ½ \cdot b \cdot h = ½ \cdot \sqrt{2} \cdot \left(D/\sqrt{2}\right) = ½ \cdot D$. It also follows that the value of the Gini Index (G) for the three point segregation curve is given by $G = A/T = \left(½ \cdot D\right)/½$ which resolves to D. This establishes that the value of D is equivalent to the value of G for a simplified segregation curve analysis in which all areas of the city are grouped into just two categories; all areas where $p \leq P$, and all areas where $p > P$.

The comparison of the three-point segregation curve with the full curve highlights two characteristics of D. One is that $D \leq G$ because the full segregation curve for G can never be "inside" the three-point segregation curve for D. Another is that D is insensitive to variations in residential distribution other than the distinction between residing in areas where $p > P$ or not. Finally, D can be understood as the minimum possible value of G for a curve containing the point (x_D, y_D) because D treats Whites and Blacks as experiencing only two relative rank scores and this maximizes ties between Whites and Blacks on relative ranks. Expanding the curve to consider more points cannot reduce the value of G as the construction principles are such that the segregation curve can only stay the same or expand outward from the three-point curve if more points are added to the curve.

D Is a Simple Difference of Group Proportions Residing in Areas Where $p \geq P$

There is an alternative computing approach for D that is simple and carries an appealing substantive interpretation. It is based on understanding D as the difference in group proportions residing in areas where $p \geq P$. This interpretation traces to the fact that the maximum vertical difference between the curve and the diagonal occurs at a particular point on the segregation curve. Specifically, it is first encountered at the end of the line segment on the curve for the last areal unit where $p < P$. It then is maintained for all subsequent points on the curve for areas where $p = P$. It is last encountered at the beginning of the line segment on the curve for the first areal unit where $p \geq P$.

When there are no areas where $p = P$, the maximum vertical difference between the curve and the diagonal will be at a single point; the point where the line segment for the last area where $p < P$ connects with the line segment for the first area where $p > P$. When some areas have $p = P$, the maximum vertical difference will be found at the beginning and end of the line segment formed for these areas. So it is correct to say that the maximum vertical distance corresponding to the value of D can be found at the following locations on the line segments that create the segregation curve.

- the end point of the line segment for the first area where $p < P$
- any point on line segments for areas where $p = P$
- beginning of the line segment for the first area where $p > P$

Because the vertical distance is at its maximum at the beginning and end of line segments where $p = P$, one can say the maximum vertical distance is found

- the end point of the line segment for the last area where $p \leq P$
- the beginning point of the line segment for the first area where $p \geq P$

This can be seen by reviewing the construction of the segregation curve in more detail. Starting at (0,0) the curve is formed by plotting line segments connecting (x,y) points for group population shares that are being cumulated over areas taken in ascending order of p. Except in the unusual case of exact even distribution, $p < P$ for the initial areas and the line segments plotted for these areas will have a slope of less than 1. Accordingly, the curve initially falls away from the diagonal and the vertical distance between the curve and the diagonal increases with each successive area so long as $p < P$ with the vertical distance being greatest at the end point of the line segment for the area. The maximum vertical distance is first reached when the sequence arrives at the first area where $p \geq P$. If the next area plotted is one where $p = P$ (*exactly*), the line segment for that area will have a slope of 1 and will run parallel to the diagonal. The maximum vertical distance is maintained for all subsequent areas where $p = P$ (exactly).[8] This changes when the sequence reaches the first area where $p > P$. At this point, the slope of the line segment plotted for that area will be greater than 1 and the segregation curve begins rising faster than the diagonal. Accordingly, the vertical distance between the curve and the diagonal will start to decline. It will continue to decline with each successive area in the sequence and the curve ultimately rises back to the diagonal to connect with the end point (1,1).

This discussion makes it clear that the value of D can be understood as a simple difference of group proportions. Specifically, the value of D is equal to the difference between the proportions of Whites and Blacks, respectively, that reside in areas where Whites are represented at or above the level for the city overall (i.e., $p \geq P$). For convenience, I designate the (x,y) pair for the beginning point of the line segment for the first area where $p \geq P$ as (x_D, y_D). Applying the subscript "D" indicates that the values of x_D and y_D determine the value of D. The values of x_D and y_D register the proportions of Blacks and Whites, respectively, that reside in areas where Whites are under-represented (i.e., areas where $p < P$). Under even distribution the value of y_D would be equal to x_D. In light of this, the value of D is given by ($x_D - y_D$), the vertical distance between the diagonal and the curve at this point. The values $(1 - x_D)$ and $(1 - y_D)$ similarly indicate the proportions of Blacks and Whites, respectively, who reside in areas where Whites are represented at parity or higher (i.e., areas where $p \geq P$). D also can be obtained from ($\left[1 - y_D\right] - \left[1 - x_D\right]$). This expression supports an appealing substantive interpretation of D; it is the White-Black difference in the proportions that reside in areas where proportion White is at or above the level of the city overall.

The example presented in Fig. C.5 shows that 82.2 % of Blacks and 13.2 % of Whites reside in areal units where Whites are under-represented (i.e., $p < P$). It

[8] These points are noted in Becker et al. (1978) and Duncan and Duncan (1955).

likewise shows that 86.8 % of Whites and 17.8 % of Blacks reside in area units where the presence of Whites equals or exceed the citywide level (i.e., $p \geq P$). The value of D can be obtained in either of two ways. It can be obtained from the Black-White difference in percentages in residing in areas where Whites under-represented (i.e., $D = 82.2 - 13.2 = 69.0$). Alternatively and more appropriately for the purposes of the present task, it can be obtained from the White-Black difference in percentages in residing in areas where Whites are represented at or above the level for the city overall (i.e., $D = 86.8 - 17.8 = 69.0$).

The Dissimilarity or Delta Index (D) – Alternative Functions for Scaling Contact

The above discussion establishes at least two viable ways to score individual residential outcomes (y) based on area group proportion scores (p) such that delta (D) can be obtained as a simple difference of group means. The first option is based on viewing D as a special case of the Gini Index (G). In this approach, y is scored as the relative rank (percentile) transformation of p applied to the two-category residential scheme for the special case of the three-point segregation curve described above. In this case delta (D) can be given by an expression comparable to Expression (C.1) introduced earlier for G. Specifically,

$$Y_W - Y_B = D/2, \text{ or, alternatively, } 2\left(Y_W - Y_B\right) = D$$

where D can be understood as a special case of G.

The second alternative involves an even simpler scoring scheme for y. This scaling function draws on the mundane fact that a proportion is equivalent to the mean for a variable that is scored 0 or 1. The above discussion established that D is equal to the White-Black difference in proportions residing in areas where $p \geq P$. Accordingly, the group proportions involved can be restated as group means on a variable that is scored 1 for individuals who reside in an area that reaches or exceeds parity on contact with whites White (i.e., areas where $p \geq P$) and 0 otherwise (i.e., when $p < P$). This provides the basis for obtaining D by scoring residential outcomes for individuals (y) as 1 for areas where proportion White are at or above parity (i.e., $p \geq P$) and 0 otherwise. Then compute the means for Whites and Blacks separately to obtain the value of D according to

$$Y_W - Y_B = D.$$

One benefit of the resulting difference of means formulation of D is that it calls attention to how segregation as measured by D is linked with individual residential outcomes. Specifically, this formulation highlights the fact that D registers group differences in average contact with Whites when contact is rescaled from its original, “natural” metric of p – which can vary continuously over the range of 0–1 (inclusive) – to a binary scoring of either 0 or 1. Seeing D formulated in this way may raise questions concerning the methodological implications and desirability of collapsing p to a dichotomy when assessing group differences in exposure. I leave these issues for discussion elsewhere.

Alternative Graphical Explorations of Relative Rank Position

Before concluding this appendix chapter, I offer additional comments on the topic of relative rank position. The preceding discussion establishes that the values of G and D reflect group differences in relative rank position on area proportion White (p). It is surprising that this is not already more widely appreciated because G and D have close relationships with the segregation curve which is an appealing graphical device for comparing group differences in distribution over areas ranked on proportion White (p). With this in mind it is instructive to directly consider group distributions on relative rank position.

To that end, Fig. C.6 presents graphs that help provide additional insight into how relative rank position relates to group distributions. The figure presents 6 graphs. Each graph plots three curves that are constructed by first ordering areas from low to high on area proportion White (p) and then plotting the cumulated proportions of the White and Black population against the cumulated proportion of the total (combined White and Black) population and then also plotting the cumulated proportion of the total population against itself to form a diagonal line rising from (0,0) to (1,1). These plotted values are designated here designated as

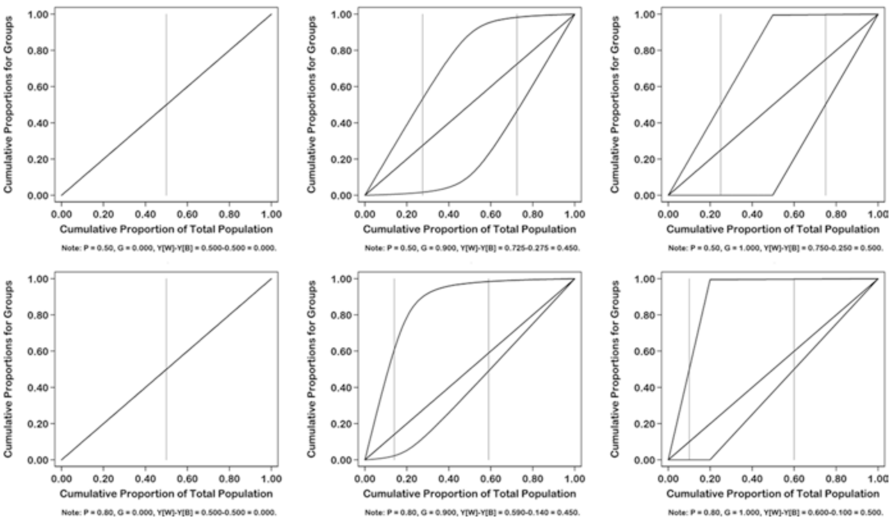

Fig. C.6 Plots of cumulative proportions of whites, blacks, and combined total by cumulative proportion of combined total

$$\begin{aligned} cpw_i &= \Sigma pw_i = \Sigma w_i / W, \\ cpb_i &= \Sigma pb_i = \Sigma b_i / B, \text{ and} \\ cpt_i &= \Sigma pt_i = \Sigma t_i / T. \end{aligned}$$

The graph that results from plotting these values as described is similar to the segregation curve in one key respect; under conditions of exact even distribution, the curves for the White and Black population will coincide with the diagonal line for the total population. So the diagonal is a reference point for even distribution. A key difference from the segregation curve is that under conditions of uneven distribution, the curve for the cumulating proportion of the Black population will rise above the diagonal and the curve for the cumulating proportion of the White population will fall below the diagonal. Like the segregation curve, the areas between the curves and the diagonal in this graph have relationships to the values of G and D. This should not be surprising since the information plotted is very similar to the information plotted in the segregation curve. However, the visual representation here is distinct.

One feature of this graphical device is that the diagonal directly reflects relative rank position on area proportion White (p). Thus, the contrast between the diagonal and the curves for Whites and Blacks provides a basis for grasping their differences in relative rank position. A curve that rises above the diagonal is skewed toward below average rank positions. A curve that falls below the diagonal is skewed toward above average rank positions. The implications of the curves for group means on relative rank position are depicted graphically by plotting two vertical lines; one indicates the value of mean relative rank for Whites (Y_W) and the other indicates mean relative rank for Blacks (Y_B). Under conditions of exact even distribution, these will necessarily coincide at the value of 0.50, the overall mean on relative rank for area proportion White (p). Where these two values differ, the value for Y_W exceeds 0.50 and is necessarily higher than the value of Y_B which falls below 0.50. As noted earlier, the logical range for Y_W is from 0.5 to $Q+(P/2)$ and the logical range of Y_B is from Q/2 to 0.5, and the maximum value for $(Y_W - Y_B)$ is 0.5 which occurs under complete segregation.

The graphs in the figure are organized by two rows and three columns. The three columns are for three conditions for segregation. The graphs in the first (leftmost) column are for the extreme condition of exact even distribution where the value of G is 0. The graphs in the third (rightmost) column are for the opposite extreme condition of complete segregation where the value of G is 100. The graphs in the middle column are for substantial, but not complete, segregation where the value of G is 0.900.[9] The two rows are for two conditions of city racial composition. The top row is for a city where P and Q are both 0.50. The bottom row is for a city where P is 0.80 and Q is 0.20.

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[9] These segregation curves are based on simulated data generated using the hyperbola model for the segregation curve described in Duncan and Duncan (1955: 214).

The graphs on both rows of the first column look the same. This is because under conditions of even distribution $cpt_i = cpw_i = cpb_i$ and the graph will necessarily consist of three identical diagonal lines rising from the lower left to the top right and this pattern holds regardless of the values of P and Q. Similarly, the vertical lines depicting the values of Y_W and Y_B coincide and both are plotted at the value of 0.50.

When segregation exists, each of the three curves will be distinct. This is seen in the two graphs in the middle column of the figure which are for examples where the value of G is 0.900. The diagonal lines in the two graphs are produced by plotting cpt_i against itself. Because areas are ordered from low to high on area proportion White (p), the curves plotting cpb_i by cpt_i rise faster than the diagonals. In contrast, the curves plotting cpw_i by cpt_i rise slower than the diagonals. The vertical lines in these graphs indicate that, as noted above, the means on relative rank (y) for Blacks (Y_B) are below 0.50 and the means on relative rank (y) for Whites (Y_W) are above 0.50. The variation in location in the top and bottom rows documents how the particular values of the group means depend not only on the level of segregation involved but also on the values of P and Q. In both cases, however, the difference of means $Y_W - Y_B$ is 0.450 and is equal to G/2.

The graphs in the third (rightmost) column depict the extreme condition of complete segregation where G is 1.00. Again the diagonal lines in the graphs reflect the curves plotting cpt_i by cpt_i. The curves plotting cpb_i by cpt_i rise from 0.0 when cpt is 0.0 to 1.0 when cpt is Q (which is 0.5 in the top graph and 0.2 in the bottom graph) and then remain at 1.0 until cpt is 1.0. The curves plotting cpw_i by cpt_i stay at 0.0 until cpt_i reaches Q, then climbs to 1.0 when cpt reaches 1.0. Here the vertical lines depicting the means on relative rank (y) for Blacks (Y_B) are at the value Q/2 which is 0.25 in the top graph and 0.10 in the bottom graph. In contrast, the vertical lines depicting means on relative rank (y) for Whites (Y_W) are at the value $Q + P/2$ which is 0.75 in the top graph and 0.60 in the bottom graph. In both of these example cases, the difference between the two means is 0.5, the maximum possible value the difference can take. This is one half of G's maximum value of 1.0, consistent with relationship in Expression (C.1).

The graphs in Fig. C.6 illustrate an important implication of expressions (C.4b) and (C.4c); namely, that the height of the curves for cpb_i and cpw_i at a given value of cpt_i will depend on two factors. One, obviously, is the extent of segregation between Whites and Blacks. That is made clear by the progression across columns for either row of the figure. The other factor is the relative sizes of the groups in the comparison; that is, the ratio of P and Q. That is made clear by how the curves for cpb_i and cpw_i, and the group means associated with these curves (plotted as vertical lines), differ with the value of P.

I offer one last set of comments on the graphs in this figure. G and D have definite relationships to the graphs in Fig. C.6. The area between the curve plotting cpb_i by cpt_i and the diagonal equals the value of G for the comparison of Blacks against total (G_{TB}). The area between the curve plotting cpw_i by cpt_i and the diagonal equals the value of G for the comparison of Whites against total (G_{TW}). The sum of these two determines the value of G for the comparison of Whites to Blacks. Specifically, G is given by the ratio of the sum of these two areas to 0.5, the maximum possible value for the sum. D is equal to the maximum vertical distance between the curves for cpb_i and cpw_i and, exactly as is the case for the segregation curve, this is value is seen at the last area where $p_i \leq P$.

One implication I stress here is that the segregation curve, while familiar and appealing in many ways, is not the only graphical device for comparing group distributions over areas ranked on area proportion White (p). The graphs presented here contain the same information as the segregation curve and like the segregation curve they support a geometric interpretation of the values of G and D. In addition, they provide a more direct basis for assessing group differences on residential outcomes (y) that are scored to reflect relative rank position on area proportion White (p).

The Nature of the Y-P Relationship for G

The nature of the y-p relationship for the Gini Index (G) is complex and difficult to summarize. Since the relationship is based on a relative rank (percentile or quantile) transformation, the y-p relationship is monotonic and positive. But few general statements beyond that can be offered.

I have explored the relationship by performing simulation studies to gain insight into the nature of the y-p relationship. I cannot provide a full review of these explorations here. But I will provide a brief summary of key points. The simulations assumed a model city with the following characteristics. It has 1000 neighborhoods with 10,000 persons in each neighborhood and only two groups – Whites and Blacks. I populated individual neighborhoods based on a model segregation curve; specifically, a segregation curve defined by the "hyperbola model" described in Duncan and Duncan (1955: 213–215). By using the hyperbola model I was able to establish particular values of G in a given simulation and thus can vary city racial composition (P) and the value of G independently across simulation trials.

Each unique combination of values for P and G produces a unique distribution of Whites and Blacks across the neighborhoods of the city. Based on the resulting distributions, I calculated the scores of p and y for each neighborhood using procedures outlined earlier. I then performed graphical analyses to gain insight into how the y-p relationship varies across different combinations of values for P and G. I offer the following to summarize key findings from my explorations.

- The relationship between y – relative rank position on p – and p is always nonlinear.
- The value of y always increases as p increases but generally rises faster (has a steeper slope) at the beginning and at the end and rises slower (has a shallower slope) in between.
- The nonlinear y-p relationship is variable, not fixed. Its exact form varies with the values of city racial composition (P) and the value of G.
- City racial composition (P) determines whether the y-p relationship is symmetrical or asymmetrical. It is symmetrical when P is 50 and increasingly asymmetrical as P departs further from 50.

- The value of G determines whether the nonlinearity in the y-p relationship describe above is mild or pronounced. When G is high, the "steeper" portions of the y-p curve occur over short ranges on p and the "flatter" portion of the y-p curve occurs over an extended range of p. As the value of G declines, the "flatter" portion of the y-p curve becomes less distinct from the "steeper" portions of the curve.

I conclude this discussion by describing how the principles just listed play out in selected example cases. I start with an example for a hypothetical "City A" where the racial composition of the city is balanced (i.e., $P = 50$) and the level of segregation as measured by G is high (i.e., $G = 90$). As shown in the top panel of Appendix Fig. C.7, the y-p relationship is symmetrical (because P is 50) and strongly nonlin-

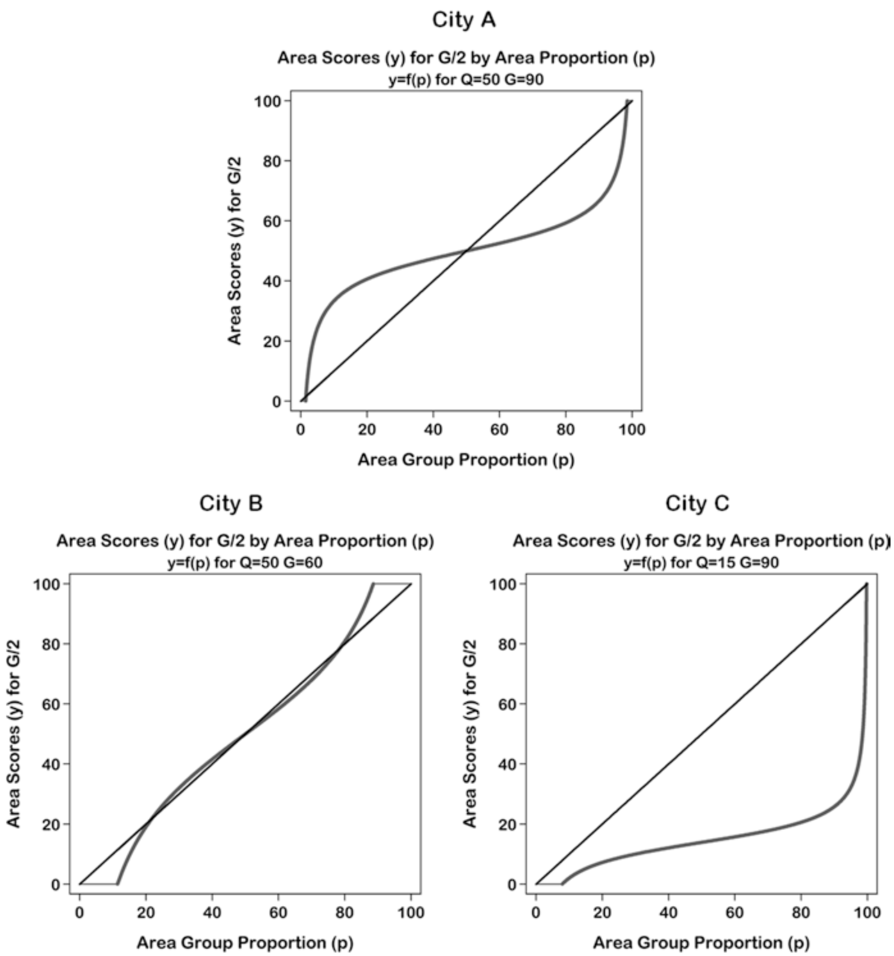

Fig. C.7 Examples of y-p relationship under varying combinations of G and P ear (because G is high). Specifically, y rises rapidly over a short portion of the lower range for p ($p = 0 - 15$); y then rises slowly over an extended portion of the intermediate range of p ($p = 15 - 85$); and y then rises rapidly again over a short portion of the upper range of p ($p = 85 - 100$). More specifically, y increases about 40 points over the range of 0–20 for p, then increases only 20 points over the range of 20–80 for p, and then increases another 40 points over the range of 80–100 for p.

The example labeled City B lowers G to 60 but leaves P unchanged at 50. The resulting y-p curve is shown in the lower left panel of the figure. The relationship remains symmetrical, as in City A, because P is 50. But the lower value for G produces a less strong nonlinear relationship evident in the fact that the differences between the steeper and flatter portions of the curve now are smaller. The example labeled City C leaves G unchanged for City A, but increases P to 85, a value more typical for US urban areas. The y-p curve continues to have distinct steep and flat portions as in City A. But now the curve is asymmetrical with most of the rise in y taking place over the last portion of the range of p ($p = 90 - 100$).

The pattern seen in City C becomes even more dramatic when relative minority group size is at low levels (i.e., below 5) and P is high. This provides a basis for understanding a finding that is discussed in Chaps. 6, 7, and 8 of the main text. The finding is that scores for G and D can be and often are much higher than scores for S when the two groups in the comparison are imbalanced in size. As the pattern for City C shows, this possibility arises because the two groups can differ by relative small amounts on p – the area outcome that determines S – and at the same time can differ by large amounts on y as scored for G and D. The pattern for City A, and especially the pattern for City B, yield insight into why discrepancies between G and D in comparison with S tend to be much smaller when city racial composition is balanced.

Appendix D: Establishing the Scaling Function $\mathbf{y = f(p)}$ Needed to Cast the Separation Index (S) as a Difference of Group Means on Scaled Pairwise Contact

In this appendix I establish the scaling function $y = f(p)$ that accomplishes the goal of scoring residential outcomes (y) from area group proportions (p) such that the scores for y fall over the range 0–1 and yield the value of the separation index (S) as a difference of means on y for the two groups in the segregation comparison. The end result is that, in the example of using S to assess White-Black segregation, $S = Y_W - Y_B$ where Y_W and Y_B are the group means for Whites and Blacks, respectively, on individual residential outcomes (y) scored from the value of the area group proportion (p) for the areas in which the individuals reside.

The value of p for an area reflects pairwise group contact or exposure. Accordingly, the value of y for an area can be described as reflecting scaled pairwise group contact or exposure and the expression ($Y_W - Y_B$) can described as the difference of group means on scaled pairwise group contact. The scaling function $y = f(p)$ that places S in the desired difference of group means framework is developed below. The scaling function is simple and substantively attractive. Specifically it is the exact one-to-one linear function $f(p_i) = p_i$ which means that S can be placed in the difference of means framework without rescaling p from its original or "natural" metric of pairwise group contact.

The separation index (S) has been known by many names including: the variance ratio index (V, James and Taeuber 1985), the correlation ratio (r, Stearns and Logan 1986; White 1986), eta squared (η^2 , Duncan and Duncan 1955; James and Taeuber 1985), the mean square deviation (MSD, White 1986; Zoloth 1976), r_{ij} (Coleman et al. 1975), and S (Zoloth 1976; Becker et al. 1978). The index is well established in the literature on segregation measurement and has been widely used in empirical segregation studies for many decades. S is particularly attractive when cast in the difference of means framework used here because S can be expressed as a difference of means on scaled pairwise group contact where group contact is based on area group proportion (p) in its "natural" metric – that is, without rescaling p as is required for the other indices considered here.

As best I have been able to determine, Becker et al. (1978: 353) were the first to show that in the two group case S can be given as the simple difference between the focal group's contact with itself (i.e., generically, P_{XX}, for White contact with Whites, P_{WW}) and the comparison group's contact with the focal group (i.e., generically, P_{YX}, for Black contact with Whites, P_{BW}) based on

$$S = P_{XX} - P_{YX} \text{ in generic form and}$$
$$S = P_{WW} - P_{BW} \text{ for White-Black segregation.}$$

Note that this relationship holds only when the population consists of only two groups and it does not generalize to situations where the population consists of three or more groups. The relationship can be adapted to all circumstances by restating contact as "pairwise" contact instead of "overall" contact as follows

$$S = P_{XX.XY} - P_{YX.XY}.$$

Here the suffix ".XY" in the subscripts contact indicates that the contact calculations are based only on the counts of the two groups in the segregation comparison. Thus, $P_{XX.XY}$ denotes the focal group's pairwise contact with itself and $P_{YX.XY}$ denotes the comparison group's pairwise contact with the focal or "reference" group.

For White-Black segregation, conventional or "overall" contact indices as introduced by Bell (1954) are given by

$$P_{WW} = 1/W \cdot \Sigma w_i p_i = 1/W \cdot \Sigma w_i (w_i / t_i)$$

for White contact with Whites and

$$P_{BW} = 1/B \cdot \Sigma b_i p_i = 1/B \cdot \Sigma b_i \left(w_i / t_i\right)$$

for Black contact with Whites. The corresponding pairwise contact indices are given as follows.

$$P_{WW.WB} = 1/W \cdot \Sigma w_i p_i = 1/W \cdot \Sigma w_i \left(w_i / \left(w_i + b_i\right)\right), \text{ and}$$

$$P_{BW.WB} = 1/B \cdot \Sigma b_i p_i = 1/B \cdot \Sigma b_i \left(w_i / \left(w_i + b_i\right)\right)$$

The difference key difference between overall and pairwise contact is that $t_i \neq \left(w_i + b_i\right)$ when the population includes groups other than Whites and Blacks.

All popular indices of uneven distribution [are usually applied as "pairwise"] measures. That is, their calculations draw only on counts for the two groups in the segregation comparison. So formulating contact indices in this way is not unusual. One simply must bear in mind that contact in this formulation is interpreted in terms of the pair of groups involved in the comparison. When the population also includes groups other than Whites and Blacks, the separation index is given by

$$S = P_{WW.WB} - P_{BW.WB}$$

where $P_{WW.WB}$ is White's average *pairwise* contact with Whites and $P_{BW.WB}$ is Black's average *pairwise* contact with Whites. When the population consists only of Whites and Blacks, the same expression obviously continues to hold but the "$_{.WB}$" subscript is not necessary.

The distinction between *overall* and *pairwise* contact is important but it is cumbersome. Since all indices of uneven distribution are based on pairwise comparisons, I drop the "$_{.XY}$" suffix notation from this point forward. Thus, for convenience, the expression

$$S = P_{WW} - P_{BW}$$

indicates a pairwise construction unless otherwise noted. Likewise, pairwise constructions are assumed for city and area proportion White (P and p_i, given respectively by $P = W / \left(W + B\right)$ and $p_i = w_i / \left[w_i + b_i\right]$) and city and area proportion Black (Q and q_i, given respectively as $Q = B / \left(W + B\right)$ and $q_i = b_i / \left[w_i + b_i\right]$). These conventions are in keeping with the literature on segregation measurement which lets context dictate when area proportion White (p_i) should computed using "overall" calculations (i.e., $p_i = w_i / t_i$) or "pairwise" calculations (i.e., $p_i = w_i / \left[w_i + b_i\right]$).

To conclude this discussion, the separation index (S) can be given as the group difference of means on average pairwise contact with the reference group. In the case of White-Black segregation, $S = P_{WW} – P_{BW}$. The terms P_{WW} and P_{BW} assess White and Black group averages on area proportion White (p_i). Setting residential outcomes (y_i) to the value of area proportion White (p_i) allows one to place S in the notation of the difference of means framework restating it as $S = Y_W – Y_B$. The next sections review terms from the "variance ratio" formulation of S and then demonstrates that the differences of means formulation of S and the variance ratio formulation of S are equivalent.

Variance Analysis

I now consider the relationship $S = \eta^2$ in more detail. I acknowledge that the expressions and relationships I introduce below are not particularly original. They have been noted elsewhere including, for example, in papers by Becker et al. (1978): 353) and White (1986:207) and also in statistical texts such as Blalock (1979: 81). The contribution of the discussion here is that it collects and calls attention to points not emphasized in most previous discussions.

Duncan and Duncan (1955) noted that the separation index (S) (which they termed the variance ratio) is equivalent to the eta squared (η^2) statistic from analysis of variance. More specifically, S is equal to η^2 for the analysis of how X, an individual-level binomial variable for race (coded 1 for Whites and 0 to Blacks), varies over areas. The value of S thus indicates the proportion of variation in race (X) that is "explained" by area of residence. Under even distribution S will be 0 because the representation of Whites and Blacks in each area will exactly reflect each group's representation in the city overall and knowledge of area will not improve the prediction of race above the baseline of assuming the overall city average. Under complete segregation S will be 1 because area of residence will be homogeneous – either all White or all Black – and thus area will perfectly predict race. Intermediate success in prediction is quantified as the ratio BSS/TSS from analysis of variance where BSS is the "between group sum of squares" for individual deviations from the overall mean and TSS is "total sum of squares" for individual deviations from the overall mean. The overall mean for X is the proportion White in the city population (P) so $TSS = \sum \left(X_k - P \right)^2$ with k used here to index individuals. Predictions for X are based on category means for X which in this case are equal to area proportion White (p_i) so $BSS = \sum \left(p_{ik} - P \right)^2$ with i here serving to index areas. Finally, for completeness, inability to explain X is quantified by WSS/TSS where WSS is the "within group sum of squares" given by $WSS = \sum \left(X_i - p_{ik} \right)^2$.

It is useful to note at this point that the value of η^2 also is equal to the square of the individual-level bivariate correlation of race (X) and area proportion White (p_i). Thus, one can interpret S as indicating the degree to which race determines area proportion White (p) for individuals as quantified by r^2 from the regression of p_i on X or of η^2 from the analysis of how p_i varies by race. Either way, it is clear that the value of S revolves around the impact of race on contact with Whites at the individual level as reflected in the White-Black difference of means in contact with Whites (p_i). Under even distribution explanation S will be 0 because all $p_i = P$ so the White and Black means for contact with Whites (p_i) are the same and knowledge of race will not improve the prediction of contact with Whites (p) above the baseline of assuming the overall city average (P). Under complete segregation S will be 1 because race will perfectly predict contact with Whites with all Whites living in areas where $p_i = 1$ and all Blacks living in areas where $p_i = 0$.

The more general relationship including intermediate outcomes is set forth in more detail below. Relevant relationships from analysis of variance can be summarized as follows.

$$TSS = BSS + WSS$$

$$\eta^2 = BSS/TSS$$

$$\eta^2 = 1 - WSS/TSS$$

$$TSS = \Sigma\Sigma\left(X_{ik} - P\right)^2$$

$$WSS = \Sigma\Sigma\left(X_{ik} - p_i\right)^2$$

$$BSS = \Sigma t_i \left(p_i - P\right)^2$$

with "i" serving as an index of areas and "k" serving as an index of individuals within areas.

The following expressions are adapted from discussions in White (1986: 207) and Becker et al. (1978: 353) and indicate how TSS, WSS, and BSS also can be obtained from terms that found in standard computing formulas for S.

$$TSS = TPQ$$

$$BSS = \Sigma t_i p_i{}^2 - TP^2$$

$$WSS = \Sigma t_i p_i q_i$$

$$BSS/TSS = 1/TPQ\left(\Sigma t_i p_i{}^2 - TP^2\right)$$

The basis for the three expressions is established as follows. First, the equivalence of TSS and TPQ can be established as follows based on Whites and Blacks being scored 0 and 1 on race (X).

$TSS = \Sigma\left(X_{ik} - P\right)^2$	(a standard formula for TSS)
$= W\left(1 - P\right)^2 + B\left(0 - P\right)^2$	(restate as separate operations for Whites and Blacks)
$= TP\left(1 - P\right)^2 + TQ\left(0 - P\right)^2$	(replace W with TP and B with TQ)
$= TPQ^2 + TQ\left(0 - P\right)^2$	(replace $(1 - P)^2$ with Q^2)
$= TPQ^2 + TQP^2$	(replace $(0 - P)^2$ with P^2)
$= TPQ\left(Q\right) + TPQ\left(P\right)$	(reorganize terms)
$= TPQ\left(Q + P\right)$	(reorganize terms)
$TSS = TPQ$	(based on $Q + P = 1$)

Next, the equivalence of WSS and $\Sigma\, t_i p_i q_i$ can be established as follows.

$WSS = \Sigma\Sigma\left(X_{ik} - p_i\right)^2$	(standard formula for WSS)
$= \Sigma\, w_i \left(1 - p_i\right)^2 + \Sigma\, b_i \left(0 - p_i\right)^2$	(restate as separate operations for Whites and Blacks)
$= \Sigma\, t_i p_i \left(1 - p_i\right)^2 + \Sigma\, t_i q_i \left(0 - p_i\right)^2$	(replace w_i with $t_i p_i$ and b_i with $t_i q_i$)
$= \Sigma\, t_i p_i \left(q_i\right)^2 + \Sigma\, t_i q_i \left(0 - p_i\right)^2$	(replace $1 - p_i$ with q_i)
$= \Sigma\, t_i p_i q_i^2 + \Sigma\, t_i q_i p_i^2$	(replace $\left(0 - p_i\right)^2$ with p_i^2)
$= \Sigma\, t_i p_i q_i \left(q_i\right) + \Sigma\, t_i p_i q_i \left(p_i\right)$	(reorganize terms)
$= \Sigma\, t_i p_i q_i \left(q_i + p_i\right)$	(reorganize terms)
$WSS = \Sigma\, t_i p_i q_i$	(based on $q_i + p_i = 1$)

Then the equivalence of BSS and $\Sigma\, t_i p_i^2 - TP^2$ can be established as follows

$BSS = \Sigma\, t_i \left(p_i - P\right)^2$	(standard formula for BSS)
$= \Sigma\, t_i \left(p_i^2 - 2p_i P + P^2\right)$	(multiply out $\left(p_i - P\right)^2$)
$= \Sigma\, t_i p_i^2 - \Sigma\, t_i 2p_i P + \Sigma\, t_i P^2$	(reorganize as multiple summations)
$= \Sigma\, t_i p_i^2 - 2P \cdot \Sigma\, t_i p_i + P^2 \cdot \Sigma\, t_i$	(move constants outside of summations)
$= \Sigma\, t_i p_i^2 - 2P\, \Sigma\, t_i p_i + TP^2$	(substitute T for $\Sigma\, t_i$)
$= \Sigma\, t_i p_i^2 - 2PTP + TP^2$	(substitute TP for $\Sigma\, t_i p_i$ based on $P = \Sigma\, t_i p_i / T$)
$= \Sigma\, t_i p_i^2 - 2TP^2 + TP^2$	(reorganize terms)
$BSS = \Sigma\, t_i p_i^2 - TP^2$	(combine terms)

From these expressions, η^2 and S can be obtained from the following computing formulas

$$S = \eta^2 = BSS/TSS$$

$$S = \eta^2 = \left(\Sigma\, t_i p_i^2 - P^2\right)/TPQ.$$

Formulation as a Difference of Means

S also can obtained from the simple difference between pairwise White contact with Whites (P_{WW}) and pairwise Black contact with Whites (P_{BW}); that is, $S = P_{WW} - P_{BW}$. Because y_i for S is scored directly from p_i, $Y_W = P_{WW}$ and $Y_B = P_{BW}$ and the following equalities hold.

$$Y_W - Y_B = BSS/TSS$$

$$P_{WW} - P_{BW} = BSS/TSS$$

I provide a derivation establishing these equivalences below. I initially developed the derivation independently. However, I later discovered that a similar derivation had been given in Becker et al. (1978: 353).

$S = Y_W - Y_B$ $= P_{WW} - P_{BW}$	(follows because $y_i = p_i$)
$= (\Sigma w_i p_i)/W - (\Sigma b_i p_i)/B$	(standard expressions for P_{WW} & P_{BW})
$= (\Sigma t_i p_i p_i)/W - (\Sigma t_i p_i q_i)/B$	(replace w_i with $t_i p_i$ and b_i with $t_i q_i$)
$= (\Sigma t_i p_i^2)/TP - (\Sigma t_i p_i q_i)/TQ$	(replace W with TP and B with TQ)
$= (Q/Q)(\Sigma t_i p_i^2)/TP - (P/P) \cdot \Sigma t_i p_i q_i)/TQ$	(introduce 1 in the form of Q/Q and P/P)
$= (Q \cdot \Sigma t_i p_i^2)/TPQ - (P \cdot \Sigma t_i p_i q_i)/TPQ$	(reorganize terms)
$= (Q \cdot \Sigma t_i p_i^2 - P \cdot \Sigma t_i p_i q_i)/TPQ$	(reorganize terms)
$= [Q \cdot \Sigma t_i p_i^2 - P \cdot \Sigma t_i p_i (1 - p_i)]/TPQ$	(reorganize terms)
$= [Q \cdot \Sigma t_i p_i^2 - (P \cdot \Sigma t_i p_i - P \cdot \Sigma t_i p_i^2)]/TPQ$	(restate $P \cdot \Sigma t_i p_i (1 - p_i)$ as $P \cdot \Sigma t_i p_i - P \cdot \Sigma t_i p_i^2$)
$= (Q \cdot \Sigma t_i p_i^2 + P \cdot \Sigma t_i p_i^2 - P \cdot \Sigma t_i p_i)/TPQ$	(reorganize terms)
$= [(P + Q) \cdot \Sigma t_i p_i^2 - P \cdot \Sigma t_i p_i]/TPQ$	(reorganize terms)
$= (\Sigma t_i p_i^2 - P \cdot \Sigma t_i p_i)/TPQ$	((P + Q = 1 and drops out)
$= (\Sigma t_i p_i^2 - P \cdot TP)/TPQ$	(substitute TP for $\Sigma t_i p_i$)
$= (\Sigma t_i p_i^2 - TP^2)/TPQ$	(reorganize terms)
$S = BSS/TSS$	(substitute BSS for $\Sigma t_i p_i^2 - TP^2$ and TSS for TPQ as established earlier)

As a last comment, I note that the discussion here shows that S simultaneously registers two separate and distinct aspects of the relationship between race and contact with Whites (p).

- Under the traditional eta squared or variance ratio interpretation, S indicates the strength of the association between race (i.e., group membership) and contact with Whites (p).
- Under the new interpretation of S as a difference of group means for contact with Whites, S indicates the "impact" or "effect" of race (i.e., group membership) on contact with Whites.

Thus, S equals both the regression coefficient (b) for race and the square of the correlation coefficient (r) from the bivariate regression analysis predicting contact with Whites (p_i) based on race (X). Interestingly, both options allow for applying significance tests for the value of S.

Appendix E: Establishing the Scaling Function $y = f(p)$ Needed to Cast the Theil Entropy Index (H) as a Difference of Group Means on Scaled Pairwise Contact

In this appendix I establish the scaling function $y = f(p)$ that accomplishes the goal of scoring residential outcomes (y) from area group proportions (p) such that the scores for y fall over the range 0–1 and yield the value of the Theil entropy index (H) as a difference of means on y for the two groups in the segregation comparison. The end result is that, in the example of using H to assess White-Black segregation, $H = Y_W - Y_B$ where Y_W and Y_B are the group means for Whites and Blacks, respectively, on individual residential outcomes (y) scored from the value of the area group proportion (p) for the areas in which the individuals reside.

The scaling function $y = f(p)$ that places H in the difference of group means framework is developed below. Discussion of this function in the main body of this monograph notes that y is a smooth continuous, nonlinear transformation of p that changes p from its original or "natural" metric to a new metric that exaggerates group differences on p over portions of the lower and upper ranges of p (i.e., roughly $p < 0.25$ and $p > 0.75$) and compresses group differences on p over middle portions of the range of p (i.e., roughly $0.30 < p < 0.70$).

Please note that the primary credit for discovering the scaling function for H should be given to Warner Henson, III. Warner derived the first version of the scaling function for H while working with me as an undergraduate research fellow completing his BS in sociology at Texas A&M University.[10] I have subsequently added refinements and extensions to his work to serve the needs of this monograph,

[10] That was in the 2007. Soon after, Mr. Henson graduated and enrolled in the Sociology doctoral program at Stanford University.

but these are minor changes. Mr. Henson established the essential features of the derivation.

Continuing with the familiar example of White-Black segregation, a basis for scoring residential outcomes (y) such that the scores of y fall over the same range as p (i.e., 0–1) and yield the Theil index (H) as the difference of means ($Y_W - Y_B$) can be established as follows. First, start with the desired equivalence

$$H = Y_W - Y_B = (1/T) \cdot \Sigma t_i (E - e_i)/E.$$

The expression on the far right side is an adaptation of the formula for H given in James and Taeuber (1985). Next replace the terms Y_W and Y_B with alternative computing expressions as follows

$$(1/W) \cdot \Sigma w_i y_i - (1/B) \cdot \Sigma b_i y_i = (1/T) \cdot \Sigma t_i (E - e_i)/E.$$

Then replace W and B with alternative expressions based on T, P, and Q. Specifically, replace W with PT and replace B with QT. Similarly, replace w_i and b_i with alternative expressions based on t_i, p_i, and q_i. Specifically, replace w_i with $p_i t_i$ and b_i with $q_i t_i$. This yields

$$(1/PT) \cdot \Sigma p_i t_i y_i - (1/QT) \cdot \Sigma q_i t_i y_i = (1/T) \cdot \Sigma t_i (E - e_i)/E.$$

Then rearrange terms as follows

$$(1/T) \cdot \Sigma (p_i/P) t_i y_i - (1/T) \cdot \Sigma (q_i/Q) t_i y_i = (1/T) \cdot \Sigma t_i (E - e_i)/E$$

$$\Sigma (p_i/P) t_i y_i - \Sigma (q_i/Q) t_i y_i = \Sigma t_i (E - e_i)/E$$

$$\Sigma t_i y_i \left[(p_i/P) - (q_i/Q) \right] = \Sigma t_i (E - e_i)/E$$

$$\Sigma t_i y_i = \Sigma t_i \left[(E - e_i)/E \right] / (p_i/P - q_i/Q) \cdot$$

From the above expression, it is evident that

$$y_i = \left[(E - e_i)/E \right] / (p_i/P - q_i/Q).$$

For actual calculations, E and e_i would be expanded to their full expressions using the following substitutions

$$E = P \cdot \ln(1/P) + Q \cdot \ln(1/Q), \text{ and}$$

$$e_i = p_i \cdot \ln(1/p_i) + q_i \cdot \ln(1/q_i).$$

Adjusting the Range to 0–1

At this point a small additional adjustment is needed. The scores for y_i will yield H as a difference of group means thus achieving one important goal of the exercise. However, the scores for y will not fall in the range 0–1. They instead will fall in the range –Q to P as p_i varies from its minimum value of 0 to its maximum value of 1. This is because, when p_i is either 0 or 1, e_i evaluates to 0 and the term $(E - e_i)/E$ evaluates to 1. This reduces the expression

$$y_i = \left[(E - e_i)/E\right] / (p_i/P - q_i/Q).$$

to

$$y_i = 1/\left[(p_i/P) - (q_i/Q)\right].$$

When p_i is 0, this expression becomes

$$y_i = 1/\left[(0/P) - (1/Q)\right]$$

which evaluates to -Q. Similarly, when p_i is 1, the resulting expression is

$$y_i = 1/\left[(1/P) - (0/Q)\right]$$

which evaluates to P.

The range for y can therefore be set to 0–1 by incorporating the constant Q in the function as follows

$$y_i = Q + \left[(E - e_i)/E\right] / (p_i/P - q_i/Q).$$

This achieves the desired solution.

A Loose End When $p = P$

There is a final issue to deal with. Interestingly, the value of y_i is undefined when p_i is *exactly* equal to P. This is because the term $(p_i/P - q_i/Q)$ will then be 0 and the same also will be true of the term $[(E - e_i)/E]$. Thus the expression $\left[(E - e_i)/E\right] / (p_i/P - q_i/Q)$ will be undefined because it involves division by zero. As a practical matter, *exact* equality of p_i and P is very rare in conventional empirical analyses of residential segregation in urban areas. Nevertheless, it is a logical possibility that it can occur in empirical studies of segregation and it is certainly likely to occur in methodological analyses and simulation studies. So it is necessary to establish a procedure for handling this situation.

The procedure I adopt is the following: when p_i is exactly P, assign a value for y based on the limiting values of y obtained by taking values of p_i that are arbitrarily close to P, but are just short of reaching exactly P. For example, the value of y can be established in this way by averaging the two values of y obtained using $p_i = P - 0.0000001$ and $p_i = P + 0.0000001$. The two values of y will be exceedingly close; so close in fact that a graph of the y-p relationship will appear as a smooth, continuous function in which y rises monotonically as p ranges from 0 to 1 with only an arbitrarily small "break" in the line at the exact point where $p_i = P$. The procedure suggested here would simply fill in this one point on the line. I offer this as a reasonable, practical strategy to follow until a better alternative is identified.

Appendix F: Establishing the Scaling Function $y = f(p)$ Needed to Cast the Hutchens' Square Root Index (R) as a Difference of Group Means on Scaled Pairwise Contact

In this appendix I establish the scaling function $y = f(p)$ that accomplishes the goal of scoring residential outcomes (y) from area group proportions (p) such that the scores for y fall over the range 0–1 and yield the value of the Hutchens Square Root Index (R) as a difference of means on y for the two groups in the segregation comparison. The result is that, in the example of using R to assess White-Black segregation, $R = Y_W - Y_B$ where Y_W and Y_B are the group means for Whites and Blacks, respectively, on individual residential outcomes (y) scored from the value of the area group proportion (p) for the areas in which the individuals reside.

The scaling function $y = f(p)$ that places R in the differences of group means framework is developed below. Discussion of this function in the main body of this monograph notes that y is a nonlinear transformation of p that changes p from its original or "natural" metric to a new metric that exaggerates group differences on p over portions of the lower and upper ranges of p (i.e., roughly $p < 0.25$ and $p > 0.75$) and compresses group differences on p over middle portions of the range of p (i.e., roughly $0.30 < p < 0.70$). The scaling function for R is very similar in shape and behavior to the scaling function for the Theil Entropy index (H). The main difference is that the nonlinearity in the scaling function for R is more pronounced; that is, it departs from linearity in the same basic manner as the scaling function for H, but the magnitude (amplitude) of the departure from linearity is consistently larger.

To establish the function $y = f(p)$, start with the desired equivalence

$$Y_W - Y_B = R.$$

Next replace R with an expression adapted from the formula for R given in Hutchens (2001, 2004).

$$Y_W - Y_B = 1 - \Sigma \sqrt{(w_i/W)(b_i/B)}.$$

Next replace Y_W and Y_B with the terms of their computing formulas as follows

$$1/W \cdot \Sigma\, w_i y_i - 1/B \cdot \Sigma\, b_i y_i = 1 - \Sigma \sqrt{(w_i/W)(b_i/B)}.$$

Then replace W and B with expressions based on T, P, and Q. Similarly, replace w_i and b_i with expressions based on t_i, p_i, and q_i to obtain

$$1/PT \cdot \Sigma\, p_i t_i y_i - 1/QT \cdot \Sigma\, q_i t_i y_i = 1 - \Sigma \sqrt{(p_i t_i/PT)(q_i t_i/QT)}.$$

Then rearrange terms as follows. First, on the right side isolate (t_i^2 / T^2) inside the radical

$$1/PT \cdot \Sigma\, p_i t_i y_i - 1/QT \cdot \Sigma\, q_i t_i y_i = 1 - \Sigma \sqrt{(t_i^2/T^2)(p_i/P)(q_i/Q)}.$$

Then move (t_i^2 / T^2) outside of the radical as (t_i /T) and then restate it as t_i (1/T) to obtain

$$1/PT \cdot \Sigma\, p_i t_i y_i - 1/QT \cdot \Sigma\, q_i t_i y_i = 1 - \Sigma (t_i\ /T) \sqrt{(p_i/P)(q_i/Q)}$$

Restate $\sqrt{(p_i/P)(q_i/Q)}$ as $\sqrt{p_i q_i/PQ}$ to obtain

$$1/PT \cdot \Sigma\, p_i t_i y_i - 1/QT \cdot \Sigma\, q_i t_i y_i = 1 - \Sigma\, t_i\ (1/T) \sqrt{p_i q_i/PQ}.$$

On the left side move P and Q inside the summations

$$1/T \cdot \Sigma (p_i/P) t_i y_i - 1/1 \cdot \Sigma (q_i/Q) t_i y_i = 1 - \Sigma (t_i/T) \cdot \sqrt{p_i q_i/PQ}$$

On the right side replace 1 with the equivalent expression $\Sigma\ t_i$ (1/T) and replace (t_i /T) with t_i (1/T)

$$1/T \cdot \Sigma (p_i/P) t_i y_i - 1/T \cdot \Sigma (q_i/Q) t_i y_i = \Sigma\, t_i\ (1/T) - \Sigma\, t_i\ (1/T) \cdot \sqrt{p_i q_i/PQ}.$$

Next reorganize on both sides

$$1/T \cdot \left[\Sigma (p_i/P) t_i y_i - \Sigma (q_i/Q) t_i y_i\right] = \Sigma\, t_i \left(1/T - 1/T \cdot \sqrt{p_i q_i/PQ}\right).$$

Next multiply both sides by T as follows

$$\Sigma(\mathrm{p_i}/\mathrm{P})\mathrm{t_i y_i} - \Sigma(\mathrm{q_i}/\mathrm{Q})\mathrm{t_i y_i} = \mathrm{T}\cdot\left[\Sigma \mathrm{t_i}\left(1/\mathrm{T} - 1/\mathrm{T}\sqrt{p_i q_i/PQ}\right)\right].$$

Then move T inside the summation on the right side to obtain

$$\Sigma(\mathrm{p_i}/\mathrm{P})\mathrm{t_i y_i} - \Sigma(\mathrm{q_i}/\mathrm{Q})\mathrm{t_i y_i} = \Sigma \mathrm{t_i}\left(1 - \sqrt{p_i q_i/PQ}\right).$$

Next reorganize terms on the left side.

$$\Sigma \mathrm{t_i y_i}\left[(\mathrm{p_i}/\mathrm{P}) - (\mathrm{q_i}/\mathrm{Q})\right] = \Sigma \mathrm{t_i}\left(1 - \sqrt{p_i q_i/PQ}\right).$$

Then divide both sides by $[(\mathrm{p_i}/\mathrm{P}) - (\mathrm{q_i}/\mathrm{Q})]$

$$\Sigma \mathrm{t_i y_i} = \Sigma \mathrm{t_i}\left(1 - \sqrt{p_i q_i/PQ}\right)\Big/(\mathrm{p_i}/\mathrm{P} - \mathrm{q_i}/\mathrm{Q}).$$

From the last expression, it is clear that

$$\mathrm{y_i} = \left(1 - \sqrt{p_i q_i/PQ}\right)\Big/(\mathrm{p_i}/\mathrm{P} - \mathrm{q_i}/\mathrm{Q}).$$

Adjusting the Range to 0–1

An additional adjustment is required. Under the last expression, the scores for y will yield R as a difference of group means. However, the scores for y will not fall in the range 0-1 as desired. Instead, values of y_i will range from –Q to P as p_i varies from its minimum value of 0 to its maximum value of 1. That is, the expression

$$\mathrm{y_i} = \left(1 - \sqrt{p_i q_i/PQ}\right)\Big/(\mathrm{p_i}/\mathrm{P} - \mathrm{q_i}/\mathrm{Q})$$

yields $-$Q when p_i is 0 and P when p_i is 1. Accordingly, the range for y can be set to 0–1 by incorporating the constant Q in the function as follows

$$\mathrm{y_i} = \mathrm{Q} + \left(1 - \sqrt{p_i q_i/PQ}\right)\Big/(\mathrm{p_i}/\mathrm{P} - \mathrm{q_i}/\mathrm{Q})$$

A Loose End When p = P

One final matter requires attention. It is that y_i is undefined when p_i is *exactly* equal to P because the term $[(p_i/P) - (q_i/Q)]$ will then be 0. Thus the expression

$$\left(1-\sqrt{p_i q_i / PQ}\right) \Big/ \left(\mathrm{p_i}/\mathrm{P}-\mathrm{q_i}/\mathrm{Q}\right)$$

will be undefined because it will involve division by zero. As a practical matter, *exact* equality of p_i and P is very rare in conventional empirical analyses of residential segregation in urban areas. Nevertheless, it is a logical possibility in empirical studies and it is especially likely to occur in methodological analyses and simulation studies. So it is necessary to establish a procedure for handling this situation.

The option I adopt is as follows: when p_i is exactly P, assign a value for y based on the limiting values of y obtained by taking values of p_i that are arbitrarily close to P, but are not exactly P. For example, the value of y can be established in this way by averaging the two values of y obtained using $\mathrm{p_i} = \mathrm{P} - 0.0000001$ and $\mathrm{p_i} = \mathrm{P} + 0.0000001$. The two values of y will be exceedingly close; so close in fact that a graph of the y-p relationship will be a smooth, continuous function in which y rises monotonically as p ranges from 0 to 1 with only an arbitrarily small "break" in the line at the exact point where $\mathrm{p_i} = \mathrm{P}$. The procedure suggested here simply fills in this one point on the line. I offer this as a reasonable, practical strategy to follow until a better solution is identified. When this approach is adopted, an interesting regularity is observed; the value of y always converges on 0.50 when p_i is set arbitrarily close to P.

An Observation

There is another interesting regularity in the y–p relationship. It is that y is always equal to Q when $\mathrm{p_i} = \mathrm{Q}$. The basis for this regularity is that the expression $\sqrt{p_i q_i / PQ}$ takes the value of 1 when $\mathrm{p_i} = \mathrm{Q}$. Accordingly, the expression

$$\left(1-\sqrt{p_i q_i / PQ}\right) \Big/ \left(\mathrm{p_i}/\mathrm{P}-\mathrm{q_i}/\mathrm{Q}\right)$$

takes the value of 0, yielding the result of $\mathrm{y} = \mathrm{Q}$. The one exception is when Q is 0.5. In that situation, P also is 0.5 and y is undefined as just described above. However, the above procedure of substituting 0.5 for y when $\mathrm{p_i} = \mathrm{P}$ also produces a result consistent with the regularity that $\mathrm{y} = \mathrm{Q}$ when $\mathrm{p_i} = \mathrm{Q}$.

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