Springer Water

Georg Meran
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The Economics of Water

Rules and Institutions

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Symbols

$AC(w)$	Average cost function
$B(w)$	Benefit function
$C(w)$	Water cost function
CM	Contribution margin
ET	Evapotranspiration
F	Fixed costs
h	Return flow factor
L	Fixed fees
$MC(w)$	Marginal cost function
p	Price
P	Precipitation
q	Water entitlement price
Q	Pollution/amount of polluted (water)
r	Run-off
R	Recharge/inflows
$R(w)$	Revenue function
s	Distance
S	Water stock
$sp_{i,\,j}$	Side-payment from riparian i to j
$U(\cdot)$	Utility/benefit function
$V(I)$	Benefit of unilateral acting riparians
$V(S)$	Benefit of (sub-)coalition S
$V(G)$	Benefit of grand coalition
w	Water abstraction/usage
W	Fresh water amount
x_i	Payoff (or assigned benefit) for riparian i
y_i	Production/output of sector i
z	Water transfer

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Introduction

1

1.1 Introduction

Water is the nature, the arch, the originating principle; water is the beginning of all things.

(Thales)

Sustainable Development Goal 6: Ensure availability and sustainable management of water and sanitation for all.

(United Nations (2015), Agenda 2030)

Water is not only the beginning of all things, as the old Greeks had already realized, but without water, no life on earth is possible, and clean water is also a precondition for any form of sustainable development. There is enough available freshwater on earth (about 91,000 km^3) to supply every individual on earth (about 7.5 billion in 2020) approx. 12,000l, more than enough to live decently. However, due to natural and man-made idiosyncracies, clean freshwater and sanitation (which we do not cover in-depth in this book) are scarce, and thus decisions need to be taken on the production, treatment, and distribution of water, given underlying technical and socioeconomic conditions. Water needs to be managed efficiently, both with respect to the growing scarcity of resources, as a natural endowment that is indispensable for the survival of mankind, but also with respect to the variety of eco-services it delivers. In fact, water is a multifunctional resource that provides people with potable water, secures landscapes in different climate zones and functions as a sink of pollutants emanating from human activities. Thus, a comprehensive approach is required, including a technical understanding of the basic hydrological principles, different economic allocation rules, but also the institutional framing of the use of water.

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Problems of water supply and demand are not new; on the contrary, they exist as long as life exists on earth. However, with rising population, environmental challenges, climate change, and adverse local conditions, and often a lack of appropriate regulatory and institutional conditions, issues of water management have become global in the last century. This has lead—amongst other goals—to the Millennium Goals of 2000, calling to halve, by 2015, the proportion of the population without sustainable access to safe drinking water and basic sanitation. Some, but not sufficient progress was made on this path, so that the successor document, the United Nations' (2015) Agenda 2030, recalls and even enhances the request, to "ensure availability and sustainable management of water and sanitation for all" by 2030; this is the Sustainable Development Goal 6 (SDG 6). But how to fulfill these requirements, given the challenges of water management?

The application of economic concepts is sometimes criticized in the (noneconomist) water community, but we believe that economics can provide useful insights. In the practical world of water, "there is a sense that economic concepts are inadequate to the task at hand, a feeling that water has value in ways that economists fail to account for, and a concern that this could impede the formulation of effective approaches for solving the water crisis" (Hanemann 2006, 61). In other words, water is too important to be left to economists. Yet, on the other hand, there are hundreds (if not thousands) of water, environmental, resource, agricultural, and other economists out there that do excellent analytical and practical work on water issues, and most of them go beyond the pure neoclassical ivory tower analysis that is sometimes full-mouthy criticized. To bridge the gap between different disciplines requires an interdisciplinary approach that respects the complexity of water: It can be a private good and a public good, is extremely mobile, very capital intensive, chemically complex, etc., after all, perhaps the most complex of all goods.

This book addresses rules and institutions of water scarcity. While the book's main contribution is the application of economic concepts, we deploy an interdisciplinary technical-economic approach. This introductory chapter provides an overview of the topics covered in the book and also defines a thread to structure the multitude of issues addressed in the various chapters. The next section provides an overview of existing literature on water economics. Section 1.3 explains the technical-economic approach of this book, followed by an outline of the topics of each chapter (Sect. 1.4). In Sect. 1.5 we provide a list of important issues that we were not able to cover in this book, and the chapter ends with acknowledgments.

1.2 State of the Literature and the Specifics of Our Approach

Water resource management is covered by a breadth of literature (economic, technical, cultural, geographic, etc.). Klaver (2012) puts water in a cultural context, and Wittfogel (1981), describes the development of the hydraulic civilization. A comprehensive account of the environmental history of water is provided by Juuti et al. (2007). Let's also recognize the "Berliner" Alexander von Humboldt, who, two centuries ago, has focussed on the water cycle in his trip to Latin America: On the way to

Caripe as part of his trip through Venezuela, he observed the immense deforestation with

> perhaps one of the main reasons for the drought and the drying up of the springs in the province of Neu-Andalusia. Forests (plants) produce not only water, give a large newly generated mass of water through their evaporation in the air, they do not only beat down, because they excite cold, water from the air and multiply the fog, but they are mainly charitable in that they prevent the evaporation of water masses fallen by periodic rain showers by providing shade. This evaporation is incomprehensibly fast here, where the sun is so high.[1]

Among the scholarly textbooks, water is part of the (important) literature on environmental and resource economics. As such, it is featured in textbooks such as Tietenberg (2005). Water is treated as an example of a renewable resource, yet the more technical aspects, such as the hydrological cycle, or issues of water quality are not extensively covered. In addition, there are some comprehensive textbooks on water economics: The introductory textbook by Griffin (2016), deals with both, basic economic concepts and their application to water resource management problems. Shaw (2007), requires some prior microeconomic knowledge, and focusses on the North American water sector; allocative questions are prioritized, while distributional and access issues are not really covered. The classical text by Hirshleifer et al. (1969), can be considered an interdisciplinary benchmark in the literature. These textbooks require some microeconomic background, and we suggest Perman (2011), as a useful and resource-oriented reference.

A third type of references are handbooks of water economics or volumes covering research contributions on the frontier of current research, amongst Dinar and Schwabe (2015), Jordan et al. (2012), Anand (2010), and Pashardes et al. (2002). Issues covered by all these volumes include pricing, consumption, and different regulatory and institutional designs. At this point, let us also mention some of the academic journals focussing on water issues, such as *Water Resources Research*, *Water Policy*, *Water Economics and Management*, *Water*, and the *Journal of Water Resources Planning and Management*. We will pick up more specific references on specific issues as we go through the chapters of this book.

1.3 A Novel Technical-Economic Approach

Why another book? We feel that the synergies from a technical-economic approach to the analysis of water have not been fully reaped. Water has distinct technical, economic, and institutional features that need to be considered jointly, but that economic tools can be usefully applied to the water sector, too: These include decisions

[1]Own translation from Humboldt, Alexander von (2000: Reise durch Venezuela. Auswahl aus den amerikanischen Reisetagebüchern. Hg. von Margot Faak. Berlin: Akademie Verlag, p. 140) http://www.hin-online.de/index.php/hin/article/view/273/513.

on the allocation of production, distribution, pricing issues and investment, as well as sustainability issues, the so-called triad of sustainability economics.

While the purely "economical" use of water has been addressed by various textbooks, and advanced texts are also available, a comprehensive treatment of the interplay between the hydrological cycle and the rules and institutions that govern today's water allocation rules is still missing. Therefore, the main endeavor of the textbook is to present a modern perspective, by combining hydrological issues (such as blue and green water, water quality, groundwater flows, river flows, etc.) with a "modern" economic approach. In this context, the adjective "modern" refers to an approach that includes distributional issues and issues of enforceability of human rights in managing water resources, instead of restricting the analysis to solely technical efficiency planning methods or the adoption of purely economic optimality criteria, e.g., the Pareto-principle. With increasing scarcity, issues of the appropriate allocation of economic goods take on an ethical dimension, which is not covered by the efficiency criterion.

The approach is based on microeconomic theory applied to the real world of water, with real technologies, thus developing a truly technical-economic approach. We assume some basic knowledge of microeconomics and try to go further in the analysis of water-specific issues. In addition to gaining more in-depth insights into the technical-economic interface, this approach also allows for more nuanced policy conclusions, which builds the second pillar of this book. Ever since the UN development goals were established, we know that the management of water is not only a matter of demand and supply but also a result from a holistic policy approach comprising constitutional aspects of the human right to water and the political governance of the water cycle as a multifunctional system that secures human livelihood. Thus, we also include an analysis of the institutional framework of water management.

Our approach also combines the technical fundamentals of the hydrological cycle and different economic approaches to resolve fundamental issues of water scarcity with an in-depth assessment of the political dimension of water management and its institutional embeddedness, such as water rights, and different approaches to water tariffs, water markets, and transboundary water management; the latter are provided through a series of case studies. Thus, the book addresses both, i/ advanced undergraduates majoring in economics, and graduate students of social sciences, engineering, natural sciences, water management, etc. (with basic knowledge of microeconomics), and ii/ practitioners, consultants, economic experts, project managers, etc., in the field of water management, interested in a deeper understanding of current-day issues and options to handle these issues conceptually. The book is thus conceived as a bridge between purely economic analysis of water, and the practical work in the field, often constrained by very concrete questions. We feel that there is a need out there, and in the university and college classrooms, too, to update and extend the technical-economic exchange, as water management issues, sometimes called water crisis, linger on.

1.4 Structure of This Book

After this introduction, each chapter covers a specific topic related to water issues. Chapter 2 provides the physical and hydrological basics of water. This includes definitions of different categories of water, such as sweet and salt water, and the differentiation into "blue" and "green" water. The chapter also discusses precipitation, interception, and evapotranspiration, and the potential impact of human activities on the water cycle.

Chapter 3 covers economic, technical, and institutional challenges of *Integrated Water Resource Management* (IWRM). In addition to a basic technical-economic model of IWRM, we discuss water management issues of a common pool resource and derive conclusions for water policy. The chapter also includes some basic economic analysis of social welfare, distribution, and the value of water, eco-hydrology and the management of water as a public good, water recycling, groundwater management, water quality, and two further IWRM issues: Water allocation along rivers, and inter-basin water transfers.

Chapter 4 covers simple and more complex issues of *water tariffication*. This includes the definition of the criteria for water pricing, tariff design, and variations thereof. An important issue discussed is the objective function, e.g., whether one aims at welfare maximization, at universal service provision, or the simple survival of the poorest parts of the population. In addition to the comparison of stylized water tariffs, such as single- and two-part tariffs, the chapter also goes into more details on increasing block tariffs, and pricing in physically unconnected water markets. Last but not least, the chapter introduces two ways to deal with very rough scarcity: pricing and rationing.

Chapter 5 addresses a broad range of questions regarding the regulation and institutional design of *water markets*, including reference to the few empirical cases where these markets were established. The chapter first sets out institutional, hydrological, and infrastructural preconditions for establishing water markets. Then a simple model of a water market along a river basin is developed, that provides insights into alternative pricing mechanisms, such as locational or uniform prices. We report the experience of a water market experiment in Australia, the Murray-Darling basin. The chapter ends with a discussion of water entitlements and water allocation.

Chapter 6 extends the discussion to *transboundary water resource management*. There are 276 international river basins worldwide that stretch over two or more countries, and about 40 percent of the world population lives in these international river basins. The first section sets the scene and describes existing transboundary water agreements and principles of international water rights. A basic model is set up to analyze benefits sharing along a river basin with two riparians first, and then extended to more than two riparians, in the context of cooperative game theory. A separate section introduces bankruptcy rules for water allocation, i.e., the physical allocation of water to consumers. In addition, rules for flexible water sharing are derived. The chapter includes two case studies on transboundary water issues along the Nile and the Euphrates.

1.5 Important Topics Not Covered

Due to constraints of time and space, we had to leave out some issues that are nonetheless important (and that we plan to pick up for the second edition of this textbook...). Amongst them are climate-related issues of water scarcity, the occurrence of floods, heavy rainfall and weather-related storm surges and their impacts on the infrastructure of an economy, and on urban water management. Water infrastructure for mega-cities is a mega subject, with respect to the use of land, infrastructure financing, and organizational models. In that context, different types of sanitation infrastructure need to be compared, for urban and rural areas, including adapted technologies that can be implemented relatively quickly, such as decentralized toilet systems. In some cases, these can be cheaper than the centralized infrastructure.

Last but not least, the theory-policy nexus needs more in-depth analysis. In fact, the microeconomic approach, even appended by distributional considerations, is a tool for analysis that can not take into account issues of implementation, of institutional regimes, and conflicting interests beyond those covered in simple models. Take the example of integrated water resources management, which can be operationalized in microeconomics and especially in welfare theory by means of optimization approaches. However, in practice, this approach should be pursued with caution if it is not to lead to technocratic malfunctioning. This comprehensive approach seems utopian in its generality and it requires reference to social and economic reality if it is not itself involved in the social process of concrete water policy. From historical science, we know that the institutional development is a process of self-organization and represents a circular process between ideas and actions. It is then like the successful effort of Baron Munchausen in the novel by Erich Raspe[2] who successfully pulled himself and his horse out of the swamp by dragging himself up by his own hair. Combining the evolutionary approach with the institutional economic approach of identifying policy options and policy gaps is left to be developed, in the realm of institutional water policy analysis of Ostrom (1990), Biswas (2004), Menard et al. (2018), and many others.

Acknowledgements This book results from a series of courses given at Berlin University of Technology (TUB) on water economics, water management, regulation, environmental economics, etc. We thank the students of these courses for their active participation, critical questions, and suggestions. Special thanks go to colleagues who have commented on earlier versions and individual sections of the book, or on the research papers related thereto. Nicole Waegner, Elisa Krammer, and Lukas Barner helped with editing, bibliography, problem sets, and also with background research on the case studies; Maximilian Reinhardt und Gero Scheck also contributed to proofreading and other tasks. We also thank our Publisher Springer, in particular Johannes Glaeser, for a regular and efficient exchange on various issues. Thanks also to TU Berlin for covering the open access fee to make the book freely available. Last but not least, we thank those who have helped us discover the water sector over the past years, researchers, stakeholders, activists, policymakers, etc.

[2]Erich Raspe: The Surprising Adventures of Baron Munchausen. The Project Gutenberg EBook 2006.

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2 Water Availability: A Hydrological View

2.1 Global Water Resources and Water Cycle

The whole amount of water on earth was generated during the earliest earth ages by volcanoes that emitted water vapor. Currently the amount of water which is allocated to the oceans, glaciers, polar ice, groundwater, lakes, and rivers stays nearly at a constant level.

The volume of the total water reserves is about 1,386 million km^3 (Table 2.1). The major part of these water reserves (about 96.5%) is located in the oceans as salt water. The total volume of freshwater stocks add up to 35 million km^3, or just 2.5% of the total stock in the hydrosphere. A large fraction of freshwater (about 24 million km^3 or 68.7% of freshwater stock) is stored in the Arctic and Antarctic regions in the form of ice and permafrost. About one-third of freshwater reserves are located in the aquifers as groundwater. Freshwater lakes and rivers, which are the most important sources for human water needs, contain on average about 90,000 km^3, or 0.26% of total freshwater reserves (Shiklomanov 1990).

Atmospheric water in the form of vapor and clouds has a volume of about 12,900 km^3, or 0.04% of total freshwater reserves. This atmospheric water is of high importance for the water cycle despite its small volume. If the atmospheric water precipitated completely, the water layer on the surface would have a height of just 25 mm. However, the annual precipitation amount is about 1,000 mm which means that the whole water stock in the atmosphere regenerates every 10 days. All other types of water also renew, but the rates of renewal differ. For instance water in the rivers regenerates every 16 days on average, but the renewal period of glaciers, groundwater, ocean water, and the largest lakes run to hundreds or thousands of years (Shiklomanov 1990).

The "regeneration" of water in rivers, lakes, atmosphere, etc., is based on the conversion of water into different types and aggregate states. Water converts from one form to another and moves to various places, for instance, from the ocean to land and

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Table 2.1 Water availability on earth. *Source* Shiklomanov (1990)

Source	Volume [10^3 km^3]	Percent of total water [%]	Percent of fresh water [%]
Total water reserves	1,385,984	100	–
Total seawater	1,338,000	96.5	–
Total groundwater	23,400	1.7	–
Soil moisture	16,5	0.001	0.05
Freshwater	10,530	0.76	30.1
Glaciers and permanent snow cover	24,064	1.74	68.7
Antarctic	21,600	1.56	61.7
Greenland	2,340	0.17	6.68
Arctic islands	83,5	0.006	0.24
Mountainous regions	40,6	0.003	0.12
Ground ice/ permafrost	300	0.022	0.86
Water reserves in lakes	176,4	0.013	–
Fresh	91	0.007	0.26
Saline	85,4	0.006	–
Swamp water	11,47	0.0008	0.03
River flows	2,12	0.0002	0.006
Biological water	1,12	0.0001	0.003
Atmospheric water	12,9	0.001	0.04
Total freshwater reserves	35,029	2.53	100

back under the influence of solar energy and gravity. An overall diagram of the global water cycle is presented in Fig. 2.1. A large amount of water, about 505,000 km^3, evaporates annually from the oceans' surface. About 90% of this evaporated amount, which is equal to about 458,000 km^3, returns directly back to the oceans in the form of precipitation while 10% of this evaporated amount, which is equal to about 50,500 km^3, precipitates on the land side. Together with evaporation and transpiration from land (about 68,500 km^3), the total precipitation falling on dry land and supplying all types of land water is 119,000 km^3. Based on this water volume, about 47,000 km^3 per year is returned back to the oceans from land in the form of rivers, ground, and glacial run-off. On the whole about 577,000 km^3 of water precipitates and evaporates on the earth. Thus, the world water balance can be considered as a closed system, such that

$$P = ET = 577000 \text{ km}^3$$

$$\text{with:} P \ldots \text{precipitation}, \ ET \ldots \text{evapotranspiration}$$

Figure 2.2 illustrates the levels of the main components of the global water circulation (Shiklomanov 1990).

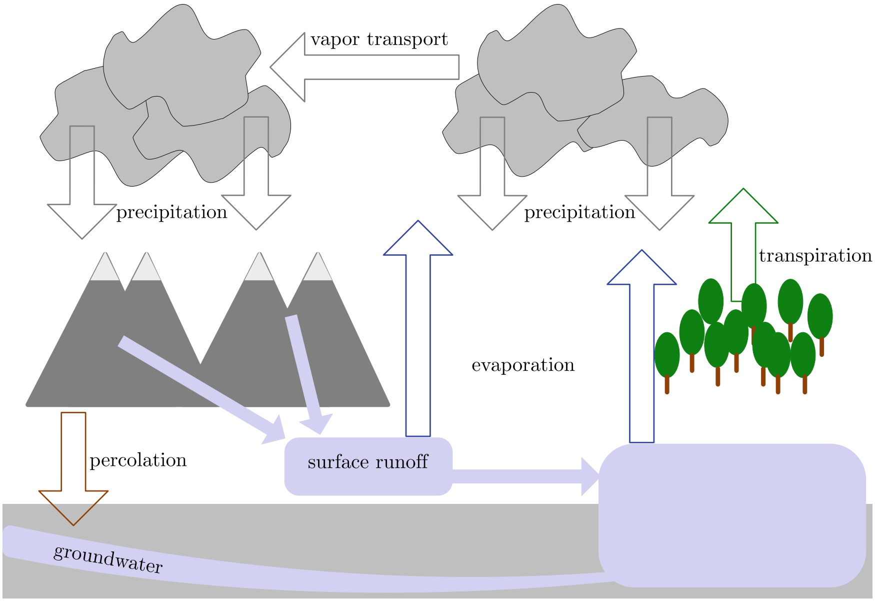

Fig. 2.1 Qualitative illustration of water cycle. *Source* adapted from Houghton (2004)

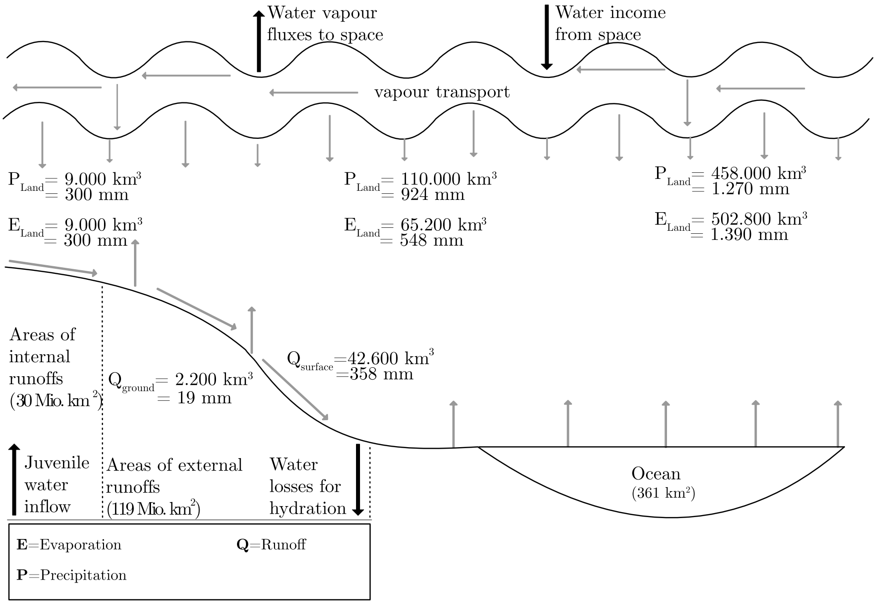

Fig. 2.2 Levels of the main components of the global water cycle. *Source* adapted from Shiklomanov (1990)

2.2 The Regional Water Cycle

Air humidity, soil surface, soil moisture storage, surface water (rivers and lakes), and groundwater are the types of water stocks that exist in each catchment. The relations and interconnections between these stocks are presented in Fig. 2.3. An external account is also introduced to illustrate the interconnection with the neighboring catchments. If water moves from a neighboring catchment to the considered catchment, the amount of water will increase in the addressed catchment. For instance, water vapor import fluxes induce an increase in air humidity; external recharges raise the amount of water in the aquifers and surface water stocks, etc. In contrast, the amount of water will decline in the respective stocks if water moves in the form of water vapor or surface and subsurface flows to neighboring catchments.

Water exchanges between the different water stocks also occur within the considered catchment. These exchanges and interconnections between the stocks are important to renew the stocks and to maintain the regional water cycle.

Precipitation, including all water in a hard or liquid state that reaches the soil surface from the atmospheric water stock (air humidity), is a very important input for plant, animal, and human life on earth. It will usually occur if the vapor pressure exceeds the saturated vapor pressure in the atmosphere. Falling precipitation, such as rainfall or snow, is usually known and the quantitatively most important kind of precipitation. Precipitation is a discontinuous and intermittent phenomenon with a high spatial and temporal variability. It is possible to distinguish between various forms of falling precipitation:

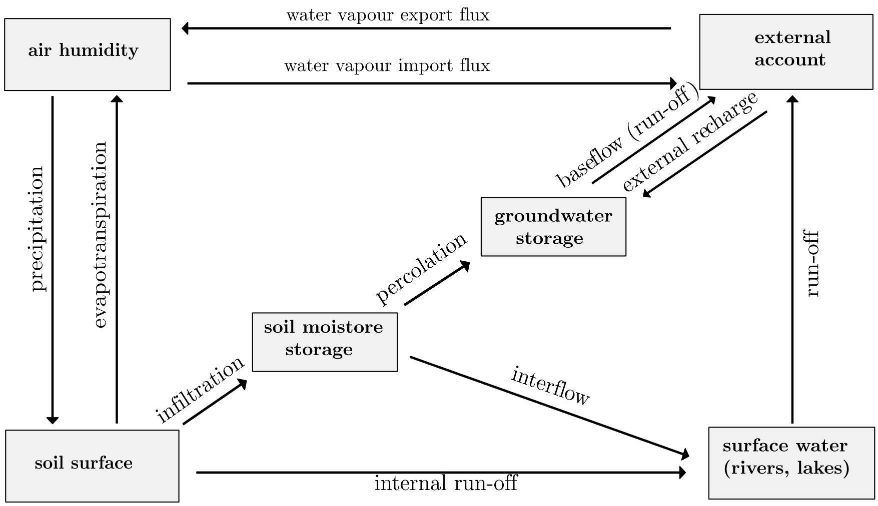

Fig. 2.3 Regional water cycle

- Convective precipitation is characterized by a high intensity, short duration, small-scale appearance, and therefore, high temporal and spatial variability. In Europe, it usually occurs in the summer months in the form of heavy rainfall and small-scale thunderstorms.
- Advective precipitation (steady rain) is more continuous than convective rainfall. It is characterized by a large-scale extent, long duration, low or medium rain intensity, and relatively low spatial and temporal variability.
- The third type of falling precipitation is the orographic one that occurs on the windward side of a mountain and is caused by rising air masses that cool down and condensate. Orographic precipitations are characterized by a long duration and a large-scale extent on the windward side.

Besides the falling precipitation, which is well known, disposing ones, such as dew, rime, and frost also exist.

Another important phenomenon that influences the available liquid water resources in a considered basin is the evapotranspiration. The atmosphere and the hydrosphere of a basin are closely linked to the existence of precipitation and evapotranspiration, because a share of liquid water, which is fed to the stocks by external inflows or precipitation, is removed by evapotranspiration, a vaporization process of water. The potential evapotranspiration, which is a hypothetical value that expresses the maximum possible amount of water that could be vaporized, depends on various meteorological conditions, such as solar energy supply, temperature, humidity, and wind. While potential evapotranspiration assumes optimal water supply, the level of real evaporation is equal to the actual vaporized water under actual water supply conditions. Therefore, the level of calculated potential evapotranspiration exceeds the amount of water vaporized by real evapotranspiration. This total real evapotranspiration includes the sum of the evaporation, transpiration, and interception. Evaporation is a pure physical process and occurs only on the surface of water and bare soil. Therefore, this kind of water vaporization influences the stored water volumes on the surface soil and surface water resources (lakes and rivers) as well as the moisture in the soil.

Evaporation accounts for only 10–15% of evapotranspiration in Central Europe while this proportion is much higher in arid regions because of less vegetation and higher solar energy supply. In Central Europe, the majority of the real evapotranspiration (about 70–75%) is related to transpiration, which is a biological process in which water vapor is released by parts of the plants. 90–95% of transpired water is released by the plants' stomata while the residual proportion is released by the cuticle. The transpiration can be regulated by opening and closing the stomata to prevent dehydration of the plant. Therefore, real evapotranspiration can deviate from potential evapotranspiration especially during hot spells. The third kind of evapotranspiration is the interception, which accounts for about 15% of total real evapotranspiration in Central Europe: It occurs on the surface of the plant; however, it is a pure physical process which cannot be influenced by the plant. Therefore, interception is often assigned to the evaporation.

Because of interception and evaporation, the quantity of surface and subsurface runoff is lower than total precipitation. The share of liquid precipitation that is not evaporated directly usually becomes surface or subsurface runoff. In contrast to precipitation and runoff which is characterized as *blue* water, water which is vaporized by transpiration is classified as *green* water. The definition of blue and green water is explained in Box 2.1. Groundwater recharge occurs if seeped water reaches the groundwater stock. The groundwater is that kind of water that completely fills all cavities in the underground and whose movement is only based on gravity. The level of groundwater recharge in a basin mainly depends on the level of precipitation, solar radiation, ground utilization, ground properties, and the distance between aquifers and surface. Infiltrated water can also drain as an interflow next to the soil surface. If seeped water does not reach an aquifer the subsurface runoff is referred to as interflow.

Box 2.1 Blue and green water

Water that is directly used for biomass production and "lost" in evaporation is termed "green water", while "blue water" is the flowing water in surface water bodies (e.g., rivers, lakes) and subsurface water bodies (aquifers). Terrestrial ecosystems (e.g., crops) are often "green water" dependent while aquatic systems are often "blue water" dependent. The management of "green water" flows holds potentials for saving water.

Source: GWP (2000)

Human activities significantly impact the water cycle. Both the quality and quantity of water stocks are influenced by discharged wastewater, climate change and water abstraction. Figure 2.4 integrates several human activities in the natural water cycle. Abstractions from the groundwater and surface water body are necessary to cover the agricultural, domestic, and industrial water demand. Wastewater that occurs after the usage of freshwater will eventually be purified in the sewage plant. The purified or non-purified wastewater will be discharged in surface water or groundwater bodies by percolation subsequently. This discharge changes the quality of water in the water stocks.

2.3 A Simplified Hydro-Economic Model

Water management is only possible on the basis of an exact consideration of the complex relationships of the water cycle. This section introduces the basic elements of the water cycle and relates them to the water use of the economy. It is important to understand the circulatory character of water. In the following, more complex nonlinear relationships that have been developed in hydrology, play no role in the analysis

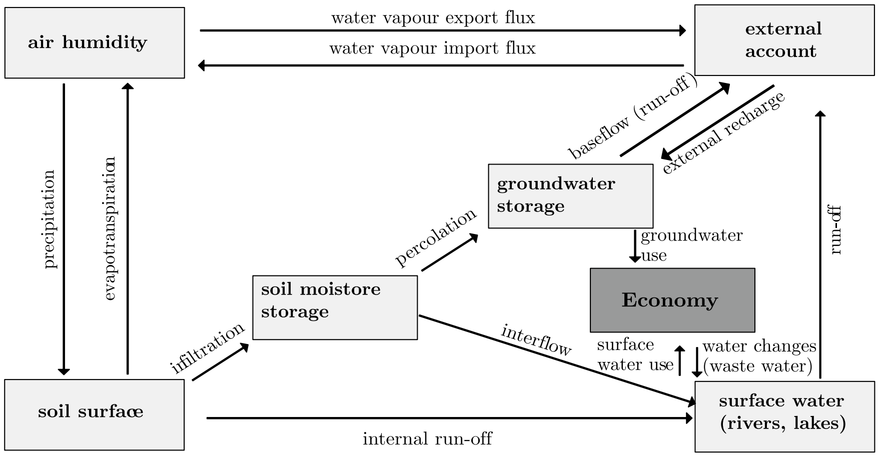

Fig. 2.4 Regional water cycle with human economy. *Source* own illustration

presented here at first. On the basis of a simple hydrological model, conclusions can be drawn which are presented in the following chapters.

A water cycle in its simplest form can be characterized by the dynamic mass balance equation, which describes the development of a water stock, including groundwater, water volume of surface water, etc., over time

$$\frac{dS(t)}{dt} = R(t) + P(t) - ET(t) - (1-h) \cdot x(t) - r(t) \tag{2.1}$$

In the balance equation, depicted in Eq. (2.1), the volume of the water stock at time t is denoted by $S(t)$. The water can be the groundwater under a catchment area, a lake or the water volume of a river.[1] $R(t)$ and $P(t)$ stand for recharge and precipitation, respectively. Both variables are taken as exogenous, i.e., they are not determined by the water management of the economy of that catchment area. Recharge may happen by a river entering the area or by subterranean groundwater flows from outside. The same applies to precipitation. Rain comes with the wind into an area and is as such exogenously given. Of course, a certain proportion of the rain can also have arisen through the local water cycle. x denotes the amount of water used in the local economy. The parameter $h \in [0, 1]$ gives the portion of x that is returned into the local watershed.[2] For simplicity, we take R, P, and x as time-independent. $r(t)$ describes the runoff at time t. Runoffs are all streams, be it on surface or underground, that leave the area. They depend, of course, on the water management of the economy and on the hydrology of the catchment area. $ET(t)$ depicts evapotranspiration. It

[1] The humidity of the soil also plays a role, but it is not considered in the following simplified model.
[2] Notice that we do not include water quality aspects into this basic model. Section 3.10 deals with water quality management.

consists of that portion of water that leaves the area as vapor. Forests, plants, and crops transpire and water evaporates on the surface of the landscape. This green water rises up and is carried with the wind in various directions. A part of it returns as rain.

To keep the model as simple as possible, we assume linearity of the various interrelations between the variables of the hydrological cycle. In the following, we assume that evapotranspiration depends linearly on the amount of water contained in a watershed, i.e.

$$ET(t) = \gamma_1 S(t) \tag{2.2}$$

If for example, the amount of water in a region or the soil moisture increases the evapotranspiration will rise groundwater or the moisture of the soil increase than the evapotranspiration will rise. Similarly, the runoff function exhibits the following relationship

$$r(t) = \gamma_2 S(t) \tag{2.3}$$

Inserting these two functions into the dynamic mass balance Eq. (2.1) yields

$$\frac{dS(t)}{dt} = R(t) + P(t) - \gamma_1 S(t) - \gamma_2 S(t) - (1 - h)x \tag{2.4}$$

Let us assume that recharge and precipitation are constant over time, i.e., $R(t) = R_0$ and $P(t) = P_0$.

The introduced equations form a dynamic hydro-economic model. The intrinsic dynamic forces can be analyzed with the help of a so-called phase diagram, a graphical method to study the properties of dynamic systems. Figure 2.5 depicts the dynamic interrelations. To begin with, the periodic abstraction of a human settlement in the catchment area is represented by a horizontal line denoted by $(1 - h)x$, where x is the raw abstraction and $h \cdot x$ are the return flows after usage. In this simple model, we assume that water use of humans does not depend on the size of the local water stock $S(t)$. Hence, $(1 - h)x$ is graphically represented by a horizontal line. The negatively sloped line in Fig. 2.5, shows the rate of replenishment of the water stock through inflows from precipitation, surface water and groundwater minus the outflows of surface and groundwater, as well as outflows through evapotranspiration (green water).

If the amount of replenished water is larger than the quantity of water used, i.e., $R_0 + P_0 - (\gamma_1 + \gamma_2)S(t) > (1 - h)x$ as indicated in Eq. (2.4), we can observe that the water stock will increase. Whether this is the case depends on the size of the water stock displayed on the horizontal axis. Let us assume that the current water stock is $S(t) = \Omega$, then the water stock will accumulate since $dS(t)/dt = R(t) + P(t) - (\gamma_1 - \gamma_2)S(t) - (1 - h)x > 0$. If $S(t)$ is somewhere on the right side of S^*, the reverse process takes place. This intrinsic dynamic behavior is identified by the arrows pointing to the intersection of both lines at S^*.

From Fig. 2.5, one cannot infer how long it will take until $S(t)$ reaches S^*, but it can be concluded that the stock will approach S^*. At the point where $S(t) = S^*$ holds, a hydro-economic equilibrium is reached, which is stationary in the sense that no further change of $S(t)$ will be observed. Additionally, S^* is also stable, i.e., if

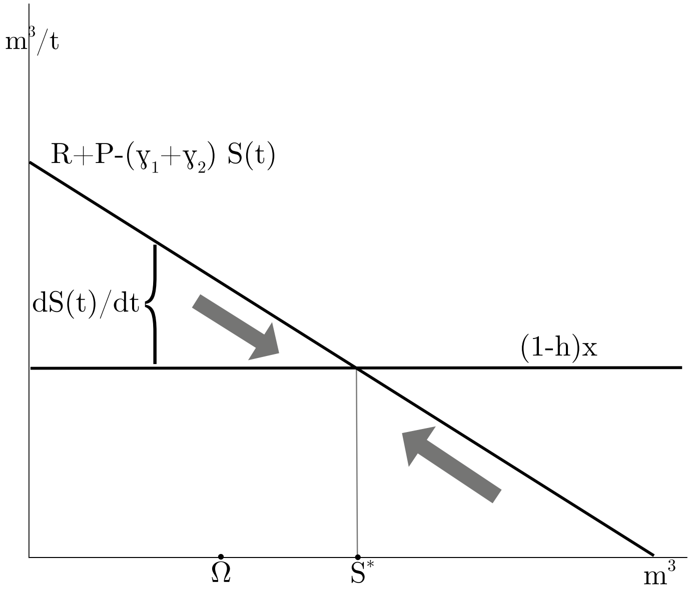

Fig. 2.5 Simple hydro-economic model. *Source* own illustration

$S(t)$ would deviate from S^*, let us say through a singular event like an unusual rain shower, then $S(t)$ would return to S^* after a while. We call this state a steady-state equilibrium.

The question remains whether the water use $(1 - h)x$ can be covered by the local water cycle, i.e., whether total water abstraction by the human settlement is sustainable. This depends on the level of net water abstraction $(1 - h)x$. From Fig. 2.5, we can infer the equilibrium water stock level S^* that corresponds to the quantity of water used by humans, i.e., $(1 - h)x$. It follows that a higher level of water abstraction is associated with a smaller water stock in the local water cycle's equilibrium. Whether the water use is sustainable depends upon the critical threshold Ω. This threshold depends on the whole ecological system and its interaction with the water cycle. We simply take this value as given. If the water stock $S(t)$ is less than Ω, severe ecological damages will occur due to a decrease of basic stabilizing functions of water beyond its economic use: micro-climate stabilization, soil control, nutrient retention, supporting habitats and diversity, and flood control through wetlands. The corresponding upper bound of sustainable water abstraction can be calculated from Eq. (2.4) by setting $dS(t)/dt = 0$ and solving for x. Inserting $S(t) = \Omega$ yields

$$x^{max} = \frac{R_0 + P_0 - g_1\Omega - g_2\Omega}{1 - h} \tag{2.5}$$

x^{max} is the upper bound of admissible water abstraction, implying that there is a quantity range $[0, x^{max}]$ of sustainable water usage. If the human water utilization is less than the level x^{max}, sustainability of the local water cycle is still assured. Of course, the change in the water table may lead to a change in the environment. But this change is not detrimental to the environment itself or its provision of ecological services nor to the people living in this catchment area. Box 2.2 describes a historic case of over-utilization of the water cycle with the help of the simple linear eco-hydrological model.

Box 2.2 The demise of the Mayas

The Mayas dominated Middle America for at least 1500 years and suddenly, around the ninth century A.D., their civilization vanished within a very short time. It is estimated that in the pre-Columbian time over 19 million people lived in Meso-America, and that after the ninth century only 10 percent were left. Archeologists and historians puzzled about the reasons for this sudden demise of this ancient civilization. Numerous explanations were presented, such as epidemic disease, warfare, and overpopulation. Today, there is reason to believe that severe droughts have caused the collapse of the agricultural system, and hence destroyed the livelihood of the Mayas. These droughts were not only the result of a long wave periodic change of the climate, but they also resulted from the deforestation that took place to gain more farmland. Dr. Thomas L. Sever, an archeologist with NASA's Marshall Space Flight Center, said that the rise of droughts in this area could be traced back to the Mayas themselves. In some recent studies, geophysicists developed complex hydrological and climatological models to reconstruct the impact of deforestation on the local climate.

Sources: Cook et al. (2012), Kuil et al. (2016)

2.4 Exercises

Exercise 2.1 Water availability in the 2020s
The source we use to describe the water availability on earth is the best one available, but it is over three decades old. Try to find reliable sources to update the values for the major categories, such as total water reserves, total seawater, freshwater, glaciers, etc., to the current times, i.e., the 2020s. Are there differences to be observed? If yes, what could be reasons for this? Is the literature unified on this issue, or are there controversies?

Exercise 2.2 The demise of Mayas
We can use our simple linear model to get an idea of how various factors were at work and led to the decline of the agricultural base as a result of increased deforestation. Our approach focusses on some pivotal interactions that cause the detrimental effects

of deforestation. To do so we extend Eq. (2.1), by introducing a coefficient β that indicates the capacity of the local climate system to return evapotranspiration as precipitation.

$$\frac{dS(t)}{dt} = R(t) + P(t) - (1-\beta)ET - r - (1-h)x \tag{2.6}$$

were $0 \leq \beta < 1$. This coefficient captures various climatological effects that are responsible for the creation of clouds through local evapotranspiration identified by climatologists: The surface albedo effect, aerodynamic effects, and chemical effects, to name some. Let us confine to the surface albedo effect. Albedo is the ratio between reflected radiation to incident solar radiation. The higher the albedo the less radiation (energy) is absorbed from the earth. The albedo rises with the deforestation because cultivated land reflects more radiation. With rising albedo, the absorption of energy from radiation decreases, which leads to less heat flux. Less heat energy causes less vapor production and results in a decrease of cloud building. Less clouds are associated with less precipitation.

This transmission chain is captured by β, which depends on deforestation. Let F be total land available in an area. This land is either covered by forest or it is utilized as cropland, whereas the latter case is denoted by A. Thus $\beta = \beta(A)$ with $\beta'(A) < 0$.

In addition, we have to distinguish between forest evapotranspiration and cropland evapotranspiration. Extending the linear model leads to

$$ET_1(t) = \gamma_1 S(t)(F - A) \tag{2.7}$$

$$ET_2(t) = \gamma_2 S(t)A \tag{2.8}$$

where Eqs. (2.7) and (2.8) represent the evapotranspiration of forest and of cropland, respectively. We assume, that $\gamma_1 \geq \gamma_2$. Finally, runoff is given by

$$r(t) = \gamma_3 S(t)A \tag{2.9}$$

assuming that runoff takes place mainly in the cultivated areas. We also assume, that $\gamma_1 < \gamma_2 + \gamma_3$, i.e., an increase of cropland A leads to an increase of evapotranspiration. If we insert these three equations into Eq. (2.6) we get

$$\frac{dS(t)}{dt} = R(t) + P(t) - (1-\beta)(\gamma_1 S(t)(F-A) + \gamma_2 S(t)A) - \gamma_3 S(t)A - (1-h)x \tag{2.10}$$

Agricultural production depends on water availability $S(t)$ and, of course, on the area A. Let us assume the simple production function

$$C = \frac{A}{F} Max[\delta(S(t) - \Omega), 0] \tag{2.11}$$

where δ is agricultural productivity. Output depends not only on the area cultivated but also on the amount of water available. This function depicts the inherent hydrological

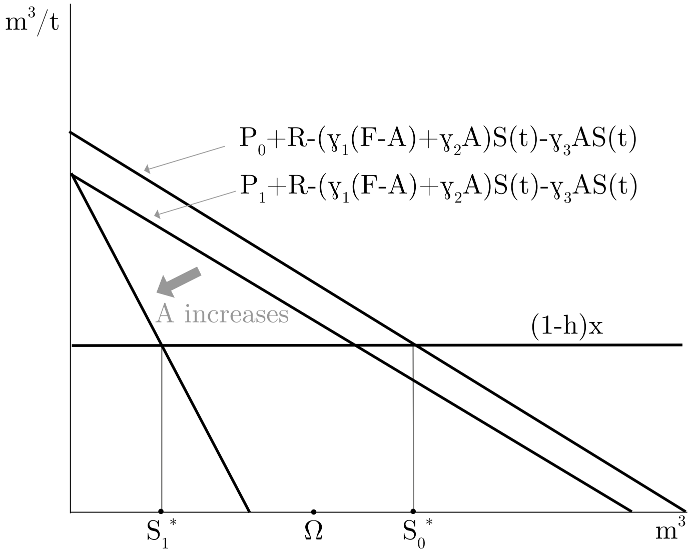

Fig. 2.6 The demise of the Maya. *Source* own illustration

and ecological preconditions of agricultural production in an extreme manner. If the water stock is above a critical threshold, agricultural production is possible. If S falls short of Ω the whole production breaks down. Figure 2.6 shows the problem of increased deforestation.

In the course of an exogenous decrease of precipitation from P_0 to P_1 the dS/dt-curve shifts downwards and the output of agriculture drops (see Eq. (2.11)). The Maya react with expanding cropland because they try to compensate the decreased productivity of cropland by increasing the size of it (see Eq. (2.11)). As a result, the increased deforestation leads to a clockwise rotation of the graph reflecting Eq. (2.10). The final hydrological equilibrium is S_1^* which is located to the left of Ω leading to severe crop failures, and finally to the demise of the Mayas.

2.5 Further Reading

A good overview about the water availability and water cycle is given by Shiklomanov (1990). More details about the components of the water balance could be found in special books which focus on meteorology or hydrology such as Brutsaert et al. (2005), Gordon et al. (2004), as well as Holton and Hakim (2012). Introductory references to geohydrological topics and groundwater are Karamouz et al. (2011) and Thangarajan (2007).

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3 Integrated Water Resource Management: Principles and Applications

3.1 What Is Integrated Water Resource Management?

3.1.1 Approaches to IWRM

The Integrated Water Resource Management (IWRM) approach goes back to the establishment of the Tennessee Valley Authority (TVA) in the year 1933, which integrated the functions of navigation, flood control and power production (Biswas 2004). Further issues, such as erosion control, recreation and public health, were also addressed by the TVA (Mitchell 1990). The Secretary-General of the United Nations Organization (UNO) addressed the topic of IWRM in 1957. The UNO's understanding of integration refers to supporting services needed to develop irrigated agriculture, but the coordination of different water-related functions was not part of this IWRM concept. This deficit was remedied at the Water Conference in Mar del Plata in 1977 where the necessity of coordination within the water sector was explicitly addressed. However, issues associated with high water demand and negative environmental impacts of irrigated agriculture were not approached sufficiently (Snellen and Schrevel 2004).

At the beginning of the 1990s, there were some observable shortcomings in traditional water management, like quality issues, overexploitation, ecosystem degradation or social concerns. Water problems also had become multidimensional, multisectoral, and multiregional and filled with multi-interests, multi-agendas, and multicauses (Biswas 2004). To overcome these issues, four important guiding principles were determined during the International Conference on Environment and Water in Dublin in the year 1992 (Xie 2006). These principles (ecological, institutional, gender, economic) became well known as the "Dublin-Principles", which are stated in the annex of this chapter.

The Dublin Guiding Principles represented an important input for the Agenda 21, which was agreed upon the United Nations Conference on Environment and

G. Meran et al., *The Economics of Water*, Springer Water,
https://doi.org/10.1007/978-3-030-48485-9_3

Development in Rio de Janeiro in 1992. Chapter 18 emphasized the need for an integrated approach to manage water resources by connecting different water services and providing good governance, appropriate infrastructure, and sustainable financing.[1]

The present understanding of IWRM with its holistic approach is strongly based on the Dublin-Principles as well as on the Agenda 21 (Chap. 18) document. There are many definitions of IWRM, for instance, in the Agenda 21.[2] A well-cited definition of IWRM is the one made by GWP (2000):

> IWRM is a process which promotes the coordinated development and management of water, land and related resources in order to maximize the resultant economic and social welfare in an equitable manner without compromising the sustainability of vital ecosystems.[3]

IWRM cannot be seen as a blueprint or product for good water management, but rather as a paradigm with a broad set of principles, tools, and guidelines that must be tailored to the specific context of a country, region, or river basin in order to implement an efficient and effective water resource management. A basic set of principles is outlined in Box 3.1.

Box 3.1 IWRM principles

- Integrate water and environmental management.
- Follow a systems approach.
- Full participation by all stakeholders, including workers and the community.
- Attention to the social dimensions.
- Capacity building.
- Availability of information and the capacity to use it to anticipate developments.
- Full-cost pricing complemented by targeted subsidies.

[1] Chapter 18.3 of Agenda 21 states:

> The widespread scarcity, gradual destruction and aggravated pollution of freshwater resources in many world regions, along with the progressive encroachment of incompatible activities, demand integrated water resources planning and management. Such integration must cover all types of interrelated freshwater bodies, including both surface water and groundwater, and duly consider water quantity and quality aspects. The multi-sectoral nature of water resources development in the context of socioeconomic development must be recognized, as well as the multi-interest utilization of water resources.

[2] A review about IWRM definitions is given by Jonker (2007).

[3] See Box 2 on page 22 in GWP (2000).

- Central government support through the creation and maintenance of an enabling environment.
- Adoption of the best existing technologies and practices.
- Reliable and sustained financing.
- Equitable allocation of water resources.
- Recognition of water as an economic good.
- Strengthening the role of women in water management.

Source: IWA/UNEP (2002)

3.1.2 The IWRM Paradigm

The IWRM paradigm contains important key concepts of integration, decentralization, participation, and sustainability (Xie 2006). Due to the holistic view of the IWRM paradigm, there is a necessity for the integrated management of horizontal sectors that use or affect water resources, e.g., water supply, sanitation, agricultural use, energy generation, industrial use, or environmental protection. In addition to horizontal integration, vertical integration is also required to coordinate efforts between local, regional, national, and international water user groups and institutions (Xie 2006). The main aspects regarding natural system integration and human system integration are listed in detail in the chapter annex Sect. 3.13.2 (GWP 2000).

Besides the necessity of integration, there is also a need for decentralized decision-making and responsibility at the lowest effective management level, to increase awareness for local and regional problems. Hence, IWRM seeks to strike a balance between top-down and bottom-up management. IWRM also wants to strengthen community-based organizations and water user associations.

The consideration of sustainability, as a main part of IWRM, is not only restricted to ecological sustainability for protecting the natural system, but it also covers aspects of financial and economic sustainability. This means, for instance, that resource allocation decisions have to be based on the economic value of water. Therefore, water must be priced at its full costs (Xie 2006).[4]

The three key policy goals of IWRM are Equity, Ecological integrity and Efficiency, which are known as the three'E's (Postel 1992):

- **Equity:** Water is a basic need and hence there is the basic right for everybody to have access to water of adequate quantity and quality.

[4]Full cost accounts for the cost of withdrawing and delivering water as well as the opportunity cost plus the cost associated with economic and environmental externalities.

- **Ecological integrity:** Water in sufficient quantities with sufficient quality should persist in the environment. Water should be used in a sustainable way, so that the future generation will be able to use it in a similar way as the present generation.
- **Efficiency:** Water must be used with maximum possible efficiency, because of its finite and vulnerable nature. Cost recovery of the water service should be attained. Water should be priced according to its economic value.

For supporting the application of IWRM principles in practice, the Global Water Partnership (GWP) has created a toolbox whose three main categories are an enabling environment, institutional roles, and management instruments (GWP 2000, 2004):

- **Enabling environment** refers to securing the rights and assets of all stakeholders and protecting public assets. This category involves the general framework of national policies, legislation, and regulation.
- The **institutional roles** involve the consideration of a whole range of formal rules and regulations, customs and practices, ideas and information, and interest and community group networks, which together provide the institutional framework or context within which decision-makers operate.
- The **management instruments**, include operational instruments for effective regulation, monitoring, and supporting decision-makers.

3.1.3 A General Framework for IWRM

For transferring the IWRM paradigm into practice, the GWP (2004) recommends an IWRM planning cycle, which is illustrated in the chapter annex. In summary, the complexity of the water cycle and interdependencies within the water sector and other sectors (e.g., food sector, electricity sector) require specific methods for integrating environmental, social, and economic issues at the level of watersheds. The paradigm of IWRM provides us with the necessary interdisciplinary tools, which come from natural water science (e.g., hydrology, geohydrology, meteorology), engineering, and social sciences like political science, sociology, and economics. Often these methods, such as optimization models or decision supportive systems, etc., utilize mathematical models as a necessary prerequisite to capture complexity. Mathematically based hydro-economic models, which can be seen as a tool of IWRM, often work with simulation or optimization models and node-link networks to replicate the spatial distribution of important system elements like natural water bodies (e.g., sea, lake, aquifer, river section, etc.), artificial water bodies (e.g., canals, etc.), infrastructure (e.g., wells, dams, pipelines, pumps, purification plants, etc.), human/artificial impacts in the water system (e.g., point of use, point pollution source, non-point pollution source). Box 3.2 gives an example for a numerical-based hydro-economic model, which is extensively used, among other applications, to establish an IWRM approach in California (Fig. 3.1).

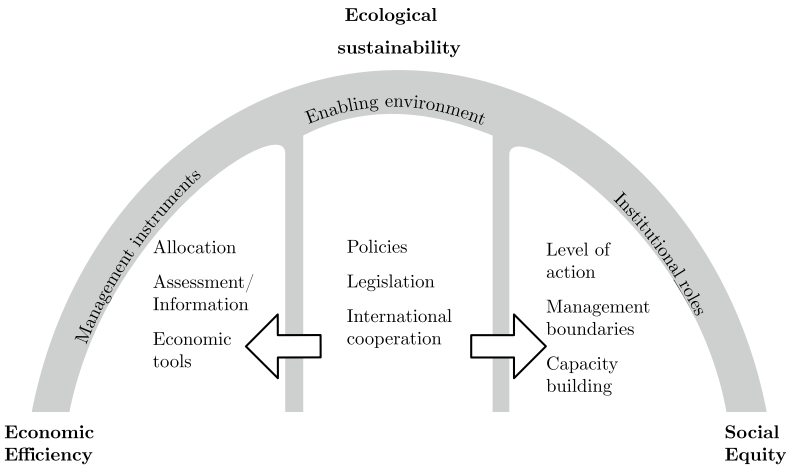


Fig. 3.1 General framework of IWRM. *Source* GWP (2000)

Box 3.2 The CALVIN Model

The CALVIN model is a numerical-based economic-engineering optimization model for water management in California. It was developed at the University of California in Davis. The data set contains a wide range of monthly parameters over the decades. The model is applicable to a variety of policy, operations, and planning problems. CALVIN manages water infrastructure and demands throughout California's intertwined water network to minimize net scarcity and operating costs statewide. Some model applications are

- Water markets,
- Capacity expansion in the water supply,
- Consequences of climate change,
- Severe sustained drought impacts and adaptation, and
- River restoration.

Source: Howitt et al. (1999), Lund et al. (2009)

The following sections introduce general but simple models that cover the major problems of IWRM step-by-step. Specific topics and economic tools of IWRM, such as the pricing policy and transboundary river management, are also addressed.

3.2 The Economic Dimension of Water

If water is scarce then inevitably the economic perspective toward water gains a particular importance. This has been stated in the aforementioned declaration of Dublin. Principle 4 states that

> water has an economic value in all its competing uses and should be recognized as an economic good. [...] Managing water as an economic good is an important way of achieving efficient and equitable use, and of encouraging conservation and protection of water resources. (Xie 2006)

Emphasizing that water is an economic good does not imply this resource is exclusively a private good. Nor does it imply that water supply should be privatized. It simply implies that the water cycle must be managed as a nonabundant resource. The main difference of this kind of scarcity to other scarce goods, like e.g., precious old paintings or fossil fuels, is that scarcity is the result of a political decision not to overexploit the water cycle. The acknowledgment of scarcity follows from the adherence to the principle of sustainability. This statement is open with respect to the institutional implementation of the necessary management steps to assure sustainability and economic efficiency as well. In the following subsection, we analyze the various functions of the water cycle from an economic perspective.

3.2.1 Types of Environmental Goods

One could imagine that water is collectively owned by a society. There are many examples worldwide of collectively owned and managed watersheds. For instance, Ostrom (1990) reports from irrigation cooperatives in Spain and the Philippines where the allotment of irrigation water has been fixed within a collective institutional setting that contains conflict resolution mechanisms as well as monitoring systems. On the other hand, there exist market-based solutions like water market institutions in the southwest of the the USA or in Australia. There, water is often owned privately according to traditional property rights and sold in spot and forward markets (see Chap. 5). Hence, saying that water is an economic good should not be confounded with the notion of water as a private good. A private good is characterized by its rivalness and by the possibility of exclusion on the basis of property rights. For example, if farmer A irrigates his fields the same water needed is not available for farmer B (rivalness). But the usage of water by farmer A requires also that he is able to get hold of this water (exclusion of other users).

But water appears not only as a private good. Indeed, the water cycle assures the livelihood of people in a watershed by satisfying many different life-supporting ecosystem functions. For environmental economists, the ecosystem functions of the water cycle interact with functions from other natural resources (soil, nutrients, vegetation, etc.), i.e., input factors that produce the total ecosystem to the inhabitants of a watershed. These ecosystem services lead to societal benefits, as they create eco-

Table 3.1 Types of environmental goods

	Rival	Non-rival
Excludable	*Private good* food, oil, gas, timber	*Club goods* swimming pool, golf club lane, national park
Non-excludable	*Open-access resources* Deep-sea fishery, ecosystem services	*Public good* carbon-absorption capacity of the rainforest, eco-system services

nomic value. As an example, a wetland mitigates flood damages and, at the same time, can be used as a recreational area. Forests contribute to the recharge of groundwater and influence the microclimate through evapotranspiration in a favorable manner. From an economic standpoint, these life-support functions represent societal and economic values far beyond, e.g., the plain use value of water for irrigation or for the water supply of households respectively.

Some of the ecosystem services of water mentioned above appear as public goods. Public goods are characterized by the absence of rivalness and the non-applicability of exclusion. The local water cycle, for instance, sustains the microclimatic stabilization of the watershed which is the base of livelihood for the inhabitants. All members of that local population reap this positive ecosystem function (non-rivalness) and nobody can be excluded (nonexclusion). Or take the example of the flood protection capability of a forest habitat or from a wetland. Here again, the advantage accrues to all neighbors sheltered. Table 3.1 shows the classification of natural resources and their services into different types of economic goods. Each of these types will require a specific approach of management to assure an efficient and environmentally sustainable supply to society.

Take natural resources as private goods. Oil, gas, and timber, for example, are resources that are traded in markets. Indeed, they are private goods due to well-defined property rights and due to their rivalness. The case of club goods is rather similar: You pay for their services, but in contrast to the private good case your consumption does not reduce the consumption opportunity of your fellow club members.[5] Now take the case of open-access resources. Deep-sea fishery is a good example. Nobody can be prevented to cast for fish outside of the exclusive economic zone 200 nautical miles from the terrestrial baseline. Hence, there is no excludability while at the same time their fishing is rivaling. Natural resources or eco-services could also assume the property of a public good: Everybody will benefit from these services and nobody can be excluded from this benefit even if one does not pay for it. A very typical example is the rainforest's capacity to absorb carbon.

Why is this classification important for economists? To explain the importance of these distinctions, let us take the example of the deep- sea fishery. The lacking excludability of fishing grounds leads to an overexploitation of fish populations. Too many trawlers are operating and do not take into account the effects of their fishing

[5]Strictly speaking, this case applies only if no congestion occurs.

Table 3.2 Economic dimensions of water

	Rival	Non-rival
Excludable	*Private good* drinking water for households, irrigation for farmers, regulated groundwater extraction	*Club goods* various types of in-stream uses, other recreational use restricted to club members
Non-excludable	*Open-access resources* ground water extraction, river as a waste water sink	*Public good* microclimate stabilization, soil control, nutrient retention, supporting habitats and diversity, flood control through wetlands

efforts on the fish population. Moreover, if one of the fishermen would be concerned about the future of fish stocks he not only would harm himself if he decided to fish less but also would not contribute to protecting the fish stock: The amount of fish he would abstain from fishing will be caught by his colleagues. Obviously, the unregulated free market is not a suitable institutional model for an efficient and environmentally sustainable fishery management system but, instead, it calls for public intervention. Similarly, the production of public goods should not be left to a market. In general, the supply of public goods through a market where each customer pays the same price leads to an under-provision of this good: As no customer can be excluded from consuming the good the provider is not able to make sufficient profits. Again, the free market solution would result in a dissatisfactory result, a situation sometimes interpreted as market failure.

3.2.2 Economic Dimensions of Water

The economic dimension of water use can further be specified by the classification presented in Table 3.2. The various types of water usage exhibit the economic dimensions of the water cycle. Different kinds of benefits arising from water use and various production structures in the four peculiar specifications call for different institutional frameworks to secure the specific water services to a satisfying extent.

Note that the notion of private good does not refer to an entitlement of owning a water resource privately. It refers only to its characteristics of being excludable and rival. The rules regarding how the user got hold of a certain amount of water are not specified so far. Perhaps she had paid for that water from a private supplier or the water had been allotted to her for free by a public agency. Or take the case of groundwater extraction. Perhaps the groundwater is under common property law, i.e., it is a common pool resource, owned by a community or municipality. Water extracted from a groundwater reservoir is a private good allotted to the members according to implemented rules. This could be accompanied by payments depending on the quantity of water retrieved or water could be obtained for free up to a certain limit (rationing). In this case, the financing of the necessary technical infrastructure (pipe, pumps, energy, etc.) has to be assured by local institutions, e.g., a municipality or a water cooperative.

3.3 Social Welfare, Scarcity, and the Value of Water

3.3.1 Fairness Criteria

As discussed above, the economic aspect of water management needs to be fully integrated into the concept of a sustainable water resources management approach.[6] Water use should not only respect the hydrological cycle and the boundaries of ecosystems but should also strive to use water in an efficient manner. Solving water scarcity problems by simply transferring water from one catchment area to another is not a sustainable approach as a rule. Integrated water resource management has to deal with the water demand side and the economic allocation of scarce water to users. Users are households, the industrial, and the agricultural sector. Water management activities refer not only to measures to enhance the efficiency of water use but also to specific rules that determine the allocation of water among users. These rules have to be institutionalized so as to make them effective.[7] This process must satisfy normative criteria or societal goals, namely, efficiency, social fairness, or equity and environmental sustainability. These criteria gain more and more importance in regions, where water gets increasingly scarce.

There exist various methods and model specifications to incorporate these goals into the management process. Let us explain the basic features with the help of an example. There are two farmers in an arid zone both exposed to water scarcity. Let us assume that the first farmer, F1, is more productive than farmer 2, F2. F1 produces an agricultural output—let us say alfalfa—according to a simple linear production function $y_1 = a_1 w_1$, where y_1 is the output of alfalfa, a_1 denotes the water productivity, and w_1 represents the amount of water used. Similarly, F2 produces the same crop according to the production function $y_2 = a_2 w_2$ with a lower productivity than F1, i.e., $a_1 > a_2$. There is a sustainable water supply of $\bar{W}$, which can be allocated to F1 and F2, i.e., $\bar{W} = w_1 + w_2$.

From a pure output view that respects the sustainability constraint $\bar{W}$, the best water allocation maximizes total output $y_1 + y_2$. In this case, all the water should be allocated to F1 leaving nothing to F2. The farm of F2 will be shut down, and F2 would lose his revenue. But would this be just? The literature mentions various allocation principles, that go far beyond the usual efficiency criterion (Johansson-Stenman and Konow 2010).

1. **Accountability principle:** This principle states that persons should be remunerated in proportion to their effort. Let us assume for a moment that both sites have the same productivity in terms of soil characteristics and geological properties. Hence, productivity differences could be traced back to different levels of effort (assuming that other reasons, like health, physical conditions, etc., are

[6]See, for instance, The Dublin-Principles of the International Conference on Water and the Environment (ICWE) in Dublin, Ireland, 1992 (GWP 2000).

[7]That is the reason that some experts prefer to speak of water governance instead of water management, which highlights the societal character of the management process.

disregarded). The optimal water allocation would then prescribe to channel a certain portion of water to F1 and the residual to the farm of F2 depending on the relative efforts of both. One possible rule splits total output proportionally, i.e., the share of total output for farmer i is $a_i/(a_1 + a_2)$. This, of course, requires suitable institutional arrangements to implement this rule. Specifically, effort must be observable.

2. **Efficiency principle:** An efficient allocation implies that water is distributed according to the users' productivities. In this case, all water goes to F1 and F2 receives nothing, which results in the highest aggregated output that can be achieved by the two farms together. This is indeed efficient in terms of maximizing total output given a certain amount of resources but the allocation seems to be at odds with fairness as nothing is left to F2. However, the economically efficient allocation, $w_1 = \bar{W}$ could lead to a fair outcome if redistributive instruments are available to secure fairness. In our case, the instrument consists of a transfer rule specifying how much of F1's output, y_1, should be transferred to F2. But how much output should be transferred if the productivity differences cannot be traced back to different effort levels? Before we deal with this issue it should be mentioned that a strong trade-off between efficiency and fairness exists only if no other management instrument than the water allocation itself is available. In the presence of other redistributive instruments, this trade-off might still exist but it is less severe.
3. **Basic need principle:** According to the basic need principle, an allocation of water has to ensure that all members of a society survive in a decent way. In the case of the two farms, the water allocation is either such that all water goes to F1 except for the amount w_2 that guarantees F2 an output sufficient to survive. Alternatively, all water goes to F1 while F1 is obligated to transfer a sufficient amount of output to F2. Again, the issue of whether the allocation is efficient or not depends on the availability of a transfer system. The transfer system might refer either to output or to water. Whatever transfer medium is chosen the basic need principle prescribes that all people have an entitlement to the provision of goods or resources so as to survive in a decent way. The basic need principle is especially important for developing countries.
4. **Strict equality:** There is a long-lasting discussion in social philosophy on distributional justice. One egalitarian view is the concept of moral arbitrariness proposed by the philosopher John Rawls.[8] In modern societies, there is a broad agreement that the social product should not be distributed according to innate entitlements as in feudal times. But John Rawls also denies that justice can be secured by the institutionalization of equal opportunities as in the case of free markets, or free markets and supporting institutions to equalize opportunities for people from all social classes. All characteristics people cannot influence by themselves shall not be decisive for the distribution of produced income. If somebody is highly gifted and utilizes this advantage in a free market then the outcome will be unjust. The

[8] Rawls (1971), see the lucid explanation in Sandel (2009).

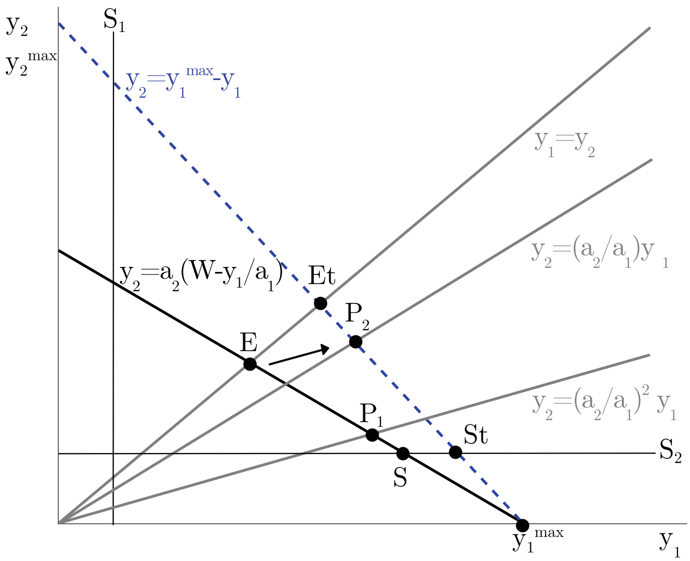


Fig. 3.2 Efficiency and fairness. *Source* own illustration

uneven distribution of innate endowments among all people is morally arbitrary and, hence, the productivity effects of these endowments shall be shared by the community as a whole. This leads to the conclusion that strict income equality is just. There is an exception: incentives. If a talented person is highly taxed then he might lessen his effort leading to less production. Here, John Rawls introduces the difference principle. The distribution of goods remains just if in the course of an income increase of a successful market participant the income of the most disadvantaged rises as well.

The four principles can be summarized with the help of Fig. 3.2. The production possibility line shows all possible combinations of output, $\{y_1, y_2\}$, as a function of the water allocation, W. The maximum potential output of F1, y_1^{max}, is reached if the whole sustainable water supply, $\bar{W}$, is allocated to F1, hence the output combination $\left\{y_1^{max}, 0\right\}$ satisfies the efficiency principle. The dotted line depicts all possible output distributions if a transfer system is available. In this case, the $y_1 = a_1\bar{W}$ will be distributed among F1 and F2 according to one of the fairness criteria. For strict equality we have point Et. Here, both farmers receive the same amount of agricultural output after the transfer has taken place. If one applies the accountability criterion, the output allocation is determined by proportional rule $y_i = [a_i/(a_1 + a_2)]y_1^{max}$. This allocation determines the proportion of both allocations, i.e., $y_2 = (a_2/a_1)y_1$. The intersection of this array with the production possibility line (dotted line) is point $P2$ that defines the allocation for this case. Alternatively, the proportional rule can be applied to total water available, i.e., $y_i = [a_i/(a_1 + a_2)]\bar{W}$, which leads to the array $y_2 = (a_2/a_1)^2 y_1$.[9] Which of the two rules should be applied depends on what the distributandum is, water or agricultural output. Points S and St indicate the distribution if the basic need principle is satisfied.

If the institutional framework does not permit a redistribution of goods, the consideration of a fair distribution is only possible with the help of water allocation. In this case, the production possibility line is given by the solid line in Fig. 3.2. The corresponding allocations are then given by the points E (strict equality), $P1$ (proportional allocation) and S (basic needs). The lines S_2 and S_1 are the respective lifelines of F2 and F1, i.e., the minimum outputs that are sufficient for survival. Moving from S to St illustrates the institutional efficiency gains that can be realized by introducing a transfer system.

The difference principle of John Rawls is visualized by the shift from E to, say, P2. If in the course of an institutional innovation transfers are introduced we could move from the strict egalitarian distribution E to P2 for instance.[10] In point P2 both incomes in terms of output quantities have increased relative to point E, therefore the difference principle is satisfied despite the deterioration of the income distribution among the users. The increase in income of F1 is accompanied by a rise in F2's income. P2 is the allocation point where the accountability principle is satisfied. Here, total output, y_1^{max}, is divided proportionally in shares of $a_i/(a_1 + a_2)$. If transfers are not possible, the accountability principle can only be applied to the water allocation. In this case allocation P1 will be chosen.

3.3.2 Social Welfare Function

3.3.2.1 Individual Utility Functions

In mathematical policy models, the optimal allocation is often derived from a social welfare function (SWF). Usually, these functions depend on the utility or the well-being of every single member of the community or society under consideration. In our simple case, the social welfare function could be written as $SWF = G(y_1, y_2)$. The well-being of F1 and F2 is indicated by their incomes, y_1 and y_2, respectively. There are various specifications of this function that can be related to the fairness principles introduced above. The SWF most prevalent in economics and also in the IWRM literature is the so-called utilitarian social welfare function, according to which social welfare is simply the sum of the individual welfare of every single member of the society. Here, individual welfare is identified as an individual's income and its consumption.

$$SWF = y_1 + y_2 \tag{3.1}$$

[9] This follows from the sharing rule $w_i = a_i/(a_1 + a_2)\bar{W}$ and, hence, $y_i = (a_i^2/(a_1 + a_2))\bar{W})$. Thus, $y_2 = (a_2/a_1)^2 y_1$.

[10] Of course, one can also select other points on the dotted line that lead to a change in the distribution ratio.

The optimal allocation of water is derived by maximizing the objective function represented by Eq. (3.1) while taking all the economic and hydrological constraints into account. This SWF adheres to the efficiency principle. What matters is the total sum of individuals' well-being without any regard of the distribution of well-being. If we insert the farmers' linear production functions into Eq. (3.1) and stick to a an ecologically sustainable solution the management's objective is

$$\max_{w_1, w_2} [a_1 w_1 + a_2 w_2] \text{ s.t. } w_1 + w_2 \leq \bar{W} \tag{3.2}$$

Utilizing the Karush–Kuhn–Tucker (KKT) conditions[11] we get the solution $w_1^* = \bar{W}$ and $w_2^* = 0$. All the water goes to F1 leading to consumption of $y_1^* = a_1 \bar{W}$ and $y_2^* = 0$.

However, as this allocation can hardly be termed fair, the program can be amended by additional constraints to include the various fairness principles. For instance, if we include the restriction of a minimum threshold for F2's consumption quantities, i.e., $y_2 \geq s_2$, the program would lead to a water allocation such that point S, or point St in the case a transfer system is in place, will be reached. The allocations in points S and St satisfy the basic need principle. Or you believe in the principle of strict equality. Then, the additional constraint to be included in the maximization program is $y_1 = y_2$, which leads to a solution indicated by point E. In the presence of a transfer system, we must include $a_1 w_1 - \tau = a_2 w_2 + \tau$ where τ is the transfer from F1 to F2, yielding the solution in point Et. The adherence to strict equality can be expressed by a Social Welfare Function (SWF), which states that the well-being of society depends exclusively on the well-being of the most disadvantaged individual.

$$SWF = \min [y_1, y_2] \tag{3.3}$$

Maximizing this SWF leads, of course, to an egalitarian solution, as depicted by points E and Et in Fig. 3.2. This SWF represents Rawls' difference principle, as the only criterion determining overall social welfare is the well-being of the poor. If in the course of an increase in income of the more advantaged, the income of the poor rises as well, social welfare has improved. The social improvement comes through the income increase of the poor, not through the increase of both incomes as in the case of the utilitarian SWF.

Thus far we have identified income or consumption as well-being. The allocation problem becomes more complicated if well-being is not directly expressed by income or consumption but by the utility these observable variables create. Thereby, the level of well-being does not follow consumption in a linear manner generally. Doubling consumption leads to less than doubling of the original satisfaction level.[12] Furthermore, this attitude toward consumption differs individually.

[11] These conditions allow to determine the optimal choice of w_1 and w_2, see appendix A.

[12] The additional value of an additional unit of a consumption good or income decreases with an increasing level of consumption. This property is called diminishing marginal utility. It makes a difference in the valuation of a consumptive item, let us say a wristwatch, whether you already have three watches on your wrist or none.

The level of satisfaction resulting from consumption can be expressed by means of a utility function that transforms consumption quantities into some utilitarian units of well-being or happiness.[13] Let us assume that the utility functions of F1 and F2, respectively, are

$$U_1(y_1) = A(a_1 w_1)^\eta \tag{3.4}$$

$$U_2(y_2) = B(a_2 w_2)^\eta \tag{3.5}$$

The heterogeneity of both farmers is indicated by the two parameters A and B, where we assume that $A < B$. Thus, the more productive farmer F1 derives less utility from consumption than the less productive farmer, F2, does.[14]

Similar to the production possibility frontier, we can construct a utility possibility frontier indicating all possible utility distributions that can be achieved by allocating water to F1 and F2. Recall that $w_2 = \bar{W} - w_1$, which is substituted in Eq. (3.5). Solving Eq. (3.4) for w_1 gives $w_1 = (B_1/A)^{(1/\eta)}$ and inserting this expression for w_1 into Eq. (3.5) yields

$$U_2 = B\left[a_2\left(\bar{W} - (1/a_1)\left(\frac{U_1}{A}\right)^{1/\eta}\right)\right]^\eta \tag{3.6}$$

Equation (3.6) is the algebraic specification of the utility possibility frontier. Note that this frontier is derived under the assumption that transfers of output between both farmers are not possible. In Fig. 3.3, it corresponds to the lower of the two convex curves.

3.3.2.2 Allocation When Transfers Are Possible

If output transfers are assumed to be feasible the derivation of the utility possibility frontier consists of two steps. First, total output is maximized by allocating all water to F1, such that $y_1^{max} = a_1\bar{W}$. Second, the (re-)distribution of utility from F1 and F2 can indirectly be accomplished by an output transfer, τ, such that

$$y_1 = a_1\bar{W} - \tau \text{ with corresponding } U_1(y_1) = A(a_1\bar{W} - \tau)^\eta \tag{3.7}$$

$$y_2 = \tau \qquad\qquad \text{with corresponding } U_2(y_2) = B(\tau)^\eta \tag{3.8}$$

[13]There exists an own branch of literature that discusses the philosophical foundation of utility functions and their application in order to derive policy options. There is also a discussion on how to fix the scale of the utility units. Some important contributions to this literature are enlisted in the bibliography of this chapter, see particularly the suggested textbook by Perman (2011), and Roemer (1996).

[14]Recall that $a_1 > a_2$.

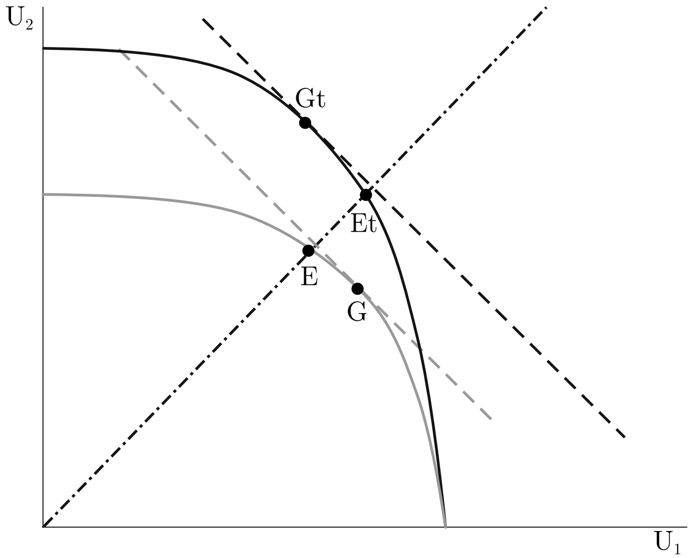


Fig. 3.3 Utility possibility frontiers with and without transfers. *Source* own illustration

Solving Eq. (3.7) for τ and inserting the expression into Eq. (3.8) yields the utility possibility frontier for the case that a transfer system can be established.

$$U_2 = B\left[a_1\bar{W} - \left(\frac{U_1}{A}\right)^{1/\eta}\right]^{\eta} \tag{3.9}$$

This function is also displayed in Fig. 3.3 (the upper convex curve). Both utility frontiers, which assume either the impossibility of output transfers or the availability of a transfer mechanism, present the maximum utility level of F2 given the utility level of F1.

The lower (light gray) possibility curve represents all utility distributions when transfers are not possible, thus F2 can only acquire utility by producing at his site. To do so, water has to be diverted from the more productive farmer F1 to F2. Hence, the choice of the utility distribution among both farmers cannot be separated from the choice of the "size of the cake". However, if an output transfer system can be installed the issues of maximizing agricultural output and of output distribution can be separated.

Figure 3.3 plots as the first proposal the equal distribution of well-being. The 45-degree line through the origin shows this equal distribution. There is one intersection with each of the two utility possibility curves. The first one is E. Here, water has to be allocated such that both farmers are equally well off. Since no transfers are possible the water allocation has to solve both tasks, production efficiency and fair distribution.

The second intersection ist Et where equality of utility is ensured. Here, the issue of distribution can be separated from the water allocation. Since F1 is more productive than F2, all water gos to F1. However, the output is distributed such that equality of well-being is achieved. Since F1 derives less utility from consumption than F2 does, F1 requires more quantities consumed than F2 in order to achieve the same level of utility for both farmers. From $U_1(y_1) = U_2(y_2)$ it follows

$$Ay_1^{\eta} = By_2^{\eta} \Rightarrow y_1 = \left(\frac{B}{A}\right)^{1/\eta} \cdot y_2 \tag{3.10}$$

and since $A < B$ it follows that $y_1 > y_2$.

Of course, to perform this calculation we must be able to compare the utilities of both farmers, i.e., interpersonal comparability must be possible. This requires a certain degree of measurability of happiness or well-being. If this is given we can not only determine the optimal total output but also its distribution according to the utility created for both farmers. This must not necessarily be the equal well-being solution, which we just have discussed. This may also depend on other fairness principles to which the policy maker or the community adheres.

Dating back to Jeremy Bentham, a philosopher of the eighteenth century, the main goal of a free society should be to organize the economy such that it leads to the *"Greatest happiness of the greatest number"* (Bentham 2008, 393). Societal welfare is defined as the sum over all individual levels of well-being. The corresponding SWF is simply the utilitarian specification, as in Eq. (3.2). The income distribution matters only insofar as it contributes to achieving the goal of the greatest happiness of all members.

3.3.2.3 Resulting Water Allocation

Again, we have to distinguish between a system without and with transfer possibilities. In the case with no transfer, the optimal water allocation can be determined by maximizing the sum of utilities, i.e., $U_{sum} = U_1 + U_2$. Graphically, this will be achieved in Fig. 3.3 at point G southeast of E. Here, U_{sum} reaches the highest value possible in an allocation system without transfer.[15] The resulting benefit distribution is not equal. The amount of water is distributed in an inequitable way. F1 receives more than F2. This result depends on two countervailing effects. On one side, F2 should receive more consumption due to its higher marginal utility, on the other side F2 is less productive than F1. Hence, shifting more water to F2 decreases total well-being.[16] It can thus be seen that Bentham's approach does indeed advocate inequality if it only leads to a maximization of aggregated well-being.

[15] The line is defined as $U_2 = U_{sum} - U_1$.

[16] If we further increase the consumer productivity of F2, it could well happen that G would be northwest of E.

The corresponding water allocation point G is based on is calculated from the following program:

$$\max_{w_1,w_2} \left[U_1(a_1 w_1) + U_2(a_2 w_2)\right], \quad \text{s.t.} \quad w_1 + w_2 \leq \bar{W} \tag{3.11}$$

The exact determination of this allocation is not necessary here. It is sufficient to see that Bentham's utilitarian approach subordinates the distribution of benefits to the criterion of aggregated welfare. As a result, the available water is allocated such that point G is realized.

In case of a system with transfer, we get a similar result. The Benthamian solution is determined graphically as in the first case. We move the line of the total benefit up to the right until we touch the outermost point Gt in Fig. 3.3. The algebraic solution is derived as follows: We start by allocating all water to F1 (see program Eq. (3.12)) and maximize total utility with respect to a transfer variable:

$$\max_{\tau} \left[U_1(a_1 \bar{W} - \tau) + U_2(\tau)\right] \tag{3.12}$$

From the optimality condition $U_1'(..) = U_2'(..)$, we can derive

$$A\eta(a_1 \bar{W} - \tau)^{\eta-1} = B\eta\tau^{\eta-1} \Rightarrow (a_1 \bar{W} - \tau) = \left(\frac{A}{B}\right)^{1/(1-\eta)} \cdot \tau \tag{3.13}$$

Since $A < B$, it follows from Eq. (3.13) that $y_1 = \left(a_1 \bar{W} - \tau\right) < \tau = y_2$. Contrary to the allocation under strict equality, F1 gets less income than F2, simply because farmer F1 is less effective in terms of generating well-being through his low valuing of consumption. Thus, point Gt is northwest of point Et. If one compares both solutions, the egalitarian and the utilitarian in Fig. 3.3, one can observe that in point Gt total utility is maximized, whereas in Et total utility is below its maximum value. The utilitarian criterion is achieved at the expense of that member of society who derives less utility from consumption.[17] If one adheres to the concept of moral arbitrariness this approach is not convincing. If the intensity of consumption pleasure is innate, then it is collectively owned by the society. Hence, the marginal utility of income attached by nature to the members of the society does not imply an entitlement to more consumption. As such, equating marginal utilities so as to maximize the SFW is morally not convincing in the view of supporters of an egalitarian standpoint.

[17]In the figure, we can make another interesting observation if we compare point G with point Gt: F1 is worse off in the case of the transfer system than in an allocation system without transfer. Perhaps this is also one reason why there is sometimes resistance to institutional innovations.

From a practical viewpoint, it is important to note that distributional issues can be separated from efficiency problems only as far as the society is equipped with the necessary institutional capacities to solve distributional requirements with instruments other than the allocation of inputs and products. In our example, irrespective of the distributional principles the water allocation was chosen by directing the whole water available to farmer F1. Many integrated water management models start from this separability assumption focusing solely on the allocation of water and other inputs while leaving distributional issues to social and distribution policy. If water is used for different purposes, e.g., as input for agricultural products and as a consumption good for households the separation of distributional and allocational issues gets more complicated. Needless to say that the weighing of distributive and allocative issues is a major challenge in the specific institutional environment, and that solutions must be tailored specific to the context, too.

3.3.3 Allocation with and without Water Scarcity

So far it was assumed that no abstraction costs incur to both farmers. Without costs, the use of water could be infinite if it was not constrained by the upper bound $\bar{W}$. If such a constraint cannot be implemented, water will be overused. However, if abstraction costs are present water overuse can be prevented or at least lessened. To analyze the relation among water utilization, abstraction costs, and sustainability thresholds, we include abstraction costs in our two-farmer model. Furthermore, it is assumed that production of the agricultural product can be captured by the production functions depicted in Eqs. (3.14) and (3.15).

$$y_1 = f_1(w_1) = a_1(w_1)^\theta \quad (3.14)$$

$$y_2 = f_2(w_2) = a_2(w_2)^\theta \quad (3.15)$$

Instead of assuming that one farmer is always more productive than the other, we now introduce production functions with decreasing marginal products. For simplicity both farmers differ only with respect to a_i, where $a_1 > a_2$. Costs of water abstraction denominated in agricultural products are determined by the cost functions in Eq. (3.16), where F denotes the fixed cost and c the marginal cost of abstracting water.

$$C(w_i) = F + cw_i, \quad i = \{1, 2\} \quad (3.16)$$

Disregarding distributional issues, the goal is to maximize the aggregated output of agricultural products, i.e.,

$$\max_{w_1, w_2} [a_1(w_1)^\theta + a_2(w_2)^\theta - c(w_1 + w_2) - 2F] \quad (3.17)$$

leading to the optimality conditions

$$\theta a_i (w_i)^{\theta-1} = c \quad \rightarrow w_i^* = \left(\frac{\theta a_i}{c}\right)^{\frac{1}{1-\theta}} \quad (3.18)$$

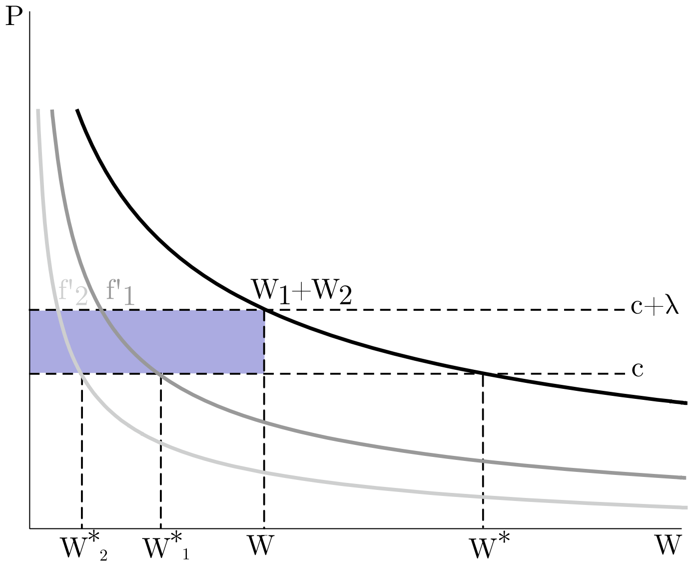


Fig. 3.4 Optimal allocation with and without water scarcity. *Source* own illustration

This allocation can also be achieved in a market economy, where farmers maximize their profits according to

$$\max_{w_i}[a_i(w_i)^\theta - qw_i] \quad \Rightarrow w_i^* = \left(\frac{\theta a_i}{q}\right)^{\frac{1}{1-\theta}}, \quad i = \{1, 2\} \tag{3.19}$$

and a water treatment plant sells water under a price regulation scheme. The price scheme for both farmers is a two-part tariff consisting of a volumetric component q and an access fee M. The price regulation authority sets $q = c$ and $M = F/2$. From Eq. (3.19), it is obvious that the market equilibrium together with the water price regulation leads to the optimal allocation. Total amount of water used, $w_1^* + w_2^* = W^*$, is determined in Fig. 3.4, where the total demand curve intersects with the constant marginal cost line. The intersections of the respective marginal product curves of each farmer with the marginal cost line yield the optimal allocation.

Since it is costly to abstract water, its use is finite. It remains to examine whether the optimal allocation, W^*, lies above the sustainable boundary, $\bar{W}$. In Fig. 3.4, two scenarios are depicted. The first scenario assumes that aggregated water use is less than the sustainability boundary leading to a water price of $q = c$. The second scenario assumes that abstracting water is sufficiently cheap such that the aggregated water use of both farmers exceeds the sustainable boundary. In this case, the presence of abstraction costs does not protect the hydrological cycle sufficiently. Hence, a hydrological constraint must be introduced leading to the optimization program

$$\max_{w_1}[a_1(w_1)^\theta + a_2(w_2)^\theta - c(w_1 + w_2) - F] \quad \text{s.t.}\, w_1 + w_2 \leq \bar{W} \tag{3.20}$$

This leads to the optimality conditions

$$\theta a_i(w_i)^{\theta-1} - c = \lambda \quad \rightarrow \; w_i^* \tag{3.21}$$

where $\lambda > 0$ is the respective Lagrangian of the constraint. This case is depicted in Fig. 3.4, where total water demand is constrained by the black line of $\bar{W}$. As a result, the marginal products exceed marginal cost by the difference which is called scarcity rent.

Consider the respective market solution. The regulatory authority must increase the volumetric component of the water tariff to $q = c + \lambda$ in order to push back total demand such that it does not exceed the sustainability threshold. Since the sustainable water supply is smaller than the amount of water that the economy would abstract, water is scarce and, as a consequence, the water price exceeds marginal costs, which yields a scarcity rent for the water supplier. Figure 3.4 shows the income of the water supplier due to the scarcity rent, which is represented by the shaded rectangular. There is a discussion about the distribution of the scarcity rent.[18] This rent income can be used to lower the access fee. But what should we do with residual (if there is one)? Some people suggest that this rent income should be taxed away and redistributed to the users. Or they take the existence of scarcity rents as an argument to claim that the water infrastructure must be owned publicly.

Box 3.3 What are the motives of the Dog in the Manger?

Alan Garcia, former president of Peru, complained that the country is poor despite its abundance of natural resources. According to the Human Development Index (HDI), Peru ranges at position 77 out of 187 countries. The HDI is an aggregated measure for the living conditions of a country with respect to life expectancy, access to knowledge and a decent standard of living. Alan Garcia identified political and cultural traits as the very source of this deplorable economic and social situation, which he referred to as the dog-in-the-manger-syndrome. The dog in the manger is a figure from a fable of the

[18] Scarcity rents can be skimmed off by suitable tariff systems, such as increasing block tariffs, see Schwerhoff et al. (2019), more literature references will be given in Chap. 4.

ancient storyteller Aesop. The beast lies in the manger full of straw and prevents other animals to take the straw from the manger. The very motive is a pure grudge, as the straw is useless to the dog. Translated into the Peruvian political environment, the dog can be seen as an analogy for poor peasants dwelling on small plots in the countryside without any access to agricultural technology and, at the same time, lacking the financial means to invest. In addition, property rights are informal thus making investments insecure. Whenever modern politicians tried to develop the traditional agriculture by consolidating the plots into plains accessible to agricultural technology, local uproar emanated, often well organized by local politicians. According to Alan Garcia, people were caught in a vicious circle of poverty and an ideological superstructure that left them in a habitual state of hostility toward modern development.

There were some attempts to modernize the agricultural sector by promoting privatization and land consolidation with the help of law amendments and even new laws. In 2009, the Peruvian parliament passed a water bill that put much emphasis on the efficient use of water. The water irrigation system of traditional agriculture was highly inefficient compared to modern technologies based on, e.g., drip irrigation. Therefore, a development framework plan was established up to attract large-scale agribusiness enterprises able to invest in efficiency-enhancing technologies. But there has been political resistance against this development agenda, which raises the question whether this opposition can only be interpreted as driven by the grudge of the dog in the manger. We can shed some light in this discussion with the help of our farmer model, thereby discussing the interrelation between water efficiency and income distribution.

Let us assume that there are n equally sized lots of land $i = \{1, 2, \ldots, n\}$ that are cropped in a traditional manner. The water productivity a_t is equal across all lots (the subscript t refers to the traditional agricultural production). In addition, labor required per lot is $l_{ti} = (1/b_t)y_{ti}$, where b_t is labor productivity which is also equal across all lots. Each peasant gets the same amount of water $w_{t1} = w_{t2} \cdots = w_{tn}$, where $\sum w_{ti} = \bar{W}$. Thus $w_{ti} = \bar{W}/n$, $\forall i$. Recall that $y_{ti} = a_t w_{ti}$. Then, total labor required for the total agricultural product is

$$L_t = \sum l_{ti} = (1/b_t)\sum y_{ti} = \frac{a_t}{b_t}\bar{W} \tag{3.22}$$

where total output amounts to

$$Y_t = a_t \bar{W} \tag{3.23}$$

We assume that in the outset there is no unemployment, i.e., the required amount of labor L_t equals the number of peasants or land laborers dwelling on the site. Production and income per peasant (laborer) is

$$\frac{Y_t}{L_t} = \frac{a_t \bar{W}}{L_t} = b_t \tag{3.24}$$

where the right-hand side follows from Eqs. (3.22) and (3.23), indicating that peasants earn their productivity.

A big investment project is proposed covering all n sites. Water productivity will increase to $a_m > a_t$ for all sites (The subscript m denotes modern agricultural technologies.). Consequently, output per site will increase to $y_{mi} = a_m w_{mi}$. Since each lot is equally productive, the water is allocated in equal portions, i.e., $w_{m1} = w_{m2} = \cdots = w_{mn}$, resulting in $y_m = a_2 \bar{W}/n$. Total agricultural output is $Y_m = a_m \bar{W} > Y_t = a_t \bar{W}$. The question remains whether there is enough labor available to produce Y_m. Assume that the new technology makes labor also more productive due to the capital intensive cultivation of the the land, such that

$$L_m = (1/b_m)Y_m = (a_m/b_m)\bar{W} \tag{3.25}$$

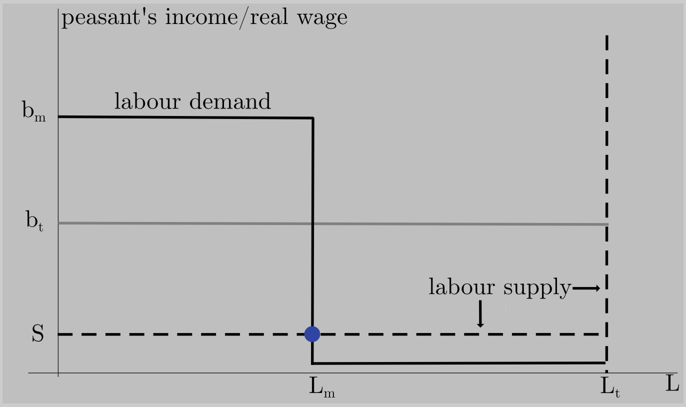
where $b_m > b_t$. Both coefficients of the modern technology are higher, leaving the question whether the new technology is labor saving. Observations in many countries document that in the course of modernizing of the agricultural sector migration into cities can be observed, which indicates that agricultural technical progress is labor saving (Bhandari and Ghimire 2016). Therefore, we assume that $L_m < L_t$.

It remains to analyze the total effect of technological progress in terms of poverty alleviation. Does the real income of peasants increase or decrease through modernizing agriculture?

Let us proceed with the analysis by introducing the investors: Profits of the investors are given by

$$\Pi = [Y_m - wL] = [b_m L - wL] \tag{3.26}$$

where w is the real wage. The labor demand function of the agro-business firm can be derived by maximizing its profits with respect to L. Since the model is linear, the demand function is a step function as depicted in the following figure below. If $b_m \geq w$, i.e., if labor productivity is not less than the real wage w, labor demand expands to $L_m = (a_m/b_m)\bar{W}$, which is the labor requirement to produce total output Y_m, given a labor productivity of b_m and water availability $\bar{W}$. If the real wage exceeds b_m, labor demand vanishes because the innovation is not profitable. The labor supply function is represented by the kinked curve in the following figure.



If real wage is higher than s, which can be interpreted as the alternative real income of the peasants leaving the countryside for employment opportunities in the urban area, all peasants L_t want to stay employed in the agricultural sector. If we let both curves intersect, the equilibrium will exactly be equal to s, implying that the efficiency-enhancing technology leads to a drop in real income for the peasants from b_t to s. Then, indeed, efficiency and distributional goals contradict.

But there are some other countervailing effects that might ease the situation of land laborers. The increased production of agricultural output from Y_t to Y_m may lead to a decrease in the price for these products, thus increasing the urban real wage s. If the price decline is such that $s > b_t$, the poverty of the peasants is reduced. The occurrence of profits for agro-business firms leads to an increase for real wage and hence to a welfare increase for the least advantaged. But even if s falls short of b_t it would be conceivable that poverty is reduced. If the poor peasants became shareholders of the firm their welfare would increase above b_t. But this requires well-defined property rights allowing peasants to sell their sites in exchange for those shares. If property rights are not well defined and well protected by sustainable institutions, it may also happen that peasants are simply expropriated (the blue intersection point in the figure). The dog in the manger knows why he defies modernization.

In summary, the political discussion of modernizing traditional production structures in the agriculture of developing countries cannot be based solely on efficiency considerations. IWRM shall not neglect the redistributive effects of efficiency-enhancing measures.

Sources: Cohen and Weitzman (1975), Boelens and Vos (2012), Bhandari and Ghimire (2016)

3.4 Eco-Hydrology and the Management of Water as a Public Good

The water cycle provides not only water as a consumption good to consumers or as a productive input to firms but it is also indispensable for environmental services like the production of vapor that stabilizes the microclimate. Or take the ecological system of water and forests as an example. Forests produce ecosystem services like flood control, water filtration, or provision of habitats for various species, whereas in turn forests need land and water. The water cycle is a fundamental part of the whole eco-system. This ecological system consists of various natural cycles which are interlinked.[19] In addition to the hydrological cycle, we have the carbon-oxygen cycle that consists of a photosynthesis part, in which carbon dioxide is converted into oxygen, and the decomposition part, where organic molecules are separated into carbon dioxide and water. There is also a nitrogen cycle that is crucial for the growth and decay cycle of plants. On an ecological level, these cycles form the nutrient cycles where all the different living systems take place. From a more holistic IWRM viewpoint, these cycles are affected by the shaping of the local hydrological cycle and the way land use is organized. Some scientists, therefore, claim that all these interactions have to be included in a comprehensive policy approach.

A management system following this holistic integrated approach is called eco-hydrology, a term introduced by Rodriguez-Iturbe (2000). It comprises the whole climate-soil-vegetation system. Thus, landscape planning and the management of water resources have to be closely linked. In this sense, IWRM goes far beyond the efficient provision of water for the private consumption of households or firms. The economic management of a river has not only to organize the abstraction of water but also to secure the water provision for the local ecosystem services. These services are sustained by assuring the viability of the various ecological cycles mentioned above. Also, these services include also more visible services, e.g., the provision of recreation in the form of, e.g., fishing, hiking, camping, or the mere presence of nature as an acoustical and visual environment that is part of the cultural landscape.

Under this perspective, we note that IWRM is much more than only managing some water flows for private use. From an economic perspective it turns out that water management, which takes these eco-services into account, considers water not only as a private but also as a public good. For instance, the stabilization of the microclimate by the water cycle is an ecosystem service that affects all inhabitants of a watershed (and beyond). In the following, we include this public good property of water into our IWRM approach. To do so we utilize our hydro-economic model introduced in Sect. 2.3.

Assume that there are two options for water management. Either water is abstracted for private purposes or water is retained for the ecosystem. To keep the

[19] An instructive description of the main ecological interdependencies can be found in O'Callaghan (1996).

analysis simple, we assume that the value of ecosystem services can be captured by a utility function $U_i(E), i = \{1, 2, \ldots, n\}$ that depends on these services. i is the index of individual i and n is the size of the population living in the watershed. E are the ecosystem services, i.e., the parameter contains the whole interdependency between the water cycle, the vegetation, and the geological structure of the watershed.[20] E depends not only on policy instruments of the IWRM but also on other economic variables that influence the ecosystem shaping the landscape (e.g., soil sealing, agricultural vegetation, …). In the following, we focus solely on the issue of allocating water to private purposes and to public services. We identify the green water, i.e., the evapotranspiration of the vegetation, as a public good, because all inhabitants are affected similarly by the vapor of green water. Thus, the utility function for ecosystem services depends on the evapotranspiration, ET, as depicted in Eq. (3.27).

$$U_i = U_i(ET) = U_i(\gamma_1 S), \quad i = \{1, 2, \ldots, n\}, \text{ and } U'' < 0 \tag{3.27}$$

The benefit of private water consumption is represented by a benefit function, B_i,[21] that represents profits or benefit from water consumption

$$B_i = B_i(w_i), \quad i = \{1, 2, \ldots, n\} \tag{3.28}$$

The hydrology can be captured by our linear dynamic mass equation

$$\frac{dS(t)}{dt} = R + P - \gamma_1 S - \gamma_2 S - \sum_{i=1}^{n} w_i \tag{3.29}$$

To keep the optimization procedure simple, we confine ourselves to a steady-state analysis, i.e., we assume that the local hydrological cycle is an equilibrium where $dS(t)/dt = 0$. Solving for S, we get

$$S = \frac{R + P - \sum_{i=1}^{n} w_i}{\gamma_1 + \gamma_2} \tag{3.30}$$

The equation shows that S depends on the water allocation to consumers, w_i with $i = \{1, 2, \ldots, n\}$. To solve the IWRM problem, the definition of the social welfare function is required and presented in Eq. (3.31).

$$SWF = \sum_{i=1}^{n} [B_i(w_i) + U_i(\gamma_1 S)] \tag{3.31}$$

[20] E can also be conceived as a multidimensional vector containing an array of ecosystem services.
[21] Notice that the costs of water abstraction are included in the benefit function so as to save on symbols.

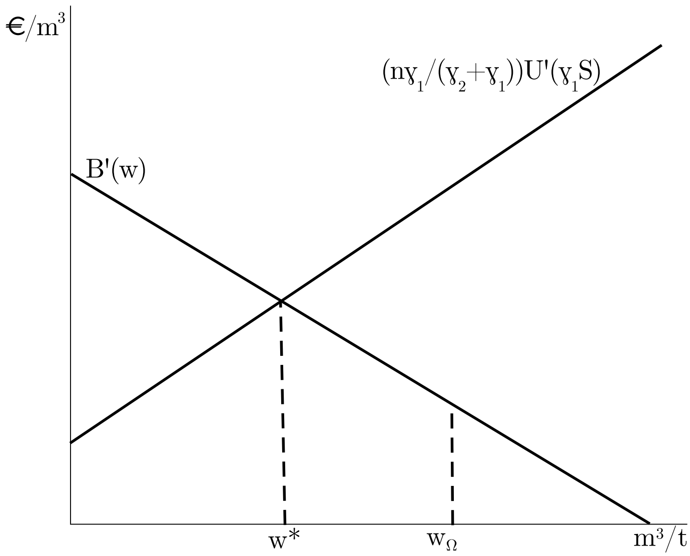


Fig. 3.5 Optimal allocation of water as a public good. *Source* own illustration

The management task is to maximize SWF with respect to w_i taking into account that private consumption reduces water available for the ecosystem (see Eq. (3.30)). It is a straightforward exercise to derive the optimality conditions for each individual:

$$B_i'(w_i) = \frac{\gamma_1 \sum_{i=1}^{n} U_i'(\gamma_1 S)}{\gamma_1 + \gamma_2}, \quad w_i, i = \{1, 2, \ldots, n\} \tag{3.32}$$

where S is represented by Eq. (3.30). This set of equations reflects the Samuelson-rule that specifies how the optimal amount of a public good should be determined. The main point is that private marginal benefit should be equal to the sum of marginal benefits over all inhabitants, as one liter of private water consumption is associated with costs that stem from the marginal loss of ecosystem services for all inhabitants. Hence, both values must be optimally balanced. If we assume that all inhabitants are identical with respect to their valuation functions we can condense the set of equations into one figure. Notice that in this case Eq. (3.32) reduces to

$$B'(w) = \frac{\gamma_1 n U'(\gamma_1 S)}{\gamma_1 + \gamma_2} \tag{3.33}$$

where S is defined in Eq. (3.30) (Fig. 3.5).

The optimal water consumption is where both marginal valuation curves intersect. Inserting w^* into Eq. (3.30) yields the optimal stream of green water, ET, the optimal evapotranspiration, which interacts with all the other natural cycle mentioned above. As a result, a micro-climate is established that sustains environmental services leading to the well-being of the local human population. Depending on cultural traits and also on the population size, this intersection of marginal valuation curves can change over time, i.e., w^* can change and move to the right, for example. There is a certain viable range of $\{ET, w\}$-combinations the water management can choose. Landscapes can be shaped in many various ways depending on cultural traditions and, of course, the biological needs of the society. However, there are boundaries. Beyond these boundaries, irreversible changes in the ecology will take place. As a result, by transgressing these ecological tipping points, the regional ecological system might switch into a state hostile to human life, like a desert for instance. This boundary is depicted in Fig. 3.5 as w_Ω. If water abstraction is higher than this tipping point, evapotranspiration decreases to an extent that triggers a complete change of the microclimate. The ecological system turns into a semiarid or arid zone with all the detrimental consequences for society.[22]

3.5 Water Allocation and the Human Right to Water

3.5.1 Millennium Goal 7 and Sustainable Development Goal 6: Water

According to the United Nations Children's Fund (UNICEF) and the World Health Organization (WHO 2019), more than two billion people in the world did not have access to safe drinking water, and another two billion people lacked access to basic sanitation in 2019. In 2010, the UN General Assembly declared the access to water, be it as drinking water or a medium for sanitation and hygiene, as a human right. Together with six additional goals, which range from halving the proportion of people living in extreme poverty to reducing the under-five mortality rate by two-thirds between 1990 and 2015, the Millennium Goal 7 called to[23]

> Halve, by 2015, the proportion of the population without sustainable access to safe drinking water and basic sanitation.

In 2015, these Millennium Development Goals were replaced by the Sustainable Development Goals consisting of 17 goals ranging from poverty and hunger eradication to strategies aiming at building peaceful and inclusive institutions. Goal 6 refers to clean water and sanitation, according to which universal access to safe and affordable drinking water should be ensured by 2030—quite an ambitious goal in the face of climate change leading to water scarcity, specifically in those areas of the world with the poorest inhabitants.

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[22] Of course, human can adapt to various climate systems. For instance, nomadic tribes have adapted to arid or desert like conditions. But this implies a very low population density and also a low living standard. We do not expand our policy discussion to include the choice of the population size.

[23] See http://www.un.org/millenniumgoals; specifically, one finds annual summaries that report on the progress made in the previous years.

Table 3.3 Water requirements for survival

Type of need	Quantity	Comments
Survival (drinking and food)	2.5–3 lpd	Depends on climate and individual physiology
Basic hygiene practices	2–6 lpd	Depends on social and cultural norms
Basic cooking needs	3–6 lpd	Depends on food type, social and cultural norms
Total	7.5–15 lpd	lpd: liters per day (per person)

Source Reed et al. (2011)

From the perspective of IWRM, the achievement of Goal 6 requires to tackle the access problem and, at the same time, to protect the catchment areas against an overutilization of water. Water scarcity translates into high water prices, which in turn brings about an optimal allocation of water use. This approach will only result in an optimal equilibrium if all market participants can afford the amount of water to cover their basic needs for a secure conduct of life. There is a broad literature on basic water needs, the lower range of which would be in the range of 15 liters per day and capita (lpd) (Reed et al. 2011). This lifeline is subdivided into various need types as displayed in Table 3.3.

In addition to water, households need a certain daily endowment of calories and nutrition as well. Therefore, poor households need a minimum income to survive in order to finance expenses that allow them to buy the subsistence basket of basic goods, containing water, food (nutrition), housing, and shelter. But often poor households do not earn enough money to secure this lifeline. It is rather obvious that price increases can affect these households in a very detrimental way. We, therefore, cannot trust in unregulated markets as institutions that secure efficiency. Classical welfare theory assumes that a market participant can make a living based on her income. Hence, the demand for goods is solely the expression of preferences following from taste and predispositions. In the case of poor households, we cannot assume that their demand for basic goods is the result of optimizing their demand according to these kinds of preferences. Often, the demand for goods is nothing else than the result of poverty management below the lifeline. The composition of food purchased is optimized with respect to calorie content. Hence, in this case, revealed preferences are based on survival strategies and not on taste.

This view coincides with social-psychological theories of need management. The famous Maslowian need hierarchy describes the stratification of human needs whose satisfaction is expressed in corresponding actions be it the demand for water and nutrition or supply of labor. [24] At the bottom is the satisfaction of physiological needs, followed by other needs such as security and social recognition. In our case,

[24] Abraham Maslow developed his concept of a need hierarchy in the 1940s, and there are a few attempts to utilize his insights for a microeconomic theory of households, see Georgescu-Roegen (1954) and Seeley (1992).

the satisfaction of physiological needs is essential. Needs at this level are undoubtedly legitimized by human rights. If markets do not guarantee their satisfaction the welfare theoretical criterion of efficiency or social optimality is irrelevant.

Take as an example the Pareto criterion economists often refer to: A reallocation of goods is said to be socially preferable to a given distribution of goods if it increases the welfare of one or more members of society without harming the well-being of others. This approach might be suitable for a middle-class society but not for an economy divided into poor people and members endowed with sufficient financial means to not only satisfy basis needs but also to buy those products and services which allow individual self-fulfillment at a higher level of the hierarchy of needs. For instance, increasing the welfare of the latter group by allowing good exchange between both classes does not increase social welfare. Here we want to refer to the Rawlsian social welfare function introduced in Sect. 3.4, where social welfare depends solely on the well-being of the poorest.

IWRM has to take into account this distinction between taste-driven consumer choices and revealed purchase behavior resulting from survival strategies. In this sense, poor households have to be included in the IWRM models that deliver allocation mechanisms that guarantee the subsistence level of drinking water, sanitation, and other basic goods and services in line with the Sustainable Development Goals.

3.5.2 Water Management for the Very Poor

In the following, we will deal with a water allocation model under the assumption that there are two categories of needs in the Maslowian hierarchy of households: physical needs and more advanced wants satisfying cultural needs. Both of these needs can be satisfied with the help of consumption goods. To keep the model simple, we restrict it to two fundamental inputs: water and nutrition. Of course, nutrition itself consists of various food products which we do not further subdivide. Let us begin with a household that has sufficient means to satisfy the first category and is also able to serve the satisfaction of cultural needs to a certain extent. The following figure identifies this household with budget line II and the respective indifference curve where the utility of the household is maximized (point O1). The corresponding budget line constraint is (see Fig. 3.6):

$$p_w w + p_n n \leq y \tag{3.34}$$

where w is water consumption and n nutrition.

Contrary to the standard household model, we distinguish between consumption which satisfies the first layer of needs $\{w_s, n_s\}$ and additional consumption which serves the cultural needs, i.e.,

$$w = (w_s + w_a) \quad \text{and} \quad n = (n_s + n_a) \tag{3.35}$$

where w_a and n_a is excess demand beyond the subsistence levels $\{w_s, n_s\}$, i.e., water and nutrition intake to assure the satisfaction of physical needs. This excess demand

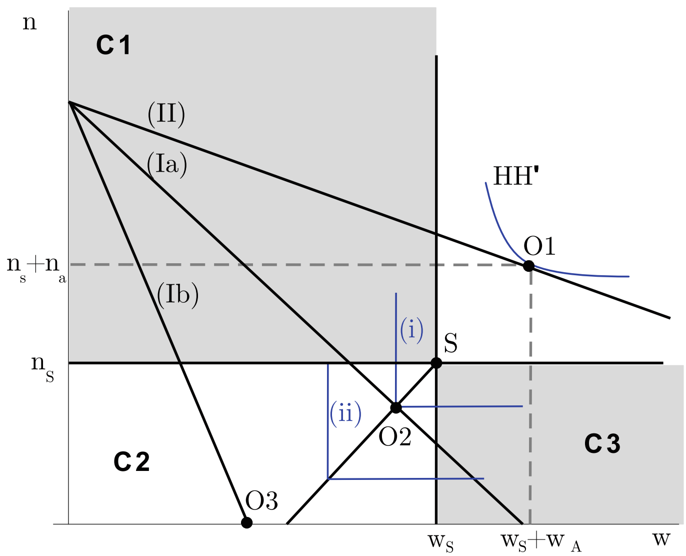


Fig. 3.6 Risk management of the poor. *Source* own illustration

can be water consumption for various other purposes than drinking, personal hygiene, cooking and cleaning, for instance, bath taking, cultivating a garden with flowers, etc. The same applies to nutrition. After having consumed the necessary calorie intake to survive, the preparation of food is a cultural need, too.

The optimal point $O1$ can be derived with the help of the standard maximization approach for households.

$$\max_{w_a, n_a} U(w_a, n_a), \quad \text{s.t.} \quad p_w w_a + p_n n_a \leq y - p_w w_s - p_n n_s \tag{3.36}$$

Figure 3.6 also depicts two further scenarios, which refer to the case that the households income does not suffice to cover the subsistence point S, i.e., the point where both goods can be purchased in an amount that guarantees the full coverage of the physical needs.[25] These are the areas C1, C2, and C3.

Here, a survival strategy is required. Poor households try to maximize their life expectancy or survival probability with the help of a household production function. This production function transforms the inputs water and basic goods into a certain health state, which can be expressed in, say, survival probability units. Similar to ordinary indifference curves derived from standard utility functions, iso-health lines can be defined with increasing survival probability to the northeast and decreasing

[25]Note that we can also assume that the full satisfaction of physical needs is an isoquant. This would be the case when both goods are substitutable in a certain range. In the following, we keep the figure simple by identifying this level solely with a point.

life expectancy to the southwest. There are various possible shapes to draw these iso-health lines. We assume for simplicity that these lines follow from a linear-limitational interrelation. Let the corresponding survival function be

$$S(w_s, n_s) = min[aw_s, bn_s + g] \tag{3.37}$$

where $a, b, g > 0$ are parameters determining the slope and the position of the expansion path connecting all corner points for various iso-health lines. As water is more important than nutrition to survive for a certain time we have assumed that the expansion path intersects the abscissa at a positive w_s-value. The expansion path, i.e., the line connecting all corner points of iso-health lines shows how households react optimally if income decreases and/or relative prices change. Assume, for instance, that the budget line (II) rotates to the southwest because the price of water has increased (line Ia). Households try to maximize their health by maximizing $S(w_s, n_s)$ subject to their budget line Ia, i.e., $y = p_w w_s + p_n n_s$. The optimal need management then leads to point O2. As the water price increases more and more, the optimal point shifts to the southwest finally reaching the horizontal axis for budget line Ib. From there, the optimum point moves along the horizontal axis and towards the origin, i.e., the household tries to use all its income for buying water (see point O3).

The specification of the survival function needs more empirical investigation. Note, however, that the main point of this model does not rest on the precise structure of iso-health lines but on the viewpoint that the extremely poor can only choose their water and basic food southwest of the point S. Whatever quantities w_s and n_s are chosen by poor households, they cannot be interpreted as instruments used to maximize well-being in the sense of an optimal management of tastes and preferences that are relevant for the upper layers of the Maslowian hierarchy. They simply represent rational survival strategies.

3.5.3 A Water Market with Extremely Poor Households

Now we are able to construct the water demand curve of poor households beginning from a price that leaves the consumption bundle in the area of needs and preferences beyond pure survival. If p_w increases, finally demand will reach w_s. From there on the water consumption decreases with increasing price ever further exhibiting an optimal survival strategy (expansion path $S - O2 - O3$). The demand curve consisting of the two parts is drawn in Fig. 3.7 from left to right. A demand curve of a second household with a higher income is drawn from right to left. This household receives such a high income that he does not get into the critical survival zone.

Supply costs are not included so as to keep the figure clearly laid out. We simply assume that there is a certain amount of water (R) given. Hence, the equilibrium water price p_M equilibrating demand supply (point M) is a scarcity rent. In this equilibrium point, demand and supply are equal. The water available is allocated to both households according to their marginal willingness to pay (demand curves).

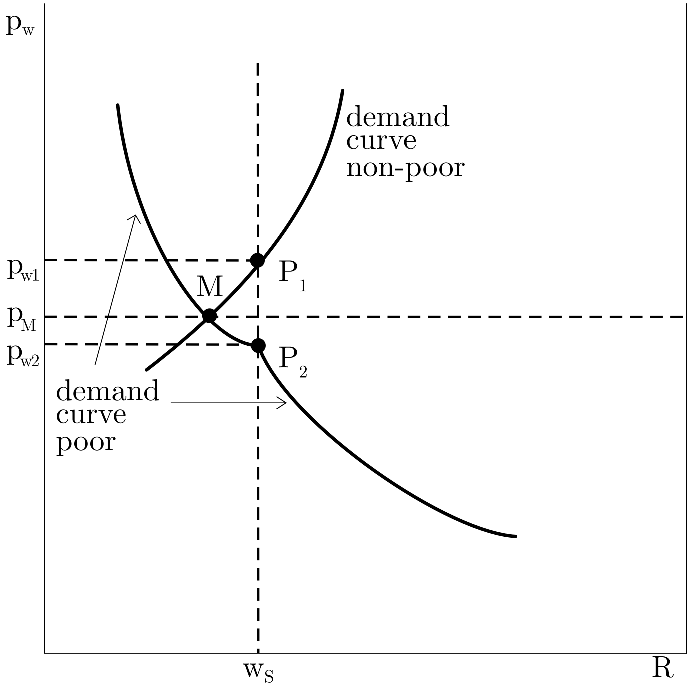


Fig. 3.7 A water market with extremely poor households. *Source* own illustration

However, the equilibrium does not result in a social optimum.[26] Obviously, the poor household operates in the survival zone and, hence, the Sustainable Development Goals are not satisfied. The market mechanism transfers too much burden to the poor due to the scarcity of water.

There are various options to secure the lifeline of the poor. One option is to introduce price discrimination. In Fig. 3.7, the high-income water demand is charged with a higher price, p_{w1}, and the poor households pay a lower price, p_{w2}. This type of price discrimination can be achieved, for example, by an increasing block tariff structure which will be presented and analyzed in depth in Chap. 4. Theoretically, another option is to subsidize the poor directly. In this case, price discrimination can be abandoned and a unique market price can prevail since poor households receive

[26]We know from the first welfare theorem that a market equilibrium is socially optimal or Pareto-efficient. In the standard microeconomic model, the equilibrium of a market system guarantees that the marginal rates of substitutions of all market participants are equalized. However, if some of these participants are very poor, it follows that the marginal rate of substitution with respect to the health production function would be equal to the marginal rate of substitution of households who could afford a consumption bundle beyond the subsistence level. If we adhere to the Sustainable Development Goals, we can not infer from this efficiency condition that the market allocation is socially optimal simply by referring to the first welfare theorem.

a lump sum transfer that lifts their income such that they can afford the subsistence point S.

3.6 Water Recycling

3.6.1 Nomenclature of Water Recycling

Water scarcity, major driver for water reuse, has been called the challenge of the twenty-first century (Miller 2006). Before we look more closely on the different types of water reuse, let us define the terminology:

- Wastewater reclamation is the treatment of wastewater to secure its reuse.
- Water reuse is the use of that treated water for beneficial purposes like irrigation in agriculture or flushing of toilets in households.
- Water recycling is water reuse where the treated water flows back into the same unit that has released the wastewater. The reused water can be utilized for the same purpose or for a purpose requiring a lower quality of water input. The former means, e.g., the reuse of treated water for drinking water (see Box 3.4) the latter refers for instance to the use of gray water for flushing toilets in households.
- Gray water is wastewater from household activities, like laundry washing or bathing.
- In contrast, black water is water from flushing toilets or from kitchen sinks with a high load of pathogens and organic content.
- Direct reuse implies that treated water is carried through pipes to its following purpose, whereas indirect reuse joins nature up in-between.
- Treated water is returned into the surface or groundwater from where it is withdrawn again.
- From a technical and economic viewpoint, a distinction between centralized and decentralized water reclamation is crucial. The former refers to reuse devices on the household or firm level, whereas the latter refers to infrastructural networks connecting the various users.

Figure 3.8 depicts the various flows of wastewater release, reclamation modes and reuse for the residential and industrial sector. It shows that water reuse is also part of the water cycle.

Box 3.4 Water recycling in Singapore

Singapore is a highly urbanized city-state. The city is characterized by a very high population density. Hence, the water demand for this state can only be met by an area several times its size, if only conventional water resources are used. For becoming more independent from water imports from Malaysia, Singapore enforces the development of innovative water technologies, e.g., desalination, but also recycling of water. Reclaimed water is an important water source in Singapore which can be produced in an innovative way. Here, the wastewater is treated using membrane filtration and reverse osmosis. The generated freshwater, after all purification steps, meets the standards of potable water. This procedure of water supply management became known under the brand name NEWater. The predominant share of the reclaimed water is used in the industry sector mainly for non-potable applications. But about 1 percent of Singapore's potable water requirement is covered with reclaimed water from the NEWater project, too.

Source: Tortajada (2012)

IWRM requires to consider all available options for water recycling. The various modes of reuse must be evaluated with respect to environmental repercussions and with respect to economic criteria. Water recycling is indeed one measure to enhance the efficiency of water use. On the other side, water reuse can also lead to environmental damages. For instance, the recycling of irrigation water into irrigation systems might lead to the salinization of the soil, thereby decreasing its fertility drastically. We will get to these kinds of quality problems in Sect. 3.10. The scheme in Fig. 3.8 depicts water reuse options in a rough manner. The civil engineering literature (such as Asano 1998) enlists a variety of reuse categories:

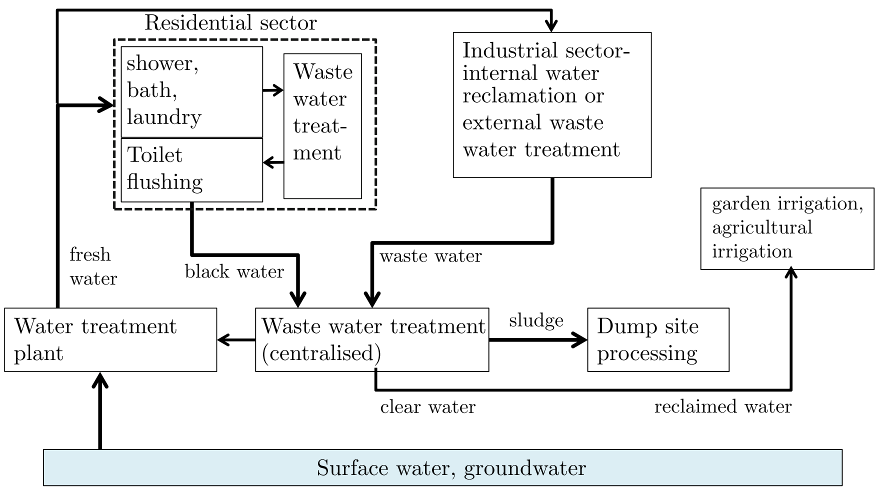


Fig. 3.8 Water reuse. *Source* own illustration

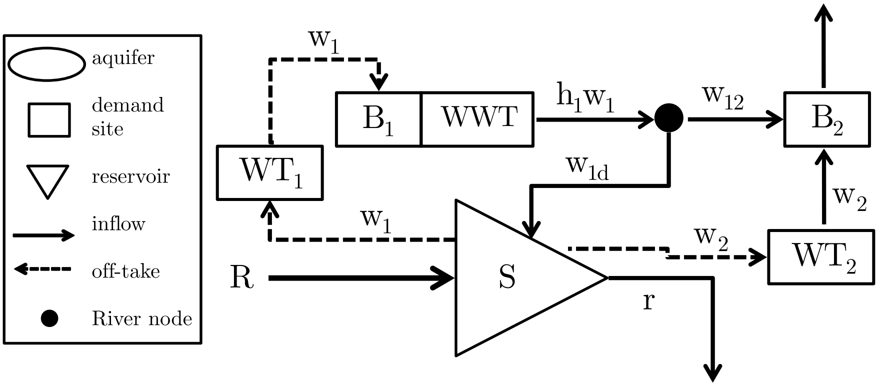


Fig. 3.9 A simple water recycling model. *Source* own illustration

- Agricultural irrigation is the largest use of reclaimed water in arid and semiarid regions. For instance, Israel currently reuses more than 65 percent of sewage water for irrigation (Friedler 2001).
- Landscape irrigation plays an important role in industrialized countries. It refers to the irrigation of parks and other areas of recreational purposes.
- Groundwater recharge belongs to the indirect mode of recycling. This kind of reuse is of high importance not only to increase the water supply but also to stabilize aquifers specifically to avoid the intrusion of salt water.
- Non-potable urban reuses refer to water for fire protection, air conditioning and toilet flushing (decentralized reuse).
- Last but not least wastewater can be purified such that it has a quality level of potable water. This is literally water recycling. Often this kind of reuse is connected with the strong resistance of people. Singapore is one exception where wastewater is purified and recharged into the freshwater distribution system (see Box 3.4).

3.6.2 Optimal Recycling

Water reuse, to whatever purpose, is part of IWRM. In its simplest form, water reuse follows from an optimal allocation procedure which takes into account return flows. Figure 3.9 shows a simple scenario for two users.

User 1 uses water w_1 from a river or lake and returns a portion $h_1 w_1$. We assume that the quality of the returned water is such that the body of water, e.g., a lake or groundwater, maintains its environmental quality. The costs of necessary purification treatments (WWT) are included in the benefit function of user 1. Water w_1 used by user 1 needs treatment (WT1) that is associated with costs, which are assumed to be c per unit. User 2 is located at the same water reservoir. She uses return flows from user 1, which is denoted by w_{12}, and diverts w_2 from the reservoir if she needs more than $h_1 w_1$. Notice that water withdrawn by user 2, w_2, also have to be treated at costs c per unit of water. If user 2 does not use the whole amount of return flow from user 1 the residual, w_{1d}, flows back into the water reservoir. To keep the model as simple as possible, we assume that user 2 returns no water. The respective dynamic balance for the water stock is

$$\frac{dS(t)}{dt} = R - r - w_1 + w_{1d} - w_2 \tag{3.38}$$

Substituting the definition of the residual flow

$$w_{1d} = h_1 w_1 - w_{12} \geq 0 \tag{3.39}$$

into Eq. (3.38) yields

$$\frac{dS(t)}{dt} = R - r - (1 - h_1)w_1 - w_{12} - w_2 \tag{3.40}$$

The model allows for both direct and indirect reuse. Exclusive indirect reuse is present if $w_{12} = 0$ is true, which implies that all return flows flow back into the reservoir, i.e., $w_{1d} = h_1 w_1$. If treatment costs occur, indirect reuse will of course be minimized because direct use avoids the additional treatment costs that accrue to w_2.

In the following, we derive the optimal allocation within our hydro-economic model. To concentrate on the allocational effects of the return flow, we disregard all the issues raised in the previous sections, i.e., fairness considerations or issues of poverty. We confine ourselves to the simple task of maximizing the aggregate benefit of both users. Let us assume that user 2 is relatively big compared to user 1, i.e., $B_1^{'}(w) < B_2^{'}(w)$. The optimization program is

$$\max_{w_1, w_2, w_{12}} \left[B_1(w_1) + B_2(w_{12} + w_2) - c(w_1 + w_2)\right] \tag{3.41}$$

subject to Eqs. (3.39) and (3.40). To keep the analysis simple, we disregard fixed costs by assuming that these costs are covered by access fees.[27] Also we assume that water is not scarce[28] which allows us to skip Eq. (3.40).

The KKT conditions are as follows:

$$B_1^{'}(w_1) - c + \lambda h_1 = 0 \tag{3.42}$$

$$\left[B_2^{'}(w_{12} + w_2) - c\right] \leq 0 \perp w_2 \geq 0 \tag{3.43}$$

$$\left[B_2^{'}(w_{12} + w_2) - \lambda\right] \leq 0 \perp w_{12} \geq 0 \tag{3.44}$$

$$[w_{1d} = h_1 w_1 - w_{12}] \geq 0 \perp \lambda \geq 0 \tag{3.45}$$

[27] In Chap. 4 we will analyze tariff systems that also cover fixed costs in depth.
[28] All the following results apply also to the case where water is scarce.

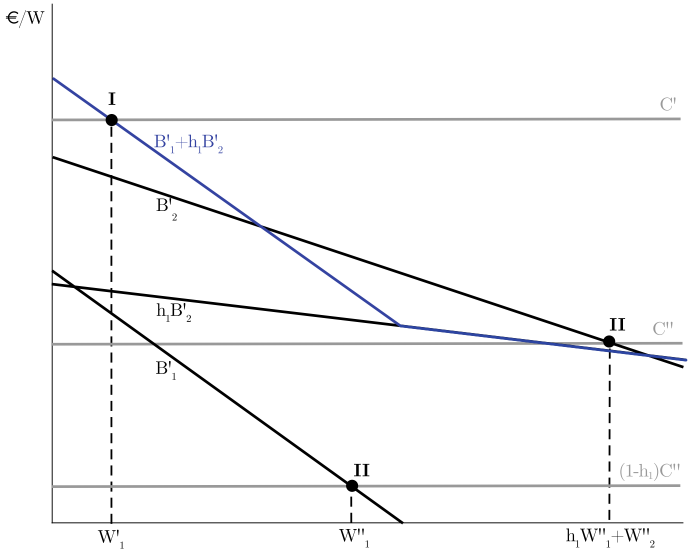


Fig. 3.10 Optimal water recycling. *Source* own illustration

where λ is the Lagrangian to the constraint, which is given by Eq. (3.39). These conditions apply under the assumption that $w_1 > 0$ (else there would be no recycling in this model). Therefore, Eq. (3.42) applies with strict equality.

Figure 3.10 depicts the solutions for two cases. The first case (case I) refers to high water treatment costs, the second to relatively low costs (case II). The figure is drawn under the assumptions that the marginal benefits of both users are declining linearly with respect to water use, i.e., "$B_i^{'}(w) = a_i - b_i w$, and that user 1 is "small" in comparison to user 2, i.e., $B_1^{'}(w) < B_2^{'}(w), \forall w \geq 0$.

Before we take a closer look at these cases, let us first note that the waste of water, i.e., $w_{1d} > 0$, cannot be the result of the optimization program. It cannot be optimal to return a portion of clarified water $h_1 w_1$ into the reservoir and withdraw it later with additional treatment costs.[29]

3.6.2.1 Case I

Here we assume that the processing costs are very high. In order to make the importance of water recycling particularly visible, we also assume that without a technical

[29]We can show this with the help of the KKT conditions. Assume, per contradiction, that $w_{1d} > 0$ and, hence, $\lambda = 0$ by Eq. (3.45). From Eq. (3.44), we have $B_2^{'}(w_{12} + w_2) \leq 0$ and, hence, $w_{12} + w_2 > 0$. Thus, we have from Eq. (3.43) $B_2^{'}(w_{12} + w_2) - c < 0$ wherefore $w_2 = 0$. Hence, to meet Eq. (3.44) we have $w_{12} > 0$, and therefore by Eq. (3.44) $B_2^{'}(w_{12}) = 0$. Since $B_1^{'}(w) < B_2^{'}(w), \forall w \geq 0$ we get the result that $w_1 < w_{12}$. But this contradicts the constraint $h_1 w_1 - w_{12} \geq 0$ (see Eq. (3.39)). Hence, $\lambda > 0$ and therefore $w_{1d} = 0$.

infrastructure for water reuse, i.e., if the used water of user 1 cannot be transferred to user 2, none of the two users would take water from the reservoir. This case is depicted in the figure by the fact that the marginal cost line lies above both marginal benefits ($c^I > a_i, i = 1, 2$).

In case I, it follows from the optimality conditions that the optimal allocation is characterized by the following compact rule (rule I)[30]:

$$B_1^{'}(w_1) + h_1 B_2^{'}(h_1 w_1) = c^I \tag{3.46}$$

The weighted aggregated marginal benefits should be equated to the marginal treatment costs (see point I in Fig. 3.10). This equation is nothing else as the allocation rule for a public good. The water use of user 1, w_1, exhibits the characteristics of a public good which serves twice as an input, the first time for user 1 and then the second time for user 2 diminished by factor h_1. Point I in Fig. 3.10 is identified by equating the aggregated marginal benefits of both users of w_1 to marginal treatment costs.

3.6.2.2 Case II

If treatment costs are very low case II, which is represented by the two points II in Fig. 3.10, applies. The corresponding optimal allocation rule is (rule II):

$$\frac{B_1^{'}(w_1)}{(1 - h_1)} = B_2^{'}(h_1 w_1 + w_2) = c^{II} \tag{3.47}$$

User 1 diverts w_1. She equates her marginal benefit to marginal treatment costs related to the *effective* water use per liter, i.e., $(1 - h_1)$ (see the left one of the two points II in Fig. 3.10). User 2 is allocated the return flow of $h_1 w_1$ and supplements her water consumption such that $B_2'(h_1 w_1 + w_2) = c^{II}$. Therefore, total water use of user 2 is $h_1 w_1 + w_2$ (see the right one of the two points II in Fig. 3.10). This rule is very well known from hydro-economic models that include return flows. Notice that this rule only applies if marginal water treatment costs are relatively low. Otherwise (case 1), we have to apply the rule for water as a public input (rule I).

Both rules are cost dependent special cases of the optimality conditions as derived in Eqs. (3.42)–(3.45), which are the result of a hydro-economic model optimizing aggregate benefits of both users.

3.6.3 Markets for Recycled Water

Let us take case II (low treatment costs) and assume that the institutional implementation of rule II should be accomplished by introducing a *water market* system, as

[30]This result can also be derived from the KKT conditions. We know that $\lambda > 0$ and that $w_2 = 0$ (due to the high marginal treatment costs c^I). Hence, $h_1 w_1 = w_{12} > 0$ and $B_2^{'}(w_{12}) = \lambda > 0$ by Eq. (3.44). Therefore, we can substitute λ for $B_2^{'}$ in Eq. (3.42).

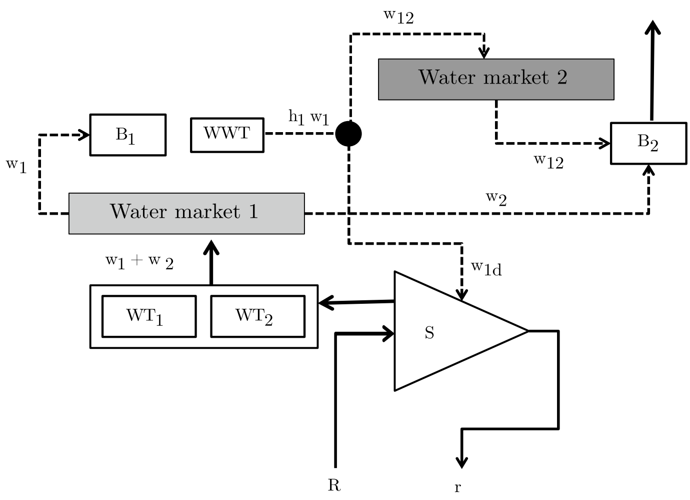


Fig. 3.11 Water recycling in two water markets. *Source* own illustration

depicted in Fig. 3.11.[31] Each user buys freshwater from the water treatment plants for a uniform price, p_1. In addition, there is a market for recycled water, in which user 1 offers treated water whereas user 2 is on the demand side. The price in water market 2 is such that supply is equal to demand.

Let us assume that both markets operate under perfect competition or, alternatively, that a regulation authority sets prices close to a competitive market. Thus, both water treatment plants will offer water for a price equal to the marginal treatment costs, i.e., $p_1 = c^{II}$.

User 1 buys water in market 1 and, at the same time, offers treated water in market 2 by solving the following optimization:

$$\max_{w_1, w_{12}} [B_1(w_1) + p_2 w_{12} - p_1 w_1] \quad \text{s.t.} \quad h_1 w_1 - w_{12} \geq 0 \tag{3.48}$$

Assuming that user 1 buys and sells water the KKT conditions are

$$B_1^{'}(w_1) - p_1 + \lambda h_1 = 0 \tag{3.49}$$

$$p_2 - \lambda = 0 \tag{3.50}$$

Merging both KKT equations yields

$$B_1^{'}(w_1) - p_1 + p_2 h_1 = 0 \tag{3.51}$$

[31] Many economists are quite skeptical about allocation rules from complex models if they are taken literally in the sense that they are prescriptions for the individual actors in the watershed. However, the derivation of allocation rules only serves as a benchmark. Economists then ask under which institutional provisions the actors would behave in such a way that these rules would be adopted.

User 2 has the option to buy water in both markets, hence the corresponding optimization program is

$$\max_{w_2, w_{12}} [B_2(w_2 + w_{12}) - p_2 w_{12} - p_1 w_1] \tag{3.52}$$

We have assumed that $p_1 = c^{II}$ is so low that user 2 buys water in both markets ($w_2 > 0$ and $w_{12} > 0$). Then the optimal water demand in both markets follows the rule:

$$B_2^{'}(w_2 + w_{12}) = p_1 = p_2 \tag{3.53}$$

If prices were to differ, the user would only operate in one of the two markets. Thus, in this scenario both prices are equal.

Both market participants set their marginal benefits equal to the respective prices. In turn p_1 is equal to marginal costs. If we put together this information by substituting prices in Eq. (3.51) by Eq. (3.53) and bear in mind that $h_1 w_1 = w_{12}$ we get

$$\frac{B_1^{'}(w_1)}{(1 - h_1)} = B_2^{'}(h_1 w_1 + w_2) = c^{II} \tag{3.54}$$

which is simply the optimality rule for case II (cf. Eq. (3.47)). The implementation of two water markets is able to replicate the optimal allocation derived in the framework of a central planning approach.

With the same approach, we can show that the system of two markets would also secure optimality in case I.[32] This is interesting because user 2 should only buy recycled water in market 2 and it is interesting because the sequential use of water makes water almost a public good.[33] It is a standard result from microeconomic textbooks or introductions to public economics that the private provision of public goods leads to a misallocation. Why not here? Users consume water one after the other and two markets are implemented (instead of only one market). The treatment plant sells water only to user 1 and user 1 sells to user 2. If then only the treatment plant sells water in market 1 to both users then both users could not afford the water in case I since $B_i^{'}(w_i) < c^I$ (see Fig. 3.10). This market result is not optimal. The reason is that we have one market missing. Inserting the second market allows to implement the optimal allocation for public goods. This is due to the hydro-technological situation implicitly endowing user 1 with property rights. He can sell the water used and treated or let it return to the reservoir S. But he will sell the water after usage to user 2. There will be a positive price less than marginal costs c^I that user 2 will accept.

[32]This case is covered in Exercise 3.4.

[33]If $h_1 = 1$ then water is a complete public good.

Box 3.5 Ecological Sanitation

Ecological Sanitation (EcoSan) is a concept standing for a potential change in the paradigm of wastewater disposal. Wastewater has been regarded only as a problem for a long time, because it involves hygienic hazards and contains organic matter and eutrophying substances in the form of nitrogen and phosphorus. These substances cause problems in seas, lakes, and streams. Due to inadequate sanitation, wastewater causes serious water-related issues in many parts of the world (e.g., Sub-Saharan Africa), as it has in the past on central Europe (e.g., cholera epidemic in Hamburg in 1892 with 8,600 deaths). However, in the framework of EcoSan the wastewater is seen as a reusable substance that contains valuable components, such as nutrients (nitrogen, phosphorous), sulfur, potassium, magnesium, and many trace elements essential for fertile soils.

The main idea of the EcoSan concept is to close the nutrient loop between sanitation and related sectors (e.g., agriculture) and hence it is quite more than simply gray water reuse or rainwater use. Closing the loop enables the recovery of organics, macro and micronutrients, water, and energy contained in wastewater and organic waste and their subsequent productive reuse mainly in agriculture, or for other reuse options. The main advantages are as follows (Werner et al. 2009):

- Promotion of recycling by safe, hygienic recovery, and use of nutrients, organics, water, and energy.
- Conservation of resources (lower water consumption, chemical fertilizer substitution).
- Preference for modular, decentralized partial-flow systems for more appropriate cost-efficient solutions.
- Possibility to integrate on-plot systems into houses, increasing user comfort, and security for women and girls.
- Contribution to the preservation of soil fertility.
- Promotion of a holistic, interdisciplinary approach (hygiene, water supply and sanitation, resource conservation, environmental protection, urban planning, agriculture, irrigation, food security, small-business promotion).

This concept was applied in a number of pilot projects, for instance, in Lübeck-Flintenbreite, Germany, for 350 inhabitants. The installed system comprises a strict separation of blackwater (wastewater from the toilet), gray water and stormwater. Blackwater together with organic waste should be treated anaerobically (producing biogas for energy and heat production) (Langergraber and Muellegger 2005). Other exemplary early EcoSan pilot projects were implemented by the “Svanholm Community” in Denmark, the Ecological Village Björnsbyn in Sweden, Ås in Norway or the Solar-City Linz-Pichling in Austria

(Fröhlich et al. 2003). In 2020, the start-up Finizio is planning to install a test equipment in the (German) city of Eberswalde (https://finizio.de/produkte/).

Source: Fröhlich et al. (2003), Werner et al. (2009)

3.7 Water Allocation Along Rivers

3.7.1 Basic Model

A simple example of a river with two users is given in Fig. 3.12. The upstream user 1 and the downstream user 2 compete for the water resources and are able to generate net benefits, designated by $B_1(w_1)$ and $B_2(w_2)$, depending on the diverted water amounts, w_1 and w_2. Furthermore, the river is fed by two inflows from headwater areas, that are located upstream of the tapping points of user 1 and user 2, respectively. These inflowing water quantities are denoted by R_1 for user 1 and by R_2 for user 2. The amount of water that leaves the addressed river system, i.e., the outflow, is represented by variable r.

3.7.2 Two Cases of Upstream Behavior with Scarcity

Based on the IWRM approach, the objective to maximize net benefits in the total river basin is formulated as presented in Eq. (3.55).

$$\max_{\{w_1, w_2, r\}} [B_1(w_1) + B_2(w_2)] \tag{3.55}$$

Any consumer can divert at most those quantities of water that are available at the respective tapping point, thus the constraints are given by Eqs. (3.56) and (3.57)

$$w_1 \leq R_1 \qquad (\lambda_1) \tag{3.56}$$

$$w_2 \leq (R_1 - w_1) + R_2 \qquad (\lambda_2) \tag{3.57}$$

Of course, if a minimum outflow quantity (r_0) of the addressed river system should be guaranteed, it is important to consider the additional constraints in Eq. (3.58),

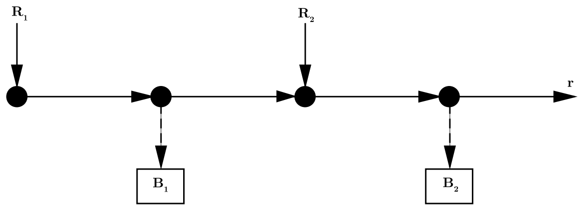


Fig. 3.12 Scheme of a simple river example with 2 consumers. *Source* own illustration

which ensures that the realized outflow r is equal or higher the obligatory minimum outflow r_0. The realized outflow is the water amount which is left in the river by both users. Therefore, it results from the difference between the headwater inflows and the abstraction amounts which means that $r = R_1 + R_2 - w_1 - w_2$. Therefore, we are able to formulate following condition for addressing the minimum outflow quantity:

$$r_0 \leq R_1 + R_2 - w_1 - w_2 \qquad (\lambda_r) \tag{3.58}$$

Based on the model formulated here, which corresponds to a maximization problem with the objective stated in Eq. (3.55) subject to the constraints defined in Eqs. (3.56) to (3.58), the KKT conditions can be derived:

$$B_1'(w_1) - \lambda_1 - \lambda_2 - \lambda_r \leq 0 \perp w_1 \geq 0 \tag{3.59}$$
$$B_2'(w_2) - \lambda_2 - \lambda_r \leq 0 \perp w_2 \geq 0 \tag{3.60}$$
$$R_1 - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{3.61}$$
$$R_1 + R_2 - w_1 - w_2 \geq 0 \perp \lambda_2 \geq 0 \tag{3.62}$$
$$R_1 + R_2 - w_1 - w_2 - r_0 \geq 0 \perp \lambda_r \geq 0 \tag{3.63}$$

It is likely that water will be diverted from the river as much as possible by the users, so that all available resources in the river are abstracted completely. This means that the outflow of the addressed river system, r, must not exceed the required outflow level, r_0, which implies $r = r_0$. Furthermore, the usable amount of water, given by $R_1 + R_2 - r_0$, has to be diverted entirely, thus the equality $R_1 + R_2 - r_0 = w_1 + w_2$ holds. Based on these relations, it follows from the KKT conditions, that

- if there exists a minimum outflow quantity ($r_0 > 0$), the Eq. (3.63) is binding which means that $\lambda_r \geq 0$. However, Eq. (3.62) is certainly nonbinding and hence $\lambda_2 = 0$.
- if there exists no minimum outflow quantity ($r_0 = 0$), it would not make sense to set up the constraint (3.58) which implies that λ_r would not exist. This has to be noticed when we set up the KKT conditions.[34] Equation (3.62) is binding which means that $\lambda_2 \geq 0$.

To conclude: if $r_0 > 0$ it follows that $\lambda_r \geq 0$ and $\lambda_2 = 0$, while if $r_0 = 0$ it follows that λ_r does not exist and $\lambda_2 \geq 0$.

[34] In case that there exists no minimum outflow quantity from the addressed river section ($r_0 = 0$) we are able to formulate the following KKT conditions:

$$B_1'(w_1) - \lambda_1 - \lambda_2 \leq 0 \perp w_1 \geq 0 \tag{3.59}$$
$$B_2'(w_2) - \lambda_2 \leq 0 \perp w_2 \geq 0 \tag{3.60}$$
$$R_1 - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{3.61}$$
$$R_1 + R_2 - w_1 - w_2 \geq 0 \perp \lambda_2 \geq 0 \tag{3.62}$$

.

By addressing Eq. (3.60) it follows that

$$B_2'(w_2) = \begin{cases} \lambda_2 & \text{for: } r_0 = 0 \\ \lambda_r & \text{for: } r_0 > 0 \end{cases} \tag{3.64}$$

Therefore the formulated conditions reduce to Eqs. (3.65) and (3.66).[35]

$$B_1'(w_1) - \lambda_1 = B_2'(w_2) \tag{3.65}$$

$$R_1 - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{3.66}$$

Based on the formulas displayed in Eqs. (3.65) and (3.66), it is possible to define optimality conditions for two different cases:

- Case 1: The upstream user diverts the whole amount of water available at his/her tapping point, i.e., $R_1 = w_1$, and hence $\lambda_1 \geq 0$ and $B_1'(w_1) \geq B_2'(w_2)$.
- Case 2: The upstream user does not divert the whole amount of water available at his/her tapping point, but passes a limited amount to his/her adjacent downstream user, i.e., $R_1 > w_1$ and therefore $\lambda_1 = 0$ and $B_1'(w_1) = B_2'(w_2)$.

The optimal case depends mainly on the headwater inflows further upstream of the the tapping points of the consumers. Figure 3.13 displays two scenarios, I and II, whereas both are subject to the same outflow requirements, such that $r_0 = r_0^I = r_0^{II}$, and they are restricted to the same amounts of useable water, i.e., $R_1 + R_2 - r_0 = R_1^I + R_2^I - r_0^I = R_1^{II} + R_2^{II} - r_0^{II}$. The amounts of useable water are represented by the lengths of the horizontal axes of both plots in Fig. 3.13. The diversions of water by both users are illustrated by the arrows segmenting the horizontal axes, respectively. The diversion of the upstream user w_1 is illustrated from the left point of origin of the diagram to the right, while in contrast the diversion of the downstream user w_2 is pictured from the right point of origin to the left.

Scenario I is characterized by the fact that the natural inflow before the upstream user 1 is low and the natural inflow before the downstream user 2 is high. This scenario is visualized in panel (a) of Fig. 3.13, where the inflow to user 1, R_1^I, is located to the left of the intersection between the marginal benefit functions $B_1'(w_1)$ and $B_2'(w_2)$. Compared to scenario I, the opposite situation is defined in scenario II (see panel b) in Fig. 3.13.[36] The headwater inflow before the upstream user, R_1^{II}, is located to the right of the intersection point between the marginal benefit functions $B_1'(w_1)$ and $B_2'(w_2)$ in the plot.

If the upstream user passed a limited amount of its inflows to the downstream user in scenario I, the situation would correspond to case 2 where $w_1^I < R_1^I$. The water allocation of the downstream user depends on the one of the upstream user and

[35] It is assumed that $w_1 > 0$, $w_2 > 0$.

[36] In scenario II, the natural inflow before the upstream user 1 is high and the natural inflow before the downstream user 2 is low.

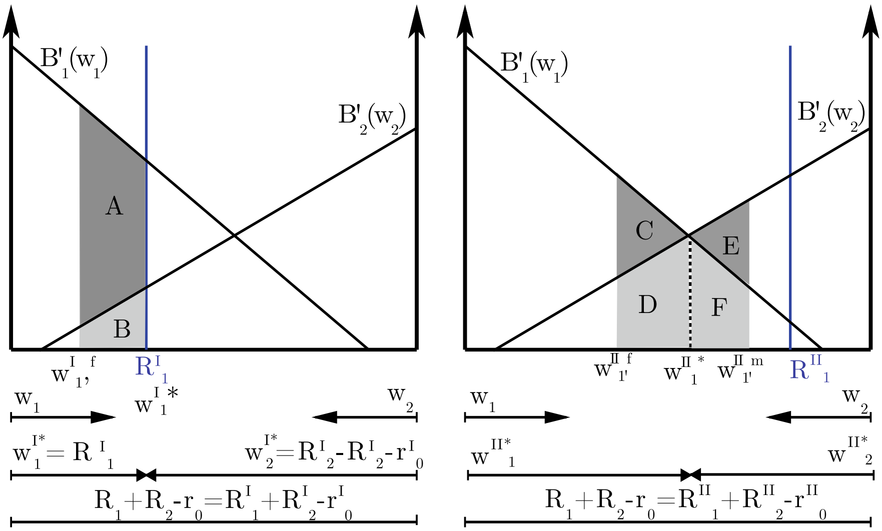


Fig. 3.13 Allocation of water in a river source under scarce conditions. *Source* own illustration

is subject to $w_2^I = R_1^I + R_2^I - w_1^I - r_0^I$. Based on the optimality condition for that case, marginal benefits of both users should be equal, i.e., $B_1'(w_1^I) = B_2'(w_2^I)$. This optimality condition cannot be fulfilled, because within the whole possible domain of $w_1^I \in [0; R_1^I]$ the marginal benefit of the upstream user always exceeds the one of the downstream user, such that $B_1'(w_1^I) > B_2'(w_2^I) = B_2'(R_1^I + R_2^I - w_1^I - r_0^I)$.

Hence for scenario I, optimality can only be assured by case 1 in which the upstream user fully diverts the available water at his/her tapping point, therefore the optimal amount of water diverted by the upstream user is equal to the inflow to the upstream player, $w_1^{I^*} = R_1^I$. In that case, the downstream user does not receive any water inflows from the upstream user. It follows that user 2 can divert the difference between the downstream headwater inflows and the necessary outflows, thus the downstream user's optimal amount of diverted water is defined by $w_2^{I^*} = R_2^I - r_0^I$. The required optimality condition of case 1 is fulfilled, because the marginal benefit of user 1 exceeds the one of user 2 for this allocation regime, i.e., $B_1'(R_1^I) > B_2'(w_2^{I^*}) = B_2'(R_2^I - r_0^I)$.

For scenario I, the inefficiency of case 2 compared to case 1 is also depicted on the left-hand side of Fig. 3.13. Any consumption level of user 1 that is below the level of available water, i.e. $w_1^I = w_1^{I,f} < R_1^I$, would result in a higher consumption of the downstream user 2 relative to the optimal case. The corresponding welfare gains for user 2 are depicted by area B, while the corresponding welfare losses for user 1 are represented by the areas A and B. Hence, a deviation from the optimal water allocation would result in a loss of social welfare equal to area A.

In scenario II (relative high upstream inflows R_1), however, if the upstream user 1 diverts its total upstream headwater inflows (case 1), $w_1^{II} = R_1^{II}$, the resulting marginal benefit of the upstream user 1 falls below the one of the downstream user 2, i.e. $B_1'(R_1^{II}) < B_2'(w_2^{II}) = B_2'(R_2^{II} - r_0^{II})$, which is illustrated on the right-hand side of Fig. 3.13. This is a violation of the optimality condition, case 1 is, therefore, the nonoptimal case, while the optimal allocation can only be implemented in case 2. To realize an optimal allocation in the river basin, the quantity of water diverted by the upstream user 1, $w_1^{II} = w_1^{II^*}$, ensures that the marginal benefits of the upstream and downstream user are equal, such that $B_1'(w_1^{II*}) = B_2'(w_2^{II*}) = B_2'(R_1^{II} + R_2^{II} - w_1^{II*} - r_0^{II})$ holds. The optimal water diversion by the upstream user, w_1^{II*}, is identical to the one implied by the intersection point between the two marginal benefit functions $B_1'(w_1)$ and $B_2'(w_2)$ in panel (b) of Fig. 3.13, and the resulting optimal diversion by the downstream user 2 is characterized by $w_2^{II*} = R_1^{II} + R_2^{II} - w_1^{II*} - r_0^{II}$. If the upstream user diverts smaller amounts than optimal, where $w_1^{II} = w_1^{II,f} < w_1^{II*}$, the downstream user can consume more water than in the optimal case, thus $w_2^{II,f} = R_1^{II} + R_2^{II} - w_1^{II,f} - r_0^{II} > w_2^{II*}$.

The corresponding welfare effects are depicted on the right-hand panel in Fig. 3.13, while the welfare gains of downstream user 2, represented by area D, are much smaller than the welfare losses upstream user 1 incurs, displayed by the areas C and D. Hence there is a loss of social welfare equal to area C, if real upstream diversion falls below the optimal upstream extraction.

Similarly, if the upstream user 1 diverts more quantities than optimal, i.e. $w_1^{II^*} < w_1^{II} = w_1^{II,m} \leq R_1^{II}$, less amounts of water are available for the downstream user 2 compared to the optimal allocation regime, thus $w_2^{II,m} = R_1^{II} + R_2^{II} - w_1^{II,m} - r_0^{II} < w_2^{II^*}$. The corresponding welfare losses for the downstream user 2 cover the areas E and F and thus outweight the corresponding welfare gains for the upstream user 1, which cover the area F. This results in a loss of social welfare equal to area E due to that kind of deviation from the optimal allocation regime.

3.7.3 Two Cases Without Scarcity in One Region

In the former analysis, water was assumed to be a scarce resource in a river basin and the useable amounts were completely diverted by the users. With respect to the allocation of the water resources, a trade-off exists between the users. The more water one user diverts, the less the other user is able to consume. This trade-off concerning the water resource is not relevant for the downstream user whose water supply is non-scarce/abundant, but if the useable amounts must not be entirely allocated among the users, the inequality $R_1 + R_2 - r_0 \geq w_1 + w_2$ will be a relevant constraint to the optimization problem. Therefore, the outflow from the river basin can exceed the minimum outflow requirements, i.e., $r \geq r_0$. From the KKT conditions, displayed in Eqs. (3.59) to (3.63), it follows that $B_2'(w_2) = \lambda_2 = \lambda_r = 0$ holds.[37] This implies for the optimal case that the downstream user has to consume at his/her saturation

[37] In case that there exists a minimum outflow ($r_0 > 0$), Eqs. (3.61) and (3.62) are nonbinding and hence $\lambda_2 = \lambda_r = 0$. In case that there exists no minimum outflow ($r_0 = 0$), we would not set up

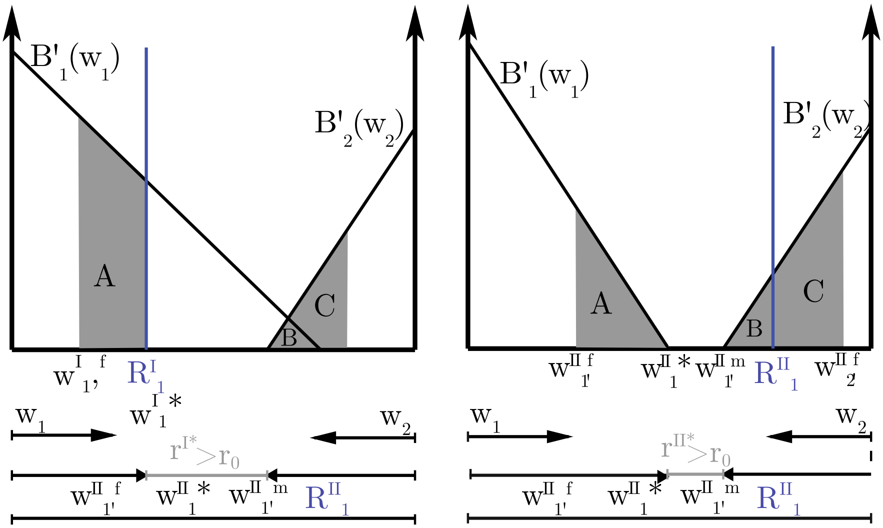


Fig. 3.14 Allocation of water in a river source under non-scarce conditions. *Source* own illustration

level, which corresponds to the null of its marginal benefit function, i.e., $B_2'(w_2*) = 0$. The user 2 chooses its optimal diversion, w_2^*, in such a way that its marginal benefit becomes zero, i.e., $B_2'(w_2^*) = 0$. For user 1, it is possible to derive algebraic characterizations of the optimality conditions from the KKT conditions, which are displayed in Eqs. (3.67) to (3.68).

$$B_1'(w_1) = \lambda_1 \tag{3.67}$$

$$R_1 - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{3.68}$$

Similar to the situation with water scarcity, it is possible again to define optimality conditions for two different cases:

- Case 1: The upstream user diverts the whole amount of water available at his/her tapping point, i.e., $R_1 = w_1$, and hence $\lambda_1 \geq 0$ and $B_1'(w_1) \geq 0$.
- Case 2: The upstream user does not divert the whole amount of water available at its tapping point, but passes a limited amount to its adjacent downstream user, i.e., $R_1 > w_1$, and therefore $\lambda_1 = 0$ and $B_1'(w_1) = 0$.

Which of the two cases is suitable to implement optimality depends on the scarcity situation of the upstream user. Illustrative examples with two different scenarios are

the constraint (3.58) and hence λ_r and Eq. (3.62) would not exist. Eq. (3.61) would be nonbinding and hence $\lambda_2 = 0$.

pictured in Fig. 3.14. In the plot for scenario I, depicted in panel (a) in Fig. 3.14, the intersection between the upstream headwater inflows, R_1^I, and the marginal benefit function of the upstream user, $B_1'(w_1)$, is characterized by a positive marginal utility for the upstream user. Hence water is scarce for the upstream user in case 1 I. If user 1 passes a limited quantity of water downstream, as in case 2, the possible domain of consumption will be $w_1^I \in [0, R_1^I]$. The marginal benefit function, $B_1'(w_1^I)$, does not become zero for any w_1^I of the defined domain. Hence there is a violation of the optimality condition for case 2 and it follows that case 1 should be the optimal one. In case 1, the upstream headwater inflows are fully consumed, i.e., $w_1^{I^*} = R_1^I$. The optimality condition of case 1 is fulfilled because the marginal benefit of the upstream user is positive for this optimal consumption level, such that $B_1'(R_1^I) \geq 0$.

In contrast to scenario I, water is not scarce for the upstream user 1 in scenario II, as displayed in panel (b) in Fig. 3.14, where the upstream headwater inflow, R_1^{II}, is located to the right of the saturation point characterized by $B_1'(w_1) = 0$ in the plot. If the upstream user 1 entirely diverted the upstream headwater inflows, as in case 1 with $w_1^{II} = R_1^{II}$, its marginal benefit would be negative, which would violate the optimality condition of case 1. Limited quantities of upstream headwater inflows should, therefore, be passed downstream, hence $R_1^{II} > w_1^{II}$ as in case 2, and user 1 diverts the water quantities in such a way that its marginal benefit becomes zero. The optimality condition of this case 2 is guaranteed to be satisfied due to the assumptions made, hence the optimal consumption, w_1^{II*}, is identical to the null of the marginal benefit function, i.e., w_1 such that $B_1'(w_1^{II^*}) = 0$, which means that the user 1 diverts the amount of water equal to its saturation point.

A loss of consumer surplus for upstream user 1 would result if the upstream user deviated from the optimal amount to be diverted, so that $w_1^{I,f} < w_1^{I^*}$ as well as $w_1^{II,f} < w_1^{II^*}$. This loss is represented by the area A in Fig. 3.14. Similarly, a loss of consumer surplus for downstream user 2, illustrated by the areas B and C, will also occur if the diverted amount of user 2 falls below its saturation quantity. Both types of deviations from the optimal allocation generate losses in social welfare.

Box 3.6 The downstream externalities of harvesting rainwater

Rainwater harvesting is a technique for providing water that has been used since ancient times. For example, Roman cities were designed and built such that the inhabitants could collect rainwater for drinking and domestic purposes. But captured rainwater can also be used for irrigation in the agricultural sector or in urban areas to provide water for the non-potable uses like a toilet flushing. One distinct advantage of rainwater harvesting is that it can be shaped in a decentralized manner, e.g., simple roof water collection systems. But also bigger projects are conceivable. In the Global South, land surface catchment systems are implemented in many rural areas. They can be used for irrigation systems or simply as a method to recharge the local groundwater. Since the technique is rather simple, rainwater harvesting investments are an integral part of rural development programs. All in all, it seems to be a highly efficient method to provide people with more water without stressing the water cycle.

From a hydrological perspective, water harvesting is nothing else as reducing the water runoff in a catchment area. But there remains one problem: If the runoff is reduced, water users downstream may suffer from less water. Hence, there exists a downstream externality, which must be included in the calculation of integrated water resource management. The optimal allocation of water to the upstream users has to take into account the opportunity costs that arise from the lower water availability of downstream users. This can easily be inferred from Eqs. (3.56) and (3.57). If precipitation is included in R_i, rainwater harvesting from upstream is equivalent to an increase of R_1 and a reduction of the same size of R_2.

Sources: UNEP International Environmental Technology Centre (2002), Boumaa et al. (2011)

3.8 Groundwater Management

3.8.1 A Simple Groundwater Model

Groundwater is a very important resource for covering water requirements in many regions of the world. In locations with sparse surface water resources due to the absence of lakes and rivers, groundwater is the only available resource. In regions with little water availability, groundwater is often used for agricultural purposes, mainly irrigation. The extraction of groundwater is an open-access problem, especially in those areas where water is quite scarce. This problem may arise due to a lack of institutions, a lack of non-enforceable water rights, a lack of legal allowance, or too high transaction cost for assessing the aquifer regulation. This open-access problem is characterized by

- Non-excludable access to the aquifer: The access to groundwater resources is unregulated. Hence anyone can potentially extract groundwater from the aquifer.
- Rivaling for the water resource: Water volumes that are extracted by someone can not be extracted or used by someone else.

There is a risk of overexploiting the groundwater resources because of the unlimited access in this specific institutional setting. This issue is quite relevant in many parts of the world, especially in regions where groundwater is the most important water source and which are characterized by a low water availability rate per capita, dry meteorological conditions like low precipitation rates combined with high potential

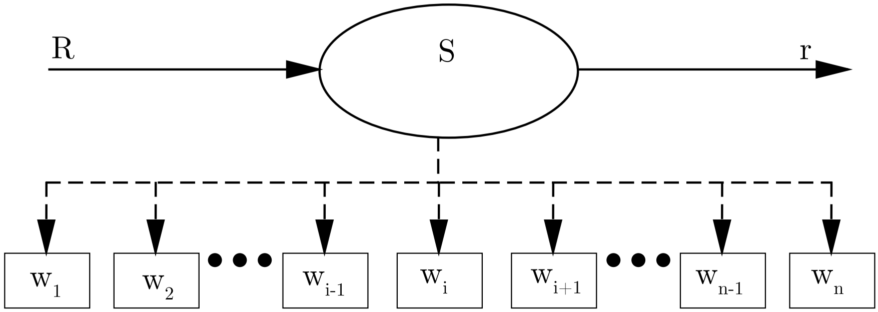


Fig. 3.15 Scheme of a simple groundwater model. *Source* own illustration

evaporation, and big water consumers in the basin, e.g., the agricultural sector. The described open-access characteristics set incentives for overexploitation of groundwater resources, which is demonstrated with the help of the following algebraic model.

As depicted in Fig. 3.15, an aquifer with an exhaustible groundwater stock, declared as S, is assumed. This stock is fed by a certain natural inflow, denoted by R. Furthermore there is also a certain amount of water, represented by r, that leaves the groundwater stock due to a natural flow processes. For reasons of simplification, it is assumed that these natural flows are constant over time.

The aquifer is commonly used by n water extractors who may use the water for covering their own demand or sell it to water consumers. For this analysis, it is irrelevant whether an extractor sells or directly uses the water to satisfy their own needs. The amount of groundwater extracted from the aquifer is represented by the variable w_i, where i is an element representing a specific groundwater extractor. The total amount of water extracted from the aquifer within one time period is equivalent to the sum of the amount extracted by each user:

$$W = \sum_{i=1}^{n} w_i \tag{3.69}$$

Under the assumption that all extractors exhibit identical properties, the equation above simplifies to

$$W = n \cdot w_i \tag{3.70}$$

The demand function aggregated for all consumers in the groundwater basin is given by the demand function in Eq. (3.71) with the demand function parameters a, defining the choke price, and b, determining the slope of the demand function.

$$P = a - b \cdot W \tag{3.71}$$

The extraction process of withdrawing water volumes from the aquifer, w_i, performed by extractor i causes costs described by the following cost function:

$$C(w_i, S) = (c - \sigma \cdot S) \cdot w_i \tag{3.72}$$

Extraction costs are not only influenced by the extracted water volumes, w_i, but also by the size of the groundwater stock in the aquifer, S, because a smaller stock of water is accompanied by a lower groundwater table, hence higher pumping heights are observed resulting in higher monetary expenses for pumping. The scope of the impact that a water stock's size has on the extraction cost depends on a level parameter, named σ, that is defined such that higher levels of σ yield higher extraction cost sensitivities on the water stock S. Furthermore, there exists a cost function parameter, declared as c, that represents the theoretical cost rate if groundwater resources in the aquifer were completely exhausted.

Due to the previously explained similarity between the characteristics of groundwater resources and of open-access goods, it is credible to assume a zero profit condition for water extraction. This assumption seems to be quite plausible: As long as positive profits can still be realized, each (potential) groundwater extractor, who competes with other (potential) extractors for the limited and unregulated groundwater resources, has an incentive to enter the market or to increase its extraction volumes. Due to the increasing extracted water volumes ($W \uparrow$), the market price for extracted groundwater would decrease ($p \downarrow$). Hence, the marginal utility for the use of extracted groundwater decreases with increasing extraction amounts. Furthermore an increase in extraction costs can be also observed ($C \uparrow$), because it becomes more expensive to extract groundwater given a decreasing groundwater table in the aquifer. Consequently, the supplier is affected by falling profits if extraction amounts rise. This process would continue until positive profits are not realizable on the market, hence, under the previously stated assumptions, the price for water equals the average extraction cost (zero profit condition), as displayed in Eqs. (3.73) and (3.74).

$$P = \frac{C(w_i, S)}{w_i} \tag{3.73}$$

$$\Rightarrow \quad a - b \cdot W = c - \sigma \cdot S \tag{3.74}$$

Based on the approach explained above, the total water extraction from an aquifer can be determined by Eq. (3.75):

$$W = \frac{a - c + \sigma \cdot S}{b} \tag{3.75}$$

3.8.2 Dynamic Stock Balance for Groundwater

It is possible to set up the dynamic stock balance for the groundwater stock, which arises from the physical paradigm that all water volumes have to be balanced (hydrological cycle). This implies that inflows into the groundwater stock exceeding the water amounts outflowing from the groundwater stock will cause the groundwater table to rise, and vice versa. Consequently, the change in the groundwater stock $\dot{S}(t)$ is equal to the difference between the in- and outflowing water volumes of the aquifer. Due to this relation, the groundwater stock in the aquifer can change over time and the parameter should be made time-dependent, such that $S(t)$. Therefore, the amount of water extracted can also change over time, as depicted in Eq. (3.75), and should be written as $W(t)$. The inflow into the groundwater stock is only determined by the natural inflow, R, which is, for reasons of simplification, assumed to be constant over time. The outflow from the groundwater stock contains two parts: On the one hand, it is determined by the natural outflow, r, which occurs due to flow processes and is assumed to be constant over time, and on the other hand it is also determined by the aggregated amount of water extraction by humans, $W(t)$. This results in the following dynamic stock balance:

$$\dot{S}(t) = (R - r) - W(t) \tag{3.76}$$

Plugging Eq. (3.75) into Eq. (3.76) yields an algebraic expression of the dynamic stock balance, denoted by $\dot{S}(t)$.

$$\dot{S}(t) = (R - r) - \frac{a - c + \sigma \cdot S(t)}{b} \tag{3.77}$$

The steady state is a specific situation in which the flows feeding and leaving the groundwater stock are balanced, i.e., $W(t) = R - r$, and, therefore, the changes in the groundwater stock amount to zero for all time periods, i.e., $\dot{S}(t) = 0$, which is also called the steady-state condition. Based on this assumption and Eq. (3.77), the size of the water stock, S^*, which satisfies the steady-state condition, $\dot{S}(t) = 0$, can be identified (see Eq. (3.78)).

$$\begin{aligned} \dot{S}(t) &= (R - r) - \frac{a - c + \sigma \cdot S(t)}{b} && (3.77) \\ \Rightarrow \quad 0 &= (R - r) - \frac{a - c + \sigma \cdot S(t)}{b} \\ \Rightarrow \quad S^* &= \frac{b \cdot (R - r) - (a - c)}{\sigma} && (3.78) \end{aligned}$$

A better understanding of the mechanism can be gained by formally proving the stability of the steady-state situation. If there exist a stable steady state, the groundwater stock, $S(t)$, must converge against the steady-state stock, S^*, in the long run, regardless of its deviation from the steady state, $S(t) - S^*$.[38] Hence, Eq. (3.77) is reformulated, which results in Eq. (3.79).

$$\dot{S}(t) = (R - r) - \frac{a - c + \sigma \cdot S(t)}{b} \tag{3.77}$$

[38] If the present level of groundwater stock is given by S_0, the deviation from steady-state groundwater stock is $S_0 - S^*$.

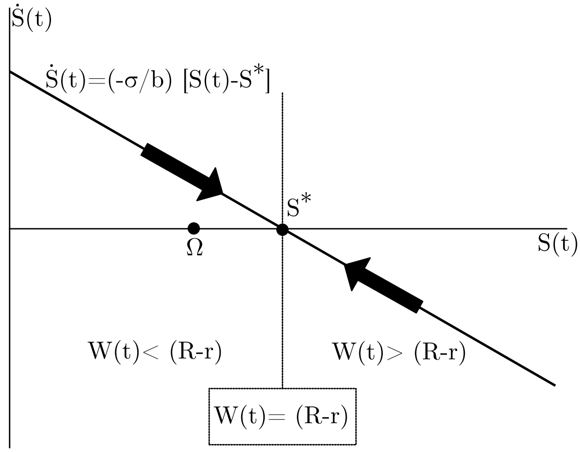


Fig. 3.16 Phase diagram of the dynamic stock balance. *Source* own illustration

$$\Rightarrow \quad \dot{S}(t) = \left(-\frac{\sigma}{b}\right) \cdot \left[\frac{a}{\sigma} - \frac{c}{\sigma} + S(t) - \frac{b}{\sigma} \cdot (R - r)\right]$$

$$\Rightarrow \quad \dot{S}(t) = \left(-\frac{\sigma}{b}\right) \cdot \left[S(t) + \frac{(a - c) - b \cdot (R - r)}{\sigma}\right] \tag{3.79}$$

By plugging Eq. (3.78) into Eq. (3.79), it is possible to derive a functional form of the dynamic stock balance, in which the temporal changes of the groundwater stock, $\dot{S}(t)$, depend on deviations from the steady-state groundwater stock, $S(t) - S^*$, such that

$$\Rightarrow \quad \dot{S}(t) = \left(-\frac{\sigma}{b}\right) \cdot \left[S(t) - S^*\right] \tag{3.80}$$

Based on Eq. (3.80), it can be proven that the steady state is a stable point to which the groundwater stock converges over time, because

- If the groundwater stock is below the steady-state stock, which means algebraically $\left[S(t) - S^*\right] < 0$, the temporal change in the groundwater stock, $\dot{S}(t) > 0$, is positive, hence the groundwater stock will increase over time.
- By contrast, if the groundwater stock exceeds the steady-state stock, i.e., $\left[S(t) - S^*\right] > 0$, the groundwater stock will decrease over time because the temporal change in groundwater stock, $\dot{S}(t) < 0$, is negative in this case.

By means of this stability analysis, it is possible to state that the groundwater stock, $S(t)$, converges to the steady-state stock, S^*, and therefore the latter can be described as the long-term groundwater stock. The higher the deviation of the current stock from its steady state, defined as $\left|\left[S(t) - S^*\right]\right|$, the higher is the temporal change in the groundwater stock, $\left|\dot{S}(t)\right|$, which implies a faster convergence in direction of the steady-state stock S^*.

The phase diagram of the dynamic stock can be found in Fig. 3.16. To calculate the steady-state stock, Eq. (3.78) is used as a starting point.

$$S^* = \frac{b \cdot (R - r) - (a - c)}{\sigma} \tag{3.78}$$

3.8.3 Hydrological and Ecologic Effects

After having studied the dynamic properties of the model, we would like to study the hydrological and ecologic effects of the open- access groundwater economy. To this end, we introduce a threshold value Ω. This threshold value represents the height of the critical groundwater level. This results in two cases:

- If $S^* \geq \Omega \quad \Rightarrow \quad b \cdot (R - r) - (a - c) > \sigma\Omega$, the long-term water stock does not fall below the threshold value. Overexploitation does not occur in the addressed aquifer. The microclimate, the vegetation, and other hydrological functions remain stable. Notice, that this case does not occur due to a common water management oriented toward sustainability goals but simply because pumping costs are high. The water cycle and the environment is protected by the low productivity of the pumping technology.
- However, if $S^* < \Omega \quad \Rightarrow \quad b \cdot (R - r) - (a - c) < \sigma\Omega$, the noncooperative use of water would lead to a regional ecologic disaster. In the long run overdraft will occur and, as a result, detrimental repercussions on regional climate, soil quality, and the local hydrological cycle will set in. These effects may be irreversible in nature. In ecology one speaks of hysteresis. Even if at a later stage common efforts are made to reverse the destruction process, it may be too late, i.e., the original environmental state can no longer be regained.

The model-based analysis conducted in this subsection allows us to draw conclusions about scarcity issues, which can result from overexploitation of an aquifer whose access is not regulated or limited. The overexploitation risk does not hinge on the cost rate parameter, σ, that characterizes the impact of the size of the water stock on the cost of extraction, but potential overexploitation relates to a multitude of other parameters. In the simple toy model presented in this subsection, the risk of overexploitation increases with the change of certain parameters as listed in Table 3.4.

3.9 Water Transfer Between Watersheds

3.9.1 Inter-basin Water Transfer Schemes

Infrastructure-based water transfer is a common instrument of water supply side management, which is applied in many regions of the world. The transfer is conducted by means of a water supply network containing pipelines, pump stations,

Table 3.4 Influence of certain parameters on risk of overexploitation

Parameter name	Parameter symbol	Effect on risk of overexploitation
Water availability spent by the nature	$(R - r)$	$\Uparrow$
Choke price of aggregated water demand function	a	$\Uparrow$
Steepness of aggregated water demand function	b	$\Downarrow$
Pumping cost rate from theoretically empty aquifer	c	$\Downarrow$

tanks, etc. It is possible to differentiate between intra-basin and inter-basin transfers, or between intra-regional and interregional/international transfers if hydrological or political boundaries are addressed, respectively.[39] The water obtained by transfers represents an additional source of water supply in the regions importing water. This additional water source is often quite necessary in the region receiving water to close the regional gap between the obtainable amount of water from local sources and the local requirements for water supply. Climate change, which expresses itself through decreasing water availability and increasing risk of drought, and increasing water requirements, resulting from population and economic growth, may further exacerbate water scarcity in some regions of the world. Consequently, water transfers, as a means for mitigating the adverse consequences of water shortage, will presumably gain importance in the future. In some cases, water transfer may be a cheaper source than alternative water supply management measures, e.g., reclaimed water or desalination.

There are many large-scale water transfer schemes around the world, most of them implemented in North America, Asia, and Australia. Important ones are, for example, the California State Water Project, the Colorado River Aqueduct, the San Juan-Charma Project (all in the US), the Lesotho Highland Water Project in Sub-Saharan Africa, the National Water Carrier in Israel, the Telugu Ganga Project in India, the South-North Water Transfer Project in China, or the Goldfield Water Supply Scheme in Australia.

Water transfers affect the local ecology, especially in the water- exporting region, due to interference with the flow regime of the water body. Furthermore, there are also some economic effects that impact water consumers and suppliers in water importing and exporting regions. These economic consequences are explained in this subsection with the help of a simple but illustrative model. Note that inter-basin transfers are often criticized by environmental organizations because they may

[39]The boundaries of the water basin are determined by a river basin or an aquifer basin. Intra-regional water transfers denote the transport within one region while interregional transfers refer to transfers from one region to another. International transfers describe water transfers between two states and, by definition, an international transfer is always an interregional transfer.

deliver much less benefits and more harm than anticipated, which is illustrated by the box in this subsection.

Box 3.7 Negative impacts of inter-basin water transfer

Inter-basin water transfers have been criticized by environmental organizations for several reasons. This is because, the development of inter-basin transfers has the potential to disturb the water balance in both the donating and the receiving region. In the past, certain inter-basin transfers have caused a disproportionate amount of damage to freshwater ecosystems in relation to the schemes' benefits. Negative social as well as economic impacts, especially for the donor basin, can also occur. Inter-basin transfers may not be the most cost-effective way of meeting water demand in the receiving region. Furthermore, inter-basin transfers do not encourage users in the receiving region to use the water more effectively, to recycle wastewater or to develop new local water sources for supply. According to WWF (2007), the following negative impacts can be observed in certain cases of inter-basin water transfers:

- Demand management in recipient basin is not sufficiently considered in preplanning for inter-basin transfer, leading to ongoing water waste.
- Inter-basin transfers can become drivers for unsustainable water use in recipient's basin-irrigation and urban water use, and create strong dependence on inter-basin transfer in the recipient community.
- The proliferation of boreholes to access groundwater can lead to overexploitation of this resource, too.
- Inter-basin transfers can become a catalyst for social conflict between donor and recipient basins or with government
- Inter-basin transfers may not help the situation of the poor affected or displaced by it.
- Governance arrangements for inter-basin transfers can be rather weak, resulting in budget blow-out or corruption

Source: WWF (2007)

3.9.2 Transfer from Water-Rich to Water-Scarce Regions

Assume a situation with one water-rich region, denoted by region 1, where water is available in abundant quantities, and one water scarce region, named region 2, where water occurs only in small amounts. Moreover, it is assumed that benefit is maximized in both regions.[40] A graphical depiction of this problem can be found in Fig. 3.17.

In both regions, the local water producers extract a specific amount of water from their regional territory, while these quantities are designated by the variables w_1 and w_2. The extraction of those water volumes is associated with total extraction costs of $C_1(w_1)$ and $C_2(w_2)$, respectively. To reduce shortage in the water-scarce region 2, region 1 exports water to the importing region 2, where the amount of transferred water is represented by z.[41] The transfer causes specific cost of γ monetary units per transferred water volume unit, hence, the total transportation costs are $\gamma \cdot z$. The consumption level of region 1 and 2 are termed as w_1^C and w_2^C, respectively. Extracted water volumes in region 1, which are not exported, are consumed in region 1, thus $w_1^C = (w_1 - z)$, whereas consumption in the water-scarce region 2 equals the amount of water extracted by the local producer and the imported volumes of water, i.e., $w_2^C = (w_2 + z)$. Hence, the consumption level in each region depends on the extraction in the region and the water transfer, i.e., $w_1^C(w_1, z)$ and $w_2^C(w_2, z)$. The corresponding benefits, which arise from water consumption in the regions 1 and 2, are $B_1(w_1^C(w_1, z))$ and $B_2(w_2^C(w_2, z))$, respectively. Based on the IWRM approach, the following objective function can be set up for the explained case:

$$\max_{w_1, w_2, z} \quad B_1(w_1^C(w_1, z)) + B_2(w_2^C(w_2, z)) - C_1(w_1) - C_2(w_2) - \gamma \cdot z \tag{3.81}$$

The abstractable amounts in any region $i \in \{1, 2\}$ are restricted by a regional-specific amount of maximum sustainable water extraction, denoted by w_i^{SUS} and defined in Eqs. (3.82) and (3.83), that may be determined according to locally varying ecological conditions, e.g., recharge rates, local precipitation, etc.[42]

$$w_1 \leq w_1^{SUS} \qquad (\lambda_1) \tag{3.82}$$

$$w_2 \leq w_2^{SUS} \qquad (\lambda_2) \tag{3.83}$$

[40]The number of suppliers and consumers is irrelevant to this problem. A situation is assumed where the benefit is maximized. This means, for instance, that a monopolist is not able to use its market power to set the monopoly price for maximizing its producer surplus because of the local price regulation.

[41]In line with the terminology used in this subsection, this situation is referred to as a water transfer from region 1 to region 2.

[42]An extraction below the maximum sustainable extraction amount ($w_i \leq w_i^{SUS}$) does not harm environment and/or (future) society and hence fulfills the intra-generation and inter-generation sustainability.

The residual KKT conditions are represented by Eqs. (3.84) to (3.88):

$$B_1'(w_1^C) - C_1'(w_1) - \lambda_1 \leq 0 \perp w_1 \geq 0 \tag{3.84}$$

$$B_2'(w_2^C) - C_2'(w_2) - \lambda_2 \leq 0 \perp w_2 \geq 0 \tag{3.85}$$

$$-B_1'(w_1^C) + B_2'(w_2^C) - \gamma \leq 0 \perp z \quad \geq 0 \tag{3.86}$$

$$w_1^{\text{SUS}} - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{3.87}$$

$$w_2^{\text{SUS}} - w_2 \geq 0 \perp \lambda_2 \geq 0 \tag{3.88}$$

If we assume that water scarcity is not present in the water-exporting region, it follows that $w_1 < w_1^{\text{SUS}}$ and hence $\lambda_1 = 0$ due to Eq. (3.87). But the producer(s) in region 2 extract the maximum sustainable amount, therefore $w_2 = w_2^{\text{SUS}}$ and hence $\lambda_2 \geq 0$ because of Eq. (3.88).

Thus, the marginal benefit from consumption should equal the marginal cost of production in the water exporting region, i.e., $B_1'(w_1^C) = C_1'(w_1)$, while the marginal benefit of the water importing region exceeds the marginal cost of production by the level of λ_2, which is $B_2'(w_2^C) = C_2'(w_2) + \lambda_2$. This shadow price, λ_2, constitutes the additional social welfare in the water-scarce region 2 that would be generated if the maximum extractable quantity of water, w_2^{SUS}, increased by one measurement unit.

If a water transfer is not feasible and cannot (or is not) realized due to technical, institutional, political, or other reasons, it is trivial to state that $z = 0$ and all the regions act self-sufficiently. In this case of self-sufficiency, the consumed amount is equal to the production level in the region, which means $w_1^C = w_1$ and $w_2^C = w_2$. We already know that the production and consumption amounts in region 1, which are depicted by w_1^A in Fig. 3.17, result from the intersection of the marginal benefit and marginal cost function, i.e., $B_1'(w_1) = C_1'(w_1)$. However, in region 2, where water is scarce by assumption, consumption quantities are equal to the maximum sustainable extraction level of the region, $w_2 = w_2^{\text{SUS}}$.

If a transfer from the water-rich to the water-scarce region is realized, we assume that $z \geq 0$. The transfer level z should at least be large enough such that the marginal benefit in the importing region 2 exceeds the marginal benefit in the exporting region 1 by the water transportation cost rate, i.e., $B_2'(w_2^C) = B_1'(w_1^C) + \gamma$. The optimal regional volumes of water extraction and the optimal transfer are illustrated by w_1^*, w_2^{SUS} and z^* in Fig. 3.17. The optimal consumption levels in the regions 1 and 2 are therefore $w_1^C = w_1^* - z^*$ and $w_2^C = w_2^{\text{SUS}} + z^*$, respectively, which are also illustrated in Fig. 3.17. In the following, we term the optimal consumption level in region 1 with $w_1^* - z^*$ and the optimal consumption level in region 2 with $w_2^{\text{SUS}} + z^*$.

Compared to the result obtained for self-sufficiency, the existence of transfers causes an increase of the water price in region 1 from $B_1'(w_1^A)$ to $B_1'(w_1^* - z^*)$, a rise in the quantity of extracted water from w_1^A to w_1^*, and a decrease in the level of consumed water from w_1^A to $w_1^* - z^*$. Hence, the surplus of consumers in that region is reduced by the area DH as a consequence of higher water prices and less water consumption, whereas the producers' profits rise by the area CDH, as illustrated in Fig. 3.17. The loss of consumer surplus is compensated completely by an increase in

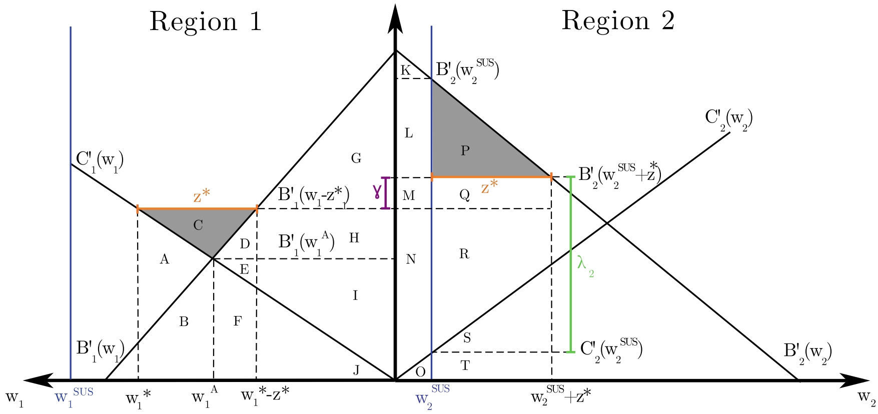


Fig. 3.17 Transfer from Water-rich to water-scarce region. *Source* own illustration

producer surplus, and hence the area C outlines the additional social welfare gained in the water exporting region due to the implementation of water transfers.

In reverse to the effects of water transfers occurring in the exporting region, in which consumers lose and producers gain social welfare, in the water importing region water transfers lead to a decrease in water prices from $B'_2(w_2^{\mathrm{SUS}})$ to $B'_2(w_2^{\mathrm{SUS}} + z^*)$, and an increase in water consumption from w_2^{SUS} to $w_2^{\mathrm{SUS}} + z^*$ compared to the case of self-sufficiency. Water extraction is not affected by transfer as the maximum sustainable amount is already extracted under self-sufficiency. Consequently, consumers gain due to lower prices and higher consumption levels, and producers lose profits because they face lower prices. The graphical depiction can be found in Fig. 3.17, where the gain of consumer surplus is illustrated by the area LP, and the loss of producer profits is represented by the area L. Therefore, the triangle P represents the overall gain in social welfare due to water transfers in the water importing region. Furthermore, the generated revenues in the water importing regions, represented by area Q, are used to cover the total transportation cost of water, which is $\gamma \cdot z^*$. The residual revenues from the water transfer, illustrated by areas RST, are used to cover the production cost of transferred water, i.e., areas ABF, to compensate the loss of producer profits from selling water to the consumers in region 1 in the self-sufficient case, which is area E, and to generate additional profits from selling exported water at increased prices, i.e., $B'_1(w_1^* - z^*) - B'_1(w_1^A)$, which are areas CD. A summarizing overview of the impacts that an implementation of a water transfer scheme has on social welfare in both regions is given in Table 3.5.

3.9.3 Transfer Between Two Water-Scarce Regions

In contrast, if a situation was assumed where water resources are also quite limited in the water exporting region the water extraction may equal the sustainable water extraction rate, $w_1 = w_1^{\mathrm{SUS}}$. Additionally, because of the scarcity in the water importing region, the extraction rate in this region is equal to the sustainable extraction,

Table 3.5 Distributional effects due to water transfers

		Area in region 1 (exporting region)	Area in region 2 (importing region)
Consumer surplus	Self-sufficient	DGH	K
	Transfer	G	KLP
	Change	$\boldsymbol{-DH}$	$\boldsymbol{+LP}$
Producer surplus	Self-sufficient	EI	MNL
	Transfer	$CDEHI$	MN
	Change	$\boldsymbol{+CDH}$	$\boldsymbol{-L}$
Change of social welfare		$\boldsymbol{+C}$	$\boldsymbol{+P}$

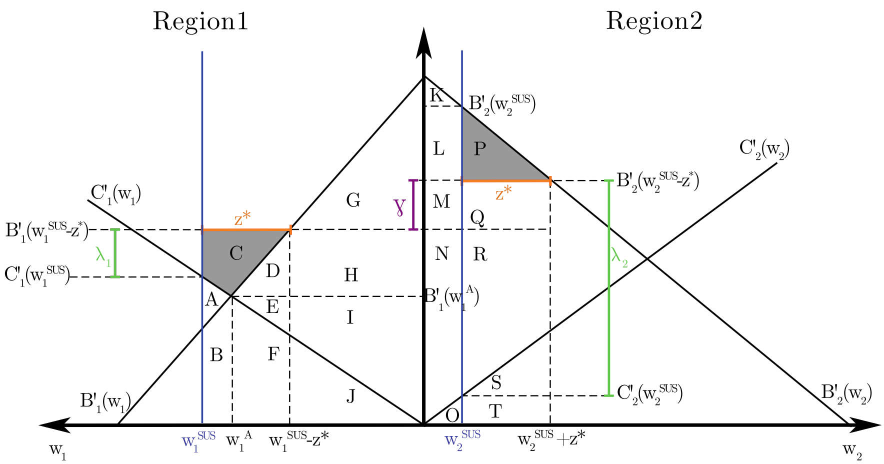


Fig. 3.18 Water transfer between water-scarce regions. *Source* own illustration

$w_2 = w_2^{\text{SUS}}$, which is equal to the former explained case. Compared to the previously explained scenario, the only alteration is the fact that the extraction amount in region 1 is restricted by the available water, i.e,. $w_1 = w_1^{\text{SUS}}$. Therefore, Eq. (3.87) is binding, this results in the fact that we assume $\lambda_1 \geq 0$. Because of Eq. (3.84), the marginal benefit of consumption exceeds the marginal cost of water extraction in the water exporting region 1, i.e., $B_1'(w_1^C) = C_1'(w_1) + \lambda_1$. The value of the shadow price λ_1 shows the increase in benefits if sustainable extraction w_1^{SUS} was to theoretically be increased by one unit. All the previously described relations, $B_2'(w_2^C) = C_2'(w_2) + \lambda_2$ as well as $B_2'(w_2^C) = B_1'(w_1^C) + \gamma$, are still valid for this addressed scenario. Due to the assumption that the amount extracted in the region is known, $w_1 = w_1^{\text{SUS}} > 0$ and $w_2 = w_2^{\text{SUS}} > 0$, the values of the variables λ_1 and λ_2 can be calculated with Eqs. (3.84) and (3.85), respectively. Finally, the value of the optimal transfer z^* can be found from Eq. (3.86). Therefore, the consumption level in region 1 and 2 is $w_1^C = w_1^{\text{SUS}} - z^*$ and $w_2^C = w_2^{\text{SUS}} + z^*$, respectively. The scenario, in which water is scarce in both regions, is illustrated in Fig. 3.18.

It becomes obvious that producer surplus, consumer surplus, costs, welfare gains, etc. (for self-sufficiency and in the transfer case) are represented by the same areas as under the former scenario, where water was a non-scarce resource in the exporting region. Hence, distributional effects are similar for both explained scenarios in this section and are summarized in Table 3.5.

3.10 Water Quality Management

3.10.1 Water Pollution: An Unresolved Issue

Even in Europe, with its highly developed infrastructure, water pollution does prevail to a reckoned extent. The Synthesis Report 2015 of the European Environmental Agency adds the water quality issue to the list of environmental problems not yet abolished:

> Much cleaner than 25 years ago, many water bodies are still affected by pollutants and/or altered habitats. In 2009, only 43% showed a good/high ecological status; the 10 points expected increase for 2015 (53%) constitutes only a modest improvement in aquatic ecosystem health.[43]

Good water quality refers not only to drinking water but also to water as a medium for recreational purposes, like fishing or swimming, and as a habitat for a healthy ecological system. There are many different sources of pollution affecting the water body negatively, be it surface water or groundwater. The main polluters are the industry, with its chemical pollutants and hazardous substances, the agricultural sector, with its runoff of nutrients (carbon, nitrogen, phosphorus), the urban sector, with households discharging mainly nutrients and fecal substances, as well as the medical sector releasing pharmaceutical residues. All these substances pollute the water through various chemical and biological chains and, as a result, deteriorate the human livelihood.

The European Parliament has enacted various directives with the purpose of protecting water. Article 4 (b) of the Water Framework Directive states

> Member States ensure, for surface water, the highest ecological and chemical status possible.

This goal shall be implemented with a regulation framework, which is established in Article 8, according to which

[43] Synthesis Report "The European environment—state and outlook 2015", see www.eea.europa.eu/soer-2015/europe/freshwater.

> Member States shall ensure the establishment of programs for the monitoring of water status in order to establish a coherent and comprehensive overview of water status within each river basin district.

Achieving an effective water regulation is a complex task. While the regulation of piped drinking water and of cleared water from waste water treatment plants is manageable, other sources of water contamination are more difficult to regulate. Specifically, agricultural non-point pollution is difficult to monitor almost by definition. The sole introduction of water quality standards is not sufficient to secure the water bodies, and therefore indirect methods of regulation must be applied. For instance, regulation and monitoring of the use of various types of fertilizers and herbicides has to be established.

Of course pursuance of these goals and the implementation of proper regulation instruments entail costs. The European Water Framework Directive is rather explicit with regard to these costs (Article 9):

> Member States shall take account of the principle of recovery of the costs of water services, including environmental and resource costs.

Box 3.8 Important parameters for identifying water quality

There exist various biological and chemical parameters to evaluate the quality of water and wastewater. For instance, the biological oxygen demand (BOD_5) or the chemical oxygen demand (COD) are important sum parameters corresponding to the concentration of organic substances in a certain water sample.

Total organic carbon (TOC), dissolved organic carbon (DOC), and particulate organic carbon (POC) are further parameters for organic bound carbon, which contains all organic substances. Nitrogen compounds are also important parameters for the evaluation of water quality. Industrial and domestic wastewater is characterized by high concentrations of reduced nitrogen (ammonium and ammonia, being NH_4, and NH_3, respectively). This form of nitrogen demands oxygen and is toxic to many aquatic and nonaquatic living organisms. The oxidation of reduced form nitrogen (ammonium and ammonia) is termed nitrification which is an autonomous biochemical process and also a treatment step in wastewater purification plants, where these reduced nitrogen compounds are oxidized to nitrite (NO_2) firstly and afterward to nitrate (NO_3). Nitrite is usually an intermediate in the nitrification process, however, it is a quite toxic substance. Nitrate is unwanted in potable water and it is also a nutrient in water bodies that causes the growth of algae, which is called eutrophication. This eutrophication can lead to the death of the aquatic livings in water bodies, hence if the concentration of nitrogen is sufficiently high it can be seen as a chronic toxic substance. Nitrate is usually emitted into water bodies by the agricultural sector because of fertilization (Sundermann et al.

2020). Nitrate can be degenerated into molecular nitrogen (N_2) during the denitrification process. Because of the harmful impact of nitrogen to water bodies, the denitrification process step should also be part of treatment in an adequate wastewater purification. Another nitrogen related sum parameter is the Kjeldahl-nitrogen which states the amount of nitrogen bound in organic substances plus ammonium. Like nitrogen, phosphorus is a nutrient that is usually the limiting factor for the growth of algae (eutrophication) in water bodies. In water, phosphorus occurs as ortho-phosphorus (salt of phosphoric acid) or as component in a nucleic acid (DNA, RNA).

A very essential physical–chemical parameter for evaluating the quality of water is the pH-value which impacts, for instance, the equilibrium of acids and bases and other chemical reactions in the water. The sensitivity of change of the pH-value, due to the addition of acids or bases, is represented by buffer capacities which are also water quality parameters.

The water hardness is a further chemical parameter which is quite important for many technical purposes and states the amount of dissolved calcium (Ca^{2+}) and magnesium (Mg^{2+}) ions in the water. Hardness of water has to be increased or decreased by specific technical processes, if the water is too soft or hard for the specific purpose, respectively.

Oxygen that is dissolved in water is the most important oxidizer for chemical processes in the water resource and impacts the kinds and composition of the aquatic livelihood. Further important physical water quality parameters are the turbidity, the electric conductivity, the temperature, the density, the viscosity, and sensory parameters (smell and flavor).

Microbiological parameters are very important indicators for pollution and for identifying the risk of water-related disease from the water source (hygienic reasons). There is a high number of various pathogens, germs, salmonellae, bacteria, etc., that can occur in water. Very important indicators for human-based pollution of freshwater sources are, for instance, the number of Escherichia coli, which is a bowel bacteria, and the number of enterococcus.

Source: Goncharuk (2014)

3.10.2 Water Quality Management

This section addresses the economic aspect of water quality. The water quality standards can be achieved with the help of an ecological-economic management approach. This requires deploying the tools of IWRM introduced in the preceding sections. In the following, we will present a simple water management model that includes some features of water quality regulation.

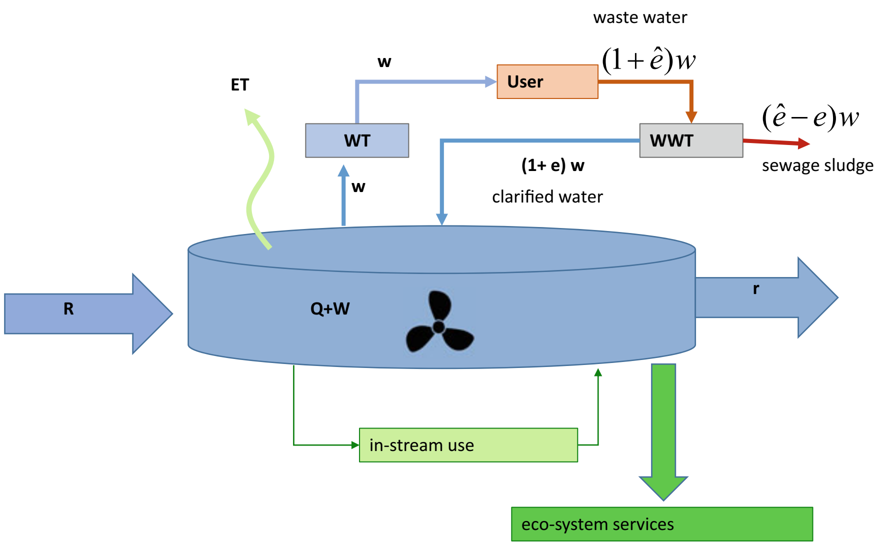


Fig. 3.19 Quantity-quality cycle. *Source* own illustration

3.10.2.1 A Model of Water Quality

Figure 3.19 displays the relationship between quantitative water flows and the discharge of pollutants. The disk depicts a water body, be it groundwater or a surface water reservoir. This water body will be recharged by a flow, denoted by R, which is assumed to consist only of clean water. The water body of volume V is of mixed quality as it contains clear water, W, and a pollutant, Q, i.e., $V = Q + W$. We assume that the reservoir is of equal quality, i.e., that the pollutant is evenly mixed in the water body, which is symbolized by the propeller. The water quality can be inversely defined with the help of the concentration of pollutants, α_Q.

$$\alpha_Q = \frac{Q}{W + Q} \tag{3.89}$$

The whole economy living in the modeled watershed is regarded as a single user that utilizes a specific water quantity, where the amount of water used is indicated by w. This water is provided by a water treatment facility that takes the water from the water body in order to clean and disinfect the water and convey it to the users as drinking water. To keep the model simple, we do not capture the quantity balance of this treatment unit, i.e., we do not determine the volume of pollutants removed from the non-treated water. After usage, the water is discharged from the user as wastewater containing a certain amount of pollutants. We assume that all water is returned. The portion of pollutants is $\hat{e}w$ where $\hat{e}$ is the concentration of the pollutant in the wastewater. The wastewater is directed into a wastewater facility, where it is treated and purified.

The total volume of clarified water is therefore $(1 + e)w$, where e, $e \leq \hat{e}$ is the pollution concentration after wastewater treatment. The residual, amounting to $(\hat{e} - e)w$, is sewage sludge, which will be dumped into a landfill. Finally, Fig. 3.19 indicates other modes of water use, namely, in-stream usage and other ecosystem services that are not explicitly modeled. The water regulation takes place in the form of a quality standard. The concentration of the pollutant shall not exceed a threshold, $\bar{\alpha}_Q$, prescribed by a water authority.

$$\alpha_Q \leq \bar{\alpha}_Q \tag{3.90}$$

The water quality of the reservoir depends not only on the performance of the waste water treatment (WWT) plant but also on the ability of the water body to self-purify, i.e., to dissolve the pollutants. This purification process is rather complex as it hinges on the water body itself, on local climate conditions and on the environmental surroundings. It is a natural process involving biological and chemical process that are interdependent and very difficult to model due to their nonlinear interconnections.[44] Here, it is sufficient to maintain the model linear just to get a basic understanding of these interdependencies. However, care must be taken when balancing pollutants and pure water.

The water leaving the WWT can be decomposed into pure water and the amount of pollutant which consists of[45] $e \cdot w$ additional discharge of pollutant and the pollutant already dissolved in the water at the time of abstraction, i.e., $\alpha_Q w$, totaling an amount of $(e + \alpha_Q)w$ which is returned into reservoir where the pollutants are partially neutralized. This process can be represented in a resorption function.[46] We introduce an resorption function

$$\dot{Q}(t) = -\pi Q(t) + (e + \alpha_Q)w - \alpha_Q r - \alpha_Q x = -\pi Q(t) + ew - \alpha_Q r \tag{3.91}$$

The volume of pollutant in the water reservoir increases by the discharge of pollutants, $ew + \alpha_Q w$, and decays with the rate π due to chemical and biological purification processes. In addition, the portion α_S of the total runoff r, which consists of clean water and pollutants, decreases the stock of pollutants. In this simple model, $\alpha_Q r$ is what hydrologists call advection, i.e., the transported mass of dissolved pollutants that is carried through a water body. Finally, one has to subtract the pollutant removed from the water body when the water of volume w is abstracted. This is the last item on the right side of the middle term $-\alpha_Q w$.

[44] An introduction into water quality modeling can be found in Loucks and van Beek (2005).

[45] Notice that the water withdrawn from the reservoir w is mixed consisting of $\alpha_Q w$ pollutant and $(1 - \alpha_Q)w$ pure water. Thus, we can decompose exactly, what portion of the redirected water is pure water and what pollutant, i.e. $(1 + e)w = (1 + e)[\alpha_Q w + (1 - \alpha_Q)w]$. Multiplying yields $\alpha_Q w + e\alpha_Q w + ew - e\alpha_Q w + (1 - \alpha_Q)w$. The first four items belong to the discharge of pollutant, reducing to $(e + \alpha_Q)w$, whereas the last term is the amount of pure water returned to the reservoir.

[46] A resorption function mathematically describes the self-purification capacity of a water body.

Similarly, we can establish a dynamic relation for pure water

$$\dot{W}(t) = R - \gamma_1 W(t) + \pi Q(t) - (1-\alpha_Q)r - (1-\alpha_Q)w + (1-\alpha_Q)w \quad (3.92)$$

The last two terms cancel each other. The first represents the removal of pure water, the latter the redirection after the purification process of the water used. The volume of clean water increases with recharge, R, and decreases with evapotranspiration. Notice that the pollutant cannot evaporate by assumption. The absorption process decomposes the harmful pollutants to clean water. Thus, the balance equation includes this process with the term $\pi Q(t)$. Finally, the run off is also carrying away clean water. Since the water body is assumed to be evenly mixed, this runoff can be captured by $(1-\alpha_Q)r$.

3.10.2.2 Policy Instruments

The analysis of the effects of various policy instruments to regulate the water quality requires dynamic optimization methods in order to satisfy the dynamic balance equations. These methods allow to derive optimal time paths of the relevant variables of the IWRM approach. Starting from given values of Q and W in the first time period, we can find optimal policy instruments, e.g., effluent charges or technology standards for the WWT plant for each point in time. In the long run, these variables would converge to constant values, which characterize the steady-state solution. For an introductory textbook, it is sufficient to confine the analysis to the steady state. Therefore, we assume that the hydrology of the watershed under consideration is in equilibrium at the outset of the analysis. Thus, setting $\dot{Q}$ and $\dot{W}$ equal to zero yields the equations

$$\pi Q = ex - \bar{\alpha}_Q r \quad (3.93)$$

$$\gamma_1 W = R + \pi Q - (1-\bar{\alpha}_Q)r \quad (3.94)$$

where the bar on $\bar{\alpha}_Q$ represents the quality standard for the water body imposed by the water authorities (see Eq. (3.89)). The water quality management must assure that the quality standard for the water body is held. This can be achieved by regulating the clarified water $(1+e)w$. From Eq. (3.89), it follows

$$W = \frac{1-\bar{\alpha}_Q}{\bar{\alpha}_Q} Q \quad (3.95)$$

Inserting Eq. (3.95) into Eq. (3.94) yields

$$Q = \frac{\bar{\alpha}_Q(R-(1-\bar{\alpha}_Q)r)}{\gamma_1(1-\bar{\alpha}_Q) - \pi\bar{\alpha}_Q} \quad (3.96)$$

which inserted into Eq. (3.93) gives

$$ew \leq \Phi := \frac{\bar{\alpha}_Q(\pi(R-r) + \gamma_1(1-\bar{\alpha}_Q)r)}{(\gamma_1(1-\bar{\alpha}_Q) - \bar{\alpha}_Q\pi)} \quad (3.97)$$

The left-hand side is the *net* pollutant load consisting of the pollutant load $(e + \bar{\alpha}_Q)w$ leaving the wastewater treatment facility minus the abstracted load $\bar{\alpha}_Q w$ that must not exceed a limit value Φ so as to secure the quality standard of the water reservoir $\bar{\alpha}_Q$.[47] The determination of the limit value requires that the hydrological relationships are known. Equation (3.97) gives the water withdrawal constraint for w that guarantees that the quality standard of the water body is met.

3.10.3 Optimal Water Quality

3.10.3.1 Model

The water quality management seeks to meet the given quality standard, $\bar{\alpha}_Q$, in an optimal way. Let us assume that the benefit of using water can be captured by the usual benefit function, $B(w)$, with the usual properties. To keep the model simple, we neglect the water treatment assuming that the water quality of the reservoir is potable. However, waste water treatment has to be taken into account.

The costs for waste water treatment are summarized in the following convex cost function:

$$C = C_{WWT}(w, e), \quad C_w > 0, \; C_e < 0 \tag{3.98}$$

Costs increase with the amount of waste water to be treated. On the contrary, if the quality of cleared water released decreases, i.e., e increases, then costs decrease. Having introduced all relevant elements the optimization program can be stated

$$\max_{w,e} \, [B(w) - C(w, e)] \quad \text{s.t.} \quad we \le \Phi \tag{3.99}$$

leading to the optimality conditions

$$B_w(w) - C_w(w, e) - \lambda e = 0 \tag{3.100}$$

$$-C_e(x, e) - \lambda x = 0 \tag{3.101}$$

Inserting Eq. (3.101) into Eq. (3.100) yields the equation

$$B_w(w) = C_w(w, e) - \frac{e}{w} C_e(w, e) \tag{3.102}$$

which can be utilized together with the constraint $ew = \Phi$ to determine the optimal values $\{w^*, e^*\}$. Figure 3.20 depicts the optimality condition given in Eq. (3.102).

[47] Notice that the denominator of the right-hand side must be positive. Otherwise, the steady-state solution would be negative which makes no sense. It can be shown that the denominator is always positive if the system of the two differential equations is stable. For the following analysis, these details are not important.

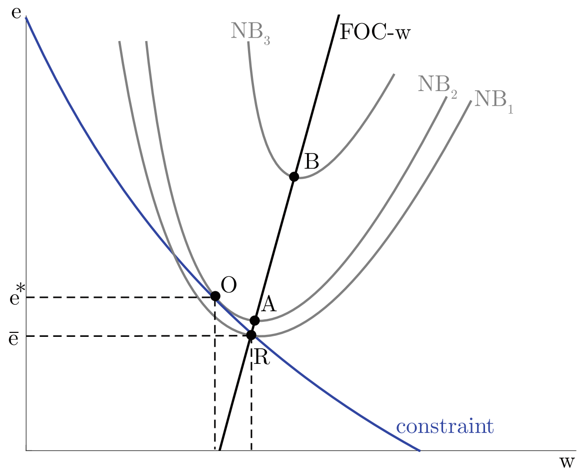


Fig. 3.20 Optimal water quality. *Source* own illustration

The NB_i-curves indicate the $w - e$-combination for constant benefit value, where $NB_1 < NB_2 < NB_3$ (iso-benefit-lines).[48] The higher e for constant w, the lower the costs and, hence, the higher are benefits. This monotonicity does not apply for w because the benefit function is concave and the cost function is convex with respect to w. Thus for every e fixed there exists an optimal value $\hat{w}(e)$ maximizing net benefit. The line $FOC - w$ shows these values. Graphically, these optimal values are at the point where the iso-net-benefit lines have their minimum (points R, A and B for example).

The black line shows Eq. (3.97), i.e., the water quality constraint. The optimal value $\{w^*, e^*\}$ can be found graphically: Increase net benefit as much as possible without violating Eq. (3.97). Obviously, this is point O.

3.10.3.2 Policy Instruments

The program defined by Eq. (3.99) is the reference point for the assessment of various policy options. These options can be evaluated with respect to their *efficacy*, i.e., their potential to secure water quality standards, and with respect to economic *efficiency*, i.e., their potential to assure the water quality standards economically. Three policy options are discussed, namely, technology standards, economic incentives, and water quality trading schemes.

To begin with the technology standard, the water authorities prescribe certain *technology standards* related to the quality of purified waste water released to the receiving waters. In our model, the clarified water $(1 + e)w$ is returned into the water body. The authority requires from the WWT plant to deploy technological measures such that the pollutant per unit of water released does not exceed a concentration

[48] The quadratic form is due to the fact that both the benefit function and the cost function are quadratic: $B(w) = aw - (b/2)w^2$ and $C(w, e) = mw^2(E - e)^2$ where a, b, m, and E, $E > \hat{e}$, are constants.

of $\bar{e}$. To guarantee an overall water quality of $\bar{\alpha}_Q$, $\bar{e}$ must be set such that $w(\bar{e} + \bar{\alpha}_Q) = \Phi$ (see Eq. (3.97)). This task requires a significant amount of information because the authorities have to anticipate how much water will flow through the water infrastructure given the technology standard.

The integrated water sector will maximize the net economic benefit for a given standard $\bar{e}$:

$$\max_{w} [B(w) - C(w, \bar{e})] \tag{3.103}$$

The optimality condition is

$$B_w(w) = C_w(w, \bar{e}) \tag{3.104}$$

From Eq. (3.104) the water quantity, $\hat{w}$, can be derived as a function of $\bar{e}$. The water authority sets the standard such that $\hat{w}(\bar{e})(\bar{e} + \bar{\alpha}_Q) = \Phi$. Thus, the technology standard approach is effective in that it secures the overall water quality standard $\bar{\alpha}_Q$. This is point R in Fig. 3.20. Here, the constraint (blue line) is satisfied and, at the same time, net benefit is maximized. However, if we compare this point with point O we see that that the allocation $\{\hat{w}(\bar{e}), \bar{e}\}$ is not optimal. The ecological constraint can also be met at O with higher net benefits. The concentration regulation leads to more water withdrawal $\hat{w}(\bar{e}) > w^*$ that is too clean $\bar{e} < e^*$.

A second management instrument is to employ *economic incentive mechanisms*. One instrument that provokes reactions from economic agents are prices. In our setting, effluent taxes serve as the price component. Let us return to the integrated water sector consisting of the water treatment (WT) plant and WWT facility and assume that the total load of pollutants, $(e + \bar{\alpha}_Q)w$, will be taxed.

The following net benefit function will be optimized by the water sector:

$$\max_{w,e}[B(w) - C(w, e) - \tau we] \tag{3.105}$$

where τ is the net effluent charge.[49] Contrary to the technology standard case, the water sector can decide on both variables, w and e. The optimality conditions are identical to those of the integrated water quality approach in Eq. (3.102) and $we = \Phi$ if the authority sets $\tau = \lambda$. Then the values $\{w(\tau), e(\tau)\}$ maximizing Eq. (3.105) are identical to the optimal solution $\{w^*, e^*\}$. Of course, the water authority has to process huge amounts of information, rather similar to the technology standard case. The authority must observe the pollutant loads released by the WWT facility and, at the same time, the amount of pollutants withdrawn during water abstraction w.

But even if this information is available, the authority cannot fix the optimal effluent charge without knowing the benefit function and the cost function. Thus, a trial-and-error approach is required, such that the regulatory authority introduces an

[49] Recall that the net pollutant discharge is $w(e + \bar{\alpha}_Q) - \bar{\alpha}_Q)w$.

initial effluent charge based on the information available. If total effluents violate the water quality constraint, the authority increases the charge and repeats this procedure until the overall water quality standard is met. Obviously, this procedure cannot last too long, because a prolonged adjustment time could lead to indirect hydrological and ecological effects associated with severe damages to the environment.

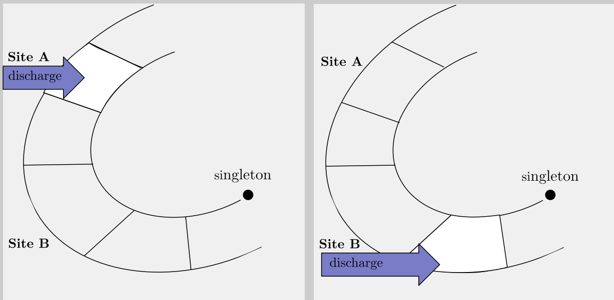
There is also another caveat to mention. Our simple model does not take the very complex diffusion process of pollutants into account. In reality, one has to tackle with stochastic fluctuations and also with complex patterns of spatial distributions of effluents. As a result, effluent charges have to be spatially differentiated and also flexible in time. If this flexibility cannot be ensured a technology standard approach might be more efficient than setting economic incentives.

Some of the problems encountered in the framework of an effluent charge can be avoided with *water quality trading schemes*. Under this policy program, tradable permits are issued to effluent chargers. These permits can be traded leading to a market equilibrium. Effluent discharges are costly because they have to be covered by the permits bought. As a result, the water sector behaves like in the case of an effluent tax. However, the advantage of this policy framework could be that no interactive trial-and-error process takes place. The permit price adjusts to an equilibrium value guaranteeing that the criteria of efficacy and efficiency are satisfied. But water markets also have their specific problems. These will be addressed in Chap. 5. One of the few existing water quality trading schemes is described below.

Box 3.9 Water quality trading: The Hunter River Salinity Trading Scheme

The Hunter River Salinity Trading Scheme (HRSTS) was introduced by the Environmental Protection Agency of New South Wales (NSW-EPA), Australia, to regulate the salinity of the Hunter River. First, it was put in operation as a pilot in 1995 and later in 2002 legally established by a regulation act of the NSW-EPA. The Hunter River drains the largest catchment area in New South Wales. Along the river, a string of heterogeneous industries are located. There is an extensive agricultural sector consisting of wineries, dairy farming, vegetable cultivation and cattle farming. In addition, the Hunter valley counts over 20 coal mines (most of which are surface mines) and three power stations. The salinity of the river comes from natural sources like rocks and soil but also from the economic activities. The river water abstracted by the mines is pumped out and additionally charged with salt. Electricity generation needs water for cooling. Thereby water evaporates leading to a high salt concentration in the remaining water that is pumped back into the river. An increased salinity leads to economic and ecologic damages. Economic damages accrue to the agricultural sector as the water cannot be used for irrigation if the salt concentration exceeds a certain threshold. The ecologic damages were quite obvious. A too high salt concentration has detrimental effects on the ecologic system of the river as a habitat for many species. As a result, there was significant conflict between the various users of the river, specifically between the agricultural sector and the mining operators.

After a long time of fruitless clashes of interests, the NSW Department of Land and Water Conservation and the NSW-EPA introduced a system with dynamic and tradeable discharge permits. In contrast to other permit systems adopted in the context of global emissions, e.g., the EU-ETS, the system here had to be adapted to the specific hydrology of a river. It is important to keep the salinity of river under a threshold along its whole course. To do so, the regulatory authorities divided the river into three sectors: the upper, middle, and lower sector. For each sector, upper ceilings of salinity were determined. These ceilings depend on the flow intensity of the river. If the river has a low flow, no salt discharge is permitted. If the flow is high, then the permitted discharge is increased; this depends on the specific hydrological properties of the river sectors. The main goal of this spatially differentiated approach is to keep the salt concentration along the whole river within justifiable boundaries. To make the HRSTS work, a string of monitoring points along the river were deployed so as to enforce the regulation of salt concentration. Following figure shows how the concentration data is transformed to allowances for the dischargers.



Hunter river salinity trading scheme. *Source* NSW-EPA (2003)

The river is divided in floating blocks that move with the flow of the river downstream. Each block is controlled with respect to its salinity which is measured by the waters electrical conductivity (micro-siemens per cubic centimeter). The allowed discharges per block are calculated on the basis of a hydrological model such that the electrical conductivity does not exceed the threshold value introduced by the authority. These allowed discharges vary with the hydrological conditions, e.g., the flow intensity of the river in the different river sectors. The allowances are distributed to dischargers as “discharge credits”. In total, there are 1000 discharge credits expressed as per mill. One credit equals one per mill of the allowed discharge in a block.

As an example take one of the blocks in above figure. Site A releases a certain amount of salt (measured in tons). As the river flows the block moves downstream and flows along site B which can also release a certain amount of salt and so forth. After the last site, the block has resumed 1000 per mil of the allowed discharge. What makes this scheme efficient is that the allotted discharge credits can be traded. We could imagine that site A does not exploit all of her credits but instead sells some of these to site B allowing B to discharge more than what was allotted to her. This trade takes place as long as the abatement costs of an additional ton of salt is lower at site A compared to site B. It is less costly to avoid one ton of salt discharge at site A than at site B. Hence, both sides can benefit from trade. The market equilibrium is reached when the marginal abatement costs of both dischargers are equalized. From an economic viewpoint, the market equilibrium sustains a discharge pattern along the river that minimizes total abatement costs of all dischargers.

If we gather the experiences of the past years one can state that the HRSTS is a success story. The average salinity of the Hunter river dropped considerably and the former conflicts between the different economic sectors have been solved within an effective institutional framework.

Sources: NSW-EPA (2003), Muschal (2006), Krogh et al. (2013)

3.11 Exercises

Exercise 3.1 Maximizing agricultural output with return flows
Assume a river basin with two riparians. If water management only looks after efficiency in the sense of maximizing agricultural output in the entire basin, then the more productive farmer should get all the water. However, this optimization rule is not correct when the return flow occurs. Let's assume the upstream farmer F1 is less productive than the downstream farmer F2. However, a ratio of water diverted by F1 flows back in the water body and is afterward available for the downstream riparian F2. How should the policymaker allocate the available water of the river?

Let us assume that the available water in the river which can be diverted is given with $\bar{W} = 100$ units (m^3, or liters, or hectoliters). The productivity of farmer F2 and F1 is $a_1 = 0.75$ and $a_2 = 1.0$, respectively. The fraction of returned water of F1 is $h_1 = 0.5$ and that of F2 is zero ($h_2 = 0$).

There are two methods for finding the optimal allocation. Either the total amount of water is allocated completely to the riparian which has the highest productivity related to net abstraction, or an optimization problem is solved. We start with explaining the first method. The parameters a_1 and a_2 represent the productivity per abstracted water, hence $\frac{a_1}{(1-h_1)}$ and $\frac{a_2}{(1-h_2)}$ stand for the productivity per net-abstracted water.[50] By applying this approach

$$\frac{a_1}{(1-h_1)} = \frac{0.75}{0.5} = 1.5, \ \frac{a_2}{(1-h_2)} = \frac{1}{1-0} = 1 \ \Rightarrow \ \frac{a_1}{(1-h_1)} > \frac{a_2}{(1-h_2)}$$

we come to the solution that the upstream riparian F1 is most productive per one unit net-abstracted water, because $\frac{a_1}{(1-h_1)} > \frac{a_2}{(1-h_2)}$. Therefore, the upstream riparian should receive the total amount of available water which means, $w_1 = \bar{W} = 100$. Therefore, the agricultural output of riparian F1 is: $a_1 \cdot w_1 = 75$. After the consumption of F1, the return flow $h_1 \cdot w_1 = 50$ flows back to the river and is available for the downstream riparian F2. The downstream riparian diverts the amount of water which is available at its abstraction point, hence: $w_2 = \bar{W} - w_1 + h_1 \cdot w_1 = 50$. Therefore, F2 is able to produce $a_2 \cdot w_2 = 50$ agricultural products.

If the relations in a river basin are more sophisticated (e.g., more complex production functions), the first method may not work and the second method (solving an optimization problem) has to be applied. The optimization problem for maximizing the agricultural production in the basin has the following form:

$$\max_{w_1, w_2} [a_1 \cdot w_1 + a_2 \cdot w_2] \ , \ \text{s.t. } w_1 \leq \bar{W} \ , \ w_2 \leq \bar{W} - (1-h_1) \cdot w_1 \tag{3.106}$$

The objective is to maximize the agricultural production in the basin, while the constraints limit the amount of water which can be diverted which is determined by the available amount at the abstraction point of the riparian. Based on the optimization problem, we can derive the Lagrangian:

$$L = [a_1 \cdot w_1 + a_2 \cdot w_2] + \lambda_1 \cdot [\bar{W} - w_1] + \lambda_2 \cdot [\bar{W} - (1-h_1) \cdot w_1 - w_2] \tag{3.107}$$

and the KKT conditions (see Appendix A):

$$a_1 - \lambda_1 - \lambda_2 \cdot (1-h_1) \leq 0 \perp w_1 \geq 0 \tag{3.108}$$

$$a_2 - \lambda_2 \leq 0 \perp w_2 \geq 0 \tag{3.109}$$

$$\bar{W} - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{3.110}$$

$$\bar{W} - (1-h_1) \cdot w_1 - w_2 \geq 0 \perp \lambda_2 \geq 0 \tag{3.111}$$

[50] If the abstraction is represented by w_i, the net abstraction is $w_i - h_i \cdot w_i = (1-h_i) \cdot w_i$, where h_i stands for the return flow factor (share of diverted water which flows back to the water body after consumption). Therefore, the net abstraction can be calculated by multiplying the term $(1-h_i)$ with the abstraction.

Inserting the numerical values gives

$$0.75 - \lambda_1 - \lambda_2 \cdot (1 - 0.5) \leq 0 \perp w_1 \geq 0 \tag{3.112}$$
$$1 - \lambda_2 \leq 0 \perp w_2 \geq 0 \tag{3.113}$$
$$100 - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{3.114}$$
$$100 - (1 - 0.5) \cdot w_1 - w_2 \geq 0 \perp \lambda_2 \geq 0 \tag{3.115}$$

From Eq. (3.113), we can infer that $\lambda_2 > 0$, for meeting the condition $1 - \lambda_2 \leq 0$. Hence, by Eq. (3.115), we can infer that F2 uses all residual water, which means $w_2 = \bar{W} - (1 - h_1) \cdot w_1$. Because of the fact that $w_2 > 0$ and due to Eq. (3.113), we know that $1 - \lambda_2 = 0$. Therefore, we can specify the level of $\lambda_2 = 1$. Inserting this numerical value into Eq. (3.112) yields $0.75 - 0.5 - \lambda_1 \leq 0$, which means that $\lambda_1 > 0$. Hence, by Eq. (3.114) we can infer that F1 diverts all its available water, $w_1 = \bar{W} = 100$. Therefore, from Eq. (3.112) and the fact that $w_1 > 0$, we know that $0.75 - \lambda_1 - 0.5 = 0$, which means that $\lambda_1 = 0.25$. Furthermore, by inserting the facts that $\lambda_2 > 0$ and $w_1 = \bar{W} = 100$ in Eq. (3.115) it is possible to find $w_2 = 50$.

Exercise 3.2 Water demand with subsistence levels (Stone-Geary Utility Function) One way to introduce life lines into the theory of households is the Stone-Geary utility function. The Stone-Geary utility function is often used in the empirical investigation of water demand in developing and emerging countries where the life line consumption plays a significant role. An example is given by Dharmaratna and Harris (2012).

$$U(w, b) = (w - w_s)^{\alpha}(b - b_s)^{(1-\alpha)} \tag{3.116}$$

where w is water and b is bread (nutrition), w_s (b_s) are the respective subsistence levels.

The respective demand functions can be derived by the usual maximization approach of a household utility function. The Lagrangian is

$$L = (w - w_s)^{\alpha}(b - b_s)^{(1-\alpha)} + \mu[y - p_w w - p_b b] \tag{3.117}$$

Notice, that due to the properties of the Stone-Geary utility function $w > w_s$ and $b > b_s$. Thus, the KKT conditions reduce to

$$\alpha(w - w_s)^{\alpha-1}(b - b_s)^{(1-\alpha)} - \mu pw) = 0 \tag{3.118}$$
$$(1 - \alpha)(w - w_s)^{\alpha}(b - b_s)^{(-\alpha)} - \mu p_b = 0 \tag{3.119}$$
$$[y - p_w w - p_b b] \geq 0 \text{ and } [\cdots]\mu = 0 \tag{3.120}$$

By inspection of the first (or second) equation we see that $\mu > 0$ and thus income is completely exhausted. Dividing Eq. (3.118) by Eq. (3.119) leads to

$$\frac{\alpha}{1 - \alpha} \frac{b - b_s}{w - w_s} = \frac{p_w}{p_b} \tag{3.121}$$

The budget constraint can be rewritten as

$$p_w(w - w_s) + p_b(b - b_s) = y - p_w w_s - p_b b_s \tag{3.122}$$

Inserting Eq. (3.121) into the budget constraint yields the demand functions for w and b, respectively. The demand function for w is

$$\hat{w} = w_s + \alpha \cdot \left(\frac{y - p_w w_s - p_b b_s}{p_w} \right) \tag{3.123}$$

Specifically, the sensitivity with respect to a change of the water price is of interest. Let us first calculate the revenue a water supplier can collect, i.e.,

$$R(p_w) = p_w \hat{w} = p_w w_s + \alpha(y - p_w w_s - p_b b_s) \tag{3.124}$$

Differentiating the revenue with respect to the water price yields

$$R'(p_w) = (1 - \alpha) w_s \tag{3.125}$$

Thus, a rising price always leads to an increased revenue. We know from basic microeconomic calculations that a rising revenue comes from a water demand elasticity less than unity. Hence, due to the subsistence level water demand is inelastic. Moreover, we can derive that the price sensitivity of water demand increases with income y and vice versa. This is intuitively clear. A rich household can escape a price increase by simply reducing water demand and consuming other goods. A poor household consumes water close to the subsistence level and, hence, cannot escape the price increase by simply substituting other goods for water.

Exercise 3.3 What is rain from an economic point of view?
A feature of the hydrological cycle is that the water cyclically changes its aggregate state: from blue water to green water and then back to blue water. We consider blue water as a private good or as a common property. In both cases, consumption is rival. But what about the rain? If blue water is rivaling in its use, what about the green water, the water vapor and the rain? What is your reasoned opinion?

The answer can be given with the help of an example from the agricultural use of rainfall. We look at farms in a region where it rains. The natural irrigation of the agricultural crops is apparently non-rivaling. Of course, the rain strength does not have to be uniformly distributed across the cultivation areas of farmers. What is decisive is rather the following: What one farmer uses does not affect the others consumption. In addition, it is difficult to exclude a farmer from the rain if he does not want to pay for it. All in all, rain is a public good. If the observation period is extended, this result may change. The rainwater, which does not enter the plants on the field, evaporates partly and partly seeps into the groundwater. It can also be carried away by rivers from the region to the seaside. When rain percolates after its journey through the earth's surface, it becomes groundwater and it changes its economic property. The public good becomes a commodity whose consumption is rivaling. Now the question remains whether this water is a private good or a common property good. The answer to this question depends on the possibility to exclude those farmers from water use who are not willing to pay for it.

Exercise 3.4 The market for recycled water

In Sect. 3.6 we have discussed a system of two water markets (see Fig. 3.11). In market 1 treated freshwater from a surface water body or from groundwater is traded, while in market 2 recycled water which is properly treated is traded. Assuming two users (user 1 and user 2) and that markets operate under perfect competition, we can show that the market allocation is identical to the optimal allocation derived from Eq. (3.41) (see Sect. 3.6).[51]

In our numerical example, user 1 operates either in both markets or neither of them as we will derive below. This user 1 is on the demand side in the freshwater market (market 1), because this he/she aims to purchase freshwater at this market. After the consumption of this freshwater, a certain share of user 1's waste water could be recycled and offered by user 1 at the market for recycled water (market 2). Therefore, user 1 is at the supply side in market 2. User 2 is at the demand side of both markets, because this user is able to purchase freshwater in market 1 or to purchase recycled water in market 2. We assume the following benefit functions:

$$B_1(w) = a_1 \cdot w - 0.5 \cdot b_1 \cdot w^2, \quad \text{with } a_1 = 50, b_1 = 1 \tag{3.126}$$

$$B_2(w) = a_2 \cdot w - 0.5 \cdot b_2 \cdot w^2, \quad \text{with } a_2 = 100, b_2 = 1 \tag{3.127}$$

with w representing the respective consumption level of each user. User 1 consumes the amount purchased at the freshwater market (w_1), while user 2 consumes the amount which results from the sum of the water purchased at the freshwater market (w_2) and the purchased water from the market for recycled water (w_{12}).

Let us assume that the price in market 1 is fixed by a regulatory authority and set equal to marginal costs of water treatment, i.e., $p_1 = c = 80$. We further assume for simplicity that the recycling quota is $h = 1$, i.e., all water used by user 1 is recycled and offered at the market for recycled water (market 2).

User 1 maximizes net benefits according to Eq. (3.48) whereas user 2 maximizes net benefits defined in Eq. (3.52).[52]

To derive the demand curves of both users and the supply curve for treated waste water of user 1, respectively, we specify the KKT conditions.

51

$$\max_{w_1, w_2, w_{12}} [B_1(w_1) + B_2(w_{12} + w_2) - c(w_1 + w_2)] \tag{3.41}$$

52 User 1 buys water in market 1 and, at the same time, offers treated water in market 2 by solving the following optimization:

$$\max_{w_1, w_{12}} [B_1(w_1) + p_2 w_{12} - p_1 w_1] \quad \text{s.t.} \quad h_1 w_1 - w_{12} \geq 0 \tag{3.48}$$

For user 1, we have

$$a_1 - b_1 w_1 - p_1 + \lambda h_1 \leq 0 \perp w_1 \geq 0 \quad (3.128)$$

$$p_2 - \lambda \leq 0 \perp w_{12} \geq 0 \quad (3.129)$$

$$h_1 w_1 - w_{12} \geq 0 \perp \lambda \geq 0 \quad (3.130)$$

Inserting the given parameter values means

$$50 - w_1 - 80 + \lambda \leq 0 \perp w_1 \geq 0 \quad (3.131)$$

$$p_2 - \lambda \leq 0 \perp w_{12} \geq 0 \quad (3.132)$$

$$w_1 - w_{12} \geq 0 \perp \lambda \geq 0 \quad (3.133)$$

From Eq. (3.131), we can infer that user 1 will not demand water in market 1 (freshwater market) without reselling it in market 2. This comes from the fact that $\lambda = 30 + w_1 > 0$ for realizing $50 - w_1 - 80 + \lambda = 0$ (see Eq. (3.131)). Due to Eq. (3.133), the parameter λ only becomes positive, if $w_{12} = w_1$.[53] So we assume that values are positive. Below we will see that this assumption is correct.

The KKTs of the optimization program of user 2 (see Eq. (3.52)) are

$$a_2 - b_2(w_2 + w_{12}) - p_1 \leq 0 \perp w_2 \geq 0 \quad (3.134)$$

$$a_2 - b_2(w_2 + w_{12}) - p_2 \leq 0 \perp w_{12} \geq 0 \quad (3.135)$$

Inserting the given parameter values:

$$100 - (w_2 + w_{12}) - 80 \leq 0 \perp w_2 \geq 0 \quad (3.136)$$

$$100 - (w_2 + w_{12}) - p_2 \leq 0 \perp w_{12} \geq 0 \quad (3.137)$$

These two equations differ only in the two prices. If $p_1 = p_2 = 80$, both markets are equally good for user 2, which means that user 2 is indifferent in purchasing water from market 1 or market 2. For different prices, we have following result[54]:

$$\begin{cases} w_2 = 0 \text{ and } w_{12} > 0 & \text{for: } p_2 < p_1 = 80 \\ w_2 > 0 \text{ and } w_{12} = 0 & \text{for: } p_2 > p_1 = 80 \end{cases}$$

User 2 has the option to buy water in both markets, hence the corresponding optimization program is

$$\max_{w_2, w_{12}} [B_2(w_2 + w_{12}) - p_2 w_{12} - p_1 w_1] \quad (3.52).$$

[53] However, simply set $w_{12} = 0$ which implies $\lambda = 0$ by Eq. (3.133). Inserting the numerical values leads to a strict inequality in Eq. (3.131) so $w_1 = 0$.

[54] If $p_2 < p_1$ it follows that $[a_2 - b_2(w_2 + w_{12}) - p_2] > [a_2 - b_2(w_2 + w_{12}) - p_1]$. If we set $[a_2 - b_2(w_2 + w_{12}) - p_2] = 0$, we know that $[a_2 - b_2(w_2 + w_{12}) - p_1] < 0$. Hence, based on Eqs. (3.134) and (3.135), we are able to derive that $w_2 = 0$ and $w_{12} > 0$, respectively.

For the contrary case of $p_2 > p_1$ it follows that $[a_2 - b_2(w_2 + w_{12}) - p_2] < [a_2 - b_2(w_2 + w_{12}) - p_1]$. If we set $[a_2 - b_2(w_2 + w_{12}) - p_1] = 0$, we therefore know that $[a_2 - b_2(w_2 + w_{12}) - p_2] < 0$. Based on Eqs. (3.134) and (3.135), we are able to derive that $w_2 > 0$ and $w_{12} = 0$, respectively.

Thus, if $p_2 > p_1$ there is no demand in market 2 and, hence, the market does not exist. This means that under this condition user 1 cannot sell its treated waste water and therefore withdraws from both markets for the given numerical values. If $p_2 = p_1 = c = 80$ we know from Eq. (3.132) that $\lambda = p_2$, and hence $\lambda = p_1$. Inserting this relation in Eq. (3.131) leads to the follwing result:

$$a_1 - b_1 w_1 = 0 \quad \Rightarrow \qquad w_1 = a_1/b_1 = 50 \tag{3.138}$$

From Eq. (3.133), we know that $w_{12} = w_1 = 50$.

For user 2 it follows from Eq. (3.136)

$$a_2 - b_2(w_{12} + w_2) = p_1 \quad \Rightarrow \quad w_{12} + w_2 = \frac{a_2 - p_1}{b_2} \quad 50 + w_2 = 100 - 80 \tag{3.139}$$

which leads to $w_2 = -30$. This is a contradiction with the specification $w_2 \geq 0$ (see Eq. (3.136)).

Thus, for an equilibrium in both markets we must have $p_1 > p_2$, which means that the price for recycled water is lower than the price for freshwater. In the following, we would like to calculate the price p_2.

From Eq. (3.132), we know that $\lambda = p_2$. Inserting this relation in Eq. (3.131), we can derive the supply curve of user 1 in the market for recycled water:

$$a_1 - b_1 w_1 - p_1 + p_2 h_1 = 0 \quad \rightarrow \quad h_1 w_1 = \frac{h_1(a_1 - c + p_2 h_1)}{b_1} \tag{3.140}$$

From Eqs. (3.136) and (3.137), we already know for the case $p_1 > p_2$ that $w_2 = 0$ and $w_{12} > 0$. This means that user 2 purchases only recycled water from market 2 since it is cheaper than buying freshwater in market 1. The demand for recycled water (w_{12}) can be derived from Eq. (3.137):

$$a_2 - b_2(w_2 + w_{12}) - p_2 = 0 \quad \rightarrow \quad w_{12} = \frac{a_2 - p_2}{b_2} \tag{3.141}$$

Please note, that the quantity supplied is equal to the quantity demanded, hence $h_1 \cdot w_1 = w_{12}$. Equating supply (Eq. (3.140)) and demand (Eq. (3.141)) will yield the equilibrium market price:

$$p_2 = \frac{b_1 a_2 - h_1 b_2 a_1 + h_1 b_2 c}{b_1 + h_1^2 b_2} \tag{3.142}$$

If we insert the numerical values we get $p_2 = 65$. Recycled water with a price of $p_2 = 65$ is, therefore, cheaper than freshwater which can be purchased for a price of $p_1 = 80$. From the supply function in Eq. (3.140) of user 1, we can calculate the supply $w_{12} = h_1 w_1 = 35$.

It remains to show that the market solution is identical to the optimal allocation of program (3.41). Simply insert the market solution $h_1 w_1 = w_{12} = 35$ into

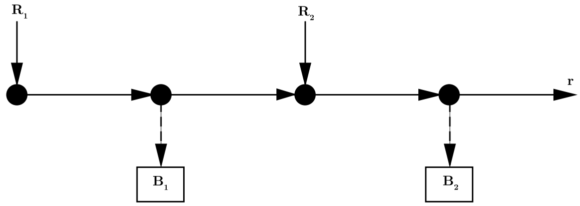


Fig. 3.21 Scheme of a river basin with 2 users. *Source* own illustration

(Eq. (3.42))—Eq. (3.45) for the assumed numerical values.[55] However, the requirement is to install a sufficient number of markets. In our case there must be two markets, one for freshwater and one for recycled water. Of course, there are some institutional intricacies. Usually piped water is offered by monopolies due to the cost advantages of a single supplier. The first welfare theorem is only valid if markets are fully competitive. Hence, to refer to the first welfare theorem is only legitimate if a regulation authority is able to control the price setting of water suppliers such that these prices are similar to those that result from a competitive market. We come back to this exercise in Chap. 4.

Exercise 3.5 Water allocation in a river

Assume there are two users who divert water from one river. The situation is illustrated by Fig. 3.21.

The marginal benefit function of the upstream user 1 is

$$B_1'(w_1) = a_1 - b_1 \cdot w_1, \text{ with: } a_1 = 100,\ b_1 = 1.5$$

while the one of the downstream user 2 is:

$$B_2'(w_2) = a_2 - b_2 \cdot w_2, \text{ with: } a_2 = 60,\ b_2 = 1$$

The variables w_1 and w_2 represent the consumption levels of user 1 and 2, respectively. We also assume headwater inflows of $R_1 = 50$ and $R_2 = 50$. The goal is to

[55] The social planner maximizes the following objective function:

$$\max_{w_1, w_2, w_{12}} [B_1(w_1) + B_2(w_{12} + w_2) - c(w_1 + w_2)] \quad (3.41)$$

$$\text{subject to: } h_1 w_1 - w_{12} \geq 0 \quad (3.39)$$

The following KKTs result from the social planner problem:

$$B_1'(w_1) - c + \lambda h_1 = 0 \quad (3.42)$$

$$B_2'(w_{12} + w_2) - c \leq 0 \perp w_2 \geq 0 \quad (3.43)$$

$$B_2'(w_{12} + w_2) - \lambda \leq 0 \perp w_{12} \geq 0 \quad (3.44)$$

$$h_1 w_1 - w_{12} \geq 0 \perp \lambda \geq 0 \quad (3.45)$$

.

calculate the optimal water allocation between the users where we maximize the benefit of the entire basin. The maximization goal is implemented by the objective function of the following optimization problem:

$$\max_{\{w_1,w_2\}} \; B_1(w_1) + B_2(w_2) \tag{3.143}$$

$$s.t. \; w_1 \leq R_1 \qquad (\lambda_1) \tag{3.144}$$

$$w_2 \leq R_1 + R_2 - w_1 \qquad (\lambda_2) \tag{3.145}$$

The constraints of the optimization problem restrict the amount of water which can be extracted. The extractable amount is limited by the water availability at the respective extraction points.

The following Lagrangian function can be set up on the basis of the optimization problem:

$$L = B_1(w_1) + B_2(w_2) + \lambda_1 \cdot [R_1 - w_1] + \lambda_2 \cdot [R_1 + R_2 - w_1 - w_2] \tag{3.146}$$

And therefore we can formulate the following KKT conditions:

$$\frac{\partial L}{\partial w_1} = B_1'(w_1) - \lambda_1 - \lambda_2 \leq 0 \perp w_1 \geq 0 \tag{3.147}$$

$$\frac{\partial L}{\partial w_2} = B_2'(w_2) - \lambda_2 \leq 0 \perp w_2 \geq 0 \tag{3.148}$$

$$\frac{\partial L}{\partial \lambda_1} = R_1 - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{3.149}$$

$$\frac{\partial L}{\partial \lambda_2} = R_1 + R_2 - w_1 - w_2 \geq 0 \perp \lambda_2 \geq 0 \tag{3.150}$$

We assume that both users consume water, i.e., $w_1 \geq 0$ and $w_2 \geq 0$, and hence

$$\frac{\partial L}{\partial w_1} = B_1'(w_1) - \lambda_1 - \lambda_2 = 0$$

$$\frac{\partial L}{\partial w_2} = B_2'(w_2) - \lambda_2 = 0$$

Furthermore we also suppose that water is scarce, which means that the available water amount will be consumed by both users completely, i.e., $R_1 + R_2 = w_1 + w_2$. This means that Eq. (3.150) is binding and therefore we assume that $\lambda_2 \geq 0$. Regarding user 1, we can distinguish between two cases:

- User 1 extracts all its available water, i.e., $w_1 = R_1$. Therefore, we would assume that $\lambda_1 \geq 0$ for this case because of Eq. (3.149). Hence, following optimality conditions result from the KKT conditions for this case:

$$w_1 = R_1 \tag{3.151}$$
$$w_2 = R_2 \tag{3.152}$$
$$\lambda_2 = B_2'(w_2) \tag{3.153}$$
$$\lambda_1 = B_1'(w_1) - B_2'(w_2) \tag{3.154}$$

- User 1 leaves some amount of water in the river, i.e., $w_1 \leq R_1$. Therefore, we would suppose that $\lambda_1 = 0$ because of Eq. (3.149). Hence, following optimality conditions result from the KKT conditions for this case:

$$B_1'(w_1) = B_2'(w_2) \tag{3.155}$$
$$R_1 + R_2 = w_1 + w_2 \tag{3.156}$$
$$w_1 \leq R_1 \tag{3.157}$$
$$\lambda_2 = B_2'(w_2) \tag{3.158}$$

Suppose we assume the second case (in which user 1 leaves water in the river), due to Eqs. (3.155) and (3.156) we can set up the system of equations:

$$B_1'(w_1) = B_2'(w_2) \;\rightarrow\; 100 - 1.5 \cdot w_1 = 60 - w_2$$
$$R_1 + R_2 = w_1 + w_2 \;\rightarrow\; 100 = w_1 + w_2$$

The solution is: $w_1 = 56$ and $w_2 = 44$.

We already know from Eq. (3.157) that $w_1 \leq R_1$. Here we found the contradiction, because the amount extracted by user 1 exceeds its available water which is $R_1 = 50$. Therefore, we cannot find the optimal solution on the base of this case 2.

Hence, we would like to check the first case where the upstream user extracts all available water. The extracted amount therefore is: $w_1 = 50$ and $w_2 = 50$, because of Eqs. (3.151) and (3.152). Hence, the marginal benefits are: $B_1'(w_1) = 25$ and $B_2'(w_2) = 10$. Therefore, it becomes obvious that the marginal benefit of the upstream user exceeds the one of the downstream, which in this case is required for optimality. The levels of the dual variables are $\lambda_1 = B_1'(w_1) - B_2'(w_2) = 15$ and $\lambda_2 = B_2'(w_2) = 10$ (see Eqs. (3.153) and (3.154)), which means that the dual variables are nonnegative which is also required. Therefore, we can not find a contradiction, and hence this case leads to an optimal solution.

Assume that we have a situation in which the downstream headwater inflow decreases to the level of $R_2 = 25$, due to for instance climate change, etc. If we suppose that the first case—in which the upstream user abstracts the total amount of available water—leads to the optimal solution as before, we find from the former explained optimality conditions (Eqs. (3.151) to (3.153)) that: $w_1 = 50$, $w_2 = 25$ and $\lambda_2 = B_2'(w_2) = 35$. When applying Eq. (3.154), we calculate that $\lambda_1 = B_1'(w_1) - B_2'(w_2) = -10$. Therefore, λ_1 is negative which contradicts the condition $\lambda_1 \geq 0$ and hence this first case does not lead to the optimal solution.

If we assume the second case where user 1 leaves water in the river, we solve the allocation amounts by applying Eqs. (3.155) and (3.156):

$$B_1'(w_1) = B_2'(w_2) \;\rightarrow\; 100 - 1.5 \cdot w_1 = 60 - w_2$$
$$R_1 + R_2 = w_1 + w_2 \;\rightarrow\; 75 = w_1 + w_2$$

which is: $w_1 = 46$ and $w_2 = 29$. Because of Eq. (3.158), we find that $\lambda_2 = B_2'(w_2) = 31$ which is nonnegative, hence the condition $\lambda_2 \geq 0$ holds. Furthermore, we also have to check Eq. (3.157) which is fulfilled because: $w_1 = 46 \leq 50 = R_1$. Hence, there exists no contradiction in this case, which means that this case leads to the optimal solution.

Exercise 3.6 Water transfers
Assume a situation with two regions, one with plentiful precipitation and non-arid conditions, while the second region is characterized by arid conditions. For preventing overexploitation in the arid region, the divertable water amounts are limited by $w_2^{SUS} = 3$. The renewed volumes of water resources in the non-arid region are very high, which means that the amount of divertable water w_1^{SUS} also becomes quite high. Hence there is no scarcity in region 1. The demand function is equal in both regions and given by

$$p_i(w_i^C) = B_i'(w_i^C) = a_i - b_i \cdot w_i^C \qquad \text{with: } a_i = 33,\; b_i = 1 \qquad i = \{1, 2\} \tag{3.159}$$

The variable w_i^C represents the amount of water consumed in the respective region.

The water extraction causes costs in the non-arid region in accordance with the linear function:

$$C_1(w_1) = F_1 + c_1 \cdot w_1 \qquad \text{with: } F_1 = 1,\; c_1 = 2 \tag{3.160}$$

To reduce scarcity in the arid region 2, water managers want to implement a water transfer scheme for exporting water from region 1 to region 2. The amount of transferred water is represented by the variable z. The transportation costs depend on the transferred amounts and are given by $\gamma \cdot z$, with $\gamma = 0.5$.

In contrast to Sect. 3.9, the marginal extraction cost function only occurs in the water-rich region 1 and has a horizontal shape ($C_1'(w_1) = c_1 = 2$). The following objective function has to be maximized to find the optimal water extraction amounts, the optimal water consumed as well as the optimal water transfer:

$$\max_{\{w_1, w_2, z\}} B_1(w_1^C) + B_2(w_2^C) - C_1(w_1) - \gamma \cdot z \tag{3.161}$$

The amount of water consumed in the regions depends on the water extraction in the respective region as well as the transferred water amount:

$$w_1^C = w_1 - z \tag{3.162}$$
$$w_2^C = w_2 + z \tag{3.163}$$

Hence, we are able to substitute the variables w_1^C and w_2^C and can finally solve the following optimization problem:

$$\max_{\{w_1,w_2,z\}} B_1(w_1 - z) + B_2(w_2 + z) - C_1(w_1) - \gamma \cdot z \tag{3.164}$$

$$s.t. \quad w_1 \le w_1^{\text{SUS}} \tag{3.165}$$

$$w_2 \le w_2^{\text{SUS}} \tag{3.166}$$

The corresponding Lagrangian function to the optimization problem is

$$L = B_1(w_1 - z) + B_2(w_2 + z) - C_1(w_1) - \gamma \cdot z + \lambda_1 \cdot \left(w_1^{\text{SUS}} - w_1\right) + \lambda_2 \cdot \left(w_2^{\text{SUS}} - w_2\right) \tag{3.167}$$

Now the KKT conditions can be set up:

$$\frac{\partial L}{\partial w_1} = B_1'(w_1 - z) - C_1'(w_1) - \lambda_1 \le 0 \perp w_1 \ge 0 \tag{3.168}$$

$$\frac{\partial L}{\partial w_2} = B_2'(w_2 + z) - \lambda_2 \le 0 \perp w_2 \ge 0 \tag{3.169}$$

$$\frac{\partial L}{\partial z} = -B_1'(w_1 - z) + B_2'(w_2 + z) - \gamma \le 0 \perp z \ge 0 \tag{3.170}$$

$$\frac{\partial L}{\partial \lambda_1} = w_1^{\text{SUS}} - w_1 \ge 0 \perp \lambda_1 \ge 0 \tag{3.171}$$

$$\frac{\partial L}{\partial \lambda_2} = w_2^{\text{SUS}} - w_2 \ge 0 \perp \lambda_2 \ge 0 \tag{3.172}$$

We can assume that water is plentiful in region 1, hence $w_1 < w_1^{\text{SUS}}$ and because of Eq. (3.171) we know that $\lambda_1 = 0$. However water is scarce in region 2 which means that water is extracted until its maximum possible limit ($w_2 = w_2^{\text{SUS}} = 3$) and hence it follows that $\lambda_2 \ge 0$ due to Eq. (3.172). Furthermore, we assume that a transfer is may be realized ($z \ge 0$).

Addressing these assumptions, we can adjust the KKT conditions to Eqs. (3.168), (3.169), and (3.170):

$$B_1'(w_1 - z) = C_1'(w_1) \to 33 - (w_1 - z) = 2 \cdot w_1 \tag{3.173}$$

$$\lambda_2 = B_2'(w_2^{\text{SUS}} + z) \to \lambda_2 = 33 - (3 + z) \tag{3.174}$$

$$B_1'(w_1 - z) + \gamma = B_2'(w_2^{\text{SUS}} + z) \to 33 - (w_1 - z) = 33 - (3 + z) + 0.5 \tag{3.175}$$

The values of the three variables w_1, z, λ_2 can be solved with the three above Eqs. (3.173), (3.174) as well as (3.175). Therefore we find the following solution:

$w_1 = 58.5$, $w_2 = 3$, $z = 27.5$, $\lambda_1 = 0$, $\lambda_2 = 2.5$.

A contradiction cannot be found in the solution, hence these are the optimal values.

If we change the assumptions such that water is not abundant in the water rich region anymore, the extractable amount is limited by its maximum, for instance

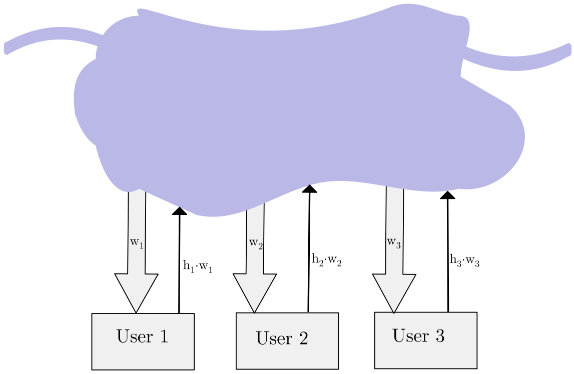


Fig. 3.22 Scheme of a lake with 3 users. *Source* own illustration

$w_1^{SUS} = 8$ for fulfilling sustainability requirements in the region. It is quite obvious that the former assumption is not suitable for solving the problem, because the optimal water extraction in the water-rich region ($w_1 = 58.5$) would exceed the maximum amount of water extractable in this region ($w_1^{SUS} = 8$) and hence there would be a violation of constraint (3.165) ($w_1^{SUS} \geq w_1$).

Therefore, we assume an extraction at each region equal to the maximum level, which means that $w_1 = w_1^{SUS} = 8$ as well as $w_2 = w_2^{SUS} = 3$. Hence $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$, because of equations (3.171) and (3.172), respectively. Similar to the assumption before, a transfer may be realized ($z \geq 0$). Entering the assumptions in the KKT conditions Eqs. (3.168), (3.169), and (3.170), we obtain the following adjusted conditions:

$$\lambda_1 = B_1'(w_1^{SUS} - z) - C_1'(w_1^{SUS}) \rightarrow \lambda_1 = 33 - (8 - z) = 2 \cdot 8 \qquad (3.176)$$

$$\lambda_2 = B_2'(w_2^{SUS} + z) \rightarrow \lambda_2 = 33 - (3 + z) \qquad (3.177)$$

$$B_1'(w_1^{SUS} - z) + \gamma = B_2'(w_2^{SUS} + z) \rightarrow 33 - (8 - z) = 33 - (3 + z) + 0.5 \qquad (3.178)$$

The unknown values of the three variables z, λ_1, and λ_2 can be calculated with the above Eqs. (3.176), (3.177) and (3.178).

$w_1 = 8$, $w_2 = 3$, $z = 2.25$, $\lambda_1 = 25.25$, and $\lambda_2 = 27.75$.

A contradiction cannot be found in the solution, hence these are the optimal values.

Exercise 3.7 Rivalry of consumption in a lake basin

Suppose a lake (see Fig. 3.22) which is the only raw water source for three users, i.e., $i = \{1, 2, 3\}$. The total amount of extractable water is determined by the natural recharge rate of the sea and is given by $R = 8$.

The users 1,2, and 3 generate benefit from abstracting water from the lake related to the following function:

$$B_i(w_i) = a_i \cdot w_i - 0.5 \cdot b_i \cdot (w_i)^2, \text{ with: } a_1 = 6,\ a_2 = 7\ a_3 = 8,\ b_1 = 1,\ b_2 = 1.5,\ b_3 = 0.5$$

We assume that after the consumption of water, there is a return flow back into the river. The return flow factor h_i indicates the proportion of consumed water which flows back to the lake. The level of the return flow from user i, which is $h_i \cdot w_i$, impacts the net abstraction of this user. The net abstraction of one user results from the difference between its abstraction (w_i) and its return flow back to the water body ($h_i \cdot w_i$), e.g., $w_i - h_i \cdot w_i$ which is, therefore, $(1 - h_i) \cdot w_i$. Hence, while the abstraction of user 1,2, and 3 are represented by the variables w_1, w_2 and w_3, the net abstraction of user 1, 2, and 3 are $(1 - h_1) \cdot w_1$, $(1 - h_2) \cdot w_2$ and $(1 - h_3) \cdot w_3$, respectively. We want to calculate the optimal water allocation to all users under the two cases that

- the return flow factors are $h_1 = h_2 = h_3 = 1$ (full return flows)
- the return flow factors are $h_1 = h_2 = h_3 = 0$ (no return flows)

Regardless of the level of the return flows, we can set up the following optimization problem for finding the optimal water allocation strategy in the basin:

$$\begin{aligned} &\max_{\{w_1,w_2,w_3\}} \quad [B_1(w_1) + B_2(w_2) + B_3(w_3)] \\ &s.t.\ w_1 \leq R - (1 - h_2) \cdot w_2 - (1 - h_3) \cdot w_3 \qquad (\lambda_1) \\ &\quad\ \ w_2 \leq R - (1 - h_1) \cdot w_1 - (1 - h_3) \cdot w_3 \qquad (\lambda_2) \\ &\quad\ \ w_3 \leq R - (1 - h_1) \cdot w_1 - (1 - h_2) \cdot w_2 \qquad (\lambda_3) \end{aligned}$$

Due to the objective, we want to maximize the total benefit in the entire basin. The constraints limit the extractable amount of water for each user. The extractable water of a specific user is determined by the natural recharge rate R and the sum of the net abstraction of the other users. Based on the optimization problem, the following Lagrangian function can be set up:

$$\begin{aligned} L = &B_1(w_1) + B_2(w_2) + B_3(w_3) \\ &+ \lambda_1 \cdot [R - (1 - h_2) \cdot w_2 - (1 - h_3) \cdot w_3 - w_1] \\ &+ \lambda_2 \cdot [R - (1 - h_1) \cdot w_1 - (1 - h_3) \cdot w_3 - w_2] \\ &+ \lambda_3 \cdot [R - (1 - h_1) \cdot w_1 - (1 - h_2) \cdot w_2 - w_3] \end{aligned}$$

And hence, the following KKT conditions can be formulated:

$$B_1'(w_1) - \lambda_1 - (1 - h_1) \cdot (\lambda_2 + \lambda_3) \leq 0 \perp w_1 \geq 0 \tag{3.179}$$

$$B_2'(w_2) - \lambda_2 - (1 - h_2) \cdot (\lambda_1 + \lambda_3) \leq 0 \perp w_2 \geq 0 \tag{3.180}$$

$$B_3'(w_3) - \lambda_3 - (1 - h_3) \cdot (\lambda_1 + \lambda_2) \leq 0 \perp w_3 \geq 0 \tag{3.181}$$

$$R - (1 - h_2) \cdot w_2 - (1 - h_3) \cdot w_3 - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{3.182}$$

$$R - (1 - h_1) \cdot w_1 - (1 - h_3) \cdot w_3 - w_2 \geq 0 \perp \lambda_2 \geq 0 \tag{3.183}$$

$$R - (1 - h_1) \cdot w_1 - (1 - h_2) \cdot w_2 - w_3 \geq 0 \perp \lambda_3 \geq 0 \tag{3.184}$$

Full return flows

In case of full return flows, which means that $h_1 = h_2 = h_3 = 1$, the net abstraction of water from the lake is zero. Therefore, the consumption is non-rivalrous. The access to the lake is also non-excludable, hence the water in the lake could be classified as a public good. By inserting $h_1 = h_2 = h_3 = 1$ in Eqs. (3.179) to (3.184), we can find the following expressions:

$$
\begin{aligned}
B_1'(w_1) - \lambda_1 \le 0 \perp w_1 \ge 0 \qquad & R - w_1 \ge 0 \perp \lambda_1 \ge 0 \\
B_2'(w_2) - \lambda_2 \le 0 \perp w_2 \ge 0 \qquad & R - w_2 \ge 0 \perp \lambda_2 \ge 0 \\
B_3'(w_3) - \lambda_3 \le 0 \perp w_3 \ge 0 \qquad & R - w_3 \ge 0 \perp \lambda_3 \ge 0
\end{aligned}
$$

which can be generalized to the following form:

$$B_i'(w_i) - \lambda_i \le 0 \perp w_i \ge 0 \qquad R - w_i \ge 0 \perp \lambda_i \ge 0 \quad \forall i$$

We know that we have to assume $w_i \ge 0$.[56] Therefore it follows, that:

$$\lambda_i = B_i'(w_i) \qquad R - w_i \ge 0 \perp \lambda_i \ge 0 \quad \forall i$$

Based on this, we can distinguish between two cases:

- The consumption is equal to the available water which is determined by the natural recharge rate R. If $w_i = R$ it becomes obvious that we have to assume that $\lambda_i \ge 0$. Therefore, it follows that $B_i'(w_i) \ge 0$, which means that the marginal benefit have to be nonnegative.
- Under the assumption that the constraint $R - w_i \ge 0$ is nonbinding it follows that we have to assume that $\lambda_i = 0$. This means that we have a consumption level where the marginal benefit is zero, i.e., $B_i'(w_i) = 0$.

User 1 and 2 consume at the level where their respective marginal benefit levels are zero, hence $w_1 = 6$ and $w_2 = \frac{14}{3} \approx 4.667$. However, user 3 abstracts the total amount of available water, hence $w_3 = R = 8$. The marginal benefit of user 3 is $B_3'(w_3) = 4$.[57]

[56] Assume that $w_i = 0$, then the constraint $R - w_i \ge 0$ is certainly not binding and hence it follows that $\lambda_i = 0$. Furthermore, we know from the assumption $w_i = 0$, that $B_i'(w_i) - \lambda_i \le 0$ which means $B_i'(w_i) \le \lambda_i$ and hence $B_i'(w_i) \le 0$. Therefore, we know that the marginal benefit would be negative. However, the marginal benefit for a consumption level of zero is nothing else than the choke price of the demand function of user i. The choke price is generally positive. In this example, the choke prices of user 1, 2, and 3 are $a_1 = 6$, $a_2 = 7$, and $a_3 = 8$, respectively. Therefore, we found a contradiction for the assumption $w_i = 0$ and hence the assumption $w_i \ge 0$ is correct.

[57] Under the assumption that $w_1 = R = 8$, the marginal benefit $B_1'(w_1) = -2$ would be negative which is a contradiction with the condition $B_1'(w_1) \ge 0$.

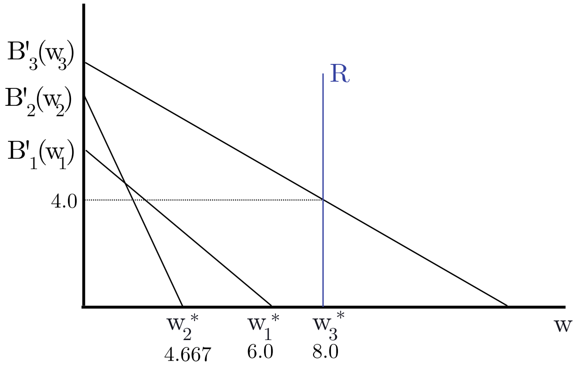


Fig. 3.23 Optimal allocation in a lake basin if there are full return flows. *Source* own illustration

The situation in the lake with full return flows is illustrated by Fig. 3.23. The zero of the marginal benefit functions of user 1 and 2 are left from the water availability level $R = 8$, which means that there are no intersection points between these marginal benefit functions and the water availability $R = 8$. Therefore the amount of water consumed by user 1 and 2 are determined by their respective zero of marginal benefit, i.e., $B_1'(w_1) = 0$ and $B_2'(w_2) = 0$, and based on this we are able to get w_1^* and w_2^*. Only the marginal benefit function of user 3 intersects the water availability $R = 8$, hence $w_3^* = R$. This intersection point determines the marginal benefit level of user 3 which is $B_3'(w_3) = 4$.

No return flows

In case of no return flows, which means that $h_1 = h_2 = h_3 = 0$, the net abstraction is equal to the abstraction. Therefore, the abstracted water amounts cannot be used by another user. Hence, the consumption is rivalrous. The access to the lake is still non-excludable as in the other case with full return flows. Therefore, the water in the lake can be classified as a common good.

By inserting $h_1 = h_2 = h_3 = 0$ in Eq. (3.182) to (3.184), we can find the following expressions:

$$R - w_1 - w_2 - w_3 \geq 0 \perp \lambda_1 \geq 0$$
$$R - w_1 - w_2 - w_3 \geq 0 \perp \lambda_2 \geq 0$$
$$R - w_1 - w_2 - w_3 \geq 0 \perp \lambda_3 \geq 0$$

Under the assumption that $w_2 = R = 8$, the marginal benefit $B_2'(w_2) = -5$ would be negative which is a contradiction with the condition $B_2'(w_2) \geq 0$.

Under the assumption that the consumption of user 3 is determined by the zero of the marginal benefit, i.e., $B_3'(w_3) = 0$, it follows that $w_3 = 16$. This is a contradiction to the condition $w_3 \leq R$, with $R = 8$.

which can be generalized in the following way:

$$R - \sum_i [w_i] \geq 0 \perp \lambda_j \geq 0 \quad \forall j$$

The term $\sum_i [w_i]$ is nothing else than $w_1 + w_2 + w_3$ and stands for the total consumption level in the basin. From this general formulation, the following aspects become obvious:

- if we assume that $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$, all available water is consumed.
- if we assume that $\lambda_1 = \lambda_2 = \lambda_3 = 0$, not all of the available water is consumed in total in the basin. Hence, water is not scarce.

By inserting $h_1 = h_2 = h_3 = 0$ in Eqs. (3.179) to (3.181), we are able to formulate

$$B'_1(w_1) - \lambda_1 - \lambda_2 - \lambda_3 \leq 0 \perp w_1 \geq 0$$
$$B'_2(w_2) - \lambda_1 - \lambda_2 - \lambda_3 \leq 0 \perp w_2 \geq 0$$
$$B'_3(w_3) - \lambda_1 - \lambda_2 - \lambda_3 \leq 0 \perp w_3 \geq 0$$

which is in a more general formulation:

$$B'_i(w_i) - \sum_j \left[\lambda_j\right] \leq 0 \perp w_i \geq 0 \quad \forall i$$

The term $\sum_j \left[\lambda_j\right]$ represents the sum of all dual variables, i.e., $\lambda_1 + \lambda_2 + \lambda_3$. Based on this formulation, we can distinguish between two cases:

- If we assume that a riparian has no consumption, i.e., $w_i = 0$, we know that $B'_i(w_i) \leq \sum_j \left[\lambda_j\right]$, which means that the choke price of the user i falls below the sum of the dual variables.
- If we assume a consumption for user i, i.e., $w_i \geq 0$, we know that $B'_i(w_i) = \sum_j \left[\lambda_j\right]$, which means that the marginal benefit of the user i is equal to the sum of the dual variables.

Under the assumption that water is scarce ($R = \sum_i [w_i]$), there may exist two kinds of users. Users for which consumption is assumed ($w_k \geq 0$) are denoted by k, while users for which consumption is not assumed ($w_m = 0$) are denoted by m. Ergo, one may state the following:

$$(\boldsymbol{\lambda_j}) : R = \sum_i [w_i] \quad \forall j \tag{3.185}$$

$$(\boldsymbol{w_k}) : \sum_j \left[\lambda_j\right] = B'_k(w_k) \quad \forall k \tag{3.186}$$

$$(\boldsymbol{w_m}) : B'_m(w_m) \leq \sum_j \left[\lambda_j\right] \quad \forall m \tag{3.187}$$

It becomes apparent that all consuming users should have an equal level of marginal benefit (see Eq. (3.186)). However, based on Eq. (3.187), the choke price of those consumers who do not consume must fall below the marginal benefit level of the consuming users.

This solution could be enforced, for instance, by pricing water. If water is priced, the users will consume in such quantities, that their marginal benefit becomes equal to the price level. Due to Eq. (3.186), we find out that the marginal benefit level of every consuming user should be the same, hence there should exist just one market price in the entire market. By pricing water, one could potentially exclude users from consuming. Hence, if a pricing policy is enforceable, water would not be a common good anymore, but a private good. If the market price for water exceeds the choke price of a specific user, this user would not consume any water from the resource. This is exactly the condition which is explained by Eq. (3.187).

There are two ways for finding the concrete solution of this optimization problem:

- Solving the system of equations
- Set the market demand function and find an intersection with water supply

Regarding the first approach, we may assume that all users consume and that water is scarce, hence: $w_1 \geq 0$, $w_2 \geq 0$, $w_3 \geq 0$, $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$. Based on this, we can define the following system of equations:

$$B'_1(w_1) = B'_2(w_2) = B'_3(w_3) \;\rightarrow\; 6 - w_1 = 7 - 1.5 \cdot w_2 = 8 - 0.5 \cdot w_3$$
$$w_1 + w_2 + w_3 = R \;\rightarrow\; w_1 + w_2 + w_3 = 8$$

The solution is: $w_1 = \frac{10}{11} \approx 0.909$, $w_2 = \frac{42}{33} \approx 1.272$ and $w_3 = \frac{64}{11} \approx 5.818$
The marginal benefits of the three users are: $B'_1(w_1) = B'_2(w_2) = B'_3(w_3) = \frac{56}{11} \approx 5.091$.

For the second approach we have to define the marginal benefit (or demand function) of the entire basin which has to meet Eqs. (3.185) to (3.187). Please note, that the demand function is nothing else than the marginal benefit function, hence:

$$B'_1(w_1) = p(w_1) = 6 - w_1 \qquad B'_2(w_2) = p(w_2) = 7 - 1.5 \cdot w_2$$
$$B'_3(w_3) = p(w_3) = 8 - 0.5 \cdot w_3$$

For finding the market demand function, we have to set up the inverse forms:

$$w_1(p) = 6 - p \qquad w_2(p) = \frac{14}{3} - \frac{2}{3} \cdot p \qquad w_3(p) = 16 - 2 \cdot p$$

and sum up these inverse forms:

$$w_M(p) = \max\{0, w_1(p)\} + \max\{0, w_2(p)\} + \max\{0, w_3(p)\}$$

We use the term $\max\{0, w_i(p)\}$ in order to address the condition of nonnegativity for w_i. If the price p exceeds the choke price, the term $\max\{0, w_i(p)\} = 0$, while if

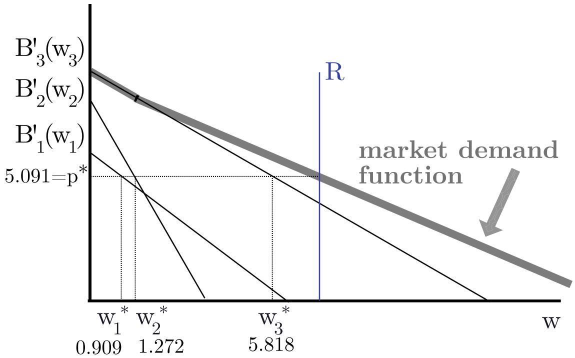


Fig. 3.24 Optimal allocation in a lake basin if there are no return flows. *Source* own illustration

the price p falls below the choke price, we get max $\{0, w_i(p)\} = w_i$. Therefore, we are able to set up following distinction of cases:

$$w_M(p) = \begin{cases} 0 & \text{for } p \geq 8 \\ w_3(p) & \text{for } 7 \leq p < 8 \\ w_2(p) + w_3(p) & \text{for } 6 \leq p < 7 \\ w_1(p) + w_2(p) + w_3(p) & \text{for } 0 \leq p < 6 \end{cases} = \begin{cases} 0 & \text{for } p \geq 8 \\ 16 - 2 \cdot p & \text{for } 7 \leq p < 8 \\ \frac{62}{3} - \frac{8}{3} \cdot p & \text{for } 6 \leq p < 7 \\ \frac{80}{3} - \frac{11}{3} \cdot p & \text{for } 0 \leq p < 6 \end{cases}$$

When setting this function equal to the water availability $w_M(p) = R = 8$, we get a price of $p = \frac{56}{11} \approx 5.091$.[58] By inserting this price in the individual demand functions we get:
$w_1 = \frac{10}{11} \approx 0.909$, $w_2 = \frac{42}{33} = 1.272$ and $w_3 = \frac{64}{11} = 5.818$.

This situation in the lake with no return flows is illustrated by Fig. 3.24. The market demand function can be set up by the summation of the marginal benefit functions of the users 1, 2, and 3 in the horizontal direction. The intersection of this market demand function with the water availability $R = 8$ determines the price for water, i.e., $p^* = 5.818$. Based on this market price and the respective marginal benefit functions of the users, the social-optimal usage of each users, symbolized by w_1^*, w_2^* and w_3^*, can be found.

3.12 Further Reading

In Sect. 3.1, we have already listed some literature sources on the IWRM. In addition, there are a number of other papers on IWRM which are worth reading from different

[58] By setting equal: $8 = \frac{80}{3} - \frac{11}{3} \cdot p$, we get the price $p = \frac{56}{11}$. The calculated price is within the allowed range $0 \leq p < 6$. Hence, this price is the optimal solution.

Regarding the other case: $8 = \frac{62}{3} - \frac{8}{3} \cdot p$, we get the price of $p = \frac{38}{3} \approx 5.818$ which is outside the allowed range $6 \leq p < 7$.

For the case $8 = 16 - 2 \cdot p$, we find a price of $p = 4$ which is outside the allowed range $7 \leq p < 8$.

perspectives: Hoekstra (1998) provides a comprehensive overview of the social perspectives of water allocation that goes beyond economic approaches. Grafton et al. (2019) view the integrated water resource management from a governance perspective; a reform process called the Water Governance Reform Framework (WGRF) is proposed.

Justice aspects are not included in many economic textbooks. Especially in the case of water, we do not believe that we can limit ourselves to questions of efficient allocation. Water is more than a private good. But which justice criteria should be taken into account in the allocation and distribution of resources? Johansson-Stenman and Konow (2010) give a structured overview of the interdisciplinary literature. In this context, the work of the philosopher John Rawls (1971) is particularly important with respect to the allocation of goods; this is where the principle of difference, which is based on the concept of moral arbitrariness, is developed and founded. Sandel (2009) dedicates a separate chapter to Rawls in his work on justice. This book takes particular account of social-philosophical approaches that are relevant to economics. In the center of the chapter on Rawls are the four theories of distribution justice: feudal system, free market with formal equality (libertarianism), free market with fair equality (meritocratic), and Rawls's difference principle (egalitarianism).

But how can a fair distribution of goods be determined? This is about the psychological and socio-philosophical foundation of the utility function. Roemer (1996) examines the question of how utility (happiness) can be measured and whether and how they can be compared between people. It takes into account the subtle question of what the consequences are for a just allocation of goods when certain resources are inalienable (e.g., talents).

Alan Garcia was a controversial president of Peru, ideologically very close to neoliberalism. The fable of the dog in the manger and its connection with the rural population was considered as polemical, as Boelens and Vos (2012) have reported. However, it is worth the exact analysis of his arguments. It turns out that the question of distributional impacts of productivity-enhancing investments depends not only on the ownership structure but also on other income options of the rural population. Here, our model uses essential elements from Cohen and Weitzman (1975), who have studied the effects of the enclosure process in England of small landholdings within common land into larger farms with private entitlements.

The human right to water is rarely addressed in the economic literature on water allocation. At the very most, the requirement of access to water as a restriction is included in the usual allocation models. Our approach explicitly includes the hierarchization of needs into the IWRM model, i.e., to place basic nutrition and water in their life-sustaining function before other consumer goods. The notion of hierarchies of needs goes back to the beginnings of utility theory: Georgescu-Roegen (1954) has written an idea-historical outline in which the concept of the irreducibility of wants is introduced as the foundation of the hierarchy of needs. Seeley (1992) extends utility theory to the Maslow triangle. Hoekstra (1998) provides a comprehensive overview of the social perspectives of water allocation that goes beyond economic approaches.

A classic contribution to water allocation along rivers is Ambec and Sprumont (2002). However, not only efficiency aspects are important, but also distribution rules that can be applied to cooperation gains. Ambec et al. (2013) analyze different distribution rules and examine them with regard to their robustness if the water supply unexpectedly decreases. This problem is particularly virulent in international water treaties and is addressed in Chap. 6.

Not only surface waters but also groundwater reservoirs are overused worldwide. The consequences are manifold. Koundouri (2004) gives an overview of how an economic approach can reconcile the use and hydrological constraints. She also deals with the difference between the use of groundwater as a common pool resource and as a co-operatively managed resource. In addition, not only are groundwater reservoirs being overused, but the interdependence of groundwater and surface waters caused by the infiltration processes means that the flow of rivers reaches ecologically critical limits. De Graaf et al. (2019) examine these relationships and conclude that the negative ecological effects of groundwater abstraction occur long before the reservoirs are overexploited. Jakeman et al. (2016) make a similar diagnosis and advocate an integrated management approach. This implies “thinking beyond the aquifer”. Surface waters and aquifers should be considered in an overarching approach (conjunctive use). Pulido-Velazquez et al. (2016) develop complex hydro-economic models that derive a sustainable and economically optimized conjunctive use of surface and groundwater storage.

Simply pumping water from one catchment area to another can certainly not be considered a result of integrated water management. A variety of ecological and social effects must be taken into account. Gupta and van der Zaag (2008) develop a system of criteria against which transfer projects should be evaluated. The different effects at the donor and at the recipient catchment areas have to be distinguished. With the help of this evaluation scheme, they examine transfer projects in India. Tian et al. (2019) develop a complex hydro-economic model that not only assesses the hydrological, ecological and social impacts of water transfer projects, but also determines optimal water allocations. The approach takes into account random fluctuations in the water supply and derives measurements for the reliability and resilience of transfer networks.

Water quality problems are only marginally addressed in this textbook, though they are of paramount importance for integrated water resource management. Olmstead (2009) gives a very instructive overview of the economic dimension of water quality regulation and analyzes various policy instruments. The literature contains a large number of articles dealing with various aspects of water quality. Zhu and van Ierland (2012) develop a hydro-economic optimization model, in which both quantity and quality problems are considered, Shortle (2013) reports on the experiences with quality trading and D’Arcy and Frost (2001) deal with the problems of diffuse pollutant inputs.

3.13 Chapter Annex: Integrated Water Resource Management

3.13.1 The Dublin Principles

Four important guiding principles were determined during the International Conference on Environment and Water in Dublin in the year 1992 with over 500 participants representing 100 countries and 80 international and nongovernmental organizations (Xie 2006). These principles are:

- **Principle No. 1 ("Ecological"): Freshwater is a finite and vulnerable resource, essential to sustain life, development, and the environment.** Since water sustains both life and livelihoods, effective management of water resources demands a holistic approach, linking social and economic development with the protection of natural ecosystems. Effective management links land and water use across the whole of a catchment area or groundwater aquifer.
- **Principle No. 2 ("Institutional"): Water development and management should be based on a participatory approach, involving users, planners, and policy-makers at all levels.** The participatory approach involves raising awareness of the importance of water among policy-makers and the general public. It means that decisions are taken at the lowest appropriate level, with full public consultation and involvement of users in the planning and implementation of water projects.
- **Principle No. 3 ("Gender"): Women play a central part in the provision, management, and safeguarding of water.** This pivotal role of women as providers and users of water and guardians of the living environment has seldom been reflected in institutional arrangements for the development and management of water resources. Acceptance and implementation of this principle require positive policies to address women's specific needs and to equip and empower women to participate at all levels in water resources programs, including decision-making and implementation, in ways defined by them.
- **Principle No. 4 ("Economic"): Water has an economic value in all its competing uses and should be recognized as an economic good.** Within this principle, it is vital to recognize first the basic right of all human beings to have access to clean water and sanitation at an affordable price. Past failure to recognize the economic value of water has led to wasteful and environmentally damaging uses of the resource. Managing water as an economic good is an important way of achieving efficient and equitable use, and of encouraging conservation and protection of water resources.

3.13.2 Integration in IWRM

It is important to bridge components of the natural systems, like availability and quality of resources, as well as characteristics of human systems, which are fundamentally determined by resource use, waste production, and resource pollution. The main aspects regarding natural system integration and human system integration are listed in detail below (GWP 2000):

- **Natural system integration**
  - **Integration of freshwater management and coastal zone management:** Requirements of coastal zones have to be considered in upstream freshwater management
  - **Integration of land and water management:** Land use influences the distribution and quality of water. Furthermore, water is a key determinant of the character of ecosystems.
  - **Distinction between "green water" and "blue water":** Water that is directly used for biomass production and "lost" in evaporation is termed "green water", while "blue water" is the flowing water in surface and subsurface water bodies.
  - **Integration of surface water and groundwater management:** An infiltration of water from groundwater bodies to surface water bodies and vice versa can occur.
  - **Integration of quantity and quality in water resources management:** Aspects of generating, abating, and disposing of waste products have to be addressed.
  - **Integration of upstream and downstream water-related interests:** Conflicts, interests, and trade-offs between upstream and downstream stakeholders using water resources have to be identified and balanced out
- **Human system integration**
  - **Mainstreaming of water resources:** The analysis of human activities have to involve the understanding of natural systems, its capacity, vulnerability, and limits.
  - **Cross-sectoral integration in national policy development:** Water policy must be integrated with economic policy. The economic and social policy needs to take into account water resource implications.
  - **Macroeconomic effects of water developments:** Water resource projects can have macroeconomic impacts (e.g., employment).
  - **Basic principles for integrated policy-making:** Assess macroeconomic conditions of effects before realizing investment; weight expected (external) costs with (external) benefits of a policy; awareness of trade-offs in short-term and long-term
  - **Influencing economic sector decisions:** Decisions impact water demands, availability, and quality.
  - **Integration of all stakeholders in the planning and decision process:** Involvement of the stakeholders in the management and planning of water resources to deal with conflicting interests between stakeholders.
  - **Integrating water and wastewater management:** Water is a reusable resource, hence wastewater flows can be a useful additional resource.

3.13.3 Implementation of IWRM

Based on the GWP, the three main pillars for implementing IWRM in practice are an enabling environment, institutional roles, and management instruments (GWP (2004)):

- **The enabling environment**

1. Policies—setting goals for water use, protection, and conservation.
2. Legislative framework—the rules to follow to achieve policies and goals.
3. Financing and incentive structures—allocating financial resources to meet water needs.

- **Institutional roles**

4. Creating an organizational framework—forms and functions.
5. Institutional capacity building—developing human resources.

- **Management instruments**

6. Water resources assessment—understanding resources and needs.
7. Plans for IWRM—combining development options, resource use, and human interaction.
8. Demand management—using water more efficiently.
9. Social change instruments—encouraging a water-oriented civil society.
10. Conflict resolution—managing disputes, ensuring sharing of water.
11. Regulatory instruments—allocation and water use limits.
12. Economic instruments—using value and prices for efficiency and equity.
13. Information management and exchange—improving knowledge for better water management.

For transferring the IWRM paradigm into practice, the GWP (2004) recommends an IWRM planning cycle which is illustrated by Fig. 3.25.
The IWRM planning cycle contains the following elements (see GWP 2004):

- *Establishing Status and Overall Goals:* The urgent water resource issues seen in a national context. Chart the progress toward a management framework in which issues can be addressed and agreed, such that overall goals can be achieved. Check if international agreements with the neighbors present potentials or constraints to developing a feasible management framework.
- *Build Commitment to Reform:* The political will is a prerequisite for a well-functioning IWRM framework. Building or consolidating a multi-stakeholder dialogue ranks high on the list of priority actions. The dialogue needs to be based on knowledge about the matter of subject and creating awareness is one of the tools to establish this knowledge and to enable participation of the broader population.

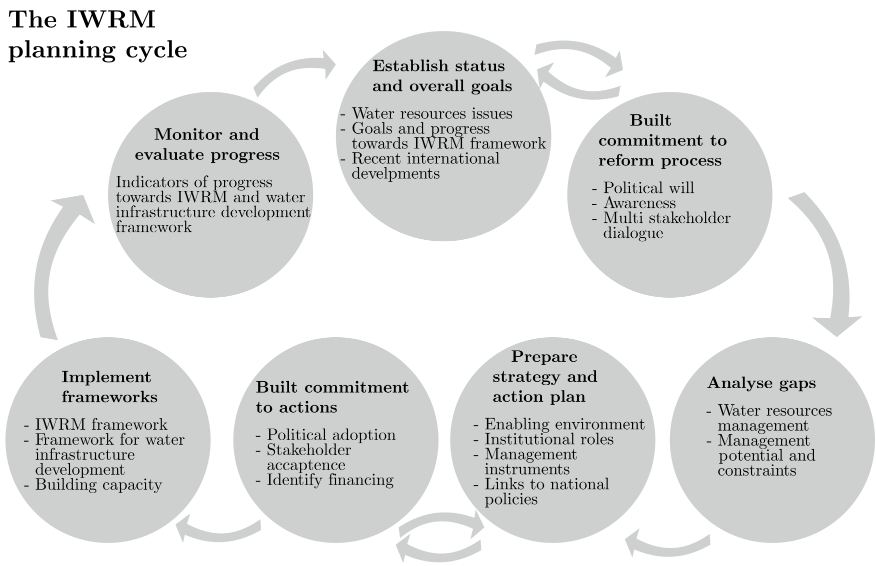


Fig. 3.25 IWRM planning cycle. *Source* GWP (2000)

- *Analyze GAP:* Given the present policy and legislation, the institutional situation, the capabilities and the overall goals; gaps in the IWRM framework can be analyzed in the light of management functions required by the urgent issues.
- *Prepare Strategy and Action Plan:* Map the road toward completion of the framework for water resource management and development as well as related infrastructural measures. A portfolio of actions will be among the outputs, which will be set in the perspective of other national and international planning processes.
- *Build Commitments to Actions:* Adaptation of the action plan at highest political levels is key to any progress; full stakeholder acceptance is essential for implementation. The long-term financial commitment is a prerequisite for taking planned actions to implementation.
- *Implement Frameworks:* Taking plans into reality, the enabling environment, the institutional roles, and management instruments have to be implemented. Changes have to be made in the present structure; building of capacity and capability also taking into account necessary infrastructure development.
- *Monitor and Evaluate Progress:* Progress monitoring and evaluation of the process inputs; Choosing proper descriptive indicators is essential to the value of the monitoring.

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4 Water Tariffs

4.1 Historical Review of the Water Pricing Debate

In the face of the deterioration of water availability in different regions of the world, the discussion on the introduction of economic policy instruments has become increasingly important (Hanemann 2004). The issue of adequate water supply for all was first addressed at the UN Water Conference in Mar del Plata (Argentina) in 1977. This convention resulted in the United Nations' commitment to a human right to drinking water in a quantity and quality appropriate to basic needs. The conference elaborated an action plan which clarified the link between water management measures and their socio-economic impact. This includes, among other things, the demand to reflect economic costs through the water price. Furthermore, economic incentives for an efficient and balanced use of water via the water price were declared to be useful. However, there were no explicit recommendations for the use of concrete instruments.

Another milestone in water policy was the 1992 International Conference on Water and Environment in Dublin, Ireland. The conference culminated in the formulation of the four important Dublin principles, which set out the conditions for sustainable water resources management. Principle 4 declares the economic value of water. Since water that was used for one process might be unsuitable to be used for other processes, a competition between different forms of use arises. Hence, water should be considered as an economic good, implying some form of cost coverage for water supply. The Dublin Conference was instrumental in a substantive and institutional reorientation of global water policy.

The outcome of the Dublin Conference formed the basis of Chap. 18 ("Water Management") of the Agenda 21, adopted at the 1992 UN Conference on Environment and Development in Rio de Janeiro. Representatives from 178 countries took part in the conference to discuss key environmental and development policy issues of the twenty-first century. The cost recovery principle was anchored in Agenda 21 as

G. Meran et al., *The Economics of Water*, Springer Water,
https://doi.org/10.1007/978-3-030-48485-9_4

a component of sustainable water resources management. In addition to production costs, external environmental costs must also be taken into account.[1]

The Agenda therefore calls for tariff systems that take the actual costs of water as well as the consumer's assumed ability to pay into account. The inclusion of social concerns in the pricing of water is therefore explicitly required. While the Dublin Conference's outcome essentially considered water to be an economic good, Agenda 21 also considered water as a social good. These different views form the basis for the subsequent discourse on water pricing policy. However, it can be noted that the content of both the Dublin Principles and Agenda 21 strongly influenced the subsequent water policy (Dinar et al. 2015). The European Water Framework Directive provides an example of this.

The European Water Framework Directive, which was adopted in 2000, provides the legal basis for securing water resources and ensuring sustainable development within the European Union. The directive was adopted as a reaction to an increasing disparity between the available water supply and water demand. The principle of cost recovery referred to in Article 9 is an important part of the directive. It aims to cover the production costs, as well as the environmental and resource costs associated with the use of water resources. Furthermore, under this directive, pricing policy must be designed in a way that incentivizes an efficient water usage.

In the General Assembly of the United Nations on August 3, 2010, the members decided that the right to clean drinking water and sanitation should be a human right (United Nations 2010). This human right is in accordance with the content of Agenda 21 and the Dublin Principles, as it does not require free water and sanitation. It rather assures affordable access to adequate water and sanitation to satisfy the basic needs. The resulting challenge is to determine the extent to which poorer sections of the population can be involved in cost recovery to ensure the affordability of water supplies.

4.2 Criteria for Water Tariffs

Water pricing policy can pursue multiple objectives, which are revenue sufficiency, economic efficiency, environmental sustainability, and social concerns, including affordability and fairness considerations. Further important aspects of water pricing policy, which are not addressed in detail in this section, are the public and political acceptance as well as the simplicity and transparency of the water pricing policy (Boland and Whittington 2000a). In the following section, we give a brief overview of the four main goals that are also described as the sustainability dimensions of water pricing policy (Massarutto 2007b).

[1] Chapter 18.16 of Agenda 21 reads: "A prerequisite for the sustainable management of water as a scarce vulnerable resource is the obligation to acknowledge in all planning and development its full costs".

4.2.1 Revenue Sufficiency

This goal is of special importance for the water suppliers because it relates to the claim that a tariff system should cover all the incurred costs. If costs are not fully covered, incoming cash flows are not sufficient to guarantee an effective and efficient operation and management of the water supply system. Furthermore, the absence of full cost recovery could result in a lack of financial resources, which would be necessary to make sufficient investments in the water supply infrastructure. This leads, in consequence, to a worse water supply service and hence an increasing dissatisfaction, which is accompanied by a decreasing willingness to pay by consumers. Hence, this goal is important for guaranteeing the long-term reproduction of the physical assets. Not only the pricing level, but also the stability is a matter of the tariff-setting process.

The full supply costs that have to be covered are those associated with providing water services to users. This contains the following types of cost (OECD 2010):

- Operation and maintenance costs, resulting from the day-to-day operations of the water supply system, such as electricity for pumping but also labor and repair costs.
- Capital costs, covering both, investments in existing infrastructure as well as capital for new investments, and servicing debt.

However, further cost components, such as opportunity cost and economic externalities, should also be addressed. In detail, these components include

- Opportunity cost that reflects the scarcity value of the water resource. They refer to the cost of not serving the next possible user.
- Economic externalities, which are benefits and costs associated with water management. It is possible to distinguish between positive external benefits (e.g., groundwater recharge benefits from irrigation or water reuse) and negative external costs (e.g., upstream diversion of water or the release of pollutants downstream within an irrigation or urban water system).

The cost components are illustrated in Fig. 4.1. The sum of the full supply cost, opportunity cost, and economic externalities is termed the full economic cost of the water supply service (OECD 2010). Furthermore, the operation and management of the water supply system could negatively impact the aquatic and non-aquatic environment, for instance, an increased water shortage in the ecosystem due to an over-exploitation of water resources. Those occurring negative externalities to the environment have to be addressed as another important cost component. The sum of the full economic cost and the environmental externalities forms the full cost of water supply service (OECD 2010).

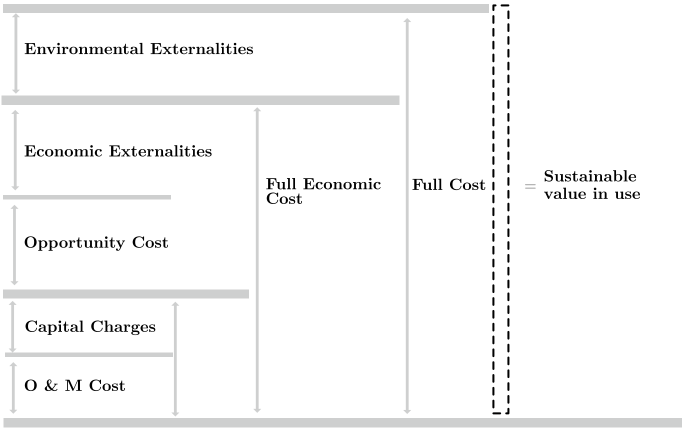

Fig. 4.1 General principles for the costs of water. *Source* Rogers et al. (1998)

4.2.2 Economic Efficiency

Water pricing policy should be conceived in a way that water is allocated to those users that benefit the most from receiving water resources. Hence, this goal implies the maximization of the aggregated economic rents of all water consumers. If water is allocated to users with low marginal benefits of water consumption, while other users with higher marginal benefits are not supplied, the principle of economic efficiency is violated. Furthermore, the pricing policy should disincentivize the wasteful usage of economic resources, because marginal benefits exceed the marginal cost of each unit consumed water.

4.2.3 Environmental Sustainability

The water resource in the environment is essential for the aquatic and non-aquatic nature and provides important ecosystem services (e.g., fishery) for the human society. As water resource conservation plays a crucial role in achieving environmental sustainability, the pricing policy should set incentives to protect the water in the nature. For example, over-exploitation from surface or subsurface water stocks has to be avoided.

4.2.4 Social Concerns

Acceptable levels of the water supply service should be accessible and affordable to all consumers, because water can be seen as a good of public interest. The focus of this goal is mainly the protection of vulnerable groups with low incomes. The reallocation of costs across different groups through the tariff structure is an important means to achieve this objective (OECD 2009).

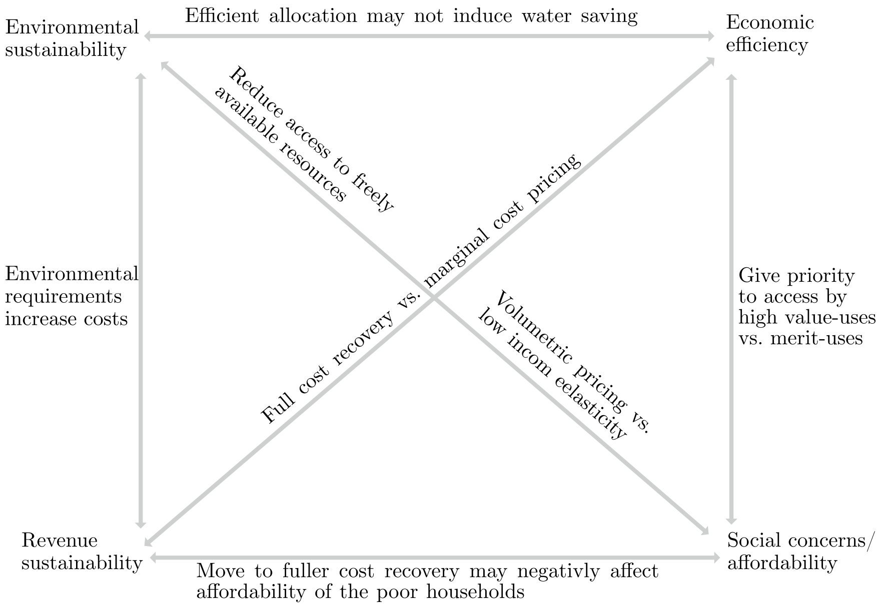

Fig. 4.2 Relation between goals of water pricing policy. *Source* Massarutto (2007a)

This means that, regardless of the budget, everybody should have access to the subsistence level. World Health Organization (2012) states the short-term subsistence level with 20 liter per capita and day. To evaluate the affordability of water supply, defining service indicators (e.g., the relation between expenditures for the water supply service and the overall budget) and determining threshold levels for these indicators are common procedures. Usually, the expenditures for the water service should not exceed three to five percent of the household income (OECD 2010; Walker 2009).

These four goals of water policy are related to one another in different ways. On the one hand, some goals can be achieved in accordance with another goal. For instance, an increase in the volumetric price may not only lead to higher revenues for the water supplier, which supports the revenue sufficiency goal, but it might also result in less exploited water resources due to a reduced water demand, which facilitates achieving the environmental sustainability goal.

On the other hand, one has to consider the trade-offs between the four objectives. Figure 4.2 illustrates these trade-offs. One important trade-off exists, for instance, between the goals of revenue sufficiency and social concerns. If water access has to be guaranteed to all consumers at low prices or maybe even for free in order to meet the affordability requirement, the revenues generated by the water supplier may not be sufficient to cover the full supply cost. Similarly, achieving the goal of economic efficiency counteracts revenue sufficiency. A pricing policy relying on the marginal cost usually constitutes the first best pricing policy because it maximizes economic efficiency. However, due to the crucial importance of fixed cost in water supply, average cost is higher than marginal cost, which is the main reason why water supply is considered a natural monopoly in most cases. If the price is equal to the marginal cost but lower than the average cost, the break-even is not reached and the water supplier faces revenue deficits. There exist a variety of examples for further trade-offs between the four main goals of water pricing policy.

4.3 Water Tariff Design

4.3.1 Tariff Structures

4.3.1.1 Overview

Various forms of water tariff systems are conceivable and implemented in practice. These tariffs generate revenues for the water suppliers and can consist of various components. The most important components applied in practice are (OECD 2010)

- A one-time **connection fee**, to gain access to the service.
- A recurrent **fixed charge** (sometimes known as a standing charge or flat fee) that can be uniform across customers or linked to the customer's characteristic (e.g., size of supply pipe, meter flow capacity, property value, or number of water-using appliances).
- If a metering system is in place, a **volumetric rate**, which, when multiplied by the volume of water consumed in a charging period, gives rise to the volumetric charge for that period.
- In some circumstances, a **minimum charge** is paid for each period, regardless of consumption.

Based on the composition of these four tariff components, various tariff structures can be implemented, which yield different expenditure functions, $R(w)$, for each tariff structure. The expenditure function describes the payments, symbolized by R, of a water-consuming household to the water supplier depending on the consumption level of the household, w.[2] Based on the expenditure function, the average expenditure function, $AR(w)$, and the marginal expenditure function, $MR(w)$, can be derived. The average expenditures are the average payments of a household per unit of water consumed (usually measured in cubic meter), while the marginal expenditure represents the payments of the household for the consumption of one additional unit (cubic meter) of water. The average and marginal expenditure function can be calculated by the following algebraic relations: $AR(w) = \frac{R(w)}{w}$ and $MR(w) = \frac{\partial R(w)}{\partial w}$. The most common tariff structures and their expenditure, average expenditure, and marginal expenditure functions are listed in the following.

[2]The expenditure incurred by the household is equivalent to the revenue obtained by the water supplier; hence, the expenditures are denoted by R.

4.3.1.2 Flat Rates

If there is no water meter available to measure the water consumption, a flat rate is usually the only feasible tariff structure. The customers pay a rate regardless of their consumption. This rate can be uniform, or differentiated with respect to the customer's characteristics, e.g., the rateable value of the property being served. The expenditure, average expenditure, and marginal expenditure functions are

$$R(w) = L$$
$$AR(w) = \frac{L}{w}$$
$$MR(w) = 0$$

where L represents the flat rate (lump sum).

4.3.1.3 Single Volumetric Rates

A single volume rate per consumed amount (e.g. cubic meter) of water is charged. The level of the volume rate does not change with consumption as it is independent of the consumption level. In addition to this single volumetric rate, a recurrent fixed charge (base price) might exist, which represents a payment to the water supplier regardless of the customer's water consumption level. The expenditure, average expenditure, and marginal expenditure functions are

$$R(w) = L + p \cdot Q$$
$$AR(w) = p + \frac{L}{w}$$
$$MR(w) = p$$

where L represents the base price and p stands for the volume price. The flat rate is a special form of this kind of tariff with $p = 0$.

4.3.1.4 Block Rates

Zone Block Rates

The volumetric charge is adjusted step-wise with increasing volumes of water consumed. In the case of increasing block rates, the volume rate rises with successively higher consumption blocks; for decreasing block rates, volume rates decline with higher consumption blocks. In addition, a recurrent fixed charge (base price) may exist in this form of tariff structure. Furthermore, it is possible to differentiate between zone tariffs and relay tariffs, while the former is the most commonly applied form of block rates. Under a zone tariff, the consumers pay the volume price of the respective block for each unit (e.g., cubic meter) of the quantity consumed. Under a relay tariff, the volume price of the highest consumption block has to be paid for the whole quantity consumed.

Given a block rate with N blocks in total, every block is separated by threshold values, denoted by $q_1, q_2, ..., q_i, ..., q_N$, with $q_1 < q_2 < ... < q_i < ... < q_N$. The ith block is defined within the range $\left[q_{i-1}, q_i\right]$ and the volume price in this block is p_i. The relation $p_1 < p_2 < ... < p_i < ... < p_N$ is valid for increasing block rates, while the contrary situation with $p_1 > p_2 > ... > p_i > ... > p_N$ occurs for decreasing block rates.

Based on the presented general case with an arbitrary number of blocks, a tariff with three blocks is specified by the two threshold consumption levels q_1 and q_2. The first block is defined for the interval $[0, q_1]$ and the relevant volume price in this first block is p_1, while the second block which is defined within the range of both threshold levels $[q_1, q_2]$ is characterized by the volume price p_2. Finally, the third block is relevant for a consumption which exceeds the second threshold level. The volume price of this third block is p_3. Based on this specified tariff with three blocks and an observed consumption level of w, the following total, average, and marginal expenditure functions could be set up for a zone block tariff:

$$R(Q) = \begin{cases} L + p_1 \cdot Q & \text{for } w \leq q_1 \\ L + p_1 \cdot q_1 + p_2 \cdot (w - q_1) & \text{for } q_1 < w \leq q_2 \\ L + p_1 \cdot q_1 + p_2 \cdot (q_2 - q_1) + p_3 \cdot (w - q_2) & \text{for } w > q_2 \end{cases}$$

$$AR(w) = \begin{cases} p_1 + \frac{L}{w} & \text{for } w \leq q_1 \\ p_2 + \frac{(p_1 - p_2) \cdot q_1 + L}{w} & \text{for } q_1 < w \leq q_2 \\ p_3 + \frac{(p_1 \cdot q_1 + (p_2 - p_3) \cdot q_2 + L}{w} & \text{for } w > q_2 \end{cases}$$

$$MR(Q) = \begin{cases} p_1 & \text{for } w \leq q_1 \\ p_2 & \text{for } q_1 < w \leq q_2 \\ p_3 & \text{for } w > q_2 \end{cases}$$

Relay Block Rates

The variable L represents the base price whose level is independent of the consumption level in this presented example. Regardless of the total observed consumption level w, the consumption within the first, second, and third blocks is priced with p_1, p_2, and p_3, respectively, for a zone block tariff. However, in contrast to a zone block rate, a relay block rate is characterized by the fact that the entire consumption is priced with p_1 or p_2 or p_3 if the entire consumption level is within the first, $w \leq q_1$, or second, $w = [q_1, q_2]$, or third block, $w > q_2$, respectively. Therefore, the expenditure for a relay block rate differs from the expenditure for a zone block rate, if the total consumption level exceeds the first block. The total, average, and marginal expenditure functions of a relay block rate with three blocks have the following form:

$$R(w) = \begin{cases} L + p_1 \cdot w & \text{for } w \leq q_1 \\ L + p_2 \cdot w & \text{for } q_1 < w \leq q_2 \\ L + p_3 \cdot w & \text{for } w > q_2 \end{cases}$$

$$AR(w) = \begin{cases} p_1 + \frac{L}{w} & \text{for } w \leq q_1 \\ p_2 + \frac{L}{w} & \text{for } q_1 < w \leq q_2 \\ p_3 + \frac{L}{w} & \text{for } w > q_2 \end{cases}$$

$$MR(w) = \begin{cases} p_1 & \text{for } w \leq q_1 \\ p_2 & \text{for } q_1 < w \leq q_2 \\ p_3 & \text{for } w > q_2 \end{cases}$$

4.3.1.5 Adjusted Block Rates

This tariff structure is quite similar to block rates, but, in contrast to conventional block rates, adjusted block rates feature volumetric rates or block sizes that are adjusted depending on the consumer's characteristics (e.g., income or household size).

4.3.2 Price Discrimination

The distinction of consumer prices with respect to the individual consumer's characteristics is termed as price discrimination. Different forms of price discrimination can be distinguished (Varian and Repcheck 2010).

4.3.2.1 First-Degree Price Discrimination

Under first-degree price discrimination, prices for each unit of a good are set such that the price charged for each unit is equal to the consumer's willingness to pay for that unit. Therefore, the consumer's surplus will be skimmed off fully by the producer. This approach is also known as perfect price discrimination. Due to information asymmetries between the players in the market (e.g., the supplier does not know the willingness to pay of each consumer), perfect price discrimination is generally not applicable in practice.

4.3.2.2 Second-Degree Price Discrimination

For second-degree price discrimination, the price depends on the bought amounts. Block rates are a common example of this form of price discrimination, as the price schedule involves different prices for different amounts of water sold.

Optional tariffs are another example of second-degree price discrimination. Here, consumers can choose from different tariff options offered (e.g., tariff options that differ in their single volumetric rates and fixed charges). The consumer's decision on the choice of tariff depends on the consumer's consumption level. The lower the consumption level, the higher the preference for choosing a tariff with a higher volume rate and lower base price.

4.3.2.3 Third-Degree Price Discrimination

Different consumers are charged with different prices based on their individual characteristics. This is a form of price discrimination commonly practiced for a multitude of products and services, e.g., discounts for students or social welfare recipients. The adjusted block rate is an application of this form of price discrimination. The price adjustment should usually be determined in the way that poor households or especially large households receive a financial relief.

A further example of third-degree price discrimination is social tariffs, under which low-income households get a discount on the volumetric rate or fixed charge they have to pay. Therefore, they get a financial relief on their water bill, relative to other non-low-income households.

4.3.2.4 Spatial/Regional Price Discrimination

Spatial price discrimination occurs when prices depend on the location of the consumer. Under this form of price discrimination, the water supplier can establish various price zones within its water supply area. Regional price discrimination is applicable if the pricing regime is based on the cost-by-cause principle. A consumer that is further away from the waterworks may cause higher water distribution network cost than a consumer closer to the water treatment facility. Similarly, water delivery to consumers located on a hill or on top of a mountain is accompanied by higher pumping costs than the supply of consumers living in a valley. Therefore, consumers which are located further away from the waterworks or that are located on a mountain have to pay a higher volume price.

4.3.2.5 Temporal/Seasonal Price Discrimination

The price for water differs depending on the time point (or season) of consumption. Based on this approach, seasonal pricing schemes can be implemented. Temporal price discrimination can be useful due to a multitude of reasons, for instance, the costs for water supply in summer months can be higher than in the winter months, because of lower groundwater tables in summer. Another reason can be higher water requirements in summer months than in the residual year, especially in regions with a high share of agricultural water demand. Capital-intensive pumping equipment may be just exhausted in the summer periods, where irrigation is intensified due to the growth of the agricultural plants. Therefore, if the tariff system based on the cost-by-cause principle is implemented, the volume prices in the summer month would be higher than in the residual months of the year.

4.3.3 Two-Part Tariff Versus One-Part Tariff

Single two-part tariffs are characterized by a single volumetric rate, i.e., the price per volume of water consumed, and a recurrent fixed rate, which is independent of the consumption level usually paid monthly or yearly. The single one-part tariff is a special form of a single two-part tariff, because just one tariff component is relevant in this kind of tariff. Either the one-part tariff is characterized by a single volumetric rate without a recurrent fixed rate (volumetric tariff) or by a recurrent fixed rate without a volumetric price (flat rate).

4.3.3.1 Two-Part Tariff

Assume that a representative consumer group has to be served with water by a supplier. The situation in the market should be cleared, which means that the volumes demanded by the consumers equal the water volumes offered by the supplier. The water supply to the consumer group causes costs, $C(w)$, where w stands for the level of consumption and supply. The average costs can be expressed as $AC(w) = \frac{C(w)}{w}$ and the marginal costs are $MC(w) = C'(w)$. We want to design a tariff that maximizes the total surplus. The total surplus is equal to the difference between the generated benefit due to consumption, $B(w)$, and the costs for supplying the consumer group, $C(w)$.

From household theory, we know that the consumers want to maximize their surplus, which is equivalent to the difference between the benefits from consumption $B(w)$ and the expenditures for water delivery which is $p \cdot w - L$. Hence, a consumer solves the following optimization problem:

$$\max_{\{w\}} [B(w) - p \cdot w - L]$$

From the consumer's perspective, water consumption, denoted by w, is the sole decision variable. By solving this optimization problem, we get $p = B'(w)$; hence, we can express water consumption in terms of volumetric prices, i.e., $w(p)$. The inverse form of $w(p)$ which is $p(w)$ is nothing else than the demand function which is determined by the marginal benefit $p(w) = B'(w)$.

If we furthermore assume that in a water supply system, total surplus should be maximized and water delivery is not restricted by any capacity or scarcity constraints, it is also possible to derive that the price is equal to the marginal cost level, $p = C'(w)$. This results from the fact that in a situation where total surplus is maximized, the demand function which is determined by the marginal benefit function should, according to household theory, be equal to the marginal cost.

The water supplier should be profitable and should generate enough revenue to cover its costs (financial sustainability goal of pricing policy). However, if we restrict the supplier (e.g., by regulation) in the way that the supplier cannot make any profits, the revenues must be equal to the total costs, which means $C(w) = L + p \cdot w$. Based on this assumption, it is possible to find the optimal recurrent fixed charge, L^*:

$$L^* = C(w^*) - p \cdot w^* \tag{4.1}$$

Based on the optimal fixed charge in Eq. (4.1), the average lump sum per consumed quantity, denoted by AL^*, can be calculated as the difference between the average costs and the price, as illustrated by Eq. (4.2).

$$AL^* = \frac{L^*}{w^*} = \frac{C(w^*) - p \cdot w^*}{w^*} = AC(w^*) - p \tag{4.2}$$

Water supply is usually a natural monopoly due to a high proportion of fixed costs to total costs. Hence, the average cost function is decreasing at the optimal consumption level w^*, i.e., $AC'(w^*) < 0$. In this case, the marginal costs are below the average cost level in the optimum, such that $C'(w^*) < AC(w^*)$ holds.[3] Furthermore, we know that $p = C'(w^*)$, and hence $p < AC(w^*)$. Therefore, it follows that the average lump sum is positive, i.e., $AL^* > 0$, which also means that the fixed charge must be positive, i.e., $L^* > 0$.[4]

4.3.3.2 One-Part Tariff with Single Volumetric Rate

If, instead of a single two-part tariff, a single one-part tariff with just a volumetric rate is implemented, the costs can only be covered by revenues from the single volumetric price. As the water supplier is assumed to be regulated, the supplier should not make any profits; hence, revenues should be equal to cost, which yields the following condition:

$$C(w) = p \cdot w \tag{4.3}$$

From Eq. (4.3) follows that the volumetric price should cover the average cost:

$$p = \frac{C(w)}{w} = AC(w) \tag{4.4}$$

In the case of a single one-part tariff with just a volumetric rate, the consumer solves the following optimization problem:

$$\max_{\{w\}} [B(w) - p \cdot w] \tag{4.5}$$

The solution is $p = B'(w)$. Because of this optimality condition, the price determines the quantity level of consumption. The marginal benefit function $B'(w)$ is nothing else than the demand function. Therefore, similar to the two-part tariff, the quantity level w^V is determined by the price p^V, hence, $w^V(p^V)$.

$$p^V = AC(w^V) = B'(w^V) \tag{4.6}$$

The optimal price and quantity, p^V and w^V, result from the intersection point of the average cost function and the demand function.

[3] We know that $AC(w) = \frac{C(w)}{w}$. Because of $AC'(w) < 0$, it is possible to write $AC'(w) = \frac{\partial \frac{C(w)}{w}}{\partial w} < 0$. Solving $\frac{\partial \frac{C(w)}{w}}{\partial w} < 0$, we get the following result: $\frac{C'(w) \cdot w - C(w)}{w^2} < 0$. This is $\frac{C'(w)}{w} - \frac{C(w)}{w^2} < 0$ and hence $C'(w) < \frac{C(w)}{w}$, which is $C'(w) < AC(w)$.

[4] $L^* = \underbrace{AL^*}_{>0} \cdot \underbrace{w^*}_{\geq 0} > 0$.

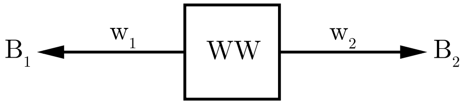

Fig. 4.3 Universal service provider: The basic setup. *Source* own illustration

4.3.3.3 Flat Rate

However, if a flat rate is implemented, only the recurrent base price, L, has to be paid for any amount of water consumption, as the volumetric price is zero, $p = 0$. For this case, the consumers solve the following optimization problem:

$$\max_{\{w\}} [B(w) - L] \tag{4.7}$$

the result of which is $B'(w) = 0$. Therefore, the consumption level w^L is determined by the maximum demand. Similar to the pricing regimes analyzed previously, revenues should cover the cost, while profits should not be generated. Hence, the revenues arising from the flat rate have to be set equal to the costs, which implies

$$L^L = C(w^L) \tag{4.8}$$

Based on the fixed rate, the average fixed rate per amount of water consumed is equal to the average cost level:

$$AL^L = \frac{C(w^L)}{w^L} = AC(w^L) \tag{4.9}$$

4.3.4 Universal Service Provider

4.3.4.1 Two Consumer Groups

The universal service provider is a service operator that offers infrastructure services such as water supply at uniform and affordable conditions. These principles are enforced by appropriate price regulation, either by the provider being a public enterprise or by a private operator being regulated by a price regulator. The concept of a universal service provider is the most common form in practice, because price discrimination based on the cost-by-cause principle is often not enforceable, whether due to political, social, economical, or fairness reasons.[5]

Figure 4.3 illustrates an exemplary situation of two consumer groups which are served with water through one water supply system. Due to delivery and consumption of water, w_1 and w_2, both consumer groups obtain benefits, which are represented by $B_1(w_1)$ and $B_2(w_2)$. The service provision to the consumers causes costs: On the one hand, there are cost components that are caused by both consumer

[5] See the survey in Cremer et al. (2001).

groups, e.g., treatment costs in the waterworks, which are symbolized by the variable $C_{12}(w_{12}(w_1, w_2))$. The variable w_{12} represents the total amount of the consumed water level, hence:

$$w_{12}(w_1, w_2) = w_1 + w_2 \tag{4.10}$$

On the other hand, there are also cost components that are incurred by only one consumer group. The specific cost of consumer group 1, $C_1(w_1)$, depends solely on the amount of water consumed by group 1. Similarly, the specific cost of consumer group 2, $C_2(w_2)$, depends only on the amount of water consumed by group 2. These specific costs which are caused by just one consumer group are, for instance, pumping costs in the water networks.[6] The optimization problem for maximizing total surplus in the water supply area is

$$\max_{\{w_1, w_2\}} [B_1(w_1) + B_2(w_2) - C_1(w_1) - C_2(w_2) - C_{12}(w_{12}(w_1, w_2))] \tag{4.11}$$

The KKT conditions resulting from the optimization problem are

$$B_1'(w_1) - C_1'(w_1) - C_{12}'(w_{12}) \cdot \underbrace{w_{12}'(w_1)}_{=1} \leq 0 \perp w_1 \geq 0 \tag{4.12}$$

$$B_2'(w_2) - C_2'(w_2) - C_{12}'(w_{12}) \cdot \underbrace{w_{12}'(w_2)}_{=1} \leq 0 \perp w_2 \geq 0 \tag{4.13}$$

It seems plausible to assume that both consumer groups consume positive amounts of water, i.e., $w_1 \geq 0$ and $w_2 \geq 0$. Under this assumption, the following optimality conditions can be formulated:

$$B_1'(w_1) = C_1'(w_1) + C_{12}'(w_{12}) \tag{4.14}$$

$$B_2'(w_2) = C_2'(w_2) + C_{12}'(w_{12}) \tag{4.15}$$

According to Eqs. (4.14) and (4.15), marginal benefit should equal marginal cost for each consumer group; hence, the optimal consumption level can be derived from the intersection point between the demand function and the marginal cost function for each addressed consumer group. Hence, for the optimal solution, the consumer groups have to pay different volumetric water prices if the marginal cost levels differ. A situation in which the marginal cost levels do not change with the output level (horizontal directed marginal costs functions) is depicted in Fig. 4.4. The component $C_{12}'(w_{12})$ is represented by the parameter c, which is, for instance, the cost rate for the treatment of 1 unit of water. However, the specific marginal (pumping) costs for serving consumer group 1 and 2 differ: $C_1'(w_1) < C_2'(w_2)$. Therefore, the volumetric

[6] We assume that the consumer group 2 has a higher geodetic level than consumer group 1. Hence, the pumping cost for serving consumer group 2 with one amount of water are higher than for the serving of consumer group 1.

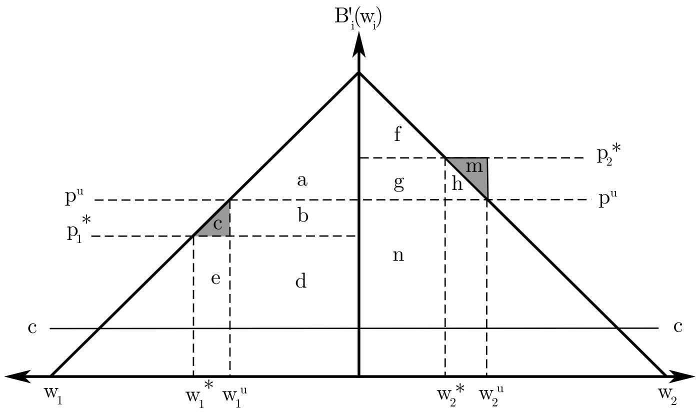

Fig. 4.4 Universal service provider with two consumer groups. *Source* own illustration

water price for consumer group 1 is lower than the one for consumer group 2, $p_1^* < p_2^*$, which means that a price discrimination has to be applied between the two exemplary consumer groups. This can be realized by, for instance, a regional price discrimination where two price zones are defined. The optimal consumption levels of both consumer groups in Fig. 4.4 are represented by the variables w_1^* and w_2^*.

4.3.4.2 Uniform Pricing

Suppose the water supplier is a universal service provider who offers water at a uniform price to all consumers, then the price p^{u} is set, which lies between the optimal price levels under price discrimination, i.e., $p_1^* < p^{\mathrm{u}} < p_2^*$, as illustrated in Fig. 4.4. Compared to the case of price discrimination, setting a uniform price induces a decrease in the consumption level of group 1 from w_1^* to w_1^{u}, while consumer group 2 experiences an increase in its consumption level from w_2^* to w_2^{u}. Under a uniform price, the changes in prices and consumption levels relative to the optimal solution result in consumer group 1 losing some of its surplus, which is represented by the areas $b + c$ in Fig. 4.4, while consumer group 2 gains additional surplus, depicted by the areas $g + h$.

With respect to the supply side, the supplier may gain or lose by supplying groups 1 and 2. The price increase for group 1 impacts the producer surplus positively, whereas the consumption level decrease influences the producer surplus negatively. The area b, which is part of the consumer surplus under an optimal pricing regime, becomes producer surplus under a uniform pricing policy due to the price increase in group 1. In total, there is a loss of social welfare in supplying price zone 1 under a uniform price, which is symbolized by the area c.

A similar analysis can be done for consumer group 2. Due to the decreased price and increased consumption in group 2, the supplier's loss in surplus amounts to the areas $g + h + m$. Therefore, there is a loss of social welfare induced by setting a uniform price in group 2, represented by area m. We conclude that a uniform pricing policy leads to economic losses in accordance with the areas $c + m$ in the whole water

Table 4.1 Distributional effects due to optimal and uniform pricing

		Group 1	Group 2
Consumer surplus (CS)	Optimum	$a + b + c$	f
	Uniform	a	$f + g + h$
	Δ CS	$-b - c$	$+g + h$
Producer surplus (PS)	Optimum	–	–
	Uniform	$+b$	$-g - h - m$
	Δ PS	$+b$	$-g - h - m$
Change social welfare		$-c$	$-m$

supply area compared to an optimal pricing policy. A more detailed overview of the distributional effects under the two addressed pricing regimes is given in Table 4.1.

Equity in pricing policy is often seen as fairer than (regional) price discrimination based on the cost-by-cause principle in a water supply area. However, the gain of fairness is associated with a loss of economic efficiency which is represented by the areas $c + m$. This is a matter of fairness preference, or of inequity aversion, which means how much loss of efficiency a society wants to accept in order to achieve a fair water allocation.

4.3.5 Optional Tariffs

4.3.5.1 The Concept

If the water supplier offers optional tariffs, consumers have the possibility to choose between various pricing options. The effectiveness of optional tariffs is explained with the help of an example: Fig. 4.5 depicts three pricing options whose expenditure functions depend on the level of consumption. Each of these pricing options contains a recurrent fixed charge and a single volumetric charge. The first pricing option, T_1, is characterized by a relatively low fixed charge and a relatively high volumetric charge, while the third pricing option, T_3, contains a relatively high fixed charge and a relatively low volumetric charge. Pricing option T_2 is characterized by a relatively moderate fixed charge and volumetric charge. If L_i and p_i stand for the fixed and volumetric charge of the ith pricing option, respectively, it is possible to characterize the optional tariff by $(L_1 < L_2 < L_3) \wedge (p_1 > p_2 > p_3)$.

The consumers have the option to choose between pricing options T_1, T_2, and T_3, whereas the decision of the consumer depends on the consumption level. If the consumer consumes less than $\tilde{w}^1$, which means $w \le \tilde{w}^1$, the consumer minimizes its expenses by choosing option T_1. Similarly, consumers who have a moderate consumption within the range $\tilde{w}^1 \le w \le \tilde{w}^2$ choose pricing option T_2 to minimize the expenditures. Consumers with a consumption above $\tilde{w}^2$, i.e., $w \ge \tilde{w}^2$, minimize their expenditures by choosing price option T_3. It becomes obvious that a higher consumption level is associated with a higher preference to choose a tariff with lower volumetric rate. This is the analogous logic to a volume discount, where the price

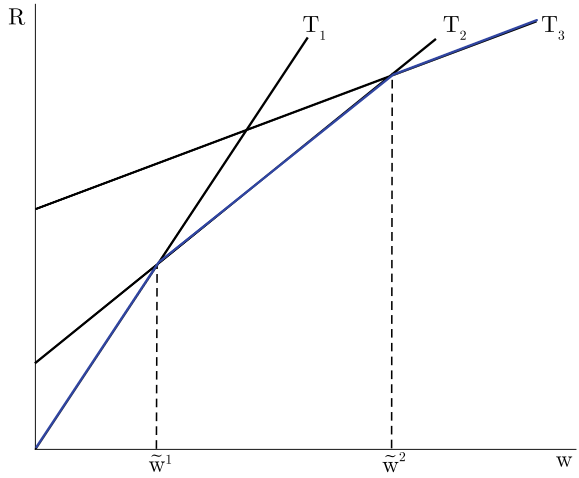

Fig. 4.5 Concept of optional tariffs. *Source* own illustration

per unit also decreases with an increase in units bought. This volume discount rule for an optional tariff can be formalized by the statement $\forall m\ \forall n : (w_m < w_n) \implies (p_m \geq p_n)$.[7]

This simple example shows that with increasing consumption the consumers choose pricing options with higher fixed fees. This is a typical characteristic of well-defined optional tariffs, which can be explained by the previously mentioned volume discount rule. Under the assumption that the volumetric price of the mth pricing option is greater than or equal to the one of the nth option, i.e., $p_m \geq p_n$, the following undesirable situations are possible:

- One or more pricing options have an absolute disadvantage compared to another/to other pricing option(s). Pricing options with an absolute disadvantage are never chosen and are, therefore, useless as an optional tariff.
- At least two pricing options are identical. If at least two options are identical, the consumer is indifferent between choosing the respective options. Hence, at least one option is useless in the optional tariff.

If one of these situations occurs, the optional tariff is not well defined. If the volumetric price of the mth pricing option is higher than the one of the nth pricing option, such that $p_m > p_n$, which means that the preference for choosing the mth pricing option instead of the nth pricing option will increase with a decreasing consumption level, the fixed charge of the mth pricing option has to be lower than the one of the nth pricing option in a well-defined optional tariff. This can be described in a formal way by the following statement: $\forall m\ \forall n : (p_m > p_n) \iff (L_m < L_n)$.

[7]If $w_n > w_m$, then $p_n \leq p_m$.

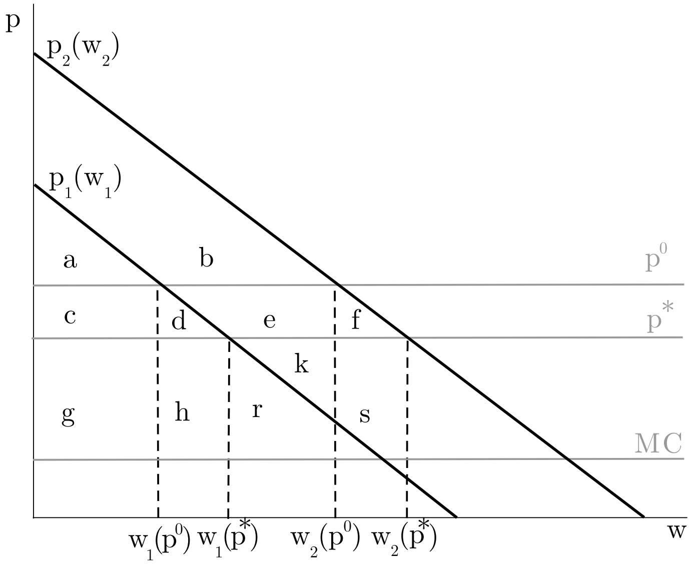

Fig. 4.6 Example with optional tariffs. *Source* own illustration

4.3.5.2 Effects on Consumers and Producers

Under the implementation of an optional tariff both, the consumer and the producer can gain social welfare. This hypothesis can be fostered with the help of a simple example illustrated in Fig. 4.6: Given there are two consumer groups $i = \{1, 2\}$ whose demand functions are known. Consumer group 1 has a lower demand than consumer group 2 at every price level. In the initial situation, only one tariff $T^0 = \left[L^0, p^0\right]$ is offered by the water supplier. L^0 and p^0 stand for the fixed and volumetric charges in the initial tariff, respectively. Based on this volumetric price, the consumer groups 1 and 2 consume the amounts of $w_1(p^0)$ and $w_2(p^0)$, respectively. The generated consumer surplus for consumer group 1 is represented by the area a, while the one for group 2 is given by areas $a + b$. The expenditures from the base price reduce the consumer surplus by L^0 for each consumer group. Therefore, the consumer surpluses for groups 1 and 2 are represented by $a - L^0$ and $a + b - L^0$, respectively. The water supplier generates a profit equal to the area $2 \cdot (c + g) + d + h$ from the volumetric charge plus $2 \cdot L^0$ from the fixed charge to cover its fixed costs.[8]

Now, we assume that the water supplier offers an optional tariff in which the initial pricing scheme $T^0 = \left[L^0, p^0\right]$ is supplemented by an alternative pricing option $T^* = \left[L^*, p^*\right]$, where L^* and p^* symbolize the fixed charge and the volumetric charge of pricing option T^*, respectively. We assume that $p^0 > p^*$, and hence $L^0 < L^*$, such that the optional tariffs are well defined. Every consumer has the choice between the pricing options T^0 and T^*. If consumer group $i = \{1, 2\}$ chooses option T^0, this

[8]The profit generated from the volumetric price paid by groups 1 and 2 is symbolized by the areas $c + g$ and $c + d + g + h$, respectively.

group consumes the amount $w_i(p^0)$ and receives the consumer surplus CS_i^0, while if it opts for option T^*, it consumes $w_i(p^*)$, which leads to a consumer surplus of CS_i^*. The consumer group $i = \{1, 2\}$ chooses the option that maximizes the consumer surplus. We assume that $CS_1^0 > CS_1^*$ and $CS_2^0 < CS_2^*$. Hence, consumer group 1 will choose option T^0, while consumer group 2 decides for option T^*, which is the option with the lower volumetric and higher base price (volume discount rule).[9] In the design of option T^*, the water supplier anticipates the reaction of group 2. Due to the reduction of the volumetric price by switching from T^0 to T^*, revenues from this charge will be lost, maximally amounting to $(p^0 - p^*) \cdot w_2(p^0)$, which is symbolized by the areas $c + d + e$. These revenue losses can be fully compensated by a higher fixed charge in option T^*, hence, $L^* = L^0 + (p^0 - p^*) \cdot w_2(p^0)$.

If the optional tariff is offered, the situation does not change for the consumer group 1, because they are still priced under T^0 and obtain a consumer surplus represented by the area $a - L^0$. However, group 2 will switch from tariff T^0 to T^* and increase consumption from $w_2(p^0)$ to $w_2(p^*)$. Therefore, the surplus increases by the areas $c + d + e + f$. Furthermore, due to the change of pricing, expenditures from the base price increase from L^0 to L^*. These additional expenditures are represented by the areas $c + d + e$.[10] Hence, the consumer surplus increases by the area f to the level $a + b + f - L^0$ because of the introduction of the optional tariff. Therefore, consumer group 2 benefits from the introduction of the optional tariff, while consumer group 1 is not affected. Offering the optional tariff is also advantageous for the water supplier, as its profits increase by s to the level $2 \cdot (c + g) + d + e + h + r + k + s$ plus $2 \cdot L^0$.

Based on the example described above, we can conclude that the introduction of optional tariffs can lead to a situation where nobody is worsened and specific actors have an advantage compared to the initial situation without an optional tariff. Tables 4.2 and 4.3 give a detailed overview of the distributional effects for consumers and the water suppliers under both addressed pricing regimes, respectively.

4.3.6 Seasonal Pricing

In the case that a seasonal pricing scheme is implemented, the price for water changes with the time period of supply. For instance, the price in the summer month could be higher than in the winter month. We would like to term the period where the price has the highest level as peak period/peak season (e.g., summer), while the residual period is termed as off-peak period/off-peak season. If we implement this form of temporal price discrimination, the different price levels between the peak and off-peak seasons can be based on various reasons, for instance,

[9] Assume that $CS_1^0 < CS_1^*$. Because of the volume discount rule, it is certain that $CS_2^0 < CS_2^*$.

[10] $L^* = L^0 + (p^0 - p^*) \cdot w_2(p^0)$ and hence $L^* - L^0 = (p^0 - p^*) \cdot w_2(p^0)$. If there is a switch from pricing T^0 to T^*, the fixed charge increased from L^0 to L^*, which is nothing else than $L^* - L^0 = (p^0 - p^*) \cdot w_2(p^0)$. The additional revenues from the fixed charge are $(p^0 - p^*) \cdot w_2(p^0)$ which is equal to the areas $c + d + e$.

Table 4.2 Effects of surplus on consumer side due to the introduction of an optional tariff

		Group 1	Group 2
Initial tariff	Surplus from fixed charge	$-L^0$	$-L^0$
	Surplus from volumetric charge	a	$a+b$
	Consumer surplus	$a-L^0$	$a+b-L^0$
		$2\cdot a+b-2\cdot L^0$	
Optional tariff	Surplus from fixed charge	$-L^0$	$-c-d-e-L^0$
	Surplus from volumetric charge	a	$a+b+c+d+e+f$
	Consumer surplus	$a-L^0$	$a+b+f-L^0$
		$2\cdot a+b+f-2\cdot L^0$	
Change consumer surplus		/	$+f$
		$+f$	

Table 4.3 Effects of surplus for water supplier due to the introduction of an optional tariff

		Group 1	Group 2
Initial tariff	Surplus from fixed charge	L^0	L^0
	Surplus from volumetric charge	$c+g$	$c+d+e+g+h+r+k$
	Profit margin	$c+g+L^0$	$c+d+e+g+h+r+k+L^0$
		$2\cdot(c+g)+d+e+h+r+k+2\cdot L^0$	
Optional tariff	Surplus from fixed charge	L^0	$c+d+e+L^0$
	Surplus from volumetric charge	$c+g$	$g+h+r+k+s$
	Profit margin	$c+g+L^0$	$c+d+e+g+h+r+k+s+L^0$
		$2\cdot(c+g)+d+e+h+r+k+s+2\cdot L^0$	
Change profit margin		/	$+s$
		$+s$	

- Less water is available in the summer (peak season) than in the residual months (off-peak season). The different prices between these time intervals result from the different scarcity price levels.
- It is more expensive to deliver water in the summer (peak season) than in the residual months (off-peak season). Therefore, the marginal cost function in the peak season is higher than in the off-peak season. This results in a higher equilibrium price in the peak season compared to the off-peak season.
- The demand for water in the summer (peak season) is higher than in the residual months (off-peak season), because in summer people are more thirsty, more water is required for plants watering, more water is needed for filling pools, etc. Therefore, the demand function of the peak season is higher than the one of the off-peak season. This results in a higher equilibrium price in the peak season compared to the off-peak season.
- The provision of capacity for delivering water is related to costs. The higher the capacity, the higher the capacity costs. These capacity costs have to be covered by the revenues from the water price. It is thinkable that the capacity is just financed in the peak season. However, it is also possible that capacity is financed during the peak and off-peak seasons. A temporal price discrimination scheme which is based on the financing of capacity costs for water delivery infrastructure is presented in the following model.

4.3.6.1 Temporal Price Discrimination for Financing Capacity Costs

Suppose the two time seasons 1 and 2 where we have the water consumption w_1 and w_2, respectively. In both seasons, benefits are generated from the water consumption related to the functions $B_1(w_1)$ and $B_2(w_2)$. We already know from household theory that the demand function is determined by the marginal benefit function. Hence, the demand function of season 1 is $p_1(w_1)$, while the demand function for season 2 is $p_2(w_2)$. The demand in season 2 is higher than in season 1 if

- for every consumption level w: $p_2 > p_1$, the price level in season 2 is higher than in season 1;
- for every price level p: $w_2 > w_1$, the consumption amount in season 2 is higher than in season 1.

Under this assumption, we term the season 2 as peak season and the season 1 as the off-peak season.

The supply of water is related to costs. On the one hand, we suppose the cost rate c for delivering water. Hence, the annual costs for water delivery are therefore represented by the term $c \cdot (w_1 + w_2)$. Furthermore, there also exist costs for the provision of capacity in the supply system. This capacity (e.g., pumping capacity) is needed for the delivery of water to the consumers. The capacity cost rate r represents the cost for the provision of one unit of capacity. The capacity level is represented by the variable k. Hence, the total annual capacity costs are therefore $r \cdot k$.

In the optimization, we want to calculate the optimal consumption and capacity levels in the way that we maximize the total surplus of 1 year, which includes the peak and off-peak seasons. The total surplus results from the difference of benefits and costs:

$$\max_{\{w_1, w_2, k\}} [B_1(w_1) + B_2(w_2) - c \cdot (w_1 + w_2) - r \cdot k]$$
$$s.t.\ w_1 \leq k \qquad (\lambda_1)$$
$$w_2 \leq k \qquad (\lambda_2)$$

Of course, the water delivery in both seasons is restricted by the chosen capacity level. Based on the optimization problem, the Lagrangian function can be set up:

$$L = B_1(w_1) + B_2(w_2) - c \cdot (w_1 + w_2) - r \cdot k + \lambda_1 \cdot [k - w_1] + \lambda_2 \cdot [k - w_2]$$

and finally the KKT conditions can be formulated:

$$B_1'(w_1) - c - \lambda_1 \leq 0 \perp w_1 \geq 0 \tag{4.16}$$
$$B_2'(w_2) - c - \lambda_2 \leq 0 \perp w_2 \geq 0 \tag{4.17}$$
$$\lambda_1 + \lambda_2 - r \leq 0 \perp k \geq 0 \tag{4.18}$$
$$k - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{4.19}$$
$$k - w_2 \geq 0 \perp \lambda_2 \geq 0 \tag{4.20}$$

We assume that we have a consumption in both seasons, hence $w_1 \geq 0$ and $w_2 \geq 0$. Because of Eqs. (4.19) and (4.20), it follows that we have to assume that $k \geq 0$. Therefore, we know from Eqs. (4.16)–(4.18) that

$$B_1'(w_1) = c + \lambda_1$$
$$B_2'(w_2) = c + \lambda_2$$
$$\lambda_1 + \lambda_2 = r$$

Regarding the conditions Eqs. (4.19) and (4.20), we have four possible options for assumption:

- $\lambda_1 = \lambda_2 = 0$: The capacity is exploited neither in the off-peak nor in the peak season;
- $\lambda_1 \geq 0, \lambda_2 = 0$: The capacity is exploited just in the off-peak season;
- $\lambda_1 = 0, \lambda_2 \geq 0$: The capacity is exploited just in the peak season;
- $\lambda_1 \geq 0, \lambda_2 \geq 0$: The capacity is exploited in the off-peak and peak seasons.

The assumption $\lambda_1 = \lambda_2 = 0$ can never lead to optimality, because of Eq. (4.18). We know that $\lambda_1 + \lambda_2 = r$ is not met, because $\lambda_1 + \lambda_2 = 0$, while capacity cost rate r is positive, i.e., $r > 0$.

The assumption $\lambda_1 \geq 0$, $\lambda_2 = 0$ can also never lead to optimality. This seems to be quite plausible, because under this assumption the capacity is only exploited in the off-peak season. This means that in the peak season the consumption is lower than in the off-peak season, even though the demand in the peak season is higher than in the off-peak season. Therefore, this assumption intuitively does not make much sense. However, there is also a mathematical way for finding a contradiction under this assumption. From conditions Eqs. (4.19) and (4.20), we know that $w_1 = k$ and $w_2 \leq k$, respectively. Hence, $w_2 \leq w_1$. From Eq. (4.18), it becomes obvious that $\lambda_1 = r$. Based on Eq. (4.17), we find that $B_2'(w_2) = c$, while from Eq. (4.16) it follows that $B_1'(w_1) = c + \lambda_1$, which is nothing else than $B_1'(k) = c + r$. Let us define the variable η which gives the difference between the consumption level in the peak season and the capacity, hence $\eta = k - w_2$. Therefore, we know that $k = \eta + w_2$. For sure, we are able to reformulate $B_1'(k)$ into the expression $B_1'(\eta + w_2)$. Furthermore, we also know that the benefit functions are concave, which means for the benefit function of user 1: $B_1''(w) < 0$. Due to this concavity condition we know that $B_1'(\eta + w_2) < B_1'(w_2)$. The marginal benefit in the off-peak season exceeds the one in the peak season, $c + r = B_1'(k) > B_2'(w_2) = c$. Combining the latter two relations, we can formulate that $B_1'(w_2) > B_1'(\eta + w_2) > B_2'(w_2)$, and hence $B_1'(w_2) > B_2'(w_2)$, which is a contradiction to a former assumption. We suppose that season 1 is the off-peak season with lower demand, while season 2 is the peak demand with higher demand. Because of this relation we know that for every consumption level w: $B_1'(w) < B_2'(w)$, the marginal benefit of the peak season 2 exceeds the one of the off-peak season 1, and therefore $B_1'(w_2) < B_2'(w_2)$.

Under the assumption $\lambda_1 = 0$ and $\lambda_2 \geq 0$, the capacity is just exploited in the peak season. Therefore, the consumption level in the peak season is higher than in the off-peak season which seems to be quite plausible. Based on Eqs. (4.16)–(4.20), we are able to set up the following optimality conditions for this case:

$$w_2 = k \tag{4.21}$$

$$w_1 \leq k \tag{4.22}$$

$$\lambda_2 = r \tag{4.23}$$

$$B_1'(w_1) = c \tag{4.24}$$

$$B_2'(w_2) = c + r \tag{4.25}$$

The situation is illustrated in Fig. 4.7. The left figure illustrates the situation where the made assumption $\lambda_1 = 0$ and $\lambda_2 \geq 0$ leads to optimality. The optimal price and consumption levels in the off-peak season result from the intersection between the marginal benefit in season 1 and the cost rate for water delivering c, while the optimal price and consumption levels in the peak season (summer) result from the marginal benefit in season 2 and the sum of the cost rate for water delivering and the capacity cost rate $c + r$.

The price in the peak season, which is equal to the marginal benefit $B_2'(w_2)$, is bigger than in the off-peak season which is $B_1'(w_1)$, because of the capacity cost rate r, which is only relevant in the peak season. Hence, the revenues for covering the

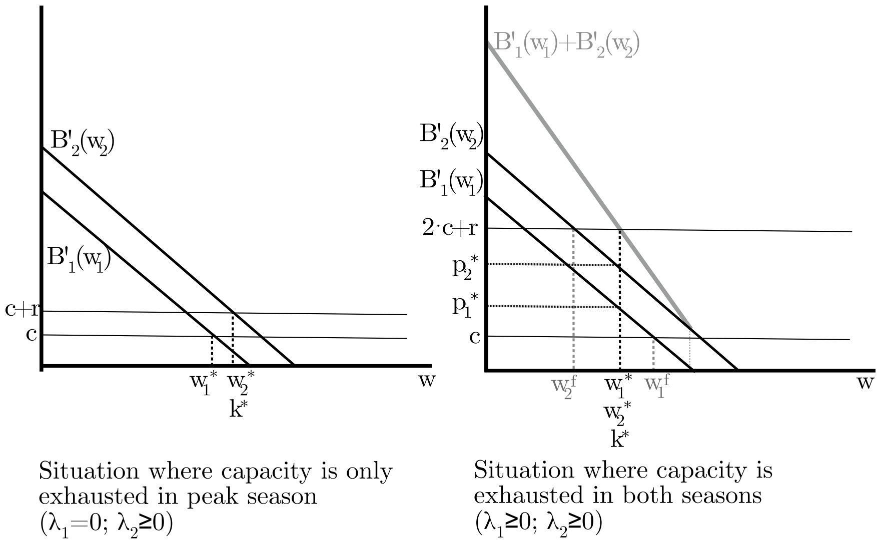

Fig. 4.7 Illustration of optimal seasonal pricing. *Source* own illustration

capacity costs are only earned in the peak season. The capacity cost rate does not impact the consumption level in the off-peak season, but the consumption level in the peak season. Of course, the higher the capacity cost rate, the lower the consumption in the off-peak season. If the capacity cost rate would be sufficiently high, the consumption in the peak season could fall below the consumption level in the off-peak season.

This situation is pictured in the right figure of Fig. 4.7. If we would set the price in the off-peak and peak season equal to c and $c + r$, respectively, it results in a consumption level w_1^f in the off-peak season which exceeds the consumption level w_2^f in the peak season. If the consumption level in the off-peak season exceeds the one of the peak season, we do not meet the condition Eq. (4.19), and hence, the other plausible case ($\lambda_1 \geq 0$ and $\lambda_2 \geq 0$) would lead to optimality.

The assumption $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ states that the capacity is exploited in both seasons, in the peak and the off-peak seasons. The consumption levels in both seasons are equal to the capacity. Based on Eqs. (4.16)–(4.18), we know that

$$\lambda_1 + \lambda_2 = r$$
$$B_1'(w_1) = c + \lambda_1$$
$$B_2'(w_2) = c + \lambda_2$$

which can be combined by summation of $B_1'(w_1)$ and $B_2'(w_2)$ to the following expression:

$$B_1'(w_1) + B_2'(w_2) = 2 \cdot c + r$$

Therefore, we can formulate the following optimality conditions:

$$w_1 = w_2 = k \tag{4.26}$$

$$B_1'(w_1) + B_2'(w_2) = 2 \cdot c + r \tag{4.27}$$

$$\lambda_1 = B_1'(w_1) - c \tag{4.28}$$

$$\lambda_2 = B_2'(w_2) - c \tag{4.29}$$

Here, we have the same consumption levels in both seasons. Because the demand in the peak season is higher than in the off-peak season, the price in the peak season is certainly higher than in the off-peak season. However, the revenues for financing the capacity costs are earned in both seasons.

The right figure of Fig. 4.7 pictures the situation for which the assumption $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ leads to optimality. The function $B_1'(w_1) + B_2'(w_2)$ results from the summation of the seasonal marginal benefit functions in vertical direction. The intersection point of the $B_1'(w_1) + B_2'(w_2)$-curve with $2 \cdot c + r$ determines the optimal capacity k^* as well as the optimal consumption levels w_1^* and w_2^* of both seasons. The seasonal prices result from the consumption levels found. The respective marginal benefit functions are illustrated in Fig. 4.7 by p_1^* and p_2^* for the off-peak and peak seasons, respectively.

4.4 Increasing Block Tariffs

4.4.1 The Concept

The increasing block tariff (IBT) is an important tariff form which is quite often implemented especially in developing countries. This tariff form is characterized by a volumetric charge that increases with rising consumption level. In some cases, even the level of the fixed charge depends on the consumption level. The characterization of IBTs can be formalized by

$$w_j > w_i \;\rightarrow\; p_j \geq p_i \tag{4.30}$$

If the jth consumption level is higher than the ith consumption level, the volumetric price of the jth consumption level must not fall below the one of the ith consumption level.

The popularity of this tariff form is attributed to the combination of some alleged advantages. Regions in developing countries are often characterized by a high proportion of poor population class with low income and low water availability leading to water shortage. Therefore, addressing social concerns and environmental protection goals is important. A well-designed increasing block tariff can enforce a simultaneous achievement of both goals. The first water volumes per household can be provided at a low price, which makes water affordable even for the poorest. Hence, the implementation of a well-defined increasing block tariff may promote the access to the public water supply for even those, with the lowest income. A secured access to water may lead to increased prosperity and well-being as well as the promotion of public health especially for the poorest people (Boland and Whittington 2000b).

Starting from a low volume price for the first consumed water amounts, the increasing block tariff is characterized by a rising volume price with increasing water consumption. A sufficiently high volume price for a certain defined consumption level sets an incentive for avoiding wasteful use and fosters the implementation of water-saving technologies. Households equate their marginal willingness to pay to the price valid in the block in which consumption falls. With increasing tariffs, water consumption can be pushed back. Hence, the tariff system meets two objectives at the same time: It prevents water wastage, thus helping to conserve resources and guarantees access to water for the poor.

Figure 4.8 displays the case of a two-block tariff where the price increase from the first to the second block leads to an implicit cross-subsidization of the poor by the middle class. We assume first that there are only two households: a poor household and a household with a middle class income. The upper part of the figure shows two blocks. The width of the first block from 0 to w_s is equal to the subsistence level of an average household. The corresponding water price is p_1, where p_1 is lower than the marginal costs of water supply (c). Water consumption beyond the lifeline falls into the second block. This demand is charged either by p_{2u} or p_{2o}. The demand for water in the second block follows the price-quantity function $p_2(w_2)$ for $w_2 \geq w_s$. While the poor household cannot afford to consume more water than the subsistence minimum, the middle-income household has sufficient income to consume more than the lifeline depending on the price in the second block.

Furthermore, we assume that the price in the second block is p_{2u}. The expense for the non-poor household is $p_1 w_s + p_{2u} \cdot (w_2 - w_s)$; the expenses for the poor households are simply $p_1 \cdot w_s$. Obviously, the first block generates a deficit of $2(c - p_1)w_s$ (2 x Area Θ) which must be covered by the contribution margin in the second block $p_{2u}(w_2 - w_s)$ (area CM_{2u}). The contribution margin is depicted on the vertical axis of the lower half of the picture as a function of the water consumption in the second block. As the price of block 2 is increased, the contribution margin (profits in the second block) rises until it reaches a maximum. If the price increases further, profits in the second block decrease until demand in the second block is chocked off. The lines are drawn such that p_{2o} and p_{2u} generate the same amount of contribution margin. In addition, these two prices are chosen such that the deficit from the first block, 2Θ, is exactly covered.

It remains to choose one of the two price options in block 2. The municipality can choose either a flat increase of the price from block 1 to block 2 or decide to implement a strong uplift which seems more egalitarian. Figure 4.8 shows that the strive for a more egalitarian outcome does not improve the situation of the poor household. The increase in the price of the second block from p_{2u} to p_{2o} does not lead to a decrease of p_1. Hence, if we follow the Pareto principle, we would choose the price in the second block such that the consumer rent of the middle class household is maximized subject to economic viability, i.e., that no deficit occurs. This is achieved if the water utility chooses p_{2u}. If a community adheres strictly to Egalitarianism, then p_{2o} is chosen at the expense of efficiency. In this case, water consumption is far

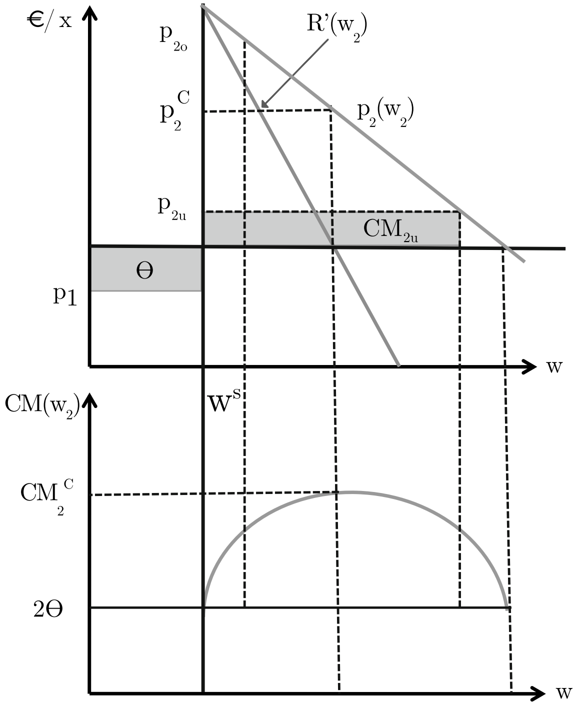

Fig. 4.8 Increasing two-block tariff. *Source* own illustration

less than the efficient level which is where the marginal willingness to pay is equal to marginal cost c. Water pricing then becomes a political issue and depends on how a society deals with justice and coherence issues.[11]

4.4.2 Potential Adverse Effects on the Poor

If a poor household consumes less water than richer households—which is quite plausible, because water is a normal good and income elasticity for water is positive—the wealthier households are burdened with more costs than the poor households. This promotes equity and the reduction of income disparities between the households. Therefore, to conclude, the wealthier households would cross-subsidize the poorer households, which can be considered as fair. However, notice that both prices p_{2u} and p_{2o} lead to the same amount of cross-subsidization. Increasing the price of the second block does not necessarily contribute to more cross-subsidization and, hence,

[11]There is a branch of cultural theories that explains the way of water management and, hence, of water allocation rules with the help of cultural configurations. In some cultural environments, the efficiency criterion in the sense economists use it (equalization of the marginal willingness to pay across all members of a society) plays only a minor role, see Hoekstra (1998).

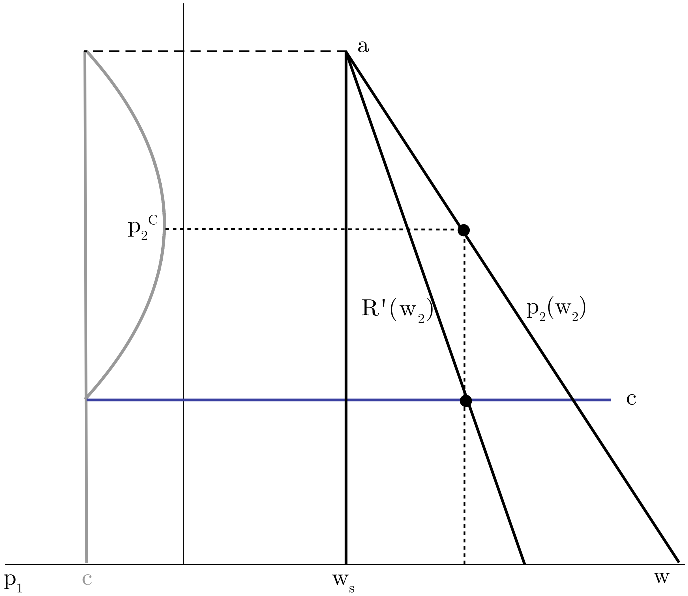

Fig. 4.9 The relation between block prices. *Source* own illustration

a lower price of the first block. Figure 4.9 shows that an exaggerated tariff progression can be detrimental to the poor.

In Fig. 4.9, it is assumed that the price-quantity function is linear

$$p_2(w_2) = a - b(w_2 - w_s) \quad \text{for} \quad w_2 \geq w_s \tag{4.31}$$

The contribution margin of the second block is therefore

$$CM_2(w_2) = (p_2(w_2) - c))(w_2 - w_s) = (a - b(w_2 - w_s - c))(w_2 - w_s) \quad \text{for} \quad w_2 \geq w_s \tag{4.32}$$

Braking even requires the water utility to set

$$2(c - p_1)w_s = CM_2(w_2) \tag{4.33}$$

The contribution margin can also be expressed as a function of p_2. Simply solve Eq. (4.31) for $w_2 - w_s$ which yields

$$w_2 - w_s = \frac{a - p_2}{b} \tag{4.34}$$

Inserting into Eq. (4.32) yields

$$CM_2(p_2) = (p_2 - c)(\frac{a - p_2}{b}) \quad \text{for} \quad c \leq p_2 \leq a \tag{4.35}$$

Finally, inserting this expression into the break-even condition and solving for p_1 gives

$$p_1 = c - \frac{(p_2 - c)(\frac{a-p_2}{b})}{2w_s} \tag{4.36}$$

This function is drawn in the left half of Fig. 4.9. We can observe that for the cases $p_2 = c$ and $p_2 = a$ the price in the first block is $p_1 = c$. For the open interval $c < p_2 < a$ we have $p_1 < c$. We observe also that the function defined in Eq. (4.36) is not monotonous. For all $p_2 > p_{2C}$, an increase of p_2 leads to an increase of p_1, i.e., strengthening the ascent of block prices goes to the detriment of the low-income customers. The reason for this is the elasticity of demand in the second block. A price increase leads to such a large reduction of demand in block 2 that the contribution margin is declining and, hence, the cross-subsidization goes back.

Instead of looking at the exact progression of the tariff, a policy dedicated to secure water access for the poor should make sure that the price of the first block is as low as possible. This can be achieved by maximizing the contribution margin in the second block. The lower half of Fig. 4.8 suggests how to choose this price optimally. It corresponds to the Cournot point p_2^C leading to the maximum cross-subsidy CM_2^C.[12]

Another potential pitfall of IBTs is that poorer households may have a larger size; thus, despite a lower per capita consumption, they could have a higher overall consumption level per household, compared to a wealthier household. In that case, the desired cross-subsidy mechanism for the support of poor households would not work anymore. Since households with larger size have a higher consumption, those families would therefore cross-subsidize households with a lower consumption level. The consequence is that poorer families have to carry a heavier burden than wealthier households which leads to the promotion of inequality.

4.4.3 Further Considerations

There are also some further issues regarding the conception of a well-defined increasing block tariff, see Boland and Whittington (2000b) and Meran and von Hirschhausen (2014):

- **Setting the initial block:** Because of political and other pressure, it is difficult for water companies to limit the initial block. A large initial block directly benefits not just the poor, but also the middle class and maybe even upper income households. If the majority of private connections to public water supply is held by middle and upper income households, these households receive the vast majority of water sold at subsidized prices. International standards for basic water needs are usually

[12] An exercise at the end of the chapter deals with the Cournot point and shows how the result depends on the demand elasticity.

in the range of 25–30 l per capita per day. It can be observed that most cities deliver households more water than the basic water need at the lowest price. Of course, also the household size is relevant for the desired sizing of the initial block. The higher the household size, the higher the required initial block size for purchasing the basic water needs at the lowest price.

- **Simplicity and transparency:** Increasing block tariffs are neither simple nor transparent. The more sophisticated the tariff structure, the harder to deduce the average or marginal price that is actually paid for a certain amount of water. The confusion about the marginal or average water price may lead to a restriction of the signal effect function of the price, whereby the consumers concerned no longer behave completely rational in accordance with expectations. Sophisticated tariffs may also create customer relation problems, making it more difficult for representatives of a water agency to explain bills. A tariff with a single volumetric rate, independent from the consumption level, is simple, transparent, robust, and easy to implement. This leads to consistent and understandable price signals.
- **Shared connections:** Increasing block rates are only implementable if customers have a metered water connection to measure the consumption quantities. In many cities of developing countries, the water meters are just available for the upper and middle-income households. However, the poor obtain water from vendors or shared connections. If several households share a metered water connection, water use by the consumers is quickly pushed to the higher priced blocks. The consequence may be that poor households pay a higher average price than the rich who have a private metered connection.
- **Reselling:** If poor households do not have a private metered water connection, so that an increasing block rate is not applicable for them, they can buy their water from households (neighbors) who have a metered connection. If a household sells water to other households, their water consumption is quickly pushed into the higher priced blocks. There is a similar situation like with shared connections: the more the water sold, the higher the average price. If this case occurs, the household which resells the water can capture the benefits from the first block and charge the resold water with a price that will recover the highest per unit charge plus some markup for inconvenience of water selling.

4.5 Pricing in Unconnected Water Markets

4.5.1 Stylized Facts

Despite some progress, there are many areas in the world, specifically in sub-Saharan Africa and South and South-East Asia, where the access to safe water and to adequate sanitation is not given. Not only in rural areas where water connections are very expensive due to the low settlement density, but also in urban regions with a growing population living in informal settlements, the rate of connected households is low. People in those areas have to rely on other sources for their water supply, such as public or private taps, and water vendors selling water door to door as well as own wells. Furthermore, there may be other available sources like leakages in water pipes, harvested rainwater, and collected surface water. Hence, many people are dependent on water of unsafe origins. Due to the poor water quality, diseases can spread easily, worsening the living conditions of those affected. People are well aware of these risks but they simply do not have the money (or the time) to secure themselves access to clean freshwater. This seems to support the comprehensive expansion of a water distribution infrastructure. Since investments in water distribution are very expensive, financing in the context of a poor population becomes a major issue.

There are additional reasons why the settlements of the poor are often not connected to a pipe-based water distribution system:

1. The lack of water infrastructure is attributed to staff incompetence and a lack of motivation due to a salary system without incentive schemes. Also, water utilities may lack the required skills.
2. There is a lack of political interest in the poor. The water supply is geared to the needs of the middle class and upper income families.
3. But even if there is a political will to improve the water supply for the poor, there are obstacles to extend pipe-borne services to low-income urban areas. Often, the ownership of land and property is not well defined. In addition, utilities have problems to collect revenue from metered customers and they cannot prevent people from illegally tapping water.
4. There are also economic motives that prevent the extension of pipe-borne water supply. The water supply system consists of many actors with divergent objectives. A close-meshed water distribution system can be against their interest.

In the following, we will describe a water distribution system in more detail that is based on a decentralized supply mode.[13] Figure 4.10 depicts the different actors.

- At the beginning of the production and distribution chain is the water utility. Utilities provide clean water through a pipe system to households. However, many empirical studies on the water conditions in urban and peri-urban areas in developing countries show that the connection rate is not very high. Households of the middle and high income (in urban areas) are connected to the water utility and the waste water treatment plant. Since the coverage of utility networks is often limited, low-income groups regularly have to rely on various other service supply systems.
- Taps and standpipes are connected to the pipes of the water utility. They are also called "water kiosks" and are often run by private or public managers. Either these managers are employed by the water utility or the kiosks are privately run. In this case, the owner/manager has to pay a license fee to the utility. The contract may include a lump sum fee as well as a volumetric part. These payments can also be

[13] We confine our analysis to the water supply system. A similar analysis can be done for the sanitation system.

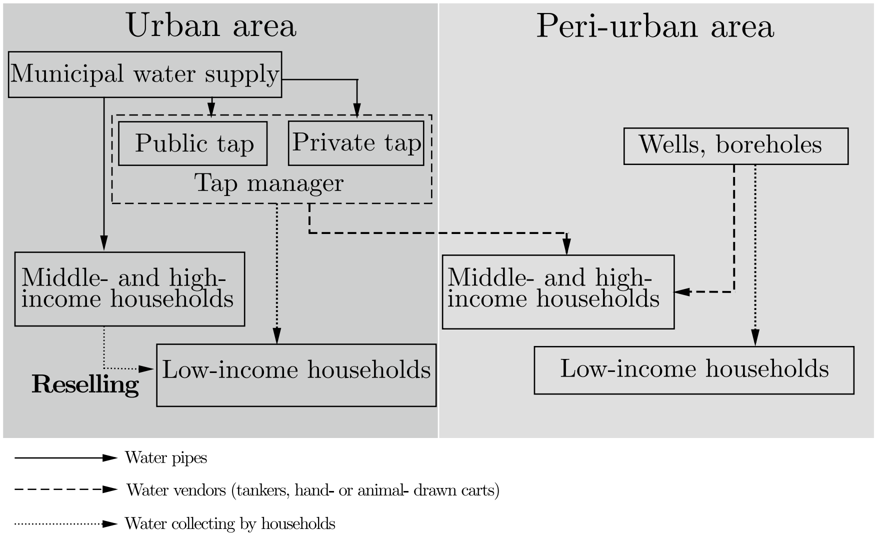

Fig. 4.10 Decentralized water sector in urban and peri-urban areas. *Source* own illustration

formal or, specifically if the prices are publicly regulated, informal, i.e., they are clandestine side payments. The manager, in turn, sells water to water vendors and to households in the vicinity of the tap as well.

- Reselling from connected households: This activity can often be observed and can be explained by the price spread between pipe-borne water and the water price in the informal market. We know from Sect. 4.4 that increasing block tariffs may be tailored in favor of the middle class offering water to a low price, sometimes even below costs. In addition, blocks are structured such that households can draw more water than needed without exhausting the block. Thus, there is scope for profitable water trading. Household water connections are therefore similar to private water kiosks.
- Water vendors: A main source of water for poor households is the services of water vendors. There exist mainly two types of vendors: wholesale vendors, often serving by truck, and distributing vendors that in turn sell water door to door. The technology of these street peddlers is rather simple. They carry water in plastic jerricans that are hauled by handcarts or bicycles. Capital costs of this mobile vending system seem to be lower than the piped water supply, at least for short and medium distances.
- Private wells and boreholes: In many urban and rural areas, households receive water from private wells. These wells can either be historic or recently built. This means that households resort to groundwater, which has decreasing quality standards due to population growth. Households are aware of this relationship. However, due to the high water price they cannot rely on any other water sources.

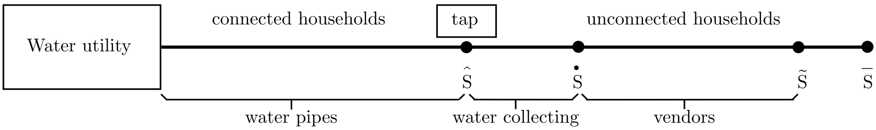

Fig. 4.11 Linear city. *Source* own illustration

4.5.2 Model

4.5.2.1 The Linear City

To analyze the characteristics of various modes of water supply, we utilize an economic model one can find in spatial economics: the linear city. It is assumed that all water customers are arranged along a line, the linear city, (Fig. 4.11). The customer density is constant along the line (identical distribution). One customer lives at each location, and each customer consumes exactly one unit of water, say 1 m^3 per month. The willingness to pay $V(s) = a - bs$ of each customer is decreasing along the linear city from the left to the right side. This property stems from the assumption that income of households decreases from left to right. On the left, the high-income households live followed by the middle class and finally the less fortunate, which settle on the right side. This model structure makes the analysis transparent and allows to identify the economic drivers that determine water prices along the linear city (Fig. 4.11).[14]

The water utility is located to the left. It conveys water to the connected households up to $\hat{s}$. At $\hat{s}$ the pipe-borne distribution ends with a kiosk or water taps that can be accessed by customers without pipe connection to fetch water, or by mobile water vendors. Water collecting customers are located in the interval $[\hat{s}, \dot{s}]$. Customers to the right of $\dot{s}$ do not fetch water at the water kiosk because collection costs are too high. Instead, they buy water from vendors operating between $\dot{s}$ and $\tilde{s}$. The length of the city is $\bar{s}$. If $\tilde{s} < \bar{s}$, some customers remain without access to safe water and they have to rely on other sources like wells or water abstraction from surface water. This scenario contradicts the sustainable development goals, and the human right to water, and must be prevented.[15]

Let us proceed by defining the costs of the various actors of the linear city. Roughly, the water utility incurs two cost components. The water supply costs depend on the total amount of water provided. We assume that the water provision and the water distribution exhibit constant returns to scale, i.e., the cost function is linear.[16] This applies also to the distribution costs, i.e., the costs of connecting households. Thus,

[14]This subsection is based on Meran et al. (2020).

[15]The simple model does not include other sources, e.g., water wells, boreholes, or the collection of surface water. Also, we do not consider reselling from connected households and illegal tapping. However, despite the simplicity we can derive some insightful results.

[16]See the literature at the end of this chapter. Our results also hold if water utilities exhibit a cost structure that cannot be approximated by a linear function.

the cost function of the water utility is

$$C_{WU} = \int_0^{\tilde{s}} m ds + \int_0^{\hat{s}} k ds = m\tilde{s} + k\hat{s} \tag{4.37}$$

where m are water treatment costs and k represents distribution costs per cubic meter. Collecting water is rather cumbersome. Often it is the women who fetch water with the help of canisters. The costs relate not only to the purchase price, but also to the lost time, which is missing for other productive activities. These opportunity costs are taken into account in the cost function. For a single household located at $s \in [\hat{s}, \dot{s}]$ to fetch 1 m^3 of water per month costs δs, where δ indicates the opportunity costs per distance walked, taking into account that the distance s has to be taken twice (to the kiosk and back). Aggregating over all households yields total fetching costs

$$D_F = \delta \int_0^{\dot{s}-\hat{s}} s ds = \frac{\delta}{2}(\dot{s} - \hat{s})^2 \tag{4.38}$$

Vendors' costs are twofold. There is the time vendors lose when filling cans at the kiosk and decanting for each household served. If we weigh this amount of time with the income per hour attainable in other occupations (opportunity costs), we can derive the first cost component which is symbolized by c_2. Besides the costs unrelated to distance, there are hauling costs which depend on the distance. It takes a certain time to haul 1 m^3 over, say, 100 meter and to return to the tap for the next delivery. Let us assume that there are two customers in distance s_i from the tap and to the left of $\dot{s}$, i.e., they do not fetch water from the tap. Total hauling costs for the vendor are $c_1(s_1 + s_2)$, where c_1 is lost income per distance unit. This calculation also takes into account that the vendor must undertake two trips per customer.[17] Then, total vending costs for two customers are $C_{ven} = 2c_2 + c_1(s_1 + s_2)$.

In the model, we have to transpose the calculation into the continuous stretch.

$$C_{ven} = c_1 \int_{\dot{s}-\hat{s}}^{\tilde{s}-\hat{s}} s ds + c_2 \int_{\dot{s}-\hat{s}}^{\tilde{s}-\hat{s}} ds = \frac{c_1}{2}\left[(\tilde{s} - \hat{s})^2 - (\dot{s} - \hat{s})^2\right] + c_2(\tilde{s} - \dot{s}) \tag{4.39}$$

The first item on the r.h.s. represents the aggregated hauling costs of all customers served in the interval $[\dot{s}, \tilde{s}]$. This term is quadratic since the aggregation takes place over distances that increase successively. The second item is the purchasing and selling costs which only depend on filling and decanting time and not on distance. Hence, this part is linear.

[17] We assume that the vendor has a limited tank capacity so that he has to cover the distance to the kiosk for each customer supplied. Alternatively, it is possible that two vendors supply one customer each.

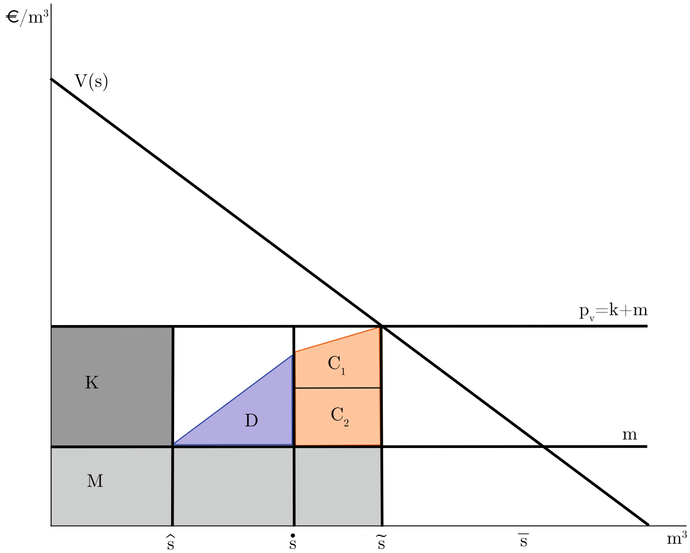

Fig. 4.12 Optimal modal split. *Source* own illustration

4.5.2.2 The Optimal Modal Split

The optimal modal split can be derived with the help of the integrated water resource management approach. How far should the pipe-borne water supply be extended, how many customers should ideally fetch water from the tap, and what distance should water vendors cover? The answer to these questions is the optimal modal split which can be derived from the following maximization program:

$$\max_{\{\tilde{s},\dot{s},\hat{s}\}} [\int_0^{\tilde{s}} V(s)ds - C_{WU} - D_F - C_{ven}], \quad \text{s.t.} \quad \hat{s} \leq \dot{s} \leq \tilde{s} \tag{4.40}$$

Assuming that all instrument variables are strictly positive and that all stretches are nonempty ($\hat{s} < \dot{s} < \tilde{s}$), the Kuhn–Tucker conditions[18] are

$$a - b\tilde{s} - m - c_1(\tilde{s} - \hat{s}) - c_2 = 0 \tag{4.41}$$

$$c_1(\dot{s} - \hat{s}) + c_2 - \delta(\dot{s} - \hat{s}) = 0 \tag{4.42}$$

$$-k + c_1\left[(\tilde{s} - \hat{s}) - (\dot{s} - \hat{s})\right] + \delta(\dot{s} - \hat{s}) = 0 \tag{4.43}$$

From these three equations, we can derive the optimal extension of the pipe-borne water supply, the optimal range of water fetchers, and the optimal operating area of vendors. This is called the optimal modal split of water supply consisting of $\{\hat{s}, \dot{s}, \tilde{s}\}$. Figure 4.12 depicts the optimal cost structure.

[18] The Kuhn–Tucker conditions are explained in Appendix A.

From 0 to $\hat{s}$, households are connected to the pipe-borne water supply. At $\hat{s}$, the water utility has installed a water kiosk where adjacent customers can fetch water. Fetching water in the range of the first yards is cheaper than having customers served by a vendor. This is due to the cost structure of both water supply modes. The vendor incurs two time-related costs, filling water into jerrycans or in canisters at the tap and at the selling point, whereas the collecting customers lose only one filling time. Of course, the household has distance-related costs δ per distance unit that are higher than the hauling costs c_1 per distance unit and m^3 of the vendor. Therefore, at $\dot{s}$ the water supply mode switches to the vendor system. Hauling water from $\hat{s}$ and selling it within the stretch $[\tilde{s} - \dot{s}]$ is less costly than having customers in this interval collect the water at the position of the tap $\hat{s}$. This can be inferred from Eq. (4.42), where both marginal costs are set equal. In turn, the optimal range of connected households, i.e., the line $[0, \hat{s}]$, is determined by equalizing the respective marginal costs. Setting Eq. (4.43) into Eq. (4.42) yields $c_1(\tilde{s} - \hat{s}) + c_2 + m = k + m$. The optimal modal split between the vendor's water supply range and the extension of pipe-borne water supply is determined by equalizing the respective marginal costs.

Once we have determined the optimal modal split, we can graphically represent the optimal cost structure. Areas K and M in Fig. 4.12 represent the distribution costs for connected households and costs of treating and providing the water to all households from $[0, \tilde{s}]$, respectively. The triangle D represents total collecting costs of households in the line section $[\dot{s} - \hat{s}]$. Vendors' costs consist of time-related purchasing and selling costs C_2 and distance depending hauling costs C_1.

The integrated water resource management usually applies a planning approach where economic rents are maximized taking into account technical constraints, e.g., hydrological laws. However, one must be careful when implementing this concept in practice. Two points are of particular importance.

- The pure maximization of the economic rent does not take into account the indispensable human right to water access. The result of Eq. (4.40) may lead to $\tilde{s} < \bar{s}$. If customers are excluded from the water supply system, we have to correct the optimization procedure by introducing the constraint $\tilde{s} \geq \bar{s}$. Then, we end up with a different optimal modal split that covers all customers in the linear city.[19]
- The planning approach sets water quantities and the line length of the various service modes in the linear city. In reality, however, consumers and also vendors are not quantity regulated, but only indirectly incentivized through prices. Therefore, it has to be clarified, which prices in the various sections of the route should be fixed or indirectly induced. The price determination in turn depends on whether the vendors are employees of the water company or whether they operate independently in a market. In the following subsection, we deal with this issue in more detail.

[19] An annotated exercise at the end of this chapter will lead the reader to the results (Problem 4.3).

4.5.2.3 Pricing and Regulation

Stylized Facts

In a decentralized water market consisting of collecting customers and vendors, it is very difficult to regulate all prices directly. Moreover, regulating prices may have repercussions on the market price at which vendors sell the water. Thus, to provide water to the customers of the linear city, the water utility has to follow a cautious regulation policy. In the context of our simple model, the water utility has the following instruments available: The extent of household water connections $\hat{s}$, the price of water at the tap point $\hat{s}$ (we assume that the utility can differentiate between usual customers and water vendors), and the water price for connected households.

The water price in the area where vendors operate cannot directly be regulated. The water utility can try to influence the market outcome by setting a proper water tap price. For this, the regulator has to take into account the degree of competition in this market. There are many examples of highly cartelized water markets in urban, peri-urban, and rural areas. These cartels can be very effective in preventing market entry. Often, they operate beyond legal limits. In addition, both the kiosk manager and the water utility may be part of cartels. However, there are also examples where the water market around taps and in the vending area is competitive. Box 4.1 provides some empirical evidence.

Box 4.1 Small-scale water providers: Pioneers or predators?

In a study entitled "Small Scale Water Providers: Pioneers or Predators?", Degol Hailu and colleagues (Hailu et al. 2011) empirically examine whether small-scale providers are an effective substitute for a missing pipe-borne water provision for the unconnected population, as proponents claim, or are simply predators, as the skeptics argue. The empirical investigation was conducted in Kenya, in a survey of nine communities within Nairobi city. The criteria for their choice are related to their settlement characteristics, i.e., urban or peri-urban locations, where a piped water supply is conceivable and their demographic characteristics. The survey sample comprised 576 households and about 159 small-scale water providers interviewed.

The supply side was structured as follows:

Types of small-scale water providers

Type of provision	Sample size	Share (%)
Pushcart vendors	17	11
Tanker truck	15	09
Borehole	28	18
Tap water vendor	62	39
Water kiosk	37	23
Total	**159**	**100**

Source Hailu et al. (2011)

The above table shows that the fixed-point water suppliers (tap water vendors and water kiosks) make up 62 percent of total water supply. The mobile suppliers (pushcart vendors and tanker tracks) make up 20 percent of providers, receiving the water from Nairobi City Water and Sewerage Company (NCWCS). Pushcart vendors supply water by manual and donkey-pulled pushcarts and obtain water mostly from boreholes, water kiosks, or through an illegal connection to the piped network. Tanker trucks supply water in bulk to end users who possess storage tanks. Some of these households resell the water. Tanker trucks obtain water either from private boreholes or directly from the utility company. The remaining water supply comes from borehole water vendors. This water is often unsafe regarding quality. However, the advantage of vendors for households is that they lead to time savings. The opportunity costs of time are very high for households.

Small-scale water providers are not price takers who take the water price for given. Rather, they set the water price directly taking into account the price behavior of competitors. The study also showed that pricing followed a cost markup approach.

Mean water price across small-scale water providers

Type of provision	Mean price	Poverty premium
Pushcart vendor	12.15	30.28
Tanker truck	7.90	18.75
Borehole	6.11	14.28
Tap water vendor	3.18	6.95
Water kiosk	2.81	6.03

Source Hailu et al. (2011)

The above table shows that push car vendors charge on average the highest water prices. If you put this price in relation to the water price for connected households, you obtain the poverty premium, calculated as $(p_i/p_{NCWCS}) - 1$ where the index i refers to the types of small-scale water providers and NCWCS is the Nairobi water utility. The official rate NCWCS is Kshs 0.40 per 201 (1 €≈ 120 Kshs). The poverty premium of push car vendors is 30, which means that low-income households are paying 30 times as much as connected households are being charged.

Competitive Versus Cartel Solution

We start with the assumption that the water market for vendors is fully competitive. They offer water in the segment $[\dot{s}, \tilde{s}]$. At each location along this segment, vendors have the same costs to supply one m^3 of water.[20] Take two points in close vicinity, s_1 and s_2, where $s_1 < s_2$ (see Fig. 4.13, left half of the picture.). Costs per m^3 of water

[20]Recall the assumption that at each point along the linear city customers want to buy one m^3 of water.

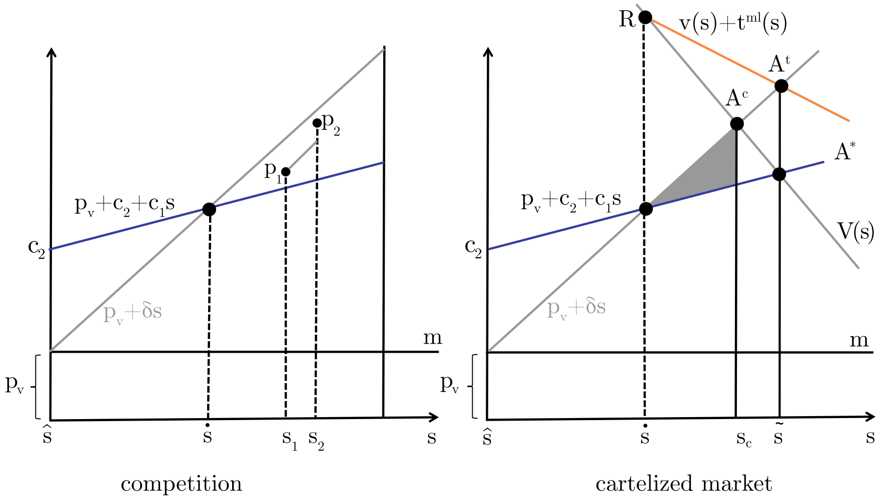

Fig. 4.13 Competition versus cartel. *Source* own illustration

sold are

$$p_v + c_2 + c_1 s_1 < p_v + c_2 + c_1 s_2 \tag{4.44}$$

where p_v is the price for the vendor at the tap. If a seller at s_2 wants to sell the water for a price higher than these costs, lets say p_2, either another seller occurs at the same point undercutting this prices[21] or she offers the water at point s_1 where the first seller does not operate. But if the price at s_1 is such that $p_1 + \delta(s_2 - s_1) < p_2$ the first seller would lose the demand at s_2 because customers at s_2 will move to s_1 to buy the water from the vendor there. Due to the mobility of customers, all selling points are in competition. It does not matter whether we have many or only two vendors in each segment. The price competition drives all prices down until

$$p(s) = p_v + c_2 + c_1 s \tag{4.45}$$

Hence, the price is depending on distance and follows the costs defined in Eq. (4.45). Notice that the highest price is reached at $p(\tilde{s}) = p_v + c_2 + c_1\tilde{s}$. Here, a customers willingness to pay is equal to the price, i.e., $V(\tilde{s}) = p(\tilde{s})$. Beyond $\tilde{s}$, there is water supply by vendors because price would be higher than the willingness to pay.

That the water prices correspond to costs per m^3 follows from the assumed openness of the water market which allows newcomers to enter the market. Openness is not only due to the absence of legal constraints but also a matter of the very nature of costs. If entry and exit are costless, markets are contestable. This is the case when no costs are sunk for firms leaving the market, and no investment into specific capital is necessary for entering the market.[22] This is not always the case in unconnected

[21] Competition in prices is called Bertrand competition.
[22] The concept of contestability was introduced and elaborated by Baumol (1982).

water markets served by vendors. Vendor's investment exhibits a certain degree of specificity (jerry cans, hand trolley with a tank mounted on it, etc.).

However, if a sunk cost structure is more or less absent, the number of firms operating in the market is not of importance. Low prices are simply the result of the competition of potential newcomers. If an incumbent charges higher prices than marginal costs (see Eq. (4.45)), newcomers immediately invade the market at s driving down the vending price until the equilibrium is reached. Since incumbent vendors anticipate the potential threat by newcomers, they keep their prices equal or close to marginal costs. Thus, even if there are only few vendors serving customers, the pressure of potential competition drives the price down to marginal costs.

The water prices customers and vendors have to pay at the kiosk are still left to be fixed. After the implementation of the optimal water supply infrastructure, i.e., setting the optimal $\hat{s}$, the policy-maker has to assure that the division of collecting and vending zones follows the optimal pattern. In other words, she has to determine the optimal $\dot{s}$. This must be achieved indirectly by fixing the water price at the kiosk.

Unconnected households decide either to buy water from water taps or to purchase it from water vendors. The marginal customer is indifferent between both options, i.e.,

$$p_{col} + \delta(\dot{s} - \hat{s}) = p(\dot{s}) = p_v + c_2 + c_1(\dot{s} - \hat{s}) \tag{4.46}$$

where p_{col} (p_v) is the water price charged to the collecting customer (vendor). From Eq. (4.42), optimality of $\dot{s}$ requires that $p_{col} = p_v$, i.e., a price discrimination policy between water collectors and vendors would be non-optimal. Finally, we can determine the level of the tap water price. From Eq. (4.41), it follows that the marginal willingness to pay must be equal to the marginal costs $c_1\tilde{s} + c_2 + m$. The water price vendors charge at $\tilde{s}$ is $p(\tilde{s}) = c_1(\tilde{s} - \hat{s}) + c_2 + p_v$. Hence, the optimal tap water price is $p_{col} = p_v = m$.

Finally, we have to examine the economic viability of the price system. Total costs of the water utility including transport and distribution costs should be covered by the revenue raised. Cost coverage is defined as

$$p_{ch}\hat{s} - k\hat{s} - m\tilde{s} + p_v(\tilde{s} - \hat{s}) \tag{4.47}$$

where p_{ch} is the water price for connected households, which can be rewritten as

$$p_{ch}\hat{s} - k\hat{s} - m\hat{s} - m(\tilde{s} - \hat{s}) + p_v(\tilde{s} - \hat{s}) = p_{ch}\hat{s} - k\hat{s} - m\hat{s} \tag{4.48}$$

since $p_v = m$. Setting $p_{ch} = k + m$ leads to cost coverage of the water utility. Notice that $p_{ch} > p_v$, i.e., water for connected households is more expensive than for collectors at the tap. This might lead to trade between the kiosk manager and the adjacent connected households.

So far we have assumed that the market for water vending is competitive. There is a growing literature from scientists, development workers, and journalists about the role of small-scale private water providers in non-competitive markets.[23] Proponents of the mobile and decentralized supply of water see vendors as pioneers and entrepreneurs that supply water to those, who otherwise would never have access to a reliable water source, even though they charge prices, well above costs. However, skeptics see the vendors solely as predators, who exploit the poor by charging high prices for water of poor quality.

Regulatory Options

Anyway, it is to be expected that water provision via a decentralized infrastructure of kiosks and mobile vendors will keep on persisting in the future. This leaves us with the following question: How can one force cartelized vendors to supply the poor at an affordable price? There are two approaches to reduce the negative economic and social effects of water supply cartels. The decision on which of the two should be adopted depends on the authority's compliance-monitoring capacity.

The first approach simply consists of introducing a zonal price cap for water vendors. The price[24] $\bar{p}(s)$ per, say, liter is optimally set, such that

$$\bar{p}(s) = p_v + c_2 + c_1(s - \hat{s}), \quad \text{for} \quad s \geq \dot{s} > \hat{s} \tag{4.49}$$

If the vendors' cartel complies with the regulated price, the optimal modal split is replicated and customers are not exploited. Of course, this type of regulation is only enforceable if the authorities are capable to monitor the price of vendors. How can it be determined whether the supplier does not charge a too high price if customers complain? One way could be to issue water coupons that people can buy for $\bar{p}(s)$ at an issuing office of the water authority.[25] They can buy as many as they want. Vendors have to accept these coupons in exchange for water. The collected coupons can be redeemed at the same public authority office for the same price. Of course, this mechanism only works if the cartel is not able to force further receipts in addition to the coupons. This would be the case if the cartel could charge an effectively higher price than the coupon price for the water. If the public institutions are not able to prevent the abuse of this system, price regulation is undermined and the water customers pay a water price that an unregulated monopolist would demand.

Thus, to guarantee access to water for an affordable price, the regulation approach should rely much more on economic incentives instead of direct price regulation, which cannot be enforced if the necessary institutional capacity does not exist. The economic incentives must be set such that the desired regulatory effect on water prices is self-enforcing. Low prices for poor customers must be in the interest of cartelized vendors. This is a second more promising approach.

To develop an incentive scheme, we first have to ask what prices prevail in a cartelized vendor market. Cartelized vendors behave like a monopoly. They fix the

[23] See Hailu et al. (2011) and the further readings at the end of this chapter.

[24] As the linear city model is continuous with respect to the distance s, zonal pricing is expressed by a continuous price function.

[25] The coupon can also be bought from authorized shops located along the stretch of the linear city.

prices along the segment beginning from $\dot{s}$ to the end of their area of operation. We have assumed in our simple model that each household along the linear supply line demands 1m^3 of water. The maximum price they are willing to pay is

$$p_{max}(s) = p_{col} + \delta(s - \hat{s}), \quad \text{for} \quad s \geq \dot{s} > \hat{s} \tag{4.50}$$

This case is depicted in the right section of Fig. 4.13. The profit depends on the supply distance:

$$\int_{\dot{s}-\hat{s}}^{s_c-\hat{s}} [p_{col} + \delta s - p_v - c_2 - c_1 s] ds \tag{4.51}$$

where s_c is the stretch the cartel is willing to supply. This distance can be derived from the maximizing behavior of the cartel. The cartel maximizes Eq. (4.51) with respect to $\{\dot{s}, s_c\}$ subject to the constraint

$$V(s_c) = a - bs_c \geq p_{col} + \delta(s_c - \hat{s}) \tag{4.52}$$

The charged price cannot be higher than the willingness to pay. The KKT conditions are

$$-\delta(\dot{s} - \hat{s}) + c_2 + c_1(\dot{s} - \hat{s}) \leq 0 \perp \dot{s} \geq 0 \tag{4.53}$$

$$\delta(s_c - \hat{s}) - c_2 - c_1(s_c - \hat{s}) - \lambda(b + \delta) \leq 0 \perp s_c \geq 0 \tag{4.54}$$

where λ is the Lagrangian to the constraint.[26]

The picture in the right half of Fig. 4.13 shows the solutions to Eqs. (4.53) and (4.54). $\dot{s}$ is chosen such that it equates the costs of water fetching to the costs of supplying customers by vending. The distance of supply s_c is chosen such that the marginal willingness to pay is equal to the price $p_{max}(s_c)$ charged by the cartel (See point A^c). In sum, the cartel will earn profits indicated as gray triangle in Fig. 4.13.

The water utility can try to reduce the cartel's prices $p_{max}(s)$ by reducing the price p_{col}, for which it sells water at the kiosk to households. However, this would lead to an inefficient cost structure. Decreasing the water price for collecting households (and at the same time leaving the purchasing price for vendors constant) leads to a reduction of consumer surplus exploitation, but at the same time results in an inefficient supply structure because the collecting segment is too large while the vending area is too narrow. Of course, it is an effective policy with regard to customer protection, but as we have argued an inefficient solution.

An alternative indirect mechanism is to subsidize customers in the vending area such that the effective marginal willingness to pay increases. This subsidy depends on the location of customers, i.e., it is a zonal subsidy[27]:

$$t_{LM}(s) = (\delta - c_1)(s - \hat{s}) - c_2, \quad \text{for} \quad s \geq \dot{s} > \hat{s} \tag{4.55}$$

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[26] Notice that $\lambda > 0$, otherwise Eqs. (4.53) and (4.54) are identical leading to $s_c = \dot{s}$. But this would imply that the monopolized vendor (cartel of vendors) does not sell water.

[27] This subsidy system is based on a regulatory mechanism proposed by Loeb and Magat (1979) to regulate monopolies.

Hence, the effective marginal willingness to pay is $V(s) + t_{LM}(s)$ (see the orange line in Fig. 4.13). This line rotates around the point R until it intersects with $p_v + \delta(s - \hat{s})$ at A^t. This subsidy driven increase in the willingness to pay expands the vending segment to $\tilde{s}$, i.e., brings the vending area back to its efficient extent.

What changes in comparison to the water market without subsidies? First, the modal split is efficient, i.e., water is supplied up to $\tilde{s}$, and, second, consumers are protected by the subsidy introduced, i.e., they effectively pay a water price under full competition. This can be seen by the following calculation utilized by Eqs. (4.50) and (4.55):

$$p_{max}(s) - t_{LM}(s) = p_{col} + \delta(s - \hat{s}) - (\delta - c_1)(s - \hat{s}) + c_2 = p_{col} + c_1)(s - \hat{s}) + c_2 \quad (4.56)$$

where $p_{col} = p_v = m$. This elegant mechanism comes at a price. The solution is rather expensive and, from a political standpoint, provocative. Customers must be subsidized and, at the same time, the cartel reaps a monopoly rent. Further, the issue of financing the subsidy remains.

4.6 Water Scarcity: Prices Versus Rationing

4.6.1 Options to Deal with Scarcity

The importance of water demand management increases in times of water scarcity. There are now many examples of how demand-side management can be designed. For example, California has developed numerous conservation strategies to reduce water demand, inter alia by utilizing pricing schemes, subsidies for water-efficient equipment, educational measures, water rationing, and water trading. Similarly, Australia has taken measures to cope with severe drought by developing a mix of water instruments to reduce effective demand for water. For instance, the Cairnes regional council has launched a campaign to use water wisely (information on water-saving behavior for households) in addition to mandatory restrictions (regulated sprinkling times).[28] In this section, we analyze the characteristics of water demand management based on prices vis a vis a non-price approach. The results of this comparison depend strongly on the evaluation criteria applied. In principle, the allocation of scarce water should comply with various criteria. The literature mentions efficiency, justice, technical feasibility, political enforceability, and environmental sustainability. The importance of each criterion depends on the situation under consideration. If, for example, water scarcity leads to a pronounced plight of the population, the criterion of just allocation of water is of greater importance than allocative efficiency, i.e., an allocation according to the marginal willingness to pay.

[28] http://www.cairns.qld.gov.au/water-waste-roads/water/save-water.

4.6.2 Rationing

We know from war periods that the distribution of basic goods is often done through food stamps that cannot be transferred. This applies also to other emergency situations, e.g., water scarcity, where personal rationing is often viewed as a just allocation procedure. There are also other forms of rationing: In many poor urban areas in developing countries, water supply to households is rationed by interruption. Often, households receive water only for about 1–2 h per day. This can be understood as a non-price demand management approach sometimes deliberately chosen by local authorities to meet fairness criteria; sometimes, it is the result of aging network pipes and weak institutional management structures. In the following, we will introduce various water rationing methods in practice and compare them with a price-based water demand management approach.

Rationing can take up various forms[29]:

- Rationing by fixed allotment: The scarce resource is distributed in fixed quantities to individuals or households depending on the household size. These allotments can vary seasonally depending on the scarcity situation. Also, the portions allotted can vary with respect to membership to specific economic sectors, e.g., industry, commerce, or the public area (school, etc.). It is important to differentiate the allotments with regard to their transferability. There exist various designs: allotments without transferability, allotments with transferability under regulated prices, and allotments completely tradeable in a free market. We will analyze these specifications below.
- Proportional allotments: Water use rights are allotted in proportion to water usage prior to the rationing. This method is easy to implement because it does not require lots of information on the characteristics of the user. The water utility simply needs a record of the historic water use profile to determine the allotment. Despite the relatively small amount of information required, this method suffers from some implementation problems. First, it is very difficult to allot water use rights to newcomers without a historic record; second, the reliance on historic water use may discourage water conservation. If water users know that the historic water use is utilized to build the distribution key for water use rights, users they will behave strategically by deliberately wasting water.
- Water rationing by increasing block tariffs: We have already analyzed increasing block tariffs. Block prices are only valid within the boundaries of a block. If a household wants to consume more water than allowed for the given block price, it exceeds the upper boundary and pays the higher prices of the following block. In this sense, increasing block tariffs also exhibit a rationing property.
- A more differentiated version of this approach is water budgets as applied in the USA. Each household gets a monthly water budget assigned, which is based on several characteristics, including the number of residents in the home, or the usage

[29] We follow Lund and Reed (1995) and also Olmstead and Stavins (2009b).

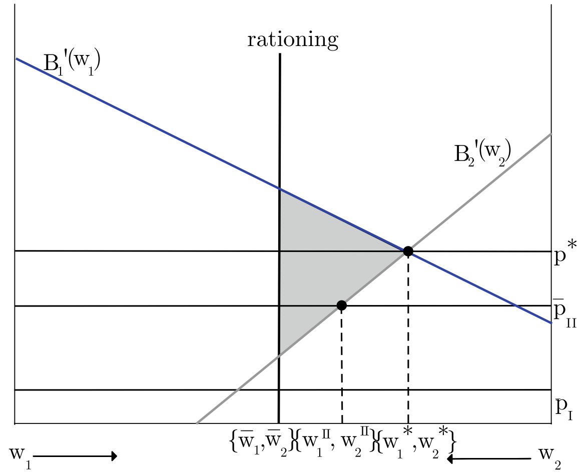

Fig. 4.14 Pricing versus rationing. *Source* own illustration

type, e.g., indoor water use, garden sprinkling, etc. The effective price depends on the percentage utilization of the allocated water budget. If the household exceeds its water budget, the respective price increases.

- Water can also be rationed with respect to the type of water use, e.g., car washing, garden watering or luxurious applications like a fountain, etc. In times of severe drought, the municipality simply forbids certain types of water use which are in the higher part of the need hierarchy.
- Rationing by outage: This method is often applied in developing countries because it needs no institutional body that gathers and calculates the necessary information from the customers. Either water is provided only few hours a day or the water pressure is reduced. There is almost no differentiation among households except, perhaps, by rotating the outage geographically among districts allowing differentiated service times.

4.6.3 Comparison

In many cases, water scarcity can be managed by a deliberate pricing policy. If water gets scarce, simply increase the water price and the water allocation will take place in an efficient manner. Or one simply introduces a market where water can be traded without any regulation. Conversely, water rationing leads to welfare losses because the water is not allocated according to the marginal willingness to pay of customers.[30] But, as mentioned above, efficiency is only one of many criteria that have to be taken into account to find an allocation which is capable of approval by residents. Figure 4.14 shows the constellation for two users.

[30]There are empirical studies estimating the exact amount of welfare losses. See, e.g., Grafton and Ward (2008).

Let us begin with the assumption that there are no water treatment costs. The sustainable water availability is given by $\bar{W}$ which is the width of the diagram. From left to right, the water allocation of user 1 can be read up and for user 2 from right to left. If water is allocated within an unregulated market allocation, the result is $\{w_1^*, w_2^*\}$. The respective equilibrium price is $p*$. We know from Chap. 3 that $\{w_1^*, w_2^*\}$ is optimal in the sense that this allocation maximizes the aggregated benefits of both users. Note that the positive water price mirrors solely the scarcity of water, whereas water treatment and distribution costs are not yet considered. However, this allocation could be regarded as unjust. In a situation of scarcity, the willingness to accept inequality is reduced. As an alternative, rationing could be implemented. In this figure, we assume that both users get the same amount of water, i.e., rationing by fixed allotment. If these allotments are not transferable, the final allocation is $\{\bar{w}_1, \bar{w}_2\}$. From a traditional welfare theoretical point of view, this allotment is connected to welfare losses (gray triangle).

Some rationing schemes are combined with tradability either under regulated prices or with completely free pricing. The figure depicts two cases of price regulation. If water rations are transferable and the regulated price is $\bar{p}_I$, we can see immediately that no trade will occur. The price is too low for user 2 to sell some of his water allotments to user 1. If the regulated price is $\bar{p}_{II}$, restricted trade takes place leading to the allocation $\{w_1^{II}, w_2^{II}\}$. Only if the price is completely free, we reach the optimal allocation $\{w_1^*, w_2^*\}$. The key difference between the pure market solution and the rationing scheme with transferability is

1. that the scarcity rent accrues to the customers. This is also the case if water treatment costs are taken into account. Let the horizontal line $\bar{p}_I$ represent marginal treatment costs and charge customers with a uniform tariff in the amount of marginal costs and distribute water coupons[31] according to the water allotment. Customers pay the tariff and get credits (debits) depending on whether they use less (more) water than their allotments. The coupon price emerges on the trading floor for scarce water. In Fig. 4.14, the scarcity rent, i.e., the trading price for water coupons, is equal to $p^* - \bar{p}_I$. Since coupons have been distributed, the scarcity rents remain with the user, for user 1 $(p^* - \bar{p}_I)w_1^*$ and for user 2 $(p^* - \bar{p}_I)w_2^*$;
2. that efficiency and equity are no longer in a tight trade-off relationship. If water rights are equitably distributed, trading leads to higher total welfare without harming the principle of fairness,[32] at least at the first glance. The final judgment depends on how fairness is defined. Do we refer to the distribution of resources, in our case water, or do we refer to the result of the allotment in terms of utility that accrues to customers? In the following, we take a closer look at this distinction.

[31]This proposal was made by Collinge (1994). Of course, these coupons need not be physically distributed to customers. They can be handled electronically on the individual account of customers.
[32]See the discussion about the various fairness criteria in Chap. 3.

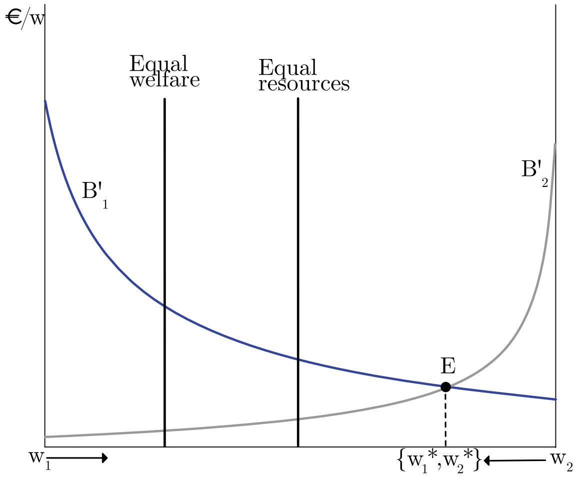

Fig. 4.15 Equality: welfare or resources. *Source* own illustration

4.6.4 Discussion

In the philosophical and in the economic literature, one can find a discussion of what is the right "equalisandum" of just distribution.[33] Do we want fairness through the equal distribution of resources, or does fairness refer to welfare equality? The issue can best be described with the help of Fig. 4.15.

Consider two farmers producing crop that needs water. From left to right, the marginal profit function of farmer 1 is depicted ($B_1'(w)$) and, vice versa, from right to left marginal profits of the second farmer ($B_2'(w)$). Obviously, farmer 1 is more productive than farmer 2. With the same amount of water, farmer 1 makes more profit than farmer 2. Since water is scarce, the question of how water should be allocated arises. If one follows the usual welfaristic approach, the allocation of point E is optimal maximizing aggregated profits leading to $B_1(w_1^*) > B_2(w_2^*)$. But if only fairness considerations matter, aggregate profits are irrelevant as a criterion of fairness. If one adheres to the principle of equality of resource distribution, each farmer gets half of it. But what if the equality of profits (welfare) counts? Then, the water allocation must be asymmetric in favor of farmer 2 who is less productive compared to farmer 1. The line named "Equal welfare" exactly depicts the resource allocation where both profits are equal. But why should profits be equal? Because it is just that both individuals bear the burden of water scarcity equally. Therefore, trade of water after allotment of water rights is often not allowed, because it can dilute fairness, even if we would observe a Pareto improvement in the case of allotment trade. This is the reason why water allotment is sometimes not transferable.[34]

But why does the lower productivity of farmer 2 entitle him to receive more water? Much depends on the causes of the lower productivity. If farmer 2 is poor without

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[33] Roemer (1996) provides a thorough analysis.

[34] In Exercise 4.5, the reader finds an analysis of the question if the efficiency criterion is always in conflict with the principle of fairness.

access to the capital market to finance a modern technology and farmer 1 has the opportunity to invest in water-saving production methods, then the uneven allocation of water might be justified. But what if farmer 2 is simply lazy or ignorant? In this case, the water allocation according to equal profits is not fair. Thus, it depends much on the water user's responsibility. If farmer 2 is not accountable for his low productivity, the fair water allocation might follow the principle of equalization of profits (or utility). If society believes in individual responsibilities, then the fair distribution will relate more to the resource side.

Box 4.2 The water-wise rules

Many municipalities in Australia have implemented so-called water-wise rules. These rules which are in fact regulations are aimed to save water in the everyday life of households. For instance, Sidney Water prescribes that households must use hoses fitted with a trigger nozzle, sprinklers, and irrigation systems when irrigating the garden. The irrigation time is restricted from 4 pm until 10 am. Breaching this rule can lead to a fine of 220 $ for households. This regulation is a typical non-price approach and can be considered as a soft form of rationing. It prescribes a technological standard. Its very aim is to save water by increasing the efficiency of water use. Trigger nozzles slow down the water current through the hose. Per time unit less water gets distributed into the garden. As a result, there is no wasted water in the form of ponds evaporating into the air or runoffs. The water from the hose gets to the roots of the plants, with less unproductive water loss.

However, one has to be careful when implementing this kind of water-saving technology. Often the water-saving effects have been smaller than expected. The reason for that lies in behavioral changes that partially offset the efficiency effect of the water-saving device. The implicit water price decreases with improved water efficiency. This leads to the so-called rebound effect which is also very well known from energy consumption. To explain this effect, we apply a very simple microeconomic model.

Households derive utility from a blossoming garden. Let us assume that the extent and the intensity of the vegetation depend positively on the amount of irrigated water (of course, we exclude over-irrigation). In turn, the quality of the vegetation, its beauty, and its range creates benefit to households. If we take into account this interrelation, we express the benefit as a function of the effective water w_e used, i.e., the water that reaches the vegetation. The relation between water from the tap w and effective water use depends on the efficiency of the irrigation technology. We have

$$w_e = w\epsilon \tag{4.57}$$

where ϵ is the technological productivity. If $\epsilon = 1$, there is no water loss. In the simplest case, households maximize their benefit with regard to the amount of water used for irrigation.

$$\max_{w} B(w\epsilon) - pw \quad \text{or} \quad \max_{w_e} B(w_e) - pw_e/\epsilon \tag{4.58}$$

where p is the water price and $p\epsilon$ is the effective price, i.e., the price per liter water reaching the plants. The optimality condition is

$$B'(w_e) = p/\epsilon \quad \Rightarrow \quad \hat{w}_e(\epsilon) \tag{4.59}$$

where $\hat{w}_e(\epsilon)$ is the effective water use. A rising ϵ leads to more consumption of water due to the decrease of the effective water price. This is the so-called rebound effect.

On the other side, the increased water efficiency lessens the water use which can be derived from Eq. (4.57). Both effects then determine the water consumption w.

$$\frac{dw}{d\epsilon} = \frac{\frac{d\hat{w}_w(\epsilon)}{d\epsilon}\epsilon - \hat{w}_e(\epsilon)}{\epsilon^2} \tag{4.60}$$

The first term on the right-hand side is the rebound effect, the second term is the counter-directed efficiency effect. That is the reason why the water-saving effect of increased water productivity is less than the calculated efficiency effect.

There are other examples where the rebound effect appears. Households with low-flow showerheads take longer showers. The "double flush" was observed when households installed low-flow toilets.

Source: Olmstead and Stavins (2009b), www.sydneywater.com.au

4.7 Exercises

Exercise 4.1 Designing an increasing two-block tariff

Increasing block tariffs are designed to allow poor income groups to access water. Often, the price of the first block is below marginal cost so as to render the access possible to even the poorest households. As a consequence, the upper income groups have to provide the necessary cross-subsidies to let the water utility break-even. There are four poor households and one household with sufficient income to consume water beyond the lifeline. The demand of the wealthy household can be captured by the price-quantity function $p_2(w_2) = 12 - (w_2 - w_s)$ where $w_s = 10$ is the lifeline. The task of water utility management is to secure the access to water for the poor by minimizing the water price in the first block and to assure at the same time that no deficit occurs. Let us assume that fix costs are $F = 20$. Marginal costs are set at $c = 2$.

First, the utility maximizes the contribution margin CM_2 in the second block. The contribution margin is defined as

$$CM_2(w_2) = (p_2(w_2) - c)(w_2 - w_s) = (12 - (w_2 - w_s) - c)(w_2 - w_s) \quad (4.61)$$

To maximize $CM_2(w_2)$, we have to set the first derivation equal to zero and solve for w_2.

$$12 - 2(w_2 - w_s) = c \quad \Rightarrow \quad w_2^* = 15 \quad (4.62)$$

Reinserting the result into the price function yields $p_2^* = 7$. The maximum surplus which can be extracted in block 2 is therefore $CM_2^* = CM_2(w_2^*) = (7 - 2)(15 - 10) = 25$.

To calculate the price of the first block , we have to ensure that the surplus of the second block covers the deficits of the first, that is to say

$$(4 + 1)(c - p_1) + F = CM_2^* \quad \Rightarrow \quad p_1^* = c - \frac{(F - CM_2^*)}{(4 + 1)} \quad (4.63)$$

Inserting the numerical values yields $p_1^* = 1$.

Exercise 4.2 Universal service provider

In many countries, the universal service obligation requires water utilities to provide water to spatially distinct customer groups at the same tariff. The tariff design does not reflect the differing connection costs of customers. Assume that marginal costs of supplying consumer group 1 are 2 € per m^3, and the marginal costs of water provision to group 2 are 4 € per m^3. To keep the calculations simple, assume that both groups have the same size $n = 1$ and that their marginal willingness to pay is identical, i.e., $p(w) = a - bw$, where $a = 10$ and $b = 0.5$. Furthermore, we assume that fixed costs can be covered by a uniform and constant access fee. Thus, it remains to fix the volumetric part of the tariff. If we follow the welfare-oriented approach of IWRM, we maximize the aggregated willingness to pay to determine the optimal allocation. This is a straightforward exercise requiring to set the marginal willingness to pay equal to marginal costs.

$$a - bw_1 = c_1 \quad \Rightarrow \quad w_1 = 16 \quad (4.64)$$

and

$$a - bw_2 = c_2 \quad \Rightarrow \quad w_2 = 12 \quad (4.65)$$

Hence, setting zonal prices such that $p_1 = c_1 = 2$ and $p_2 = c_2 = 4$ will maximize total economic rent. However, this is in contrast to the principle of universal service obligation that requires an equal treatment of both groups. Thus, we have to find the cost covering volumetric price, i.e., the price that covers all operating costs (fixed costs are covered by the access fee). This price can be calculated by deriving the marginal willingness to pay from the demand functions $\hat{w}_i(p) = (a - p)/b$, $i = \{1, 2\}$. Cost coverage requires

$$(p - c_1)\hat{w}_1(p) + (p - c_2)\hat{w}_2(p) = (2p - c_1 - c_2)\hat{w}_2(p) = 0 \tag{4.66}$$

which yields $p = 3$. Each group is charged 3 € per m^3 whereby group 1 (2) pays more (less) than their marginal costs. Hence, group 2 is cross-subsidized by group 1. From a welfare theoretical viewpoint, this price is suboptimal. If one places emphasis on equal treatment of customers and if no other redistributing instruments are available, the implementation of the universal service obligation principle has its price in the form of welfare losses.

Exercise 4.3 Optimal modal split
Imagine a village with residents living along a straight road that runs from west where high- and middle-income people live to east where poor people live. In the west, there is a water utility, which processes water and distributes it to the inhabitants through an underground pipeline. However, the waterworks are still in an investment phase, and it is necessary to consider how many households are to be provided with a water connection and how many households are to be supplied by water kiosks and mobile water sellers. Residents are distributed evenly along the main road. The length of the road (linear city) where households dwell is $\bar{s} = 200$ length units, say, measured in 100 m. Water consumption per household is assumed to be 1 m^3/month. The willingness to pay for water depends in our example solely on income. We assume that water demand is completely price inelastic. The income distribution is reflected in the geography (west: upper incomes, east: lower incomes). The willingness to pay for water is geographically distributed according to the function $V(s) = a - bs$, where a = 50 and b = 0.25. Let us assume that treatment cost per m^3 water is m = 0.5 and distribution cost for connected households is k = 16 per length unit. Collecting costs refer to time costs and are $\delta = 1.5$ per length unit. Vendors' costs consist of purchasing/selling costs $c_2 = 4$ per m^3 and hauling costs $c_1 = 0.5$ per length unit.

The first task is to derive the optimal modal split as specified in Eq. (4.40). If you insert the numerical values of the parameters and solve the equation system Eqs. (4.41)–(4.43), you get the optimal values $\{\tilde{s} = 134, \dot{s} = 114, \hat{s} = 110\}$. From these values, we can derive the length of the collecting segment, i.e., $\dot{s} - \hat{s} = 4$ and the vending area $\tilde{s} - \dot{s} = 20$. The pure economic approach based on the marginal willingness to pay leads to an under-supply of the village. Households along the stretch of $200 - 134 = 66$ length units are not provided with water from the utility and have to take care of themselves.

If we follow the Social Development Goal 6, and its aim to give access to safe water sources for everybody, the water infrastructure has to be enlarged. From Eq. (4.42), it follows that $\dot{s} - \hat{s} = c_2/(\delta - c_1) = 4$, i.e., the collecting area is independent of the total length of the village. To calculate the length of the segment where households are connected, we take Eq. (4.43) and substitute for $\tilde{s}$ the total stretch of the village, i.e., 200. Solving the equation yields $\hat{s} = 176$. Since $\dot{s} - \hat{s} = 4$ we have in this case

$\dot{s} = 180$. Thus, the vendor stretch $\bar{s} - \dot{s}$ is still 20. What we see is that the full coverage of water supply was solely achieved by enlarging the area of connected households.

What happens when vendors are cartelized and act like a monopolist? We have shown that in this case water prices in the vending area $s \geq \dot{s} > \hat{s}$ follow exactly the marginal collecting costs, i.e., $p_c = m + \delta(s - \hat{s})$. The cartel will equate the willingness to pay of the marginal customer at s_c with the collecting costs, i.e., $V(s)a - bs = m + \delta(s - \hat{s})$ which yields $s_c = 122.571$ which is less than the optimal value $\tilde{s} = 134$ (see point A^c in Fig. 4.13).

The subsidy that makes $s_c = \tilde{s}$ can be calculated from Eq. (4.55): Inserting the given numerical values of all parameters yields $t_{LM} = (s - \hat{s}) - 4$.

Exercise 4.4 Seasonal pricing

Suppose there are two time periods 1 and 2 where we have the water consumption w_1 and w_2, respectively. Period 1 is the winter period (off-peak period), while period 2 is the summer period (peak period). The demand functions (which are equal to the marginal benefit functions) in both periods are

$$p_1(w_1) = 110 - w_1 \qquad p_2(w_2) = 150 - w_2$$

The cost rate for the delivery of one amount of water is given with $c = 10$, while the cost rate for the provision of one unit of capacity is $r = 20$. In the optimization, we want to calculate the optimal consumption and capacity levels in the way that we maximize the total surplus of 1 year, which includes the peak and off-peak period:

$$\max_{\{w_1, w_2, k\}} [B_1(w_1) + B_2(w_2) - c \cdot (w_1 + w_2) - r \cdot k]$$
$$s.t.\ w_1 \leq k \qquad (\lambda_1)$$
$$w_2 \leq k \qquad (\lambda_2)$$

Of course, the water delivery in both periods is restricted by the chosen capacity level which is addressed in the constraints of the optimization problem. Based on the optimization problem, the Lagrangian function can be set up:

$$L = B_1(w_1) + B_2(w_2) - c \cdot (w_1 + w_2) - r \cdot k + \lambda_1 \cdot [k - w_1] + \lambda_2 \cdot [k - w_2]$$

and finally the KKT conditions can be formulated:

$$B_1'(w_1) - c - \lambda_1 \leq 0 \perp w_1 \geq 0 \tag{4.16}$$
$$B_2'(w_2) - c - \lambda_2 \leq 0 \perp w_2 \geq 0 \tag{4.17}$$
$$\lambda_1 + \lambda_2 - r \leq 0 \perp k \geq 0 \tag{4.18}$$
$$k - w_1 \geq 0 \perp \lambda_1 \geq 0 \tag{4.19}$$
$$k - w_2 \geq 0 \perp \lambda_2 \geq 0 \tag{4.20}$$

We suppose that capacity is only exploited in the summer month; hence, we have to assume $w_1 \geq 0$, $w_2 \geq 0$, $k \geq 0$, $\lambda_1 = 0$ and $\lambda_2 \geq 0$. Therefore, we can calculate

$$(k): \ \lambda_2 = r = 20$$
$$(w_1): \ B_1'(w_1) = c \ \rightarrow \ 110 - w_1 = 10 \ \rightarrow \ w_1 = 100$$
$$(w_2): \ B_2'(w_2) = c + r \ \rightarrow \ 150 - w_2 = 30 \ \rightarrow \ w_2 = 120$$
$$(\lambda_2): \ k = w_2 = 120$$

The solution does not violate the constraint Eq. (4.19), which states that $w_1 \leq k$ (hence: $100 \leq 120$). Therefore, we found no contradiction in the KKT conditions and this case leads to optimality. The prices in the winter/(off-peak) period (p_1) and summer/(peak) period (p_2) are

$$p_1 = B_1'(w_1) = c = 10 \qquad p_2 = B_2'(w_2) = c + r = 30$$

For this case, the capacity is completely financed by the revenues from the summer month (peak period).

Suppose now that the capacity cost rate increases to the level of $r = 50$. Having the same assumption as before, $w_1 \geq 0$, $w_2 \geq 0$, $k \geq 0$, $\lambda_1 = 0$ and $\lambda_2 \geq 0$, where we suppose that capacity is only exploited in the peak period, we get the following results:

$$(k): \ \lambda_2 = r = 50$$
$$(w_1): \ B_1'(w_1) = c \ \rightarrow \ 110 - w_1 = 10 \ \rightarrow \ w_1 = 100$$
$$(w_2): \ B_2'(w_2) = c + r \ \rightarrow \ 150 - w_2 = 60 \ \rightarrow \ w_2 = 90$$
$$(\lambda_2): \ k = w_2 = 90$$

The change of the capacity cost rate does not impact the consumption level in the winter period ($w_1 = 100$). However, due to the increase of the capacity cost rate, the consumption in the summer period decreases to the level of $w_2 = 90$. Therefore, we do not meet the constraint Eq. (4.19), which states that $w_1 \leq k$, because the consumption level in the winter period ($w_1 = 100$) is higher than the chosen capacity level ($k = 90$). Due to the contradiction, this case does not lead to optimality.

Therefore, we suppose that capacity is exploited during the entire year. Hence, we assume that $w_1 \geq 0$, $w_2 \geq 0$, $k \geq 0$, $\lambda_1 \geq 0$, and $\lambda_2 \geq 0$. Because of this assumption, we are able to set up the following system of equations based on the KKT conditions:

$$(\lambda_1) \wedge (\lambda_2): \ w_1 = w_2 = k$$
$$(w_1) \wedge (w_2): \ B_1'(k) + B_2'(k) = 2 \cdot c + r \ \rightarrow \ 110 - k + 150 - k = 2 \cdot 10 + 50$$

The solution is $w_1 = w_2 = k = 95$. The value of the dual variables λ_1 and λ_2 can be calculated from constraint Eqs. (4.16) and (4.17):

$$(w_1): \ \lambda_1 = B_1'(w_1) - c = 110 - 95 - 10 = 5$$
$$(w_2): \ \lambda_2 = B_2'(w_2) - c = 150 - 95 - 10 = 45$$

The dual variables are non-negative, hence, we do not find a contradiction and this case leads to optimality. The price in the winter/(off-peak) period, which is p_1, is lower than in the summer/(peak) period, being p_2:

$$p_1 = B_1'(w_1) = c + \lambda_1 = 15 \qquad p_2 = B_2'(w_2) = c + \lambda_2 = 55$$

Therefore, the capacity is financed by 90% during the summer period, because $\frac{\lambda_2}{r} = \frac{45}{50} = 0.9$, while the capacity is also financed by 10% during the winter period, because $\frac{\lambda_1}{r} = \frac{5}{50} = 0.1$.

Exercise 4.5 Proportional water right allotments
The allotment of tradable water rights may be a promising instrument to reconcile efficiency and fairness. Let us assume that the entitlements to water use are reduced in a proportional way along with increased water scarcity. In this exercise, we want to analyze how the proportional allotment is able to fulfill the criteria of efficiency and fairness. Let us assume that there are two water users, say, firms. Marginal profits are

$$B_i' = a_i - b_i w_i, \quad a_1 = 40, a_2 = 20, b_1 = b_2 = 1 \tag{4.67}$$

If water is abundant and there are no treatment costs of water, firms set marginal profits equal to zero which leads to

$$\hat{w}_1 = a_1/b_1 = 40 \quad \text{and} \quad \hat{w}_2 = a_2/b_2 = 20 \tag{4.68}$$

What are the profits? The calculation needs the profit function which can be achieved by integrating the marginal profit function with respect to w.

$$B_i = \int_0^{w_i} [a_i - b_i \upsilon] d\upsilon = w_i (a - (b_i/2) w_i) \tag{4.69}$$

Inserting the calculated water usage yields profits $B_1 = 800$ and $B_2 = 200$. The profit of firm 1 is four times as high as that of firm 2.

In the course of increasing water scarcity firms total water use of 40 + 20 = 60 cannot be covered any more. Instead there is only water available of a total of $\bar{W} = 30$, i.e., half of the former total use. Water entitlements $\bar{w}_i$ will be allotted proportionally[35]:

$$\bar{w}_i = \frac{\hat{w}_i}{\hat{w}_1 + \hat{w}_2} \bar{W} = \hat{w}_i \frac{\bar{W}}{\hat{w}_1 + \hat{w}_2} = \frac{1}{2} \hat{w}_i \tag{4.70}$$

[35]Notice that $\bar{W} = (1/2)(\hat{w}_1 + \hat{w}_2)$.

Utilizing Eq. (4.68) yields $\bar{w}_1 = 20$ and $\bar{w}_2 = 10$. If these allotments are not tradable, the resulting profits can be calculated by halving the former water use, and inserting the results into the profit function Eq. (4.69) which yields $\bar{B}_1 = 600$ and $\bar{B}_2 = 150$. The distributional effects of the proportional allotment can be captured by the profit ratio $\bar{B}_1/\bar{B}_2 = 4$. Hence, water rationing has not changed the profit distribution, according to the proportionality rule. However, the water allocation is not optimal if one takes the efficiency criterion into account. Hence, we maximize total profits under the constraint that total water use does not exceed the sustainability constraint $\bar{W} = 30$.

$$\max_{w_1, w_2} [B_1(w_1) + B_2(w_2)], \quad \text{s.t.} \quad w_1 + w_2 \leq \bar{W} \tag{4.71}$$

The first-order conditions require w_i^* to be set in a way that $B_1' = B_2'$ while meeting the sustainability constraint. Inserting all relevant values leads to the solution $w_1^* = 25$ and $w_2^* = 5$. We see immediately that these values differ from the proportional allotments $\bar{w}_1 = 20$ and $\bar{w}_2 = 10$.

The efficient solution can be implemented by introducing a market for water entitlements. The market equilibrium is characterized by an excess demand for water by firm 1 of $w_1^* - \bar{w}_1 = 25 - 20$. Firm 2 is net seller of entitlements $\bar{w}_2 - w_2^* - = 10 - 5$. The equilibrium price can be calculated by inserting w_1^* into the marginal profit function of firm 1, i.e.,

$$p^* = B_1'(w_1^*) = a_1 - b_1 w_1^* = 15 \tag{4.72}$$

To calculate net profits, we insert the market solution into the profit functions and deduct the net demand for water

$$B_i^n = w_i^*(a - (b_i/2)w_i^*) - p^*(w_i^* - \bar{w}_i) \tag{4.73}$$

which yields $B_1^n = 612.5$ and $B_2^n = 162.5$. If we compare these values with profits under the rationing system without trading, we see that both profits have risen. This is not surprising, because trade is voluntary and therefore only takes place when both parties are better off after a trade. By close inspection, we also see that the profit of the small firm 1 has risen stronger than that of the bigger firm 2. The profit ratio is now $B_1^n/B_2^n \approx 3.8$. Hence, in our example, the enhancement of efficiency by allowing to trade water allotments leads to a profit distribution in favor of the smaller firm. Obviously, the principle of efficiency and the principle of justice need not always be in conflict. In addition, the introduction of trade fulfills Rawls' principle of difference: The worst placed user improves her situation under the observed regime.

4.8 Further Reading

Water is a multidimensional resource that not only serves as a private good. Hanemann (2004) gives an instructive historical outline of the economic dimension of water, taking into account not only efficiency aspects but also the primary supply based on human rights. The design of tariffs for water services (drinking water, sanitation) should take into account various criteria. Boland and Whittington (2000a) and Massarutto (2007b) present the various criteria and evaluate different tariff structures on the basis of these criteria. In particular, IBTs are taken into account (see also OECD 2010; Walker 2009). Rogers et al. (1998) review the tariff building criteria underlying the Dublin Water Principles. Designed as practical guides, OECD (2009) and OECD (2010) provide a problem-oriented introduction and an overview of the financing of water infrastructures with special attention to tariff policy. Not only internal costs but also social costs of water supply (environmental costs, etc.) are taken into account.

There are a number of tariff structures that are used in the supply of infrastructure goods (water, energy, transportation). The basic analysis techniques of the effects of tariff variants are introduced in the microeconomic textbook of Varian and Repcheck (2010). But there are not only economic aspects to be considered. We often observe political and legal requirements, which have to be taken into account in the tariff structure, particularly in the case of network services. Cremer et al. (2001) analyze the universal service provider which is subject to the universal service obligation, i.e., the provision to serve all customers at affordable (and equal) rates.

IBTs are widespread in the water sector, especially in Asia. There is a large number of studies on how these tariffs work, some more practical, others more theoretical, among the latter Boland and Whittington (2000a, b), Whittington (2003), Dahan and Nisan (2007), and Monteiro and Rosetta-Palma (2011). Meran and von Hirschhausen (2017) develop a microeconomic model with social preferences where a strong inequity aversion leads to IBTs as tariff system.

Unconnected water markets play a major role in developing countries. There are many case studies investigating the precise institutional, cultural, and political characteristics. A comprehensive study is Kjellén and McGranahan (2006), which examines the water supply in Dar es Salaam, Tanzania, Hailu et al. (2011) for Nigeria, Kenya, and for Cochabamba, Bolivia, Wutich et al. (2016). An analytical economic analysis of rent extracting behavior is given by Lovei and Whittington (1991). Baumol (1982) analyzes cost structures that allow for competitive behavior of suppliers even if only few actors operate in the market.

Rationing is often used in the case of water scarcity. Lund and Reed (1995) and Olmstead and Stavins (2009a) provide an overview of the different types and an economic assessment. An analysis that takes into account aspects of equity and efficiency in times of severe scarcity (e.g., war times) is Tobin (1970).

4.9 Chapter-Annex: Overview of Water Tariff Structures

Table 4.4 Tariff structures for water supply and sanitation and policy objectives: a synthesis based on OECD (2010)

Tariff structure	Examples	Ecological sustainability	Economic efficiency	Financial sustainability	Equity/affordability
Uniform flat fee	Sub-areas of two water supply companies in the United Kingdom. Still used by many sampled non-OECD utilities	Very poor. No incentives to water saving nor to other aspects of sustainable water use	Poor for drinking water (no linkage between fee structure and behavior that may help minimize investment). OK for water-borne sanitation (costs do not depend on water consumption)	Potentially OK, but commitment to cost recovery is what really matters. Avoid political determination of fees	Very regressive (unless properly integrated with other elements of a social security system)
Non-uniform flat rate linked with specific aspects of households, e.g., (i) property value or other income proxy, (ii) dwelling characteristics linked with water use	Still used by 70% of UK households, common in the former Soviet Union	Poor if linked with income-related variable. Good if linked with dwelling characteristics linked with water use (e.g., use of water recycling devices) or with specific behavior that wants to be encouraged (e.g., rainwater harvesting)	As above	As above, provided that total revenues are guaranteed	Potentially good effects, provided that criteria used correspond to personal wealth. Regressive otherwise (unless properly integrated with other elements of a social security system)

(continued)

Table 4.4 (continued)

Tariff structure	Examples	Ecological sustainability	Economic efficiency	Financial sustainability	Equity/affordability
Uniform volumetric rate + 0 fixed charge	Still present in numerous OECD countries. Most recurrent in sample of non-OECD utilities	As above; higher, since 0 fixed charge means a larger marginal rate (for the same revenue levels)	Efficient if water is scarce or infrastructure nearing capacity (i.e., if there is rivalry in consumption) or if variable costs are high compared to fixed costs. Not very efficient if otherwise it would discourage users but this would reduce societal benefits. Inefficiency depends on demand elasticity (the lower the elasticity, the lower the inefficiency)	Good potential for financial recovery. Can have (temporary) negative impact on revenue in case of a sudden move from flat charges due to impact on demand (e.g., Berlin experience)	Depends on income elasticity. If this is low, it can hit large poor households hard
Uniform volumetric rate + fixed charge > 0	Classic, e.g., Germany (structure enshrined in law)	High, depending on the marginal rate (impact on demand only if it is high enough) + individual metering	Optimal provided the following applies: volumetric rate = SRMC (short-run marginal cost) and fixed charge = lump sum. Particularly suited in case SRMC is constant (e.g., electricity)	As above	Depends on size of fixed charge, but tends to be regressive (not so only if marginal cost is high and income elasticity is high which is rare). Size of fixed charge can be differentiated based on income

(continued)

Table 4.4 (continued)

Tariff structure	Examples	Ecological sustainability	Economic efficiency	Financial sustainability	Equity/affordability
Uniform volumetric rate + rebate (fixed charge < 0)	No known application. May have been applied in municipalities in the United States	As above. Highest if rebates take into account specific circumstances (e.g., use of water recycling devices, drip irrigation or water-saving sprinklers in gardens) or with specific behavior that wants to be encouraged (e.g., rainwater harvesting, use of less pollutant detergents)	As above. In turn, could be efficient in combination with a positive fixed fee (idea: r = SRMC; fixed cost redistributed including a rebate for the poor)	As above	Progressive and useful for reducing impact on poor. But only if rebate is targeted; otherwise, distributive effect depending on income elasticity, just like with IBTs
Traditional IBT (both block widths and prices fixed) + fixed charge	Italy. Increasing number of developing countries	Highest, provided that metering is individual and marginal rates in the upper blocks are high	Potentially the best solution provided r = SRMC and fixed charge = lump sum. Particularly suited in case SRMC is increasing (e.g., costly extra supply to be purchased)	As above	Can be very regressive if: (i) low demand elasticity to income; (ii) resulting average tariff is below cost recovery levels and this discourages extension of network; (iii) many households sharing the same tap

(continued)

Table 4.4 (continued)

Tariff structure	Examples	Ecological sustainability	Economic efficiency	Financial sustainability	Equity/affordability
IBT + fixed charge + exact occupancy amendment	Flanders, Brussels Malta, some communes in Luxembourg	As above, but reduced incentives for large families	Depends on how closely the resulting average volumetric charge reflects SRMC. Rest as above	As above	Reduces impact on large families (best if accompanied by reduction of leaks and improved efficiency of appliances). Depends on correlation of size and income of households. Problem (ii) above remains
IBT + fixed charge + low-income households may apply for extension	Proposed Social Tariff Plan in Portugal	As above, but reduced incentives for low-income households that apply for extension of blocks	Good for reducing demand in peak periods and optimizing capacity use	Uncertainty about number of households applying (may be reduced over time)	Successful, if all eligible claim and block width reflect consumption patterns of the poor. Problem (ii) above remains
IBT + fixed charge + larger households (e.g., N = 4) may apply for extension	Some Spanish cities. Greek DEYA, cities. Proposed option in Portugal	As above, but reduced incentives for large families that apply for extension of blocks	Depends on whether there is a fixed charge or not	As above	Depends on correlation of size and income of households. Problem (ii) above remains
IBT + fixed charge + targeted subsidies to low income	Chile	Highest, provided that metering is individual and marginal rates in the upper blocks are high	As above	As above	Depends on the capacity to target the poor. Problem (ii) above remains

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5 Water Markets

5.1 Institutional, Hydrological and Infrastructural Preconditions

5.1.1 Design of Water Markets

5.1.1.1 Design Options

Water markets are one possible institutional option to deal with water management. Currently, a few formal water markets are established in countries where water is scarce and governmental organization is fairly effective. The enforcement of basic laws and rules is essential for an effectively working formal water market, because they are needed for the registration of water rights and to specify conditions for the trading of water and water rights. Hirshleifer et al. (1969) illustrate in their reference, how water laws can promote or hinder the implementation of water markets. They focus their analysis on the riparian and appropriation rights regime in the United States.[1]

There exist a number of legal preconditions for the promotion of water markets. Based on Endo et al. (2018), these are

[1]Under the riparian water rights regime, the water is allocated among those who possess land along the water body. All landowners whose properties adjoin a water body have the right to make reasonable use of the water source as it flows through or over their properties. However, the appropriation water rights regime originated in California, during the time of Gold Rush (Grompe and Hansjürgens 2012). The idea of this right regime is that the first person who takes the water for beneficial use is allowed to continue the water usage of this quantity for that purpose. We can differ between the senior and junior water rights, with senior rights being emitted earlier than junior rights. The water is first allocated to the senior rights owner and afterward to the junior rights owner.

G. Meran et al., *The Economics of Water*, Springer Water,
https://doi.org/10.1007/978-3-030-48485-9_5

- the existence of laws and rules that allow the reallocation of water (see Grafton et al. 2012)
- the separation of water rights and landownership; (see Chong and Sunding (2006) as well as Grafton et al. (2012))
- rules for the case that water rights are non-used. Here the non-used water rights should not be canceled.
- the predictability of the available water before the irrigation periods
- public control of groundwater pumping throughout the jurisdiction

For the implementation of formal water markets the establishment of water rights are quite essential. Regarding these water rights, a number of characteristics—such as the duration, the conditions for renewing and restrictions for trading water rights—have to be determined by rules and laws:

- On the one hand, there is the duration of a water right and on the other hand, there are the conditions for renewing an expired right. These two characteristics determine the value of the water right and also affect the level of infrastructure investments in the water supply system. The higher the duration and the higher the assurance for renewing an expired water right, the higher is the incentive for investments in water (delivery) infrastructure.
- It has to be determined which party is allowed to buy a water right. It has also to be clarified whether the buyer is able to use the water in just a certain location and for just selected purposes. Furthermore, the feasibility of water rights' divisibility has to be specified, which means whether it is possible to sell just a selected proportion of the owned water right.
- It has to be specified whether a water allocation under an entitlement must be used. This may oblige a water right owner to use it for a specified time. Also, the consequences of non-using water entitlements have to be clarified. The allocated amount not used during a specified time period could either be extinguished, or may be used in later periods. The more the non-usage of water entitlements is penalized, the less is the incentive to save water for dry years.

Organizations and institutional arrangements such as water user organizations, water courts or even state courts are also important to resolve conflicts either between various water right holders or between water right holders and third parties. Here, third parties are those who may be (negatively) affected by the water trade. The best-known water markets which currently exist are in the western USA, Australia, and Chile. Evidence is mixed thus far, but one may expect that due to climate change and the resulting increase in water shortages in these countries, water markets may become more important in the future (Easter and Huang 2014b). Endo et al. (2018) analyze the countries regarding their applicability of water markets on the base of their water current laws.

Apart from the formal water markets, there also exist informal ones. Water markets are operated at a local level, for instance allowing neighboring farmers to trade water. For example, these forms of markets may make groundwater available for those farmers who cannot afford the installation of their own wells. Usually, the rules at this type of market are informal and there is no requirement for large investments in management and infrastructure capacities. Furthermore, rent-seeking issues, as well as high transaction cost, may lead to the emergence of informal markets instead of formal ones (Easter and Huang 2014b).

In addition to the distinction between formal and informal water markets, it is also possible to differentiate between markets for water rights and markets for water. Property rights are transferred to a new user (buyer) in a market for water rights, while in markets for water (which are also termed as leasing markets), water is transferred to the buyer and the seller retains the ownership of the asset (water right) (Goemans and Pritchett 2014).

5.1.1.2 Permanent Transfers: Water Right Markets

The transaction in water rights markets could either be a direct or an indirect transfer of water rights' ownership. Direct transfer means that the ownership of water rights moves during the transaction from the seller to the buyer so that the buyer obtains the right to divert water. Indirect transactions occur when a water user buys shares of a ditch company to gain water resources and the ditch company retains the right to divert. These transactions are governed by the ditch companies bylaws (Goemans and Pritchett 2014). Direct transfer of ownership becomes more complex when the location of diversion is changed or if the water is used in a different way after the transaction.

The direct transfer may involve two steps for the buyer: the purchase of the water right and the change of use. There is no fixed order in this two step-process which means that the right can be changed before selling, or the right can be sold first. Furthermore, the change in use may occur at a much later date than the selling date. This becomes especially relevant for municipal water providers which expect a high growth of water demand for the future, and thus buy water rights for covering future water demand. In the interim, the municipal water provider leases the rights back to the original water right owner (Howitt and Hansen 2005). For approving the change in use by the state administrative, in some markets it has to be demonstrated that no right owner is adversely affected by the change of use, which means that the quantity of available water must not decrease for other right owners.

5.1.1.3 Temporary Transfers: Leasing of Water Rights

For a temporary transfer of rights, the seller leases the water right to another user, but retains the ownership of the water right for future use. There exist three types of leasing water rights: water banks, single-/multi-year leases, and interruptible water supply agreements.

Water banks reallocate water on a short-term basis. They are quite often formed to fulfill a specific need, for instance, maintaining water supply during drought periods, ensuring an instream flow for habitats, or augmenting flows for future use. The water bank serves as a facilitator of exchange by matching buyers and sellers. It is a clearinghouse for transactions and it provides services to realize transactions which include the determination of the type of water right and the adherence to the regulation regime. By depositing rights into the bank, potential sellers make them available for potential buyers. The water banks differ in various categories (see Goemans and Pritchett 2014):

- the organization of the agency: it could be an organization of the federal agency, the state government, special districts or interested parties.
- the determination of prices: water banks may post a fixed price or use options to determine a market clearing price.
- contract types: there exist supplier contracts (that are used to organize specific entitlements in a bank), storage contracts (allowing the deposit of water in a physical storage), and contingent claim contracts (which permit the buyer to use water from the bank under specific circumstances such as a drought period).

While water banks are organizations where many buyers and sellers are able to exchange water rights, water leases are bilateral agreements between individuals in which the water right owner agrees to lease a specific amount of water. The bilateral negotiations make it possible to customize the contract. Typical contract stipulations include the contract term, the pricing policy (determining fixed and volume prices per unit), as well as the integration of a leaseback option, which means that the lessor has the first right to use the water if it is not needed by the lessee. A special type of water leases are the interruptible supply agreements which last for a multitude of years, but where water delivery is just made when it is needed. The interruptability could be realized by, for instance, an option agreement in which the lessee pays a baseline fee for the option to use a water right. This option does not need to be exercised each year. If the option is not exercised the lessor has the first right to use the water, while for the contrary case that the option is used the lessee pays an additional, pre-negotiated fee to exercise the option and get the water right for the year. Therefore, the lessor receives a secure revenue stream from the lessee, while the lessee in return receives the guarantee that there will be additional water supply when needed at a pre-negotiated price (Goemans and Pritchett 2014).

5.1.1.4 Limitations of Water Markets

Just as any other markets, real-world water markets are no perfect representations of theory. Rather, they are subject to transaction costs and issues of implementation (Western Governor Association 2012). Transaction costs include the search costs of a willing buyer/seller, negotiations, navigating institutional requirements (permits, water courts proceedings), and the physical expense for collecting, storing and treating of water (McCann and Easter 2004; Furubotn and Richter 2005). High transaction costs can reduce the frequency of water transfers, make it difficult to match water supplies to changing use and to limit the participants in the market. In empirical studies, transaction costs range from 3 to 70% in water markets (Garrick et al. 2013). The level of these costs mainly depends on various physical and institutional factors. A detailed overview regarding these factors is provided by McCann and Garrick (2014).

5.1.2 Transaction Costs and Institutional Factors

5.1.2.1 Physical Factors

Physical, biological, and technical factors which are subsumed under the term physical factors are important drivers for transaction and transformation costs. These physical factors are

- **Scale:** Quantity and quality issues may have to be addressed on the watershed location which means that the geographical scale of intervention is needed for resolving water market issues. This involves, for instance, the transfer of pollutants in space. If the water market issue is linked with the geographical scale, more coordination is required which results in higher transaction costs.
- **Time lags:** Time lag between a measurement and its impact, for instance, a lag between improved management and noticeable improvements in the water market.
- **Magnitude of changes:** The higher the changes in water quality standards or water consumption levels, the higher the resistance against the new policy if the change is related to an economic loss for stakeholders.
- **Heterogeneity:** Property rights are more difficult to establish if the property rights are poorly specified and fail to account for different sources of water and their hydrological interactions (Young and McColl 2009).
- **External effects:** Downstream water rights are often dependent on the return flows of upstream users. A reallocation at the upstream may affect the return flow, and therefore, lead to third-party effects for downstream water right owners.
- **Excludability:** Excluding people from consumption of a non-excludable good (e.g., groundwater) requires strict monitoring and enforcement.
- **Measurability/Observability:** Measurability and observability impact the cost for monitoring and enforcement, and determine which kind of policy is feasible.
- **Economics of scope:** Market design becomes more complex in a setting with multiple outputs, e.g., the multi-purpose design of infrastructure to optimize irrigation, flood control, hydropower, urban water use, etc.
- **Number of agents:** Transaction cost increases with the number of agents that are involved (Cacho et al. 2013). Water banks and water trading registry systems standardize policy and procedures to reduce transaction costs even for a large number of buyers and sellers.
- **Uncertainty:** Time lags, natural variability in space and time, biological diversity, heterogeneity of agents, etc., impact uncertainty. Higher uncertainty leads to incomplete contracts, and thus, increasing ex-post transaction costs (Williamson 1985).
- **Asset specificity:** Asset specificity refers to a situation in which a resource is unique to a transaction partner and cannot be easily redeployed for transactions with other partners. The design and scale of water infrastructure, as well as the heterogeneity of water rights, contribute to asset specificity and complex institutional arrangements.

5.1.2.2 Institutional Factors

Some institutional factors that affect transaction and transformation costs are

- **Culture:** Culture affects the socialization of people, their fundamental values, the level of trust within the society, notions of fairness, interest in common goods, etc. (Schmid 2004; Vatn 2005). Concerns of irrigation communities regarding the long-term effects of water trades on their economic and cultural viability have slowed down the emergence of spot markets (Howitt 2014; Bjornlund et al. 2014; Hearne and Donoso 2014).
- **Institutional environment:** The institutional environment consists of constitutions, legal systems, laws, and policies (Williamson 2000). Especially constitutional provisions related to water are quite difficult to change. This could result in a fragmented institutional framework limiting water trade (Bjornlund 2004). The legal system and the courts also affect transaction costs. The less effective the legal system is able to enforce contracts, the higher are transactions costs. The existence of conflict resolution mechanisms in water markets can avoid costly and cumbersome administrative hearings and court cases, see Ostrom (1990).
- **Physical versus administrative boundaries:** Administrative boundaries that do not coincide with environmental areas of interest make cooperation difficult (Perry and Easter 2004). Coordination costs rise with the number of agents involved in specific transactions (Laurenceau 2012).
- **Lobbying:** Transaction costs at the enactment stage may be higher than transaction costs to implement a policy (Krutilla et al. 2011).
- **Property Rights:** The exchange of property rights implies transaction costs. With changing technology and changes of preferences, the transaction cots of exchanging property rights is likely to increase (Demsetz 1967; Garrick et al. 2013; Crase et al. 2013). Also agents who do not have property rights may incur costs to change the property rights structure (Bromley 1992; Stavins 1995). Furthermore, if governments create new rights, transaction costs are incurred to allocate those rights (Krutilla et al. 2011).
- **Market structure:** A monopsony market structure may facilitate bargaining, while bilateral monopoly can impede it (McCann and Garrick 2014).
- **Sequencing and timing:** The implementation of a draconian policy may cause more transaction and transformation costs than a policy which is less restrictive (McCann and Garrick 2014). Transaction costs of multiple policies are incurred if it is required that a policy is changed subsequently. For supporting water trade, for instance Garrick et al. (2013), note the importance of multiphase sequencing of institutional transformation. This involves three phases: market emergence, market strengthening, and adjustment.
- **Intermediaries:** The use of intermediaries (e.g., brokers) may reduce transaction costs, especially for infrequent transactions that require specific knowledge (see Coggan et al. (2010)). For instance, water banks provide a clearinghouse function to decrease transaction costs of administrative reviews or price discovery for buyers and sellers.

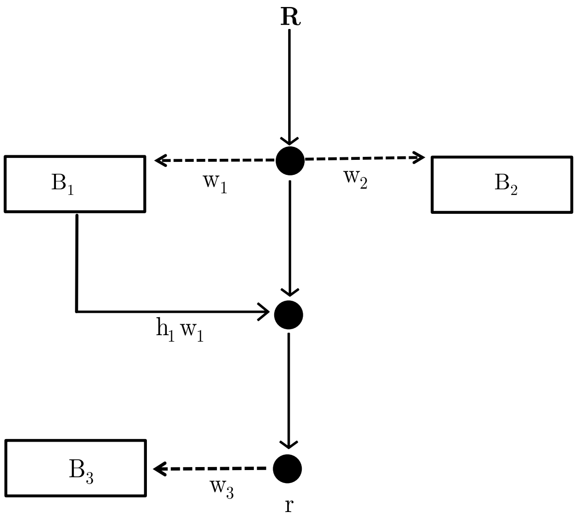

Fig. 5.1 A simple river basin model. *Source* own illustration

5.2 A Water Market Model

5.2.1 Water Markets and Return Flows

With the help of a water market model, we now derive the problems of implementing water markets. As a normative starting point we use the approach of the optimal allocation of water along a river as presented in Sect. 3.7. Figure 5.1 depicts a simple hydrological scenario.

In the river basin dealt with here, there is an inflow R and a prescribed runoff r.[2] There are three users, say farmers, who want to irrigate their plantations located along the river. Farmers 1 and 2 are located upstream. Farmer 3 is situated further below. Here we also take into account the return flows that occur in agriculture. For simplification, it is assumed that only farmer 1 has return flows.[3] The water diversion of farmer 2 and 3 is, therefore, identical to their water consumption. Regarding farmer 1 we have to distinguish between water diversion and water consumption. Diversion is captured by the variable w_1 and water consumption is $(1 - h_1)w_1$. The fraction $h_1 w_1$ returns to the river and is available for farmer 3. The reference point for an assessment is the water allocation that follows from an integrated water resource management approach. Here, we limit ourselves to the criterion of efficiency on the implicit assumption that distribution issues are solved by parallel transfer payments.

$$\max_{w_i} [B_1(w_1) + B_2(w_2) + B_3(w_3)] \quad (5.1)$$

[2]In the following, we will assume that $r = 0$ for simplicity. All results also apply to the more general and realistic case of $r > 0$.

[3]Our results are also valid for the more general case where all three farmers have return flows.

under the constraints

$$w_1 + w_2 \leq R - r \tag{5.2}$$
$$w_3 \leq R - r - w_1 - w_2 + h_1 w_1 \tag{5.3}$$

Assuming that all farmers get a portion of the sustainable amount of available water which is $R - r$, we derive the following optimality conditions

$$B_1^{'}(w_1) - \lambda_1 - \lambda_2(1 - h_1) = 0 \tag{5.4}$$
$$B_2^{'}(w_2) - \lambda_1 - \lambda_2 = 0 \tag{5.5}$$
$$B_3^{'}(w_3) - \lambda_2 = 0 \tag{5.6}$$

From Sect. 3.7 we know that we have to distinguish two cases of optimal water allocation that depend on the farmers' marginal benefit functions and on the extent of water scarcity: In the first case, all available water is used up by farmer 1 and 2.[4] In the second case a portion of water flows to farmer 3, so that this amount and the return flow $h_1 w_1$ is available to farmer 3. In this second case we have $\lambda_1 = 0$ and the optimality conditions condense to

$$\frac{B_1^{'}(w_1)}{(1 - h_1)} = B_2^{'}(w_2) \tag{5.7}$$
$$B_2^{'}(w_2) = B_3^{'}(w_3) \tag{5.8}$$
$$w_3 = R - r - w_1 - w_2 + h_1 w_1 \tag{5.9}$$

The water allocation equates the marginal benefits of water consumption, taking farmer 1's return flows into account. The return flows increase the water productivity. Therefore, farmer 1 is assigned more of the water than in the case of no return flows.

In order to examine the problems of implementing water markets as a means of optimal water allocation in the presence of return flows, we focus on the second scenario, where Eq. (5.2) is not binding. To further simplify the algebra, we assume simple numerical values. For the marginal benefit of water we assume $B_i^{'}(w) = a - bw = 300 - w$, i.e., all farmers are identical. Further: $h_1 = 0.5$, R = 300 and $r = 0$. Inserting these parameter values into Eqs. (5.7)–(5.9) yields the optimal water allocation[5]: $\{w_1^* = 200, w_2^* = 100, w_3^* = 100\}$. Farmer 1 gets twice as much as farmer 2, so the available water R is completely allocated to them. Farmer 3 receives farmer 1's return flow. Note that the optimal allocation does not violate the constraint Eq. (5.2).

[4] This implies that constraint Eq. (5.2) is binding and hence, $\lambda_1 > 0$.

[5] The exact calculation is presented in Exercise 5.1 in Sect. 5.4.

Now we introduce a water market in which water withdrawals are traded. The property rights to water are distributed in such a way that they protect the river basin. This implies that total water rights do not exceed $R + h_1 w_1$. Whatever the distribution of water rights between farmers, in sum they must comply to this constraint to ensure hydrological sustainability. The exact key of the distribution of water rights follows fairness criteria or is historically given.

Each farmer maximizes the net benefit

$$\max_{w_i}[B_i(w_i) - q(w_i - T_i)] \quad \Rightarrow \quad B_i^{'}(w_i) = q \tag{5.10}$$

where q is the price of, say, one m^3 of water diverted and T_i are the water rights assigned. Solving this optimization program with respect to w_i yields the individual market demand of farmer i. Taking our example we have

$$a - bw_i = q \quad \Rightarrow \quad \hat{w}_i(q) = \frac{a-q}{b} = 300 - q \tag{5.11}$$

It should be noted here that we assume a competitive market in which there is no strategic behavior. No market participant can manipulate the price of water. We, therefore, rule out collusion, monopolistic or oligopolistic behavior.

Total demand is $\hat{W} = \sum_{i=1}^{3} \hat{w}_i$. The equilibrium price q^* can be calculated by equating total demand with the given supply $R - r + h_1 w_1$. The market auctioneer has a difficult task to solve: He must not only determine the equilibrium price, but he also has to calculate the effective water supply at each price. This presupposes that he can compute the return flows of farmer 1 which is only possible if a reliable water accounting system of the river basin exists. In the following, we will assume that he is able to do so.

Since all demand functions are identical, all farmers buy the same amount of water, i.e., $\hat{W} = 3(a - q)/b$. The equilibrium price can be calculated from the market clearing condition

$$3\left[\frac{a-q}{b}\right] = R + h_1\left[\frac{a-q}{b}\right] \quad \Rightarrow \quad q^* = \left[a - \frac{bR}{3-h_1}\right] \tag{5.12}$$

Inserting the numerical values yields $q^* = 180$. If q^* is inserted in the demand functions, we obtain the market allocation $\{\hat{w}_1 = 120, \hat{w}_2 = 120, \hat{w}_3 = 120\}$. If one compares the market allocation with our reference allocation, one sees that the introduction of the market leads to a suboptimal water allocation. The water market allocates too little water to farmer 1. This is because farmer 1 does not base her demand decision on net water flows. The market refers to water diversion, not water consumption. This result is well-known in water economics and various institutional designs have been proposed to remedy this market failure. One of these proposals suggests to introduce a water market where water consumption is traded, not water diversion. Of course, if water trading is based on water consumption the return flows must be observable. In addition to the water accounting system, a hydrological model must be implemented to predict the price sensitivity of return flows.

Let us assume that this informational requirement is fulfilled. Then, our model has to be changed slightly. Farmers 2 and 3 behave as before because their water diversion is not related to return flows. Farmer 1's water demand is dependent on his water consumption. He only pays for water consumption $(1 - h_1)w_1$. Hence

$$\max_{w_1}[B_1(w_1) - q(1 - h_1)w_1 + qT_i] \quad \Rightarrow \quad B_1^{'}(w_1) = q(1 - h_1) \tag{5.13}$$

From Eq. (5.13), we can calculate the water demand of farmer 1

$$\hat{w}_1(q) = \frac{a - q(1 - h_1)}{b} = 300 - 0.5q \tag{5.14}$$

The water demand of farmer 2 and 3 remains the same. Thus, the equilibrium price of the water market follows from equating total demand to supply

$$(1 - h_1)\hat{w}_1(q) + \hat{w}_2(q) + \hat{w}_3(q) = R - r \quad \Rightarrow \quad \frac{a - q(1 - h_1)}{b} + 2\left(\frac{a - q}{b}\right) = R - r \tag{5.15}$$

Inserting the numerical values yields $q^* = 200$. Reinserting q^* into the respective demand functions leads to the final market induced water allocation $\{\hat{w}_1 = 200, \hat{w}_2 = 100, \hat{w}_3 = 100\}$. This allocation is identical to the optimal allocation derived from our IWRM approach.

5.2.2 Water Markets and Instream Constraints

Even if there were no return flows, the optimal allocation is not necessarily ensured by a single water market covering the river basin. This is the case when instream flows have to be taken into account. For various reasons a minimum of running water along the course of a river is necessary. Examples are ecological reasons, recreation of the local population, navigability, or yet other reasons. These instream flows are called environmental flows. Our model captures this inflow instream constraint by requiring that in the first flow section a flow rate $\bar{v}_1$ must not be undercut. For the second section a lower limit of $\bar{v}_2$ applies accordingly.[6] Thus, the hydrological constraints from Eqs. (5.2) and (5.3) have to be changed to[7]

$$w_1 + w_2 \leq R - \bar{v}_1 \tag{5.16}$$

$$w_3 \leq R - w_1 - w_2 - \bar{v}_2 \tag{5.17}$$

[6]Notice, that we must have $\bar{v}_1 > \bar{v}_2$. Otherwise, upstream farmers cannot divert water from the river.
[7]Again, we assume as before that $r = 0$.

5.2.2.1 Insufficiency of a Single Market

For simplicity, let us assume that farmers are identical. In addition, we assume that the first stretch of the river is regulated, i.e., there is a minimum river flow needed, say, $\bar{v}_1 = 150$. For the second river section we assume, that $\bar{v}_2 = 0$, i.e., farmer 3 can use up all water available.

First, we calculate the optimal water allocations using the maximization program (5.1) under the new hydrological constraints. The optimal conditions consist of Eqs. (5.4)–(5.6) for $h_1 = 0$, Eqs. (5.16) and (5.17). From these conditions we can infer that the first constraint must be binding, i.e., for $\lambda_1 > 0$ this constraint was not binding and, hence, $\lambda_1 = 0$ the optimality conditions (5.4)–(5.6) would imply that $w_1^* = w_2^* = w_3^* = R/3$. But this violates constraint (5.16) since $w_1^* = w_2^* = (2/3)R = 200 > R - \bar{v}_1 = 150$. Therefore, the constraint (5.16) is binding and $w_1^* + w_2^* = R - \bar{v}_1$. Since both water allotments for farmer 1 and farmer 2 must be equal (see Eqs. (5.4) and (5.5)) we have $w_1^* = w_2^* = (R - \bar{v}_1)/2 = 75$. From Eq. (5.17) follows $w_3^* = R - w_1^* - w_2^* = 150$. The environmental instream regulation brings an advantage for farmer 3.

We now show that this allocation cannot be achieved with a single water market for the entire catchment area, although there are no return flows. If a single market is implemented, a single water price exists equilibrating total demand with supply. To secure the instream constraint in the first stretch of the river total supply is equal to $R - \bar{v}_1$ we have $\hat{w}_1 = \hat{w}_2 = \hat{w}_3 = (R - \bar{v}_1)/3 = 150/3 = 50$. This allocation does not correspond to the optimal solution. If instead total inflow R is offered, the market allocation for each farmer amounts to $R/3 = 100$. Again, this violates the hydrological constraints, since $\hat{w}_1 + \hat{w}_2 = 200 > R - \bar{v}_1 = 150$. Hence, a single market cannot provide the optimal solution.

5.2.2.2 A System of Local Markets

Therefore, a system of local markets must be introduced. We establish two markets, one for the water of the upstream section and one for the downstream section of the river. The upstream market extends from the inflow to before the lower withdrawal point of farmer 3. The lower market encompasses the flow from this withdrawal point to the end of the river. The upper stretch is regulated by the instream constraint $\bar{v}_1$, the lower section has no regulation (for simplicity). Before trade takes place, the public water authority assigns locational property rights of water withdrawal to the farmers. Upstream property rights are in total $(R - \bar{v}_1)$, guaranteeing the ecological solidity of the upper stretch. These rights are distributed to the farmers according to a given key, which we will not discuss further. Justice aspects, power structures or historically given rights can play a role here. These rights can be utilized to divert water or to sell the rights in the market. The same applies to the downstream water market. Here, total property rights cover the remaining water $\bar{v}_1$. In contrast to the upper market, there are some constraints on the part of farmer 1 and 2. Both can sell their downstream property rights, but they cannot use these rights to buy water due to the unidirectionality of the river flow.

We are now in a position to determine the demand and supply behavior of farmers in both markets. For farmer 1 and 2 we have the following net benefit functions:

$$\max_{w_i, w_{i,1}} [B_i(w_i) - q_1(w_{i1} - t_{i1}) + q_2 t_{i2}] \quad \text{s.t.} \quad w_i \leq w_{i,1} \tag{5.18}$$

where w_i is water consumption and w_{i1} is the use of water rights of farmer i in market 1. The difference $(w_{i1} - t_{i1})$ indicates the net position of farmer i. If it is positive (negative) she sells (buys) water rights in market 1. t_{i1} and t_{i2} are the respective water rights in both markets assigned to farmer i. Since both farmers cannot buy water rights to use for water consumption from market 2, they only have the option to sell their rights t_{i2}. Thus, we have included the revenue from these sales in the net benefit function. The demand function for each farmer follows from maximizing Eq. (5.18) with respect to $\{w_i, w_{i,1}\}$ subject to the constraint that water diversion cannot be more than water rights used. From the optimality conditions

$$B_i^{'}(w_i) - \lambda = 0 \tag{5.19}$$

$$-q_1 + \lambda = 0 \tag{5.20}$$

we can calculate the demand functions for the assumed specification of $B_i^{'} = a - bw_i$ which yields

$$\hat{w}_i(p_1) = \frac{a - q_1}{b} = 300 - q_1, \quad i = \{1, 2\} \tag{5.21}$$

Determining farmer 3's demand behavior is somewhat more comprehensive because farmer 3 is a buyer of water rights in both markets. She maximizes

$$[B_3(w_3) - q_1(w_{31} - t_{31}) - q_2(w_{32} - t_{32})], \quad \text{s.t.} \quad w_3 \leq w_{31} + w_{32} \tag{5.22}$$

with respect to $\{w_3, w_{31}, w_{32}\}$ where $w_{3,j}$ are water rights demanded and utilized in market j and t_{31} and t_{32} are water rights assigned in market 1 and 2 before trade takes place. Thus, we have the following assignments of water rights for all farmers and both markets.

$$t_{11} + t_{21} + t_{31} = (R - \bar{v}_1) \quad \text{and} \quad t_{12} + t_{22} + t_{32} = \bar{v}_1 \tag{5.23}$$

We assume that farmer 3 consumes water as well, i.e., $w_3 > 0$, and that he buys water rights from the second market, i.e., $w_{32} > 0$ but not from the first market.[8] The optimality conditions are

$$B_i^{'}(w_i) - \lambda = 0 \tag{5.24}$$

$$-q_1 + \lambda \leq 0 \tag{5.25}$$

$$-q_2 + \lambda = 0 \tag{5.26}$$

If the overall market equilibrium leads to water prices such that $q_1 > q_2$, then farmer 3 does not buy in the first market (Eq. (5.25) applies with strict inequality). The scenario is shown in Fig. 5.2.

[8]Subsequently, we will show why this scenario takes place with the assumed numerical values.

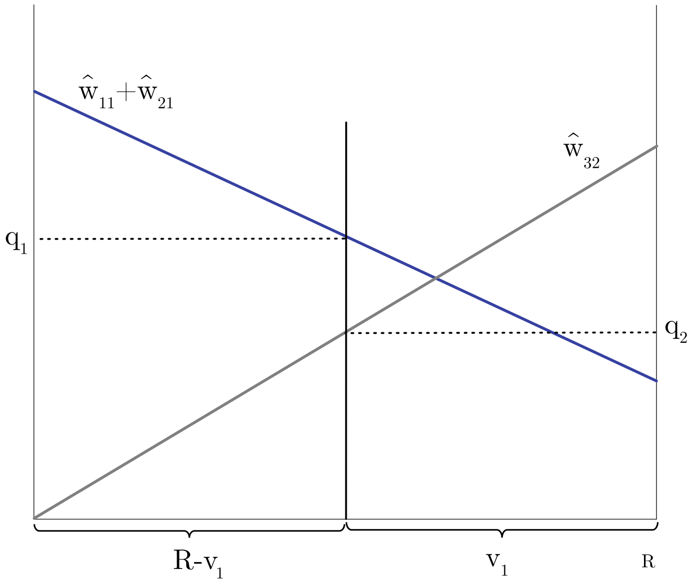

Fig. 5.2 Equilibrium of locational water markets. *Source* own illustration

Total water demand in the upper market is equal to $\hat{w}_{11}(q_1) + \hat{w}_{21}(q_1)$ since farmer 3 does not participate in this market, i.e., $w_{31} = 0$. This demand is equal to total water available in this stretch of the river, i.e., $R - \bar{v}_1 = 150$. The resulting equilibrium price is $q_1 = 225$.[9]

Similarly, the equilibrium of the second water market can be determined, i.e., $\hat{w}_{32}(q_2) = t_{12} + t_{22} + t_{32} = \bar{v}_1$. The resulting water price q_2 is lower than q_1.[10] One can see that our assumption has proven to be correct. If we insert the numerical values into the demand functions we get $\hat{w}_{11} = \hat{w}_{21} = (R - \bar{v}_1) = 150/2 = 75$ and $\hat{w}_{32} = \bar{v}_1 = 150$, which is identical to the optimal water allocation. Hence, to achieve the optimal allocation two separate markets are required.

One can see that the implementation of water markets has to be done with caution. If return flows or ecological concerns have to be taken into account, it is not enough to simply set up a water market for a catchment area. Rather, an institutionally complex system of interdependent markets must be established.

[9]Inserting the numerical values into the market equilibrium equation $2(a - q_1)/b = R - \bar{v}_1$ gives $2(300 - q_1) = 150$.

[10]The equilibrium price can be derived from the equilibrium condition in market 2 $(a - q_2)/b = \bar{v}_1$ which yields $q_2 = 150$.

Box 5.1 Water recovery management in the Murray–Darling Basin MDB

Australia is among the first countries that have implemented water markets. In particular, the Murray–Darling Basin (MDB) has been regulated by market-oriented instruments, i.e., a cap-and-trade approach, in recent decades. This experiment is assessed very differently in the literature, and it has been criticized, amongst others, as piracy, organized theft of water, and mismanagement. The background to this debate is the history of water reforms in the MDB over the last 50 years, which became necessary due to increasing drought and severe ecological damages. The Federal Government seized power under the Water Act 2007 which, in 2012, led to a ten-year basin plan specifying so-called sustainable diversion limits (SDLs). These were based on hydrological and ecological limits. The water level of a river should not fall below 2/3 of its natural height. The necessary restriction of water abstraction, however, would have led to a substantial loss of income for the agricultural sector and would have encountered much resistance. The government, therefore, decided to buy back water entitlements and grant subsidies for technical measures to increase irrigation efficiency: 2.5 bn. dollars were earmarked for the purchase of the water entitlements and 3.5 bn. dollars. dollars for the subsidy measures.

Critics considered the unilateral granting of water rights to agribusiness as theft. There were also other remarks:

- The implementation of the water plan was intransparent. The responsibilities were unclear. This led to a high loss of confidence in government action.
- There was a serious lack of monitoring of water consumption. The National Water Commission, which was responsible for the monitoring, fulfilled the implementation requirements only partly. Just about 70% of water consumption has been metered. One of the reasons for this is that return flows are difficult to calculate.
- Subsidies for the upgrade of the irrigation infrastructure have been rather inefficient. This is not astonishing. We know from the analysis of the rebound effect that price-oriented instruments (water price) can be more efficient than fostering water saving indirectly by subsidy schemes.

The example of the Murray River shows how difficult it is to implement a management model in practice which is functional from a theoretical point of view.

Sources: Grafton and Wheeler (2018), Grafton et al. (2019)

5.3 Water Entitlements and Water Allocations

As mentioned above, some jurisdictions such as Australia and California, have established water markets based on a cap-and-trade system. In a case in Australia, the water in a catchment area is divided into a consumptive pool and water for the environment. The latter is water which must not be withdrawn from the water cycle. The consumptive pool instead defines the water that can be privately owned. A single share of this consumptive pool is called water access entitlement. It is a perpetual or ongoing entitlement to exclusive access to the water of the respective catchment area. Notice, however, that this entitlement is defined in nominal volumes and does not imply a perpetual allocation of water of this amount. The actual volume of water allocated to an entitlement depends on the scarcity of available water in a given season. The level of water allocation thus depends on the seasonal conditions of the water cycle. As a rule, due to increasing water scarcity, the total of all annual allocations is now likely to be below 100% of the consumptive pool.

Water users can use different instruments to cover their water use. They can directly use the water assigned to their entitlements (water allocations), they can buy additional water allocations or sell part or all of their allocations. Or they buy or sell entitlements. Water rights are more long-term in nature. They entitle their holders in each period (season) to a certain allocation of water. Water rights represent an asset, such as shares in a company. Trading of entitlements is, therefore, also called permanent trading. Thus, it is not surprising that empirical studies of the water market have identified a distinct dependence of the entitlement price on interest rates. Also, the price for water entitlements is higher than the price for short-term water allocations because entitlements do not expire (although their actual water claims are subject to seasonal fluctuations).

In the following, we will analyze the relationship between the market for entitlements and the market for allocations in more detail. Since water entitlements are assets with long-lasting validity, the interrelations should be examined in a dynamic model context. However, we can also investigate the essential peculiarities in a simple, quasi-dynamic model.[11]

Let us introduce a representative water user. She derives benefit (or profits) from the seasonal water use. At the same time she has to decide how much water to use and how to handle her long-term entitlements, as well as her seasonal allocations. This can be summarized in the following approach:

$$\max_{w_t, N} \sum_{t=0}^{T} \beta^t \left\{ E[B(w_t) - \tilde{p}_t(w_t - \tilde{\alpha}_t N)] \right\} - qN \tag{5.27}$$

[11]Meant by this is a model which, although it has a multi-period planning horizon, does not apply dynamic optimization methods. The optimal demand for water allocations is determined for each period, while the demand for water rights is determined only at the beginning of the planning period (period 0).

where $E[..]$ is the expectation operator. w_t are the seasonal water allocations of the water user, $\tilde{p}_t$ is the price for allocated water in period t, N are the water entitlements that are bought at the beginning of the planning process in period 0. $\tilde{\alpha}_t N$ are the water allocations allotted in each period to the owner of entitlements. This portion is stochastic due to the seasonal weather conditions and their repercussions on the water cycle. If we define $\bar{\bar{W}}$ as the total entitlements, i.e., the size of the consumptive pool, we can define

$$\tilde{\alpha}_t = \frac{\tilde{W}_t}{\bar{\bar{W}}} \tag{5.28}$$

where $\tilde{W}_t$ is total water available in period t. Finally, we have the discount rate $\beta = 1/(1+r)$, where r is the interest rate. Discounting takes place because the water rights can be claimed in all periods.

The water user, e.g., a farmer, in the catchment area that is covered by the market system first chooses the water allocations per season she wants to buy. This leads to the usual optimality condition

$$B^{'}(w_t) = \tilde{p}_t \tag{5.29}$$

From this equation, the allocation demand $\tilde{w}_t = w_t(\tilde{p}_t)$ for each period can be derived. In our simple model, this demand does not depend on the decision with respect to water entitlements.[12] The decision on water consumption is independent of the ownership of water rights. If she needs more water than assigned to her by water allocations $\tilde{\alpha}_t N$, she buys additional water. In the reversed case, she sells part of her water allocations. The question remains as to how many water entitlements should be bought or sold. We have taken the long-term nature of this decision into account in our model by making this decision ex ante, i.e., before the realization of the actual water allocations are known. If we derive Eq. (5.27), with respect to N, we obtain

$$\Pi E[\tilde{p}_t \tilde{\alpha}_t] - q \tag{5.30}$$

where $\Pi = ((1+r)^T - 1)/(r(1+r)^T)$.[13] We see that the objective function is linear in N. This is because the participant in the water market only looks at average values. She does not assess the risk herself. Whether the volatility of allocations and prices is high or low is irrelevant for the valuation of water rights. In reality, the risk should play a role in the decision to buy or sell water entitlements, but to keep the calculations simple, we ignore it here.

If a market equilibrium exists, Eq. (5.30) must be equal to zero. Otherwise, the market participants could materialize arbitrage gains. Profits are made by either

[12] We have assumed that the water user is risk neutral, i.e., she does not care about the riskiness of her decision.

[13] On average, each period produces the same profit. This makes it possible to write the discounting formula more compactly. The derivation can be found in any introductory textbook on financial economics $\sum_{t=0}^{T} \beta^t = ((1+r)^T - 1)/(r(1+r)^T)$. This expression is greater than 1 for all $t > 1$.

selling and buying back, or purchasing and reselling entitlements. Thus, utilizing Eq. (5.28) leads to

$$\Pi[\tilde{p}_t\tilde{\alpha}_t] - q = 0 \quad \Rightarrow \quad \frac{BE[\tilde{p}_t\tilde{W}_t]}{\bar{\bar{W}}} = q \tag{5.31}$$

For the economic interpretation it is instructive to rewrite this equation. In doing so, we make use of a simple factorization of covariances[14]

$$\Pi\left[\bar{p}\frac{\bar{W}}{\bar{\bar{W}}} + \frac{\text{cov}(\tilde{p}, \tilde{W})}{\bar{\bar{W}}}\right] = q \tag{5.32}$$

Equation (5.32) summarizes the essential relationships between the two markets in a compact way

- If one recalls the definition of Π, one sees that the relationship between the two prices depends on the interest rate r. Since $\bar{p}$, $\bar{W}$, and the covariance are determined solely in the market for allocations, i.e., are exogenous to the entitlement market, the interest rate affects q alone. Assume that the planning horizon is infinite, then $\Pi = 1/r$. It is intuitive that with rising r the price q decreases and vice versa. This is exactly what empirical studies have shown and it is rather plausible. We know that this inverse relationship is observable in the stock market. High interest rates decrease the value of shares and vice versa.
- When comparing the time series of both prices, it becomes evident that q is greater than p. This is because the water entitlements are assets, while the water allocation is only valid for one period.
- Without discounting, the average allocation price $\bar{p}$ would be higher than q. This can be seen from the expression in square brackets in Eq. (5.32). First of all, $\bar{W}/\bar{\bar{W}}$ is less than 1 because the average seasonal allocation is less than the consumption pool. Also, the covariance is negative because the price and the seasonal supply of allocations are negatively related. If the allocation is high, the price is low and vice versa. Therefore, for B = 1 it holds that $\bar{p} > q$. That is plausible. The average supply of water allocations is less than the amount of water entitlements ($\bar{W} < \bar{\bar{W}}$). On average, water rights cannot be converted 100% into water allocations due to water scarcity.
- It is also interesting to note that the price difference between q and $\bar{p}$ decreases with increasing variability (covariance) in the allocation market because the variability leads to a devaluation of water entitlements. This is not due to the valuation of the risk (we have assumed risk neutrality), but due to the fact that with higher volatility of $\tilde{W}$ the ownership of water rights must be worthless. If the negative

[14] The covariance of two stochastic variables $\tilde{x}$ and $\tilde{y}$ is defined as $\text{cov}(\tilde{x}, \tilde{y}) = E[(\tilde{x} - \bar{x})(\tilde{y} - \bar{y})]$ where $\bar{x} = E[\tilde{x}]$ and $\bar{y} = E[\tilde{y}]$ are the respective means. Multiplying yields $\text{cov}(\tilde{x}, \tilde{y}) = E[\tilde{x}\tilde{y}] - \bar{x}\bar{y}$.

correlation between the water allocations and prices increases in absolute value, an increase in allocations ($\tilde{\alpha}$) is countervailed by a sharp price decrease ($\tilde{p}$) and vice versa.

5.4 Exercises

Exercise 5.1 Optimal water allocation for the simple river basin model
We have chosen parameter values such that the unidirectionality of the river does not play a role, i.e., the allocation of water to farmer 1 and 2 is not constrained by Eq. (5.2). Thus, we can take the optimality conditions Eqs. (5.7)–(5.9) to calculate the optimal values and check whether they violate the constraint (5.2). Inserting $B_i^{'} = 300 - w_i$ and the numerical values $R = 300$ and $h = 0.5$ it follows from Eq. (5.7) that

$$(300 - w_1)/(1 - 0.5) = 300 - w_2 \tag{5.33}$$

From Eq. (5.8), we have $w_2 = w_3$ such that Eq. (5.9), can be written as

$$2w_2 = 300 - (1 - 0.5)w_1 \tag{5.34}$$

From Eqs. (5.33) and (5.34), it follows that $\{w_1^* = 200, w_2^* = 100\}$. If we insert these values in Eq. (5.2) we have $w_1^* + w_2^* = 300 \leq R = 300$. The optimal values do not violate the constraint. Hence, our assumption that $\lambda_1 = 0$ was plausible. Finally, we can calculate the optimal allocation for farmer 3 which is simply the return flow $0.5w_1^* = 100$.

Exercise 5.2 NGO intervention in the water market
There are some initiatives in the European carbon market to buy up CO_2 certificates and then cancel their validity. Of course, this strategy assumes that NGOs are allowed to participate in trading or have an accredited trader who makes purchases on their behalf in the market. We want to transfer this idea to a water market. We assume that a water market has been implemented in a water catchment area. The water authorities provide a fixed amount of water rights for purchase (water supply) that can be bought by the local economy (farmers, industry, municipalities). The members of a local NGO find that too many water rights have been emitted and decide to buy and cancel water rights on the basis of donations in the market.

We want to derive the water demand of the local economy from the usual approach of benefit maximization.

$$\max_w [B(w) - pw] \quad \Rightarrow \quad B'(w) = p \tag{5.35}$$

As in Sect. 5.3, we assume a quadratic benefit function. The demand function is, therefore, linear (see Eq. (5.11)).

$$w = (a - p)/b \tag{5.36}$$

From the NGO's point of view, the assessment of water use leads to environmental damage, which can be expressed by a damage function. From the NGO's point of view, the damage lies in the fact that the abstraction of water for economic and consumption purposes damages the local ecosystem. We summarize this assessment by a quadratic damage function, from which the demand for water rights can also be derived. We assume that the purchases are covered by donations.

$$\min_{v}[(D/2)(\bar{W}-v)^2+pv] \quad \Rightarrow \quad D(\bar{W}-v)=p \tag{5.37}$$

where $D>0$ is a constant, $\bar{W}$ the amount of water entitlements issued by the local water authority and v the water demand of the NGO. From Eq. (5.37), the water demand function of the NGO follows:

$$v=\bar{W}-p/D \tag{5.38}$$

Adding both demand functions to total demand and equating to the regulated water supply allows the calculation of the equilibrium price

$$\frac{a-p}{b}+\left[\bar{W}-p/D\right]=\bar{W} \quad \Rightarrow p^*=\frac{Da}{(D+b)} \tag{5.39}$$

The intervention of NGOs in the water market apparently leads to the fact that the water price is independent of the regulated supply of water rights. The NGOs react to every change in the water supply with compensatory purchases. This can be seen from Eq. (5.38). Thus, if NGOs are allowed access to the water market, they take over the political control of the water supply displacing the local authorities. This may be a problem from a democratic point of view. However, note that our model's result is only valid as long as the financing of the purchases is secured by donations. If their budgets are limited, the effective purchases might be less than v.

Exercise 5.3 Markets for entitlements and allocations
This problem is about calculating the prices for the market for water entitlements and for the market for water allocations. We assume that two identical farmers have water rights corresponding to the full amount of the water pool, say $\bar{\bar{W}}=60$. The benefit function of both farmers is identical and quadratic, so the first derivative is linear, $B_i^{'}(w_i)=a-bw_i$, whereby by assumption $a=615$ and $b=1$. From Eq. (5.29), the demand function follows

$$w(\tilde{p})=615-\tilde{p} \tag{5.40}$$

The equilibrium price can be determined by setting total demand equal to the seasonal water supply $\tilde{W}$

$$2w(\tilde{p})=2(615-\tilde{p})=\tilde{W} \quad \Rightarrow \quad \tilde{p}=615-\frac{1}{2}\tilde{W} \tag{5.41}$$

The average price is calculated by taking the expectation of both sides, yielding

$$\bar{p} = E[\tilde{p}] = 615 - \frac{1}{2}\bar{W} \tag{5.42}$$

where $\bar{W} = E[\tilde{W}]$. Let us assume that $\tilde{W}$ is independent and identically distributed, i.e., the probability density is identical for all $\tilde{W}$ and independent across all periods supported by a finite interval $I = [0, \bar{\bar{W}}]$, where $\bar{\bar{W}} = 60$. From statistics textbooks we know

$$\bar{W} = \frac{\bar{\bar{W}}}{2} \quad \text{and} \quad \mathrm{Var}[\tilde{W}] = \frac{\bar{\bar{W}}^2}{12} \tag{5.43}$$

where $Var[\tilde{W}]$ is the variance of the periodical water supply. Due to our assumption, the mean water allocation to both farmers is half of total entitlements leading to $\bar{\alpha} = (1/2)$ (See Eq. (5.28)).

Now we are able to calculate the average price for water allocations. From Eq. (5.42), it follows

$$\bar{p} = 615 - \frac{\bar{W}}{2} = 615 - \frac{\bar{\bar{W}}}{4} = 615 - 15 = 600 \tag{5.44}$$

In order to calculate the price for water rights, we have to determine the covariance in Eq. (5.32). Utilizing Eqs. (5.41) and (5.42), we have

$$\mathrm{cov}[\tilde{p}, \tilde{W}] = E\left[\left(615 - \frac{\tilde{W}}{2} - 615 + \frac{\bar{W}}{2}\right)(\tilde{W} - \bar{W})\right] = -\frac{1}{2}E[(\tilde{W} - \bar{W})^2] = -\frac{1}{2}\mathrm{Var}[\tilde{W}] \tag{5.45}$$

Inserting the numerical values yields $\mathrm{cov}[\tilde{p}, \tilde{W}] = -(1/2)60^2/12 = -3600/24 = -150$.

Assuming that the horizon T is infinite, we know that $\Pi = 1/r$. Taking $r = 0.1$ it is straightforward to calculate the entitlement price q. Simply insert the numerical values in Eq. (5.32). This yields

$$q = (1/0.1)[(1/2)\bar{p} - 150] = 10(600 \times (1/2) - 150) = 1500. \tag{5.46}$$

Due to the discount factor and the infinite planning horizon the price for water entitlements is much higher than the price for the seasonal water allocations.

5.5 Further Reading

The economic analysis of water markets started quite early at a time when water allocation did not follow economic criteria but was determined solely by ownership structures. Certainly, the increasing scarcity of water in many regions of the world has led to an increased focus on economic efficiency criteria in water allocation.

Olmstead and Stavins (2009) provides an overview in which the welfare effects of price-oriented allocations are compared to those of quantity allocations based on rights. The functioning of water markets is subject to certain conditions, which Endo et al. (2018) further specify. In establishing these conditions, they examine in which countries of the world markets could be introduced in principle. Some examples are presented in the volume (Easter and Huang 2014a). Australia provides the first experience with water markets, and Turral et al. (2005) and Grafton and Wheeler (2018) provide an overview about the evolution of the case of the Murray–Darling Basin. They also analyze the effects of a policy mix (water market, subsidies). Grafton and Wheeler (2018) and Grafton et al. (2019) examine further management approaches in Mexico, Tanzania, USA, and Vietnam.

In water markets, specific hydrological relationships must be taken into account. Griffin and Hsu (1993) have examined these interrelationships in detail within the framework of a market model. Return flows, in particular, are taken into account here. Ansink and Houba (2012) deal with competition problems. How do water markets allocate scarce water when the water supply along a river is monopolized? Finally, Wheeler et al. (2008) empirically investigate the determinants that explain the price difference between water allocations and water entitlements.

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6 Transboundary Water Resource Management

6.1 Water Rivalry, Agreements, and International Water Rights

There are 276 international river basins worldwide which stretch over two or more countries (De Stefano et al. 2012). About 40% of the world population lives in international river basins (Water 2008). A major issue in transboundary rivers arises when claims for water exceed the available water quantity. Therefore, rules and legal paradigms are required to prevent tensions between competing consumers.

There exist two extreme legal paradigms:

- First is the principle of Absolute Territorial Sovereignty (ATS). Every state has the right to abstract and use the water in the basin on the basis of a souvereign decision of the state within its territory. This approach favors the upstream country which is able to fully cover its claims as long as enough water is available in the river.
- Second, the principle of Absolute Territorial Integrity (ATI) concerns the allocation of water between two states which are ordered sequentially along the course of the river. In that case, the downstream country must not be negatively affected by the upstream actor. For scarce water resources in the river, a diversion of water by the upstream state may increase the shortage and, therefore, shrinks the availability of water for the downstream country. Thus, a negative impact would occur for the downstream state and hence the diversion of water by the upstream state would not be allowed under the ATI principle. The ATI principle, therefore, favors, if any, the downstream state.

Currently, these two extreme approaches are diametral to each other and are commonly rejected in international water policy. Therefore, the "Territorial Integration of All Basin States" (TIBS) as well as the approach of Limited Territorial Sovereignty (LTS) are compromises between the conflicting ATS and ATI principle.

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- The TIBS principle states that the water in the river is a common resource and any riparian has the right to divert an appropriate share regardless of its river position and its river inflow contribution.
- The LTS principle enables each riparian to use the water while any other riparian is not harmed by the usage. Due to its flexibility and its room for interpretation, this LTS principle is widely accepted in international water policy (Moes 2013).

International laws for allocating water of transboundary sources mainly developed in the second half of the twentieth century. These range from a multitude of bilateral contracts to a number of UN conventions which are valid at a global scale. In this textbook, we focus on the development of the most important conventions.[1] These contain the

- Helsinki Rules on the Use of International Rivers agreed upon in 1966;
- UN Convention on the Protection and Use of Transboundary Watercourses and International Lakes (1997);
- Berlin Rules on Water Resources from the year 2004.

The Helsinki rules were agreed upon at the 52nd Conference of the International Law Association (ILA) in August 1966, and they regulate the usage of transboundary rivers and their connected groundwaters. The Helsinki Rules consist of a total of 37 articles which are split into six chapters (International Law Association 1966). Articles 4 and 5 are most relevant for transboundary river management. Article 4 entitles any riparian to a reasonable and equitable share in the use of water,[2] while Article 5 defines the criteria to estimate this reasonable and equitable share of water usage. These criteria are, for instance, the geography and hydrology of the basin, past utilization, economic and social needs, and comparative costs of an alternative.[3] The Helsinki Rules were a quite important inspiration for the UN Convention on the

[1]For a more detailed overview of the rules, we recommend, for instance, Van Puymbroeck (2003).
[2]Article 4 of Helsinki Rules: "Each basin State is entitled, within its territory, to a reasonable and equitable share in the beneficial uses of the waters of an international drainage basin."
[3]Article 5 of Helsinki Rules:

1. What is a reasonable and equitable share within the meaning of Article 4 is to be determined in the light of all the relevant factors in each particular case.
2. Relevant factors which are to be considered include, but are not limited to
   (a) the geography of the basin, including, in particular, the extent of the drainage area in the territory of each basin State; (b) the hydrology of the basin, including, in particular, the contribution of water by each basin State; (c) the climate affecting the basin; (d) the past utilization of the waters of the basin, including, in particular, existing utilization; (e) the economic and social needs of each basin State; (f) the population dependent on the waters of the basin in each basin State; (g) the comparative costs of alternative means of satisfying the economic and social needs of each basin State; (h) the availability of other resources; (i) the avoidance of unnecessary waste in the utilization of waters of the basin; (j) the practicability of compensation to one or more of the co-basin States as a means of adjusting conflicts among uses; and (k) the degree to which the needs of a basin State may be satisfied, without causing substantial injury to a co-basin State.

Protection and Use of Transboundary Watercourses and International Lakes in 1997, and were superseded by the Berlin Rules on Water Resources in the year 2004.

On May 21, 1997, the General Assembly of the UN passed the Law of the Non-Navigational Uses of International Watercourses, which is also known as the UN Watercourses Convention.[4] Until the present, it is the only treaty under international law with global validation which rules the non-navigational usage of international water sources including both surface and groundwater (Wehling 2018). It mainly aims to further the optimal and sustainable usage as well as to ensure the development and conservation of international water sources (Salman 2007). Because of their wide acceptance, the former explained principle of Limited Territorial Sovereignty (LTS) is incorporated in the convention. Based on Article 5, the water utilization has to be equitable and reasonable. The factors for such an equitable and reasonable usage are listed in Article 6 of the convention. These factors are, for instance, natural characteristics such as geographical and hydrological conditions, social and economic needs as well as the population dependent on the watercourse. Furthermore, the following articles oblige the riparian states to take appropriate measures to prevent significant harm to other watercourse states, to cooperate with each other on the basis of sovereign equality, territorial integrity, mutual benefit, and good faith, as well as to a regular exchange of available data and information on the condition of the watercourse (e.g., hydrological, meteorological, and water quality conditions).

The Berlin Rules on Water Resources—which replaced the Helsinki Rules—were passed at the 71st Conference of the International Law Association (ILA), August 21, 2004. While the Helsinki Rules and the UN Convention established the right for each riparian state to a reasonable and equitable share, the Berlin Rules emphasize the obligation to manage the shared watercourse in an equitable and reasonable manner (Salman 2007). The term manage is specified in Article 3 (14) of the Berlin Rules and contains the development, use, protection, allocation, regulation, and control of the waters. In contrast to the former principles, the Berlin rules are not only valid for international watercourses, but also relevant for national water sources. Furthermore, the Berlin Rules have downgraded the principle of equitable and reasonable utilization and have equated this principle with the obligation of not causing significant harm to other riparians (Salman 2007; Bourne 2004).

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3. The weight to be given to each factor is to be determined by its importance in comparison with that of other relevant factors. In determining what is a reasonable and equitable share, all relevant factors are to be considered together and a conclusion is reached on the basis of the whole.

[4]This convention entered into force when the minimum quorum for ratification was reached on August 17, 2014, after Vietnam signed the ratification document as the 35th state.

6.2 Benefit Sharing Between Two Riparians

6.2.1 Principles of Benefit Sharing

In Sect. 3.7, we have analyzed the IWRM approach for water allocation along rivers in which the generated benefit in the entire basin was maximized. However, this analysis ignores the fact that generated benefits could be arbitrarily assigned between the riparians by realizing side payments between the riparians. This question about an efficient and incentive-compatible assignment of the basin's benefit is the main focus of the benefit sharing problem.[5]

In this section, we focus on the case with just two riparians at an international water body. This is quite common, as the majority of international rivers are shared by just two riparian states (De Stefano et al. 2012). There exist two possible cooperation scenarios in such a basin:

- Either the riparians act unilaterally in a noncooperative way where they maximize their individual benefit from water usage,
- or they form a joint arrangement where they act in a cooperative manner which means that both riparians consume the water in such a way that the common benefit in the entire basin is maximized.[6]

If both states act in a noncooperative manner, country 1 diverts the amount w_1^{NC} and generates the benefit $B_1(w_1^{NC})$, while country 2 receives the water amount w_2^{NC} and generates the benefit $B_2(w_2^{NC})$. Thus, the benefit generated in the entire basin is $B_1(w_1^{NC}) + B_2(w_2^{NC})$.

However, if both riparians make an agreement where they act and share the water in a cooperative manner, we assume that states 1 and 2 receive the water amount w_1^C and w_2^C, respectively. The resulting benefit in the entire basin is $B_1(w_1^C) + B_2(w_2^C)$. The cooperation gain Δ is the additionally generated benefit in the entire basin compared to the noncooperation scenario (see (6.1)):

$$\Delta = B_1(w_1^C) + B_2(w_2^C) - B_1(w_1^{NC}) - B_2(w_2^{NC}) \tag{6.1}$$

From an economic perspective, there is only an incentive for forming a joint arrangement if the cooperation gain is positive ($\Delta > 0$), which means

$$B_1(w_1^C) + B_2(w_2^C) > B_1(w_1^{NC}) + B_2(w_2^{NC}) \tag{6.2}$$

The generated benefit from consumption $B_1(w_1^C)$ and $B_2(w_2^C)$ results from the optimal water allocation in the joint arrangement. However, the assignment of benefit to

[5]In this context, incentive compatible means that each riparian has an incentive for realizing the social-optimal solution.

[6]A joint arrangement is only achievable if both riparians are willing to form a joint arrangement where they cooperate. However otherwise, if one or both riparians do not want to form a joint arrangement, both riparians act unilaterally in a noncooperative way.

the riparians is the focus of benefit sharing problems. The benefit of each riparian is not only affected by the benefit from consumption, but also by side payments paid or received.[7]

The benefit of the riparians 1 and 2 in a joint arrangement which are represented by the variables x_1 and x_2 results, therefore, from the benefit of consumption ($B_1(w_1^C)$ and $B_2(w_2^C)$) and the level of the side payments, with $sp_{1,2}$ representing the side payments made by riparian 1, while $sp_{2,1}$ stands for the side payments made by riparian 2:

$$\begin{aligned} x_1 &= B_1(w_1^C) + sp_{2,1} - sp_{1,2} \\ x_2 &= B_2(w_2^C) + sp_{1,2} - sp_{2,1} \end{aligned} \tag{6.3}$$

Of course the assignment of benefits to the riparians has to be equal to the generated benefit in the joint arrangement which is represented as (6.4)

$$x_1 + x_2 = B_1(w_1^C) + B_2(w_2^C) \tag{6.4}$$

A solution in which the sum of the assigned benefits exceeds the total generated benefits in the joint arrangement ($x_1 + x_2 > B_1(w_1^C) + B_2(w_2^C)$) is not realizable and would therefore violate the feasibility condition. However, the contrary case in which the sum of assigned benefits falls below the total generated benefits ($x_1 + x_2 < B_1(w_1^C) + B_2(w_2^C)$) is Pareto-inefficient and would therefore violate the Pareto-efficiency condition. The determination of the assigned benefits to the riparians in a joint arrangement is the main focus of the benefit sharing problem.

If we assume that side payments are made by just one riparian, it is possible to derive from Eq. (6.3) that riparian 1 has to make side payments if its assigned benefit x_1 falls below its benefit from consumption $B_1(w_1^C)$, while similarly, riparian 2 has to make side payments if its assigned benefit x_2 falls below its benefit from consumption $B_2(w_2^C)$. The level of the side payments results from the difference between the assigned benefits (x_1 and x_2) and the benefit from consumption ($B_1(w_1^C)$ and $B_2(w_2^C)$) (see Eq. 6.5).

$$\begin{aligned} &\text{If: } x_1 < B_1(w_1^C) \Leftrightarrow x_2 > B_2(w_2^C) \text{ then: } sp_{1,2} = B_1(w_1^C) - x_1 = x_2 - B_2(w_2^C) \\ &\text{If: } x_1 > B_1(w_1^C) \Leftrightarrow x_2 < B_2(w_2^C) \text{ then: } sp_{2,1} = x_1 - B_1(w_1^C) = B_2(w_2^C) - x_2 \end{aligned} \tag{6.5}$$

In a cooperative arrangement, the riparians have to receive at least the benefit which they would have gained if they acted unilaterally. This requirement is also known as individual rationality, which has the following algebraic formulation:

$$\begin{aligned} x_1 &\geq B_1(w_1^{NC}) \\ x_2 &\geq B_2(w_2^{NC}) \end{aligned} \tag{6.6}$$

[7]Side payments are payment transactions between the riparians; the side payment is beneficial for the receiving riparian, while it is a financial burden for the paying one.

A riparian whose individual rationality condition is not met has an incentive to leave the joint arrangement and act in a noncooperative way.

Hence, due to the feasibility and Pareto-optimality conditions (see Eq. (6.4)) as well as the individual rationality condition (see Eq. (6.6)), sharing of the cooperation gain Δ is the main focus of the benefit sharing problem with two riparians.

6.2.2 UID, DID and the Shapley Solution

There are two extreme solutions for assigning the cooperation gain to the riparians, either the first or the second riparian receives the entire cooperation gain. Assuming riparian 1 is the upstream and riparian 2 is the downstream riparian, it is possible to distinguish between two extreme scenarios regarding the allocation of the cooperation gain:

- **Upstream incremental distribution (UID)**: When applying the UID approach, the cooperation gain is completely assigned to the upstream riparian 1, while the downstream riparian 2 just gets enough benefit to meet its individual rationality condition:

$$\begin{aligned} x_1^{UID} &= B_1(w_1^{NC}) + \Delta \\ x_2^{UID} &= B_2(w_2^{NC}) \end{aligned} \tag{6.7}$$

  If we assume that either riparian 1 makes side payments ($sp_{1,2} > 0 \wedge sp_{2,1} = 0$) which is the case if $x_1^{UID} < B_1(w_1^C)$ or riparian 2 makes side payments ($sp_{2,1} > 0 \wedge sp_{1,2} = 0$) which is the case if $x_2^{UID} < B_2(w_1^C)$, it is possible to determine the level of side payments:

$$sp_{1,2} = \begin{cases} B_1(w_1^C) - B_1(w_1^{NC}) - \Delta = B_2(w_2^{NC}) - B_2(w_2^C) & \text{if: } x_1^{UID} < B_1(w_1^C) \Leftrightarrow x_2^{UID} > B_2(w_2^C) \\ 0 & \text{if: } x_1^{UID} \geq B_1(w_1^C) \Leftrightarrow x_2^{UID} \leq B_2(w_2^C) \end{cases}$$
$$p_{2,1} = \begin{cases} B_1(w_1^{NC}) + \Delta - B_1(w_1^C) = B_2(w_2^C) - B_2(w_2^{NC}) & \text{if: } x_1^{UID} > B_1(w_1^C) \Leftrightarrow x_2^{UID} < B_2(w_2^C) \\ 0 & \text{if: } x_1^{UID} \leq B_1(w_1^C) \Leftrightarrow x_2^{UID} \geq B_2(w_2^C) \end{cases} \tag{6.8}$$

- **Downstream incremental distribution (DID)**: In the DID approach, the cooperation gain Δ is completely assigned to the downstream riparian 2. The upstream riparian 1 receives benefit, such that its individual rationality condition is fulfilled.

$$\begin{aligned} x_1^{DID} &= B_1(w_1^{NC}) \\ x_2^{DID} &= B_2(w_2^{NC}) + \Delta \end{aligned} \tag{6.9}$$

Hence, the following side payments result:

$$p_{1,2} = \begin{cases} B_1(w_1^C) - B_1(w_1^{NC}) = B_2(w_2^{NC}) + \Delta - B_2(w_2^C) & \text{if: } x_1^{DID} < B_1(w_1^C) \Leftrightarrow x_2^{DID} > B_2(w_2^C) \\ 0 & \text{if: } x_1^{DID} \geq B_1(w_1^C) \Leftrightarrow x_2^{DID} \leq B_2(w_2^C) \end{cases}$$
$$p_{2,1} = \begin{cases} B_1(w_1^{NC}) - B_1(w_1^C) = B_2(w_2^C) - B_2(w_2^{NC}) - \Delta & \text{if: } x_1^{DID} > B_1(w_1^C) \Leftrightarrow x_2^{DID} < B_2(w_2^C) \\ 0 & \text{if: } x_1^{DID} \leq B_1(w_1^C) \Leftrightarrow x_2^{DID} \geq B_2(w_2^C) \end{cases} \quad (6.10)$$

Based on these two extreme cases, it is possible to find all possible realizations for x_1 and x_2 for the benefit sharing problem based on the linear combination of the extreme scenarios (see Eq. 6.11):

$$\begin{aligned} x_1 = \beta \cdot x_1^{UID} + (1-\beta) \cdot x_1^{DID} &\Leftrightarrow x_1 = B_1(w_1^{NC}) + \beta \cdot \Delta \\ x_2 = \beta \cdot x_2^{UID} + (1-\beta) \cdot x_2^{DID} &\Leftrightarrow x_2 = B_2(w_2^{NC}) + (1-\beta) \cdot \Delta \\ &\text{with: } 0 \leq \beta \leq 1 \end{aligned} \quad (6.11)$$

The value of parameter β is defined within the range [0, 1]. It becomes obvious from Eq. 6.11 that we get the UID or DID solution if β is set equal to 0 or 1, respectively. The higher the value of β, the more advantageous the benefit sharing solution for the upstream user 1, while the profit for the downstream user 2 raises with a decreasing value of β.

A further specific case is the determination of β with $\beta = 0.5$ which means that each riparian receives half of the cooperation gain. This solution results from applying the Shapley value approach for the case with two riparians (Shapley 1953). Therefore, this could be termed the Shapley solution. The Shapley solutions for x_1^{SH} and x_2^{SH} are

$$\begin{aligned} x_1^{SH} &= B_1(w_1^{NC}) + 0.5 \cdot \Delta \\ x_2^{SH} &= B_2(w_2^{NC}) + 0.5 \cdot \Delta \end{aligned} \quad (6.12)$$

For this Shapley solution, the following side payments result on the basis of Eqs. 6.5 and 6.12:

$$sp_{1,2} = \begin{cases} B_1(w_1^C) - B_1(w_1^{NC}) - 0.5 \cdot \Delta = B_2(w_2^{NC}) + 0.5 \cdot \Delta - B_2(w_2^C) & \text{if: } x_1^{SH} < B_1(w_1^C) \Leftrightarrow x_2^{SH} > B_2(w_2^C) \\ 0 & \text{if: } x_1^{SH} \geq B_1(w_1^C) \Leftrightarrow x_2^{SH} \leq B_2(w_2^C) \end{cases}$$
$$sp_{2,1} = \begin{cases} B_1(w_1^{NC}) - B_1(w_1^C) + 0.5 \cdot \Delta = B_2(w_2^C) - B_2(w_2^{NC}) - 0.5 \cdot \Delta & \text{if: } x_1^{SH} > B_1(w_1^C) \Leftrightarrow x_2^{SH} < B_2(w_2^C) \\ 0 & \text{if: } x_1^{SH} \leq B_1(w_1^C) \Leftrightarrow x_2^{SH} \geq B_2(w_2^C) \end{cases} \quad (6.13)$$

The benefit sharing problem in a basin with two riparians is also illustrated by Fig. 6.1. We draw riparian 1's assigned benefit (x_1) on the horizontal axis (abscissa), while the benefit of riparian 2 is illustrated on the vertical axis (ordinate). The benefits of the riparians 1 and 2 when acting unilaterally in a noncooperative way ($B_1(w_1^{NC})$ and $B_2(w_2^{NC})$) are pictured in this graph by the vertical and horizontal functions, respectively. In this diagram, these two functions have the algebraic expression: $x_1 = B_1(w_1^{NC})$ and $x_2 = B_2(w_2^{NC})$. The benefit generated in the basin when both riparians

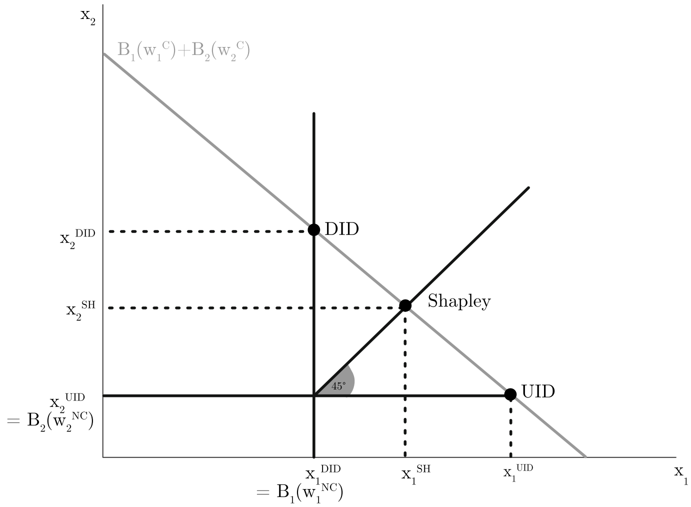


Fig. 6.1 Benefit sharing in a basin with two riparians. *Source* own illustration

form a joint arrangement is illustrated by the monotonous-decreasing diagonal line. In this diagram, the function has the algebraic expression: $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ due to Eq. 6.4. This means the higher the assignment of benefits to riparian 1, the lower the assignment to riparian 2 and vice versa. For meeting the feasibility and Pareto-efficiency conditions, the benefit sharing solution has to be located on the function.[8]

The function $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ intersects with the horizontal and vertical axes at the level $B_1(w_1^C) + B_2(w_2^C)$. These points can be interpreted as the full assignment of the basin's benefit to just one riparian. Of course, these two benefit sharing solutions would meet the feasibility and Pareto-efficiency conditions, but would fail the individual rationality condition which states that the assignment of benefits to each riparian must be as high as the benefit the riparians would generate if they acted unilaterally in a noncooperative manner, which means $x_1 \geq B_1(w_1^{NC})$ and $x_2 \geq B_2(w_2^{NC})$ (see Eq. 6.6). Therefore, for meeting the individual rationality condition, the UID solution with $x_1 = x_1^{UID} = B_1(w_1^{NC}) + \Delta$ and $x_2 = x_2^{UID} = B_2(w_2^{NC})$ limits the assigned benefit to the upstream riparian 1 to the maximum level $B_1(w_1^{NC}) + \Delta$, while the DID solution with $x_1 = x_1^{DID} = B_1(w_1^{NC})$ and $x_2 = x_2^{DID} = B_2(w_2^{NC}) + \Delta$ determines the maximum possible assigned benefit for riparian 2 to the level $B_2(w_2^{NC}) + \Delta$. Therefore, the solutions which are on

[8]A solution which is located in between the area which is spanned by the axis and the function $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ would fail the Pareto-efficiency condition, because fewer benefits are allocated than generated, while a solution beyond the function $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ would fail the feasibility condition, because more benefits are allocated to the riparians than generated.

the function $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ and which are in between the UID and DID solution, which means $B_1(w_1^{NC}) \leq x_1 \leq B_1(w_1^{NC}) + \Delta$ and $B_2(w_2^{NC}) \leq x_2 \leq B_2(w_2^{NC}) + \Delta$, form the set of solutions for the benefit sharing problem with two riparians.

A specific focal point solution of this benefit sharing problem is the Shapley solution with $x_1 = x_1^{SH} = B_1(w_1^{NC}) + 0.5 \cdot \Delta$ and $x_2 = x_2^{SH} = B_2(w_2^{NC}) + 0.5 \cdot \Delta$. This solution could be found in Fig. 6.1 at the intersection point between the monotonous-decreasing diagonal function $x_2 = B_1(w_1^C) + B_2(w_2^C) - x_1$ and the 45°-degree line whose origin is at the intersection of the vertical and horizontal lines that represent the benefit for noncooperative acting riparians $x_1 = B_1(w_1^{NC})$ and $x_2 = B_2(w_1^{NC})$.

6.3 Benefit Sharing Between More Than Two Riparians

In this section, we focus on methods to allocate the generated benefits in a basin with more than two riparians.[9]

6.3.1 Model of a River Basin

6.3.1.1 Superadditivity Condition

If the solution of the allocation problem is beneficial for all riparians, they have an incentive to form a cooperation scheme where they decide together about water management plans, water allocation strategies, and the allotment of benefits. An important condition for forming joint arrangements is the superadditivity of benefits which can be expressed by the following relation:

$$V(S) + V(T) \leq V(S \cup T) \text{ with: } S \cap T = \emptyset \qquad (6.14)$$

The sets S and T stand for coalitions where participants, which are represented by elements of these sets, act in a cooperative manner. The sets could contain just one element or a multitude of elements. If the set S or T contains just one element, the corresponding riparian is not participating in a joint arrangement and, therefore, acts unilaterally in a noncooperative manner. However, if there are a multitude of elements in the set, the corresponding riparians act together in a sub-coalition. For the analysis of superadditivity, one riparian cannot be part of both sets S and T, hence $S \cap T = \emptyset$. In the case of a grand coalition, all riparians act in one joint arrangement, which means that the union set $S \cup T$ would contain all riparians.[10] The $V(\ldots)$ stands for the benefit which can be generated in the respective coalition, therefore, it is the

[9]The chapter-annex (Sect. 6.9) provides a full account to the mathematical derivations.

[10]The grand coalition cannot be represented by S or T, while a unilaterally acting player can not be represented by $S \cup T$. Sub-coalitions can be represented by S, T as well as by $S \cup T$.

value of the coalition. The superadditivity condition (6.14) states that the value of a joint arrangement between the sub-coalitions S and T must be at least as high as the sum of values of arrangements S and T.

If the superadditivity condition is fulfilled, the additional benefit due to the joining of S and T to a mutual arrangement can be expressed by the following equation:

$$V(S \cup T) - V(S) - V(T) \tag{6.15}$$

If the superadditivity condition holds for all cooperation arrangements, the highest benefits for the entire basin can be generated by forming the grand coalition.[11] Finally, for finding adequate solutions for sharing the benefit between the riparians of a coalition, methods from cooperative game theory can be applied.[12]

6.3.1.2 General Procedure for Solving a Benefit Sharing Problem

For solving the benefit sharing problem within a river basin, the first step is to set up a model of the river containing the riparians, their benefit and cost functions as well as the main hydrological conditions such as natural external inflows into the river and the flow direction. Afterwards, various options for cooperation in the river basin are addressed. Every riparian has the option either to cooperate and form an arrangement with other riparians, or not to cooperate and act unilaterally. Therefore, it is usually possible to form either one joint arrangement (grand coalition), or to arrange different forms of sub-coalitions between selected riparians. Further, it is also possible that no arrangement occurs in the basin, so that every riparian acts unilaterally. These different options of cooperation are quantified by calculating the joint benefit within the cooperation arrangement and by finding the individual benefit of each unilaterally acting riparian for any cooperation scenario. Finally, it is important to find a way to allocate the benefit generated in a joint arrangement between the participating riparians.

For this benefit sharing issue, various techniques from cooperative game theory can be applied. In this context, we focus on the concept of the core which gives a set of possible solutions to the question of how to share the benefit of a joint arrangement between its member riparians. Based on this, the bargaining power of each riparian can be found. While the core gives a set of possible solutions, there also

[11] In this textbook, we focus on sharing benefits. However, if there is a cost game (e.g., realizing a dam project under different coalition scenarios), the subadditivity condition has to be fulfilled which can be expressed by the following formulation:

$$C(S) + C(T) \geq C(S \cup T) \text{ with: } S \cap T = \emptyset$$

The $C(\ldots)$ represents the cost for the related coalition. The subadditivity condition means that the cost under a cooperative arrangement between coalition S and T must not exceed the sum of costs if coalition S and T act separately. If this subadditivity condition is fulfilled for all coalition scenarios, the lowest cost is caused by forming the grand coalition.

[12] In contrast to the concepts of noncooperative game theory which are more popular in the economic literature, interaction and payments between the relevant actors are possible in cooperative game theory.

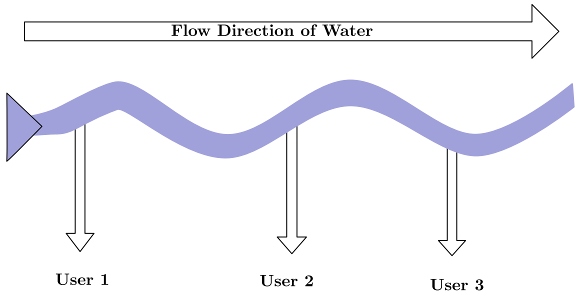


Fig. 6.2 Network of a hypothetical river basin. *Source* own illustration

exist focal point solution concepts for calculating concrete results for sharing the benefit: We focus on the concept of the Shapley value, the Nash-Harsanyi solution, and the nucleolus. The Shapley value allocates the benefits according to the strength of each player in the joint arrangement, while the Nash-Harsanyi solution maximizes the additional utility from cooperation in a joint arrangement compared to the non-cooperative case for each riparian. The nucleolus is a procedure for minimizing the maximum objection against the benefit sharing solution. Finally, parameters indicating the acceptability of a benefit sharing solution can be calculated. Nonacceptance of a solution results if one riparian views its payoff as unfair against the payoff of other players in the coalition. The higher the nonacceptance, the higher the risk that the unsatisfied player will leave the coalition.

The procedure which was explained for the general case above is now applied to a hypothetical river basin whose water is shared by three riparians. We assume a river basin (see Fig. 6.2) which consists of upstream, midstream, and downstream riparians, represented by index i with $i = \{1, 2, 3\}$. The river is fed by an inflow upstream of the first user; there are no other external inflows. By consuming the water, the riparians generate benefits. Assuming a constant marginal benefit, the benefit increases linearly with increasing consumption. If the available water amounts are consumed completely by the upstream or midstream or downstream user, the generated benefit is α or β or γ, respectively. Furthermore, we suppose the relation $\alpha < \beta < \gamma$, which means that the upstream riparian is the least productive one while the downstream user is the most productive one. Backflows to the river after withdrawing and consuming do not occur for this hypothetical case, hence the consumption of water is completely rivalrous. Furthermore, there are no limitations regarding the access to and extraction of water.

Options of Cooperation

For the hypothetical river network, there are different options of cooperation:

- All riparians act unilaterally which means that an arrangement in the river basin does not exist. The set which states the occurring coalition is, therefore, an empty set, $\emptyset$. The unilateral acting of riparians is stated with sets which just contain one element. This means if riparians 1, 2, and 3 act unilaterally, this is stated by the formulation $\{1\}$, $\{2\}$, and $\{3\}$, respectively.

Table 6.1 Generated benefits for different cooperation scenarios

Coalition	Benefit for ...							
	Non-cooperating			Coalition				Entire Basin
	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}	{1, 2, 3}	
∅	α	0	0	–	–	–	-	α
{1, 2}	–	–	0	β	–	–	–	β
{1, 3}	–	0	–	–	α	–	–	α
{2, 3}	α	–	–	–	–	0	–	α
{1, 2, 3}	–	–	–	–	–	–	γ	γ

- Two of three actors cooperate with each other. Hence, the following arrangements between the riparians are possible: {1, 2}, {1, 3}, and {2, 3}. The residual riparian that is not a member of the coalition acts unilaterally.
- All actors cooperate in one joint arrangement and thus form a grand coalition, which is symbolized by {1, 2, 3}.

Value of Coalitions

For the different options of cooperations, different levels of generated benefits can be found which are summarized in Table 6.1.

If no arrangement is formed between the riparians, the available water is completely consumed by the upstream user who is able to generate a benefit α. Further, downstream users do not receive any amount of water from the river and therefore cannot generate any benefit. The same water consumption pattern in the basin is also observable, if the mid- and downstream riparians form a coalition {2, 3} without the upstream user.[13] However, if the up- and downstream users arrange a cooperation without the midstream riparian, {1, 3}, there seems to be an incentive for the upstream one to leave water in the river, because the downstream riparian could generate the highest benefit for the coalition. However, the intermediate midstream riparian would seize the full amount of water, which is also known as leakage. (Ansink et al. 2012) This reaction of the midstream is anticipated by the upstream, hence the upstream would withdraw the total amount of water to maximize the benefit for the formed coalition. The generated benefit for the coalition would be α, while the midstream does not receive any amount of water.

If all the riparians form a joint cooperative arrangement, {1, 2, 3}, the upstream and midstream leave the total amount of water in the river, hence the downstream user is the only one who uses the water and is able to generate a benefit of γ for the grand coalition.

[13]This means that for the coalition {2, 3}, the same situation occurs as for the case that all riparians act unilaterally. The total amount of water is consumed by the upstream user who generates a benefit of α. The coalition of the mid- and downstream users {2, 3} does not receive any amount of water and gets a benefit of 0.

Superadditivity Conditions
For the presented case, the superadditivity condition holds for all cooperation scenarios:

$$\underbrace{V(\{1\})}_{=\alpha} + \underbrace{V(\{2\})}_{=0} \leq \underbrace{V(\{1,2\})}_{=\beta} \qquad \rightarrow \alpha \leq \beta \checkmark$$

$$\underbrace{V(\{1\})}_{=\alpha} + \underbrace{V(\{3\})}_{=0} \leq \underbrace{V(\{1,3\})}_{=\alpha} \qquad \rightarrow \alpha \leq \alpha \checkmark$$

$$\underbrace{V(\{2\})}_{=0} + \underbrace{V(\{3\})}_{=0} \leq \underbrace{V(\{2,3\})}_{=0} \qquad \rightarrow 0 \leq 0 \checkmark$$

$$\underbrace{V(\{1\})}_{=\alpha} + \underbrace{V(\{2\})}_{=0} + \underbrace{V(\{3\})}_{=0} \leq \underbrace{V(\{1,2,3\})}_{=\gamma} \qquad \rightarrow \alpha \leq \gamma \checkmark$$

$$\underbrace{V(\{1,2\})}_{=\beta} + \underbrace{V(\{3\})}_{=0} \leq \underbrace{V(\{1,2,3\})}_{=\gamma} \qquad \rightarrow \beta \leq \gamma \checkmark$$

$$\underbrace{V(\{1,3\})}_{=\alpha} + \underbrace{V(\{2\})}_{=0} \leq \underbrace{V(\{1,2,3\})}_{=\gamma} \qquad \rightarrow \alpha \leq \gamma \checkmark$$

$$\underbrace{V(\{1\})}_{=\alpha} + \underbrace{V(\{2,3\})}_{=0} \leq \underbrace{V(\{1,2,3\})}_{=\gamma} \qquad \rightarrow \alpha \leq \gamma \checkmark$$

Due to this fulfillment of the superadditivity condition, the grand coalition is the best option in order to maximize the total benefit in the entire basin. The result is also visible from the rightmost column of Table 6.1, which illustrates the aggregated benefit in the river basin for all possible cooperation scenarios.

Sharing the Benefits
After forming the grand coalition, the generated benefit with the level of γ has to be shared between all riparians. For this bargaining problem, different methods from cooperative game theory are available such as the core or various focal point solutions, for instance, the Shapley value, the Nash-Harsanyi solution as well as the nucleolus. For this analysis, the sets I, S, and G are defined. I contains as set elements those riparians which act unilaterally in a noncooperative way. Therefore, the value of the characteristic function for the set I, being $V(I)$, is based either on the benefits for the non-cooperating case in the entire river basin (see line Ø of Tables 6.1 and 6.2) or on the minimum benefit which is gained under all coalition scenarios in which the relevant riparian is not a member (see Table 6.2).

Regardless of the applied approach, both procedures result in the same solution for this example. In the unilateral acting case, riparian 1 generates a benefit of α, while riparians 2 and 3 are not able to divert any water amounts and generate, therefore, a benefit of 0. The set S represents all possible sub-coalitions, which means that a cooperative scheme is formed between 2 or more riparians but does not contain all riparians of the basin. For the presented example, the three tuples $\{1, 2\}$, $\{1, 3\}$,

Table 6.2 Generated benefits of unilaterally acting riparians for different cooperation scenarios

Coalition	Benefit for ...		
	Non-cooperating		
	{1}	{2}	{3}
Ø	α	0	0
{1, 2}	–	–	0
{1, 3}	–	0	–
{2, 3}	α	–	–
MINIMUM	$\boldsymbol{\alpha}$	**0**	**0**

Table 6.3 Value of cooperations

$V(I)$			$V(S)$			$V(G)$
$V(\{1\})$	$V(\{2\})$	$V(\{3\})$	$V(\{1,2\})$	$V(\{1,3\})$	$V(\{2,3\})$	$V(\{1,2,3\})$
α	0	0	β	α	0	γ

and {2, 3} form the set S. However, the set G stands for the grand coalition, which means that all riparians of the basin form a coalition. For the presented example, the riparians 1, 2, and 3 would form a common cooperative scheme, hence the tuple $\{1, 2, 3\}$ is element of the set G. The characteristic functions of the cooperation scenarios, $V(I)$, $V(S)$, and $V(G)$, are based on the analysis concluded in Table 6.1 and are listed in Table 6.3.

The grand coalition is the optimal coalition due to the fulfillment of the superadditivity condition. Therefore, we assume that a grand coalition is formed and the basin's benefit of γ has to be shared between the riparians. The payoff (or imputation)[14] that each riparian i receives is symbolized by the variable x_i in the following. All the variables x_i develop the vector x, which contains for the presented example: $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. The determination of x is the main focus of the benefit sharing problem.

6.3.2 Benefit Sharing in the Grand Coalition: Four Approaches

6.3.2.1 The Core

The core is a set of payoffs which meets the four following conditions (Gillies 1959):

- Feasibility: Only benefits which are received can be allocated, which means $\sum_i [x_i] \leq V(G)$.

[14]The term imputation is used as a synonym of the term payoff.

- Pareto-efficiency: Nobody's payoff can be improved without worsening the payoff of another riparian. Therefore, there must not be an under-allocation of the gained benefits to the riparians, $\sum_i [x_i] \geq V(G)$.
- Combining the feasibility and Pareto-efficiency conditions leads to the requirement that the allocated benefits are equal to the generated benefits, $\sum_i [x_i] = V(G)$.
- Individual rationality: Each riparian rejects a payoff which is below its benefit when acting in a noncooperative way. Hence, $x_i \geq V(I)$. If this condition does not hold, the riparian would have an incentive to leave the grand coalition and to act unilaterally.
- Group rationality: Each sub-coalition of the grand coalition rejects a payoff which is below the benefit which is gained when the riparians act in a sub-coalition. Hence, $\sum_{i \in S} [x_i] \geq V(S)$. If this condition is not met, the relevant riparians $i \in S$ would have an incentive to leave the grand coalition and form the sub-coalition S.

Based on the presented example, it can be indicated whether the core is empty and has no solution ($Z < 0$), or there is only one solution ($Z = 0$) or there is a multitude of solutions ($Z > 0$) by solving the following optimization problem:

$$\begin{aligned}
&\max\ [Z = V(\{1,2,3\}) - [x_1 + x_2 + x_3]] \\
&x_1 \geq V(\{1\})\ ,\ x_2 \geq V(\{2\})\ ,\ x_3 \geq V(\{3\}) \\
&[x_1 + x_2] \geq V(\{1,2\})\ ,\ [x_1 + x_3] \geq V(\{1,3\})\ ,\ [x_2 + x_3] \geq V(\{2,3\})
\end{aligned}$$

which is equivalent to the following formulation:

$$\begin{aligned}
&\max\ \left[Z = \gamma - [x_1 + x_2 + x_3]\right] \\
&x_1 \geq \alpha\ ,\ x_2 \geq 0\ ,\ x_3 \geq 0 \\
&[x_1 + x_2] \geq \beta\ ,\ [x_1 + x_3] \geq \alpha\ ,\ [x_2 + x_3] \geq 0
\end{aligned}$$

Here, it is possible to calculate $Z = \gamma - \beta > 0$, hence, there are various payoff combinations which fulfill the conditions of the core.

Based on the core, the range of payoffs for each riparian can be indicated. There exists a lower bound and an upper bound for each riparian in the core:

- The riparian does not have an incentive to stay in the coalition until its payoff falls below the lower bound.
- However, if the payoff of an riparian exceeds its upper bound, another riparian certainly has an incentive to leave the coalition.

The lower and upper bounds of each player can be derived by solving the following optimization problems:

$$\begin{aligned}
&\textit{Lower Bound} \text{ or } \textit{Upper Bound} \text{ of riparian } i: \\
&\min [x_i] \text{ or } \max [x_i] \\
&s.t.\ [x_1 + x_2 + x_3] = V(\{1,2,3\}) \\
&\quad x_1 \geq V(\{1\})\ ,\ x_2 \geq V(\{2\})\ ,\ x_3 \geq V(\{3\}) \\
&\quad [x_1 + x_2] \geq V(\{1,2\})\ ,\ [x_1 + x_3] \geq V(\{1,3\})\ ,\ [x_2 + x_3] \geq V(\{2,3\})
\end{aligned}$$

Table 6.4 Lower and upper bounds of payments which are in the core

Riparian	Lower bound	Upper bound
Upstream riparian (User 1)	α	γ
Midstream riparian (User 2)	0	$\gamma - \alpha$
Downstream riparian (User 3)	0	$\gamma - \beta$

which is, for the presented river basin example, equivalent to the following formulation:

$$\begin{aligned}&\textit{Lower Bound} \text{ of riparian } i:\\ &\min\,[x_i]\\ &s.t.\ [x_1 + x_2 + x_3] = \gamma\\ &\quad x_1 \geq \alpha\ ,\ x_2 \geq 0\ ,\ x_3 \geq 0\\ &\quad [x_1 + x_2] \geq \beta\ ,\ [x_1 + x_3] \geq \alpha\ ,\ [x_2 + x_3] \geq 0\end{aligned}$$

$$\begin{aligned}&\textit{Upper Bound} \text{ of riparian } i:\\ &\max\,[x_i]\\ &s.t.\ [x_1 + x_2 + x_3] = \gamma\\ &\quad x_1 \geq \alpha\ ,\ x_2 \geq 0\ ,\ x_3 \geq 0\\ &\quad [x_1 + x_2] \geq \beta\ ,\ [x_1 + x_3] \geq \alpha\ ,\ [x_2 + x_3] \geq 0\end{aligned}$$

The objective contains the payment of the considered riparian i, which has to be minimized or maximized for finding the lower or upper bound of the core, respectively. The constraints of the optimization problem contain the conditions that have to be fullfilled for a payoff to be in the core:

- Feasibility and pareto-efficiency, $[x_1 + x_2 + x_3] = \gamma$;
- individual rationality, $x_1 \geq \alpha\ ,\ x_2 \geq 0\ ,\ x_3 \geq 0$;
- and group rationality, $[x_1 + x_2] \geq \beta\ ,\ [x_1 + x_3] \geq \alpha\ ,\ [x_2 + x_3] \geq 0$.

For the presented example, the lower and upper bounds of payments are listed in Table 6.4.

The lower bound of the riparians is determined by their individual rationalities. Therefore, the upstream user has to receive at least a payment of α, while the lower bounds of the mid- and downstream users are determined at the value 0.

Regarding the upper bound of payments, the upstream user could postulate a claim for the total generated benefits γ, because all constellations without the upstream user generate a benefit of zero,

$$\overline{x_1^{\text{Core}}} = \min\,[\gamma - \underbrace{V(\{2\})}_{0}\ ,\ \gamma - \underbrace{V(\{3\})}_{0}\ ,\ \gamma - \underbrace{V(\{2,3\})}_{0}] = \gamma\ .$$

However, due to the fact that the upstream user must receive at least a payment of α, the midstream user could get a maximal payment of $\gamma - \alpha$,

$$\overline{x_2^{\text{Core}}} = \min\,[\gamma - \underbrace{V(\{1\})}_{\alpha}\ ,\ \gamma - \underbrace{V(\{3\})}_{0}\ ,\ \gamma - \underbrace{V(\{1,3\})}_{\alpha}] = \gamma - \alpha\ .$$

The downstream riparian cannot claim more than $\gamma - \beta$, because of the threat that the up- and midstream riparians can form a sub-coalition $\{1, 2\}$ where they would generate a benefit of β,

$$\overline{x_3^{\text{Core}}} = \min\,[\gamma - \underbrace{V(\{1\})}_{\alpha}\,,\ \gamma - \underbrace{V(\{2\})}_{0}\,,\ \gamma - \underbrace{V(\{1,2\})}_{\beta}] = \gamma - \beta\,.$$

To sum up, the concept of the core gives a set of possible payments, which meets the feasibility, Pareto-efficiency, individual and group rationality conditions. By considering these conditions, a proposed payment which is in the core provides an incentive for each riparian to join the grand coalition. However, the core may be impractical in practice, because of the large amounts of possible solutions. Therefore, it might be advantagous to use focal point solution concepts for calculating a concrete payment vector x.

6.3.2.2 The Shapley Value

The Shapley Value which is based on Shapley (1953) shares the benefits in terms of the incremental value of the respective player for the coalition. The solution can be calculated using Eq. 6.16:

$$x_i = \sum_{\substack{I:i\in I\,\vee\\ S:i\in S\,\vee\\ G}} \left[\frac{(\#G - \#ISG)!\cdot(\#ISG-1)!}{\#G!}\cdot[V(...) - V(... - i)]\right] \qquad (6.16)$$

The $\#ISG$ sign in Eq. (6.16) represents the number of riparians which form a coalition. These coalition scenarios can be

- unilaterally acting riparians, represented by set I with $I = \{\{1\}\ ;\ \{2\}\ ;\ \{3\}\,\}$;
- sub-coalitions, represented by set S with $S = \{\{1,2\}\ ;\ \{1,3\}\ ;\ \{2,3\}\,\}$;
- the grand coalition, which is represented by set G with $S = \{\{1,2,3\}\}$.

We already discussed that the set I stands for unilaterally acting riparians which act in a noncooperative way. Therefore, the set I contains the tuples: $\{1\}$, $\{2\}$, and $\{3\}$. There is just one element in these tuples, hence, $\#ISG = 1$ for these types of coalition scenario. The set S stands for the sub-coalitions which represent all coalitions between the riparians except the grand coalition. Therefore, the tuples $\{1,2\}$, $\{1,3\}$, and $\{2,3\}$ form the set S. It is obvious that there are two riparians which form a sub-coalition, hence, $\#ISG = 2$ for all sub-coalition-constellations. In case of a grand coalition G, all riparians of the basin form a common cooperation scheme, which means that the tuple $\{1,2,3\}$ is an element of set G. This coalition includes three riparians, hence $\#ISG = 3$. Furthermore, the parameter $\#G$ also represents the number of riparians in the grand coalition, hence $\#G = 3$.

Due to the formulation $\sum_{\substack{I:i\in I\,\vee\\ S:i\in S\,\vee\\ G}} [...]$ in Eq. (6.16), only those coalitions are addressed in which riparian i is a member. These involve the unilateral action situation for riparian i ($i \in I$), appropriated sub-coalitions ($i \in S$) as well as the grand coalition (G). Therefore, the following constellations are relevant for solving the Shapley solution of the riparians:

- Riparian 1: $\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}$
- Riparian 2: $\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}$
- Riparian 3: $\{3\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$

The term $\frac{(\#G - \#ISG)! \cdot (\#ISG - 1)!}{\#G!}$ in Eq. (6.16) is a weighting factor which comes from mathematical permutation, based on the normative assumption that every player in the coalitions is randomly ordered, with every ordering equally possible (Wu and Whittington 2006).

The term $V(...)$ in Eq. (6.16) represents the generated benefit for the addressed coalition, while $V(... - i)$ symbolizes the benefit of the coalition which is formed on the basis of the addressed coalition without the riparian i. Therefore, the difference $V(...) - V(... - i)$ could be interpreted as the incremental benefit of riparian i for the coalition.

The application of Eq. 6.16 for the presented simple example is concluded in Table 6.5. Because of the three users (upstream, midstream, and downstream), it becomes obvious that $\#G = 3$. Regarding riparian 1, there is the realization probability of $\frac{1}{3}$ each that this riparian acts unilaterally or works in a grand coalition. Furthermore, there exists the realization probability of $\frac{1}{6}$ each that riparian 1 forms a sub-coalition either just with riparian 2 or 3 (see column (IV) of Table 6.5). If we observe the cooperation scenarios $\{1\}$, $\{1, 2\}$, $\{1, 3\}$, and $\{1, 2, 3\}$, the benefits α, β, α, and γ could be generated in the coalition scenarios, respectively (see column (V) of Table 6.5). A coalition without the unilaterally acting riparian 1, does not exist and therefore generates a benefit of 0. The coalitions $\{1, 2\}$, $\{1, 3\}$, and $\{1, 2, 3\}$ without riparian 1 result in the coalition constellations $\{2\}$, $\{3\}$, and $\{2, 3\}$ (see column (VI) of Table 6.5) which are each characterized by the generated benefit of 0 (see column (VII) of Table 6.5). Hence, the incremental benefit of riparian 1 for the coalitions $\{1\}$, $\{1, 2\}$, $\{1, 3\}$, and $\{1, 2, 3\}$ is α, β, α, and γ, respectively, which can also be found as the difference between the columns (V) and (VII) of Table 6.5 (see column (VIII) of Table 6.5). The incremental benefit of riparian 1 for each coalition scenario has to be multiplied with the realization probability of each coalition scenario, which can be found by the product of columns (IV) and (VIII) in Table 6.5 (see column (IX) in Table 6.5). By summing up all these weighted incremental benefits of riparian 1, we get the Shapley solution of riparian 1 which is $\frac{3 \cdot \alpha + \beta + 2 \cdot \gamma}{6}$.

This procedure could be applied for riparians 2 and 3, analogously.

Therefore, it is possible to formulate the following payment vector as the focal point solution for the bargaining problem:

$$x^{SH} = \left(x_1^{SH} \; x_2^{SH} \; x_3^{SH}\right) = \left(\tfrac{3 \cdot \alpha + \beta + 2 \cdot \gamma}{6} \;\; \tfrac{\beta + 2 \cdot \gamma - 3 \cdot \alpha}{6} \;\; \tfrac{\gamma - \beta}{3}\right)$$

Table 6.5 Calculation of Shapley value for simple river basin example

(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	(IX)	(X)
Riparian i	Coalition	$\#ISG$	$\frac{(\#G-\#ISG)!\cdot(\#ISG-1)!}{\#G!}$	$V(\dots)$	Coalition without i	$V(\dots - i)$	(V) - (VII)	(IV) · (VIII)	Shapley Value
User 1	{1}	1	$\frac{1}{3}$	α	Ø	0	α	$\frac{1}{3}\cdot\alpha$	$\frac{3\cdot\alpha+\beta+2\cdot\gamma}{6}$
	{1, 2}	2	$\frac{1}{6}$	β	{2}	0	β	$\frac{1}{6}\cdot\beta$	
	{1, 3}	2	$\frac{1}{6}$	α	{3}	0	α	$\frac{1}{6}\cdot\alpha$	
	{1, 2, 3}	3	$\frac{1}{3}$	γ	{2, 3}	0	γ	$\frac{1}{3}\cdot\gamma$	
User 2	{2}	1	$\frac{1}{3}$	0	Ø	0	0	0	$\frac{\beta+2\cdot\gamma-3\cdot\alpha}{6}$
	{1, 2}	2	$\frac{1}{6}$	β	{1}	α	$\beta-\alpha$	$\frac{\beta-\alpha}{6}$	
	{2, 3}	2	$\frac{1}{6}$	0	{3}	0	0	0	
	{1, 2, 3}	3	$\frac{1}{3}$	γ	{1, 3}	α	$\gamma-\alpha$	$\frac{\gamma-\alpha}{3}$	
User 3	{3}	1	$\frac{1}{3}$	0	Ø	0	0	0	$\frac{\gamma-\beta}{3}$
	{1, 3}	2	$\frac{1}{6}$	α	{1}	α	0	0	
	{2, 3}	2	$\frac{1}{6}$	0	{3}	0	0	0	
	{1, 2, 3}	3	$\frac{1}{3}$	γ	{1, 2}	β	$\gamma-\beta$	$\frac{\gamma-\beta}{3}$	

It becomes obvious that

$$x_1^{SH} > x_2^{SH} > x_3^{SH}$$

which means that the upstream riparian 1 receives the highest benefits, while the downstream riparian receives the lowest payoffs, which is reasoned by the hydrological power of the respective riparians. The more upstream the riparian is located in the basin, the earlier the riparian is able to abstract (or control the water amounts), which makes an upstream riparian more (hydrological) powerful than the downstream one. For instance, a coalition with just riparians 2 and 3 generates a benefit of 0. When riparian 1 joins this arrangement, which means that the grand coalition would be formed, it would generate the benefit of γ. Therefore, the incremental benefit of riparian 1 for this coalition is γ. The upstream riparian receives a higher proportion of the generated benefit than the downstream riparian.

6.3.2.3 The Nash-Harsanyi Solution

The Nash-Harsanyi solution maximizes the product of assigned benefits in excess to the benefits generated in the noncooperative case (Harsanyi 1958). Furthermore, the feasibility, Pareto-efficiency, individual rationality, and group rationality are also addressed, hence it is guaranteed that the solution is within the core. Therefore, the following general optimization problem can be formulated for finding the Nash-Harsanyi solution:

$$\begin{aligned}
&\max \left[\prod_i (x_i - V(I))\right] && \\
&s.t. \sum_i [x_i] = V(G) && && \text{(Feasibility and Pareto-Efficiency)}\\
&\quad x_i \geq V(I) && \forall I && \text{(Individual rationality)}\\
&\quad \sum_{i \in S} [x_i] \geq V(S) && \forall S && \text{(Group rationality)}
\end{aligned}$$

which is for the presented river basin example:

$$\begin{aligned}
&\max\, [(x_1 - \alpha) \cdot x_2 \cdot x_3] && \\
&s.t.\ [x_1 + x_2 + x_3] = \gamma && \text{(Feasibility and Pareto-efficiency)}\\
&\quad x_1 \geq \alpha\ ,\ x_2 \geq 0\ ,\ x_3 \geq 0 && \text{(Individual rationality)}\\
&\quad x_1 + x_2 \geq \beta\ ,\ x_1 + x_3 \geq \alpha\ ,\ x_2 + x_3 \geq 0 && \text{(Group rationality)}
\end{aligned}$$

When applying this solution procedure for the presented river basin example, the following optimality conditions can be found:

$$\begin{aligned}
&x_2 \cdot x_3 = (x_1 - \alpha) \cdot x_3 = (x_1 - \alpha) \cdot x_2\\
&x_1 + x_2 + x_3 = \gamma
\end{aligned}$$

Table 6.6 Additional benefits for Nash-Harsanyi solution in the Grand Coalition for the simple river basin example

	Upstream riparian (User 1)	Midstream riparian (User 2)	Downstream riparian (User 3)
Nash-Harsanyi solution	$\frac{1}{3} \cdot (2 \cdot \alpha + \gamma)$	$\frac{1}{3} \cdot (\gamma - \alpha)$	$\frac{1}{3} \cdot (\gamma - \alpha)$
Benefit for noncooperative acting	α	0	0
Additional benefits in grand coalition	$\frac{1}{3} \cdot (\gamma - \alpha)$	$\frac{1}{3} \cdot (\gamma - \alpha)$	$\frac{1}{3} \cdot (\gamma - \alpha)$

The equation $x_2 \cdot x_3 = (x_1 - \alpha) \cdot x_3 = (x_1 - \alpha) \cdot x_2$ can be reformulated to the following expression:

$$x_1 - \alpha = x_2 = x_3$$

which is nothing else than

$$x_1 - V(\{1\}) = x_2 - V(\{2\}) = x_3 - V(\{3\})$$

Hence, the assigned benefit in excess of the respective noncooperative benefit is equal for each riparian, which is a typical characteristic of the Nash-Harsanyi solution. This characteristic results mainly from the objective function of the Nash-Harsanyi optimization problem.

The concrete Nash-Harsanyi solution of the presented river example is

$$x^{NH} = \left(x_1^{NH} \; x_2^{NH} \; x_3^{NH}\right) = \left(\tfrac{1}{3} \cdot (2 \cdot \alpha + \gamma) \; \tfrac{1}{3} \cdot (\gamma - \alpha) \; \tfrac{1}{3} \cdot (\gamma - \alpha)\right)$$

Therefore, the additional benefit in the grand coalition compared to the noncooperative case is the same for every player $\frac{1}{3} \cdot (\gamma - \alpha)$, which is also illustrated by Table 6.6.

For the unilateral acting case, the upstream riparian generates the highest benefit with the level α, while the mid- and downstream ones do not receive any water and therefore generate a benefit of 0. Hence, when applying the Nash-Harsanyi solution, the upstream riparian receives the highest payoffs while the mid- and downstream users receive equal payoffs:

$$x_1^{NH} > x_2^{NH} = x_3^{NH}$$

The total benefit in the basin when all riparians act unilaterally is α, because $V(\{1\}) + V(\{2\}) + V(\{3\}) = \alpha$, while the basin's benefit in case of a grand coalition is γ, because $V(\{1, 2, 3\}) = \gamma$. Therefore, the term $\gamma - \alpha$ can be interpreted as the cooperation gain in the basin. The Nash-Harsanyi solution shares this cooperation gain equally among the riparians, hence each riparian obtains the benefit $\dfrac{\gamma - \alpha}{3}$ in excess to those benefits the riparian would generate when acting unilaterally.

Table 6.7 Objection against the Shapley solution in the Grand Coalition

	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}
Benefit of Coalition	α	0	0	β	α	0
Payoff in {1, 2, 3} based on Shapley solution	$\frac{3\cdot\alpha+\beta+2\cdot\gamma}{6}$	$\frac{\beta+2\cdot\gamma-3\cdot\alpha}{6}$	$\frac{\gamma-\beta}{3}$	$\frac{\beta+2\cdot\gamma}{3}$	$\frac{3\cdot\alpha-\beta+4\cdot\gamma}{6}$	$\frac{4\cdot\gamma-3\cdot\alpha-\beta}{6}$
Objections of coalition against Shapley solution	$\frac{3\cdot\alpha-\beta-2\cdot\gamma}{6}$	$\frac{3\cdot\alpha-\beta-2\cdot\gamma}{6}$	$\frac{\beta-\gamma}{3}$	$\frac{2\cdot\beta-2\cdot\gamma}{3}$	$\frac{3\cdot\alpha+\beta-4\cdot\gamma}{6}$	$\frac{3\cdot\alpha+\beta-4\cdot\gamma}{6}$

6.3.2.4 The Nucleolus

The main goal of the nucleolus is to find a solution in which the maximum objection against a benefit sharing solution is minimized. This concept was first presented by Schmeidler (1969). The objection of a coalition against a benefit sharing solution results from the difference between the generated benefit of this coalition (if it was formed in the basin) and the payoff for this coalition due to the benefit sharing solution.[15]

For instance, the objections of the various cooperation constellations against the Shapley solution in the grand coalition are illustrated in Table 6.7.

The maximum objection against the Shapley solution is therefore

$$\max\left[\frac{3\cdot\alpha-\beta-2\cdot\gamma}{6};\ \frac{\beta-\gamma}{3};\ \frac{2\cdot\beta-2\cdot\gamma}{3};\ \frac{3\cdot\alpha+\beta-4\cdot\gamma}{6}\right]=\frac{\beta-\gamma}{3}$$

For minimizing the maximum objection under the consideration of the feasibility, Pareto-efficiency, individual rationality, and group rationality conditions, the following general optimization problems have to be solved to find the nucleolus solution (see Wang et al. (2003)):

[15]For the presented river basin example, the objections of the coalition constellations are

- Objection of riparian 1, {1} : $V(\{1\})-x_1$
- Objection of riparian 2, {2} : $V(\{2\})-x_2$
- Objection of riparian 3, {3} : $V(\{3\})-x_3$
- Objection of coalition {1, 2} : $V(\{1,2\})-x_1-x_2$
- Objection of coalition {1, 3} : $V(\{1,3\})-x_1-x_3$
- Objection of coalition {2, 3} : $V(\{2,3\})-x_2-x_3$

$$
\begin{aligned}
&\min\,[e] \\
&s.t. \sum_i [x_i] = V(G) && && \text{(Feasibility and Pareto-efficiency)} \\
&\quad e + x_i \geq V(I) && \forall I && \text{(Individual rationality)} \\
&\quad e + \sum_{i \in S} [x_i] \geq V(S) && \forall S && \text{(Group rationality)}
\end{aligned}
$$

which for the presented river basin example is equivalent to the following formulation:

$$
\begin{aligned}
&\min\,[e] \\
&s.t.\ [x_1 + x_2 + x_3] = \gamma && \text{(Feasibility and Pareto-efficiency)} \\
&\quad e + x_1 \geq \alpha\ ,\ e + x_2 \geq 0\ ,\ e + x_3 \geq 0 && \text{(Individual rationality)} \\
&\quad e + [x_1 + x_2] \geq \beta\ ,\ e + [x_1 + x_3] \geq \alpha\ ,\ e + [x_2 + x_3] \geq 0 && \text{(Group rationality)}
\end{aligned}
$$

The maximum objection against a benefit sharing solution is represented by the variable e. This variable e is a free variable, therefore, it is defined within the domain $[-\infty, \infty]$. If the variable e becomes zero or negative, the individual and group rationality conditions are certainly fulfilled. However, regardless of the value of variable e, the feasibility and Pareto-efficiency conditions are always met. The objective of the optimization problem sets the goal to minimize the value of e, which means that the maximum objection against the benefit sharing solution has to be minimized. As we already discussed, this is the main motivation of the nucleolus solution concept. The value of the variable e can not be set arbitrarily low, because the lowest possible value of e is restricted by the individual and group rationality conditions. If the superadditivity condition is met, the variable e takes a positive value. Therefore, the individual and group rationality conditions are fulfilled.

When applying the optimization problem to identify the nucleolus solution, we have to differentiate between two cases of parameter specifications α, β, and γ:

- Case 1: We specify β and γ such that $\beta \leq \frac{\gamma}{3}$. However, if in contrast $\beta > \frac{\gamma}{3}$, this case is also relevant for the specification $\frac{3 \cdot \beta - \gamma}{2} \leq \alpha$. Therefore, the compact description of this case is $(\beta \leq \frac{\gamma}{3}) \vee ((\frac{\gamma}{3} < \beta) \wedge (\frac{3 \cdot \beta - \gamma}{2} \leq \alpha))$.
- Case 2: This case becomes relevant when α, β, and γ are specified such that case 1 does not fit. Therefore, we know that $\frac{\gamma}{3} < \beta$. Furthermore, we also know that $\alpha < \frac{3 \cdot \beta - \gamma}{2}$. Hence, the compact description of this case is $(\frac{\gamma}{3} < \beta) \wedge (\alpha < \frac{3 \cdot \beta - \gamma}{2})$.

The optimality conditions which result from the application of the optimization problem are

$$
\begin{aligned}
&\text{For case 1: } e = \alpha - x_1 = -x_2 = -x_3 \\
&\text{For case 2: } e = \beta - x_1 - x_2 = -x_3 \\
&\text{For cases 1 and 2: } x_1 + x_2 + x_3 = \gamma
\end{aligned}
$$

Table 6.8 Objection against the nucleolus solution in the Grand Coalition (for case 1)

	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}
Benefit of Coalition	α	0	0	β	α	0
Payoff in coalitions based on nucleolus solution	$\frac{2\cdot\alpha+\gamma}{3}$	$\frac{\gamma-\alpha}{3}$	$\frac{\gamma-\alpha}{3}$	$\frac{\alpha+2\cdot\gamma}{3}$	$\frac{\alpha+2\cdot\gamma}{3}$	$\frac{2\cdot\gamma-2\cdot\alpha}{3}$
Objections against nucleolus solution	$\frac{\alpha-\gamma}{3}$	$\frac{\alpha-\gamma}{3}$	$\frac{\alpha-\gamma}{3}$	$\frac{3\cdot\beta-\alpha-2\cdot\gamma}{3}$	$\frac{2\cdot\alpha-2\cdot\gamma}{3}$	$\frac{2\cdot\alpha-2\cdot\gamma}{3}$

For *case 1*, it follows from the optimality conditions that the unilaterally acting riparians, denoted by {1}, {2} and {3}, state the maximum objections against the proposed nucleolus solution. Therefore, the level of the variable e which stands for the maximum objection is limited in its lowest value by the individual rationality conditions. The nucleolus solution for this case is

$$x^{nuc,1} = \left(x_1^{nuc,1}\ x_2^{nuc,1}\ x_3^{nuc,1}\right) = \left(\frac{2\cdot\alpha+\gamma}{3}\ \frac{\gamma-\alpha}{3}\ \frac{\gamma-\alpha}{3}\right)$$

This solution is equal to the Nash-Harsanyi solution. This means that the equal sharing of cooperation gains between the riparians (which is done by the Nash-Harsanyi solution) minimizes the maximum objections which are stated by the unilaterally acting riparians. The objections under this case 1 are listed in detail in Table 6.8.

The maximum objection under this case 1 is

$$e = \max\left[\frac{\alpha-\gamma}{3}\ ;\ \frac{3\cdot\beta-\alpha-2\cdot\gamma}{3}\ ;\ \frac{2\cdot\alpha-2\cdot\gamma}{3}\right] = \frac{\alpha-\gamma}{3}$$

which is stated by the unilaterally acting riparians 1, 2, and 3. Of course, this maximum objection determines the value of the variable e.

Under *case 2*, the sub-coalition between the riparians 1 and 2, denoted by {1, 2} and the unilaterally acting riparian 3, represented by {3}, state the maximum objection against the nucleolus solution which becomes apparent by the relevant optimality conditions. Therefore, the level of the variable e is limited in its lowest value by the group rationality of coalition {1, 2} as well as by the individual rationality of riparian 3.

Based on the assumption under this case 2 ($e = \beta - x_1 - x_2 = -x_3$) as well as the formerly presented optimality conditions, the following relations are valid:

$$\begin{aligned}
&e = \beta - x_1 - x_2 = -x_3 \\
&x_1 + x_2 + x_3 = \gamma \\
&e + x_1 \geq \alpha,\ e + x_2 \geq 0,\ e + x_1 + x_3 \geq \alpha,\ e + x_2 + x_3 \geq 0
\end{aligned}$$

When solving this problem, we can find an explicit solution for x_3, which is

$$x_3^{nuc,2} = \frac{\gamma - \beta}{2}$$

Furthermore, we find that the sub-coalition containing riparians 1 and 2 has to receive a payoff in the level:

$$x_1^{nuc,2} + x_2^{nuc,2} = \frac{\beta + \gamma}{2}$$

However, there is no concrete solution regarding the assignment of benefits to riparians 1 and 2. However, for meeting the optimality conditions, we know that the solutions of x_1 and x_2 have to be in the following intervals:

$$x_1^{nuc,2} = \left[x_1^{nuc,2,min} \ , \ x_1^{nuc,2,max}\right] = \left[\frac{2 \cdot \alpha - \beta + \gamma}{2} \ , \ \beta\right]$$
$$x_2^{nuc,2} = \left[x_2^{nuc,2,min} \ , \ x_2^{nuc,2,max}\right] = \left[\frac{\gamma - \beta}{2} \ , \ \beta - \alpha\right]$$

Of course, we would like to have a focal point solution, which means that we want to find an explicit assignment of payoffs for riparians 1 and 2.

A possible opportunity for assigning the payoffs is just to apply the nucleolus procedure for the sub-coalition $\{1, 2\}$.[16]

In Exercise 6.2, we present the solution steps of the nucleolus procedure for a basin with 2 riparians in detail. Every riparian has to receive the benefit it would get when acting unilaterally (individual rationality condition). On top of that, the riparians get a share of the cooperation gain, which results from the difference between the payoffs the sub-coalition would receive in the nucleolus solution , which is $0.5 \cdot (\beta + \gamma)$, and the sum of the benefits the riparians would receive when acting unilaterally which is

[16]The optimization problem for finding the nucleolus solution of the sub-coalition $\{1, 2\}$ is

$$\min_{\{e, x_1, x_2\}} [e]$$
$$s.t. \ x_1 + x_2 = 0.5 \cdot (\beta + \gamma)$$
$$e + x_1 \geq \alpha$$
$$e + x_2 \geq 0$$

Table 6.9 Objection against the nucleolus solution (case 2)

	{1}	{2}	{3}	{1, 2}	{1, 3}	{2, 3}
Benefit of Coalition	α	0	0	β	α	0
Payoff based on nucleolus solution	$\frac{2\cdot\alpha+\beta+\gamma}{4}$	$\frac{\beta+\gamma-2\cdot\alpha}{4}$	$\frac{\gamma-\beta}{2}$	$\frac{\beta+\gamma}{2}$	$\frac{2\cdot\alpha-\beta+2\cdot\gamma}{4}$	$\frac{3\cdot\gamma-2\cdot\alpha-\beta}{4}$
Objections of coalition against nucleolus solution	$\frac{2\cdot\alpha-\beta-\gamma}{4}$	$\frac{2\cdot\alpha-\beta-\gamma}{4}$	$\frac{\beta-\gamma}{2}$	$\frac{\beta-\gamma}{2}$	$\frac{2\cdot\alpha+\beta-2\cdot\gamma}{4}$	$\frac{2\cdot\alpha+\beta-3\cdot\gamma}{4}$

α. When applying the nucleolus procedure in a coalition or basin with 2 riparians, we have to share the cooperation gain equally between the riparians.[17]

Therefore, we can set the following payoffs for the riparians 1 and 2:

$$x_1^{nuc,2} = \frac{2\cdot\alpha+\beta+\gamma}{4}$$
$$x_2^{nuc,2} = \frac{\beta+\gamma-2\cdot\alpha}{4}$$

Hence, the nucleolus solution for this case is

$$x^{nuc,2} = \left(x_1^{nuc,2}\ x_2^{nuc,2}\ x_3^{nuc,2}\right) = \left(\frac{2\cdot\alpha+\beta+\gamma}{4}\ \frac{\beta+\gamma-2\cdot\alpha}{4}\ \frac{\gamma-\beta}{2}\right)$$

The objections under case 2 are listed in detail in Table 6.9.

The maximum objection is:

$$e = \max\left[\frac{2\cdot\alpha-\beta-\gamma}{4};\ \frac{\beta-\gamma}{2};\ \frac{2\cdot\alpha+\beta-2\cdot\gamma}{4};\ \frac{2\cdot\alpha+\beta-3\cdot\gamma}{4}\right] = \frac{\beta-\gamma}{2}$$

[17]In case of unilateral acting, riparians 1 and 2 generate a benefit of α, i.e., $V(\{1\}) + V(\{2\}) = \alpha$. The sub-coalition between riparians 1 and 2 should receive a benefit of $0.5\cdot(\beta+\gamma)$ in the nucleolus solution, i.e., $x_1^{nuc,2} + x_2^{nuc,2} = 0.5\cdot(\beta+\gamma)$. Therefore, the cooperation gain is

$$\Delta = x_1^{nuc,2} + x_2^{nuc,2} - V(\{1\}) - V(\{2\}) = 0.5\cdot(\beta+\gamma) - \alpha$$

Half of the cooperation gain is $0.5\cdot\Delta = 0.25\cdot(\beta+\gamma) - 0.5\cdot\alpha$. Therefore, the riparians receive the following benefits:

$$x_1^{nuc,2} = V(\{1\}) + 0.5\cdot\Delta = \alpha + 0.25\cdot(\beta+\gamma) - 0.5\cdot\alpha = 0.5\cdot\alpha + 0.25\cdot(\beta+\gamma)$$
$$x_2^{nuc,2} = V(\{2\}) + 0.5\cdot\Delta = 0.25\cdot(\beta+\gamma) - 0.5\cdot\alpha = 0.25\cdot(\beta+\gamma) - 0.5\cdot\alpha$$

The maximum objection $e = \frac{\beta - \gamma}{2}$ is stated by the unilaterally acting riparian 3 and the sub-coalition between riparians 1 and 2.

Because of the hydrological power in the river basin, which results in the highest payoff for riparian 1 due to its position in the river basin, and the fact that the sub-coalition $\{1, 2\}$ states the maximum objection, the payoff for riparian 2 exceeds the one of riparian 3, hence

$$x_1^{nuc,2} > x_2^{nuc,2} > x_3^{nuc,2}$$

Box 6.1 Benefit sharing in the Nile river basin

With a total length of 6700 km, the Nile is the longest river in the world; its basin stretches over 11 countries: Egypt, Sudan, South Sudan, Ethiopia, Uganda, Kenya, Tanzania, Burundi, Rwanda, Democratic Republic Congo, and Eritrea. Similar to other international rivers (such as Euphrates and Tigris, Syr Darya, Amu Darya, and Ganges), there is a gap between the water quantity available in the basin and the water quantity claimed by the riparians. (Wu and Whittington 2006) Therefore, the riparians compete for the scarce water sources in the river. Wu and Whittington (2006) state a water defecit of about 50 billion cubic meter per year in the basin. Egypt which is the most downstream country in the basin contributes essentially nothing to the flow of the Nile, however, it currently consumes more than 80% of the Nile water due to its political and military power in the region (Wu and Whittington 2006). Ethiopia which is located in the upstream of the basin contributes over 85% of the water flow. It claims significantly more water resources than its current consumption for realizing its dam and irrigation project which became necessary to meet the needs of an increasing population.

There exist a number of Nile river models in the scientific literature which are explained in detail by, for instance, Nigatu and Dinar (2011). The Nile Economic Optimization Model (NEOM, see Wu (2000)) is a basin-wide economic optimization model which quantifies the economic benefits from water usage. Wu and Whittington (2006) use the NEOM to study conflict incentive-compatible resolution strategies based on various cooperation scenarios in the basin. Block and Strzepek (2010) set up the Investment Model for Planning Ethiopian Nile Development (IMPEND) which focuses on the impact of dams constructed for irrigation and hydropower purposes. The model which is applied by Nigatu and Dinar (2011) as well as by Dinar and Nigatu (2013) is based on the NEOM model and takes into account additional features such as the resource degradation, various climate change scenarios, and the possibility of introducing basin-wide water trade. It just focuses on the basin of the Blue Nile which involves the countries Ethiopia, Sudan, and Egypt.

Dinar and Nigatu (2013) distinguish different scenarios of allocation between the riparians in the Blue Nile. The scenario WRA-I, which allocates 12.2, 22.0, and 65.8%, respectively, to Ethiopia, Sudan, and Egypt, is based on the notion of Egypt's long-term use pattern. The scenario WRA-II, which allocates 38.4, 14.1, and 47.5% to Ethiopia, Sudan, and Egypt, respectively, is based on the notion of equitable access as reflected in the UN Water convention from 1997. The generated benefit for the different allocation scenarios are listed in the following table, with ETH, SDN, and EGY representing Ethiopia, Sudan, and Egypt, respectively. In cases of cooperation, the allocated water amounts are traded to another country of the cooperation scheme if increasing benefits result from this transfer. Therefore, the highest basin's benefit is generated for all allocation scenarios in the cooperation arrangement which involves all three riparians. The highest basin's benefit with 9.21 is generated under the allocation scenario WRA-II, followed by the allocation scenario WRA-I with 8.77.

Benefits under different allocation and cooperation scenarios. *Source* Dinar and Nigatu (2013)

Allocation Scenario	Unilateral Acting			Sub-Coalitions			Grand Coalition
	ETH	SDN	EGY	ETH+SDN	ETH+EGY	SDN+EGY	ETH+SDN+EGY
WRA-I	1.29	2.62	4.83	3.94	5.71	7.55	8.77
WRA-II	2.21	2.56	3.91	4.62	6.60	6.80	9.21

Assuming the three riparians form a joint arrangement, we focus on the question of how to allocate the common benefit to the individual riparians. This means that for the scenarios WRA-I and WRA-II, the basin's benefits of 8.77 and 9.21, respectively, have to be allocated to the riparians. The incremental benefit for any riparian results from the difference between its received benefit in the joint arrangement and the benefit which the riparian would generate when acting unilaterally. Applying the Nash-Harsanyi solution, the benefits are allocated in a way that the incremental benefits become equal for all riparians. For the scenario WRA-I, the benefits when acting unilaterally for Ethiopia, Sudan, and Egypt are 1.29, 2.62, and 4.83, respectively, which results in the basin's benefit of 8.74. The basin's benefit in the joint arrangement is 8.77, hence when applying the Nash-Harsanyi solution, the incremental benefit of every riparian becomes 0.01, because $\frac{8.77-8.74}{3} = 0.01$. This means that Ethiopia, Sudan, and Egypt are assigned a benefit of 1.30, 2.63, and 4.84, respectively, for the Nash-Harsanyi solution. For scenario WRA-II, acting unilaterally holds benefits of Ethiopia, Sudan and Egypt, respectively, which results in a basin's benefit of 8.68. The basin's benefit in the joint arrangement is 9.21. Hence, the incremental benefit of each riparian amounts to about 0.18 ($\frac{9.21-8.68}{3} \approx 0.18$) when applying the Nash-Harsanyi solution. Hence Ethopia, Sudan and Egypt receive a benefit of 2.39, 2.74 and 4.08, respectively. The Shapley and Nash-Harsanyi solution for the Benefit Sharing problem of the Blue Nile basin is illustrated in the following table.

Shapley and Nash-Harsanyi solution under different allocation scenarios

Allocation Scenario	Shapley			Nash-Harsanyi		
	ETH	SDN	EGY	ETH	SDN	EGY
WRA-I	1.20	2.79	4.78	1.30	2.63	4.84
WRA-II	2.33	2.61	4.27	2.39	2.74	4.08

Regardless of the allocation scenario, the upstream riparian Ethiopia prefers the Nash-Harsanyi to the Shapley solution, because $1.3 > 1.2$ and $2.39 > 2.33$. For the allocation scenario WRA-I, Sudan and Egypt prefer the Shapley ($2.79 > 2.63$) and Nash-Harsanyi solution ($4.84 > 4.78$), respectively, while for the other allocation scenario WRA-II, the contrary situation becomes obvious because Sudan and Egypt prefer the Nash-Harsanyi ($2.74 > 2.61$) and Shapley solution ($4.27 > 4.08$), respectively. The Shapley solution of the allocation scenario WRA-I is not in the core, because the assigned benefits to Ethiopia and Egypt with 1.20 and 4.78, respectively, are lower than the benefits Ethiopia and Egypt would generate under unilateral acting which are 1.29 and 4.83, respectively (see Table 6.10). Therefore, the Shapley solution for the allocation scenario WRA-I violates the individual rationality of Ethiopia and Egypt. However, the Shapley solution of the allocation scenario WRA-II is within the core. The Nash-Harsanyi solution is also in the core for both the scenarios WRA-I and WRA-II.

Source Dinar and Nigatu (2013)

6.3.3 Concluding Remarks on the Benefit Sharing Problem

The benefit sharing problem focuses on the question of how to assign benefits to riparians which are generated in a joint arrangement. The assigned benefit in this context is termed as payoff or imputation. The first important concept is the core which contains all payoffs which meet the feasibility, Pareto-efficiency, individual and group rationality conditions. Due to the feasibility and Pareto-efficiency conditions, the generated benefit in the joint arrangement has to be assigned to the riparians of this arrangement in total. The individual rationality condition means that any riparian of the joint arrangement has to receive at least as much payoffs as it would generate when acting unilaterally in a noncooperative way. However, the group rationality condition means that a coalition which could be formed by a subset of riparians acting cooperatively in the joint arrangement must receive at least as much benefits in the joint arrangement as it would generate if the respective coalition was formed in the basin. Usually, either the core is empty—which means that it is not incentive-compatible to form this joint arrangement—or there are a multitude of payoffs in the core. Therefore, we discussed the lower and upper bounds of the core for each riparian. The lower bound of each riparian is the minimum payment the respective riparian has to receive to have an economic incentive to join the cooperative arrangement, while the upper bound is the maximum payment

Table 6.10 Payoffs for riparians regarding the presented focal point solution concepts

	Rriparian 1	Riparian 2	Riparian 3
Shapley solution	$\frac{1}{6} \cdot (3 \cdot \alpha + \beta + 2 \cdot \gamma)$	$\frac{1}{6} \cdot (\beta + 2 \cdot \gamma - 3 \cdot \alpha)$	$\frac{1}{3} \cdot (\gamma - \beta)$
Nash-Harsanyi solution	$\frac{1}{3} \cdot (2 \cdot \alpha + \gamma)$	$\frac{1}{3} \cdot (\gamma - \alpha)$	$\frac{1}{3} \cdot (\gamma - \alpha)$
Nucleolus (case 2)	$\frac{1}{4} \cdot (2 \cdot \alpha + \beta + \gamma)$	$\frac{1}{4} \cdot (\beta + \gamma - 2 \cdot \alpha)$	$\frac{1}{2} \cdot (\gamma - \beta)$

the respective riparian would get without setting an incentive for another riparian to leave the cooperative arrangement.

Furthermore, we also discussed some focal point solution concepts which state concrete solutions (a specific payment for each riparian): The Shapley value may be in the core, while the Nash-Harsanyi and nucleolus solutions are certainly in the core when the superadditivity condition is fulfilled. The Shapley value assigns the benefits depending on the (weighted) incremental benefit of the respective riparian for the coalitions. Particularly powerful riparians, e.g., those riparians that are located upstream in the river basin and, therefore, have hydrological power, benefit from this concept. The more the benefit in a coalition increases due to the joining of the respective riparian in this coalition—which is nothing else than the incremental benefit of the respective riparian for the coalition—the higher the proportion of benefits of the joint arrangement that goes to this respective riparian. However, the Nash-Harsanyi and nucleolus solutions do not focus as much on the power situation of a riparian, but more on aspects of justice. The Nash-Harsanyi solution allocates the benefits to the riparians in a way that the assigned benefit in excess to the respective noncooperative benefit is equal for each riparian. This means that the cooperation gain is shared equally between the riparians. The nucleolus is a solution concept in which the maximum objection against a payment solution is minimized. The objection of a coalition against a benefit sharing solution results from the difference between the generated benefit of this coalition (if it was formed in the basin) and the payoff for this coalition due to the benefit sharing solution.

For the presented river basin example, the realized benefit sharing solutions are illustrated in Table 6.10. Please note that we differentiate between two cases of parameter specification regarding α, β, and γ. The nucleolus and Nash-Harsanyi solutions just differ for the second case. For the first case, the nucleolus solution and Nash-Harsanyi solutions are the same.

The upstream riparian prefers the Shapley solution, because due to its upstream position (hydrological power) it generates high incremental benefit for the possible coalitions. For example, a coalition with just riparians 2 and 3 generates a benefit of 0. If riparian 1 joined this arrangement, it would generate the benefit of γ. Hence, the incremental benefit of riparian 1 for this coalition is γ. This relation illustrates why riparian 1 gets such a high proportion of benefit when applying the Shapley value.

Case 1: $x_1^{SH} > x1^{NH}$ Case 2: $x_1^{SH} > x_1^{nuc,2} > x_1^{NH}$

The midstream riparian also prefers the Shapley solution, because the Nash-Harsanyi solution and nucleolus solution are based on the low benefit level of 0, if this riparian acted unilaterally.

$$\text{Case 1: } x_2^{SH} > x_2^{NH} \qquad \text{Case 2: } x_2^{SH} > x_2^{nuc,2} > x_2^{NH}$$

The downstream riparian has the lowest hydrological power due to its position in the basin. Because of its limited hydrological power, the riparian has the highest aversion against the Shapley solution, while it prefers the Nash-Harsanyi solution:

$$\text{Case 1: } x_3^{NH} > x_3^{SH} \qquad \text{Case 2: } x_3^{NH} > x_3^{nuc,2} > x_3^{SH}$$

In the grand coalition, the benefit of the entire basin which is γ is generated by the water consumption of the downstream riparian. The less productive riparians 1 and 2 leave the water in the river, hence, they generate no benefit in the grand coalition. However, they have to receive payoffs for all presented benefit sharing solutions. Therefore, riparian 3 has to make side payments:

$$\text{Side payments made by riparian } 3 = \gamma - x_3$$

which means for the focal point solution concepts:

$$\text{Side payments made by riparian } 3 = \begin{cases} \frac{\beta+2\cdot\gamma}{3} & \text{Shapley solution} \\ \frac{\alpha+2\cdot\gamma}{3} & \text{Nash-Harsanyi and nucleolus solutions} \end{cases}$$

Riparians 1 and 2 receive the following side payments from riparian 3 in the level of their respective benefit sharing solution:

$$\text{Side payments received by riparian } 1 = x_1$$
$$\text{Side payments received by riparian } 2 = x_2$$

6.4 Bankruptcy Rules for Water Allocation

6.4.1 Principles of Bankruptcy Rules

The last two sections were based on the construction of monetary or utility-measured characteristic functions of cooperative game theory. Now, we turn to the so-called bankruptcy methods that distribute scarce water quantities *directly* to riparian states without calculating the economic value they create. Thus, the distributandum is not the monetary benefit, but the water itself. These methods have been developed in a completely different context: If a firm goes bankrupt, how should the residual liquidated wealth be distributed among its creditors? There is a plethora of rules and principles on how to allocate the insolvency assets to the creditors.[18] Should the residual assets be divided, for example, proportionally or equally? Suppose the first creditor lent 200 € to the company, a second 100 €. However, the remaining goodwill is only 200 €, so not all claims are covered. If we apply the proportional rule, then the first creditor receives two-thirds of the residual value, i.e., 133 €, while the second creditor receives only one third, i.e., 67 €. It is also possible to allocate equal shares to both creditors, in which case both would receive 100 €. Hence, it is necessary to develop bankruptcy rules according to which the residual value is distributed.

These methods have been applied to transboundary water issues as well, both theoretically and empirically (Box 6.2). The application of the bankruptcy rules is based on the Principle of the Territorial Integration of all Basin States (TIBS) as explained in Sect. 6.1. The whole catchment area is collectively owned by the riparian countries. If water becomes scarce, the allocation should not be based on the geographical position of these countries. Their claims are equally legitimate and are the sole information that will be taken into account within the allocation process.

On the one hand, it is advantageous that we do not need any complex models that reflect the link between water usage and economic welfare. It is all about water quantities that are measurable. On the other hand, by restricting water distribution alone, we give up the possibility of combined contracts in which other goods and services are specified in addition to water, which makes trade possible. As will be shown subsequently, one has to be careful when transferring these rules from the context of credit markets to water issues, because the very nature of claims in both sectors is rather different.

Suppose a set of $N = \{1, 2, 3, \ldots, n\}$ countries share a water resource R. Their claims can be summarized by a claim vector $c = \left[c_1\ c_2\ \ldots\ c_n\right]$. Water is scarce, hence, we assume that

$$\sum_{i=1}^{n} c_i > R \tag{6.17}$$

Bankruptcy rules specify the allocation of R to the countries by a sharing rule function $x(R, c)$, where x is an n-dimensional vector. There is a variety of properties that are met by different distribution rules to varying degrees. These properties refer to consistency criteria that follow the principle of rationality and to normative criteria that take fairness considerations into account.

Basic properties are the following requirements that represent plausibility:

1. *Feasibility*: The implementation of bankruptcy rules must be feasible, i.e., the sum of water allotments required by the rules should not exceed the amount of

[18] A concise survey is Thomson (2002).

water available, i.e.,

$$\sum_{i=1}^{n} x_i(R, c) \leq R \tag{6.18}$$

2. *Efficiency*: Efficiency excludes waste. There is no water loss, i.e.,

$$\sum_{i=1}^{n} x_i(R, c) = R \tag{6.19}$$

3. *Claim boundedness*: Claim boundedness is not only a basic property that relates to plausibility but also to fairness considerations. The water allotment of a rule shall never exceed the claim stated, i.e.,

$$x(R, c) \leq c \tag{6.20}$$

   A rationing scheme would be considered very unfair if it were to allocate more water than the claim of the respective riparian state.

There are additional, more specific properties to make a bankruptcy rule considerable for implementation. These rules refer to fairness considerations, and what the Helsinki Rules (Article 4) call "a reasonable and equitable share in the beneficial uses of the waters of an international drainage basin".

4. *Consistency*: Consistency refers not only to rationality but also to fairness. A water allocation which is considered as fair for all countries in a water treaty remains fair also if a subgroup shares the water allotted to them. Formally,

$$\text{for all } S \subset N \;\; x_i(R, c) = x_i\left(R - \sum_{i \in (N/S)} x_i, c_S\right) \tag{6.21}$$

   where c_S is the vector of claims of the countries in S.
5. *Equal Treatment of Equals:* This condition is central to a fair water allocation. The same claims should lead to the same water allocation. Formally,

$$\text{for all } i, j \in N \text{ if } c_i = c_j \Rightarrow x_i(R, c) = x_j(R, c) \tag{6.22}$$

6. *Order Preservation*: Fairness also requires that those countries claiming more water receive more water under the sharing rule.

$$\text{for all } i, j \in N \text{ if } c_i \geq c_j \Rightarrow x_i(R, c) \geq x_j(R, c) \tag{6.23}$$

   Of course, this requirement presumes that claims are legitimate and justifiable.

7. *Regressivity*: If a certain degree of inequity aversion prevails, regression may be required:

$$\text{for all } i, j \in N \text{ if } c_i \geq c_j \Rightarrow \frac{x_i(R,c)}{c_i} \leq \frac{x_j(R,c)}{c_j} \tag{6.24}$$

In other words, the relative fulfillment of the claims decreases with the amount of the claims. Whether this criterion makes sense depends heavily on the nature of the claims.

8. *Claim monotonicity*: Division rules should not be static. This means that they should not only refer to actual values of claims and water quantities but also be flexible with regard to changes in framework conditions. Fairness must also apply to changed input data. It is fair to say that the allocation of water resources increases for those riparian states whose justified claims increase. Formally,

$$\text{for all } i, j \in N \text{ if } c_i' \geq c_i \Rightarrow x_i(R, c_i', c_{-i}) \geq x_i(R, c) \tag{6.25}$$

where $c_{-i} = [c_1, c_2, c_{i-1}, c_{i+1}, c_n]$.

9. *Resource monotonicity*: The supply of water varies heavily depending on weather and climate conditions. The distribution rules must be fair for all possible water scarcity scenarios. Resource monotonicity is considered as fair. If there is less (more) water, every riparian state should get less (more):

$$\text{if } R' \geq R \Rightarrow x(R', c) \geq x(R, c) \tag{6.26}$$

6.4.2 Hydrologically Unconstrained Allocation Rules

The fulfillment of the individual properties defined above does not determine a unique allocation. In the literature, it is rather the case that different rules, seen as reasonable and fair, are proposed. The application of these rules leads to different allocations. In the following, we will present the most important ones, examine their properties and their practical applicability. We assume that the rules can be implemented hydrologically, i.e., the calculated allocations can also be physically transferred to the water users. This is the unconstrained case.[19]

Proportional Rule

Let us start with the most widely known and used rule. This rule is already mentioned in Aristotle's Nikomachian ethics.[20] The rule divides the available water in proportion to the claims, i.e.,

$$x_i^P = \frac{c_i}{\sum_{j=1}^{n} c_j} R, \quad i = \{1, 2, \ldots, n\} \tag{6.27}$$

[19]The constrained cases refer to river basins where the unidirectionality of the water flow may lead to the case that the calculated allocations cannot be physically realized. See the next subsection.

[20]Nikomachian ethics, book V. See the explanation in Young (1994), p. 64 ff.

This rule is self-evident and fulfills, it seems, the sense of justice at once. This is certainly due to the principle of accountability[21] that underlies this rule. Subsequently, we will compare this rule with other rules for different types of claims.

Adjusted Proportional Rule
This rule puts more weight to those countries with higher claims. It is derived in a two-step procedure. First, the so-called minimum rights have to be determined as

$$m_i = max\left[0, R - \sum_{j \neq i} c_j\right], \quad i = \{1, 2, \ldots, n\} \tag{6.28}$$

Minimum rights refer to the water allocation which is not contested. All $j \neq i$ concede this residual to i. It is simply the water that is left after serving the claims of all the other water users.[22] After the minimum rights have been distributed, the second step follows:

$$x_i^{AP} = m_i + \frac{c_i - m_i}{\sum_{j=1}^{n}(c_j - m_j)}\left(R - \sum_{j=1}^{n} m_j\right), \quad i = \{1, 2, \ldots, n\} \tag{6.29}$$

Here, the water division consists of the minimum rights plus the proportional portion of the residual water supply, which will be left after deduction of the minimum rights. The proportionality factor is formed with the help of the claims adjusted for the minimum rights. One should be careful with the concept of minimum rights. This is not a minimal provision of water in terms of human rights for water. It refers only to the amount of water the other competitors would leave without the request for negotiation.

Constrained Equal Award (CEA)
The CEA rule sets the water allocation in a very egalitarian way. Each country receives the same portion of available water regardless of its claims. Claims only play a role insofar as the equal shares apportioned may be higher than these. In this case, only the claims will be covered.

Formally,

$$x_i^{CEA} = min[E, c_i] \quad \text{where} \quad \sum_{i=1}^{n} min[E, c_i] = R \tag{6.30}$$

where E is the equal share provided E is less than both claims.

Constrained Equal Loss (CEL)
Instead of focussing on the distribution of the water, it is also possible to turn the observation around and look at the distribution of the water loss defined as the number

[21] See Sect. 3.3.
[22] Of course, if the residual $R - \sum_{j \neq i} c_j$ were negative, then the other claims are not met because negative minimum rights are excluded. That is reasonable and therefore acceptable.

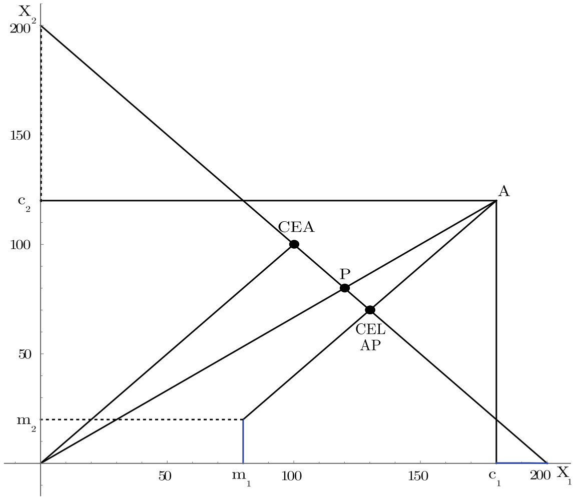


Fig. 6.3 Bankruptcy rules. *Source* own illustration

of claims not fulfilled, i.e., the allocation of the total loss $\sum c_i - R$. The CEL rule is defined as follows:

$$x_i^{CEA} = max[0, c_i - E] \quad \text{where} \quad \sum_{i=1}^{n} max[0, c_i - E] = R \tag{6.31}$$

where E is the equal share of loss.

All the rules introduced do satisfy the properties with the exception of regressivity.[23]

Which rule is fair now? As we will see, this question cannot generally be answered, but depends on the circumstances[24]. By circumstances, we understand the object of allocation, in this case water, and the type and degree of legitimacy of claims. With the help of an example, we will discuss the appropriateness of sharing rules depending on the type of legitimacy of claims. At first, however, we want to look at a numerical example in order to examine the distributional effects of the four rules. Assume the following numerical values: $R = 200$, $c_1 = 180$, and $c_2 = 120$. Straightforward application of the rules allows to calculate the corresponding allocations and to insert the numerical values in Fig. 6.3.[25]

The budget line $x_2 = R - x_1$ represents all possible water distributions. The claims are plotted with a vertical and a horizontal line. Due to the scarcity of water, its intersection lies north-east of the budget line. The allocation according to the

[23]For the case of two countries, the proof is left to the reader; for the n-country case, the proof is more extensive, see, e.g., Thomson (2002).

[24]There are interesting studies which empirically determine the assessment of fairness of bankruptcy rules within the framework of experiments, see, e.g., Gaechter and Riedl (2006)

[25]Details can be found in Exercise 6.3.

proportional rule is the intersection of the budget line with the line connecting the origin with the claim point A, the slope of which is c_2/c_1 (see Fig. 6.3). Similarly, the CEA rule can be identified by the intersection of a line with a slope of 1 that starts from the origin. The available water is shared in half. This is due to this numerical example where the water allocation according to this rule falls short of both claims. The CEL rule is constructed by running a 45°-line from point A to the budget line. This line implements the requirement that the loss should be apportioned on a fifty-fifty base between the two countries. In the two country case, this allocation is identical to the adjusted proportionality rule. Here, we start from the minimal right point $\{m_1, m_2\}$ with a line of 45° degrees.

What we can see from this figure is that there is an ordering of rules with respect to the degree of equality. The CEA rule distributes the available water equally. Only if claims are fully covered, the rule deviates from the equal share principle (see point A). The proportional rule prefers the country with the higher claims somewhat whereas the CEL rule and the adjusted proportionality rule favor countries with higher claims.

But what is just, the complete equality of water allocations or the equality of the individual water losses suffered, measured by the degree of regressivity x_i/c_i? It turns out that only the proportional rule weakly satisfies this property in general, i.e., the ratio is constant with respect to different values of R. The CEA rule exhibits progressivity, i.e., the percentage of fulfilled claims rises with the number of claims and the CEL rule is undetermined. Progressivity and degressivity depend on the amount of water supplied. However, the question remains as to whether the allocation should be made more evenly distributed or whether the claims should be taken into account in the allocation. This question depends on the very nature of these claims or on the attributes claimants have. The following scenarios show that a sharing rule should only be decided upon once the legitimacy of claims has been clarified.

Scenario I

There are two countries with different population sizes. Country 1 (L1) is large compared to country 2 (L2), i.e., n_1 is larger than n_2, where n_i is the population size. The (culturally determined) subsistence level of water per person is υ. We assume that this subsistence level is the same in both countries. The claims are therefore

$$c_i = \upsilon n_i, \quad i = \{1, 2\} \tag{6.32}$$

We assume that the water available is less than the aggregated claims, i.e., $c_1 + c_2 > R$. Applying the proportionality rule immediately results in $x_i = (n_i/(n_1 + n_2))R$. Dividing the allocation by the respective population size yields the water allocation per capita, which is equal in both countries:

$$\frac{x_1}{n_1} = \frac{R}{(n_1 + n_2)} = \frac{x_2}{n_2} \tag{6.33}$$

Of course, due to water scarcity, this allocation is less than υ.

For water allocation under the CEA rule, we have to differentiate between two cases because of the claim boundedness: If $R/2 < c_2$, i.e., the water allocation is less than the lesser claim, both countries receive the same amount of water R/2. It follows that the water allocation per capita in L1 is lower than in L2. In other words, the more populous country gets less water per capita than the country with a smaller population. This result follows also for the case that $R/2 > c_2$. In this case, people from L2 receive $c_2/n_2 = (\upsilon n_2)/n_2$ (see Eq. (6.32)). Because water is scarce, i.e., $c_1 + c_2 > R$, per capita allocation in L1 is less than υ.

Both claims are well-founded, as they are derived from people's elementary needs. They should, therefore, not be called into question when drawing up a water contract. Fundamental needs should be met as good as possible. The more fundamental a need, the greater the role of equality that applies here to people, not countries. Therefore, the P rule is likely to be preferred to the CEA rule, because it is not acceptable that the more populous country has a lower per capita water supply due to the basic need property of water.

Scenario II

Let us resume the water allocation problem within the same mathematical structure, but with a different economic context. Again, there are two countries L1 and L2. Both countries have the same national product y and the same population size. However, the water consumption of the first country is higher than that of the second country, because L1 is more inefficient than L2, which leads to different water claims:

$$c_i = \epsilon_i y, \quad i = \{1, 2\} \tag{6.34}$$

where ϵ_i is the water intensity per unit of social product of the respective country. Since L1 is less efficient than L2, we have $\epsilon_1 > \epsilon_2$. Again, we assume water scarcity, which makes it necessary to apply a bankruptcy rule. Applying the P rule leads to

$$x_1 = (\epsilon_1/(\epsilon_1 + \epsilon_2))R \quad \text{and} \quad x_2 = (\epsilon_2/(\epsilon_1 + \epsilon_2))R \tag{6.35}$$

Hence, the inefficient country L1 gets more water than country 2. Dividing x_i by ϵ_i yields the national product under rationing, i.e.,

$$\frac{x_1}{\epsilon_1} = (1/(\epsilon_1 + \epsilon_2))R = \frac{x_2}{\epsilon_2} \tag{6.36}$$

Both countries end up with less affluence. However, the P rule allocates the water such that both countries bear the scarcity equally.

Again, if we apply the CEA rule, we have to distinguish between two cases: In the first case, i.e., $R/2 < c_2$, both countries are allocated the same amount of $R/2$. This implies that L2 can sustain a larger national product than L1:

$$\frac{x_2}{\epsilon_2} = \frac{R}{2\epsilon_2} > \frac{R}{2\epsilon_1} = \frac{x_1}{\epsilon_1} \tag{6.37}$$

The same result occurs if $R/2 > c_2$. In this case, the water allocation to L2 is $c_2 = y\epsilon_2$. Hence, L2 can sustain the social product of y. This implies that the affluence in L1 decreases due to water scarcity, i.e., the social product is less than y:

$$\frac{R - c_2}{\epsilon_1} < \frac{c_1}{\epsilon_1} = y \tag{6.38}$$

since $R - c_2 < c_1$.

Is it fair that the efficient country can maintain its standard of living, while L1, due to its inefficient water economy, has to accept a loss of welfare if the CEA rule is applied? Or should the proportional rule be applied, which will lead to an equal decrease in GDP in both countries? In contrast to scenario I, this might require more inquiries about the reasons for the different water efficiencies. Are countries accountable for that, or do these different efficiencies reflect geological properties countries cannot influence? In the latter case, they are not responsible in the sense of the principle of moral arbitrariness (as introduced in Sect. 3.3) and they should bear the scarcity equally. If the different water intensities are rooted in mismanagement, then the principle of accountability will apply with the consequence that the CEA rule is to be applied.

A comparison of both scenarios shows that bankruptcy rules should be weighted carefully before being adopted. Beyond the question of which rule has to be applied, the nature of the claims must also be examined. Are those derived from existential needs or are they economic wants? In analogy to Maslow's hierarchy of needs[26], we can put the claims in a prioritized order. This naturally means that the introduced water allocation rules have to be adapted to these hierarchies of needs. Let us go back to our numerical example and assume that the claims can be subdivided into basic needs and secondary needs. Let us assume that L1 is a developed country with a relatively small population ($n_1 = 4$) while L2 is a developing country with a large population ($n_2 = 10$). The subsistence minimum of water per capita is $w_s = 10$. Claims are the sum of basis needs and secondary wants. For L1, we have

$$c_1 = 180 = c_1^1 + c_1^2 = w_s n_1 + c_1^2 = 40 + 140 \tag{6.39}$$

and for L2:

$$c_2 = 120 = c_2^1 + c_2^2 = w_s n_2 + c_2^2 = 100 + 20 \tag{6.40}$$

We first look at the undifferentiated water allocation to the two countries (see Fig. 6.3). The numerical values are given in Table 6.11 for each country and for the three distribution rules considered.[27] The quantities of water per capita are calculated for each rule: These values differ considerably between the different rules. For the P rule and CEL rule, the water allocation per capita is below the subsistence minimum w_s.

[26]See Sect. 3.5 for Maslow's hierarchy.

[27]Notice, that in the two country case, the water allocation under the AP-rule is identical to the allocation under the CEL rule.

Table 6.11 Non-differentiated water allocation

	P Rule		CEA Rule		CEL Rule	
Countries	x_i^P	x_i^P/n_i	x_i^{CEA}	x_i^{CEA}/n_i	x_i^{CEL}	x_i^{CEL}/n_i
L1	120	30	100	25	130	32.5
L2	80	8	100	10	70	7

Table 6.12 Differentiated water allocation

	P Rule		CEA Rule		CEL Rule	
Countries	x_i^P	x_i^P/n_i	x_i^{CEA}	x_i^{CEA}/n_i	x_i^{CEL}	x_i^{CEL}/n_i
L1	92.5	23.125	80	20	100	25
L2	107.5	10.75	120	12	100	10

Needs of different priority make the direct application of bankruptcy rules to the aggregated claims questionable. Basic needs should definitely be met.[28] Thus, we should adopt a sequential approach. First, water should be distributed according to basic needs $\{c_1^1, c_2^1\}$. After that, the residual water $R - c_1^1 - c_2^1$ should be allocated taking into account the secondary claims $\{c_1^2, c_2^2\}$. The water allocation rule is then $x_i(R - c_1^1 - c_2^1, \{c_1^2, c_2^2\})$. Total water assignments are

$$c_1^1 + x_1(R - c_1^1 - c_2^1, \{c_1^2, c_2^2\}) \quad \text{and} \quad c_2^1 + x_2(R - c_1^1 - c_2^1, \{c_1^2, c_2^2\}) \qquad (6.41)$$

In Table 6.12, we have calculated these allocations for the three rules.

The sequential treatment of the requirements leads to the fact that the subsistence level is also fulfilled for the more populated country. The remaining water is then distributed to the two countries on the basis of the remaining entitlements, without taking into account the population figures. As shown in Table 6.11 and in Fig. 6.3, the CEA- and CEL rule take greater account of the asymmetry of claims than the P rule.

6.4.3 Sequential Allocation Rules

If we want to apply bankruptcy rules to a river system, its specific hydrological characteristics must be taken into account. The unmodified application of bankruptcy rules could lead to the fact that the resulting water allocations are not feasible. Imagine L1 is upstream, L2 downstream. The river system is characterized by a relatively low inflow in L1 and a big inflow in L2. A direct application of the CEA rule, for example, could lead to an equal distribution of the available water, i.e., the sum of all water

[28] Of course, if the sum of basic needs exceeds the water quantity available, we can apply the rules directly taking the basic needs as claims.

Table 6.13 Claims and inflows along a river

Countries	Inflows	Claims	Rule-based allocation	Downstream availability	Downstream excess claims
L1	$R_1 = 100$	$c_1 = 180$	x_1	$E_1 = R_1$	$cd_1 = (c_2 - R_2) = 20$
L2	$R_2 = 100$	$c_2 = 120$	x_2	$E_2 = R_2 + (R_1 - x_1)$	$cd_2 = 0$
Sum	200	300	200	–	–

inflows. This would not be possible considering the geographical position of the two countries and the low inflow in L1. For this constrained case, a sequential sharing rule has been proposed.[29] This bankruptcy rule takes the hierarchical order of countries and the unidirectionality of the river into account. As before, the sum of claims $c_1 + c_2$ is higher that the sum of inflows $R_1 + R_2$. However, due to the specific geography of the river, the feasibility of a water allocation must also be ensured. For this purpose, we define total available water in the two territories:

$$E_1 = R_1, \quad E_2 = R_2 + (R_1 - x_1) \tag{6.42}$$

where x_i is the water allocation (per country) according to a sharing rule to be specified. Equation (6.42) shows the typical water availability structure of a river depending on the position of the countries. Finally, to apply the bankruptcy rules to a river system, we have to define the downstream excess claims

$$cd_1 = (c_2 - R_2), \quad cd_2 = 0 \tag{6.43}$$

Equation (6.43) is of crucial importance for the sharing rules. Downstream country's claims are adjusted by its inflow R_2. The very reason for this approach is the insight that R_2 is always with country two due to the unidirectional flow of the river.

Starting with the water allocation for L1, we have to compare the available water for L1 with the excess claims of L2. In the following table, all the relevant variables are summarized. In addition to the numerical values in the example above, we have added numerical values for the inflows R_1 and R_2, so we can calculate the water allocations proposed by the sequential sharing rules. Table 6.13 lists all relevant parameters to apply bankruptcy rules. In the following, we will consider the sequential variants of the P rule, the CEA rule, and the CEL rule. Let us start with the P rule.

P Rule

In the first step, the water allocation for L1 is calculated, as shown in the first row of Table 6.14. x_1^{S-P} is a fraction λ_1 of the claim c_1. The amount of water remaining is the same fraction of the residual net claims of L2, the downstream excess claims. The proportionality factor λ_1 is simply the ratio between the availability of water

[29] See Ansink and Weikard (2012).

Table 6.14 The sequential proportionality rule

Countries	Rule-based water allocation to L_i	Rule-based downstream allocation	Proportionality factor
L1	$x_1^{S-P} = \lambda_1 c_1$ $x_1^{S-P} = 0.5 \cdot 180 = 90$	$x_{cd1}^{S-P} = \lambda_1 cd_1$ $= 0.5 \cdot 20 = 10$	$\lambda_1 = E_1/(c_1 + cd_1)$ $= 100/(180 + 20) = 0.5$
L2	$x_2^{S-P} = E_2 =$ $R_2 + (R_1 - x_1^{S-P})$ $x_2^{S-P} =$ $100 + (100 - 90) = 110$	$x_{cd12}^{S-P} = 0$	——

Table 6.15 The sequential CEA Rule

Countries	Rule-based water allocation to L_i	Rule-based downstream allocation	Award calculation
L1	$x_1^{S-CEA} = Min[c_1, \lambda_1]$ $x_1^{S-CEA} = 80$	$x_{cd1}^{S-CEA} = Min[cd_1, \lambda_1]$ $x_{cd1}^{S-CEA} = 20$	$x_1^{S-CEA} + x_{cd1}^{S-CEA} =$ E_1 $Min[180, \lambda_1] +$ $Min[20, \lambda_1] = 100$ $\rightarrow \lambda_1 = 80$
L2	$x_2^{S-CEA} = E_2$ $= 100 + (100 - 80) =$ 120	$x_{cd2}^{S-CEA} = 0$	—-

and the sum of claims $c_1 + cd_1$. After having determined the water allocation for L1, we go one step downstream and determine the water allocation for L2. In our simple two country case, we only have to allocate the remaining water supply E_2.

CEA Rule
The water shares that follow from the CEA rule are also calculated in a sequential way. Again, we begin with the upstream country L1. We start in the first line (L1) of Table 6.15 and split the available water R_1 to L1 and L2 downstream according to the CEA rule. The upper bounds (see the principle of claim boundedness Eq. (6.20)) are the claims c_1 and the excess claims of the downstream country cd_1. For the given numerical values, it follows that L1 receives 80 and the downstream country 20. We proceed to the second country L2 and calculate the residual water available, $E_2 = R_2 + (R_1 - x_1^{S-CEA})$ which yields 120. Finally, we have to calculate the water allocation sequentially for the CEL rule.

CEL Rule
Again, we begin with the first row for country L1 (Table 6.16), and calculate λ_1. First, we assume that there exists an equal share of loss. This is not feasible, because under this assumption the second term of the right-hand side would get negative. Hence, we assume that this term is nil which yields that $\lambda_1 = 80$. Having calculated x_1^{S-CEL}, we can compute $x_2^{S-CEL} = E_2$.

Discussion
The sequential sharing rules are somewhat exhaustive with regard to the calculation effort even if there are only two riparian countries. However, their logic is clear. The

Table 6.16 The sequential CEL Rule

Countries	Rule-based water allocation to L_i	Rule-based downstream allocation	Loss calculation
L1	$x_1^{S-CEL} = Max[0, c_1 - \lambda_1]$ $x_1^{S-CEL} = 100$	$x_{cd1}^{S-CEL} = Max[0, cd_1 - \lambda_1]$ $x_{cd1}^{S-CEL} = 20$	$x_1^{S-CEL} + x_{cd1}^{S-CEL} = E_1$ $Max[0, 180 - \lambda_1] + Max[0, 20 - \lambda_1] = 100$ $\rightarrow \lambda_1 = 80$
L2	$x_2^{S-CEL} = E_2 = 100 + (100 - 100) = 100$	$x_{cd2}^{S-CEL} = 0$	—-

calculation begins upstream and first divides the available water on the basis of the existing claims between the first country and the remaining countries. If the water allocation for L1 is calculated, the calculation moves downstream and the remaining water is allocated to L2. The reason for the sequential approach is justified by the unidirectionality of the river system flow. However, one must be careful with the direct application of the sequential sharing rules, whenever water is to be distributed along a river.

Figure 6.4 shows the water allocation calculated in the last three tables graphically and compares it with the actual application of pure bankruptcy rules. The figure displays the water allocation from the bankruptcy rules $\{P, CEA, CEL\}$ depicted in Fig. 6.3. The P rule and the CEL rule are not feasible since these rules allocate more water to L1 than available by R_1. The river structure is depicted by the blue dotted water budget line with its kink at $\{R_1, R_2\}$. Feasible water allocations are only those along the blue dotted budget line. The sequential rules are by construction feasible. The P Rule (see Table 6.14) allocates the water on the basis of proportional shares. The proportional distribution is calculated with respect to inflow R1. In addition,[30] country 2 also receives inflow R2, which for hydrological reasons cannot be split between L1 and L2.

One has to be careful: The fact that the algorithm of the sequential sharing rule takes the hydrology of a river into account does not imply that these rules should always be applied for river basins. This is a normative decision. The consideration of the flow direction has the consequence that the claims from upstream are no longer considered in the calculation of the water allocation downstream. This follows from the construction of downstream excess claims (see, e.g., Table 6.14). Thus the calculated water allocations are compatible with hydrology, but at the expense of the lower weighting of upstream claims. The modification of the bankruptcy rules for unidirectional running waters is therefore not only of a technical nature, but also involves a normative adjustment. Thus, the decision as to which of the two sets of rules, the direct or the sequential, is to be applied remains not only a

[30]The line from $\{0, R_2\}$ to the point S-P and $\{c_1, c_2\}$ can be constructed from the L1-row in Table 6.14. Since, $x_1^{S-P} = \lambda_1 c_1 = R_1[c_1/(c_1 + (c_2 - R_2))]$ and $x_2^{S-P} - R_2 = R_1 - x_1^{S-P} = R_2 + R_1((c_2 - R_2)((c_1 + (c_2 - R_2))$ we can calculate $(x_2^{S-P} - R_2)/x_1^{S-P} = (c_2 - R_2)/c_1$.

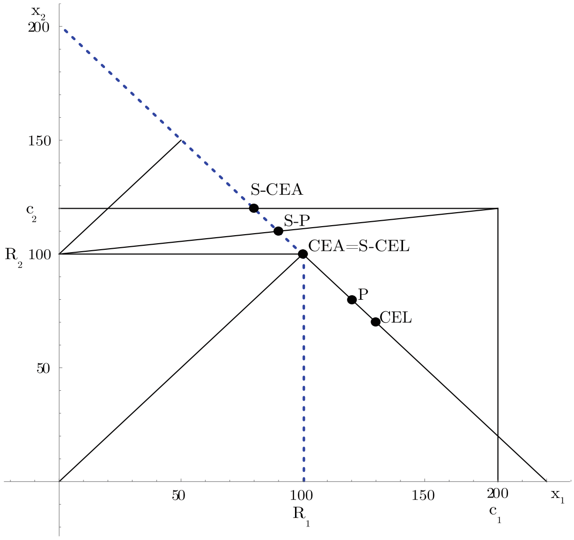


Fig. 6.4 Bankruptcy rules and sequential sharing rules. *Source* own illustration

technical-hydrological one, but also a normative one. Table 6.14 clearly shows that the sequentiality favors the downstream country.

If the application of this sequential rule is perceived as unfair, one can return to the direct rule. Since it cannot be implemented in our example, a second best option is available, namely to come as close as possible to the desired allocation. This allocation is $\{x_1 = R_1, x_2 = R_2\}$. In Fig. 6.4, we see that this second best option is closest to the P rule and the CEL rule. At the same time, it is the water allocation according to the principle of absolute territorial sovereignty (ATS). Both riparian countries make full use of the water that originates in their territory. Notice, however, that this result is not based on the principle of sovereignty, but on fairness considerations.

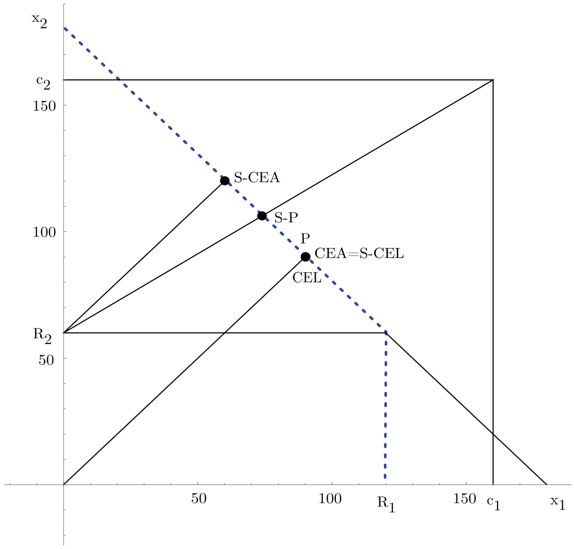
The normative problem of the sequential rule is even clearer in Fig. 6.5. Here, we have chosen numerical values of the relevant variables such that both sets of sharing rules, the direct ones and the sequential ones, are hydrologically feasible.[31]

In this scenario, the choice of allocation only depends on normative criteria. The hydrology does not constrain the choice. The sequential sharing rules favor the downstream country with exception of the sequential CEL rule. In our two country example, both CEL rules, together with the direct CEA rule, lead to the same water allocation. This is due to the assumption that both countries have the same claims. The sequential versions of the P- and CEA rule do not satisfy the principle of Equal Treatment of Equals.[32] Thus, whenever the hydrology allows the direct application of bankruptcy rules, the application of the sequential rules cannot be justified. But even if the direct rules cannot be applied for hydrological reasons, the sequential rules cannot be applied automatically. It may turn out that second-best solutions are preferred for reasons of fairness, as shown above.

[31] The numerical values are $\{R = 180, c_1 = 160, c_2 = 160, R_1 = 120, R_2 = 60\}$.

[32] See the above-introduced properties. If $c_1 = c_2$ then $x_1 = x_2$. However, this might not be possible due to the hydrological conditions.

Fig. 6.5 Bankruptcy rules. *Source* own illustration



The discussion shows that we have to be careful when applying bankruptcy rules. The mere application of mathematical rules does not solve the sharing problem. These methodological tools are helpful but cannot substitute the intrinsic fairness problems of sharing scarce water resources. If the distribution rules are discussed in principle, the geographical position of the countries poses a principle problem. Geography could be regarded as morally arbitrary.[33] This implies that the actual order along the river must not result in any disadvantages for the individual countries. If this argument is valid, it is might be necessary to talk about the legitimacy of claims before applying a rule of division. Moreover, if moral arbitrariness is the basis of the TIBS principle, we might end up with the insight that water treaties have to be constructed in a more complex way. The pure allocation of water might not suffice to compensate for geographical disadvantages. This leads back to our discussion of welfare-based approaches which allow for side payments and other in-kind compensations. Of course, with this step we have to face all the problems discussed in Sect. 6.3.

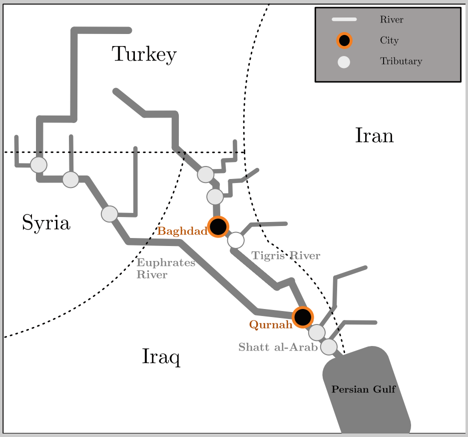
Box 6.2 Applying water bankruptcy rules to the Euphrates River

The Euphrates flows through three countries: Turkey, Syria, and Iraq. Together with the Tigris, it forms the water catchment area, which has been known as Mesopotamia ever since. From its springs in Turkey to the Persian Gulf, the Euphrates River stretches over a length of 2,786 km. The average annual water flow is 25 billion m^3 serving 23 million people in the transboundary catchment area. The water use in all three riparian countries relates mainly to irrigation

[33] See the fairness principles in Sect. 3.3.

(70%), hydropower, and drinking water supply. Data records of the past 70 years indicate a negative trend of water availability measured as a decrease in mean annual flows. The need for sustainable water treaties is, therefore, becoming increasingly important. Currently, there are two bilateral accords in force: an agreement between Syria and Turkey specifying the minimum average flow at the Syrian-Turkish border and another treaty between Iraq and Syria determining the water allocation of Euphrates water between these two countries.

The linear arrangement of neighboring states and the simple geography of tributaries make the Euphrates a good example of the sequential sharing rule. The following map shows the geographical structure. The main water inflow is provided from Turkey. In Syria, there are three tributaries contributing water to the Euphrates (the Sajur, the Balikh, and the Khabur). Iraq does not contribute to the watercourse.



Source: Jarkeh et al. (2016).

The following table summarizes all necessary information to apply the sharing allocation rules.

Claims and contributions				
Riparian Countries	Claim (MCM/year)	Claim %	Contribution (MCM/year)	Contribution %
Turkey	14,000	25.6	31,580	88.8
Syria	12,600	23	4000	11.2
Iraq	28,100	51.4	0	0

While water inflows are well measurable, the determination of the claims requires an estimate of the water demand components from the various economic sectors of the riparian countries. There are several studies in the literature, the results of which are gathered by Jarkeh et al. (2016) and then entered into the table as a best guess. The application of the sequential sharing rule yields the following water allocation for the three countries (as a percentage of the claims).

Sequential sharing rule. *Source* Jarkeh et al. (2016)

Riparian	Sequential sharing rule		
	P Rule	CEA Rule	CEL Rule
Turkey	62	100	32
Syria	66	86	62
Iraq	66	38	83

It is interesting to see that the percentage satisfaction is almost the same for all three countries in the case of the sequential P rule, despite Iraq's lack of inflow and its high claims. The application of the CEA rule leads to a complete coverage of Turkish water demand, while Iraq only receives about 40% of its claims. The CEL rule would yield exactly the opposite: Iraq achieves the highest fulfillment of claims while Turkey is allowed to use only extremely little water, 70% of its claims would not be covered. The remaining water is to cross the Turkish border for the benefit of downstream states. The question remains as to whether this water allocation has any chance of being implemented...

Sources UN-ESCWA and BGR (2013), Jarkeh et al. (2016)

6.5 Flexible Water Sharing

If one examines the emergence of water agreements between riparian states on an international water body, it becomes apparent that it often takes years to reach a successful conclusion. The Indus Waters Treaty, for instance, took over 6 years of bargaining until it was concluded with the assistance of the World Bank. Agreements are rather difficult to alter in response to unexpected changes of underlying hydro-climatological conditions. Specifically, if the volume and the pattern of the regional water inflow into an international catchment area changes, conflicts may occur. This instability is the result of the inflexibility of water agreements. New hydrological framework conditions are difficult to be taken into account in the treaties. Compliance with a treaty on the basis of outdated framework conditions can lead to a situation in which the conflict is more advantageous for some partner states than compliance with the treaty concluded. In the following, we will, therefore, investigate how different contract types can influence the behavior of the contracting parties in the event of unexpected changes in the hydrological conditions. We limit ourselves to investigating the case of decreasing water inflows into an international river.

Contract Types

Roughly, we can distinguish between three contract types:

- Complete contingent contracts: This complex type of contract would be the best answer to the variability of the water supply. For every conceivable hydrological and climatological scenario, the water quantities are allocated ex ante, possibly with corresponding non-water transfers. However, the amount of information required is very high. The concept of complex contingent contracts actually leaves out the problem of unexpected events. Thus, we subsequently only focus on the following two contract types.

- Fixed flow allocation: This type of water sharing rule is most common. A fixed amount of water for the downstream country is stipulated. In the following analysis, we assume that this fixed amount is accompanied by a non-water transfer from the downstream country to the upstream one.
- Proportional allocations: The water allocation follows a percentage rule. The downstream country is entitled to a certain percentage of the water supply available upstream. Again, we combine this type of agreement with a non-water transfer which is also proportional to the water received.

To analyze these two contract forms, we take up our example of a river with two riparian countries from Sect. 6.2. Upstream is labeled 1 and downstream is denoted by 2. In the following, we assume that the river is fed only by water upstream. There is no downstream tributary (i.e., $R_2 = 0$). The optimal allocation then can be derived from the following maximization program.

$$\max_{w_1, w_2} \quad B_1(w_1) + B_2(w_2) \quad w_1 + w_1 \leq R_1 \tag{6.44}$$

Let the water supply be scarce. Then, the optimal fixed water supply for downstream w_2^* satisfies the following condition

$$B_1^{'}(R_1 - w_2) = B_2^{'}(w_2) \tag{6.45}$$

Similarly, the proportional sharing rule can be fixed. The allotted amount of water downstream is expressed as percentage α^* of the total water available:

$$w_2^* = (1 - \alpha^*)R_1 \quad w_1^* = R_1 - w_2^* = \alpha^* R_1 \tag{6.46}$$

Thus if the water supply is constant, both allocations are identical. However, if the water supply R_1 decreases unexpectedly, the effects on both contract types are rather different. To show this, we first have to determine the non-water transfers in both contracts, the level of which depends on the bargaining power of both riparian countries. Of course, whatever the amount of money (or other non-water transfer vehicles) will be, the solution must lie in the core as defined in Sect. 6.2:

$$B_1(w_1) + T \geq B_1(R_1) \quad \text{and} \quad B_2(w_2) - T \geq 0 \tag{6.47}$$

where T is the non-water transfer. In the case of a fixed flow agreement, T is also a fixed amount. In the case of a proportional allocation of the water supply, T varies with the amount of water transferred from upstream to downstream whereby the price of water t is fixed.

$$T(r) = t(1 - \alpha)r \tag{6.48}$$

where $r \leq R_1$ is the actual water supply and t is the fixed water price. The proportional water agreement makes the non-water transfer contingent on the actual amount of water delivered to the downstream country. The fixed water price is calculated as

$$t = \frac{T}{R_1(1-\alpha^*)} \tag{6.49}$$

that is, t are the average payments per amount of water delivered to the downstream riparian at the time of the conclusion of the contract, i.e., when $r = R_1$. Inserting Eq. (6.49) into Eq. (6.48) yields

$$T(r) = \frac{r}{R_1}T \tag{6.50}$$

Robustness to Changing Hydrological Conditions
Now, the question needs to be answered on how the two types of contracts perform if, let's say as a result of climate change, the water inflow unexpectedly decreases. Three criteria are important here: efficiency, robustness, and fairness. Before the unexpected water reduction, both contracts are efficient and fair by construction. The sum of the benefits is maximized, both parties have agreed to the contract by appropriate choice of a transfer T in the core and the resulting distribution of the benefits is considered fair.

What happens now, if the water supply decreases, that is $r < R_1$? In both types of contracts, the quantities of water allocated differ from the quantities originally negotiated, with the result that the efficiency properties change. This also applies to the distribution of benefits. Further, it is unclear whether the two parties have an incentive to comply with the contract, i.e., how robust the contract is. In the following, we focus on this issue.

Let us assume that the water supply of the river r is falling, i.e., $r < R_1$. In the case of a fixed contract, the stipulated water allocation at the outset is w_2^* ($R_1 - w_2^*$) for downstream (upstream). Let us assume that the fixed non-water transfer is calculated such that both countries are better off than in the case of no agreement, i.e.:

$$B_1(R_1 - w_2^*) + T^* > B_2(R_1) \quad \text{and} \quad B_2(w_2^*) - T^* > 0 \tag{6.51}$$

For example, we could calculate T^* such that total welfare of both riparian countries is distributed according to the Shapley values.[34] In this case,

$$T^* = \frac{1}{2}\left[B_2(w_2^*) - B_1(R_1 - w_2^*) + B_1(R_1)\right] \tag{6.52}$$

Now, let us analyze the robustness of this contract with the help of a numerical example: $B_i^{'} = a - bw_i$, $a = 100, b = 1$, and $R_1 = 100$. Let's start with the upstream country. The country will stick to the contract as long as it is better off than in the

[34] See Sect. 6.2 and Exercise 6.4.

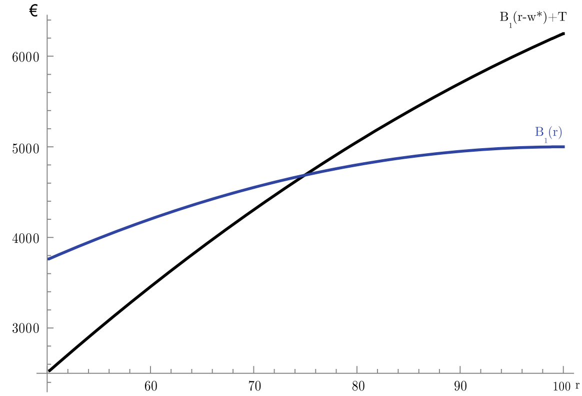


Fig. 6.6 Robustness of a fixed contract. *Source* own illustration

stand-alone case. This can be observed in Fig. 6.6. The blue line shows the welfare (utility) in the stand-alone case, i.e., in the case of conflict. In this case, country 2 does not receive water from country 1. The upstream country uses the whole water supply R_1. Of course, it does not receive any transfer because the downstream country has stopped to pay due to the breach of contract. The black line represents total utility for the case that the upstream country complies with the contract. It delivers the fixed amount of water w_2^* and receives in exchange T^*. As the water supply is decreasing, the water available for upstream decreases because the downstream country receives the fixed amount of water. There is a critical threshold $\hat{r}$, the intersection of both lines, where it does not pay for the upstream country to comply any more with the contract if r continues to drop. The length of the range $R_1 - \hat{r}$ indicates the robustness of this type of contract.

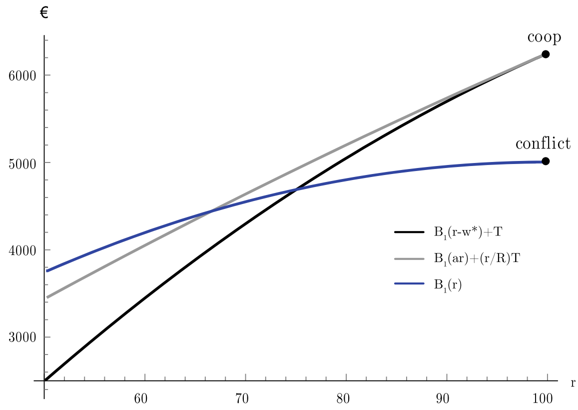
Let us turn to the proportional contract. At the outset, i.e., $r = R_1$, the agreement provides that downstream country receives w_2^* which can be expressed as a proportion of R_1 (see Eq. (6.46)). The amount of non-water transfer to the upstream country depends on the actual amount of water delivered downstream whereas the price is fixed according to Eq. (6.48). Thus, the countries' benefits including transfers are

$$B_1(\alpha^* r) + t(1-\alpha)r \quad \text{and} \quad B_2((1-\alpha^*)r) - t(1-\alpha)r \tag{6.53}$$

Figure 6.7 shows the stand-alone benefit of upstream as a function of the variable water supply r (blue line) again. The black line depicts benefits plus transfers under the fixed agreement and the gray line is net benefits of upstream under the proportional contract. The black and gray lines begin at $r = R_1$ at the same point "coop" where the difference to point "conflict" indicates the benefit increase for upstream to conclude a contract with downstream. However, this difference shrinks as r decreases. Then, comparing the intersections of both benefit lines of the two contract forms with the benefit lines in the conflict case (blue line) shows that the proportional contract is more robust than the contract with fixed quantities.

Durability of Agreements

Fig. 6.7 Comparing fixed and proportional contracts. *Source* own illustration



The robustness of a water agreement depends on the flexibility of its construction. But the flexible design is not sufficient for the durability of an agreement:

- Water agreements are more viable if the participating countries share the risk associated with unexpected water shortage. A proportional contract provides a rule to share the risk. A fixed water agreement shifts the risk to the upstream country. Its only incentive to comply with the agreement is the anticipated threat of downstream to cancel the non-water transfer if no water is delivered. The stability of a water agreement depends on the flexibility of its items stipulated, specifically, the transfers agreed upon. If these non-water transfers are to be provided on a periodical basis, downstream can cancel its payment in reaction to a breach of contract by upstream. Here again, the fixed term agreement is less flexible than the proportional rule. If upstream does not deliver the amount of water agreed upon, downstream stops the payments. In contrast, the proportional contract allows (a bit) more flexibility. Less water delivered by upstream leads only to less payment from downstream according to the internal water price stipulated.
- The analysis so far assumes that the contract parties assess the advantage of the agreement by comparing the outcome under compliance with the stand-alone benefits. As long as the contract leads to more utility compared to the conflict situation (breach of contract), the terms and conditions agreed upon will be respected. However, whether to comply with the contract under a shrinking water supply might not be the only consideration of the parties. Even if the benefits under a contract are higher than in the conflict case (breach of contract), the distribution of benefits changes with less water supply. If the resulting benefit distribution is considered unfair, the contract might be broken even if the resulting conflict situation worsens the economic situation of one or both parties. We know from experimental economics that people do not only look at their benefits but also at the relative position, i.e., the distribution of benefits.[35]

[35] The ultimatum game has shown this with astonishing evidence, see, e.g., Thaler (1988).

- Even if a certain degree of flexibility is built into the treaty, there may be renegotiation because the countries are not satisfied with the scheduled outcome of the agreement when the supply of water has declined. Then it is important that the institutional framework of the treaty is operational. Regular contacts between representatives of the two countries create a basis of trust which makes successful renegotiation likely.

6.6 An Institutional Perspective on Transboundary Water Agreements

6.6.1 An Institutional Approach

In the previous sections, we have investigated designs to divide a transboundary water resource among riparians. It is evident that the solutions, such as the Shapley value, will not be translated into a real-world treaty as such. An institutional economic approach suggests that real-world transboundary management would not follow such a technocratic top-down approach. The study of the theoretical principles of water allocation, however, allows to clarify which division rules can be qualified as fair in principle and worthy of approval. These concepts are deeply connected with the fundamental principles of justice and ethics and have also shaped international water law. They certainly belong to what institutional economics calls the institutional environment, traditions, and informal institutional framework conditions shaped by cultural configurations (Ostrom 1990; North 1990). For example, the ancient Talmud's garment rule already contains a simple version of the constrained equal awards rule.

The two approaches presented here, benefit sharing and bankruptcy rules, differ with regard to the weighting of two traditions of thought in social philosophy and political theory. The concept of the core and Shapley value can be assigned to the concept of the social contract as the constitution of cooperation. Rational people come together and agree to divide the advantages of cooperation in a fair and acceptable way.[36] The underlying fairness concept of the Shapley value is certainly the accountability principle.[37] The Shapley value is calculated on the basis of the average marginal productive contributions of the individual partners to the overall result. Those who do not contribute to the cooperation receive nothing. The consideration of individual productivity also leads to the fact that the allocation is acceptable. In

[36]Remember that $\alpha < \beta < \gamma$ where the latter number is the outcome with cooperation.
[37]See Sect. 3.3.

this way, the Shapley value combines the concept of fairness (accountability) with the concept of rationality as exploiting mutual cooperation advantages.

The nucleolus is close to the concept of the "veil of ignorance" by John Rawls.[38] This approach implies that the allocation of resources should follow those who are most disadvantaged. The nucleolus first determines all "disappointments" that follow from a proposed allocation and selects the largest one. Then, in an iterative procedure, a new proposal is put forward with the aim of reducing the largest disappointment. This leads to disappointments of other sub-coalitions. Thus, the search process is continued until the maximum disappointment has been minimized. This approach is also in the tradition of the social contract: it is fair (in the sense of Rawls) and worthy of acceptance, i.e., it lies in the core and puts all partners in a better position.

However, these approaches have their limits in practice. They abstract from too many complex relationships that have to be considered if one wants to successfully conclude water contracts. To begin with, the contracting parties are not simply individuals, but state entities which themselves consist of a number of social groups with differing interests. We, therefore, consider the game theoretical allocation rules as an element of a comprehensive holistic approach to understand the development of international water treaties. Institutional economics[39] allows this broad perspective to be built up scientifically. Here, we distinguish between institutional environment (blue area in Fig. 6.8) and institutional arrangements (yellow bottom area).

The fundamental considerations of justice and its game theory specifications certainly belong to the first area together with culturally determined concepts of justice, religious belief systems, and grown principles of law, written and unwritten. In contrast, the institutional arrangements are the structure within which the members of a society act politically and carry out economic transactions (production, consumption). These structures have grown historically, a development process that is not solely the result of planning, but is often predetermined by the past. Historians speak here of path dependencies or lock-in effects. Socio-technical structures often exhibit an inertia that resembles a lock-in, such as an energy system based on fossil resources that does not change to a system of renewable energy production without deliberate energy policy measures. Similar retarding forces of grown institutional structures inhibit the further development of spatially bound infrastructures, such as waterways. Changed geographical settlement structures, for example, require new waterways instead of simply preserving the historic ones.[40] The interaction between environment (vegetation, landscape, terrestrial eco-system) and man-made infras-

[38] This is the concept underlying the social welfare function: If no one knew in advance how he would fare on earth because he had no prior information about it (veil of ignorance), he would argue in favor of improving the situation of the worst off in the world. In the context of the nucleolus, the worst off is the one who is most disappointed with respect to the difference of the utility apportioned to him in the grand coalition and the welfare level he can achieve by himself.

[39] Saleth and Dinar (2004) explain the importance of the institutional economics approach to understand the water sector.

[40] See Willems and Busscher (2019) for an analysis of the Dutch national waterways.

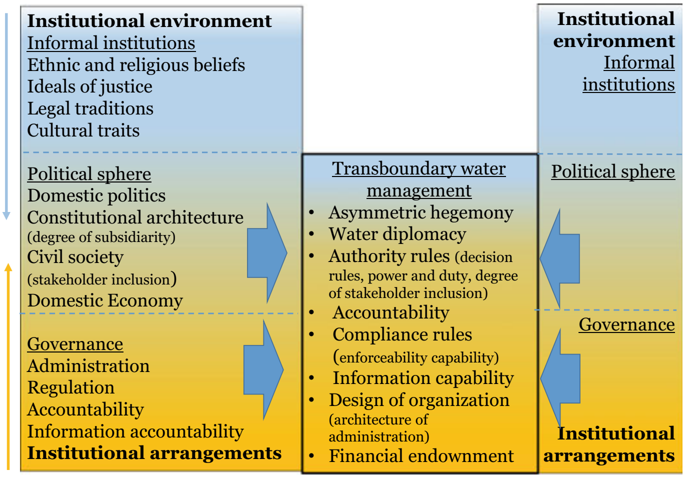


Fig. 6.8 The institutional embeddedness. *Source* own illustration

tructure determines the political level in which social and economic developments are embedded. Interventions in the water cycle can lead to ecological effects that extend beyond the boundaries of the local infrastructure. It is then not enough to assess infrastructure investments at the lower local political levels, but also higher levels must be involved in the decision-making process. This is entirely in the spirit of integrated water resource management, linked to the principle of subsidiarity. The integrative approach does not only refer to the geographical dimension, but also to the water users and indirect users of the water cycle. The latter are, for example, farmers who not only use water directly (irrigation), but also depend on a functioning ecosystem to ensure soil fertility. An inclusive approach to water management should be pursued at national level. All stakeholders should be taken into account in the sustainable shaping of the water cycle.

The problem of incomplete inclusion of all social groups in water management is not only due to an asymmetry of political power. Even if access to co-determination is guaranteed constitutionally and politically, it depends on the executive implementation of water policy plans. The level of governance is thus addressed. The effectiveness and functioning of water management institutions depend on an adequate design that takes into account the political environment, the inclusion of stakeholders and the incentives of employees at different levels of the institution.

6.6.2 Principles for Effective Institutional Development

In the following, some principles are presented that are important for the development of effective institutions, both for national authorities and for transboundary institutions. However, this should not give the perception that effective water management institutions can simply be assembled from a design toolbox. Institution building is always a laborious singular historical process, which refers to the respective individual case. Nevertheless, general principles should be considered in that context.

- **Purpose, objectives, and role**: It must be clear from the outset why management institutions have been set up. What are the actual objectives and purposes of the institution? This question must be asked at every administrative level. For example, is the purpose confined to the mere collecting and analyzing of relevant data? Is it about coordination of decentralized decision-makers (passive management), of advice, or does the institution established also have decision-making power (active management)?
- **Power and duties**: If water management is endowed with power, it is particularly important to precisely define its competences and, to communicate its limits. These boundaries may be of an economic or geographical nature. Is the authority able to take planning initiatives or does it only have a monitoring function to enforce the regulatory measures laid down by other institutions (regional parliaments, etc.)?
- **Decision rules** : The exercise of power requires legitimacy, otherwise, the implementation of water management measures will not be enforceable. It must be transparent how decisions have been taken, and this with reference to the constitutional legal basis.
- **Accountability and responsibility** The responsibilities of all participants must be clearly defined. This applies on the one hand to the managers or civil servants employed in the administrative units and on the other hand to the water users. In the course of the institutional implementation of water management, the assignment of duties and the takeover of responsibilities must be clearly communicated. This also includes the definition and description of sanction measures in the event that those involved do not comply with them.
- **Mediation**: Integrated water management is often about competing claims. Conflicts will inevitably arise. As a rule, these cannot be decided top-down. The institution must, therefore, build up the competence and capacity to resolve these conflicts in an orderly communication and negotiation process.
- **Competence and expertise**: Institutions do not function abstractly. Design alone does not ensure their effectiveness. It is important to build up a personnel development with regard to competence and expertise right from the start (capacity building). Administrative and decision-making units must have a critical mass of a well-trained and competent core staff. In the long term, an institution cannot rely on external consultancy (Biswas 1996).

6.6.3 Idealtypes of Governance

If the international water catchment area is regarded as a common complex ecosystem, the institutional structure should take into account the specific complex interre-

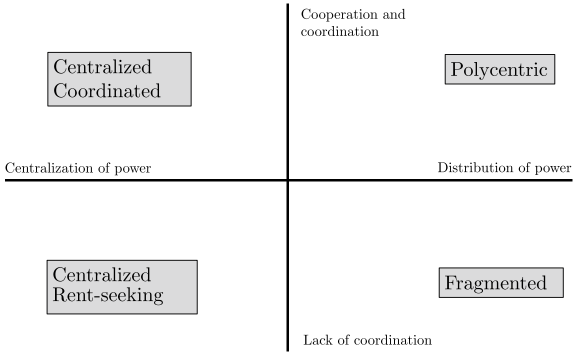


Fig. 6.9 Idealtypes of governance. *Source* Pahl-Wostl and Knieper (2014)

lations. The fitting of the management structure to the hydro-ecological conditions is called *adaptive management*.[41] For this approach, it is particularly important that the institutional structure must "mirror" the geographical, hydrological, and ecological complexity of a catchment area. An institutionalized top-down approach, for example, is usually not effective because decision-makers at the national level make water management decisions without necessarily taking into account regional impacts, leading to a so-called spatial scale mismatch. The spatial scale runs from the global level to the regional level, then to the level of regional rivers (lakes) and finally to subwater catchment areas. At all levels, effects can arise that must be perceived by suitable institutions (government agencies, NGOs, municipalities, etc.). The information must then be brought together promptly and effectively so that it can be processed at the respective institutional levels.

Basically, international waters should only be managed as a multilevel common pool within the framework of co-management of all open operational units. This can lead to problems of sovereignty, problems that must be solved in the underlying treaty. The increasing use of regional water systems and the increasing volatility of weather events (heavy rainfall, drought, etc.) require a very high degree of flexibility in the institutional structure, to be able to react effectively to these unforeseeable events and should therefore be polycentric in nature. This idealtypical structure has certain characteristics, which are illustrated by Fig. 6.9, based on Pahl-Wostl and Knieper (2014).

Along the horizontal axis, the degree of power decreases from the left with centralized power to the right pole, where power is equally distributed among all institutional units involved. These could be, for example, regional water authorities that are located at the same level without hierarchies. The vertical axis indicates the degree of cooperation or coordination between the sub-institutions. This can refer, for example, to the coordination of decisions or to the exchange of information. Coop-

[41] There is an extensive literature on this concept, see, e.g., Akamani and Wilson (2011).

eration/coordination is strongly pronounced at the upper end; it ends in a completely uncoordinated coexistence.

Four idealtypes result from this coordination system. At the top left is the coordinated, centralized water institution as found in top-down approaches. Its adaptability is low. The transparency of information is low, the degree of participation is just as low, and it derives its legitimacy only from the national level. At the bottom left, we find ourselves in a completely disintegrated situation. A few players, equipped with comparatively much power, pursue their own interests to the detriment of the international water catchment area. Economic literature refers to this constellation as rent seeking. This system is also very susceptible to corruption. The situation improves a little on the lower right because the un-cooperating institutions are endowed with little power. They cannot effectively implement their interests. However, this does not mean that the catchment area will be managed sustainably. Fragmentation does not allow the development of a targeted sustainability strategy. The polycentric structure is the only idealtype that has the prerequisites for adaptive management. There are no dominance structures, such that the various user interests can be balanced. The individual stakeholders are well networked and coordinated. It is therefore possible to react quickly to changing environmental conditions.

However, the polycentric configuration can only be understood as an ideal type. Whether the structure can be implemented at all in the respective political gravitational fields is a completely different question. It may be that, due to historical path dependency and cultural conditions, certain forms of adaptive management can only be implemented in the course of a long reform process. The development of typologies is nevertheless useful because it elaborates the necessary institutional prerequisites for successful transboundary water management. This makes it clear that there is a long way between fundamental considerations about the allocation of scarce water, as described in the previous sections, and practical implementation as recognized institutional structures.

6.6.4 Application to Transboundary Agreements

Institutional design of integrated water resource management is even more challenging once cooperation between sovereign states is required. Figure 6.8 highlights some institutional issues of transboundary water management. The political spheres of both countries play a role in the joint management. At the political level, negotiations are first held on the allocation of water, which is restricted by geographical patterns of watercourses (tributaries, lakes, direction of streams, etc.). Of particular importance is whether a multidimensional contract or only a contract for the quantities of water should be negotiated. The compromise space is much larger in the case of the multipurpose contracts because different economic sectors can be combined, for example, water use can be traded against energy supply.[42] This negotiation might be conducted in the presence of power asymmetries, whether due to the position of the riparian states (upstream, downstream) or due to economic and military dominance. These initial strategic positions become increasingly important as water scarcity increases. In some circumstances, riparians may not be prepared to negotiate the use of water because they feel strong enough to use water without taking into account the needs of other riparian states. This does not necessarily mean, however, that conflicts must arise. There may be something like a status quo under customary law in which the management of a transboundary water body takes place. However, these undefined floating conditions are likely to vanish as water scarcity increases.

When a contract becomes ready to be signed, implementation is an issue. This raises the question of the institutional nature of transboundary water management. Here, similar aspects to those described for the national or regional level must be considered. The organization to be formed is located in the gravitational area of sovereign states, which makes the institutionalization and administrative work considerably more difficult. The Damocles' sword of unilateral termination or simply noncompliance with the treaty by the contracting parties is hovering over the institution established.

The effectiveness of a transboundary organization depends not only on the principles introduced above, but also on the depth of cooperation granted to it by the contractual partners. A distinction can be made between different degrees of cooperation (see Vollmer et al. (2009)):

- *Shallow cooperation*: There is only a loose connection between the contracting parties. The cooperation is not "visible", i.e., there are no formalized structures, like joint committees, task forces, or established partnerships. There is only a loose direct contact with the respective national organizational entities of the riparian countries enclosed in a treaty. This minimal institutionalization is, of course, the result of a contract that does not explicitly regulate much, but rather represents a declaration of intent for cooperation.
- *Intermediate cooperation*: The operational level is visibly structured here. There are regular meetings between the responsible representatives of the state authorities, and a secretariat organizes this interaction, which also requires its own staff. However, there is no budget sovereignty.
- *Deep cooperation*: Within this framework, the established authority has a certain autonomy. It has an independent budget and its powers go far beyond preparatory work (information, organization). It has decision-making powers.

The varying degrees of cooperation reflect the level of allocative power conferred on the established institution. The wider the field of competence and organizational depth of the institution, the greater its clout. Within the framework of the Sustainable

[42] Benefit sharing takes account of this exploitation of exchange gains, while bankruptcy rules restrict themselves to water as a means of distribution.

Development Goal, UN Water has defined the effectiveness of the institutionalization of transboundary basin management (indicator 6.5.2).[43] A transboundary management institution is called "operational" if it meets the following criteria:

- *There is a joint body, joint mechanism, or commission (e.g., a river basin organization) for transboundary cooperation;*
- *There are regular (at least once per year) formal communications between riparian countries in [the] form of meetings (either at the political or technical level);*
- *There is a joint or coordinated water management plan(s), or joint objectives are set, and;*
- *There is a regular exchange (at least once per year) of data and information.*

UN Water collects data on the organizational implementation of formalized cooperation.[44] For surface water projects, 84 of the 155 international contractual cooperations responded to the survey 2018. 42 of these have a very high degree of organizational structure, mainly Europe and Northern America, and sub-Saharan Africa. However, some caution is called for when evaluating the empirical results. As UN Water notes, the degree of organization cannot be used automatically to draw conclusions about the results, such as better water quality or an improvement in the livelihood of people living in the international waters under the organizational structures implemented. There is a critical literature on this indicator.[45] The mere fact that an organizational structure has been established does not necessarily mean that the underlying contract is fair and inclusive in terms of sustainability, i.e., involves the various stakeholders in transboundary water management.

6.7 Exercises

Exercise 6.1 Benefit sharing in a river with two riparian states
Assume there are two riparians at one river, riparian 1 which is upstream and riparian 2 which is downstream. The riparians are indexed with the indices i, with $i = \{1, 2\}$. The natural inflow into the river upstream of riparian 1 is given with $R_1 = 100$, while the natural inflow downstream of riparian 1 but upstream of riparian 2 is given with $R_2 = 50$. Due to the diversion and consumption of water from the river with the level w_i, the riparians generate a benefit of B_i. The benefit functions are specified with

[43] McCracken and Meyer (2018) analyze the methodology of the SDG indicator 6.5.2 and report on empirical results.
[44] See Bertule et al. (2018).
[45] See Hussein et al. (2018).

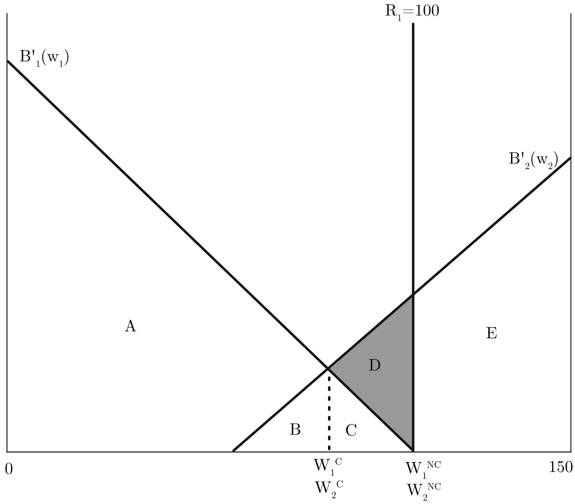


Fig. 6.10 Benefit sharing in a river basin with two riparians. *Source* own illustration

$$B_i(w_i) = a_i \cdot w_i - 0.5 \cdot b_i \cdot (w_i)^2$$

The parameters of the benefit functions are assumed with $a_1 = 100$, $a_2 = 60$, $b_1 = b_2 = 1$.

The situation in the river is shown by Fig. 6.10. The marginal benefit of riparian 1 is illustrated from the left vertical ordinate to the right direction, while we plot the marginal benefit of riparian 2 from the right vertical ordinate to the left direction. The length of the abscissa stands for the water amount $R_1 + R_2$, while the upstream external inflow R_1 is represented by the distance between the left origin of the diagram and the vertical line named with R_1. Therefore, the downstream external inflow R_2 is represented by the distance between the right origin of the diagram and the vertical line R_1. In the following explanation, we would like to find the UID, DID, and Shapley solution of the benefit sharing problem.

In the river basin, two cooperation scenarios are possible:

- The riparians act unilaterally in a noncooperative way. The water consumption amounts of the riparians are symbolized with w_1^{NC}. The consumption level of the upstream riparian (w_1^{NC}) is represented in Fig. 6.10 by the distance from the left origin of the diagram to the position of w_1^{NC}, while the consumption level of the downstream riparian (w_2^{NC}) is represented in Fig. 6.10 by the distance between the position of w_1^{NC} and the right origin of the diagram.
- The riparians form a joint arrangement and act in a cooperative manner. The water consumption amount for this scenario is represented by w_1^C. The consumption level of the upstream riparian (w_1^C) is represented in Fig. 6.10 by the distance from the left origin of the diagram to the position of w_1^C, while the consumption level of the downstream riparian (w_2^C) is represented in Fig. 6.10 by the distance between the position of w_1^C and the right origin of the diagram.

The first step of the benefit sharing problem is the calculation of the benefits under each cooperation scenario. Let's start with the unilateral acting. If the riparians act in a noncooperative way, any riparian would like to maximize its own specific benefit. Riparian 1 is upstream of riparian 2, hence riparian 1 will receive the natural inflow R_1 first. Therefore, we start with the benefit maximization problem of riparian 1. However, we have to note that the diverted amount w_1 is restricted by the water availability R_1. Therefore, we are able to formulate the following optimization problem:

$$\max_{\{w_1\}} [B_1(w_1)] \qquad s.t.\ w_1 \leq R_1 \tag{6.54}$$

Therefore, the following Lagrangian function can be formulated:

$$L_1 = B_1(w_1) + \lambda_1 \cdot (R_1 - w_1) \tag{6.55}$$

The resulting KKTs are

$$\begin{aligned} \frac{\partial L_1}{\partial w_1} &= B_1'(w_1) - \lambda_1 \leq 0 \perp w_1 \geq 0 \\ \frac{\partial L_1}{\partial \lambda_1} &= R_1 - w_1 \geq 0 \perp \lambda_1 \geq 0 \end{aligned} \tag{6.56}$$

Both assumptions, on the one hand $w_1 \geq 0$ and $\lambda \geq 0$ and on the other hand $w_1 \geq 0$ and $\lambda = 0$, are leading to the optimal solution. For the assumption $w_1 \geq 0$ and $\lambda \geq 0$, it is possible to find the following solution:

$$\begin{aligned} (\boldsymbol{\lambda_1}):\ R_1 - w_1 = 0 \\ \rightarrow\ w_1^{NC} = R_1 = 100 \\ (\boldsymbol{w_1}):\ B_1'(w_1) - \lambda_1 = 0 \\ \rightarrow\ \lambda_1 = B_1'(w_1^{NC}) = a_1 - b_1 \cdot w_1^{NC} = 0 \end{aligned} \tag{6.57}$$

Based on the other assumption $w_1 \geq 0$ and $\lambda_1 = 0$, we find the same solution:

$$\begin{aligned} (\boldsymbol{w_1}):\ B_1'(w_1) = 0 \\ \rightarrow\ a_1 - b_1 \cdot w_1 = 0 \\ \rightarrow\ w_1^{NC} = \frac{a_1}{b_1} = 100 \\ (\boldsymbol{\lambda_1}):\ R_1 - w_1 \geq 0 \\ \rightarrow\ R_1 = 100 \geq 100 = w_1 \end{aligned} \tag{6.58}$$

Therefore, the benefit of riparian 1 for unilateral acting is

$$B_1(w_1^{NC}) = a_1 \cdot w_1^{NC} - 0.5 \cdot b_1 \cdot (w_1^{NC})^2 = 5000 \tag{6.59}$$

which is represented by the illustration in Fig. 6.10 as the areas $A + B + C$.

After the consumption of riparian 1 (w_1) and the downstream headwater inflow R_2, the riparian 2 is able to divert and consume the water from the river. Due to the former water abstraction by riparian 1, just $R_1 + R_2 - w_1$ amounts of water are available for riparian 2. The optimization problem of the downstream riparian 2 is therefore

$$\max_{\{w_2\}} [B_2(w_2)] \quad s.t.\ w_2 \leq R_1 + R_2 - w_1 \tag{6.60}$$

Hence, the following Lagrangian function can be formulated:

$$L_2 = B_2(w_2) + \lambda_2 \cdot (R_1 + R_2 - w_1 - w_2) \tag{6.61}$$

which leads to the following KKT:

$$\begin{aligned} \frac{\partial L_2}{\partial w_2} &= B_2'(w_2) - \lambda_2 \leq 0 \perp w_2 \geq 0 \\ \frac{\partial L_2}{\partial \lambda_2} &= R_1 + R_2 - w_1 - w_2 \geq 0 \perp \lambda_2 \geq 0 \end{aligned} \tag{6.62}$$

Based on the assumption that riparian 2 will have a consumption ($w_2 \geq 0$) and that the available water for riparian 2 is fully used ($\lambda_2 \geq 0$), the solution of the optimization problem can be found as

$$\begin{aligned} (\boldsymbol{\lambda_2}):\ & R_1 + R_2 - w_1 - w_2 = 0 \\ \rightarrow\ w_2^{NC} &= R_1 + R_2 - w_1^{NC} = 50 \\ (\boldsymbol{w_2}):\ & B_2'(w_1) - \lambda_2 = 0 \\ \rightarrow\ \lambda_2 = B_2'(w_2^{NC}) &= a_2 - b_2 \cdot w_2^{NC} = 10 \geq 0\ \checkmark \end{aligned} \tag{6.63}$$

Therefore, the benefit of riparian 2 for the noncooperative acting in the basin is

$$B_2(w_2^{NC}) = a_2 \cdot w_2^{NC} - 0.5 \cdot b_2 \cdot (w_2^{NC})^2 = 1750 \tag{6.64}$$

which is represeted by area E in Fig. 6.10.

If the riparians form a joint arrangement, in which they allocate the water in a way that the benefit in the entire basin is maximized, the following optimization problem can be formulated:

$$\max_{\{w_1, w_2\}} [B_1(w_1) + B_2(w_2)] \quad s.t.\ w_1 \leq R_1,\ w_2 \leq R_1 + R_2 - w_1 \tag{6.65}$$

Similar to the problems (6.54) and (6.60), the water consumption of any riparian is restricted by the available water at the respective abstraction point (see constraints of problem (6.65)). The available water for riparians 1 and 2 is R_1 and $R_1 + R_2 - w_1$, respectively. Based on problem 6.65, the following Lagrangian function can be set up:

$$L = B_1(w_1) + B_2(w_2) + \lambda_1 \cdot (R_1 - w_1) + \lambda_2 \cdot (R_1 + R_2 - w_1 - w_2) \tag{6.66}$$

Therefore, it is possible to formulate the following KKT:

$$\begin{aligned}
\frac{\partial L}{\partial w_1} &= B_1'(w_1) - \lambda_1 - \lambda_2 \leq 0 \perp w_1 \geq 0 \\
\frac{\partial L}{\partial w_2} &= B_2'(w_2) - \lambda_2 \leq 0 \perp w_2 \geq 0 \\
\frac{\partial L}{\partial \lambda_1} &= R_1 - w_1 \geq 0 \perp \lambda_1 \geq 0 \\
\frac{\partial L}{\partial \lambda_2} &= R_1 + R_2 - w_1 - w_2 \geq 0 \perp \lambda_2 \geq 0
\end{aligned} \tag{6.67}$$

The assumptions $w_1 \geq 0$, $w_2 \geq 0$, $\lambda_1 = 0$, and $\lambda_2 \geq 0$ lead to the following optimality condition[46]:

$$\begin{aligned}
(\boldsymbol{w_1}):\ & B_1'(w_1) - \lambda_2 = 0 \\
(\boldsymbol{w_2}):\ & B_2'(w_2) - \lambda_2 = 0 \\
(\boldsymbol{\lambda_1}):\ & R_1 - w_1 \geq 0 \\
(\boldsymbol{\lambda_2}):\ & R_1 + R_2 - w_1 - w_2 = 0
\end{aligned} \tag{6.68}$$

It is therefore possible to find the optimal level of consumption based on the following system of equations:

$$\begin{aligned}
(\boldsymbol{w_1}) \wedge (\boldsymbol{w_2}) :\ & B_1'(w_1) = B_2'(w_2) \\
\rightarrow\ & a_1 - b_1 \cdot w_1 = a_2 - b_2 \cdot w_2 \\
(\boldsymbol{\lambda_2}):\ & R_1 + R_2 - w_1 - w_2 = 0
\end{aligned} \tag{6.69}$$

The solution of the system of equations is

$$w_1^C = 95\ ,\ w_2^C = 55$$

This solution is optimal, because there are no contradictions within the optimality conditions or assumptions:

$$\begin{aligned}
(\boldsymbol{w_1}) \wedge (\boldsymbol{w_2}) :\ & \lambda_2 = B_1'(w_1^C) = B_2'(w_2^C) \\
\rightarrow\ & \lambda_2 = a_1 - b_1 \cdot w_1^C = a_2 - b_2 \cdot w_2^C = 5 \geq 0 \\
(\boldsymbol{\lambda_1}):\ & R_1 \geq w_1\ \rightarrow\ 100 \geq 95
\end{aligned} \tag{6.70}$$

The benefits which result from consumption are therefore

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[46] We assume that both riparians consume water and therefore, $w_1 \geq 0$ and $w_2 \geq 0$. Furthermore, we assume that the upstream riparian 1 does not consume the entire available water at its abstraction point and leaves water in the river, hence, $\lambda_1 = 0$, while the downstream riparian 2 abstracts the total amount which is available, hence, it can be assumed that $\lambda_2 \geq 0$.

$$B_1(w_1^C) = a_1 \cdot w_1^C - 0.5 \cdot b_1 \cdot (w_1)^2 = 4987.5\,,$$
$$B_2(w_2^C) = a_2 \cdot w_2^C - 0.5 \cdot b_2 \cdot (w_2)^2 = 1787.5$$

The benefit from consumption for riparians 1 and 2 are represented in Fig. 6.10 by the areas $A + B$ and $C + D + E$, respectively. Based on these benefits from consumption, it is possible to calculate the cooperation gain, which is

$$\Delta = B_1(w_1^C) + B_2(w_2^C) - B_1(w_1^{NC}) - B_2(w_2^{NC}) = 25 \tag{6.71}$$

which is represented in Fig. 6.10 by area D.

In the joint arrangement, any riparian has to receive at least as much benefits as it would generate for the unilateral acting case, which means $z_1 \geq B_1(w_1^{NC})$ and $z_2 \geq B_2(w_2^{NC})$. Therefore, the question of how to share the cooperation gain is the main focus of the benefit sharing problem for a basin with 2 riparians.

For the *UID approach*, the total cooperation gain is assigned to the upstream riparian, hence,

$$x_1^{UID} = B_1(w_1^{NC}) + \Delta = 5000 + 25 = 5025,\, x_2^{UID} = B_2(w_2^{NC}) = 1750$$

The benefit of the riparians 1 and 2 are represented by the area $A + B + C + D$ and E in Fig. 6.10, respectively. For realizing the UID approach, the upstream riparian has to receive side payments from the downstream riparian:

$$sp_{2,1}^{UID} = B_2(w_2^C) - z_2^{UID} = 1787.5 - 1750 = 37.5$$

which is represented by the area $C + D$ in Fig. 6.10.

However, the total cooperation gain is assigned to the downstream riparian 2 for the *DID approach*, therefore,

$$x_1^{DID} = B_1^{w_1^{NC}} = 5000\,,\; x_2^{DID} = B_2^{w_2^{NC}} + \Delta = 1750 + 25 = 1775$$

These benefits for riparians 1 and 2 are represented in Fig. 6.10 by the areas $A + B + C$ and $E + D$, respectively. For realizing this solution, the downstream has to make side payments to the upstream riparian of the level:

$$sp_{2,1}^{DID} = B_2(w_2^C) - x_2^{DID} = 1787.5 - 1775 = 12.5$$

which is represented by the area C in Fig. 6.10.

The UID and DID solution sets the minimum and maximum bound for the assigned benefits to the riparians in the joint arrangement (see Eqs. 6.72 and 6.73). Furthermore, the generated benefit has to be assigned to the riparians in total to meet feasibility and pareto-efficiency conditions (see Eq. 6.74).

$$x_1^{DID} \leq x_1 \leq x_1^{UID} \rightarrow \; 5000 \leq x_1 \leq 5025 \tag{6.72}$$

$$x_2^{UID} \leq x_2 \leq x_2^{DID} \rightarrow \; 1750 \leq x_2 \leq 1775 \tag{6.73}$$

$$x_1 + x_2 = B_1^{w_1^C} + B_2^{w_2^C} \rightarrow \; x_1 + x_2 = 4987.5 + 1787.5 = 6775 \tag{6.74}$$

The Shapley solution is a specific focal point solution of the benefit sharing problem, in which both riparians receive half of the cooperation gain:

$$x_1^{SH} = B_1(w_1^{NC}) + 0.5 \cdot \Delta = 5000 + 12.5 = 5012.5\,,\; x_2^{SH} = B_2(w_2^{NC}) + 0.5 \cdot \Delta = 1750 + 12.5 = 1762.5$$

The assigned benefit for riparian 1 is represented by the areas $A + B + C + 0.5 \cdot D$ in Fig. 6.10, while the benefit of riparian 2 is represented by areas $E + 0.5 \cdot D$. This Shapley solution could be realized by side payments made by riparian 2:

$$sp_{2,1}^{SH} = B_2(w_2^C) - x_2^{SH} = 1787.5 - 1762.5 = 25$$

which is represented by areas $C + 0.5 \cdot D$ in Fig. 6.10.

Exercise 6.2 Applying the focal point solution concepts of benefit sharing to a water body with two riparians

Assume a water body with two riparians (1 and 2). Both riparians can either act unilaterally (noncooperation scenario) or they can form a joint arrangement where they act in a cooperative way:

- If both act unilaterally, we assume that the water consumption of riparians 1 and 2 is w_1^{NC} and w_2^{NC}, respectively. Based on the consumption levels, the riparians 1 and 2 generate a benefit of $B_1^{NC}(w_1^{NC})$ and $B_2^{NC}(w_2^{NC})$. For simplification reasons, we will term the benefit in the case of noncooperation of riparian 1 by B_1^{NC} and the one of riparian 2 by B_2^{NC} in the following. The benefit generated in the entire basin is $B_1^{NC} + B_2^{NC}$.
- If both form a joint arrangement, the riparians allocate the water in a way that the benefit in the entire basin is maximized. Therefore, the riparians 1 and 2 receive the water w_1^C and w_2^C, respectively. Based on the consumption, they generate a benefit of $B_1^C(w_1^C)$ and $B_2^C(w_2^C)$, which can be simplified as B_1^C and B_2^C. The generated benefit in the basin is $B_1^C + B_2^C$. The cooperation gain Δ results from the difference of the benefit in the entire basin between cooperation and noncooperation:

$$\Delta = B_1^C + B_2^C - B_1^{NC} - B_2^{NC}$$

Therefore, the benefit in the entire basin under the case of cooperation $B_1^C + B_2^C$ can be also formulated as

$$B_1^{NC} + B_2^{NC} + \Delta$$

Assume the joint arrangement is formed and the generated benefit in the basin $(B_1^{NC} + B_2^{NC} + \Delta)$ should be assigned to the riparians 1 and 2. For solving this benefit sharing problem, we want to apply the three formerly explained focal point solution concepts which are presented in Sect. 6.3, for finding the Shapley, Nash-Harsanyi, and nucleolus solutions.

The Shapley Solution

We apply Eq. (6.16) for finding the Shapley solution, which is explained in detail in Sect. 6.3.[47]

The Shapley value solution of one riparian is affected by the weighting factor and the incremental benefit of this user for various cooperation scenarios.

Regarding the weighting factor, there are just two cooperation scenarios which can be realized, either the unilateral acting or the joint arrangement. We assume in the Shapley approach that both cooperation scenarios have the same realization probability, which is a purely normative assumption from the Shapley approach (see (Wu and Whittington 2006)). Hence both cooperation scenarios have a realization probability of 0.5:

- Unilateral acting: this cooperation scenario is represented by the sets $\{1\}$ and $\{2\}$. There is of course per definition just one riparian in these sets, hence $\#ISG = 1$. We have two riparians in the basin, hence the grand coalition $\{1, 2\}$ consists of these two riparians and therefore $\#G = 2$. Inserting these parameters in the weighting factor, we get

$$\frac{(\#G - \#ISG)! \cdot (\#ISG - 1)!}{\#G!} = \frac{(2-1)! \cdot (1-1)!}{2!} = \frac{1! \cdot 0!}{2!} = 0.5$$

- Joint arrangement: this situation is represented by the set $\{1, 2\}$ which consists of two riparians, hence $\#ISG = 2$. We already discussed the level of $\#G$, which is $\#G = 2$. Inserting these parameters in the weighting factor, we get

$$\frac{(\#G - \#ISG)! \cdot (\#ISG - 1)!}{\#G!} = \frac{(2-2)! \cdot (2-1)!}{2!} = \frac{0! \cdot 1!}{2!} = 0.5$$

[47]The formula for finding the Shapley solution is

$$x_i = \sum_{\substack{I: i \in I \vee \\ S: i \in S \vee \\ G}} \left[\frac{(\#G - \#ISG)! \cdot (\#ISG - 1)!}{\#G!} \cdot [V(\ldots) - V(\ldots - i)] \right] \qquad (6.16)$$

The incremental benefit of riparian i, which is the second main element of the Shapley approach, is represented in Eq. (6.16) by the term $[V(...) - V(... - i)]$. This incremental benefit is

- in case of unilateral acting for riparians {1} and {2}: the level of the respective benefit the unilaterally acting riparian generates.
- in case of a joint arrangement $\{1, 2\}$: the difference between the generated benefit in the joint arrangement and the level of benefit the other riparian generates under the situation of unilateral acting. Hence, the incremental benefit of riparian 1 is the difference between the generated benefit in the joint arrangement and the generated benefit of the unilaterally acting riparian 2, $V(\{1, 2\}) - V(\{2\})$, while the incremental benefit of riparian 2 is the difference between the generated benefit in the joint arrangement and the generated benefit of the unilaterally acting riparian 1, $V(\{1, 2\}) - V(\{1\})$.

We know that the benefit in the grand coalition is $V(\{1, 2\}) = B_1^{NC} + B_2^{NC} + \Delta$, while the benefit of the unilaterally acting riparian 1 is $V(\{1\}) = B_1^{NC}$ and the benefit of the unilaterally acting riparian 2 is $V(\{2\}) = B_2^{NC}$.

The riparian 1 is just part of the coalition scenarios {1} and $\{1, 2\}$, hence the Shapley solution is

$$\begin{aligned} x_1^{SH} &= 0.5 \cdot V(\{1\}) + 0.5 \cdot [V(\{1, 2\}) - V(\{2\})] \\ &= 0.5 \cdot B_1^{NC} + 0.5 \cdot (B_1^{NC} + B_2^{NC} + \Delta - B_2^{NC}) \\ x_1^{SH} &= B_1^{NC} + 0.5 \cdot \Delta \end{aligned}$$

while the riparian 2 is just part of the coalition scenarios {2} and $\{1, 2\}$, hence, its Shapley solution is

$$\begin{aligned} x_2^{SH} &= 0.5 \cdot V(\{2\}) + 0.5 \cdot [V(\{1, 2\}) - V(\{1\})] \\ &= 0.5 \cdot B_2^{NC} + 0.5 \cdot (B_1^{NC} + B_2^{NC} + \Delta - B_1^{NC}) \\ x_2^{SH} &= B_2^{NC} + 0.5 \cdot \Delta \end{aligned}$$

The following table at the next page can be also used as an auxiliary tool for finding the Shapley solution:

The Nash-Harsanyi solution

The optimization problem of the Nash-Harsanyi solution concept is (see Sect. 6.3)

$$\begin{aligned} &\max_{\{x_1, x_2\}} \left[(x_1 - B_1^{NC}) \cdot (x_2 - B_2^{NC}) \right] \\ &s.t.\; x_1 + x_2 = B_1^{NC} + B_2^{NC} + \Delta \\ &\quad B_1^{NC} \leq x_1 \\ &\quad B_2^{NC} \leq x_2 \end{aligned}$$

(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	(IX)	(X)
Riparian i	Coalition	$\#ISG$	$\frac{(\#G-\#ISG)!\cdot(\#ISG-1)!}{\#G!}$	$V(\ldots)$	Coalition without i	$V(\ldots - i)$	(V) - (VII)	(IV) · (VIII)	Shapley Value
User 1	$\{1\}$	1	0.5	B_1^{NC}	$\emptyset$	0	B_1^{NC}	$0.5 \cdot B_1^{NC}$	$B_1^{NC} + 0.5 \cdot \Delta$
	$\{1,2\}$	2	0.5	$B_1^{NC} + B_2^{NC} + \Delta$	$\{2\}$	B_2^{NC}	$B_1^{NC} + \Delta$	$0.5 \cdot (B_1^{NC} + \Delta)$	
User 2	$\{2\}$	1	0.5	B_2^{NC}	$\emptyset$	0	B_2^{NC}	0	$B_2^{NC} + 0.5 \cdot \Delta$
	$\{1,2\}$	2	0.5	$B_1^{NC} + B_2^{NC} + \Delta$	$\{1\}$	B_1^{NC}	$B_2^{NC} + \Delta$	$0.5 \cdot (B_2^{NC} + \Delta)$	

Therefore, the following Lagrangian function results:

$$L = (x_1 - B_1^{NC}) \cdot (x_2 - B_2^{NC}) + \mu \cdot (B_1^{NC} + B_2^{NC} + \Delta - x_1 - x_2) + \lambda_1 \cdot (x_1 - B_1^{NC}) + \lambda_2 \cdot (x_2 - B_2^{NC})$$

And hence, we are able to set up the following KKT conditions:

$$(x_2 - B_2^{NC}) - \mu + \lambda_1 \leq 0 \perp x_1 \geq 0 \tag{6.75}$$

$$(x_1 - B_1^{NC}) - \mu + \lambda_2 \leq 0 \perp x_2 \geq 0 \tag{6.76}$$

$$B_1^{NC} + B_2^{NC} + \Delta - x_1 - x_2 = 0\ ,\ \ \mu \text{ is free} \tag{6.77}$$

$$x_1 - B_1^{NC} \geq 0 \perp \lambda_1 \geq 0 \tag{6.78}$$

$$x_2 - B_2^{NC} \geq 0 \perp \lambda_2 \geq 0 \tag{6.79}$$

Suppose that both riparians receive benefits which exceed their respective individual rationality conditions. Hence, we have to assume that $x_1 \geq 0$, $x_2 \geq 0$, $\lambda_1 = 0$, and $\lambda_2 = 0$.[48] Based on Eqs. (6.75)–(6.77), we can set up the following system of equations:

$$x_1 - B_1^{NC} = x_2 - B_2^{NC}$$
$$x_1 + x_2 = B_1^{NC} + B_2^{NC} + \Delta$$

The solution is

$$x_1^{NH} = B_1^{NC} + 0.5 \cdot \Delta$$
$$x_2^{NH} = B_2^{NC} + 0.5 \cdot \Delta$$

This solution meets the conditions (6.78) and (6.79).[49]

<u>The nucleolus solution</u>

The nucleolus solution can be calculated on the basis of the following optimization problem (see Sect. 6.3):

$$\min_{\{x_1, x_2, e\}} [e]$$
$$s.t.\ x_1 + x_2 = B_1^{NC} + B_2^{NC} + \Delta$$
$$e + x_1 \geq B_1^{NC}$$
$$e + x_2 \geq B_2^{NC}$$

[48]The variable μ is a free variable, because it is related to an equality constraint.
[49]Based on condition 6.78, $x_1 \geq B_1^{NC}$. Due to $x_1 = B_1^{NC} + 0.5 \cdot \Delta$, Eq. 6.78 is met.
Based on condition 6.79, $x_2 \geq B_2^{NC}$. Due to $x_2 = B_2^{NC} + 0.5 \cdot \Delta$, Eq. 6.79 is met.

Therefore, we can formulate the following Lagrangian function:

$$L = e + \mu \cdot (x_1 + x_2 - B_1^{NC} - B_2^{NC} - \Delta) + \lambda_1 \cdot (B_1^{NC} - e - x_1) + \lambda_2 \cdot (B_2^{NC} - e - x_2)$$

and hence, we are able to set up the following KKT conditions:

$$\mu - \lambda_1 \geq 0 \perp x_1 \geq 0 \tag{6.80}$$

$$\mu - \lambda_2 \geq 0 \perp x_2 \geq 0 \tag{6.81}$$

$$1 - \lambda_1 - \lambda_2 = 0\ , \ \ e \text{ is free} \tag{6.82}$$

$$x_1 + x_2 - B_1^{NC} - B_2^{NC} - \Delta = 0\ , \ \ \mu \text{ is free} \tag{6.83}$$

$$B_1^{NC} - e - x_1 \leq 0 \perp \lambda_1 \geq 0 \tag{6.84}$$

$$B_2^{NC} - e - x_2 \leq 0 \perp \lambda_2 \geq 0 \tag{6.85}$$

Please note that e and μ are free variables, which means that they can have a positive or negative value. Under the assumption that $x_1 \geq 0$, $x_2 \geq 0$, $\lambda_1 \geq 0$, and $\lambda_2 \geq 0$, we are able to formulate the following system of equations[50]:

$$e = B_1^{NC} - x_1 = B_2^{NC} - x_2$$
$$x_1 + x_2 = B_1^{NC} + B_2^{NC} + \Delta$$

The solution is

$$x_1^{nuc} = B_1^{NC} + 0.5 \cdot \Delta$$
$$x_2^{nuc} = B_2^{NC} + 0.5 \cdot \Delta$$

The maximum objection which is minimized by applying the nucleolus approach is $e = -0.5 \cdot \Delta$.

Comparison of Focal Point Solutions

It becomes obvious from this analysis, that in a basin with just two riparians the three presented focal point solution concepts lead to the same results:

$$x_1^{SH} = x_1^{NH} = x_1^{nuc} = B_1^{NC} + 0.5 \cdot \Delta$$
$$x_2^{SH} = x_2^{NH} = x_2^{nuc} = B_2^{NC} + 0.5 \cdot \Delta$$

This means that each riparian receives the benefit it would generate when acting unilaterally in a noncooperative way and furthermore half of the cooperation gain. Therefore, the cooperation gain is shared equally between the two riparians.

[50]We suppose that both riparians receive benefits, hence we assume $x_1 \geq 0$ and $x_2 \geq 0$. If we furthermore assume that the maximum objection in the nucleolus solution, denoted by e, is based on the payoff of the unilaterally acting riparian 1 (which means $e = B_1^{NC} - x_1$) as well as on the payoff of the unilaterally acting riparian 2 (which means $e = B_2^{NC} - x_2$), it becomes obvious that the conditions 6.84 and 6.85 become binding, and hence, we have to assume that $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$.

Exercise 6.3 Water allocation under bankruptcy rules
The importance of rationing rules will probably increase in the next few years. In many international waters, inflows are decreasing due to climate change. In particular, justice issues will become even more important in the discussion. Thereby, it is difficult to determine which bankruptcy rule leads to a fair distribution of water. We have seen that this question is closely related to the legitimacy of claims. But even if an agreement has been reached on what a justified level of claims is, the question remains as to which of the bankruptcy rules is fair. We cannot answer this question a priori here. That remains to be decided on a case-by-case basis.

What we can do, however, is to investigate how water allocations develop as a function of the scarcity of water. Thereby, we are primarily interested in how the relative allocation of scarce water as a function of R develops. To do so, we first define the degree to which water claims are met[51]:

$$\gamma_i^{BR} = x_i^{BR}/c_i, \quad i = \{1, 2\}, \;\; BR = \{P, CEA, AP\} \tag{6.86}$$

To compare this degree, we construct the relative fulfillment:

$$\Gamma^{BR} = \gamma_1^{BR}/\gamma_2^{BR}, \;\; BR = \{P, CEA, AP\} \tag{6.87}$$

From Eq. (6.27), it is easy to derive the claim satisfaction of both countries for the proportional rule:

$$\gamma_1^P = x_1^P/c_1 = \frac{R}{c_1 + c_2} = \gamma_2^P = x_2^P/c_2 \tag{6.88}$$

From Eq. (6.88), it is clear that the relative fulfillment of claims does not change with respect to R. This bankruptcy rule is obviously fairness-stable, i.e., the relative claim fulfillment does not vary with R.

The same applies to the CEA rule, as can easily be shown. In our example in Sect. 6.4, the CEA allocation led[52] to $x_1^{CEA} = R/2 = x_2^{CEA}$. Thus,

$$\Gamma^{CEA} = \gamma_1^{CEA}/\gamma_2^{CEA} = \frac{(R/2)/c_1}{(R/2)/c_2} = \frac{c_2}{c_1} \tag{6.89}$$

which shows that the CEA rule is also fairness-stable with regard to a decrease of the water supply.

It remains to analyze the Adjusted Proportional Rule. From Eq. (6.28), we have

$$x_1^{AP} = (R - c_2) + \frac{c_1 - R + c_2}{2(c_1 + c_2 - R)}\{R - (R - c_2) - (R - c_1)\} \tag{6.90}$$

---

[51] The superscript BR refers to bankruptcy rules.
[52] We have assumed the following numerical values: $R = 200$, $c_1 = 180$, and $c_2 = 120$. This has led to water allocation as depicted in Fig. 6.3.

which can be reduced to

$$x_1^{AP} = \frac{R + c_1 - c_2}{2} \quad \Rightarrow \quad \gamma_1^{AP} = x_1^{AP}/c_1 = \frac{R + c_1 - c_2}{2c_1} \tag{6.91}$$

Finally, utilizing the claim fulfillment of country 2, we get

$$\Gamma^{AP} = \gamma_1^{AP}/\gamma_2^{AP} = \left(\frac{c_2}{c_1}\right)\left[\frac{R + c_1 - c_2}{R + c_2 - c_1}\right] \tag{6.92}$$

It is left as an exercise to the reader to prove that

$$\frac{\partial \Gamma^{AP}}{\partial R} = \left(\frac{c_2}{c_1}\right)\frac{2(c_2 - c_1)}{(R + c_2 - c_1)^2} \tag{6.93}$$

Our numerical example $c_1 = 180 > c_2 = 120$ implies that with increasing R, the relative claim fulfillment for the upstream country gets worse. Thus, with a lower water supply the relative claim fulfillment decreases for the downstream country.

What is the lesson of this task? It shows that the riparian countries choose bankruptcy rules not only depending on the outcome for a certain water supply currently provided by the regional hydrological cycle but also on the characteristics of these rules when the water supply changes.

Exercise 6.4 The robustness of water agreements

In many international waters, water inflow has declined in recent years. This is partly due to climate change. This unexpected change in the water cycle is often not taken into account in international water treaties. We have addressed this problem in the section on the robustness of water contracts. In the following, we will examine the stability of fixed and proportional contracts with the help of a very simple numerical example.

Assume two identical countries, country 1 (upstream) and country 2 (downstream). The benefit function of each is $B_i(w_i) = aw_i - (b/2)w_i^2, i = \{1, 2\}$ Let $a = 100$ and $b = 1$. As in Sect. 6.5, we assume that water inflow takes place only upstream and is $R = 100$. Since both countries are identical, the optimal allocation would be simply $w_1^* = w_2^* = R/2 = 50$, i.e., the upstream country allows half of the water supply to flow through to country 2. It remains to analyze the non-water transfer of the downstream country to the upstream country. Let us assume, that this transfer is constructed such that the joint benefit of cooperation is distributed according to the Shapley value. We know that the Shapely value lies for the case with just two riparians in the core. The formula is

$$s_1 = B_1(R) + \frac{1}{2}[B_1(R/2) + B_2(R/2) - B_2(0) - B_1(R)] \tag{6.94}$$

$$s_2 = B_2(0) + \frac{1}{2}[B_1(R/2) + B_2(R/2) - B_2(0) - B_1(R)] \tag{6.95}$$

This benefit division can be accomplished with the help of a transfer of country 2 to country 1. We have

$$s_1 = B_1(R) + \frac{1}{2}[B_1(R/2) + B_2(R/2) - B_2(0) - B_1(R)] = B_1(R/2) + T \tag{6.96}$$

where T is the non-water transfer from downstream to upstream. From this equation it is easy to calculate T:

$$T = \frac{1}{2}[B_1(R) + B_2(R/2) - B_1(R/2)] \tag{6.97}$$

Notice thereby that $B_2(0) = 0$. Since we have assumed identical countries $B_2(R/2) = B_1(R/2)$ and, hence, the transfer is simply

$$T = \frac{1}{2}B_1(R) \tag{6.98}$$

the downstream country ends up with

$$s_2 = B_2(R/2) - T \tag{6.99}$$

It is now easy to determine the degree of robustness of both types of contracts analyzed in Sect. 6.5 for the Shapley value. We begin with the fixed contract.

The upstream country delivers the fixed amount of water $R/2$ for a fixed payment of T. As the water supply drops, the net benefit of the upstream country decreases. Note that the benefit of country 2 is not affected by the water decrease. The contract is robust as long as

$$B_1(r - (R/2)) + T = B_1(r - (R/2)) + (1/2)B_1(R) \geq B_1(r) \tag{6.100}$$

If we insert the quadratic benefit function, we can find the critical value of r for which Eq. (6.100) is an equality:

$$r = (3/4)R \tag{6.101}$$

This result can be found in Fig. 6.6. If the water decrease is less than 25% of R, the water contract is stable. However, a larger water decrease would lead to a dissolution of the agreement if the parties feel that the decrease will be long term.

To derive the r-range of the proportional contract, we start with the benefit of the upstream county. The contract specifies that half of the available water flows downstream. Utilizing Eq. (6.50), total benefit of country 1 is

$$B_1(r/2) + T(r) = B_1(r/2) + (r/R)T \tag{6.102}$$

where T is defined in Eq. (6.98).

To determine the range where total benefit defined in Eq. (6.102) within the contract is higher than or equal to the conflict option, we have to set it equal to $B_1(r)$. If we substitute the quadratic utility function into Eq. (6.102), it is an easy task to calculate the critical r-value[53]:

$$r = (2/3)R \tag{6.103}$$

It remains to check whether the downstream country sticks to the contract as the water supply drops. The respective constraint is

$$B_2(r/2) - (r/R)T = B_2(r/2) - \frac{r}{2R}B_1(R) \geq B_2(0) = 0 \tag{6.104}$$

Substituting the quadratic benefit function into this constraint yields after some algebraic manipulation $r \leq 2R$ which is always satisfied, since $r \leq R$. The downstream country never has an incentive to break the contract. It is always worse off without water from upstream.

This analysis assumes that both countries only compare their benefits with respect to the conflict case, i.e., the situation without a cooperative solution. However, we know from the history of the bargaining process preceding the conclusion of a water agreement that the distribution, i.e., the relative position of the contract partner, is of high importance. Let us assume that in our example both countries were satisfied with the distribution of the benefits at the outset. The allocation of water and the transfer determined by the Shapley approach is deemed fair. As the water supply drops, both net benefits decrease and the question remains whether the resulting distribution of benefits continues to be regarded as just if the contract concluded for $r = R$ still applies. If not, it may happen that the contract will be broken even if the conflict position makes the parties worse off. Such behavior due to an injured sense of justice may well occur, as we know from experimental economics and also from everyday life.

6.8 Further Reading

International environmental agreements as well as international water agreements are often analyzed in economics as a sequence of strategic negotiation steps. Noncooperative game theory is particularly suitable for this purpose. Each negotiating participant tries to maximize his advantages, taking into account the behavior of the other participants. In cooperative game theory, the main purpose is to determine joint action and the distribution of cooperative gains. The theory assumes that binding contracts can be concluded, i.e., the parties to the contract adhere to the agreed arrangements. A very useful introduction to cooperative game theory with a number of relevant examples from international water agreements is Dinar et al. (2007). Wu

[53] See in Fig. 6.7, the intersection between the blue and the gray line.

and Whittington (2006) apply the concepts of the Shapley value and the nucleolus to a water-sharing game of the Nile. In addition to the calculation of the benefits of cooperation, the authors have also included hydrological constraints.

Contrary to cooperative game theory, bankruptcy rules deal with zero-sum games in a noncooperative setting. What one gets, the other does not have, and vice versa. It is obvious that with zero-sum games, considerations of justice are of particular importance. Thomson (2002) gives a very instructive overview. Which rules of division fulfill which axioms or criteria of justice? The relationship with cooperative game theory is made. This concept can also be applied in the context of zero-sum games (e.g., the Shapley value). Dagan and Volij (1993) compare different bankruptcy rules with the bargaining approach of game theory. This is an interesting approach because the criteria and properties of bankruptcy rules are interpreted as the result of cooperative negotiations. For example, the authors show that the cooperative Nash solution leads to CEA. One must be careful when applying bankruptcy rules to transboundary water agreements. This applies in particular to river basins where there is a hierarchical structure of claims due to the unidirectionality of watercourses. It is possible that a water allocation resulting from the direct application of a bankruptcy rule cannot be implemented at all for hydrological reasons. Ansink and Weikard (2012) have therefore developed modified bankruptcy rules (sequential sharing rules) that take hydrological restrictions into account.

The effects of climate change will also affect international waters. For some time now, scientists have been working on the question of how the increasing water scarcity and variability of the water supply should be taken into account in international water agreements. Cooley and Gleick (2011) analyze how existing contracts can accommodate these changes. The allocation rules must be made dependent on the amount of water available. This requires a functioning monitoring system embedded in a well institutionalized transboundary management system. Ansink and Ruijs (2008) analyze the exact effects of sharing rules in a formal model when the average available water quantity decreases and the variability increases at the same time. They conclude that the increasing scarcity of water reduces the stability of international treaties, but that increasing variability can even lead to a strengthening of contractual cooperation. This analysis is deepened in Ambec et al. (2013). Different contract formats (fixed and variable) are examined with regards to their vulnerability to increasing water scarcity. An important finding of the authors is that contracts are stable when their contractual components (water and compensatory transfers) are contingent on a variable water supply. In this case, the contract becomes self-enforcing, i.e., there is no incentive for the contracting parties to violate the contract.

The literature on the institutionalization of international treaties on water use is very extensive. It is interdisciplinary and transdisciplinary. One of the latter is Biswas (1996), who has worked on transboundary water management both as a scientist and as an expert and practitioner. This also includes Draper (2007), who is active as a researcher and political planner of water infrastructures. He developed criteria as a necessary prerequisite for effective water-sharing agreements.

In addition to more descriptive studies, such as Vollmer et al. (2009), which bring the institutional diversity of transboundary management into a taxonomic order, there are analyses based on institutional economics, such as the comprehensive work of Saleth and Dinar (2004). Here, the network of institutional structures at different administrative and political levels is examined with regard to their effectiveness. This depends primarily on the goals of the institutional units and the existing incentive mechanisms.

The interdependence of institutional units, e.g., between water authorities at the regional level and transboundary institutions, which are made up of representatives of different riparian states, is also the focus of research dealing with adaptive management. However, the question is broader: How can the complexity of the ecological system of an international water catchment area be combined with a correspondingly adapted design of water management to form a sustainable and resilient ecological-social integrated system? Akamani and Wilson (2011) and Pahl-Wostl et al. (2008) give an overview. The basic philosophy is presented in Folke et al. (2005), and Karkkainen (2004) provides two very instructive examples of how joint water management goes far beyond the rigid implementation of a treaty at a national level (Chesapeake Bay Program, US-Canadian Great Lakes Program).

6.9 Chapter-Annex: Step-by-Step Solution of Optimization Problems of Sect. 6.3

The Core: Identify the Number of Solutions in the Core

Optimization problem:

$$\max\left[\gamma - x_1 - x_2 - x_3\right] \qquad s.t.\; x_1 \geq \alpha\,,\; x_2 \geq 0\,,\; x_3 \geq 0$$
$$x_1 + x_2 \geq \beta\,,\; x_1 + x_3 \geq \alpha\,,\; x_2 + x_3 \geq 0$$

Lagrangian Function:

$$\begin{aligned} L = \gamma - x_1 - x_2 - x_3 &+ \lambda_{\{1\}} \cdot (x_1 - \alpha) + \lambda_{\{2\}} \cdot x_2 + \lambda_{\{3\}} \cdot x_3 \\ &+ \lambda_{\{1,2\}} \cdot (x_1 + x_2 - \beta) + \lambda_{\{1,3\}} \cdot (x_1 + x_3 - \alpha) + \lambda_{\{2,3\}} \cdot (x_2 + x_3) \end{aligned}$$

KKT Conditions:

$$\begin{aligned} -1 + \lambda_{\{1\}} + \lambda_{\{1,2\}} + \lambda_{\{1,3\}} \leq 0 \perp x_1 \geq 0 \\ -1 + \lambda_{\{2\}} + \lambda_{\{1,2\}} + \lambda_{\{2,3\}} \leq 0 \perp x_2 \geq 0 \\ -1 + \lambda_{\{3\}} + \lambda_{\{1,3\}} + \lambda_{\{2,3\}} \leq 0 \perp x_3 \geq 0 \\ x_1 - \alpha \geq 0 \perp \lambda_{\{1\}} \geq 0 \\ x_2 \geq 0 \perp \lambda_{\{2\}} \geq 0 \\ x_3 \geq 0 \perp \lambda_{\{3\}} \geq 0 \\ x_1 + x_2 - \beta \geq 0 \perp \lambda_{\{1,2\}} \geq 0 \\ x_1 + x_3 - \alpha \geq 0 \perp \lambda_{\{1,3\}} \geq 0 \\ x_2 + x_3 \geq 0 \perp \lambda_{\{2,3\}} \geq 0 \end{aligned}$$

Assumption:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 = 0$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0,\ \lambda_{\{1,2\}} \geq 0$$

Solution: (Please note, the objective $\gamma - x_1 - x_2 - x_3$ is denoted as Z)

$$x_1 = \alpha + \delta,\ x_2 = \beta - \delta,\ \lambda_{\{1,2\}} = 1$$
$$Z = \gamma - x_1 - x_2 - x_3 = \gamma - \beta \geq 0$$

The core: The Lower Bound of Each Player in the Core

Optimization problem:

$$\min\left[x_j\right] s.t.\ x_1 + x_2 + x_3 = \gamma,\ x_1 \geq \alpha,\ x_2 \geq 0,\ x_3 \geq 0$$
$$x_1 + x_2 \geq \beta,\ x_1 + x_3 \geq \alpha,\ x_2 + x_3 \geq 0$$

Lagrangian Function:

$$L = x_j + \lambda_{\{1,2,3\}} \cdot (x_1 + x_2 + x_3 - \gamma) + \lambda_{\{1\}} \cdot (\alpha - x_1) - \lambda_{\{2\}} \cdot x_2 - \lambda_{\{3\}} \cdot x_3$$
$$+ \lambda_{\{1,2\}} \cdot (\beta - x_1 - x_2) + \lambda_{\{1,3\}} \cdot (\alpha - x_1 - x_3) + \lambda_{\{2,3\}} \cdot (-x_2 - x_3)$$

KKT Conditions of Dual Variables:

$$x_1 + x_2 + x_3 - \gamma = 0\ ,\ \lambda_{\{1,2,3\}} \text{ is free}$$
$$\alpha - x_1 \leq 0 \perp \lambda_{\{1\}} \geq 0$$
$$-x_2 \leq 0 \perp \lambda_{\{2\}} \geq 0$$
$$x_3 \leq 0 \perp \lambda_{\{3\}} \geq 0$$
$$\beta - x_1 - x_2 \leq 0 \perp \lambda_{\{1,2\}}$$
$$\alpha - x_1 - x_3 \leq 0 \perp \lambda_{\{1,3\}} \geq 0$$
$$-x_2 - x_3 \leq 0 \perp \lambda_{\{2,3\}} \geq 0$$

KKT Conditions of Primal Variables (primal variable is in the objective):

$$1 - \lambda_{\{1\}} - \lambda_{\{1,2\}} - \lambda_{\{1,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_1 \geq 0$$
$$1 - \lambda_{\{2\}} - \lambda_{\{1,2\}} - \lambda_{\{2,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_2 \geq 0$$
$$1 - \lambda_{\{3\}} - \lambda_{\{1,3\}} - \lambda_{\{2,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_3 \geq 0$$

KKT Conditions of Primal Variables (primal variable is not in the objective):

$$-\lambda_{\{1\}} - \lambda_{\{1,2\}} - \lambda_{\{1,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_1 \geq 0$$
$$-\lambda_{\{2\}} - \lambda_{\{1,2\}} - \lambda_{\{2,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_2 \geq 0$$
$$-\lambda_{\{3\}} - \lambda_{\{1,3\}} - \lambda_{\{2,3\}} + \lambda_{\{1,2,3\}} \geq 0 \perp x_3 \geq 0$$

Assumption for Problem of Riparian 1:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$\lambda_{\{1\}} \geq 0,\ \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0$$

Solution for Problem of Riparian 1:

$$x_1 = \alpha$$
$$\lambda_{\{1\}} = 1,\ \lambda_{\{1,2,3\}} = 0$$
$$x_2 = \delta_2,\ x_3 = \delta_3$$
with $0 \leq \delta_2 \leq \gamma - \alpha, 0 \leq \delta_3 \leq \gamma - \alpha$ and $\delta_2 + \delta_3 = \gamma - \alpha$

Assumption for Problem of Riparian 2:

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$
$$\lambda_{\{1\}} = \lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0,\ \lambda_{\{2\}} \geq 0$$

Solution for Problem of Riparian 2:

$$x_2 = 0$$
$$\lambda_{\{2\}} = 1,\ \lambda_{\{1,2,3\}} = 0$$
$$x_1 = \alpha + \delta_1,\ x_3 = \delta_3$$
with: $0 \leq \delta_1 \leq \gamma - \alpha, 0 \leq \delta_3 \leq \gamma - \alpha$ and $\delta_1 + \delta_3 = \gamma - \alpha$

Assumption for Problem of Riparian 3:

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0,\ \lambda_{\{3\}} \geq 0$$

Solution for Problem of Riparian 3:

$$x_3 = 0$$

The core: The Upper Bound of Each Player in the Core

Optimization Problem:

$$\max \left[x_j\right] \qquad s.t.\ x_1 + x_2 + x_3 = \gamma,\ x_1 \geq \alpha,\ x_2 \geq 0,\ x_3 \geq 0$$
$$x_1 + x_2 \geq \beta,\ x_1 + x_3 \geq \alpha,\ x_2 + x_3 \geq 0$$

Lagrangian Function:

$$L = x_j + \lambda_{\{1,2,3\}} \cdot (\gamma - x_1 - x_2 - x_3) + \lambda_{\{1\}} \cdot (x_1 - \alpha) + \lambda_{\{2\}} \cdot x_2 + \lambda_{\{3\}} \cdot x_3$$
$$+ \lambda_{\{1,2\}} \cdot (x_1 + x_2 - \beta) + \lambda_{\{1,3\}} \cdot (x_1 + x_3 - \alpha) + \lambda_{\{2,3\}} \cdot (x_2 + x_3)$$

KKT Conditions of the Dual Variables:

$$\begin{aligned}\gamma - x_1 - x_2 - x_3 = 0,\ \lambda_{\{1,2,3\}} \text{ is free}\\ x_1 - \alpha \geq 0 \perp \lambda_{\{1\}} \geq 0\\ x_2 \geq 0 \perp \lambda_{\{2\}} \geq 0\\ x_3 \geq 0 \perp \lambda_{\{3\}} \geq 0\\ x_1 + x_2 - \beta \geq 0 \perp \lambda_{\{1,2\}} \geq 0\\ x_1 + x_3 - \alpha \geq 0 \perp \lambda_{\{1,3\}} \geq 0\\ x_2 + x_3 \geq 0 \perp \lambda_{\{2,3\}} \geq 0\end{aligned}$$

KKT Conditions of Primal Variables (primal variable is in the objective):

$$\begin{aligned}1 + \lambda_{\{1\}} + \lambda_{\{1,2\}} + \lambda_{\{1,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_1\\ 1 + \lambda_{\{2\}} + \lambda_{\{1,2\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_2 \geq 0\\ 1 + \lambda_{\{3\}} + \lambda_{\{1,3\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_3 \geq 0\end{aligned}$$

KKT Condition of Primal Variables (primal variable is not in the objective):

$$\begin{aligned}\lambda_{\{1\}} + \lambda_{\{1,2\}} + \lambda_{\{1,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_1 \geq 0\\ \lambda_{\{2\}} + \lambda_{\{1,2\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_2 \geq 0\\ \lambda_{\{3\}} + \lambda_{\{1,3\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_3 \geq 0\end{aligned}$$

Assumption for Problem of Riparian 1:

$$\begin{aligned}&x_1 \geq 0, x_2 = 0, x_3 = 0\\ &\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0\end{aligned}$$

Solution for Problem of Riparian 1:

$$x_1 = \gamma,\ \lambda_{\{1,2,3\}} = 1$$

Assumption for Problem of Riparian 2:

$$\begin{aligned}&x_1 \geq 0,\ x_2 \geq 0,\ x_3 = 0\\ &\lambda_{\{1\}} \geq 0,\ \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0\end{aligned}$$

Solution for Problem of Riparian 2:

$$\begin{aligned}&x_2 = \gamma - \alpha\\ &x_1 = \alpha,\ \lambda_{\{1\}} = 1,\ \lambda_{\{1,2,3\}} = 1\end{aligned}$$

Assumption for Problem of riparian 3:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0,\ \lambda_{\{1,2\}} \geq 0$$

Solution for Problem of Riparian 3:

$$x_3 = \gamma - \beta$$
$$\alpha \leq x_1 \leq \alpha + \delta,\ 0 \leq x_2 \leq \beta - \alpha - \delta$$
with: $0 \leq \delta \leq \beta - \alpha$ as well as $\lambda_{\{1,2\}} = 1$ and $\lambda_{\{1,2,3\}} = 1$

The Nash-Harsanyi Solution

Optimization Problem:

$$\max\,[(x_1 - \alpha)\cdot x_2 \cdot x_3] \qquad s.t.\ x_1 + x_2 + x_3 = \gamma,\ x_1 \geq \alpha,\ x_2 \geq 0,\ x_3 \geq 0$$
$$x_1 + x_2 \geq \beta,\ x_1 + x_3 \geq \alpha,\ x_2 + x_3 \geq 0$$

Lagrangian Function:

$$L = (x_1 - \alpha)\cdot x_2 \cdot x_3 + \lambda_{\{1,2,3\}}\cdot(\gamma - x_1 - x_2 - x_3) + \lambda_{\{1\}}\cdot(x_1 - \alpha) + \lambda_{\{2\}}\cdot x_2 + \lambda_{\{3\}}\cdot x_3$$
$$+ \lambda_{\{1,2\}}\cdot(x_1 + x_2 - \beta) + \lambda_{\{1,3\}}\cdot(x_1 + x_3 - \alpha) + \lambda_{\{2,3\}}\cdot(x_2 + x_3)$$

KKT Conditions:

$$x_2 \cdot x_3 + \lambda_{\{1\}} + \lambda_{\{1,2\}} + \lambda_{\{1,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_1 \geq 0$$
$$(x_1 - \alpha)\cdot x_3 + \lambda_{\{2\}} + \lambda_{\{1,2\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_2 \geq 0$$
$$(x_1 - \alpha)\cdot x_2 + \lambda_{\{3\}} + \lambda_{\{1,3\}} + \lambda_{\{2,3\}} - \lambda_{\{1,2,3\}} \leq 0 \perp x_3 \geq 0$$
$$\gamma - x_1 - x_2 - x_3 = 0,\ \lambda_{\{1,2,3\}} \text{ is free}$$
$$x_1 - \alpha \geq 0 \perp \lambda_{\{1\}} \geq 0$$
$$x_2 \geq 0 \perp \lambda_{\{2\}} \geq 0$$
$$x_3 \geq 0 \perp \lambda_{\{3\}} \geq 0$$
$$x_1 + x_2 - \beta \geq 0 \perp \lambda_{\{1,2\}} \geq 0$$
$$x_1 + x_3 - \alpha \geq 0 \perp \lambda_{\{1,3\}} \geq 0$$
$$x_2 + x_3 \geq 0 \perp \lambda_{\{2,3\}} \geq 0$$

Assumption:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0$$

Optimality Conditions:

$$\lambda_{\{1,2,3\}} = x_2 \cdot x_3 = (x_1 - \alpha)\cdot x_3 = (x_1 - \alpha)\cdot x_2$$
$$x_1 + x_2 + x_3 = \gamma$$

Solution:

$$x_1 = \frac{1}{3} \cdot (2 \cdot \alpha + \gamma),\ x_2 = x_3 = \frac{1}{3} \cdot (\gamma - \alpha)$$

The Nucleolus

Optimization Problem:

$$\min\,[e] \qquad s.t.\ x_1 + x_2 + x_3 = \gamma,\ x_1 + e \geq \alpha,\ x_2 + e \geq 0,\ x_3 + e \geq 0$$
$$x_1 + x_2 + e \geq \beta,\ x_1 + x_3 + e \geq \alpha,\ x_2 + x_3 + e \geq 0$$

Lagrangian Function:

$$L = e + \lambda_{\{1,2,3\}} \cdot (x_1 + x_2 + x_3 - \gamma) + \lambda_{\{1\}} \cdot (\alpha - x_1 - e) + \lambda_{\{2\}} \cdot (-x_2 - e) + \lambda_{\{3\}} \cdot (-x_3 - e)$$
$$+\lambda_{\{1,2\}} \cdot (\beta - x_1 - x_2 - e) + \lambda_{\{1,3\}} \cdot (\alpha - x_1 - x_3 - e) + \lambda_{\{2,3\}} \cdot (-x_2 - x_3 - e)$$

KKT Conditions:

$$\lambda_{\{1,2,3\}} - \lambda_{\{1\}} - \lambda_{\{1,2\}} - \lambda_{\{1,3\}} \geq 0 \perp x_1 \geq 0$$
$$\lambda_{\{1,2,3\}} - \lambda_{\{2\}} - \lambda_{\{1,2\}} - \lambda_{\{2,3\}} \geq 0 \perp x_2 \geq 0$$
$$\lambda_{\{1,2,3\}} - \lambda_{\{3\}} - \lambda_{\{1,3\}} - \lambda_{\{2,3\}} \geq 0 \perp x_3 \geq 0$$
$$1 - \lambda_{\{1\}} - \lambda_{\{2\}} - \lambda_{\{3\}} - \lambda_{\{1,2\}} - \lambda_{\{1,3\}} - \lambda_{\{2,3\}} = 0,\ e \text{ is free}$$
$$x_1 + x_2 + x_3 - \gamma = 0,\ \lambda_{\{1,2,3\}} \text{ is free}$$
$$\alpha - x_1 - e \leq 0 \perp \lambda_{\{1\}} \geq 0$$
$$-x_2 - e \leq 0 \perp \lambda_{\{2\}} \geq 0$$
$$-x_3 - e \leq 0 \perp \lambda_{\{3\}} \geq 0$$
$$\beta - x_1 - x_2 - e \leq 0 \perp \lambda_{\{1,2\}} \geq 0$$
$$\alpha - x_1 - x_3 - e \leq 0 \perp \lambda_{\{1,3\}} \geq 0$$
$$-x_2 - x_3 - e \leq 0 \perp \lambda_{\{2,3\}} \geq 0$$

In accordance with the specification of α, β, and γ, we have to differentiate between two cases:

- Case 1 is valid if $(\beta \leq \frac{\gamma}{3}) \vee ((\frac{\gamma}{3} < \beta) \wedge (\frac{3 \cdot \beta - \gamma}{2} \leq \alpha))$
- Case 2 is valid if $(\frac{\gamma}{3} < \beta) \wedge (\alpha < \frac{3 \cdot \beta - \gamma}{2})$

Assumption under Case 1:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$\lambda_{\{1\}} \geq 0,\ \lambda_{\{2\}} \geq 0,\ \lambda_{\{3\}} \geq 0,\ \lambda_{\{1,2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0$$

Optimality Conditions Under Case 1:

$$\alpha - x_1 - e = -x_2 - e = -x_3 - e = 0$$
$$x_1 + x_2 + x_3 - \gamma = 0$$
$$\lambda_{\{1,2,3\}} = \lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}}$$

Solution Under Case 1:

$$x_1 = \frac{2 \cdot \alpha + \gamma}{3},\ x_2 = x_3 = \frac{\gamma - \alpha}{3}$$
$$e = \frac{\alpha - \gamma}{3}$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{3\}} = \lambda_{\{1,2,3\}} = \frac{1}{3}$$

Assumption Under Case 2:

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$
$$\lambda_{\{1\}} = \lambda_{\{2\}} = \lambda_{\{1,3\}} = \lambda_{\{2,3\}} = 0,\ \lambda_{\{3\}} \geq 0,\ \lambda_{\{1,2\}} \geq 0$$

Optimality Conditions Under Case 2:

$$\beta - x_1 - x_2 - e = -x_3 - e = 0$$
$$x_1 + x_2 + x_3 - \gamma = 0$$
$$\lambda_{\{1,2,3\}} = \lambda_{\{1,2\}} = \lambda_{\{3\}}$$

Solution Under Case 2:

$$x_1 + x_2 = \frac{\beta + \gamma}{2},\ x_3 = \frac{\gamma - \beta}{2}$$
$$e = \frac{\beta - \gamma}{2}$$
$$\lambda_{\{3\}} = \lambda_{\{1,2\}} = \lambda_{\{1,2,3\}} = 0.5$$

The nucleolus for the sub-coalition $\{1, 2\}$ can be solved using the following optimization problem:

$$\min_{\{e, x_1, x_2\}} [e]$$
$$s.t.\ x_1 + x_2 = 0.5 \cdot (\beta + \gamma),\ e + x_1 \geq \alpha,\ e + x_2 \geq 0$$

The step-by-step nucleolus solution for a coalition with two riparians is explained in detail in Exercise 6.2.

Solution:

$$x_1 = \frac{2 \cdot \alpha + \beta + \gamma}{4},\ x_2 = \frac{\beta + \gamma - 2 \cdot \alpha}{4}$$

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Appendix: Karush–Kuhn–Tucker Conditions

Given is the following optimization problem:

$$\max_{\{x_1,\ldots,x_I,y_1,\ldots,y_J\}} f(x_1,\ldots,x_I,y_1,\ldots,y_J) \tag{A.1}$$

$$s.t. g_m(x_1,\ldots,x_I,y_1,\ldots,y_J) \leq b_m \qquad (\lambda_m) \qquad \forall m \tag{A.2}$$

$$h_n(x_1,\ldots,x_I,y_1,\ldots,y_J) = c_n \qquad (\mu_n) \qquad \forall n \tag{A.3}$$

$$x_i \geq 0 \qquad \forall i \tag{A.4}$$

The values of the variables $x_1,\ldots,x_I, y_1,\ldots,y_J$ have to be set in that way that the function value $f(x_1,\ldots,x_I,y_1,\ldots,y_J)$ is maximized (see Eq. (A.1)). The values of the positive variables $x_1,\ldots,x_I$ are defined within the range $[0,\infty]$ (see Eq. (A.4)); while the value of the free variables $y_1,\ldots,y_J$ are defined in the range $[-\infty,\infty]$.

The solution space of the optimization problem is restricted by M inequality constraints (Eq. (A.2)) and N equality constraints (Eq. (A.3)). Due to the mth inequality constraint of Eq. (A.2), the function value of $g_m(x_1,\ldots,x_I,y_1,\ldots,y_J)$ must not exceed the value b_m.

Every inequality constraint is related with one dual variable, hence the mth inequality constrained is related with the dual variable λ_m. The value of the dual variable variable depends on the bettering of the function value $f(x_1,\ldots,x_I,y_1,\ldots,y_J)$, if the related mth inequality constraint was relaxed by for instance one unit. A relaxation of the mth inequality constraint by one unit means that we formulate $g_m(x_1,\ldots,x_I,y_1,\ldots,y_J) \leq b_m + 1$ instead of Eq. (A.2). The relaxation increases the solution space, hence the functional value $f(x_1,\ldots,x_I,y_1,\ldots,y_J)$ becomes higher. This increase of $f(x_1,\ldots,x_I,y_1,\ldots,y_J)$ due to the relaxation of mth inequality constraint is stated by the value of the dual variable λ_m.

Furthermore, there are also N equality constraints in the optimization problem; the nth equality constraint (Eq. (A.3)) guarantees that the value of function $h_n(x_1,\ldots,x_I,y_1,\ldots,y_J)$ becomes equal to the value c_n. Similar to the inequality constraints, the value of the dual variable (shadow price) μ_n represents the change

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of the objective value $f(x_1, \ldots, x_I, y_1, \ldots, y_J)$, implied by a relaxation of the nth equality constraint by one unit.[1]

For finding the optimal values of the maximization problem Eqs. (A.1)–(A.4), the corresponding Lagrangian function can be set up

$$\begin{aligned} L = {} & f(x_1, \ldots, x_I, y_1, \ldots, y_J) \\ & + \lambda_1 \cdot [b_1 - g_1(x_1, \ldots, x_I, y_1, \ldots, y_J)] + \cdots + \lambda_M \\ & \cdot [b_M - g_M(x_1, \ldots, x_I, y_1, \ldots, y_J)] \\ & + \mu_1 \cdot [c_1 - h_1(x_1, \ldots, x_I, y_1, \ldots, y_J)] + \cdots + \mu_N \\ & \cdot [c_N - h_N(x_1, \ldots, x_I, y_1, \ldots, y_J)] \end{aligned} \tag{A.5}$$

The first term on the right hand side (RHS) of the Lagrange function (see Eq. (A.5)) is the objective function ($f(x_1, \ldots, x_I, y_1, \ldots, y_J)$). Furthermore, all the inequality and equality constraints are also added as separate terms in the Lagrange function. All these constraints are multiplied with their corresponding dual variables. For using a more compact illustration, the Lagrangian function (A.5) can be rewritten as follows:

$$\begin{aligned} L = {} & f(x_1, \ldots, x_I, y_1, \ldots, y_J) \\ & + \sum_m \lambda_m \cdot [b_m - g_m(x_1, \ldots, x_I, y_1, \ldots, y_J)] \\ & + \sum_n \mu_n \cdot [c_n - h_n(x_1, \ldots, x_I, y_1, \ldots, y_J)] \end{aligned} \tag{A.6}$$

Based on the Lagrange function, the KKT optimality conditions can be formulated by the first derivatives of the Lagrange function with respect to all the primal $(x_1, \ldots, x_I, y_1, \ldots, y_J)$ and dual variables $(\lambda_1, \ldots, \lambda_M, \mu_1, \ldots, \mu_N)$ in the following way:

$$\frac{\partial L}{\partial x_i} \leq 0 \perp x_i \geq 0 \qquad \forall i \tag{A.7}$$

$$\frac{\partial L}{\partial y_j} = 0 \qquad \forall j \tag{A.8}$$

$$\frac{\partial L}{\partial \lambda_m} = b_m - g_m(x_1, \ldots, x_I, y_1, \ldots, y_J) \geq 0 \perp \lambda_m \geq 0 \qquad \forall m \tag{A.9}$$

$$\frac{\partial L}{\partial \mu_n} = c_n - h_n(x_1, \ldots, x_I, y_1, \ldots, y_J) = 0\,; \ \mu_n \text{ is free} \qquad \forall n \tag{A.10}$$

The Eqs. (A.7) and (A.8) illustrate the derivative of the Lagrange function with respect to the positive and free primal variables, respectively; while Eqs. (A.9) and (A.10), are the first derivatives of the Lagrange function with respect to the dual variables

[1]Respective equality constraint would become after relaxation: $h_n(x_1, \ldots, x_I, y_1, \ldots, y_J) = c_n + 1$.

related to the inequality and equability constraints, respectively. It becomes obvious that the derivative of the Lagrange function with respect to a dual variable results in the corresponding constraint.

The $\perp$-sign in Eqs. (A.7) and (A.9), implies that either the inequality before or after the $\perp$-sign has to be equal to zero. Hence, instead of the $\perp$-sign in Eq. (A.7), it is also possible to write $\frac{\partial L}{\partial x_i} \cdot x_i = 0$. Therefore, if we assume that $x_i = 0$, it follows that $\frac{\partial L}{\partial x_i} \leq 0$; while if we assume that $x_i \geq 0$, it is necessary that $\frac{\partial L}{\partial x_i} = 0$ in order to the condition $\frac{\partial L}{\partial x_i} \cdot x_i = 0$.

This is also valid for Eq. (A.9), therefore, instead of the $\perp$-sign, we could also write $\frac{\partial L}{\partial \lambda_m} \cdot \lambda_m = 0$. If the mth inequality constraint (Eq. (A.2)) becomes an equality, which means that the functional value of $g_m(x_1, \ldots, x_I, y_1, \ldots, y_J)$ is equal to the value of parameter b_m, i.e., $g_m(x_1, \ldots, x_I, y_1, \ldots, y_J) = b_m$, this mth inequality constraint could be termed as a binding constraint. This formulation $b_m - g_m(x_1, \ldots, x_I, y_1, \ldots, y_J) = 0$ is exactly: $\frac{\partial L}{\partial \lambda_m} = 0$. For this case, it results from the complementarity condition $\frac{\partial L}{\partial \lambda_m} \cdot \lambda_m = 0$, that we can assume $\lambda_m \geq 0$. Hence to conclude, in case that we would assume $\lambda_m \geq 0$, it follows that $\frac{\partial L}{\partial \lambda_m} = 0$ which is nothing else than $b_m = g_m(x_1, \ldots, x_I, y_1, \ldots, y_J)$ which means that the mth inequality constraint is binding.

In contrast, if we would assume that $\lambda_m \geq 0$, it follows from the complementarity condition $(\frac{\partial L}{\partial \lambda_m} \cdot \lambda_m = 0)$ that $(\frac{\partial L}{\partial \lambda_m} = 0)$ which is nothing else than $b_m \geq g_m(x_1, \ldots, x_I, y_1, \ldots, y_J)$ which means that the mth inequality constraint is non-binding.

Relaxing the mth equality constraint in Eq. (A.2) by one (which changes this respective constraint to: $g_m(x_1, \ldots, x_I, y_1, \ldots, y_J) \leq b_m + 1$) may lead to an increase (but never a decrease) of the objective level $f(x_1, \ldots, x_I, y_1, \ldots, y_J)$. Hence, the dual variables of all inequality constraints must be positive and hence $\lambda_m \geq 0 \quad \forall m$ (see Eq. (A.9)).[2] However, if we increase the level of parameter c_n by one in Eq. (A.3) (which changes this respective constraint to: $h_n(x_1, \ldots, x_I, y_1, \ldots, y_J) = c_n + 1$), the value of the objective function $f(x_1, \ldots, x_I, y_1, \ldots, y_J)$ may decrease, may increase or may stay constant. Therefore, the corresponding dual variables of all equality constraints are free, and thus are defined in the range $[-\infty, \infty]$ (see Eq. (A.10)).

[2] $\lambda_m \geq 0 \quad \forall m$ means $\lambda_1 \geq 0, \ldots, \lambda_M \geq 0$.

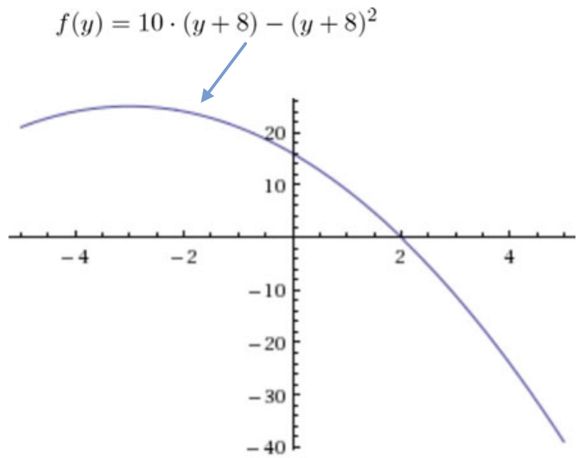

Fig. A.1 Illustration of first example. *Source* own illustration

We would like to illustrate the procedure with the help of three examples. Assuming as first example the following maximization problem:

$$\max_{\{x\}} f(y) = 10 \cdot (y+8) - (y+8)^2 \tag{A.11}$$

Here the objective function $f(y)$ should be maximized by finding an optimal value for the variable y. We assume that variable y is a free one, whose domain is defined within the range $[-\infty, \infty]$. The corresponding Lagrange Function of this mathematical problem contains simply the objective function

$$L = 10 \cdot (y+8) - (y+8)^2 \tag{A.12}$$

The Lagrange function in Eq. (A.12) has to be derived with respect to the only endogenous parameter y for finding the corresponding KKT conditions of the problem given in Eq. (A.11)

$$\frac{\partial L}{\partial y} = 10 - 2 \cdot (y+8) = 0 \tag{A.13}$$

The solution of Eq. (A.13), is the optimal value of variable y which maximizes the objective function $f(y)$

$$y^* = -3 \tag{A.14}$$

A plot of the objective function $f(y)$ is given in Fig. A.1.

The second example is characterized by a similar functional form as the first example; however, here we have a positive variable x instead of a free variable. The domain of the variable x is defined within the range $[0, \infty]$ (see Eq. (A.16))

$$\max_{\{x\}} f(x) = 10 \cdot (x+8) - (x+8)^2 \tag{A.15}$$

$$x \geq 0 \tag{A.16}$$

Based on the optimization problem containing Eqs. (A.15) and (A.16), the following Lagrange function can be set up:

$$L = 10 \cdot (x + 8) - (x + 8)^2 \tag{A.17}$$

A KKT condition, which simply results from the derivative of the Lagrange function with respect to the variable x, would be ***wrong***, because the resulting optimal value of variable x would be negative and hence would violate the non-negativity condition (A.16)

$$\frac{\partial L}{\partial x} = 10 - 2 \cdot (x + 8) = 0 \qquad \rightarrow \; x = -3 \tag{A.18}$$

We address the non-negativity of variable x by the following KKT condition which is correct:

$$\frac{\partial L}{\partial x} = 10 - 2 \cdot (x + 8) \leq 0 \perp x \geq 0 \tag{A.19}$$

Because of the $\perp$-sign, which means: $\frac{\partial L}{\partial x} \cdot x = 0$, two cases are possible:

- *Case 1:* $x \geq 0$ and hence $\frac{\partial L}{\partial x} = 10 - 2 \cdot (x + 8) = 0$:

  Solving the equation $\frac{\partial L}{\partial x} = 10 - 2 \cdot (x + 8) = 0$ leads to the result $x = -3$ which violates the non-negativity condition. Due to this contradiction, the optimal solution can not be found by this case.

- *Case 2:* $x = 0$ and hence $\frac{\partial L}{\partial x} = 10 - 2 \cdot (x + 8) \leq 0$:

  Setting $x = 0$ in the equation $\frac{\partial L}{\partial x}$ leads to the result

$$\frac{\partial L}{\partial x} = -6 \leq 0 \tag{A.20}$$

  There is no contradiction in case 2, hence the optimal solution is $x = 0$.

Compared to the second example, in the following third example two additional constraints are addressed (Fig. A.2)

$$\max_{\{x\}} f(x) = 10 \cdot (x + 8) - (x + 8)^2 \tag{A.21}$$

$$3 - x^2 - 2 \cdot x \leq 0 \qquad (\lambda_1) \tag{A.22}$$

$$x \leq 2 \qquad (\lambda_2) \tag{A.23}$$

$$x \geq 0 \tag{A.24}$$

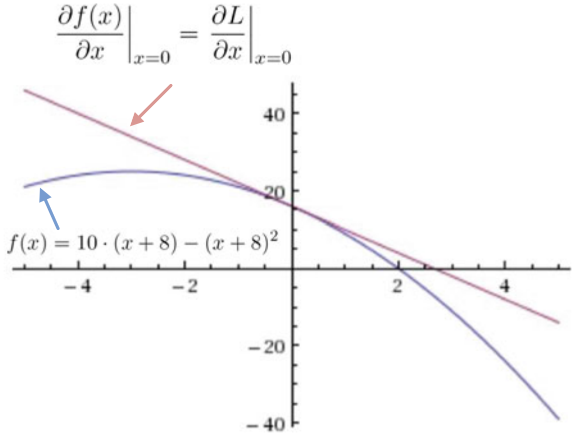

Fig. A.2 Illustration of second example. *Source* own illustration

These two additional constraints limit the solution space; their corresponding dual variables are λ_1 and λ_2.

Based on this optimization problem (A.21)–(A.24), the following Lagrange function can be set up:

$$L = 10 \cdot (x+8) - (x+8)^2 + \lambda_1 \cdot \left[x^2 + 2 \cdot x - 3\right] + \lambda_2 \cdot [2 - x] \quad \text{(A.25)}$$

The resulting KKT conditions of this problem are

$$\frac{\partial L}{\partial x} = 10 - 2 \cdot (x+8) + \lambda_1 \cdot (2 \cdot x_1 + 2) - \lambda_2 \leq 0 \perp x \geq 0 \quad \text{(A.26)}$$

$$\frac{\partial L}{\partial \lambda_1} = x^2 + 2 \cdot x - 3 \geq 0 \perp \lambda_1 \geq 0 \quad \text{(A.27)}$$

$$\frac{\partial L}{\partial \lambda_2} = 2 - x \geq 0 \perp \lambda_2 \geq 0 \quad \text{(A.28)}$$

Due to the $\perp$-sign in Eqs. (A.26)–(A.28), we know that $\frac{\partial L}{\partial x} \cdot x = 0$, $\frac{\partial L}{\partial \lambda_1} \cdot \lambda_1 = 0$ and $\frac{\partial L}{\partial \lambda_2} \cdot \lambda_2 = 0$, respectively, which are the complementarity conditions.[3] In the following we analyze different possible cases (for different made assumptions) to find the optimal solution of the problem:

- *Case 1:* $\underline{x = 0:}$

[3] Either the inequality to the left or to the right of the $\perp$-sign has to be equal to zero. Hence, for each $\perp$-sign, two different cases are possible. Here we have 3 $\perp$-signs in the KKT conditions, therefore, a maximum of $2^3 = 8$ cases is possible.

- If we set $x = 0$ in Eq. (A.27) we get $\frac{\partial L}{\partial \lambda_1} = -3 < 0$. The condition that $\frac{\partial L}{\partial \lambda_1} \geq 0$ is violated, and hence $x = 0$ is not the optimal solution.
- Therefore, we know that we have to assume $x \geq 0$, which means that we can specify Eq. (A.26) in the following way: $10 - 2 \cdot (x + 8) + \lambda_1 \cdot (2 \cdot x_1 + 2) - \lambda_2 = 0$.

• *Case 2:* $\underline{\lambda_2 \geq 0:}$
  - In this case, we assume that the inequality (A.28) is a binding constraint, hence $x = 2$.
  - If we set $x = 2$ in Eq. (A.27), we know that $\frac{\partial L}{\partial \lambda_1} = 5 > 0$ and hence $\lambda_1 = 0$.
  - If we set $x = 2$ and $\lambda_1 = 0$ in Eq. (A.26), we get $\lambda_2 = -10 < 0$. This violates the condition that $\lambda_2 \geq 0$.
  - Therefore, we know that we have to assume $\lambda_2 = 0$, which means that due to Eq. (A.28): $x \leq 2$.

• *Case 3:* $\underline{\lambda_1 \geq 0:}$
  - In this case, we assume that constraint (A.27) is the binding constraint, which implies that $x = 1$.
  - We know from our analysis (see Case 2) that we have to assume $\lambda_2 = 0$, which means $\frac{\partial L}{\partial \lambda_2} \geq 0$ (see (A.28)), which is met because $\frac{\partial L}{\partial \lambda_2} = 1$.
  - Because of Eq. (A.26) and $x = 1$ as well as our former made analysis (see Case 1), we can state that $\frac{\partial L}{\partial x} = 0$. Calculating λ_1, by setting $x = 1$ and $\lambda_2 = 0$ in equation $\frac{\partial L}{\partial x} = 0$, we get $\lambda_1 = 2$.
  - A contradiction can not be found in this case, hence the problem is maximized for $x = 1$.