New ICMI Study Series

Yoshinori Shimizu
Renuka Vithal *Editors*

Mathematics Curriculum Reforms Around the World

The 24th ICMI Study

International Commission on Mathematical Instruction

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New ICMI Study Series

NISS Aims and Scope

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This Series is indexed in Scopus.

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Foreword

The curriculum is central to schooling, and curriculum reform, including reform in the mathematics curriculum, is a phenomenon as old as schooling itself since it first existed. Every stakeholder seems to have an opinion on what should go into the curriculum, and on how the curriculum should be reformed. But curriculum reform in mathematics remains a much talked about and yet, as the editors of this Study Volume pointed out, under-researched topic. The aim of the book is therefore to take stock of the work that has been done in this area, and forge theoretical understanding and guide practice in curriculum reform in mathematics.

Curriculum reform entails at least three multi-faceted aspects: the development process of the reform, the content of the reformed curriculum, and implementation of the reformed measures. Each of these aspects involves a host of complex, interwoven elements. In addition, there are the important issues of the driving forces and barriers that shape the different aspects of curriculum reform. So it is hard to imagine that one book would be able to capture all these different aspects and elements of mathematics curriculum reform and their complexities. But this is exactly what this Study Volume intends to do. As such, this is perhaps one of the most ambitious studies within the series of ICMI Studies in terms of the scope and depth of the topic covered. And the authors of the book, under the leadership of the editors, have achieved this aim remarkably well.

To understand the development process, content and implementation of curriculum reform in mathematics, this book documents and critically examines major mathematics curriculum reforms in various countries from both historical and international perspectives. Major curriculum projects in the past from countries of a diversity of geographical and cultural backgrounds around the world are analyzed judiciously, and the impact of globalization on curriculum reform in selected countries is also examined. International perspectives have assumed particular importance in recent years in the wake of international studies of mathematics achievement such as TIMSS and PISA, which have a major impact on curriculum reforms in different countries. Learning from past reforms and from the impact of globalization on the curriculum does not only help us to understand the present curriculum better - how it has ended up in the state it is and why - and plan for the future curriculum better, it also contributes to theoretical understanding of the determinants of curriculum reform, issues on the underlying factors, causes, and agents of the curriculum.

As far as the content of curriculum reform is concerned, this book examines issues of coherence and relevance of the curriculum, illustrated with the curricula in different countries. Internal issues of coherence and relevance include relation with other disciplines (e.g., other STEM subjects), and external issues include the role of mathematics in society (e.g., mathematics citizenship). These eventually hinge on fundamental issues about the nature of mathematics and the purpose of mathematics education, such as: what is mathematics? what mathematical knowledge should students of this generation learn? what are mathematics and mathematics education for?

The above issues relating to the process and content of curriculum reform are extensive and complicated enough, but even more complex is the issue of curriculum implementation. The case studies on implementation of curriculum reforms in different countries provided in the book illustrate clearly the complexity of the issues involved in the implementation of the reformed curriculum in different contexts and traditions, and at the theoretical level, the authors put forward a generalization of the "laws" of curriculum implementation.

On the determinants of curriculum reform, a significant part of the book touches on cultural factors, an area that I am particularly interested in personally. Mutual influences of curriculum reforms in different countries and the impact of globalization on curriculum reform in individual countries result in a lot of similarities in both the reform processes and the reformed curricula among countries. Yet, mathematics curriculum reforms in different countries still differ in subtle and significant ways. Without resorting to explanations at the cultural level, sometimes it is hard to understand the deep-rooted reasons for the subtle differences. The development process of the reformed curriculum, how the content of the reformed curriculum is formulated and organized, and the way curriculum changes are implemented, are all profoundly influenced by cultural factors. These are important theoretical issues, both for researchers and for policy makers.

This informative book brings forth a rich and in-depth discussion of the various issues pertaining to curriculum reform in mathematics, offering valuable information and analysis for practitioners, policy makers and researchers in education. For front-line teachers, teacher educators and other practitioners, the book provides materials for a deeper understanding of the mathematics curriculum. For policy makers, it provides insights into what works and what doesn't work in curriculum reforms, and why. For researchers, the book provides theoretical understanding on the various aspects of curriculum reform, and points to future research directions in the field.

I congratulate the editors and authors on their remarkable efforts and achievement in putting together such an informative and insightful book. The book is an important addition to the series of ICMI Studies, and I am sure it will prove to be an invaluable reference in the area of mathematics curriculum reform, and in the field of curriculum reform at large.

Pokfulam, Hong Kong
March 2022

Frederick K. S. Leung

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Part I
Introduction to ICMI Study 24

Chapter 1
School Mathematics Curriculum Reforms: Widespread Practice But Under-Researched in Mathematics Education

Yoshinori Shimizu and Renuka Vithal

Introduction

The International Commission on Mathematical Instruction (ICMI) studies are unique in that they create opportunities to bring together large numbers and a diversity of scholars across countries and all kinds of practitioners to discuss a particular topic and generate publications deemed important and useful for different communities. An international study on school mathematics curriculum reforms, some might argue, is overdue given its long history and given that it is a practice engaged by a large number of countries, states or regions around the world.

Typically, ICMI Studies are commissioned when there is a considerable scholarship in particular area and the studies serve to pull these together in providing a 'state of the art' discourse on a particular topic and to shape the direction going forward. This ICMI Study 24 is especially singular in that while school mathematics curriculum reforms are a widespread and enduring practice internationally and globally, as the Study Conference (Shimizu & Vithal, 2018) revealed, its in-depth study is arguably less so in comparison with, say, ICMI Study 23 on primary mathematics study on whole numbers or ICMI Study 22 on mathematics task design.

This became particularly evident to the International Program Committee (IPC) as a search was undertaken for leading mathematics education scholars in the broad area of school mathematics curriculum reforms to provide keynotes and plenary lectures for the conference. That is to say, there are a number of leading mathematics educators participating in or even leading major school mathematics curriculum

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reforms in their countries, (many of whom participated in the study conference), but arguably a dearth of scholars or researchers in school mathematics curriculum reforms in the traditional sense of expert scholars with a wealth of publications about mathematical reforms. In fact, it would appear that those mathematics educators involved in, say, national school mathematical curriculum reforms seldom seem to develop an extensive and deep scholarship of the intense practice of the curriculum reform itself that they engaged in driving, leading or participating.

Not only are those who may be considered insiders to mathematics curriculum reforms not generating scholarship in this critical area, but there is also a general lack of in-depth study of those very reform processes, which are often highly contested within governing structures that are tasked with making curriculum reforms. Even for those considered to be outsiders to these processes, such as independent researchers, clearly one challenge would be the issue of access to government curriculum policy review and development environment to study all aspects of how, what and who makes and shapes curriculum reforms. This is alluded to and long recognised as a gap in mathematics education research in the broad area of policy studies (Lerman et al., 2002; Vithal & Volmink, 2005).

This suggests that, while there are considerable scholarly developments in general education on curriculum reform studies, these have not crossed boundaries to the same extent into mathematics education as other areas or have not been developed by mathematics educators themselves to the same extent. One consequence of this lack of in-depth research in school mathematics curriculum reforms, especially at macro levels, is that there is equally a lack of theories to explain the many different aspects and consequences of school mathematics curriculum reforms as well as limited writings about appropriate research methodologies for studying this topic in all its manifestations, dimensions, levels and components.

This ICMI Study is, therefore, a part of a process of taking stock of the work that has been done and *forging an un- or under-explored area of research in which there is diverse and considerable practices, but not as much scholarly work to understand and guide these practices.* Furthermore, literature reviews show that, while there are rich descriptions and analyses of individual cases of mathematics curriculum reforms at macro levels (e.g. Li & Lappan, 2014; Thompson et al., 2018), drawing out key messages or learning points from across these is less common. Therefore, while school mathematics curriculum reform is a well-established and entrenched practice in countries, districts or regions around the world, it begs questions about research – methodologies and theories – informing those diverse practices that eventually impact schools, teachers and learners on the ground. Many reasons may be advanced for this state of affairs.

This study volume seeks to contribute to such gaps (among others) and open up for new directions in research, theory, policy and practice. It points to an emergent but very necessary area of deeper study requiring re-conceptualisation of research questions, and the development of appropriate theories and methodologies that are able to take account of the enormous contextual variation within which school mathematics reforms take place around the world.

Background and Rationale of the Study[1]

School mathematical reforms have taken place in many countries around the world in contexts that vary significantly on many different dimensions - culturally, politically, geographically, economically, historically – to name but a few. It is evident that much could be learnt from deeper and more substantial reflections and research about all different aspects of these reforms. Reforms have been large-scale involving an education system as a whole, at a national, state, district or regional level in which mathematical curricula, standards or frameworks have been developed and implemented. Changes have taken place at all levels of mathematics in the school educational system from pre-primary through senior secondary.

School mathematics reforms are often conducted with changes in all different aspects of the curriculum: mathematics content, teaching and learning methodologies and resources (e.g. texts and technologies), as well as assessment and examinations. It is possible to observe different influences on school mathematics curriculum reforms over time. During the mid-twentieth century, school mathematics curriculum reforms were shaped by developments within the discipline of mathematics and by the ideas of some mathematicians. This is captured in an address by the French mathematician Jean Dieudonné, one of the proponents of what was then termed the 'New Math' in 1959:

> In the last fifty years, mathematicians have been led to introduce not only new concepts but a new language, a language which grew empirically from the needs of mathematical research and whose ability to express mathematical statements concisely and precisely has repeatedly been tested and has won universal approval.
>
> But until now the introduction of this terminology has been steadfastly resisted by secondary schools, which desperately cling to an obsolete and inadequate language. And so, when a student enters the university, he will most probably never have heard such common mathematical words as set, mapping, group, vector space, etc. (cited in Howson et al., 1981, p. 102)

The New-Math reform, took place in a particular historical context of the 'cold war' and colonialism. It became a mathematical movement that spread to many countries around the world with different influences on national curricula and practical implementations in schools. The character of this reform and its challenges was a departing point for many developments and discussions in the teaching of mathematics. Since then, with the lessons from the New-Math reform movements, the field of mathematics education has progressed immensely.

Another major influence on school mathematics curriculum reforms in the second half of the twentieth century has been from outside mathematics, that is, developments in other disciplines, most notably, psychology. Studies and theories in behaviourism, the rise and development of cognitive science and constructivism, to

[1] This introductory chapter integrates the text from the ICMI Study 24 Discussion Document, which called for the Study Conference entitled *School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities* authored by the International Programme Committee (2017) for the study.

name a few, have especially impacted content and approaches advocated in mathematics curriculum reforms. Other trends in mathematics curriculum reforms included problem solving and back-to-basics (among others).

More recent influences on mathematics curriculum reforms, in this twenty-first century, have come from other areas, such as large international studies, especially those focusing on student achievements. These studies have enabled comparisons of mathematics curricula (such as intended and attained curricula) across many countries and generated particular conceptions (such as mathematics literacy), which have found their way into mathematics curriculum reforms. Nowadays international comparative studies like the Trends in International Mathematics and Science Study (TIMSS) (Mullis et al., 2016) and the Organisation for Economic Co-operation and Development Programme for International Student Assessment (PISA) (OECD, 2016), which attract a great deal of public attention and media focusing on student and teacher performance in mathematics education (and to which politicians and policy makers are especially responsive), are impacting and shaping school mathematics curriculum reforms as countries or regions both compete and share curriculum policies, materials and approaches.

These studies have raised the stakes significantly, and arguably, entrenched a focus on student performance and better test scores as opposed to better student learning within mathematics curriculum reforms. There are a diversity of studies and findings from international experience and research that can and does influence the nature of curriculum changes, and the possibilities of educational reform and its implementation: curricular design results; a revised role for components in the teaching of mathematics (e.g. mathematics content, pedagogy and assessment); the role of technology; new cognitive, socio-cultural and socio-political perspectives.

In recent years, the internationalisation and globalisation of the economy, universality of technological development and related needs for new skills and knowledge play the role of strong motivations for curriculum reforms that have brought calls for unified standards for mathematics in school. In the international debate, many scholars, teachers and policy-makers speak of the "*twenty-first-century competencies*" and consider important items like: "critical and inventive thinking; communication, collaboration and information skills; and civic literacy, global awareness and cross-cultural skills".[2] In many countries, the so-called "21st-century competencies framework"[3] is being worked on, in order to guide the development of the national curriculum and to design school-based programmes to nurture these competencies.

In relation to this, new mathematics curriculum discourses have emerged and taken hold. Notions of mathematical 'competences' and that of mathematical 'literacy' are important examples that have been raised, from different perspectives around the world (Niss, 2015). In particular, from the approach of OECD's PISA, several notions (and their underpinning theoretical framework) have become very

[2] See, for example, the Singapore Ministry of Education website: https://www.moe.gov.sg/education/education-system/21st-century-competencies

[3] See, for example, *Towards Defining 21st-Century Competencies for Ontario, Canada*: http://www.edugains.ca/resources21CL/About21stCentury/21CL_21stCenturyCompetencies.pdf

influential in many countries in the changes being made in local curricula and standards: for example, in Denmark, Germany, Japan, South Korea, Costa Rica, Spain, Norway, Mexico, Sweden. PISA stresses the role of mathematical literacy as a central goal in school mathematics education, because it improves the life chances of most students, and justifies why mathematics is essential to describe, explain and predict the world. According to the PISA 2015 Mathematics Framework:

> Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD, 2016, p. 65)

However, the word *literacy* itself is ambiguous with multiple meanings, and trying to translate it into different languages and cultures has proved to be a difficult, if not sometimes impossible, task. In the literature, one finds different names and definitions; and many changes over the years showing the notion of literacy to be:

> a socially and culturally embedded practice, and [… its] conceptions [… vary] with respect to the culture and values of the stakeholders who promote it. (Niss & Jablonka, 2014, p. 395)

The differences in approach are directly linked to the goals that are pursued in mathematics education in individual countries. Its inclusion in curriculum reforms identifies new demands about what citizens are obliged to know (or not allowed to ignore). Hence, a careful analysis of this notion is relevant in order to focus its rationale in a curriculum.

International studies that examine the successes and failures in achieving the promised aims from different reforms, across countries need to be undertaken. The aftermath of the New-Math reform alludes to the importance of reflecting on the requirements for a new curriculum, suitable to escape the causes of the complete, or partial, rejection of this reform in so many countries.

The challenges of this particular reform and others that followed opens a discussion about different aspects of curriculum reforms, which go beyond content, such as:

- the existence of different epistemological and cultural positions concerning mathematics and its relevance in different societies;
- the distance of the proposed reform from the mathematical, educational and material conditions and possibilities in different countries, including teacher quality, their preparation, knowledge, beliefs and expectations;
- the relationships with the social, cultural and personal contributions brought by the students in the classroom, so relevant to avoid students' alienation from their social and cultural environment and to allow students to engage in learning in a productive way;
- the influence of political and institutional scenarios within educational systems, that can promote, discourage or weaken curricular reforms.

A consequence of these reflections is that the communities of researchers, teachers and policy-makers need to become more aware that considerations of curriculum reforms from various perspectives and constructs (mathematical literacy or competences, for instance) raises many issues, from a scientific, political and cultural point of view.

This ICMI study topic invokes not only questions about changes in curriculum design but – with force – questions about the implementation of these changes across an educational system. A curriculum reform will be influential or have impact in so far as it can be implemented. What has functioned (or not) at the time of implementing a curricular change? What are the limitations? How have resources (e.g. textbooks and technology) influenced the reforms and their enactment? How must large-scale teacher preparation be conducted to achieve the reform goals? How do diverse social, economic, cultural and national contexts condition the nature and extent of curricular reforms – especially teacher expectation, attitudes and beliefs – as well as the social and cultural background of students? How are assessments of students' learning influential in curriculum reforms? An ICMI Study offered an opportunity to provide a synthesis and meta-analysis of different aspects of school mathematics reforms historically, geographically and globally.

There are many studies conducted in different parts of the world about these issues of mathematics curriculum reforms and their implementation with findings that can be systematised, compared and studied. The way curricula are elaborated, proposed, changed and reorganised is, however, still a rather under-explored area in mathematics education. This ICMI Study allows for a more informed and comprehensive analysis of the roles of different actors and of the many aspects influencing and shaping mathematics curriculum reforms that are taking (or have taken) place; and of the possibilities and means to tackle a curricular reform in the current scenario we live in and unfolding future developments.

It is as crucial an issue for developing countries as it is for developed ones, given by the global changes taking place in societies, as they confront different challenges of growing inequality, unemployment, poverty, mass migration, environmental disasters, various form of discrimination and conflicts (to name but a few), and within which school mathematics reforms must take place. However, the processes of curriculum reform may differ in developed versus developing countries, due to different protocols followed, different intentions and agendas, resources and conditions as well as policy and political rhythms. Other comparative fault lines are, for example, East–West differences in mathematics curricula and reforms which have gained much interest, largely from the results of international studies and the migration of students. New global phenomena such as the spread and actions against infectious diseases (for example, COVID-19), and the massive shift to on-line teaching and learning creates new urgent imperatives related to all aspects of technology in mathematics curricula.

A further rationale for this study is to stimulate further research and publications that explore mathematics curriculum reforms, especially at a policy level and across multiple and diverse contexts. Some recent volumes, such as by Thompson et al. (2018) and Li and Lappan (2014), point to the growing need for further research in this area and its potential for more evidence-based mathematics curriculum policy generation and implementation.

An ICMI Study offers a unique opportunity to examine past and present mathematics curriculum reforms in different parts of the world, from a macro- perspective and meta-level and to investigate larger questions of who or what in a society drive and most influence curriculum reforms, what reforms precisely are taking place, how are these being implemented and, whether they are deemed successful (or not), what counts as success. Hence, this study has the potential to build understanding of the implications – current and future – of these larger questions for school mathematics, for different aspects of teaching and learning mathematics, and for its role in the broader society. By studying curriculum reforms across multiple contexts, key messages and lessons may be derived to inform, improve and better conduct future mathematics curriculum reforms.

Themes and Questions

The overarching question of this ICMI Study was to explore what school mathematics curriculum reforms have been or are taking place, at all levels, but especially at a macro- or system level: to learn about and from the many aspects of mathematics curriculum reforms from past experience; to gain an understanding of the current status and issues in school mathematics curriculum reforms around the world; and to identify gaps silences and glean possible directions for the future of school mathematics.

School mathematics curriculum reform is a very broad topic comprising many different perspectives and inter-related components. The diverse range of possible questions have been clustered around five themes, described below. They were selected for the study to address the research questions in a systematic manner, notwithstanding the challenges of overlap.

A. **Learning from the past: driving forces and barriers shaping mathematics curriculum reforms**

School mathematics curriculum reforms are contested spaces with many different vested interests because of the multiple goals and intentions they are expected to serve. Therefore, in any curriculum reform, there are both driving forces and barriers in shaping mathematics curricula. This first theme sets a general background and the context, and invites studies of school mathematics curriculum reforms in the past decades.

A1. Which aspects of school mathematics curriculum reform carried out in the past decades are considered to be the most important (for example, in content, pedagogy and the underpinning theoretical approaches)? What potentially crucial aspects of mathematical curricula have not been considered and, even less, touched upon?

A2. Which goals and values in school mathematics curriculum reforms, carried in the past decades, have been the most important (for example, in the selection and organisation of mathematics contents, or process aspects of mathematical activities)?
A3. How have the questions of content become linked to the notions of mathematical competencies, capabilities, and literacy? How have these evolved to become a driving force in the curriculum development and reform initiatives?
A4. What has been the role and function of curriculum resources, materials and technology, including digital curricula and textbooks in curriculum reforms and their implementation as driving forces or barriers?

B. **Analysing school mathematics curriculum reforms for coherence and relevance**

The role, content and importance of mathematics as a school subject are examined in each educational system from time to time. All mathematics curricula set out the goals expected to be achieved in learning through the teaching of mathematics; and embed particular values, which may be explicit or implicit. Recent emphases on STEM (science, technology, engineering and mathematics) education in many countries raises both the question of the place of mathematics among these subjects, and the discussion of introducing an integrated or interdisciplinary subject. Questions about the study of school mathematics curriculum reforms are raised in this context for their coherence and relevance.

B1. What is the extent of coherence within and among different aspects of reformed curricula such as values, goals, content, pedagogy, assessment and resources? How are curriculum ideas organised and sequenced for internal coherence in a curriculum reform? What are the effects of a lack of coherence? For example, regarding relations between high-stakes examinations and curriculum reforms.
B2. How are mathematics content and pedagogical approaches in reforms determined for different groups of students (e.g. in different curriculum levels or tracks) and by whom? How do curriculum reforms establish new structures in content, stakeholders (e.g. students and teachers) and school organisations, and what are their effects?
B3. What interrelation between mathematics and other disciplines, or movement toward integrated or interdisciplinary curricula, can be observed in mathematics curriculum reforms, given the current emphases on STEM education? What is the relationship between school mathematics and mathematics as a discipline in school mathematics curriculum reforms?
B4. What curriculum materials development and technology are or have been engaged, and what are their roles, goals and underlying values in school mathematics curriculum reforms?
B5. What theories and methodologies are appropriate for studying phenomena related to mathematics curriculum reforms?

C. **Analysing school mathematics curriculum reforms for coherence and relevance: Implementation of reformed mathematics curricula within and across different contexts and traditions**

The cultural, social, economic and political contexts and positions for the implementation of the school mathematics curriculum are important considerations. The processes of implementing new or reformed curricula may differ according to the cultural and historical contexts and traditions, due to different protocols followed and the processes of political decision-making.

C1. What processes, models or best/common practices can be identified from the experiences in the implementation of new or reformed school mathematics curricula?
C2. What are examples of successful or unsuccessful reforms and what are the reasons for their success or failure? What criteria are used for assessing curriculum reforms and their degree of success or failure?
C3. How is the implementation of new or reformed curricula monitored, evaluated and acted upon? What are models or mechanisms of continuous improvement in school mathematics curricula? How does the existence of such a mechanism affect the frequency, (dis-)continuity and perceived challenges and successes of curriculum reforms?
C4. What models or processes for professional teacher preparation and continuous development have been carried out in different countries in the implementation of new or reformed curricula. What are their influences, effectiveness, successes or failures?
C5. What are the types of resources and what are their roles (e.g. textbooks, materials, technology) in the implementation of reformed curricula?

D. **Globalisation and internationalisation, and their impacts on mathematics curriculum reforms**

There are a number of factors that advance globalisation and internationalisation through rapid changes in the nature of communication and availability of information. This internationalisation and globalisation of life in the twenty-first century seem to affect mathematics curriculum reforms. These influences appear to lead increasingly toward a ‘convergence’ in school mathematics curriculum reforms. Commonalities and diversity may be observed through comparative studies.

D1. How have results of international experience and research in the teaching and learning of mathematics influenced curricula changes? To what extent can local curriculum reforms be examined against an emergent ‘international’ mathematics curriculum?
D2. How have particular international studies become drivers for school mathematics curriculum reforms? What new discourses with dominant theoretical and conceptual underpinning have emerged? And how have these been taken up in curriculum reforms in different contexts? For example, how have the OECD’s PISA notions of mathematical literacy and mathematical competencies been interpreted and expressed in curriculum reforms?

D3. How are mathematics curriculum reforms varied (or similar) in different social, cultural, economic and political contexts such as developing versus developed countries or East versus West? How do selected curriculum components such as content, pedagogy, materials technology and teacher preparation vary from one reform, tradition, country or context to another?
D4. How can comparative or meta-analyses of curriculum reform processes and implementations shed light on what works or does not work in mathematics curriculum reforms in contemporary societies?

E. **Agents and processes of curriculum design, development and reforms in school mathematics**

Curriculum reform processes are as much an educational matter as they are a political one. Nowadays, they involve a broad range of stakeholders with vested interests. Educational, social and political actors influence and shape curriculum reforms – from business, industry, media, teacher unions and parents, on the one hand, to those with different expertise, such as curriculum policy makers, educators, mathematicians and researchers, on the other.

E1. What are the processes, and how are they deployed, in the development of and during a mathematics curriculum reform? What agents lead or dominate and what is their influence on the aspects of curriculum reforms?
E2. What different roles do mathematics teachers, teacher educators, (education) researchers and mathematicians play in curriculum reforms? What kind of influences do these role players have in mathematics curriculum reforms?
E3. How (if at all) is public engagement with the mathematics curriculum reforms organised and managed; and who takes or is given this responsibility? What is the role and influence of different media in curriculum reforms?
E4. To what extent does or could research inform or influence curriculum design and development processes in reforms?

The ICMI Study began with these themes and questions, which served as the programme for the related ICMI Study Conference in calling for papers and the format of the proceedings. These themes and questions have also been retained as an organising structure to frame this study volume. However, from the Study Conference discussions and as will be evident in each theme constituting this volume and its concluding chapter, there was a greater focus on some questions than others.

The ICMI Study 24 Conference

A feature of any ICMI Study is that a Study Conference is convened as a part of the process of developing the study volume. The study conference was held in November 2018 in Japan and the Conference Proceedings (Shimizu & Vithal, 2018) was produced which provided the foundation for the study volume.

The study conference of ICMI Study 24 attracted ninety-six participants from twenty-eight different countries or regions. Participants came from all six different continents and from educational systems with different cultural, economic and historical backgrounds. This diversity provided rich discussion on the state-of-the-art scenarios of school mathematics curriculum reforms around the world. The conference participants shared specific cases of reforms. The study conference provided a singular opportunity to the researchers to juxtapose different cases highlighting commonalities and differences.

The conference proceedings document includes sixty-eight papers from diverse countries: Algeria, Australia, Chile, China/Hong Kong, Costa Rica, Denmark, France, Hungary, Indonesia, Iran, Ireland, Israel, Italy, Japan, Lebanon, Malaysia, Mexico, the Netherlands, Peru, Philippines, Portugal, Serbia, South Africa, Spain, Thailand, United Kingdom, United States of America and Vietnam. The diversity provided at the study conference enriched the discussion on school mathematics curriculum reforms, demonstrating variety in cases planned and implemented in societies with different socio-cultural backgrounds. This diversity also extended to the conference delegates who were not only from different contexts, but also brought a variety of positionalities in respect of mathematics curriculum reforms as receivers or drivers of a particular reform and differing knowledge, skills, expertise and experiences in mathematics curriculum reforms.

Defining Key Concepts Related to School Mathematics Curriculum Reforms

In each of the themes, the issue of definition of key concepts in the study became important to enable coherence in the study. There are a number of challenges in this respect. Different countries or parts of the world develop or draw on different meanings of curriculum, reforms or even of mathematics itself. It is also important to observe that, as much as mathematics education scholars and practitioners develop their own concepts or definitions, equally there are advances in general education or curriculum studies that are drawn on and influence these meanings.

The dictionary meaning of curriculum, as well as in common everyday discourse, is typically used to refer to a syllabus, set of courses or a programme of study. This meaning has shifted and been extended, over time, to a much broader conception, to refer to all the teaching and learning content, activities and experiences that are organised by an educational institution to achieve particular outcomes. In this respect, definitions of curriculum may be viewed as being on a continuum from a specific meaning to a very wide concept.

It is evident from the literature that the term ‘curriculum’ is used with many different meanings and scholars have noted that it seems almost impossible to give a universally acceptable definition of ‘curriculum’. For example, in the US it often means a textbook series, and in the UK the set of experiences a child has in school classrooms, according to Burkhardt (2014), while in other countries 'curriculum' is taken to mean some national document referred to as standards or frameworks.

From a general education perspective, Ornstein and Hunkins argue that, "the way we define curriculum reflects our approach to it" (2018, p. 26). For example, for them, curriculum defined as a plan for achieving educational goals may be aligned to a behavioural or system approach to curriculum, while curriculum defined as comprising the experiences learners have in school reflect humanistic approaches.

In mathematics education, the same could be observed. In the New Maths approaches of the mid-twentieth century, it could be argued that curriculum was defined primarily in terms of mathematics subject matter or content, while the rise of psychological approaches such as behaviourism led to the meaning of curriculum to shift to a much sharper focus on learners, learning experiences and learner outcomes.

It can also be noted from the literature that how curriculum is defined is related to its purpose. An enduring and widely deployed conceptualisation of curriculum is the one that has been used in the Trends in International Mathematics and Science Study (TIMSS) over several decades. The TIMSS Curriculum Model for 2019 continues to defines curriculum in terms of three levels: the intended, implemented and attained curriculum – as its core major organising conceptualisation (Mullis et al., 2016) since its earliest studies, the second IEA mathematics study (Garden, 1987).

This conceptualisation of curriculum can be expanded to integrate other meanings of curriculum. It is also the conceptualisation that is taken as a point of departure for this ICMI Study in the associated Discussion Document, which formed the basis for the Study Conference.

The *intended curriculum* can be variously described as the planned, official curriculum at an educational system level, for example at a national, district or state level. It is the overt, explicit curriculum in the form of written documents, texts and materials chosen to support the intended curriculum agenda. The intended curriculum is the formal curriculum which typically specifies what the relevant educational authority (and society) expect students will learn in terms of knowledge, skills, values and attitudes. Embodied in curriculum frameworks, guides, regulatory documents, polices or standards are the various curriculum components – goals, content, pedagogy, materials, assessments, examinations and other aspects. It may be the intended institutional curriculum.

The *implemented curriculum* is the actual curriculum delivered at the level of the school and the classroom. It is also defined as the curriculum-in-action or the enacted curriculum, through which the teacher in the classroom interprets and translates the intended curriculum in practice within a school. While the intended curriculum is contextualised with reference to broader national, social and educational contexts, the implemented curriculum is realised with reference to the context of the school and classroom, and the teachers' knowledge and experience. Also defined as the taught curriculum, it is the actual teaching and learning activities which take place through interactions between teachers and students – the teaching methodologies and resources through which the content and assessments are delivered in practice.

The third level, the *attained curriculum*, can also be referred to as the received curriculum or the curriculum as experienced by the students and manifested in their achievement and attitudes. The attained curriculum indicates the knowledge, skills, values and attitudes acquired by learners as a result of teaching and learning through different means and materials and demonstrated in practice. It has been documented and shown that there can be great disjuncture between each level of the intended, implemented and attained curriculum. The power of this definition is that, while a government or educational authority, in very unequal societies, can claim to offer the same curriculum when in fact they are referring only to the intended official curriculum, the implemented and/or the attained curriculum can reveal deep inequalities given by different resources, etc.

It is evident from the above that, at each curriculum level (intended, implemented and attained), attention can be paid to a range of different curriculum components (goals, content, teaching approaches, materials and assessment). For the purposes of this study on mathematics curriculum reforms, these two dimensions – curriculum levels and curriculum components – offer a means for the core conceptualising of the notion of curriculum.

Mathematics educators have long recognised the challenge of defining what is meant by the notion of curriculum. Some four decades ago, Howson, Keitel and Kilpatrick (1981) underscored that, "[curriculum] must mean more than syllabus – it must encompass aims, content, methods and assessment procedures" (p. 2). Along similar lines, Niss (2016) defines curriculum with respect to an educational setting, (which may be as broad as all public schools in a country to a single institution or a particular mathematics course), as a "vector with six entries": goals; content; materials; forms of teaching; student activities; assessment. These could be regarded as key components of a curriculum, especially within the context of school mathematics curriculum reforms, and are defined by Niss (2016, p. 241) as follows:

- *goals* (the overarching purposes, desirable learning outcomes, and specific objectives and aims of the teaching and learning taking place under the auspices of this curriculum);
- *content* (the topic areas, concepts, theories, results, methods, techniques, and procedures dealt with in teaching and learning);
- *materials* (the instructional materials and resources, including textbooks, artefacts, manipulatives, and IT-systems employed in teaching and learning);
- *forms of teaching* (the tasks, activities and modes of operation of the teacher of this curriculum);
- *student activities* (the activities of and tasks and assignments for the students taught according to this curriculum);
- *assessment* (the goals, modes, formats and instruments adopted for formative and for summative assessment in this curriculum).

Given the complexity of notions of curriculum reforms, this definition of curriculum resonated with delegates at the Study conference and was taken up by a number of authors as is reflected in several of the themes of this volume as part of the working definition of curriculum that was adopted.

There are, however, other dimensions that may be especially relevant for a study on mathematics curriculum reforms, given the high status and gate-keeping role mathematics plays in how it opens or closes curriculum pathways for students. In this context, curriculum definitions must also consider the unplanned, informal or hidden curriculum. The hidden curriculum, for instance, is the curriculum that is not planned or intended, but is nevertheless acquired by students and reflected in their performance, perspectives, attitudes, behaviour and feelings about mathematics and mathematics education.

Also relevant for conceptualising curriculum in a study on mathematics curriculum reforms is a definition of curriculum as process and as product. The term ‘curriculum’ can be used both as a product and as a process. A curriculum is a product: a set of instructional guidelines and materials for students’ acquisition of certain culturally valued knowledge and skill. A curriculum can also be viewed as a process. In this sense, the curriculum is not a physical thing, like textbooks, but rather the interaction of teacher, students and knowledge (Cai & Howson, 2013). Both of these – curriculum as process and as product – can be applied to each level of the intended, implemented and attained curriculum. For example, curriculum as process can refer to processes in curriculum development – at the intended level, this could refer to mathematics curriculum policy-making processes.

International studies on curriculum shows that the notion of curriculum takes on a variety of meanings in different contexts. The context within which any level or component of a curriculum finds expression is shaped and influenced by a range of dimensions – social, cultural, historical, political, technological and economic. Furthermore, a curriculum may be mainly descriptive, prescriptive or somewhere in-between (Suurtamm et al., 2018).

Mathematics curriculum can be defined in a variety of ways, according to Suurtamm et al., and can be conceptualised from different theoretical perspectives pointing to multiple facets. From an international perspective, they point to the development of mathematics curricula relative to a particular context that includes multiple influences which may be political, linguistic, cultural and ideological. Curriculum definitional debates demonstrate that the language of those involved in them is neither philosophically nor politically neutral and this presents particular challenges of definition (Ornstein & Hunkins, 2018). Moreover, they assert that, “the more precise one’s definition of curriculum […] the greater the tendency to omit or miss relevant (or hard to observe) sociopsychological factors” (p. 27), such as interactions, feelings, attitudes and behaviour.

For ICMI Study 24 on mathematics curriculum reforms, ‘curriculum’ is defined broadly as multi-dimensional and as inclusive of several core components. While most authors have drawn on the definitions discussed here, some in this volume have also used other specifications that may be relevant to the exposition in their respective chapter or theme. This acknowledges that curriculum reforms continually need to take adequate account of the wider context in which they occur and with reference to aspects being explored and explained.

Nevertheless, however curriculum is defined, it is necessary to acknowledge that there will be blank and blind spots. Definitions, theories, methods, perceptions that "we know enough to question but not to answer" are our blank spots; while "what we do not know well enough to even ask about or care about" are our blind spots (Wagner, 1993; Gough, 2002). Different definitions of curriculum offer different vantage points, which illuminate some aspects and hide others. We may recognise our blank spots in curriculum reforms through some definitions of curriculum, but our blind spots are more difficult – that is, what are the questions that we do not even know to ask? Recognising this point reminds us that we will always have an incomplete picture and incomplete understanding that we are continually attempting to unravel and uncover.

The notion of *reform* within mathematics curriculum reforms is also a contested concept. While curriculum reforms in some contexts is used to refer to almost any changes in the curriculum, especially at more macro-levels, for others this notion does not capture the depth of a reform when a curriculum is undergoing a major overhaul. Hence, different concepts have emerged to refer to the extent of a curriculum reform. Relatively minor changes can sometimes be referred to curriculum *revisions* while significant changes may be referred to as curriculum *transformations*. In this study, the notion of curriculum reforms has been used from the outset to refer to all degrees of changes and kinds of changes made to curricula.

In each of the sections in this volume, the authors have endeavoured to elaborate definitions and meanings of the key concepts being engaged in the theme. What is meant by a historical perspective, coherence and relevance in a mathematics curriculum reforms, implementation in curriculum reforms, agents or processes in curriculum development, and globalisation and internationalisation in mathematics curriculum reform discourses and practices are explored and explained.

Structure of the ICMI Study 24 Volume

The entire volume is organised around the five major themes that were first identified in the Discussion Document for ICMI Study 24 and later expanded in the Study Conference and Proceedings (Shimizu & Vithal, 2018).

Part I provides the **introduction** chapter by the editors followed by a chapter from Jeremy Kilpatrick, one of the authors of the first seminal volume on *Curriculum development in mathematics* (Howson et al., 1981). It captures a historical perspective but also includes his reflections on the current status and future trends in school mathematics reforms based on his long and considerable scholarship in this broad area. It is included in this introduction as it gives a context and perspectives for the practice and study of school mathematics curriculum reforms. Kilpatrick tells of the necessity both of applied and of pure mathematics, and that school mathematics curriculum is necessarily a mixture of ideas from these two poles. For him, after the New Math era, the movement appears to have been largely in the direction of the applied mathematics pole, partly because students are more attracted by using mathematics than they are by learning it, but also because computers and other developments are making applications more accessible.

The next five Parts correspond with and are based on the five themes that the ICMI Study was conceptualised around in the Discussion Document and Study Conference Proceedings. Each Part comprises a short introduction to the theme and follows with chapters that address the research questions in the theme and then concludes with key messages and what was learned from the theme.

Part II provides chapters on the ***Historical Perspectives*** in school mathematics curriculum reforms. The chapters in this theme A has highlighted key issues relating to past curriculum reform movements, thereby ensuring that the lessons and challenges that emerged from these past reforms can serve to inform future movements. After the introduction Chap. 3, Chap. 4 presents four cases of national reforms in the period since the 1960s. Chapter 5 extends the empirical landscape framed by the four cases of Chap. 4. It aims to address the research questions about the aspects of mathematics teaching, and learning processes certain international reforms attend to and to identify the key stakeholders in curriculum reforms, factors that underpinned curriculum reforms and barriers that inhibited reform efforts. Chapter 6 analyses the relationship between curriculum reforms and cultural values of countries or regions. Chapter 7 focuses on the question of mathematical content and how it is treated and affected by past curriculum reforms. The theme is concluded in Chap. 8 summarizing the drivers and barriers of school mathematics curriculum reforms.

Chapters in Part III discuss theme B on the ***Coherence and Relevance*** of school mathematics curriculum reforms. The theme is introduced in Chap. 9 and then follows with Chap. 10 that examines the notion of coherence in depth, within and between components of curricula, and between the curriculum and curriculum system in which it is enacted. Chapter 11 focuses on the relations between mathematics and other disciplines through the lens of school curricula, and describes the central role mathematical modelling plays in transdisciplinary approaches to school curriculum in integrating learning in the disciplines of Science, Technology, Engineering and Mathematics (STEM).

Chapter 12 identifies the increasing range of physical and digital curriculum resources that have been developed to support particular curriculum reforms. The analysis suggests that these resources can be both coherent with the intent of the reforms and relevant to the intended user. Messages about the characteristics of such resources, and the constraints that weigh on achieving the goals of coherence and relevance, are identified. Chapter 13 examines the evidence in the study and literature to find that there is a lack of conscious and careful application of theory to analyses of mathematics curriculum reforms. As a result, there are limited systematic theories and methodologies in the literature for researching and analysing reforms and to identify those likely to be successful as well as the conditions required for that success. Some guiding principles deriving from the theme are set out in the concluding Chap. 14.

Chapters in Part IV report on theme C related to ***Implementation*** of school mathematics curriculum reforms by sharing various experiences or examples about the implementation of curriculum reforms in different countries or regions. Following the introduction of the theme in Chap. 15, presents the plenary panel papers, which demonstrate how reforms are diverse, multi-factorial, non-linear, uncertain and require both top-down and bottom-up strategies. Chapter 17 examines factors that intervene within mathematics curriculum reforms and precisely seeks 'processes, models, or best/common practices' that can be relevant for the progress or success of a reform.

Chapter 18 analyses the factors in the initial preparation and professional development of teachers that act in curriculum implementation, and the interrelation between reform and teachers' actions. The participation of diverse resources is also studied – material, social, based on technologies or multimedia. The chapter additionally points out the role of assessment at the national or international level as a conditioning factor and, at the same time, as a potential instrument in curricular development. The conclusion to the theme in Chap. 19, proposes several 'laws' that emerged from the studies carried out in the previous chapters.

Chapters in Part V examine the influence of ***Internationalisation and Globalisation*** on school mathematics curriculum reforms. The introduction to this theme D in Chap. 20, starts with an exposition on the definition of key concepts. Chapter 21 analyses the emergence of understandings about numeracy and mathematical literacy, and compares their relationship with curriculum reform processes in selected countries. Chapter 22 compares the impact of TIMSS and PISA in economically and geographically diverse countries, being concerned not simply with reform of the intended curriculum, but also supporting teacher professional development offered.

Chapter 23 identifies the emergence of new areas in mathematics curriculum reforms of the inclusion of algorithmic/computational thinking, which is likely to be accelerated within the context of shift to on-line and e-learning in the current COVID-19 pandemic which has led to remote teaching and learning. Part V concludes with Chap. 24 in which future visions of the impact of internationalisation and globalisation on school mathematics curriculum reforms and some general recommendations important for future curriculum reform are provided.

Chapters in Part VI focus on the role of different ***Agents and Processes*** of school mathematics curriculum reforms. This last theme E begins with an introductory Chap. 25 and then presents papers from another plenary panel in Chap. 26 with four contributions from prominent leaders of mathematics curriculum reforms in different cultures, countries and contexts. Chapter 27 proposes a model of curriculum reform as a system of agents (who is involved), objects (what documents, materials etc. they are working with) and processes (how do agents work with objects and other agents?), characterised in terms of arenas (where reform takes place).

Chapter 28 examines communication and negotiation between stakeholders in different communities of practice in curriculum reform discussions and identifies factors which support more constructive boundary crossing, according to available evidence, and analyses outcomes of the cases. Chapter 29 addresses the professional dynamics stemming from the relationship between the stakeholders leading the development or refinement of an official curriculum and the stakeholders responsible for translating the official curriculum into the classroom. These dynamics contribute to the degree of agency and intellectual freedom afforded to, and felt by, the teachers responsible for translating curriculum documents into action in classrooms, which in turn has an impact on how effectively an official curriculum can be implemented at a local level. The concluding Chap. 30 draws on the earlier chapters to elaborate the implications for active curriculum reform work, challenges for conducting curriculum reform research and identifies future research directions.

Part VII invokes **contrasting curriculum perspectives** from two major influences that have had international impact in school mathematics reforms not only in their own country or region, but in many other countries. Two chapters are presented, firstly by Miho Taguma et al. (Chap. 32), who is leading the OECD Learning Compass 2030 framework, (which has implications for mathematics curricula), and secondly byWilliam McCallum (Chap. 33), who was involved in the Common Core State Standards in Mathematics in the USA. Both were keynote speakers at the study conference. From a very different context, Berinderjeet Kaur (in Chap. 34) has contributed a reaction to these two chapters and added her reflections based on her experience and involvement in the Singapore school mathematics curriculum reforms.

Two **commentaries on the volume** as a whole constitute Part VIII. Chapters are included from two independent leading scholars in mathematics education research with a keen interest in school mathematics curriculum reforms, and who did not participate in the ICMI Study 24 conference. Anjum Halai (Chap. 36) and Paola Valero (Chap. 37) bring two very different perspectives and reflections on the volume given the contexts of their own research and experience, which spans different continents of Asia and Africa and South America and Europe, respectively.

The volume finally concludes with a **closing chapter** by the editors (Chap. 38) that distils key messages and lessons from the themes and their respective chapters, as well as from the keynotes and plenary panels, that may be of use and value to school mathematics curriculum reform researchers, practitioners and policy makers.

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Chapter 2
School Mathematics: A Bipolar Subject

Jeremy Kilpatrick

For centuries, mathematics has been taught in schools around the world. Although school mathematics has changed in content and emphasis over the years and across national borders, the school mathematics curriculum has until recently been rather constant and stable. From a distance, one sees school mathematics in the primary grades as essentially concerned with numbers, simple figures, and concrete operations with numbers, whereas in the secondary grades, it deals with more abstract material to prepare students for the liberal arts. The primary grades have tended to focus on problems involving practical arithmetic, measurement, and geometric figures. In contrast, the focus of school mathematics in the secondary grades has historically been more on theoretical problems from algebra, geometry, and analysis. In other words, the early focus has tended to be on applied mathematics, and the later focus on pure mathematics.

During the century from 1850 to 1950, secondary school enrolments expanded, with more and more students all around the world studying secondary mathematics. Before that time, the secondary curriculum had been mostly for those few students who were going on to universities. Therefore, it was rather pure and rather removed

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Editors' Note: This chapter by Jeremy Kilpatrick is based on an interview that was conducted with him by the editors as a special keynote session at the ICMI Study 24 conference. The full transcript is available in the ICMI Study 24 Conference Proceedings (Shimizu & Vithal, 2018). As one of the authors of the seminal work, *Curriculum development in mathematics* (Howson et al., 1981), which is the inspiration for this ICMI Study volume, he was invited to reflect on the theme: Learning from the past, the driving forces and barriers shaping mathematics curriculum reforms; and to share his views on future school mathematics curricula (before the COVID-19 pandemic). We therefore include his chapter as part of the introduction to the volume and because it is much broader touching on several of the other themes.

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from real problems. The effect of the enrolment expansion was to increase the mathematical preparation of those students but also, because university mathematics was becoming more formal and abstract, to increase the differences between secondary and tertiary mathematics courses. In response, a number of projects launched in the mid-twentieth century attempted to change the school mathematics curriculum.

> These efforts arose from various sources and took many forms, but they tended to have in common a desire to bring school mathematics closer to the academic mathematics of the twentieth century—to eliminate inane jargon and make it better preparation for the mathematics being taught in the university. (Kilpatrick, 2012, p. 563; see also Howson et al., 1981, and Kilpatrick, 1997/2009)

The New Math Era

The result of those efforts has been called “the new math” (Kilpatrick, 2012), a term used “to describe the multitude of mathematics education concerns and developments of the period 1955–1975” (National Advisory Committee on Mathematical Education, 1975, p. 137). The term does not characterise a single approach to, or style of, curriculum development. Instead, as the projects outlined by Howson et al. (1981) clearly demonstrate, the new math projects in the United States and Great Britain alone spanned a variety of approaches.

Nonetheless, the curriculum development projects of the new math era did have some common features. Many of the projects, for example, turned to university mathematicians for ideas about revising the school curriculum, and many of those mathematicians considered the twentieth-century concepts of sets, functions, groups, rings, and fields to be appropriate fodder for young learners. They thought more children would be attracted to the study of mathematics if it were organised around those abstract ideas. They soon discovered, however, that changing school mathematics is far from simple. It requires attention to local conditions and teachers’ preparation and attitudes (Kilpatrick, 1997/2009).

During the new math era, some mathematicians who played a strong and influential role in shaping the curriculum got a bit burnt. They thought they knew what primary school children should learn, and they wrote books about that content. Teachers and students alike, however, had trouble with some of the approaches taken in those books. Their response was not what the mathematicians expected. It is one thing for mathematicians to address the secondary curriculum, because the connections to what is happening in university classes are clear. What mathematicians have to say about the elementary or primary school curriculum, however, is a different story. Some mathematicians have stayed with that issue, but in general, not many feel comfortable working on the school curriculum. It is not a rewarding thing for them to spend time on school mathematics. They have their own area to work in, and they get their rewards from proving theorems and doing other things like that. In mathematics, there are not many rewards for mathematicians to spend time on the school curriculum.

In the past, individual mathematicians like Felix Klein and some others looked at the secondary school curriculum. They said that it needs to be made more like the university curriculum, and that was a part of what their contribution was. Felix Klein probably did the best job by introducing functions as a concept and making calculus the endpoint of secondary education. He really had a strong impact on the school curriculum. Throughout recent history, we have had mathematicians who helped us understand how the secondary curriculum could be made more like what the university curriculum was becoming. The question of what kind of help mathematicians could provide the primary curriculum, however, has proved to be much more difficult, and we have had fewer mathematicians working on that. On the topic of modelling, statistics, and that sort of thing, few mathematicians want to work. Many do not consider statistics to be mathematics. They do not really see the point of it. It is something, however, that students need to know; and most countries want to make it part of school mathematics. Therefore, we have to get more statisticians to help us understand what the mathematics of statistics should be in the curriculum.

Critique

Not all the mathematicians who worked on the new math curricula approved of the direction the reforms were taking. In an article signed by 64 mathematicians that was published in both the *Mathematics Teacher* and the *American Mathematical Monthly* in 1962, a strong critique was made of that direction. The article gave some guidelines for judging school mathematics curricula:

> It warned against focusing the curriculum too exclusively on future mathematicians, urged that abstract concepts be built on concrete examples, and recommended greater attention to connecting mathematics with science. (Roberts, 2004, p. 1063)

Despite differences of opinion about the new math, it had the effect of awakening mathematicians and mathematics educators to the curriculum as a phenomenon to be studied, understood, and changed – at least potentially.

> Before the new math era, no one thought of school mathematics as something to be reformed or updated; it simply was what it was. The new math reformers knew almost nothing about the school mathematics curriculum in other countries or, in some cases, in their own country. By the time the new math era ended, in contrast, everyone concerned with school mathematics had a much better sense of what was going on around the world. (Kilpatrick, 2012, p. 569)

Bipolarity

The school mathematics curriculum has two foci – two poles. The elementary curriculum did not originally have many pure aspects to it. It was mostly applied mathematics: arithmetic with some simple geometry. Over time that changed, and during the new math era, some abstractions and other ideas from pure mathematics were introduced into the earlier grades. The pure mathematics pole moved into those grades. The other pole, the other part of the bipolarity, is that pure mathematics had always dominated the secondary curriculum, and that curriculum was not intended for every child. As enrolments grew in secondary mathematics over the last century, however, the applied mathematics pole moved into the secondary level.

Subsequent Changes

By the 1980s, many were convinced that the new math had been a failed effort (Kilpatrick, 1997/2009, 2012, 2017). That verdict, however, depends upon one's criteria for success. Certainly, the curriculum looked different from before even if it did not change in all the ways reformers wanted.

Since the 1980s in response to some of the changes fostered by the new math, there have been a number of projects to build school mathematics around the more applied parts of the subject matter and to take social and cultural aspects of the subject into account. Reformers have wanted to include such topics as statistics and other ways of looking at representations of practical problems. They have also wanted to connect the mathematics being taught more closely to the social-cultural context in which the students are learning.

An Applications Turn

One of the big arguments against the new math was that the pure mathematics being taught did not have applications – or at least that students were not being introduced to applications. In response, a number of projects in various countries sought to build the curriculum around the more applied parts of the subject matter, including statistics and other ways of looking at representations of mathematical problems. A special effort was made to look at how children might approach practical problems. Today we have many applications for the earlier grades that we did not have during the new math era.

When I was teaching junior high school mathematics in Berkeley, California, in the late 1950s, I took a summer school course at Stanford whose instructor was Morris Kline. Kline, the author of *Why Johnny Can't Add* (Kline, 1973) and one of the authors of the critique cited above (for details, see Roberts, 2004), was probably the strongest U.S. critic of the new math. He was a professor of applied mathematics at New York University and wanted to build, if he could, a curriculum of applications of mathematics. He considered applications a better way than using pure mathematics to get into the subject matter.

Kline attempted to have us students collect applications that he might use, but we were not very successful. For the elementary algebra course that some of us taught, for example, we were able to develop a few problems involving projectile motion – to illustrate uses of parabolas – but because our students had no access to calculators or computers at that time, the calculations needed would have been too complicated for them to perform. Few of the applications we came up with used real world data that did not involve such calculations. Even when we had some good applications, we could not handle them very well in our classrooms because the students would get bogged down in the work. Today we can let the computer do the calculations. Then the students can go much farther, and I think that we are moving in that direction. I would guess that school mathematics is going to become a much more applied subject over the next few years.

School mathematics is likely to become more applied as teachers learn more about how to handle the applications of mathematics. I expect that programs that include modeling, statistics, and other applications of mathematics will grow once teachers learn what they want to do with applications. The focus of the push will be in that direction because technology is allowing us to deal in the classroom with applications that were never possible before.

There are many problems associated with bringing applications into the curriculum. Parents may say, "Why is this in here? I didn't study this when I was in school. Why are you having students do this? This is not mathematics." And some mathematicians may agree: "This is not mathematics. These are applications. They are not part of mathematics." For those mathematicians, it ruins the subject to bring in applications. Even if it makes students happy, it is not staying true to what mathematics really is. If we stick with pure mathematics with no applications, however, students are likely to say, "When will I ever use this?" And it is not surprising, therefore, that those students do not pursue more mathematics. I think that for self-preservation, mathematicians and mathematics educators should work on the question of: how we orchestrate the curriculum so that applications play a major role.

One of the things that will slow any change is that teachers do not necessarily know about applications, and they are not always sure how to handle them in class, especially if they have not seen that done. When an application works, however, it can work very well. After having seen a mathematical topic put into practice, the students can say, "Oh! Now I understand where I would use this mathematics."

There is even a problem with the word *applications* because it implies that first you do the mathematics and then you apply it. In class, however, it can actually go the other way. You can start with a good application, with a situation where mathematics can be applied, and then students can learn how mathematics can be brought into the situation. "I'm learning quadratic functions, and now I see what good that might do me." If you have a good application, then you can convince students that the mathematics does work, and they need to know that.

The whole idea of trying to organise the applications into a coherent curriculum is a special problem of its own. In a sense, pure mathematics is easy to organise into a curriculum because everything is sort of logical, connected, and so on. In a more applied curriculum, there are some big questions: Where do we start with applications? In what order do we take them? Which ones do we use where? Nonetheless, in a project we did with an upper secondary precalculus course that we studied in several places in the United States (Kilpatrick et al., 1996), the teachers told us that their students loved the examples of applications of mathematics and that it really helped them understand why they were doing this mathematics. The students understood much more about functions, for example, than they would have from just a pure mathematics approach.

I think there are pedagogical values in working with applications even though it is difficult to put together a sensible curriculum made up largely of applications. How do we weave together the pure mathematics and the applied mathematics? Whatever we construct needs to be some kind of coalescence of pure and applied. We can downgrade the applied part, and we have done that in the past. There are good pedagogical reasons, however, for raising the level of the applications and the number of applications. It is just that we have to be careful about how we choose and arrange them.

Today, teachers can look online for some problems, but they may not be comfortable with that. Any change will likely be a slow process. Teachers, however, are the ones who know the students in front of them. They know what these students can do or cannot do, and we need to trust the teachers to bring in the applications that these students will be able to learn from.

A Social and Cultural Turn

A second movement that one finds in curriculum projects today has to do with what has been called the 'social turn' in mathematics education (Lerman, 2000), or more precisely, the 'sociocultural turn' (Lerman, 2004). Rather than just looking at how individual children are learning, curriculum developers are looking at how classes of students learn and how we can incorporate the socially and culturally relevant aspects of mathematics learning into our work. The sociocultural turn has been a focus of many recent projects because people recognise that the situation in which you learn mathematics affects the mathematics that you learn. That idea was not well understood or even thought about much before the 1980s. (For a recent critique of the sociocultural movement, see Jorgensen, 2014.)

One of the most difficult lessons learned during the new math era (see the last chapter of Howson et al., 1981) was to recognise that the teacher was the critical person in curriculum reform. That is, if the teacher did not understand why the change was being made or what the change was, it did not matter what materials you gave to the teacher. The teacher had to be part of the process of understanding what is going on and fitting it into the culture of the classroom.

> Many new math reformers began their efforts with the view that the curriculum would be brought up to date mathematically if they could simply get their new syllabuses and textbooks into the hands of students and teachers. By the end of the era, they had come to see that much more was required. At the crux of any curriculum change is the teacher. The teacher needs to understand the proposed change, agree with it, and be able to enact it with

> his or her pupils—all situated in a specific educational and cultural context. (Kilpatrick, 2012, p. 569)

Another lesson was that every country has a unique classroom culture when it comes to the teaching of mathematics. Some countries have connections to each other's classroom cultures. Around the world, however, there are many differences between cultures. In some cases, for example, the teacher is expected to pose all of the problems, and in other cases, the book is supposed to have the problems; all the teacher does is help the students work. Countries differ quite a bit on that question.

Another part of the sociocultural turn relates to whether teachers work together on mathematics instruction. In some countries, each teacher just closes the door and does what she or he wants to do. In other countries, teachers, at least in principle, work together and help each other change. In our study of a precalculus course in the United States (Kilpatrick et al., 1996), we found that only when groups of teachers worked together did one see good curriculum change. When the teachers tried to make the change individually and alone, there were so many barriers and problems that it was not successful. It was teachers working together that made the difference.

Another lesson that I hope has been learned is that people who want to research the curriculum cannot do it without engaging with the people in the classroom. The work of those educators who are going to be doing the reforms, creating the materials, and creating the teacher development plans cannot be separated from the research and has to be tied into it. I think some researchers have made the mistake of going to study the curriculum as if it was out there. But they need to be a part of the change in order to study it.

One theme of the book *Mathematics Curriculum in School Education* (Li & Lappan, 2014) is that mathematics educators have not done a good job of studying how the curriculum change process works or could work in schools around the world. We just do not know, and that is a sort of a first step. Despite an enormous amount of curriculum development work, we do not have an enormous amount of curriculum development research. That is a challenge for the future.

The Context of the United States of America

During the new math era, when we wrote Howson et al. (1981), U.S. politicians did not have any connection to the school mathematics curriculum. There were almost no cases of politicians anywhere saying, "Vote for me, and we will have this curriculum in the schools." An exception was West Germany, where there were politicians who took different sides on the school mathematics curriculum. That was, however, the only case I ever heard of. In the United States today, however, there are politicians who say, "If you elect me, we will go back to that curriculum; we will not follow this curriculum." And in particular, the proposed Common Core State Standards in Mathematics (Li & Lappan, 2014, pp. 38–40; see Chap. 33)) have been debated.

We have a movement to privatise school education, and that movement is caught up with some politicians on one side and other politicians on another side. Somehow the mathematics curriculum gets connected with that conflict. It started largely with the question: Should we teach mathematics to everyone, should we teach it just to the people who deserve it, or should we have different curricula for different pupils? Politicians have gotten into that conversation to say, “Well, these people are trying to teach the same mathematics to everybody; they are ruining mathematics.” There are some mathematicians who say that, too. Somehow U.S. politicians, mathematicians, and mathematics educators are involved in discussions today that they were never involved in during the new math era. It was not a political issue at the time.

The United States is almost unique in the fact that we do not have a ministry of education that establishes the school curriculum. One of the articles of faith for the U.S. public is that we do not want Washington telling us what our curriculum should be and what we should be teaching. All of our efforts in recent years have been to bring some structure into the school mathematics curriculum across the country, and having to face up to a public that says, “We don’t want this,” and “Who are you to tell us what to do?”

The fact that we have a National Council of Teachers of Mathematics (NCTM) setting up a curriculum standards program is very unusual. I do not know of any other country that has something like that happening. I have heard people saying, “Who chose the NTCM to do this work?” Well, they decided to do it on their own, and the government did not set it up.

The government has, however, in some cases embraced it. That is one of our problems. We have had political problems that can be attributed mostly to the fact that we do not have a national curriculum. Some people think we should have one, and other people say no. We have never had a national curriculum except informally. Therefore, there are a lot of divisions about that. If you start offering something as a core curriculum that everybody should work on, you will get some politicians saying, “Go ahead,” and parents and others saying, “Don’t do that.” We have a somewhat special situation.

Elsewhere around the world, there seems to be more acceptance of a national curriculum. There is a wonderful quotation in the book by Howson et al. (1981, p. 58) that I am fond of citing. Essentially, it goes back to the time when the United Kingdom did not have a national curriculum. At that time, a French school inspector was quoted as saying that in the UK, every teacher was supposed to be going his or her own way, but nobody was, whereas in France, everyone was supposed to be doing the same thing, but nobody was. That about sums up the difference between what politicians say and what teachers do.

The United States continues to grapple with the question of whether we should teach the same mathematics to everyone. Can everyone learn the same mathematics? One of the ideas during the new math was that we ought to have a standard curriculum. It might take some students longer than others to learn that mathematical material, but it ought to be the same for everybody. That was the general idea proposed in the new math era. That idea is, however, not widely accepted in the United States today. We have many cases in which students are given a test at the end of a certain grade. If they do not do well on the test, they are put into one set of classes; and if they do well, they are put into another set of classes. So, we have layers of school mathematics. If you do well on a test, you get a certain mathematics, and if you do not do well, you get something else.

This sorting happens in different ways in different parts of the country. There are schools that have different primary courses in mathematics for different students, but the differentiation is usually made in the middle grades. Typically, a line is drawn around Grade 8, and if you pass, you go into one program, and if you do not pass, you go into another. But in some cases, it happens earlier than that, in the primary grades. It almost never happens that students are kept together as a group all the way through to the twelfth grade. So, we have not figured out what we as a country want to do. Some reformers say, "We should keep kids together in the same class to learn mathematics regardless of what mathematics we are teaching." But there are others who say, "No, we have to separate them. Some of them are going to do well, and others are not going to do well. We should not put those students into the same class." It is a political issue in many places.

Each country has to deal with the question of when to start differentiating the curriculum. How do we give students choices, how do we give anybody a choice, and who chooses? The teacher? The parents? The students? What are the paths that students can take? When do they start taking mathematics, and do they have to take it every year? Those are all questions that each school system, or each nation, has to decide. Are we going to teach the same mathematics all the way through school? Most places say no, we are not.

International Comparisons in Mathematics Performance

The rise of international comparative studies of mathematics performance – such as the Trends in International Mathematics and Science Studies (TIMSS) and the Programme for International Student Assessment (PISA) studies (Li & Lappan, 2014, see theme D) – has had both positive and negative impacts on the school mathematics curriculum. One positive impact has been that it has made some countries more aware of what is happening in other countries, and what their curriculum looks like. For all of us, it has allowed us to see across the world what students can do and what they cannot do. I think it has, in general, been positive for people to see what students in their country can do, and then to compare that with the performance of students in other countries.

One negative impact stems from the problem that all these studies make use of artificial curricular frameworks that have been drawn up for a different purpose. I have criticised efforts by American educators to try to use a mixture of data to make points about U.S. schools (Kilpatrick, 2011), because TIMSS is one thing, and PISA is another. You cannot mix the two – that is one issue. Another is that these frameworks are pretty arbitrary. PISA is trying primarily to get a picture of how fifteen-year-olds can deal with applications of mathematics, whereas TIMSS is trying to get a picture of how well kids at different levels, say eighth grade, come out of the mathematics program. What can they do, and what can they not do? All of that is somewhat arbitrary.

I remember a recent conference in Malaysia where I heard a mathematics educator from Singapore say that they were going to look at how the Singapore students did on the different kinds of questions in PISA, and then they were going to change their curriculum to deal with the places where the students were not doing so well. That struck me as completely backwards. You do not want to use such a framework to say this is how our curriculum should be. You should decide what your curriculum is, and if it does not match what PISA has, okay, it does not match it. I do not accept the idea that the people from Singapore should be taking the PISA framework as the gold standard.

I have worked with measurement people in putting some of these framework documents together. Those documents reflect judgments as to what content questions should be included on the assessment instruments and what should not.

I remember an international content experts' meeting for PISA in which at one point we discussed questions about conversion from Fahrenheit to Celsius units. For U.S. educators, such questions would be reasonable to include. We still use both the imperial system and the metric system, and U.S. students are expected to be able to convert measurements from one system to the other. Further, such conversion is a worthwhile mathematical exercise. For most of the rest of the world, however, items dealing with conversion do not make much sense. They are not part of the school curriculum. The experts threw those questions out of the PISA pool because they applied to only one country.

These frameworks and item pools are arbitrary constructions by experts. Who says, however, that they should be what the people in a given country are using as their gold standard – as their framework? That is a problem, I think, with these international comparative studies. They are being misused when the framework is taken as the thing that we want students to be able to do. A framework can be helpful. It can give some general idea of how your students are doing on this topic or that topic. To use it, however, as an overall evaluation of what your country is doing is, I think, a big mistake. Many of the concepts you are treating in school mathematics may not be on the test, but they might be important concepts that your students are learning. So why not keep those concepts there?

I understand that in order to make comparison you have to have a common measuring stick, but you do not have to take that measuring stick as the endpoint for your curriculum. That is where I think the problem is. If you using the measuring stick to say this is what we want, you have not solved the curriculum problem for your country. The frameworks are a kind of consensus documents. You and your country may be teaching something important and very good, and getting good outcomes. But it may not be measured on TIMSS or PISA. Does that mean you should throw it away? I do not think so.

I understand that TIMSS and PISA can a strong influence on educational policy, but that has its downside as well. In the United States, our students do not do well on some problems, but we very seldom look closely at the PISA results. TIMSS seems to dominate our attention when compared with PISA. That is kind of crazy, too, because I think both projects have something to tell us. It is just that the PISA message does not come through very clearly. People get into comparisons between states, for example, or between school systems on the basis of these tests. That is not a good idea.

We have not yet learned to put a distance between ourselves and these results. I think that as the results pile up, and as people get used to these situations, it may get better, because then they may stop being attracted by the disparities. The results tend to stay relatively constant, so there is not much to be gained from the way the results are being reported. I think there is a kind of lack of attention to what is happening, which is probably a good thing.

A potential contribution of PISA is to increase attention to mathematical literacy as an educational outcome related to effective citizenship. In some countries – including Japan, Korea, and Denmark – the curriculum is based on competencies: not contents but processes. By considering mathematical literacy rather than specific content knowledge, mathematics educators in these countries are looking for better outcomes from school mathematics, which is a good thing. To the extent that PISA gives us some ideas about students' mathematical literacy, it can be quite helpful. Unfortunately, however, what usually happens when the results are reported, at least in the United States, is that all we get are numbers in the newspapers: Japan was here, and the U.S. was there. We do not get any discussion of the mathematical literacy of the U.S. students or the Japanese students.

Final Comments

The book *Adding It Up* (Kilpatrick et al., 2001) talks about mathematical proficiency and offers a framework for studying mathematical proficiency. It was an attempt to say that if you are aiming for mathematical proficiency, you need to think about more than just content and process, you need to think about other dimensions that are being dealt with in school mathematics. The metaphor of a braid for proficiency – strands that are being developed along the way – is a metaphor for how the curriculum might work that is different from the metaphors discussed by Howson et al. (1981).

So, I think the idea of curriculum as a process, and one that needs to be shaped by the situation in the school, the situation in the country, the situation in the classroom – all of that has changed from what it was in the 1980s. Today, I would say we are moving much more toward recognising that the goals for school mathematics may be different across different school systems, countries, and situations. Each country has to figure out what its goals are, and in what directions it wants to go.

The school mathematics curriculum is diverse and multi-dimensional, which makes it impossible to capture well in a single study, framework, or trend (Li & Lappan, 2014, pp. 6–9). The levels of school mathematics range from the *intended* curriculum (goals prescribed in policy documents) to the *implemented* curriculum (what is taught in classrooms) to the *achieved* curriculum (as seen in students' mathematics performance). A single research study can address only some aspects of those curriculum levels.

The bipolar nature of school mathematics, in contrast, shines through regardless of the curricular context or level. We have learned since the new math era that school mathematics is complicated, contextualised, not easily changed, and not easily studied. The bipolarity of school mathematics offers a possible entryway into studying it in context and retaining much of its complexity.

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Part II
Theme A – Learning from the Past: Driving Forces and Barriers Shaping Mathematics Curriculum Reforms

Chapter 3
Introduction

Marianna Bosch and Niamh O'Meara

Most of the studies presented in this book describe past experiences of curriculum reforms. What is then the specificity of theme A chapters? What kind of particular approach to past reforms do they propose? Two aspects can be distinguished. In what corresponds to the object of study considered, either they refer to reforms with a particular role or transcendence in the evolution of mathematics education across the world, or they approach more local reforms over a sustained period. In what concerns the types of questions addressed, these sometimes 'large' empirical objects are approached to identify some driving forces that explain the conditions and outcomes of the reforms' implementation, and also some barriers that seem to have constrained their development. Theme A uses the past as a *learning* strategy and as an experience from which one can draw some lessons.

Our approach is therefore not purely historical, even if a specific account of the different reforms is presented in each case: we are always relying on empirical material or previous studies – previous *narratives* – about them. The issue of the narrative, the perspective adopted about the reform is crucial for many reasons. Curriculum reforms are changes that are undertaken under certain conditions, to modify a given aspect of a country's or region's educational system. They are breaks or interruptions in the established curricula, motivated by a particular diagnostic of these curricula to improve or guide them in a new and revised direction. Once implemented, and before being modified by another reform, their effects are also assessed, to be used as rationales for subsequent change. Therefore, we do not only deal with curricula that have been changed, but also with the perspective of the

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different agents that took place in the change – politicians, mathematicians, educators, teachers, students, journalists, parents, etc. Their viewpoints, diagnosis and evaluations are also part of the unit of analysis.

The questions that motivated theme A were formulated in the following terms:

> What aspects of school mathematics curriculum reform carried out in the past decades have been taken to be the most important, for example, in content, pedagogy, and in the underpinning theoretical approaches?
>
> What potentially crucial aspects of mathematical curricula have not been considered, and even less, touched upon?
>
> Which goals and values in school mathematics curriculum reforms, carried out in the past decades, have been the most important (for example, in the selection and organisation of mathematics contents, or process aspects of mathematical activities)?
>
> How have the questions of content become linked to the notions of mathematical competencies, capabilities, and literacy; and how have these evolved to become a driving force in the curriculum development and reform initiatives?
>
> What has been the role and function of curriculum resources, materials, and technology, including digital curricula and textbooks in curriculum reforms and their implementation as driving forces or barriers?

These questions are strongly intertwined and did not suggest a set of problems to be approached independently. It was not surprising that the nine contributions to theme A addressed some of them and each one to a different extent. In preparing this section, we decided to organise them in four chapters. The first two chapters give an account of past curriculum reforms and provide an overview of the empirical field of study, with many shortcomings but enough diversity. In the other two chapters, specific perspectives are adopted: first, the cultural dimensions of curriculum reforms and the values they convey, related to the societies' specificities and the particular vision of mathematics proposed by each reform; and second, the question of the "content" and what is related to it is addressed. We present these four chapters in broad outline while the description of the main learnings from theme A are presented in the concluding chapter.

International Co-operation and Influential Reforms

Chapter 4 presents four cases of national reforms. The first three were initiated in the 1960s in European countries, namely The Netherlands, Hungary and France, after the Royaumont conference in 1959 organised by the Organisation for European Economic Co-operation, which would soon become the OECD. It is interesting to see how the same initial starting point, motivated by what was seen at that time as, "a desire to bring school mathematics closer to the academic mathematics of the twentieth century" (Kilpatrick, 2012, p. 563), led to such diverse outcomes. Especially when they were all being piloted by prestigious mathematicians, like Hans Freudenthal and Tamás Varga for the case of the Netherlands and Hungary and sustained by newly created government commissions or institutions. The specific personality of the mathematicians who took part in the reform movements, alongside the mathematical culture, epistemology or tradition they conveyed, seem to have been a critical driving force for the concrete implementation of the reform and its divergent continuations.

There are also two other significant elements that the brief accounts given in the chapter do not show in detail, but can be found in the original contributions of the authors (Doorman et al., 2018; Gosztonyi et al., 2018; Van Zanten et al., 2018). On the one hand, there were some previous local innovations in mathematics education during the beginning of the twentieth century, like Ehrenfest's and Van Hiele's developments for the teaching of geometry in the Netherlands, or the "teaching of mathematics by guided discovery" in Hungary. They can be seen as specific original conditions for the basis of the reforms, both for the educational resources they had already provided and for the tradition they represented.

On the other hand, even if the Royaumont seminar served as an impulse for European governments to launch curriculum reforms in mathematics, what made the difference is the concrete work organised by leading mathematicians in each country that resulted in the production of particular mathematical tasks and content organisations. The case of Spain, as portrayed by Ausejo (2010), is a good counter-example. Pedro Puig-Adam, the leading mathematician involved in the renewal of mathematics education who participated in the first meetings of the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM) (and even organised the eleventh one in Madrid in 1957), died prematurely in 1960. The absence of leadership, together with the complex political situation of the country, ended up with a series of 'hybrid' textbooks where the most new-math oriented proposals in terms of sets and applications side-by-side coexisted with the more traditional knowledge organisations based on the classic arithmetic of quantities, ratios and proportions.

As in many other countries, the cases of France and Hungary show reforms that were somehow interrupted or 'counter-reformed', for reasons or principles that were not as explicit as those claimed in the New Math reforms. As a contrast, the work of Freudenthal in the Netherlands, the Realistic Mathematical Education (RME) movement has been described as a longstanding reform movement. To this respect, the 'period of educational engineering' in the 1970s that followed the emergence of the RME reform in the 1960s can be seen as a sustained effort made by the Institute for Development of Mathematics Education (IOWO, which later became the Freudenthal Institute) to create a wide variety of teaching materials to support curriculum development, and also teacher education and professional development. This effort continues to the present day, with some highs and lows in its original country but also with sustained dissemination internationally.

The chapter ends with a description of the Japanese case to counterbalance the European ones. What we can see through this case is continuous curriculum development, based on explicit principles, involved in and influenced by the international reform movements and organisations. By going back to the period before World War II, this account recalls that the domain of mathematics is also the fruit of a reform process that appears as a way to unify the traditional school distinction between arithmetic, algebra, geometry and analysis. It also shows how the curriculum is developed in line with international movements, such as the Meran project, the New Math, the NCTM Common Standards or the subsequent OECD-PISA framework but is never a mere adaptation of outside proposals. Each reform period corresponds to the will of emphasising different learning goals linked to specific aspects of mathematical activities, along with the social evolutions of the country.

Case Studies of Past 'Local' Curriculum Reforms in Mathematics

Chapter 5 extends the empirical landscape that is initially framed by the four cases of this chapter. It aims to address the research questions about the aspects of mathematics teaching and learning processes certain international reforms attend to and to identify the key stakeholders in curriculum reforms; factors that underpinned curriculum reforms, and barriers that inhibited reform efforts. It also aims to identify the universal lessons that can be taken from international reform efforts so that future curriculum reform movements can learn and build on past efforts.

To gain an insight into curriculum reform efforts in different countries internationally, a survey was designed by some theme A participants. The survey sought to gather information in relation to:

- how and why the reform movement came about;
- the ideologies underpinning the reform movement;
- the aim of the reform movement;
- the agents or stakeholders involved in the reform movement;
- the impact of the reform on mathematical content, mathematical teaching and mathematical assessment;
- the lessons learnt from the reform movement.

In total, six research colleagues from different countries (Brazil, Japan, Ireland, Italy, Serbia and South Africa) responded to the survey, ensuring a geographical spread across four continents. Three survey responses were analysed in detail for the purpose of this chapter. The countries selected were Ireland, Serbia and South Africa. These were selected for consideration due to the commonalities in some aspects of the curriculum reform efforts as well as quite unique differences. Many of the responses from these countries also reflected in part the reactions from the other respondents, and so the authors believed a comprehensive overview could be achieved with this limited sample.

The analysis showed that while the reasons behind the reform, the nature of the reform and the stakeholders involved in the reform differed slightly, many commonalities could also be found. Many of these commonalities were described as factors affecting curriculum reform in the framework proposed by Memon (1997). It led to the authors outlining a series of lessons that could be taken from these reform movements to inform future reform efforts, hence highlighting how we can learn from the past and use past reform movements to overcome and avoid challenges or barriers in the future.

The Role of Values and Culture in Past Mathematics Curriculum Reforms

Chapter 6 analyses the relationship between curriculum reforms and cultural values of countries or regions. The cases considered correspond to Italy and its most recent reform movement (2001–2018), with the implementation of 'mathematical laboratories'; Serbia and the changes brought in in relation to the nature of mathematics as a teaching subject (1970–1985); and Iran since the establishment of a formal educational system in 1920.

In the first two cases – Italy and Serbia – curricula from different periods illustrate how a reform conveys not only a body of content knowledge (notions, concepts, procedures, etc.) but also a specific way of considering mathematics, or of *valuing* it. Furthermore, they show that this conception cannot only be understood as it appears in the official texts and guidelines but in the specific proposals that are made. For instance, in the most recent global curriculum reform in Italy (2001), mathematics is conceived as having two fundamental functions, an instrumental one (for understanding reality and everyday life) and a cultural one (a coherent and systematic knowledge with a robust cultural unity). These two values assigned to the subject can just be stated, and they can be part of the discourse accompanying the reform. However, as is the case here, it can also give rise to a concrete instructional proposal – the mathematics laboratory – with its specific activities, where students will live mathematics as an empirical activity linked to many other disciplines (Bartolini Bussi et al., 2018).

The case of Serbia illustrates the changes of values about what mathematics is and how it is conceived through two examples of definitions (polygons and functions) in 1970 and 1985. It also shows how general principles about mathematics – under the New Math influence or in the 'counter-reform' – cannot be understood unless one approaches the concrete activities and tools that are proposed to the students and the way these activities and tools are structured (Milinkovic, 2018).

The case of Iran enlarges the perspective. It illustrates an interesting evolution of a curriculum that has been subjected to many political and cultural influences since 1920. It also shows the (positive and negative) effects of some of the decisions made. For instance, the centrality inherited from the French political influence in the 1920s resulted in the adaptation of a single national textbook that ensured students' access to educational resources. The study presents a rich illustration of how international movements can impact on a country with a strong cultural tradition, by adopting a specific shape and creating peculiar effects (Gooya & Gholamazad, 2018).

For instance, and this is a case that has been repeated in other countries, when the New Math curriculum was imported, the traditional geometry and trigonometry were maintained. More recently, Iran's participation in TIMSS opens the way for international co-operation and dissemination of recent movements in mathematics education. The way these global trends can permeate the educational system is always subject to political decisions and cultural circumstances. The chapter illustrates how driving forces and barriers that are not directly of a mathematical nature – but rather political, economic or social – can explain many of the changes that occur within educational systems and the way these changes are concretely operated.

The Effects of Past Reforms on the Construction of the Knowledge to Be Taught

Chapter 7 focuses on the question of the mathematical content and how it is treated and affected by past curriculum reforms. It addresses this question from a concrete perspective, the anthropological theory of the didactic, and by modelling curriculum reforms in terms of *didactic transposition processes* (Chevallard, 1985; Chevallard & Bosch, 2020). A crucial element in this perspective is the notion of the *knowledge to be taught*, which can be approximated to the notions of 'intended' or 'official' curriculum. The notion of didactic transposition points to the existence of a complex process undertaken to elaborate the knowledge to be taught, usually from a raw material that is called *scholarly knowledge*. Selecting, structuring, labelling and elaborating on the concrete mathematical activities and conceptual organisations that are proposed to be carried out by the students corresponds to the *transposition work* that is undertaken – even if not always visible – any time a curriculum reform is proposed. Its analysis helps highlight the driving forces but also the difficulties met during this endeavour.

The first case study (Wijayanti & Bosch, 2018) illustrates the complexity of the transposition work by considering a "piece of content", the notion of proportionality, and by looking at the different mathematical organisations that have composed the knowledge to be taught. An interesting phenomenon appears, showing how reforms are very much indebted to the past. It is, of course, a titanic enterprise to elaborate from scratch the knowledge to be taught for a given discipline – like mathematics – and for a whole curriculum. This was partly what the New Math reform undertook in some countries, as a result of pressure from politicians and the impulse of some mathematicians: they built up new content organisations, with new topics, new definitions, new types of tasks and exercises, new procedures, etc.

Before the New Math reforms, the knowledge to be taught was the result of centuries of construction and remained rather stable, especially in the lower educational levels. The question is about what happened after the New Math reform when educational systems applied a 'back to the basics' strategy. How was the new knowledge to be taught elaborated and by whom? Where did the knowledge resources come from? What we find when digging into it is a patchwork of pieces of knowledge coming from different layers in time: a particular structure resulting from the New Math remains, but many elements have been taken over from the previous organisation, which suddenly appears as a newly discovered world. However, the final result is not always a coherent construction, as the underdeveloped treatment of quantities in school mathematics shows.

The second case study of the chapter (Vu-Nhu, 2018), focuses on an even smaller piece of knowledge, the notion of integral, and shows its destiny across different curriculum reforms in Vietnam before and after the reunification of the country. The phenomenon described can be seen as a simple anecdote. Still, it illustrates a rather general situation, where big decisions are made by the high authorities as if the details of the transposition work were to follow naturally. When looking at the concrete activities that students are required to carry out, and the mathematical means we propose them to do so, we find important barriers that can explain the difficulties met in the implementation of curriculum reforms and in its outcomes. As in many other situations, involving curriculum reforms, the devil is in the details.

Chapter 7 illustrates another transposition phenomenon related to the elaboration and reception of the knowledge to be taught by the teachers. In a study about the most recent (2008–2010) curriculum reform in Ireland using Memon's (1997) framework, O'Meara et al. (2018) identify "the instruction time" – hours devoted to the content to be taught – as one of the barriers hindering the implementation of the reform from the perspective of the teachers. According to the didactic transposition approach, in teaching and learning processes *didactic time* is created when new elements of the knowledge to be taught are introduced in the sequence of elements that define it. When, in reform, the sequence and its elements are totally transformed or newly built, teachers cannot easily identify the new milestones that mark the path of the learning process and show the advance of time. Curriculum evolutions necessarily materialise in changes in the knowledge to be taught that are sometimes taken too much for granted by the curriculum developers themselves and might end up creating difficulties in the very concrete activities teachers and students carry out in their classrooms.

Driving Forces and Barriers

Theme A chapters present key elements of reforms that took place in the distant and recent past, in countries of different cultural and educational traditions, with differing degrees of influence in the time and the space, with some still being in force and others reaching their conclusion many years ago. We approach these reforms from different perspectives, trying to understand the reasons that prompted them, the strategies used to implement them, their local and sometimes external effects, the values they conveyed, and the constraints that hindered their development. The chapters point at the commonalities and specificities of these reforms, from different perspectives and using various methodological tools. General descriptions about the historical and social situations are complemented with a detailed analysis of the mathematical content organisations and the specific conditions of implementation. The gathering of all these experiences constitutes a valuable endeavour that can be used to draw some important lessons that we expect will be useful for the planning of future mathematical reforms.

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Chapter 4
International Co-operation and Influential Reforms

Katalin Gosztonyi, Marja van den Heuvel-Panhuizen, Naomichi Makinae, Shizumi Shimizu, and Marc van Zanten

International Reform Movements in the Twentieth Century

At the beginning of the twentieth century, reforms of mathematics education took place in various European countries, concerning principally secondary and higher education. They aimed to develop a better transition between these levels and to adapt mathematics education to the increasing technological and scientific needs of the period. Mathematicians played leading roles in these reforms: for example, Felix Klein in Germany led the so-called "Merano Syllabus", Henry Poincaré and Emil Borel among others in France contributed to the 1902 curricular reform (Weigand et al., 2017).

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International exchanges promoted these reform movements and offered a basis to international co-operation for the development of mathematics education. Soon after its creation in 1908, the International Commission on Mathematics Instruction (ICMI) was engaged in facilitating this co-operation under the direction of Felix Klein. International comparative reports were created on the development of the teaching of various topics, contributing to the dissemination of reform ideas in Europe. World War 1 broke this collaboration, which was restarted only after World War II, in the 1950s.

Significant curriculum changes took place in many countries at this period. In the Western block, the so-called 'Sputnik shock' (the launch of the first Sputnik by the USSR in 1957) is often defined as a starting point: the technical competition of the cold war would give political motivation to invest in mathematics education in Western countries, and especially in the US. However, historical studies underline its multiple motivations. Mathematicians were actively discussing the necessity of adapting mathematics education to the development of modern mathematics since the beginning of the 1950s. Socio-economic changes due to industrial development in both the Eastern and Western block also created a need for specialists educated in mathematics. At the same time, the massification and democratisation of lower secondary education required to define a new function for primary education (Gispert & Schubring, 2011; Kilpatrick, 2012).

Various international organisations structured the debates on the reform of mathematics education. The CIEAEM, created by Caleb Gattegno in France in 1952, focused on the psychology of mathematics education, based on Piaget's work and Bourbaki's conception of modern mathematics. These two entries were connected by the notion of 'structure': mental structures on the one hand and mathematical structures on the other. ICMI, reorganised in 1952, focused more on the role of mathematics in post-war societies, on the increasing importance of applied mathematics and the consequences of the massification of educational systems. The OECD emphasised the economic needs of a revised mathematics education. It opened an office in Paris in 1958 to favour the development of mathematics and science teaching and organised several international meetings at the beginning of the 1960s. UNESCO contributed to the international discourse by holding conferences and publishing recommendations for mathematics curricula and teaching methods.

Several key elements can be identified in these discussions, which impacted reforms all around the world. One is the idea of aligning the school mathematics curriculum to the contemporary development of research mathematics, in its content as well as in its organisation. Emphasis was put on mathematical structures and sets as the basis of the construction of mathematics. Finally, the impact of Piaget's results on the psychological development of children; and, especially concerning the lower grades, the intention was to implement methods of active pedagogy, mathematical games and manipulatives. However, the curricular reforms implemented in different countries are various and reflect local specificities. As Stanic and Kilpatrick (1992) suggest, New Math is "a label not so much for a cohesive set of reform proposals and activities as for an era during which a variety of reforms were undertaken" (p. 413).

In a report published by ICMI (Freudenthal, 1978) in *Educational Studies in Mathematics*, entitled "Change in mathematics education in the late 1950's", reforms of countries from five continents are discussed: Australia, Bangladesh, France, Great Britain, Hungary, India, Iran, the Netherlands, Nigeria, Poland, Sierra Leone, Sri Lanka, Sudan, Thailand, the United States and the West Indies. Some of these developmental projects were based on completely original material, others were mainly based on the implementation of other countries' materials. In other cases, an international effort was made to support the development of original materials for some countries or regions, as in the case of the UNESCO Arab State Project and the Entebbe project in Africa. In most of the cases, these projects are characterised by Western dominance:

> Even when attempts were made to produce original materials specifically for the countries concerned, for example, the Entebbe Project and the UNESCO Arab States Project, the writing teams were dominated mathematically and professionally, if not numerically, by Western authors who lacked any prior understanding of the educational systems of the countries concerned and, more importantly, of the social ethos that was manifested in the schools. (Kilpatrick, 2012, p. 567)

The Cold War political background played a role in certain international co-operation, for example, in the US projects implemented in Latin-America. The USSR had its own reform in the period, known as the 'Kolmogorov-reform', parallel with the Western reforms in many senses. However, some Eastern European countries were also engaged in the collaborations structured by the CIEAEM or the UNESCO during the 1960s and 1970s (Freudenthal, 1978; Karp & Schubring, 2014). Some countries integrated lessons from the international exchange with specific local developments; in some later reforms, especially from the 1970s, conclusions from the vivid social and professional debates and the experienced failures of the first wave 'New Math' reforms were also considered.

The '*Mathématiques Modernes*' Reform in France

The complexity of the motivations and driving forces behind the New Math movement can be well observed in the case of the French reform. This reform appeared in the context of a unification and democratisation process of lower secondary school. In 1959, the age limit of obligatory schooling was prolonged to age sixteen; then, between 1959 and 1975, the lower secondary school was progressively transformed into a unified education for all until grade 9 (15–16-year-old students). In addition to this transformation of the educational system, discussions were held on the role of mathematics in society: the discourse on the technical, industrial and scientific needs met with the influence of structuralism, suggesting that mathematics can offer a universal language and a model of thinking for all (d'Enfert & Kahn, 2010). In this context, modern formal mathematics is considered not only as the best tool to form the students' mental structures but also as a tool of democratisation, assuring that each student receives the same education, independently of their social background.

The ambition of modernising the content of mathematics education is particularly emphasised, and French mathematicians played a leading role in the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM) activities and the discussions about the reform of mathematics education. Among those involved were Jean Dieudonné, the leader of the Bourbaki group; Gustave Choquet, the first president of the CIEAEM who introduced the 'new mathematics' in his lectures at the University of Paris in the 1950s; and André Lichnerowicz, the future leader of the French reform commission (Gispert, 2010).

The French mathematics teachers' association, the APMEP,[1] actively promoted the reform with the establishment of a dedicated commission and with the publication of recommendations during the 1960s. Whilst the 1950s heard from different voices in the debate and the axiomatic school became dominant in the 1960s, the APMEP emphasised the teaching of the same mathematics from the kindergarten until the university, based on the notion of set, on the modern algebraic language of mathematics, on the structures and the axiomatic method (Barbazo & Pombourcq, 2010).

From a pedagogical point of view, the emphasis was made on the promotion of active pedagogical methods (d'Enfert, 2010). These ambitions were influenced by Piaget's psychological work and the aspirations to extend the renovation of mathematical content to the primary school level. The coherence of the project was confirmed by the conviction that the development of the children's mental structures, described by Piaget, correlates with the mathematical structures described by Bourbaki.

Thus, a diversity of actors anticipated the reform, with various and sometimes conflicting ambitions. Many of these actors were represented by the Ministerial Commission, created in 1966 to prepare the reform, under the direction of André Lichnerowitz. It includes mathematicians and secondary school mathematics teachers and also physics and technology teachers, representatives of the industry and primary school teachers. Several experimentations accompanied the project since 1968. The new curriculum was introduced progressively, starting from 1969 for grade 6 and 10 (respectively the first year of the lower secondary and the high school) and from 1970 on for primary education. A general agreement accompanied the introduction of the first two years in lower and upper secondary education, but those of grades 8 and 9 provoked serious tensions and led to a crisis of the reform.

The coherence of the curriculum was assured by a hierarchic structure of the different mathematical domains, based on set theory and showing the influence of contemporary mathematicians' work, especially the structure of Bourbaki's *Éléments de mathématique*. However, while in the lower grades, these principles appeared in combination with activities like 'practical exercises' and 'observations

[1] *Association des Professeurs de Mathématiques de l'Enseignement Public.*

of physical objects', a critical break can be observed between the curricula of grades 7 and 8.

The most representative example and also the theme which caused the most controversies was the curriculum of geometry. In the lower grades, the geometry curriculum contained observations and activities related to the physical reality, but the theme had minor importance, and was not recognised as 'veritable mathematics'. The curriculum underlined that the study of 'veritable geometry' started from the eighth grade, as an example of axiomatic thinking. Classical synthetic geometry was completely eliminated and the main aim was not to study geometrical figures but to construct an algebraic tool to describe first the affine, then the Euclidian plane and space. Principal notions were projections, vectors, frames, transformations, etc.

According to the instructions, axioms and notions had to be introduced via physical observations, but once they were admitted, they had to be clearly distinguished from the physical word and every further theorem had to be deduced by formal demonstrations. The textbooks however barely gave help to this introduction, they contain principally an axiomatic–deductive treatise of geometry, where figures are only illustrations of the theorems described in a formal algebraic language (d'Enfert & Gispert, 2011).

The curriculum was soon criticised inside the Commission as well as by the general public. It was accused to serve only the interest of the elite, that is the future mathematicians, and not a wider audience, not even future engineers or students of experimental sciences. At the same time, the majority of mathematics teachers came from the earlier popular school system and were familiar with a practical approach to mathematics. For them, the mathematical content of the new curriculum, as well as the radical epistemological change, posed serious problems. Furthermore, because of the growing number of students in the lower secondary school, many teachers at this level were former primary school teachers without specific training in mathematics.

The commission anticipated the necessity of in-service teacher education. The network of IREMs (*Institut de Recherche sur l'Enseignement des Mathématiques*) was founded in 1969 with the intention to contribute to the continuous development of mathematics education, to organise in-service teacher training and to prepare resources for teachers. A variety of media, including television, was also deployed to promote the reform and prepare the teachers. However, these efforts were insufficient compared to the needs (Barbazo & Pombourcq, 2010).

The '*mathématiques modernes*' reform process came to an end in 1972, when Lichnerovicz resigned, and the Commission finished its work. It was often considered as a failure. However, it exerted a long-term impact on French mathematics education in several senses. A new curriculum was introduced in 1977, eliminating the most controversial aspects of the recent reform, but the main structure of the curriculum and many elements of it remained. More radical changes were introduced in the 1980s whereby many characteristics of the New Math period disappeared. Problem solving and applications of mathematics became progressively more to the forefront (Gispert, 2014).

Several projects and institutions, created in the 1960s in order to support the reform process, continued to work, and became the main contributors to the discussion and debate around the problems of the reform as well as the first centres for didactics research. The experimentations of the National Pedagogical Institute went on and led to the publication of an innovative resource for primary schools, the ERMEL series. The IREM network continues to work today, and these institutes offer a forum for teachers and researchers from several domains to work in thematic groups, develop new material and teacher training sessions. Many of the first generations of French researchers in mathematics education started their work in the frame of this network. The Theory of Didactical Situations, created by Guy Brousseau in the late 70s (Brousseau, 1997), which is considered one of the early 'big' theories of mathematics education research, can be understood in many aspects as a reaction to the discourse and debates around the '*mathématiques modernes'* (Dorier, 2018).

This French reform exerted significant influence abroad. As we saw earlier, several actors of the French reform played a leading role in international organisations and meetings. The documents of the French reform were disseminated in and beyond Europe; for example, the new curriculum and the related textbooks were adopted by former French colonies of Africa (Khôi, 1986). However, this influence was reduced in time and soon 'counter-reforms' were implemented, trying to come back to a more traditionalist view of school mathematics. But many elements remained, like the replacement of the old 'arithmetic' by the strand of 'numbers and operations' and the disappearance of quantities in favour of sets of numbers.

A Reform Movement from the Eastern Bloc: Varga's 'Complex Mathematics Education' Reform

In the 1950s, Hungary was part of the Eastern bloc, under the political influence of the Soviet Union. This alliance also determined the development of the educational system. In the 1960s, however, a certain liberalisation and political opening towards the Western bloc increased the possibilities for educational developments. A reform movement in mathematics education was stimulated by a series of workshops given by Zoltán Dienes[2] in 1960, and by a UNESCO conference on mathematics education organised in 1962 in Hungary (Halmos & Varga, 1978). The leader of the Hungarian reform movement, Tamás Varga, engaged in the international 'New Math' discourse following this conference. For example, he was invited to co-edit the report of the UNESCO conference with the Belgian Willy Servais (Servais & Varga, 1971), and was invited to various countries (the Soviet Union, Germany, France, Italy, the USA, Canada, etc.). He also regularly participated in international

[2] Dienes was of Hungarian origin, but grew up and lived abroad in several different countries.

conferences and published in international mathematics education journals during the 1960s and 1970s.

Varga started an experimental project in 1963 in two classes of grade 1. In the following years, the experiment was progressively expanded to other schools and the lower secondary school level, reaching more than a hundred classes in the country. The project was conducted by a group within the National Pedagogical Institute and involved close collaboration with another group, namely the Hungarian Mathematical Research Institute, on the preparation of the newly created (high school level) special mathematics classes curriculum. In the early 1970s, a ministerial commission evaluated different experimental projects concerning mathematics education. They chose Varga's project as the basis of the planned new curriculum. An optional version of the reform curriculum was introduced in 1974 before the reform became obligatory in 1978, in the framework of a general reform of Hungarian curricula.

In the Hungarian case, the frames of the educational system in which this reform arrived were established since 1946. Compulsory education was provided by the eight-grade, single-structure 'basic schools', comprising elementary (grades 1–4) and lower secondary (grades 5–8) education. During the 1950s and 1960s, the regulation of the educational system was extremely centralised, with detailed curricular instructions, while the communist ideology was imposed. From the late 1960s, the most significant change concerning the educational system was the launch of a slow liberalisation process (Báthory, 2001). The influence of the ideology was pushed into the background, pedagogical and psychological considerations were taken into account and differentiation, as well as teachers' autonomy and liberty, began to be emphasised. This in turn played a crucial role in the preparation of the 1978 reform, and Varga's project can be considered as pioneering in this sense.

The impact of the New Math movement can be observed on the Hungarian reform in many aspects. For example, the introduction of a coherent subject termed 'mathematics' instead of 'arithmetic and measurement'; new mathematical domains introduced in early ages like sets or logic; the reference to Piaget's psychology and Dienes's mathematical games; the role of manipulative tools, etc. However, Varga was also critical of some aspects of other countries' New Math reforms, especially with, what he considered, the excessive emphasis on mathematical formalism. His project is based on an epistemology of mathematics which is significantly different from the 'Bourbakian' epistemology, and rather influenced by Hungarian mathematicians' 'heuristic' view on mathematics (Gosztonyi, 2016).

This tradition existed originally in the teaching of young mathematical talents[3] and went back at least to the beginning of the twentieth century. Varga himself was in intensive personal contact with some representative mathematicians of this tradition (L. Kalmár, R. Péter, A. Rényi, J. Surányi, among others) since the 1940s; and they all supported, more or less actively, Varga's later reform movement, which extended this approach for all students. These mathematicians, together with

[3] Nowadays, its most important representative is L. Pósa. See: http://agondolkodasorome.hu/en/

well-known thinkers like George Pólya or Imre Lakatos, represented a quite coherent, 'heuristic' epistemology of mathematics, closely related to questions of mathematics education.

This epistemological approach emphasised that mathematics is a human activity, developed through a dynamic of problems and attempted solutions, based on intuition and experience. Mathematical activity was seen as dialogical, and teaching mathematics as a joint activity of the students and the teacher, where the teacher acts as an aid in students' rediscovery of mathematics. These mathematicians rejected excessive formalism, seeing formal language also as a result of a development. They described mathematics as a creative activity close to playing and to the arts.

The pedagogical and psychological background of the reform seems to be more complex. Together with Piaget's influence, several Hungarian thinkers, representing a socio-constructivist approach, impacted on Varga's conception, stressing the importance of visual intuition, among other things (Gosztonyi et al., 2018).

As with other New Math reforms, Varga sought to integrate new topics in mathematics education, and to present mathematics as a coherent science, organising the curriculum in accordance with modern mathematics. It involved basing notions on sets and relations, or the strengthened role of algebra. However, for him, it also meant introducing logic, combinatorics, probability or algorithmic thinking in primary and lower secondary school. He was internationally recognised for his work on the teaching of logic, combinatorics and probability – the specific domains studied by the Hungarian mathematicians supporting his movement.

The internal coherence of the curriculum was ensured by the parallel, spiral presentation of 5 big domains, all being present throughout the entire curriculum, with frequent and various internal connections amongst them. These were: (1) sets and logic; (2) arithmetic and algebra; (3) relations, functions and series; (4) geometry and measure; (5) combinatorics, probability and statistics. Another significant characteristic of Varga's curriculum was its flexible structure with 'suggested' and 'compulsory' topics distinguished from 'requirements'. This organisation essentially gave liberty to teachers, allowed differentiation amongst students, provided a rich and varied experimental basis to the progressive generalisation and abstraction of mathematical notions, and supported a learning process based on mathematical discovery, while elements of mathematical knowledge can emerge as tools during problem-solving situations.

Teachers' adaptations to the new curriculum and to the related pedagogical expectations were supported by a series of textbooks and teachers' guides prepared by those responsible for developing the curriculum. At the time it was the only available textbook series in Hungary. For the primary school, similarly to other countries in the New Math period, worksheets were available, intended for use only as partial resources alongside various activities. Official teacher's guides served as primary resources for teachers. For middle school, there were textbooks provided, with (much less detailed) teacher's manuals.

According to the handbooks, teachers had an important responsibility in the construction of long-term teaching processes, which were based principally on ordered series of problems. Mathematical concepts were constructed on a broad experimental basis, by discovering links and analogies among apparently different problems and by generalising progressively the knowledge related to concrete problem contexts. The importance of the use of various manipulative tools and representations was underlined (some of these tools being widespread at the time including the Dienes blocks or the Cuisenaire rods).

Various forms of classroom organisation were promoted in the guides (including individual and group work), but collective classroom dialogue was particularly emphasised. The guides offered advice regarding teacher questions and interventions that would enable teachers to react efficiently to students' contributions. It was envisaged that this would help the advancement of the collective research project while leaving an important responsibility to students in the problem-solving process and the construction of mathematical knowledge.

As with many other reforms of the period, Varga's reform provoked vivid public debates and was followed by an important correction during the 1980s. His former colleagues interpreted this as a failure, and they considered the obligatory introduction of the reform for all as the main reason of its rejection. According to them, the approach should have been disseminated progressively in the frame of a bottom-up process, as had happened during the (generally successful) experimentations – but this kind of slow diffusion was not politically supported. Although teacher education media were offered, these efforts were far from enough to prepare teachers for this radical reform and to settle the resistance. While a narrow circle of teachers (mostly colleagues of Varga and their disciples) continued to follow the approach in their teaching practices in the following decades and until today, the majority of Hungarian teachers did not adopt it or integrated only partial elements of the approach in their practice.

Despite that, Varga's work remained influential in Hungarian mathematics education. An important continuity can be observed in the current curricula's conception: the main structure and the content of the curriculum remained quite stable until today. Some of the textbook authors from his team were active until the 2010s, and their textbooks demonstrate continuity with the original versions of the 1970s – although other manuals are also available now. Most of the teacher trainers consider his 'guided discovery' conception still relevant and find inspiration in it, especially for primary level in-service teacher training (Gosztonyi et al., 2018).

Varga's work also exerted a particular international impact, although not comparable to the leading Western European and American projects. Some of his works were translated in many countries in the Eastern bloc, and several of his publications also appeared in France, Canada, and the USA, especially in the domains of teaching combinatorics and probability in lower grades (Glaymann & Varga, 1973; Varga 1967, 1982; Varga & Dumont, 1973). His worksheets were translated into Italian and used mainly for teacher education,[4] and more recently, a Finnish primary school teacher association was created, inspired by his work.[5]

[4] Information given by Maria Bartolini Bussi.

[5] Varga-Neményi Association (https://varganemenyi.fi/).

At the same time, and in contrast to the French case, Varga's reform did not lead to a dynamic research culture in mathematics education. After he died in 1987, only a few Hungarian researchers remained active. His work became, however, the catalyser of a newly emerging research movement in Hungary in recent years.

Realistic Mathematics Education in the Netherlands

Realistic Mathematics Education (RME) has become the main approach to mathematics education in the Netherlands and has also left its mark on mathematics education in other countries (Van den Heuvel-Panhuizen, 2020a, b). In this section, we give a short sketch of the development of the RME reform movement in the Netherlands and we take the perspective of the reform that happened in primary school mathematics education.

When from the early 1960s on, New Math gained world-wide influence, the Netherlands chose another direction to change the rather mechanistic approach to mathematics education that was common at that time. This approach was characterised by teaching fixed calculation procedures in a step-by-step manner with the teacher demonstrating for each step how to proceed, with real-world problems only used for the application of previously learned calculation procedures, and little or no attention for developing insight in the underlying mathematics of these procedures.

The reform that was an answer to this approach was initiated by the inception of the Wiskobas project in 1968 and was further enhanced by the establishment of the IOWO (Institute for Development of Mathematics Education) of which Freudenthal was the first director. The IOWO produced a broad variety of materials making this change to a new mathematics education possible, including rich tasks, themes, lessons, teaching sequences, and complete programs for various topics within arithmetic, measurement and geometry. The Special Issue of Educational Studies in Mathematics titled "Five Years IOWO" (Freudenthal, Janssen & Sweers, 1976) reflects the outburst of ideas in the initial period of the reform movement.

In addition to these design activities carried out in the early days of RME, the underlying theory was also given much attention. Freudenthal (1973) published his ground-breaking book *Mathematics as an educational task*, and Treffers (1978) brought out his first work on the goals and approaches to mathematics education according to Wiskobas. Other important research work that started at the end of the 1970s involved carrying out textbook analysis. Existing textbooks were commented on and critically examined from the perspective of the intended reform, which had a guiding function for the innovation.

In 1981, the Wiskobas work came to an end as a result of a decision of the government. The work on RME was continued by OW&OC (Mathematics Education Research and Educational Computer Center) and the newly established SLO (the Netherlands Institute for Curriculum Development). Characteristic for the 1980s were the many research activities and the various national and international publications that resulted from them. For primary education, important work was done for example by Adri Treffers (progressive schematisation), Leen Streefland (context and models for fractions), and Jan van den Brink (mathematical language and representations for early number). Furthermore, a new boost to theory development was given by Freudenthal's (1983) *Didactical phenomenology of mathematical structures* and Treffers' (1987) *Three dimensions*.

An important impetus for implementing RME in curriculum documents, textbooks and school practice was the establishment of the Netherlands Association for the Development of Mathematics Education (NVORWO) in 1982. One of the first actions of NVORWO was to prepare a national plan for primary mathematics education. In 1984, a draft version of this plan was submitted for consultation to almost three hundred experts in the field of primary school mathematics. It was proposed to give algorithmic digit-based calculation a less central position in favour of mental calculation, estimation and number sense, to aim more at applicability, and not to start with teaching students the most shortened forms of standard algorithms immediately, but begin with a notation using whole numbers.

This plan received much acclaim. In 1987, the findings resulted in the first blueprint for a national programme for mathematics education in primary school, the so-called 'Proeve publications' (e.g. Treffers, De Moor & Feijs, 1989). Later, the goals as described in the Proeve publications were officially given approval by the government by adopting as the national core goals for primary education (MoE, 1993). A further implementation in curriculum documents was possible through the development of the TAL teaching-learning trajectories for primary mathematics education commissioned by the Ministry of Education (e.g. Van den Heuvel-Panhuizen, 2001). Also, the Proeve and TAL publications with their descriptions of goals, examples of tasks and teaching methods served as beacons for textbook authors. This resulted in a noticeable change in the nature of textbooks. The market share of RME-oriented textbooks increased from 15% in 1987 to 75% in 1997 and reached 100% around 2004.

However, the educational climate changed remarkably around 2007 (see Van den Heuvel-Panhuizen, 2010). This change was prompted by the results from the 2004 PPON survey (the National Assessment of Educational Achievement) by CITO, the Netherlands Institute for Educational Measurement. The results showed that student performance in number sense, mental calculation and estimation had substantially improved since 1987, but that achievements for written algorithmic calculation had decreased. Although this was to a certain degree in line with the performance profile opted for twenty years earlier, these findings evoked much protest against the RME reform.

The complaints particularly came from a few mathematicians, who were in favour of returning to the mechanistic approach to mathematics education of forty years ago. A fierce debate arose in newspapers. After a commission established by the Royal Netherlands Academy of Arts and Sciences (KNAW) (2009) concluded there was no convincing empirical evidence for the claims on the effectiveness of traditional methods versus RME, the peace returned. However, the debate was not without consequence.

The market share of RME-oriented textbooks lost a few percentage points of its 100% position, and new editions of the RME-oriented textbooks that were still on the market included features of the mechanistic approach of the past by putting more emphasis on written calculation. Nevertheless, to date they do not focus on blindly training of procedures but aim at understanding by starting with a phase of transparent whole-number-based written calculation. By and large, the RME characteristics are still upheld in most current textbooks.

Another recent movement toward the past is the return to the original ideals of RME to give much attention to mathematical reasoning and problem solving (Wiskobas team, 1980). One example of this revival is the Beyond Flatland project, set up in 2015, to investigate how the Dutch primary school mathematics curriculum can be made 'more mathematical' by including activities on mathematical reasoning in the context of early algebra, dynamic data modelling, and probability.

The idea of already starting in primary school with mathematical problem solving through modelling and reasoning is also reflected in the advice given by the project team of teachers recently commissioned by the Ministry of Education to rethink and revise the current mathematics curriculum to have students better equipped for their future personal and professional life. The resulting plans are supported by NVORWO (2017) and also clearly show elements of the spirit of Wiskobas.

Mathematics Curriculum Reform in Japan

Reforms of mathematics education in European countries at the beginning of the twentieth century affected mathematics education in Japan. Mathematics had, until that time, been taught separately, broken down into domains of mathematics. In elementary school, the curricular focus was on arithmetic such as calculation and conversion of units of measurement, followed by the study of algebra, geometry (especially Euclidean) and analysis in secondary school. Japan was in the midst of modernisation in the early twentieth century and had just established an educational system and curriculum modelled on those of Western countries.

These reforms, such as the 'Meran project', were introduced in the 1910s and 1920s, but it took time for these to be reflected in practice, and they truly came into effect after the 1930s. These reforms sought to update traditional mathematics education to suit the development and interests of children. The curriculum was structured without dividing the academic domains of mathematics (Monbu-syo, 1931). Arithmetic and geometry were studied at the same time during elementary school and were not presented via axiomatic treatment by proof (Monbu-syo, 1935). The national textbooks in this age were introduced in the tenth International Congress of Mathematicians (ICM) convened in Oslo in 1936 (Kunieda, 1936). In the new teaching method, the so-called Life Arithmetic, which relates mathematics to daily life and experience, was taught at schools affiliated with the national normal schools and advanced private schools.

In the 1950s, in the years after World War II, US progressive education was introduced by the Occupation Command and incorporated into mathematics education as part of post-war educational reform. Thus, the relationship with children's daily life and experience was emphasised for its teaching. Teaching content was reduced or taught in later grades. However, they were revised during Japan's independence.

In the 1960s, the influence of New Math was also seen in Japanese mathematics education. New material such as set theory, algebraic structures and linear transformations was incorporated during secondary school lessons. This resulted in new difficulties, which were not limited to relating the new material to the old grade distribution but included structural changes and improvement to the mathematics curriculum as a whole. More specifically, the idea of set theory reconfigured conventional learning on the range of numbers and the meaning of operations from a different perspective, which required changes in elementary school mathematics. The New Math curriculum was incorporated into the revision of the national curriculum in the late 1960s (Monbu-syo, 1968).

In the 1970s, the mathematics curriculum was reorganised in reaction to New Math. Set theory and algebraic structure were removed from elementary and secondary school mathematics, and the teaching content was carefully selected to present foundational knowledge and skills. Elementary school mathematics content was divided into four categories: (a) numbers and calculations; (b) quantities and measurements; (c) geometrical figures; (d) mathematical relations (Monbu-syo, 1977). The first three categories corresponded to the subjects handled in mathematics that is, number, quantities, and shapes. Mathematical relations summarise the methods and ideas dealt with in mathematics and include functional concepts, statistical methods, and mathematical expressions related to other contents such as number, quantities, and shapes. This framework based on separately describing the subjects and methods was a characteristic way to systematise the content in mathematics curricula, and it was close to the categorisation of common standards of the National Council of Teachers of Mathematics (NCTM, 1989).

After 1990, the focus of the Japanese mathematics curriculum shifted from the content to learning processes and purpose. Objectives now included the new term 'mathematical activity' and the principle that students should learn math through 'mathematical activities' (Monbu-syo, 1998). It also includes new category types that fall under the heading 'mathematics activity': (a) activities to discover and develop the properties of numbers and figures based on learned mathematics; (b) activities using mathematics in everyday life and society; (c) activities that use mathematical expressions to clarify the basics, to explain them reasonably and to communicate them to others (Monbu-kagaku-syo, 2008). Table 4.1 summarises the changes in the post-war mathematics curriculum in Japan.

The national curriculum was revised and announced in 2016 for elementary and junior high school, and in 2017 for high school. This revision will go into effect in 2020 in elementary schools, in 2021 in junior high schools, and in high school staring in the first year from 2022. This revision emphasises qualifications and literacies as the purpose of education. It is based on the idea of key competencies and the OECD-PISA framework. Kyouiku-Katei Kikaku Tokubetsu Bukai

Table 4.1 Changes in the post-war mathematics curriculum in Japan

Time-division	Characteristics	Notes				Total hours in 9 years (1 h = 50 min)
1st period: 1950s	Improvement of daily life as the reconstruction of experiences (influence of progressivism in the US)	Emphasis on daily life and experiences				1346
2nd period: 1960s	Emphasis on the system of each subject and enrichment of basic ability (movement toward economy-based and technology-based country)	Quantitative enlargement of education	Mathematical ways of observing and thinking			1292–1467
3rd period: 1970s	Advancement of educational content and new math (influence of Western countries during the 1960s)					1362
4th period: 1980s	Modification based on new math; selection of educational content; balance of intellect, virtue and physical strength					1295
5th period: 1990s	Fostering the ability to think, to judge and to represent; emphasis on students' will to learn and their autonomy	Qualitative enrichment of education		Value of mathematics	Mathematical activities	1295
6th period: 2000s	Careful selection of educational content and nurturing of zest for living without pressure and allowing room to grow					1097
7th period: 2010s	Fostering students' power to live in a knowledge-based society; enrichment of language activities and international validity and currency of science and mathematics education					1295

(KKKTB – Curriculum Study Group of National Council for Education), an advisory board for Ministry of Education, Culture, Sports, Science and Technology (MEXT), compiled three pillars of qualification and literacies for the foundation of the new curriculum in 2014 (KKKTB, 2014). The three pillars suggested to MEXT are: (a) what you know and what you can do (individual knowledge and skills)? (b) how do you use what you know and what you can do (application of thought, judgment, expression)? (c) how do you live a better life and relate to society and the world (promoting individuality, diversity, co-operativeness, attitudes toward learning and humanity)?

These qualifications and literacies became watchwords and were guided by both the desire to examine what children can do as a result of school education and the reason for teaching it, rather than focusing on what to teach. The mathematics curriculum has been organised using these three pillars as a framework. The objectives for this subject area have thus been rewritten from these three perspectives, and the teaching content is presented separately for the categories (a) individual knowledge and skills and (b) ability to think, judge, and express. This curriculum framework will be used in the further development of mathematics education in Japan.

Conclusions

Reforms of mathematics education are national phenomena, and they have been presented here as such. Nevertheless, as the examples of the movement of the beginning of the twentieth century and the reforms that originated after the Royaumont conference show, they are also closely related to international exchanges. International exchanges were both a resource and a consequence of national reform movements, and international organisations, especially ICMI, played a crucial role in this process since its foundation.

The cases included in this chapter have also been chosen because of their importance beyond their strict national sphere. The first two cases – France and Hungary – correspond to reforms that took place over a short period and were followed by counter-reforms, even if they deeply marked the mathematics curricula established after them, in their content, organisation and resources. The remaining two cases – The Netherlands and Japan – embrace a more extended period and show a continuity in the reform processes, with some back-and-forth movements but presenting a clear sustainability.

In all cases, well-known and highly regarded mathematicians were involved in the first steps of the process and, what seems more relevant but maybe remains less visible in the descriptions provided, is the intensive mathematical 'engineering work' (to use Freudenthal's expression) undertaken to launch the reforms. This work was a collective enterprise leading to the construction of new mathematical contents, curriculum organisations and teaching resources. The efforts put by researchers and teachers to carry out this enterprise and the means initially provided to do so appear as a common trait of the four cases considered. They seem to have also generated a fertile milieu for the emergence of mathematics education research.

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Chapter 5
Learning from the Past: Case Studies of Past 'Local' Curriculum Reforms

Niamh O'Meara and Jasmina Milinkovic

Introduction

This chapter presents some results of a survey designed by some of the theme A participants to gain an insight into curriculum reform efforts in different countries internationally. The survey sought to gather information in relation to:

- how and why the reform movement came about;
- the ideologies underpinning the reform movement;
- the aim of the reform movement;
- the agents or stakeholders involved in the reform movement;
- the impact of the reform on mathematical content, mathematical teaching and mathematical assessment;
- the lessons learnt from the reform movement.

In total, six research colleagues from different countries (Brazil, Japan, Ireland, Italy, Serbia and South Africa) responded to the survey, ensuring a geographical spread across four continents. Three survey responses were analysed in detail for the purpose of this chapter. The countries selected were Ireland, Serbia and South Africa. These were selected for consideration due to the commonalities in some aspects of the curriculum reform efforts as well as quite unique differences. Many of the responses from these countries also reflected, in part, the reactions from the other respondents, and so the authors believed a comprehensive overview could be achieved with this limited sample.

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Case Study A: Ireland

The Irish secondary school mathematics curriculum experienced reform over the past ten years. The revised curriculum, which was piloted between 2008 and 2010, was implemented on a national basis in 2010. This reform movement was initiated for a multitude of reasons. Firstly, there was a need for improved alignment between primary and secondary mathematics education. In 1999, the primary school mathematics curriculum was reformed in Ireland while the previous reform at secondary level was conducted in the 1970s. As such, the alignment between the two curricula was tenuous, and so it appeared logical to reform the secondary school mathematics curriculum to modernise a dated mathematics curriculum and to improve the alignment between the primary and secondary curriculum. Ireland's poor performance in international assessments such as TIMSS and PISA was another reason for the introduction of a revised curriculum.

At the time the curriculum was first conceptualised in 2005, Irish students were classified as 'average' performers in these international assessments in mathematics. Many believed that this deficiency in students' mathematical capabilities stemmed from an over-reliance on rote learning in the Irish mathematics classroom (State Examinations Commission, 2003, 2005) and the declining attitudes towards mathematics among secondary school students (National Council for Curriculum and Assessment, 2005). This led to calls from employer groups, such as the Irish Business and Employers Confederation [IBEC], as well as groups such as the Project Maths Implementation Support Group [PMISG] for a better-qualified workforce equipped with the mathematical skills necessary to ensure "Ireland's future economic growth and competitiveness" (PMISG, 2010, p. 4). A combination of these reasons resulted in the reform of secondary mathematics education in Ireland, a reform known locally as Project Maths.

The Realistic Mathematics Education (RME) movement inspired Project Maths, as it was seen as the "most fashionable approach among mathematics educators" (NCCA, 2005, p. 6) at the time. However, Project Maths was not solely based on this international reform movement. The rationale behind this decision to not align the new Irish mathematics curriculum entirely with the broader RME movement was as a result of the lessons learned from previous curriculum reform efforts. The previous mathematics curriculum reform in Ireland, which took place in the 1970s, was based on the ideologies proposed by the New Maths movement and the curriculum was entirely modelled on this broader movement. This was deemed to be a naive approach to local curriculum reform. As a result, it was strongly advised by all stakeholders that Project Maths should not come from one single ideological standpoint but instead be inspired by appropriate aspects of the RME movement and aligned with international best practice and the needs of the Irish economy.

This standpoint led to the PMISG (2010) outlining that the principal aim of Project Maths was to "teach mathematics in a way which promotes real understanding, where students can appreciate the relevance of what they are learning and its application to everyday life, and how mathematics can be used to solve problems"

(p. 12). The reform effort also aimed to place a much stronger emphasis on active learning and teaching for understanding. Hence, Project Maths not only sought to change the content being taught but also the manner in which mathematics was taught, learned and assessed (Cosgrove et al., 2012). This aim to change the focus to problem-based, student-centred mathematics education was in line with the teaching and learning of mathematics internationally (Eurydice Network, 2011), as well as many of the principles proposed in the RME movement.

The three most influential agents/stakeholders involved in this local reform movement were international agencies, as the results from international assessments acted as key points of reference when revising the curriculum, government agencies and partners in industry. For the reform effort to be successful, there had to be a convergence of national Government agencies so that they could withstand any resistance from other stakeholders. Government agencies, most notably the Department of Education and Skills [DES], are considered to have played the most significant role in the instigation, development and implementation of the new curriculum. The DES initiated the reform movement and the responsibility of developing the new curriculum, based on a consultation with all stakeholders and advice from a steering group,[1] lay with the National Council for Curriculum and Assessment [NCCA] (NCCA, 2012).

During the implementation phase, the NCCA worked closely with the DES and the State Examinations Commission, the state body responsible for designing the summative assessment for each curricular subject. This unified approach from a multitude of government agencies allowed for an alignment between the goals of the curriculum reform and the mathematical content and assessment structures. Industry and employer groups, as well as international agencies such as the Organisation for Economic Co-operation and Development (OECD), via TIMSS and PISA, also played a significant role in instigating the reform movement (OECD, 2004; NCCA, 2006). Pressure came on the Government from employer groups to review the mathematics curriculum and improve students' proficiency in mathematics, due to Irish students' 'average' performance in international tests such as PISA and TIMSS (NCCA, 2005). As a result, the nature of the PISA assessment strongly influenced the curriculum reform in many ways.

Finally, prior to the reform effort, the NCCA released a discussion paper as well as a document outlining international trends in mathematics education (Conway & Sloane, 2005). Upon the release of these documents a consultation process commenced and allowed "those with an interest in the issues raised to respond to these and to raise any other concerns which they considered should be addressed under the review" (NCCA, 2006, p. 2). This provided other stakeholders including teacher unions, mathematicians, teachers, parents, textbook publishers, students as well as universities and other higher education institutes with the opportunity to contribute in some way to the reform efforts.

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[1] The steering group was made up of representatives from within the Department of Education and Science and relevant Government agencies.

An on-line questionnaire was made available to all stakeholders on the NCCA website, while postal responses were also welcomed. The questionnaire sought respondents' opinions in relation to the role of mathematics education; concerns regarding the teaching and learning of mathematics; current trends in mathematics education; the provision and uptake of mathematics; attitudes towards mathematics; the mathematics syllabus and assessment and student achievement in mathematics.

The NCCA also held focus group discussions as part of the consultation process. These discussions were held with parent associations; mathematics teacher associations; the Irish Union of students and mathematics teachers. One of the most prominent messages to emerge from this consultation process and one which was echoed by the majority of stakeholders was that any reform effort had to "make mathematics more related to the lives of students" (NCCA, 2006, p. 44). This recommendation, along with many others outlined in the consultation report, were acknowledged and resulted in meaningful change across mathematics content, pedagogy and assessment.

The reform movement which followed the publication of the consultation findings simultaneously affected the mathematical content taught; the pedagogical approaches employed and the structure of the assessment used. However, the reform did not, in any way, alter the discipline structure, as mathematics was a stand-alone, all-encompassing curricular subject before and after the reform. The main changes across each of the three aforementioned areas are documented in Table 5.1.

With such substantial changes recommended another key aspect of the Irish curriculum reform related to teacher training. In order to equip teachers with the knowledge and skills necessary to deliver a very different mathematics curriculum, continuous professional development for practising teachers was deemed essential. Professional development workshops were developed and delivered by the Project Maths Development Team to all teachers involved in piloting the new curriculum, while the National Centre for Excellence in Mathematics and Science Teaching and Learning were tasked with delivering three summer schools to teachers during the pilot phase (NCCA, 2012). Additionally, when the new curriculum was introduced nationally, a series of ten compulsory workshops were made available to all teachers.

To conclude, Project Maths appears to have promoted a fundamental shift in the teaching and learning of mathematics in Ireland. The emphasis changed from examination-driven, procedural teaching that promoted rote learning to student-centred teaching that promoted conceptual understanding. While there was initially some resistance to aspects of the reform movement, as reported by Cosgrove and colleagues (2012), other researchers have found that the reform effort has resulted in positive changes in the teaching styles used in the mathematics classroom and in students' attitudes towards mathematics (Jeffes et al., 2013). Furthermore, while Jeffes and colleagues found that, "the revised mathematics syllabuses taken as a whole does not appear to be associated with any overall deterioration or improvements" (p. 71), they did conclude that improvements were noted across individual strands, most notably in the area of Statistics and Probability.

Despite these perceived successes, numerous challenges were encountered but it is from these challenges that lessons can be learnt. Firstly, teachers have deemed the

Table 5.1 Changes to content, pedagogy and assessment as a result of Project Maths

Area	Pre-2010	Main changes post-2010
Content	Core curriculum containing a list of topics such as matrices; differential calculus; algebra; vectors; trigonometry; statistics etc. In addition to these core concepts, teachers could also select one advanced topic to teach from the following list: Advanced Calculus and Series; Advanced Probability and Statistics; Group Theory; Geometry.	Change in how mathematical content was structured. The new curriculum was divided into five inter-related strands that aligned with the primary school curriculum, namely Statistics & Probability; Geometry & Trigonometry; Number; Algebra and Functions (which includes Calculus at upper secondary level).
		Increased emphasis on Probability & Statistics, particularly inferential statistics, and this strand now constituted one-fifth of the new curriculum.
		Increased emphasis on Euclidean and Synthetic Geometry.
		Financial mathematics was a new topic introduced.
		Content relating to linear algebra (e.g. vectors, matrices) was removed.
Pedagogy	The pedagogical approaches favoured were teacher-led, with a strong emphasis on rote learning.	Promotion of a collaborative culture where teachers and students construct new knowledge together.
		Fundamental shift to a more co-operative teaching and learning environment that promoted active learning; an investigative, problem-solving approach to learning and encouraged students to develop a meaningful understanding of mathematics.
Assessment[a]	Students sat two papers. At Junior Cycle, there were 8 questions on Paper 1 and 2 and students selected six. The questions were all procedural in nature. A similar picture emerged at Senior Cycle but on Paper 2 there were an additional 4 questions from which students selected one based on the optional elements of the curriculum.	In 2010 external, summative examinations remained the chosen mode of assessment at both Junior and Senior Cycle.
		Choice was removed from both papers at Junior and Senior Cycle.
		The nature of the questions included on the examinations also underwent significant change. The new examination was divided into two sections one focussing on procedures and concepts and the second on contexts and problem-solving. This led to examinations being more unpredictable.

[a]In Ireland, students, at the end of their first three years in secondary school (end of Junior Cycle) sit a state examination known locally as the Junior Certificate. Likewise, at the end of their two years in upper secondary school (end of Senior Cycle) they sit a second state examination, known locally as the Leaving Certificate. The Leaving Certificate acts as a gatekeeper to third level education

time allocated to the revised curriculum insufficient (Prendergast & O'Meara, 2017). In the study carried out by Prendergast and O'Meara (2017), teachers stated that the new curriculum required significantly more time than was the case previously, but the vast majority of these teachers reported that no additional time was afforded to the subject in the aftermath of the reform. As such, O'Meara and Prendergast (2019) state that one lesson that can be taken from the Irish reform effort is that those responsible for instigating and implementing reform must "work with curriculum developers to specify a time allocation that is feasible and aligned with the curriculum" (p. 509).

Another outstanding issue is in relation to the transition from primary to post-primary mathematics education. Project Maths sought to better align the primary and post-primary curriculum. However, this alignment of curricula has not led to improvements in the transition as documented by Prendergast et al. (2019) and O'Meara et al. (2018, 2020), indicating a mismatch between the actual reform movement and one of the motivating factors which led to its conceptualisation. These authors found that the main reason for the issues around transition was a lack of 'horizon knowledge' (Ball et al., 2008) on the part of teachers at either side of the transition. As such, it is recommended that when curriculum reform occurs at one educational level, it is critical that teachers at other levels (in this case primary and tertiary level) are fully informed about these developments so they too can be prepared for any possible positive or negative implications.

Case Study B: Serbia

The case in Serbia is somewhat different to that reported in Ireland. According to Milinkovic (2018), there have been numerous changes made to the mathematics curriculum in Serbia since the 1960s. Many of these have been regarded as minor changes; however, the particular reform that will be discussed in this chapter is a significant change and is the most recent of all reform efforts. This reform of the mathematics curriculum was conceptualised in 2015 and the implementation process began in 2018.

There were two primary motives for the instigation of this reform. Firstly, many people in the Serbian education community, and among the wider public, believed that the previous curriculum was overcrowded and needed to be updated. They purported that the curriculum was too demanding and time-consuming. Hence, it was proposed that a new curriculum be introduced that relaxed "curriculum demands and the number of math lessons per week" (p. 145). The second reason for recent curriculum reform in Serbia was because of pressure from public opinion, as well as from Government authorities who were dissatisfied with Serbian students' performance in international tests such as PISA and TIMSS.

When the 2012 PISA results were analysed, mathematical literacy among Serbian students was found to be below average, with Serbia's score 45 points below the OECD average (Pavlović-Babić & Baucal, 2013). While analysis of the results from the 2015 TIMSS study yielded more positive findings, with Serbian students' scoring higher than many European countries, Serbia remained a hundred points lower than many of the most successful countries. These findings, combined with a desire for Serbian students to excel in the area of mathematics, led to a call from authorities for a reformed curriculum that reflected the content assessed in both the PISA and TIMSS studies.

The overarching goal of this reform was to enhance the mathematical literacy levels of students across the entire educational system. In order to achieve this goal, there were a number of objectives associated with the new curriculum, namely:

(1) to connect conceptual and procedural knowledge so that students develop a deep, connected and meaningful understanding of mathematical concepts as proposed by Skemp (1976);
(2) to highlight the inter-related nature of mathematics by connecting different mathematical fields and concepts;
(3) to use multiple representations in the teaching and learning of mathematics as proposed by Wagner & Kieran (1989).

These aims all point to the fact that RME is central to the latest reform efforts in Serbia. In addition to this, the ideas of Skemp, Bruner and Freudenthal are reflected in the reform movement, particularly in the pedagogical approaches advocated by the new curriculum. By adopting aspects from the work of leading mathematics educators and psychologists, and framing the curriculum around RME, it was anticipated that the latest mathematics curriculum offered to Serbian students would be current, relevant and achieve the desired outcome of improved mathematical performance.

As with all curriculum reform movements, in order for the revised Serbian curriculum to be conceptualised and implemented, contributions were required from a number of different stakeholders. The main stakeholders in Serbia were teacher unions, local and international mathematicians, teachers, parents, textbook publishers, students and institutes. However, in Serbia, international organisations such as the OECD were instrumental in instigating curriculum reform and helping curriculum developers identify what aspects of the old curriculum needed to be changed.

As mentioned previously, it was the results of PISA in 2012 and TIMSS in 2015 that resulted in calls for change to the Serbian curriculum, but these international assessments also played a significant role in determining the content that would be added, altered, or removed under the reform. As such, the revised curriculum in Serbia now reflects many aspects of the TIMSS and PISA assessments. The second most influential stakeholder in the reform was two Government agencies, namely The Bureau for Improvement of Education and Upbringing and The National Council for Education. These agencies were primarily responsible for defining reform policy and making decisions in relation to how content, pedagogy and assessment structures would change.

In addition to this, teacher training is seen as a key component of any curriculum reform in Serbia. All primary and post-primary mathematics teachers received formal teacher training as part of this reform effort, and Government agencies were responsible for assigning expert groups to organise, coordinate and deliver this in-service training. Teacher unions also played a role in the upskilling of teachers. They provided professional development opportunities to teachers to prepare them for the new content they were expected to teach and the new teaching methodologies they were expected to employ. As a result, they too were seen as a very influential stakeholder.

In addition to their work on teacher professional development, these teacher unions played a key role in facilitating changes in teaching practices so that teachers were better positioned to adopt the pedagogical practices and principals advocated in the curriculum documentation. This was facilitated through semi-annual meetings of the teacher unions and through workshops and seminars, accredited by the Government Bureau.

The Serbian curriculum reform resulted in simultaneous changes to the mathematics proposed to be taught and the way it was taught and assessed. It also resulted in a change in the structure of the discipline of mathematics within schools, with a new mathematical discipline called *informatics* introduced as a specialist subject, distinct from the subject of mathematics (Milinkovic, 2018). This new subject sought to equip students with the skills necessary to manage information, communicate securely in a digital environment and develop computer programmes for solving various problems in a rapidly changing digital society.

This subject was offered at elementary and high school levels. As mentioned previously, many of the changes made to mathematical content under this reform were influenced by PISA and TIMSS assessments. For example, according to Milinkovic (2015) many topics that featured on the TIMSS 2015 assessment, including addition and subtraction of simple fractions; decimal numbers; axial symmetry and three-dimensional shapes, had been removed from the primary school mathematics curriculum in the 1970s. Under the reform effort, many of these topics were reintroduced.

At the post-primary level many of the topics that featured on the previous curriculum were preserved, with some explored in more detail. Other aspects of the previous curriculum were removed or undermined. For example, at the primary school level, prior to the introduction of the revised curriculum, the topic of 'sets' was a core component of the arithmetic section of the course. However, the emphasis on this particular concept has been diminished significantly in the revised curriculum.

On the other hand, topics in the areas of data analysis and financial mathematics were included for the first time in the revised curriculum. There was also a fundamental change to how mathematics content was structured. Prior to the reform, it was generally the case that different topics were taught at different grade levels. However, under this reform it was proposed that the curriculum be organised in a 'spiral' manner, meaning that the same topics were taught across different grades with the scope and depth of the topic increasing as students progress.

In addition to these significant changes to mathematical content, pedagogical changes were also detailed in the new curriculum. In line with the RME movement, Serbian teachers were now encouraged to utilise more problem-based learning strategies using real-world contexts. Milinkovic (2018) states that the topics being reintroduced to the primary curriculum will now be taught in a less formal or procedural manner and, instead, "the emphasis is on understanding in context and […] problem solving" (p. 6). In addition to the increased focus on problem solving, teachers were also encouraged to utilise more resources in the mathematics classroom to improve students' understanding. In particular, the use of technology and digital resources was encouraged.

Finally, for smooth curriculum reform, it was necessary for the way in which the students were assessed to be altered in line with the changes to content and pedagogy. Under this reform, it was advocated for assessment to be viewed as a source of information for planning instruction and evaluating individual progress along the individual's line of development. As a result of this, continuous assessment feature much more heavily in the new curriculum and teachers are expected to use multiple sources of evidence throughout the school year.

The Serbian curriculum reform is still very much in its infancy. While the ideals underpinning the curriculum are in line with many of the ideals promoted in the RME movement, it is too early to determine if such ideals have been realised. It is also too early to determine if the revised curriculum has had the desired effect on Serbian students' mathematical performance. It is believed that the long-term effects of this reform will only be evident in 2027, when the curriculum has been in place for an entire twelve-year cycle, hence highlighting one of the issues that often arise with the evaluation of curriculum reform – time. A lesson one must take from the Serbian curriculum reform effort is that it is critical, with any curriculum reform, that a sufficient amount of time is allowed for implementation and rollout before conclusions are drawn on the perceived success or failure of the movement.

Case Study C: South Africa

To date in this chapter, the authors have discussed curriculum reform in two European countries, both of which have been largely driven by students' performance in international comparison tests and a desire to improve students' mathematical capabilities. However, the reasons for mathematics curriculum reform in South Africa are quite different and much more political. While many in South Africa acknowledge that TIMSS did play some role in the curriculum reform, Parker (2006) determined that, "radical school curriculum changes implemented since 1997 […] are explicitly aimed at overturning the unjust distribution of power and control relations characterising South African society" (p. 59).

In essence, reform movements across all curriculum subjects, including mathematics, were instigated to address the challenges presented by the post-apartheid era and those presented by the need for South Africa to be seen as a competitor in the global market (Vithal & Volmink, 2005). The reform efforts sought to move away from all aspects of apartheid education to a new curriculum that could serve the vast majority of South African people. Associated with the issues of inequality that arose from the apartheid era were high drop out and failure rates in the area of mathematics and assessment structures that placed a strong emphasis on rote learning and "unimaginative teaching methods" (Botha, 2002, p. 361).

These were also issues that the revised mathematics curriculum sought to address. In this chapter, the authors will discuss the initial reform from the mathematics curriculum that was in place pre-apartheid to the short-lived outcomes-based education (OBE) movement. However, this OBE paradigm was also the underlying philosophy behind the subsequent reform effort known as 'Curriculum 2005' or C2005 (Botha, 2002), which will also be discussed. de Waal (2004) clearly differentiated between these two key movements by stating that OBE is the framework through which the aims of a curriculum can be realised while C2005 is the actual curriculum that has been derived from the framework. Pudi (2006) described this series of curriculum reforms using the following analogy:

> This can be likened to a car that starts to move from first gear through to higher gears. Once the car has been engaged in first gear (likened to OBE) to start the car moving, the need exists to change to second gear (likened to C2005) (p. 104)

When the South African mathematics curriculum was first changed in 1997, the ideas or theories underpinning the new curriculum came from a multitude of sources. The new South African curriculum did not specifically align with one of the major reform movements previously discussed but instead took inspiration from a mixture of educational positions including:

- socio-constructivism;
- ethnomathematics;
- critical mathematics education.

Adopting this stance resulted in the new curriculum promoting the idea that mathematics is a cultural product. Furthermore, with the principles of the aforementioned educational positions in mind, the revised curriculum aimed to ensure equal educational opportunities for all students (DoE, 2003) and "to construct new pedagogic identities in teachers and learners" (Bernstein, 2000; cited in Parker, 2006, p. 59). The discourse in which the curriculum policy was expressed allowed for different theoretical orientation and approaches to be used in the mathematics classroom.

In addition to this, the curriculum reform post-apartheid sought to change what was regarded as valuable mathematical knowledge, hence resulting in a significant change in the way the subject of mathematics was structured and in the content being taught. In essence, the reform sought to radically change the way mathematics was taught and assessed with much more emphasis being placed on student-centred learning (De Waal, 2004).

Historically, there has been a large number of stakeholders involved in curriculum reform efforts in South Africa (Vithal & Volmink, 2005). Chisholm (2005) ascertained that this too was the situation when OBE and C2005 were being developed. A consultation process was held prior to the re-design of the curriculum, and this allowed input from a vast number of stakeholders including

Government agencies; teacher unions; local mathematicians and textbook publishers. Following this consultation process, several committees were formed by the Minister for Education. These committees were broadly representative of all aforementioned stakeholders, and they worked to derive a revised curriculum that would be both educationally and politically acceptable.

Once they had developed a draft version of a revised curriculum, there was much public commentary before it was approved. For example, in 2001, there were public hearings in relation to the proposals put forward by committee members and presentations were also made by Government officials. This allowed all key stakeholders and members of the public to have further input into the revised curriculum, and based on these public processes, there was a further refinement of curriculum documents. This approach to curriculum reform in South Africa meant that many stakeholders felt that they had some ownership over the new curriculum and the curriculum was the product of the efforts of an entire nation.

Once the refined mathematics curriculum was implemented, many changes to mathematics education in South Africa occurred. First and foremost, there was a change in the way the subject of mathematics was structured. Under OBE and C2005, all students were required to complete the same mathematics curriculum between grade 1 and grade 9,[2] but once they reached grade 10 they got the option to select either mathematics or mathematical literacy. The introduction of a specific mathematical literacy programme indicated a much stronger focus on mathematical literacy in South Africa. This was a significant change to the way mathematics was structured, and the choice students made in this regard determined their “right of access to jobs and further education” (Vithal & Volmink, 2005, p. 17).

In addition to this fundamental change in the structure of the subject, there was also changes to the mathematical content to be taught, however, this change in content was not as drastic as that reported in the previous two case studies. Under the subject of mathematics, new topics including statistics, probability and modelling were introduced. However, what many consider the most notable change in relation to content was the fundamental shift from a very prescribed curriculum in the pre-apartheid era to a curriculum where teachers have much more autonomy over the content being taught. According to de Waal (2004), C2005 also resulted in a significant shift in the pedagogical approaches advocated in South Africa.

Under the C2005 movement, the learner was placed at the centre of learning, and the teacher was seen as a facilitator. Furthermore, much more emphasis was placed on co-operative learning and the use of relevant and contextualised problems in the mathematics classroom (De Waal, 2004). This was a stark change from the previous curriculum, which promoted rote learning, teacher-led approaches and the accumulation of isolated facts and knowledge (Killen & Vandeyar, 2003; Pudi, 2006).

[2] Grade 1 in South Africa generally comprises of students aged approximately 6 years of age, while students are approximately 15 years old by the time they reach grade 9.

Finally, in addition to a change in structure, content and pedagogical approaches, changes in the assessment were also a core component of the revised curriculum. According to Killen & Vandeyar (2003), "The most obvious change in assessment was in its general focus – away from a fixed body of content that was to be remembered towards a set of outcomes that were to be demonstrated" (p. 125). Teachers were also encouraged to employ more continuous assessment in their teaching and incorporate assessment strategies that were cross-disciplinary in nature.

OBE and C2005 resulted in major changes to the teaching and learning of mathematics. This reform movement was initiated when the apartheid regime was abolished in 1994, and so there has been a substantial amount of time for the revised curriculum to be implemented and lessons to emerge. Firstly, many teachers resisted both OBE and C2005 in the early years. One of the primary reasons for this was that teachers felt ill-prepared to deliver this new curriculum (Killen & Vandeyar, 2003). Without adequate professional support, teachers were overwhelmed by the radical changes proposed and so while many were in favour of the ideologies underpinning the reform, many resisted the reform itself. This led to many problems during the implementation stage, with many teachers continuing to rely on direct instruction and overlooking the new pedagogical approaches (De Waal, 2004).

As such, it is critical that teacher professional development is seen as a core component of any curriculum reform movement. Reform efforts that are not accompanied by teacher training have little hope of succeeding as one of the key agents, namely teachers, will not have the skills, knowledge or confidence to deliver the revised curriculum in the manner envisaged.

Secondly, key stakeholders in the education system in South Africa struggled to differentiate between OBE and C2005 and this too led to some issues at the inception and implementation phase. According to the C2005 Review Report (2000), the distinction between C2005 and OBE was often blurred with many stakeholders, particularly teachers, struggling to recognise the difference between the two movements. De Waal (2004) determined that this was one of the factors that led to "misinterpretations concerning teachers understanding and practice of C2005" (p. 44). This finding shows that it is important to ensure that all reform movements are unambiguous, and there are clear distinctions between one reform effort and subsequent efforts. Only then will teachers, students and other key stakeholders fully appreciate the aims and objectives of the curriculum reform and truly understand what is expected of them.

The final lesson to be discussed here is one that has arisen in the Irish case study and relates to time. De Waal reported that the revised South African curriculum placed a lot of time pressures on teachers, and this was not considered by policymakers. He declared that the main issue relating to time, particularly among primary school teachers, was that with so much new content and expectations outlined in C2005, teachers were required to put a lot more time and effort into planning to incorporate all the new aspects of the curriculum and this affected the time available for them to teach core concepts in reading, writing, mathematics and science.

The South African case highlights the fact that curriculum developers need to be cognisant of the fact that additional time needs to be allocated to support teachers in the planning and preparation of units of learning and lesson plans during the initial stages of reform. In conclusion, while the reasons for driving this particular curriculum reform were quite different to the driving factors behind the previously discussed reform movements, there are still many lessons that can be learnt from this reform movement. Some lessons echo the lessons that emerged from the other case studies, while some are unique to South Africa and stem from the waves of reform that occurred post-apartheid.

Conclusion

We briefly reflect on the six research questions addressed in this chapter. Starting from the question "how and why did the reform movement come about?", we presented evidence that international comparisons and unsatisfactory achievements fostered reforms (in Ireland and Serbia). Political context and changes in ideology sometimes effectively influence reforms in mathematics education as it happened in South Africa. We found out that although the proclaimed aim of three reform movements discussed in the chapter differed, the resulting curriculums have had shared characteristics, like the adoption of an RME approach and the effort for gaining applicable knowledge.

As the three cases suggest, the agents or stakeholders involved (besides mathematicians) in the reform movement significantly influenced not only the direction of the reform but also its results. The involvement of teachers and the support from government agencies seemed to improve the reform implementation but, in some cases, it also undermined the achievement. The impact of the reform on mathematical content was more or less prominent, depending on the main reform agenda. Apparently, in the case of South Africa, it was not in the focus of the reform. In all cases, the support and professional development of in-service teachers were of critical importance. The impact of the reform was dominantly directed to mathematical teaching, but also to mathematical content and assessment. The ongoing assessment was recognised as an important indicator, but only long-term assessment actually provided valid evidence of the results of reform.

To conclude, many lessons have emerged across each of the three individual case studies presented in this chapter. However, the authors now summarise what they perceive as the five main lessons to emerge across all three reform movements.

Lesson 1: The Influence of International Assessments

The case studies presented in this chapter outline the process of curriculum reform in three different contexts. However, despite the different geographical locations, different political landscapes and different educational structures, the manner in which curriculum reform was initiated and implemented across the three countries draws many parallels. Firstly, one of the most striking similarities across the reform movement in each of the three countries was the influence of TIMSS and PISA in instigating curriculum reform efforts. Both in Ireland and in Serbia, students' performance in TIMSS and PISA led to employment groups and government agencies calling for change, while this was also cited as a factor in initiating curriculum reform in South Africa, albeit not the most influential factor. This suggests that TIMSS and PISA may be used to call for change to education systems and to justify reform efforts among the wider public. As such, if countries intend to continue to act upon the results of international comparison tests to instigate reform movements, it is critical that these tests offer a precise and accurate representation of the mathematical competencies of young people internationally.

In addition to the role of TIMSS and PISA in instigating curriculum reform, these international assessments also impacted upon the change to mathematical content reported in the three countries. Many of the test items and categories that appear in TIMSS and PISA test instruments are now reflected in the revised curricula. For example, the five strands introduced in the revised Irish curriculum closely align with the PISA mathematical content categories as depicted in Fig. 5.1.

In addition to this alignment, the authors firmly believe that the increased focus on financial literacy reported across all three curriculum reform movements is reflective of international assessments. In 2012, PISA first introduced its financial literacy assessment instrument. This move indicated that the OECD recognised the importance of this strand of mathematics in the Twenty-First Century and so too have curriculum developers across the three countries. Similarly, it has been recognised internationally that statistical literacy is a critical skill in the twenty-first century due to the increased presence of numbers and quantitative data in our everyday lives (Watson, 2006; Ben-Zvi & Garfield, 2008). Hence, it was not surprising to see that all three case studies reported that this was another topic on which more emphasis was placed as a result of the reform movement.

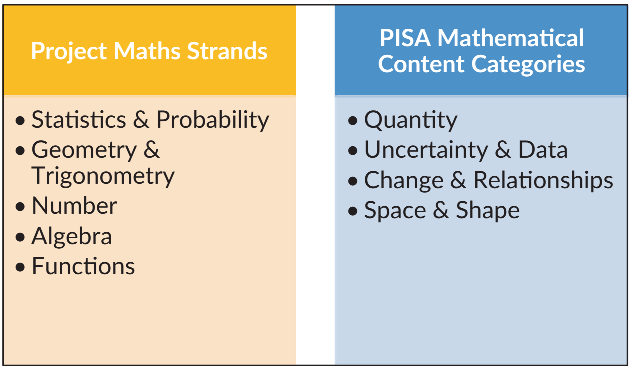

Fig. 5.1 Alignment between PISA categories and revised Irish curriculum

Lesson 2: The Influence of RME and Importance of Consultation

In addition to the important role of TIMSS and PISA, the RME mathematics movement also appeared to be influential in curriculum reform efforts in South Africa, Serbia and Ireland. This was reflected in the fact that all three case studies reported a shift from didactical teaching to more student-centred teaching, with a greater emphasis on developing conceptual understanding and problem-solving skills. Furthermore, the aims and objectives of curriculum reform in Serbia clearly aligned with the aims of RME while curriculum developers in Ireland clearly specified that RME had an influence on the changes made to the mathematics curriculum.

The RME movement was mainly led by mathematicians and groups of mathematics teachers but there was a much wider range of stakeholders involved in each of the local reform movements outlined here. While mathematicians still had some input into curriculum reform in the three countries, many more stakeholders including Government agencies, teacher unions, employer groups, industry partners and textbook publishers also played significant roles. Involvement of such a diverse mix of stakeholders was facilitated in two of the case studies through the inclusion of a consultation process. In South Africa, this consultation process allowed input from a variety of sources and many people, whose voices had not been listened to before, got the opportunity to advise on the future direction of mathematics education.

In Ireland, a similar situation was reported, and the authors have previously described how the voices and opinions of the Irish public were listened to when shaping the new curriculum. This meant that the local reform effort could accommodate and incorporate recommendations from different groups in society, thus developing a revised curriculum that was reflective of societal needs and acknowledged and valued the opinions of those who would be central in the successful implementation of the new curriculum. The authors propose that meaningful consultation of this nature should be a key feature of all future reform efforts.

Lesson 3: Time

Time manifested itself as a key lesson across all three case studies. The case of Serbia highlighted that time must be allowed to elapse before judgement is passed on the success of a reform movement. South African research pointed to the need to allow time for teachers to plan for the new content to be taught and for curriculum developers to be cognisant of the additional time pressures new curricula place on teachers (De Waal, 2004). In Ireland, the misalignment between the recommended time in syllabus documents and the actual time needed to achieve the objectives of the new curriculum was described by O'Meara and Prendergast (2017). They found that the revised curriculum required the allocation of more instructional time but very few Irish teachers reported an increase in the time available to them since the new curriculum was introduced. Hence, these case studies show that multiple manifestations of time need to be considered when engaging with curriculum reform if the goals of the reform movement are to be realised.

Lesson 4: Teacher Professional Development

In any reform movement teachers are key agents in the effective delivery of the new curriculum, and so the potential success of the reform is heavily dependent on them. The South African case study highlighted how teacher resistance can lead to reform efforts being compromised. Such strong resistance to revised curricula among teachers did not feature as prominently in either of the other two case studies and one potential reason for this was the provision of teacher training. Killen and Vandeyar (2003) argued that South African teachers opposed many of the changes because they felt ill-prepared to teach the new curriculum.

Despite the new curriculum advocating for significant changes to the way mathematics was taught and the content to be taught, South African teachers did not receive adequate guidance in this regard. This contrasted with the situation in both Ireland and Serbia where teacher training was seen as an integral part of the reform movement. Hence, a lesson emanating from the case studies is the importance of teacher training during the implementation of a new curriculum. Only when such support is made available will teachers have the opportunity to develop the knowledge, skills and confidence necessary to deliver the new curriculum.

Lesson 5: The Need for Continuous Research

The Irish case study highlighted the importance of research in relation to curriculum reform. Much research was conducted in Ireland before the reform movement was initiated (e.g. Conway & Sloane, 2005; NCCA, 2005). However, the research did not stop there. Since the inception of the revised curriculum many researchers have continued to investigate key curriculum issues in an attempt to determine how successful the reform movement was and how certain aspects could be modified and improved (PMISG, 2010; Cosgrove et al. 2012; Jeffes et al., 2013; O'Meara & Prendergast 2017; O'Meara et al., 2020a, b; Prendergast et al., 2019).

Much of this research focused on teachers' perspectives and was another way to facilitate the teacher's voice being heard throughout the reform movement. In addition to this, such research allows evidence to be gathered that enables all stakeholders to evaluate the impact of the curriculum and to determine the new challenges facing mathematics education. As the Serbian curriculum reform efforts continue in earnest this is one lesson that they and others could take from the Irish case study. Research is key to evaluating the reform efforts and needs to take place on a continuous basis so that mathematics curricula evolve regularly in response to the needs of society.

Despite the valuable lessons to emerge from these case studies, the authors firmly believe that many more lessons could be learned if more reform efforts were critically analysed and evaluated. However, such an endeavour presents many challenges. For example, what methodological strategy is best suited to such an undertaking? How can reliable and valid data be collected? How can we obtain accurate insights into the thought processes behind the instigation of curriculum reform? How do we, as researchers, distance ourselves from the discourse to accurately analyse the impact of curriculum reform? However, if researchers can overcome these challenges there is potential for an array of research projects to be conducted to analyse mathematics curriculum reform efforts across many countries. Such projects would yield rich data and insights that would undoubtedly inform future reform movements.

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Chapter 6
The Role of Values and Culture in Past Mathematics Curriculum Reforms

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Mathematics is a subject taught in every country. It is interesting to note that the word mathematics is plural in many western languages (among the ones that distinguish singular and plural names). This is the case, for instance, in French, German, Spanish, Catalan, etc. In Italian, the plural *Matematiche* was common at the beginning of the twentieth century, but now the singular *Matematica* is more popular. The plural name may hint to general facts. First, the fact that mathematics is conceived as a collection of different fields, distinguishing, for instance, pure mathematics and applied mathematics. Second, that mathematics is conceived in different ways in different cultures. Some examples of the second case are discussed in Bartolini Bussi and Sun (2018). The influence of values and cultures on the definition of curricula and curriculum reforms will be reconsidered in the cases reported in this chapter.

Although mathematics is usually conceived as a universal scientific subject, this is not true in general. Western mathematics is derived from the Greek approach to knowledge, and the structure of Indo-European languages. An astonishing example, mentioned in Bartolini Bussi and Sun (2018), is the fact that, in Maori language, whole numbers are verbs (actions) and not names or adjectives (Barton, 2008). The different ways of conceiving mathematics as a scientific subject have a strong

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influence on the way of teaching and learning mathematics. The national standards are not the same all over the world but point to different ways of considering the foundations of mathematics. We can illustrate this fact by referring to some exemplary cases.

Whole numbers are usually introduced as a multifaceted concept addressing cardinal, ordinal and measure aspects (Ma & Kessel, 2018; Bass, 2018). However, in Australia the Standards focus on a pattern-based approach (English & Mulligan, 2013). As defined in our studies, mathematical pattern involves any predictable regularity involving number, space, or measure. Examples include friezes, number sequences, measurement, and geometrical figures. By structure, we mean the way in which the various elements are organised and related. Thus, a frieze might be constructed by iterating a single 'unit of repeat'; the structure of a number sequence may be expressed in an algebraic formula; and the structure of a geometrical figure is shown by its various properties What we call structural thinking is more than simply recognising elements or properties of a relationship; it involves having a deeper awareness of how those properties are used, explicated, or connected (p. 30).

This statement is mirrored in the Australian Curriculum: Mathematics of 2019, as from the Foundation stage, patterns and algebra are integral part of the curriculum. For instance, the Australian Curriculum reads for the foundation year:[1]

> patterns and algebra;
> sort and classify familiar objects and explain the basis for these classifications;
> copy, continue and create patterns with objects and drawings:

- observing natural patterns in the world around us;
- creating and describing patterns using materials, sounds, movements or drawings.

This focus is different from that of other countries, where patterns and algebra are introduced in higher grades. This means that in Australia, foundations of mathematics are reconceptualised around algebra and patterns rather than on other approaches that align with Piaget's ideas on whole numbers. Algebra is considered a part of the foundation of mathematics rather than a development of arithmetic.

Algebra also features in the early mathematics curriculum in the Soviet Union (after Davydov) and China. This has been discussed by Mellone and colleagues (2019) during their elaboration on the construct of *cultural transposition,* as a process activated by researchers, educators, and teachers who deconstruct those educational practices adopted in other cultural contexts in order to reconsider the issues of educational intentionality, which is the background of any educational practice (p. 201).

A first example of educational activity focuses on the 'problems with variation', which is considered one of the most significant mathematics education tools in Chinese primary schools (Bartolini Bussi et al., 2014). The second example relates to the visionary mathematics curriculum for pupils attending the first grade, which is proposed by Davydov (1982). In both cases an algebraic approach through

[1] https://www.australiancurriculum.edu.au/

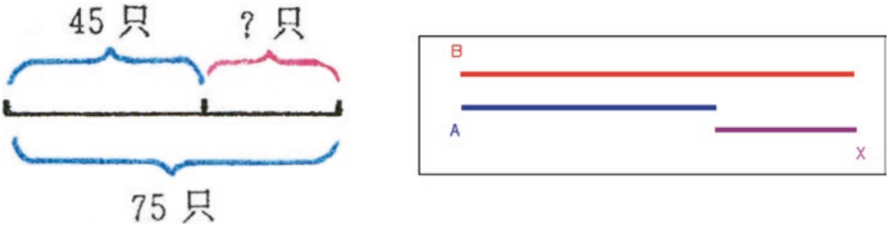

Fig. 6.1 Two common examples of representing subtraction in Chinese and Russian curricula

quantities is presented from the very beginning with extended references to pictorial equations. Figure 6.1 shows a representation from a Chinese textbook (45 + X = 75) in Bartolini Bussi & Sun (2018, p. 65) and from a standard Davydov representation of the relationship A + X = B.

The two representations are very similar, and one may wonder whether Davydov's curriculum and the Chinese curriculum have been developed independently of each other. The answer is no. In Shao, Fan, Huang, Ding and Li (2012), a careful historical reconstruction shows that a Russian educator, Ivan Andreevich Kairov, had a strong influence on the development of the Chinese curriculum in the second half of the twentieth century. Prior to the Revolution Russian schools were similar to schools in Germany. However, after revolution John Dewey's ideas were introduced and for a short period had a strong influence on building new school systems. This approach was abandoned in the mid-thirties of the nineteenth century.

The importance of Algebra in the Chinese approach to whole numbers is so strong that some Chinese scholars (e. g. Liping Ma, personal communication) prefer to address the problems of variation theory, algebraic in fact, as a part of arithmetic. This approach to algebra is fundamentally different from the pattern-based approach, mentioned in the Australian curriculum.

The short examples provided above show that there are very complex relationships between the mathematics curriculum and different cultures. Figure 6.2 summarises some different variables involved (Bartolini Bussi & Martignone, 2013).

In what follows, we briefly report on some examples that have been presented in the Conference, drawing on the papers submitted and published in the proceeding (Bartolini Bussi et al., 2018; Gooya & Gholamazad, 2018; Milinkovic, 2018). In all three cases, mathematics is considered very important in the three cultures, although the curricular choices may be different.

Italy: The Most Recent Global Curriculum Reform

The most recent global curriculum reform in mathematics in Italy took place at the beginning of the new century. In this section, a short summary of the process is given, as it was complex and intertwined with the political processes of the last twenty years (see Ciarrapico, 2002). In the year 2001, the UMI (*Unione Matematica*

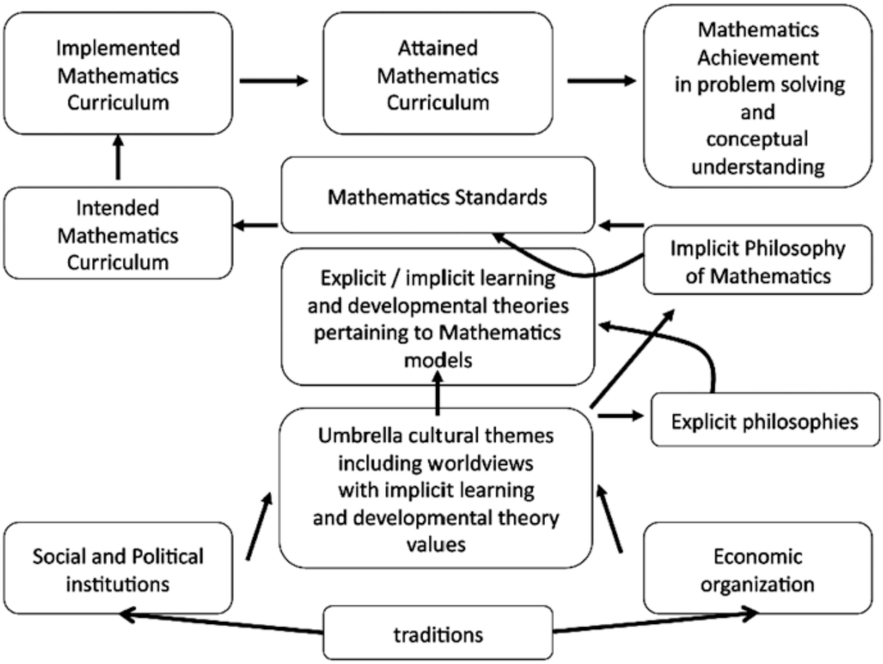

Fig. 6.2 A general scheme adapted from Xie and Carspecken (2008), as represented in Bartolini Bussi & Martignone (2013)

Italiana), in collaboration with the Italian Commission for Mathematical Instruction (CIIM), published a mathematics curriculum (Matematica, 2001) for primary and junior secondary school (grades 1–8) that had some influence on the curricula issued by the government in the following years. A few years later the UMI published a mathematics curriculum for grades 9–13 (Matematica, 2003, 2004). In 2001, the Ministry of Education Tullio De Mauro, an internationally acknowledged scholar of Italian language, introduced to Parliament a proposal of curriculum for pre-school, primary school and junior secondary school.

The document for mathematics was based on Matematica 2001. Three guidelines were taken into account in the elaboration: the *essentiality*, that is the identification of the fundamental epistemological aspects of mathematics (founding nuclei), with the intention of a quantitative reduction of the contents in favour of a better quality of learning, the *progressiveness* of the objectives along the entire primary and secondary school trajectory, since the mathematical goals are reached only in the long term, the *continuity* with the recent past, which takes into account successes and failures of past experiences.

The curriculum was organised by four thematic nuclei each with specific content (number, space and figures, relationships, data and forecasts), and three others, called process ones, which do not have their own content, because they are transversal to the first four: arguing and conjecturing, measuring, posing and solving problems. Mathematics, an essential component of the formation of the citizen, highlights two fundamental functions, the instrumental and the cultural: mathematics, therefore, is an essential tool for understanding reality and for everyday life. A formal mathematics, devoid of reference to reality, would in fact be a pure 'play of signs', but even a purely instrumental mathematics, without a global vision, would risk of being fragmentary and not very incisive. These two aspects, although with different nuances, have been recognised for many years as fundamental goals of mathematical teaching. De Mauro's decree was issued but not implemented because of new elections with a change in the political majority. However, for mathematics, the influence of the debate around the decree was strong and strengthened by the subsequent publication of *Matematica* 2003 and *Matematica* 2004.

A paradigmatic example of the influence of the UMI curricula can be seen in the *Indicazioni Nazionali National Guidelines (2012)* that mentioned the construct of mathematical laboratory, elaborated in the intended curriculum *Matematica* 2003:

> In mathematics, as in other scientific disciplines, the laboratory is a fundamental element, understood both as a physical place and as a moment in which the student is active, formulates his hypotheses and monitors the consequences, designs and experiments, discusses and argues his own choices, learn to collect data, negotiate and build meanings, leads to temporary conclusions and new openings the construction of personal and collective knowledge. In primary school it possible to use the game, which has a crucial role in communication, in the education to respect shared rules, in the development of strategies suitable for different contexts. (Indicazioni Nazionali, 2012, p. 60; *translated by the authors*)

The quotation above is taken from the Italian National Guidelines from pre-primary to grade 8. Laboratory activity is considered a general methodology not only for the scientific disciplines but for every subject matter as "it is the working method that best encourages research and planning, involves pupils in thinking, creating and evaluating shared and participated experiences with others, and can be activated both in the different spaces and occasions within the school and by enhancing the territory as resource for learning" (translated by the authors, Indicazioni Nazionali, 2012, p. 35).

A recent document (*Indicazioni Nazionali e Nuovi Scenari, National Guidelines and New Scenarios*) prepared by the Committee for the implementation of the National Guidelines has again focused on the importance of the laboratory and, in particular, of the Mathematical Laboratory:

> the laboratory can also be a gym to learn how to make informed choices, to assess the consequences and therefore to assume responsibility, which are central aspects for the education to an active and responsible citizenship. (Indicazioni Nazionali e nuovi scenari, 2018, p. 12; *translated by the authors*)

In the Indicazioni Nazionali (2012), the term 'laboratory' is a reference to the teaching of scientific disciplines: the spirit of the laboratory activity is maintained, with reference to ICT, to the history of mathematics, to mathematical modelling and to students' agency.

Bartolini Bussi et al. (2018) reported the features of the Mathematical Laboratory in the Italian intended curriculum, as stated in different documents, together with an example of implementation, that had some effects at a broader level.

> A mathematics laboratory is not intended as opposed to a classroom, but rather as a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects. A mathematics laboratory activity involves people (students and teachers), structures (classrooms, tools, organisation and management), ideas (projects, didactical planning and experiments). We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other, in a word: practicing. In the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together (without distinguishing between teacher and students). It is important to bear in mind that a tool is always the result of a cultural evolution, and that it has been made for specific aims, and insofar, that it embodies ideas. This has a great significance for the teaching practices, because the meaning cannot be only in the tool *per se*, nor can it be uniquely in the interaction of student and tool. It lies in the aims for which a tool is used, in the schemes of use of the tool itself. The construction of meaning, moreover, requires also to think individually of mathematical objects and activities. (Matematica, 2003, p. 26; *translated by the authors*)

The reference to the Renaissance workshop is clearly taken from the history of art, that is evident in many museums and exhibitions all over the country, hence is part of the *umbrella cultural themes* mentioned in the Fig. 6.2. A question arises: to what extent is the idea of a mathematical laboratory implemented across the country, at different school levels? A report of the effects of the *Indicazioni Nazionali* in a large sample of schools (grades 1–8) was prepared in December 2017 by the National Committee in charge of monitoring the experiments for all subjects. The conclusions are realistic and strongly support the need for investment in teacher development (Indicazioni nazionali e nuovi scenari, 2018).

This document is just the starting point of a necessary reflection on teacher development in Italian schools. Teacher development had not been compulsory but realised on a voluntary base in the Italian system of instruction for decades. Only recently, for the first time in all schools, a mandatory three-year programme (2016–2019) of teacher development has been issued. The issue of laboratory (including the mathematical laboratory) needs to be a major focus of teacher development to overcome the transmissive attitude and to foster students' agency in the near future.

The lack of an institutional teacher education program for secondary schools had strong influences also on the teachers' perception on the need for continuing education. The situation is very different from primary school where an institutional University program has been realised for more than twenty years.

Serbia: Changing the Perspective of Mathematics as a Subject

In the Serbian education system, the position of mathematics as a school subject was and continues to be extremely high. In grades Kindergarten to 10th, all students have mathematics classes with variations on the number of lessons per week in grades 9 and 10. In their final years (grades 11 and 12), math is present in all Gymnasiums, Technical Vocational Schools and Economical Vocational Schools. It has a special position in Mathematical High Schools. Finally, there are Mathematical Grammar Schools, enrolling about 1% of the student population. In the present reform, numerous high schools are forming special classes directed toward Informatics. Following worldwide trends (OECD, 2019), Informatics is taught from the first grade.

Reforms of curriculums in Serbia followed worldwide trends often with a few years delay. We present here two examples.

Example 1

In 1970, geometry teaching was predominantly based on Set Theory, starting with definitions of all geometrical objects and relations between them. Geometrical objects were defined as sets of points, while facts concerning them were defined as relations. Geometric objects (line, line segment, angle etc.) and facts concerning them (shapes, relations, measurement, etc.) were defined and analysed using language and apparatus of the theory of Sets.

In the 1985 state textbook (exclusively used at the time), one can read the following definition:

> Polygon is a set of points in a plane which is a union of the set of points of a polygonal line without self-intersecting points and a set of points within that line. The polygonal line is called the boundary of the polygon. […] The domain of the polygon is a set of polygon's points which do not belong to the boundary. Thus, the domain of a polygon is a difference between the set of points of the polygon and the set of its boundary. (Adnadjevic et al., 1985, pp. 30–31)

In the following reform – called mid reform – a new textbook proposed a somewhat less formalised introduction of the concept: "a polygon is a figure defined by a polygonal line, even if in the preceding lines in the textbook, the figure was introduced as a union of a closed line and a domain determined by it" (Micic & Jockovic, 2002, p. 35).

So far, the concept of the polygon was taught in Grade 5. As a result of the current reform, the approach to the concept of polygon is significantly relaxed without proposed formal definitions in the curriculum. Its introduction is postponed to Grade 7. It is expected that pupils explore and form the idea of the polygon through the practice of drawing a closed broken line without intersections and a plain domain determined by it.

Example 2

Serbia had the same historical pattern of development to the concept of function in the school curriculum. In 1985, first the concept of a binary relation was introduced in grade 5 as a subset of the Cartesian product. The function was defined as a special type of binary relation. In the next period, the general concept of a function was postponed until secondary school. In grade 7, the concept of dependency of two variables was introduced (direct and indirect proportionality) within realistic problem situations (e.g. change of temperature during the day). In the following grade, the linear function was introduced as a special type of functional dependency. In the current reform, in the curriculum for grades 5 to 8, the approach taken in the preceding curriculum was preserved, including a categorical demand that the general concept of function should not be mentioned. These are the facts that illustrate directions of changes in succeeding reforms of the math curriculum is Serbia.

We might summarise the situation as follows: referring to Fig. 6.2, we might say that, on the one hand, the curricular reforms depended on background ideas and, on the other hand, contributed to changing the perspective on mathematics as a subject in society.

Iran: Two Aspects of the Educational Systems Affected by Curriculum Reforms

Traditionally, mathematics has always had special place in the heart and mind of the Iranian people. It is assumed that the main essence of the mathematics that has been created and developed there was to solve practical problems that society asked for and gradually move towards more and more abstraction. Thus, mathematicians have served as one of the main pillars of Iranian culture and civilisation. This is evident in Iranian art, architecture, literature and poetry.

The formal education system in Iran was established in 1920 by adapting the French education system that was highly centralised. The centralisation comprised of all aspects of schooling including the mathematics curriculum. Further, since the 1960s, there has been one single national textbook for each school subject and with some tolerance, the textbooks were mainly considered as curriculum guides at the national level (Gooya, 1999). Despite the possible limitations that such centrality could pose for the system, it assured the accessibility of all students to educational resources.

Another characteristic of the education system in Iran is that, in general, schools are segregated from grade 1 to 12 – with some exceptions, including rural and nomad schools. Within this structure, mathematics plays different roles compared to a co-education system. The reason is that girls have never been compared with boys and thus in recent history, gender has not played a similar role in Iran as was the case in many other systems which kept girls away from mathematics.

The Driving Forces Behind Various Mathematics Curriculum Changes

Each education system has been, and will continue to be driven by various forces that are not necessarily rooted in education (Furinghetti et al., 2013). Iran is not an exception. Since the establishment of the formal education system in Iran, there has always been strong political and social forces behind almost all changes in education systems in general and mathematics curriculum changes in particular. As an example, the first mathematics curriculum of the first formal education system went through various contextual and content modifications, to accomplish the societal needs and political wills (Gooya, 2010). After World War II and the emergence of the "New Math era", many education systems, regardless of their different cultural and social backgrounds and needs, adopted the New Math curriculum.

The approach of the New Math was to move towards internationalised mathematics curriculum or as Bishop (1990) and Clemens and Elerton (1996) have pointed out, an implicit form of modern colonisation. Another feature of the New Math *was that its* target population was not *all* students, but mainly designed for elites (Clements & Ellerton, 1996). Therefore, the majority of students showed resistance to that. Nevertheless, the history of mathematics curriculum provided much convincing evidence to show that internationalised school mathematics curriculum is more at the theoretical level than real world of schools which means 'neutral" or 'value and culture free' mathematics curriculum cannot exist in the practical world of schooling (Chevallard, 2007; Bishop, 1997).

The social readiness and the new political establishment were two main driving forces for another major mathematics curriculum change. The expectation was to design a more meaningful curriculum by looking at the cultural, societal and national needs, and try to make a well-rounded integration between new findings in the mathematics education field at the global level, and having better understanding of the local opportunities to design a whole new curriculum.

After the revolution of 1979, the educational branches at secondary level and its mathematics curriculum was revised in 1992 by a study covering 10% of volunteer students, and the process went on until full implementation of the new curriculum in 1998. The theoretical perspective of the new curriculum was based on ideas taken from constructivism along with 'integrated approach' from different perspectives. The mathematics curriculum of the first-year senior secondary was designed and developed for all students. After the first year of secondary school, students had to choose their branch and strand, so the mathematics textbooks[2] of each strand were written based on that. The major change happened in the mathematics curriculum/textbooks in the Human Science strand. The focus of this curriculum was mathematics for those who had not much experience of enjoying and seeing the usefulness and applicability of mathematics in their own field. The main purpose was to provide students with opportunities to experience the beauty and usefulness of mathematics in a practical sense. The first year of Mathematics and Physics strand (grade 10) had two mathematics textbooks as Mathematics 2 and Geometry 2. The second year (grade 11) had two textbooks including pre-calculus, and algebra and probability. The integrated approach in the latter showed how deterministic and stochastic aspects of mathematics are related.

During mathematics curriculum change, one of the driving forces was the mathematics performance of the Iranian students in TIMSS- 2007[3] that was much lower

[2] In this piece, we intentionally use 'curriculum' and 'textbook' as two different words for one meaning. This is because since by that time there was no formal mathematics curriculum and national textbooks served this purpose.

[3] Iran has constantly participated in TIMSS since 1995 to 2015.

than that expected. Along with this, a new tendency was shaped at policy-making level to look at the factors contributing to the school mathematics curriculum of the "successful" countries as well. Due to the rush for full implementation within a short period of time, it was decided to do parallel evaluations of grade 1 (Kabiri, 2011), grade 2 and grade 6 (Kabiri, 2012), grade 3 (Kabiri, 2013) and grade 7 (Gholamazad, 2013) at the intended and implemented curriculum levels. This decision was made to partially compensate for the lack of the trial implementation of the newly written textbooks. The findings of these evaluations identified major challenges associated with these textbooks (Gholamazad, 2015). However, the findings were only used for cosmetic revisions of the first drafts of these textbooks.

Yet, another major education change started in 2011 and consequently, school mathematics curriculum underwent a radical change, including approach, content organisation, and context. This sudden decision for change caused the Iranian education system to face a number of unexpected challenges; those that might explicitly or implicitly affect school mathematics curriculum in other situations as well. The forces behind these challenges were not necessarily educational in nature, yet have had enough power to distract the direction of change.

Considering Values and Goals

Values and culture have played a strong role in mathematics curriculum changes since the establishment of the formal education system in Iran. The very first mathematics curriculum included specific skills that the traditional workforce and the new bureaucracy requested. For instance, the traditional Iranian accounting system called 'siagh'[4] arithmetic and teaching base 10 abacus was included in the mathematics (arithmetic) curriculum, as well as simple concepts of banking and book keeping (Gooya, 2010). Finally, the pedagogy was mainly teacher-centred and drill and practice. Overall, the major driving force of the first mathematics curriculum was political; believing that joining the international community and moving towards modernisation is not possible with a largely illiterate society.

Another salient feature is that despite the internationalised mathematics of New Math, the new reform in Iran was not limited to the imported approach of it and, instead, the content organisation and pedagogy adapted were influenced by the traditional and national style. Within the New Math curriculum, two parts of the Iranian mathematics curriculum, namely Euclidian geometry and trigonometry, remained in the same traditional manner, with separate national textbooks for each. This showed that despite the great hegemony that the New Math imposed on almost all mathematics curriculum around the globe, the influence of Iranian values and culture, was still significant.

[4] 'Siagh' is a special counting system for commercial purposes hat instead of numbers or symbols uses 'words' and until 20 to 30 years ago still was used by some professions such as traditional trading.

To summarise, in Iran there was no single driving force behind the mathematics curriculum changes during the last hundred years; however, political determination is visible. Any extremism for curriculum change and ignorance towards local values and cultural context may lead to challenges. The Iranian experience shows that main driving forces for mathematics curriculum reform included political determination, international mathematics curriculum movements, international studies (TIMSS), new theories of learning, and new research findings in the fields of mathematics and mathematics education.

The lesson we learned in Iran from various mathematics curriculum changes is to avoid ‘radicalism’ or ‘extremism’ and choose a moderate approach to include local culture and tradition, as well as being connected with the global scene and international research findings.

Concluding Remarks

The three cases presented in this chapter show the important place that mathematics had in formal education of grades 1 to 12 in various education systems. These cases have also depicted some major driving forces behind mathematics curriculum changes that are not necessarily mathematical in nature and yet, should be carefully considered. We refer to the position of mathematics in the structure of educational systems; the traditions; the changes in the social, political and economic institutions; and the implicit or explicit models of mathematics teaching and learning.

The scheme introduced in Fig. 6.2 hints at many of the above variables and may be used to study a curriculum reform, analysing the deep roots of the teaching and learning mathematics in a given culture, but also used to study the effect of a curriculum reform on society. Hence the scheme, by reverting the arrows, may suggest how a curriculum reform may impact on the general background for instance influencing the implicit/explicit philosophies of mathematics and the learning theories.

Looking at all the three examples, it was clear that differences were to be considered. The important message for the international community is that the global perspective and local production are different from ‘internationalisation’ of mathematics curriculum which considers school mathematics as culture and value free. Any sort of extremism in mathematics curriculum design brings about a heavy and sometimes very costly load on education systems of every country. Theories have an important role to play in developing and designing mathematics curriculum. However, it is necessary to modify global theories to fit the local situations by considering the cultural, societal, and values at local levels.

This chapter has reviewed mathematics curriculum reforms in three education systems. In these cases, values and cultures, either explicitly or implicitly, have been crucial. The analysis of these reforms allows us to speculate that values and culture have and will continue to play a salient role in mathematics curricula globally.

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Chapter 7
Curriculum Reforms and the Construction of the Knowledge to Be Taught

Marianna Bosch, Thu-Huong Vu-Nhu, and Dyana Wijayanti

Approaching curriculum from a research perspective requires defining units of analysis that include an extended part of the educational reality: not only what happens in a classroom, but also the decisions made – and the decision-making process – by many instances and institutions surrounding the school system. In this chapter, we focus on a specific part of past curriculum reforms, the one related to the content. We are also adopting a particular perspective: the anthropological theory of the didactic and its approach to curriculum reforms in terms of the didactic transposition processes.

The variety of meanings associated with the term 'curriculum' (Shimizu & Vithal, this volume) might help us remember that the curriculum first appears as an agreement between different social agents – including teachers – about what kind of activities schools have to organise for students. In our societies, this educational contract is usually established around a set of pieces of knowledge and activities structured in disciplines, subject areas, strands or domains, and notions and tasks, which have recently been enriched with new entities such as skills, competencies and values. By adopting a broad notion of knowledge, all these entities form what has been called the *knowledge to be taught* in the theory of the didactic transposition (Chevallard, 1985; Chevallard & Bosch, 2014). Analysing the knowledge to be

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taught includes considering the conditions under which such knowledge is elaborated – usually over long periods of time – the main agents taking part in it, and the criteria and assumptions underlying the decisions made.

This analysis is especially important because in the management of educational processes – and even in the studies about curriculum – the process of elaboration of the knowledge to be taught tends to be underestimated as if there was only one possible way to select, organise and name the specific content and activities proposed to be learnt at school. Moreover, one runs the risk to omit the significant amount of work needed in order for a given piece of knowledge to exist as a 'teachable' entity. Looking to the past is a productive way to break the 'illusion of transparency' that appears in this regard.

The core of the educational contract is the result of complex historical processes where different agents (members of the educational system and scholars from various fields of knowledge) elaborate bodies of knowledge and resources that concretise what has to be taught and learnt at school. The notion of didactic transposition was introduced to take into consideration this type of processes. The agents are part of what is called the *noosphere*, a layer surrounding the educational system, a kind of "membrane" of those who think (*noos*) about the educational system.

The elaboration of the knowledge to be taught, by (and within) the noosphere, is not a creation that starts from scratch. What students have to learn is not an invention of the school. It comes from knowledge – always in the broad sense that includes activities, skills and values – existing outside the school, within what is called *scholarly* institutions. Therefore, the educational contract proceeds by the identification of some pieces or bodies of *scholarly knowledge* students have to learn. The selected scholarly knowledge is then *transposed* to fulfil the requirements to be taught at school.

This process corresponds to the *external* didactic transposition (see Fig. 7.1) (Bosch & Winsløw, 2020). The term 'transposition' includes the assumption that the bodies of scholarly knowledge are not just disseminated into school but need to be transformed into something 'teachable', adapted to the school conditions. In particular, they have to be assigned a number of activities, organised in specific ways, sometimes with the focus put on the notions that structure them and the tools needed to carry them out, other times on the types of problematic situations they belong to. This process leads to the elaboration of the *knowledge to be taught*. The *internal* didactic transposition corresponds to the transformations applied to the knowledge to be taught inside the educational system until it becomes *knowledge actually taught* and also *learnt knowledge*.

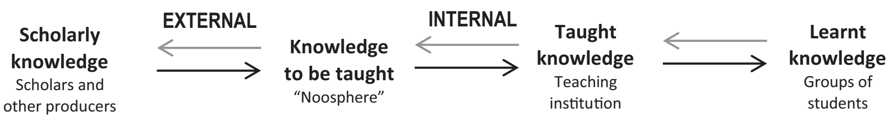

Fig. 7.1 *The* external *and* internal *steps of the didactic transposition process*

The didactic transposition analysis poses a methodological problem about the kind of evidence that has to be gathered and the scope of the object of study considered. The entity ‘knowledge to be taught’ is not easy to understand since it is not an official entity in the education system. Therefore, it has to be delimited and shaped by researchers, and it has to be understood as a model to study didactic phenomena, not as an empirical reality in itself.

In this chapter, we present three case studies about the effect of past curriculum reforms that can be analysed from the perspective of the didactic transposition. The first is a general analysis of the evolution of proportionality since the beginning of the 20th century, and the effects of the New Math reform in its current organisation in school knowledge. Even if the description is general and certainly depicts the situation of many countries, the empirical material used refers to the education systems in France, Spain and Indonesia. The second study refers to two curricular reforms which took place in Vietnam. This study delves deeper into the details of some small pieces of knowledge affected by them. Finally, the approach in terms of didactic transposition provides a possible reason for the emergence of a barrier identified in the implementation of the last curriculum reform in Ireland.

The Evolution of the Knowledge to Be Taught: The Case of Proportionality

Wijayanti and Bosch (2018) illustrate the general evolution of the knowledge to be taught in the case of proportionality, based on previous analysis of the Indonesian and Spanish education systems (Bosch, 1994; Wijayanti, 2015; Wijayanti & Winsløw, 2017). They show how what is taught today as “proportionality” and the position of this content in the global mathematical curriculum can be explained as the result of choices and elaborations based on different historical constructions of scholarly mathematical knowledge. This highlights the importance for research about curriculum to consider, not only the process of didactic transposition, but the different entities it brings into the open, and that provides the raw material of curriculum reforms.

The proposed analysis considers three main periods of time that correspond to three main types of curriculum organisations related to proportionality: ‘classical mathematics’, the New Math or Modern Math period and the post-modern period that began in the 1980s and is still in place today. These three periods correspond to three very different organisations of the mathematics to be taught about proportionality. However, while in the first two, the relationship between the scholarly knowledge and the knowledge to be taught might seem clearer, the post-modern situation appears to be more complex (Hersant, 2005). The analysis in terms of didactic transposition helps identify some incoherence in the knowledge to be taught. It also sheds light on the intricacies of the process of elaboration of this knowledge, that does not always seamlessly shift from the scholarly knowledge, but also relies on previously constructed curriculum organisations.

Classical Mathematics

In the scholarly knowledge in classical mathematics, ratio and proportions were a crucial tool in all domains. They appeared as the ‘basic language’ to work with relationships between arithmetical (including commercial), geometrical and physical entities. They were the main language of scientists before being replaced by functions and functional relationships. The ‘theory of ratios and proportions’ included the definition of different types of (arithmetical and geometrical) ratios, together with their main properties and transformations, which in some cases led to a real “algebra of proportions”.

When we look at the knowledge to be taught at that same period, we find a transposed version of the theory of ratios and proportions, usually under the same name. This organisation appears in a simplified version in Arithmetic books, with few theoretical elements and many practical cases to be solved using different versions of the rule of three (simple direct or inverse, multiple or compound). The practical cases had names: commission, brokerage, and insurance; discount; equation for payments; stocks; bankruptcy; partnership; exchange (Hotson, 1842). Theory of ratios and proportions also appears in Algebra books, which provide a more general presentation, using letters and developments of the theoretical elements (properties of the ratios and proportions).

In certain cases, one can find highly developed calculations, similar to the way proportions were used by scientists in the seventeenth and eighteenth centuries. These calculations with proportions remind us of today’s school work with equations (Wijayanti & Bosch, 2018, p. 177). The theory of ratios and proportions was a core part of classical mathematics related to the arithmetical and algebraic work. For instance, a typical textbook of elementary mathematics or arithmetic would include numbers and operations, fractions and operations, ratio and proportion and practical problems or measure and geometry. This kind of content organisation is found in many school and college textbooks of arithmetic and algebra in the late 19^{th} and the first half of the 20^{th} century, until the New Math reform.

Typically, the notion of ratio and proportion and the techniques of the rule of three (reduction to the unit and “cross product”) were introduced at the primary level in arithmetic courses, while the theory of ratios and proportions addressed more generally in the algebra domain was taught at a higher level. It also played an important role in geometry, the third domain of classical mathematics.

The Effects of the New Math Reform

In the mathematical curriculum that was elaborated during the New Math reform, the organisation around ratios and proportions disappeared. It was replaced by linear maps between numerical sets. The interpretation in terms of the didactic transposition, is that the New Math reform produced a complete transformation of the mathematical knowledge to be taught in order to make it more compatible with the scholarly knowledge (see Chap. 3). The idea of structure and structure building was the driving force of the organisation of contents; the construction of sets of numbers replaced the ill-defined notion of quantity, and the notion of map or function was introduced to represent relationships between variables. The "practical applications" that were at the core of the classical organisation disappeared.

Proportionality in Today's School Mathematical Organisations

Jeremy Kilpatrick describes the knowledge to be taught that was elaborated during the 'counter-reform' in the following terms:

> In no country did school mathematics return to where it had been before the new math movement began […] many of the ideas brought into school mathematics by the new math have remained. For example, textbooks still refer to sets of numbers and sets of points […] Terms such as *numbers*, *numeral, unknown, inverse, relation, function*, and *graph* are given reasonably precise definitions and used to clarify notions of quantity, space, and relationships. (Kilpatrick, 2012, p. 569; *italics in original*)

In the case of proportions, the rule of three and its theoretical environment reappeared, but they now have to coexist with functions and equations. The role played by quantities in classical mathematics are replaced by sets of numbers and functions of numerical variables. As Hersant (2005) noticed, since 1977 in France (and in many other countries), the return to the study of concrete situations did not mean the return of proportional quantities and their productivity as modelling and calculation tools, but rather on sequences of numerical measures that end up being only numbers.

The knowledge to be taught consists of hybrid organisations made up of pieces taken from different mathematical periods, mixing elements from different mathematical organisations that maintain redundancies and some incoherence in the kind of tools used. Some of them come from the classical organisation of ratios and proportions – the old knowledge to be taught – and others from the modern organisation of functions between numerical variables – the current scholarly knowledge.

In summary, the curriculum is formed with elements that come from the updating of both the scholarly knowledge and the old elements of the knowledge to be taught (see Table 7.1).

Table 7.1 Summary of the didactic transposition process related to proportionality

<table>
<tr><th>Period</th><th>Scholarly knowledge</th><th>Knowledge to be taught</th></tr>
<tr><td>Classical mathematics</td><td>Ratios and proportions are the main tools to describe and establish relationships between quantities</td><td>Ratios and proportions between quantities.
Practical problems (especially in commerce) solved using proportionality between quantities</td></tr>
<tr><td>New math reform</td><td rowspan="2">Functions are the main tool to describe relationships between variables. The construction of the set of real numbers avoids the use of the notion of quantity, which is relegated to its use in experimental sciences</td><td>Sets, numbers, maps, numerical variables, functions (no quantities)</td></tr>
<tr><td>Counter- reform (current situation)</td><td>Proportionality between numerical variables; coexistence of ratios and proportions with equations and linear functions
Quantities are not properly addressed</td></tr>
</table>

Wijayanti and Bosch (2018, p. 179)

Didactic Transposition Constraints

The analysis of curriculum reforms needs to scrutinise and question not only the general principles that determine what should be taught and learnt at school, but also consider the choices made with respect to the more specific ingredients that form the knowledge to be taught, together with the criteria adopted for these choices. In the teaching of relatively new subjects such as statistics or algorithmics, the knowledge to be taught is more regularly contested or, at least, put under question, with periodic renewal of the transposition work. When considering traditional domains, the knowledge to be taught appears as an entity that tends to become transparent, invisible because assumed as obvious, natural and unquestionable. The analysis in terms of didactic transposition aims at bringing it back into the debate.

Didactic Transposition Phenomena in Curriculum Reforms in Vietnam

Vu-Nhu (2018) offers two examples of the effect of curriculum reforms in Vietnam in the very concrete organisation of the knowledge to be taught concerning integrals and probabilities. She describes two curriculum changes in secondary school Vietnamese mathematics curricula and textbooks since 1975, which occurred in tandem with the political unification of the country.

In 1990, a unique and compulsory curriculum for twelve-year general education was established. However, the modifications of the knowledge to be taught were not radical, since it solely comprised of two sets of textbooks compiled by two different author groups, one used in the North and the second in the South. The *integral concept* was introduced in grade 12 across all of Vietnam, a novelty for the North educational system that initially only reached Grade 10.

In 2006 a reform implemented two new programs and two sets of textbooks, distinguishing two sections: the *basic* and the *advanced* one. According to the pathway chosen (natural sciences, social sciences, language, etc.), students have to use the set of basic or advanced textbooks. The highlight of the mathematic program at this period was that the concept of probability was introduced in the school mathematics curriculum in Grade 11 for the first time, together with other basic statistical content (in Grade 10) and complex numbers (in Grade 12), with the aims of including more applied mathematics knowledge, to link it to other subjects and to align with international mathematics curricula.

Based on two doctoral research works that present a comparative study of the knowledge to be taught in French and Vietnamese secondary schools (Vu-Nhu, 2009; Tran-Luong, 2006), Vu-Nhu presents the choices made in Vietnam programs about the type of activities that are proposed in the classrooms related to the notions of integral and probability in the different curriculum reforms. From the perspective of the didactic transposition, she describes how the decisions made to introduce the concept of integration or equiprobability did not always coincide with the concrete mathematical activities students and teachers carry out in the class.

Discrepancies were identified between the formal description of the knowledge to be taught – the formal or prescribed curriculum – and the knowledge actually taught as proposed by the textbooks – the implemented curriculum. In the case of probability, for instance, it appears that all the random experiments mentioned in the manuals are experiments with equiprobable events. Therefore, students do not check if the outcomes are equiprobable when they have to calculate the probability of an event.

What Vu-Nhu (2005) shows in her description of curriculum reforms in Vietnam are examples of the second step of the didactic transposition process, the *internal didactic transposition*, the one that happens between the inclusion of new knowledge objects into a given educational system through to its concrete specification in classroom activities. She also shows how some theoretical elements that are not explicitly included in the knowledge to be taught can affect the specific types of problems that are proposed by textbooks and the implicit assumptions underlying the solutions that are expected from the students.

Is It Time or Content? Lessons Learnt from an Irish Curriculum Reform

The previous section describes a study of the effects of curriculum reforms in the very concrete pieces of knowledge that are proposed to be taught to students, and those that are actually taught. We can see that the assumptions made by curriculum designers when they decide to include or remove a given notion are not always followed by a scrutinised analysis of the new knowledge organisations that are proposed to the students. Consequences appear at the lower levels of specification of these organisations, related to the concrete types of tasks students are required to solve, the tools that can be used for it and the theoretical assumptions that not only justify but also give meaning to the school mathematical activities.

The framework of the didactic transposition aims at providing research tools to bring into light the number of assumptions that are made regarding the knowledge to be taught in curriculum reforms. In particular, the fact that teaching and learning processes require – while they contribute to – the construction of a body of activities that our society relates to in a given discipline or knowledge domain and that are proposed to be carried out in the classroom. These activities can be more or less repetitive or open, short or long, mono- or multidisciplinary, related to daily life or to formal study, content or competence-based, but in any case, they must be elaborated, constructed, defined and delimited.

To what extent and in what detail remains an open question and a matter of negotiation with teachers. However, this body of activities cannot be considered as a given. The example of the teaching of proportionality during the 'counter-reform' after the New Maths shows that, when these activities are not properly elaborated, they end up being comprised of elements taken from different knowledge organisations of the past, giving rise to many inconsistencies. In the case of proportionality, the main one is the instable status of quantities in secondary school mathematics and the isolation of proportionality from other elementary functional models (García et al., 2006).

The interesting study presented by O'Meara, Fitzmaurice, Johnson, Prendergast and Freemyer (2018) show the effects of the last post-primary mathematics curriculum reform in Ireland that took place in 2010. Through a large empirical study among the teachers who participated and guided the implementation of the reform, the authors identify enablers and inhibitors of curriculum reform drawing on Memon's (1997) framework for curriculum change. One of the main findings of the study is the identification of *instruction time* as a key barrier to the successful implementation of the curriculum. The new curriculum encouraged more active learning and problem-solving activities, which were considered by teachers to be more time consuming than the old traditional teaching methodologies.

This assumption led the designers to reduce the 'level of mathematical content' on the old curriculum, for instance, by removing linear algebra and part of the calculus content. However, when analysing Irish teachers' views of curriculum factors which affect curriculum reform, the authors found that time was initially mentioned by teachers as a barrier, a factor that was not explicitly stated in Memon's framework. They, therefore, propose to include time as a core factor "of the *mismatch between official curriculum and actual curriculum* dimension" (O'Meara et al., 2018, p. 151).

A second important factor identified by the authors is phrased in terms of the 'breadth of the curriculum', related to the alignment between the primary and the post-primary mathematics, even if the reform already considered this dimension and proposed new strands for the new curriculum to reinforce the co-ordination between the two educational levels.

How can this phenomenon be related to the process of didactic transposition? In the first formulation of the framework, Chevallard (1985) introduces the notion of *didactic time* to describe the evolution of teaching and learning processes. According to Sensevy et al. (2002):

> In order to describe the development of the teacher and the student mathematical works, Chevallard (1991) proposed the concept of didactic time. That is the time of the teaching progression through the study of knowledge. In fact, the teacher's action is constrained by the necessity of presenting to his students a body of knowledge, part by part, shaped for teaching. So, in their action, teachers have to give a certain amount of time to pieces of knowledge, in order to cover its content. When teachers give up some item of knowledge, replacing it by a new one, they produce a unit of didactic time. This type of monitoring implies an efficient pacing. (pp. 423–424)

Based on the provided curriculum resources, teachers create the didactic time by introducing new elements of the knowledge to be taught and, especially, by marking the progress of the teaching and learning process using appropriate marks or references: "We have seen this", "we are now doing that", etc. In the case of curriculum reform, the change in the description and composition of the knowledge to be taught – which not only consists of theoretical elements but also includes activities of any kind – leads to the need of new references to identify the different elements of the teaching and learning process.

It is worth mentioning, in this regard, one of the first works of Guy Brousseau (1980) where he mentions that he became interested in mathematics education in 1975, in the period of the New Math reform, when he discovered the strong decrease of dyscalculia diagnoses at Saint-André Hospital in Bordeaux. It was not the students or the teachers who had changed, but the way difficulties were appreciated and, especially, the type of requirements that were introduced in the new curriculum.

In the study about the Irish reform efforts reported by O'Meara et al. (2018), it is not clear what do teachers refer to when they talk about time difficulties in the implementation of the new curriculum. On the one side, it is about the "clock-time" devoted to mathematics but, on the other side, it is about the new system of reference introduced by the new definition of the knowledge to be taught, that is still not enough developed nor easily referenced to by the teacher.

The notion of knowledge to be taught helps identify the content at stake in a curriculum reform as a constructed object, initially elaborated by curriculum designers and further detailed by other *noosphere* agents – especially curriculum writers, textbook authors and finally teachers themselves. It is the result of collective work, with several authors moved by different motives and who resort to material resources that are also different, some of these previously elaborated in similar collective processes.

When this process becomes transparent to the analyst, it runs the risk of being substantiated into an entity that appears as unique and unquestionable. Many of the criticisms that appear during educational reforms come from this fixed vision of the curriculum, which ends up using the old referents to reprove the new construction of the knowledge to be taught. The case of (Kirkland, 2012) seems to be an illustration of this unfortunately too common situation.

Conclusions

This chapter presents some short analysis based on the notion of didactic transposition processes and, especially, on the identification of the knowledge to be taught as an evolving construction crucially affected by school curriculum reforms. During the big reforms of the last century, reported in Chap. 4, many mathematicians carried out a considerable transposition work to provide schools with new knowledge organisations to be taught, considered more in tune with the needs of the time. The failure of some of these initiatives was mainly due to a certain illusion of transparency about the educational conditions and phenomena that rule teaching and learning processes.

It is certain that, when educational processes are defined in terms of competencies, active learning, and problem-based or inquiry-based activities, we need a broader perspective about their content and goals. However, it sometimes seems that the pendulum has moved to the opposite side as if the construction of the knowledge to be taught, the craftsmanship it requires from different agents of the *noosphere* – including mathematics educators and also mathematicians – was a natural process without major obstacles and difficulties.

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Chapter 8
Conclusion Final Considerations: Driving Forces and Barriers Affecting Curriculum Reforms

Marianna Bosch and Niamh O'Meara

Theme A presents two types of studies. The first type are cases of reforms undertaken in various countries or regions that are considered special for different reasons. Some were chosen because they produced a significant change in the curriculum, sometimes for a short period of time – like reforms related to the New Math movement – and sometimes as a long-standing process that is still in force today and even expanding internationally – as is the case with the Realistic Mathematics Education movement. In both cases, however, they left remarkable effects and can be located at the origin of the research communities in mathematics education, the local as well as the international ones. The second type of cases and choices of past reforms, illustrate how they could be affected by political movements in the countries, by international movements like those promoted by the OECD, by UNESCO or, more significantly, by ICMI itself, and also by cultural values or visions about the nature and role of mathematics in our societies.

The second type of study relies on the previous one, which constitutes, in a way, its empirical landscape. These studies focus on the driving forces and barriers that appear to have had some effect on the considered reforms. The most significant such force can be summarised in four points. One corresponds to the international organisations and assessments, such as those conducted by PISA and TIMSS. It would, however, be very interesting to see how they have affected local reforms in different ways, depending on the specific political, economic, cultural and social conditions of each country or region. Two other driving forces are the cultural values and the political movements that can shape entire reforms or a specific aspect of them.

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Finally, we have seen how some major reform movements can plant substantial ideas, but some countries have been reluctant to model their own curriculum reform efforts solely on these bigger reforms and only adopt some aspects of them. The motivation and criteria for these choices can be as varied as the choices themselves.

The chapters presented in theme A have identified at least two barriers that seem to have hindered past reforms movements. One is related to the levels or cycles that divide educational systems and assume different roles in the education of future citizens: primary school to prepare for life; secondary school to prepare for future studies; vocational school to prepare for specific professions. Many reforms have been motivated by some changes in this structure of educational systems and the need to improve transitions between levels. However, what is usually implemented as a general reform in a school system entails important consequences in what concerns the specific mathematical content and activities that are proposed to the students.

These consequences are not always foreseen. Kilpatrick (Chap. 2) mentions in this respect, the undefinition – characterised as "a bipolar nature" – of school mathematics, that still exists nowadays, despite the huge amount of research in mathematics education approaching this educational level. From a research perspective, educational reforms raise the most fundamental question: what mathematics (in the broad sense, including activities, notions, skills, competences, etc.) is to be taught and learnt by a given group of students? The elaboration of the concrete knowledge resources and activities – the 'knowledge to be taught' in terms of the didactic transposition theory – shows up consistently as another barrier that appears under different forms: for instance, as the lack of time for teachers or as an over-reliance on textbooks.

We just mentioned that research on curriculum reforms addresses the fundamental question of mathematics education: what should be taught and learnt concerning mathematics? In mathematics education, many studies tend to consider 'small' units of analysis, like the teaching of a given topic in a short period or the students' recurrent difficulties in its learning. Analysing a whole curriculum and its evolution over time, whether in the shorter or the longer term, requires the development of specific research tools.

In theme A, authors have used three main approaches: the theory of didactic transposition (Chevallard, 1985; Chevallard & Bosch, 2014), Memon's (1997) framework for curriculum change and a general scheme represented by Bartolini Bussi and Martignone (2013). Specific research tools are necessary to help researchers maintain a neutral perspective in the face of curriculum reforms they are analysing, and in which they are always partially involved – as citizens at the very least. It is also necessary to consider large units of analysis – including various countries over long periods – while also digging deep into the concrete knowledge activities that form teachers' and students' realities in classrooms.

From the outset, theme A sought to address research questions relating, on the one hand, to the motivating factors, values and beliefs that contributed to different mathematics curriculum reform movements worldwide and, on the other hand, to the enabling and inhibiting factors that affected such reform efforts. Whilst addressing these research questions, the authors shared insights and lessons learnt from different mathematics curricula reforms worldwide. We hope that this opening series of chapters has highlighted key issues relating to past curriculum reform movements, thereby ensuring that the lessons and challenges that emerged from these past reforms can serve to inform future movements.

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Part III
Theme B – Analysing School Mathematics Curriculum Reforms for Coherence and Relevance

Chapter 9
Introduction

Will Morony

The focus of theme B is "analysing school mathematics curriculum reforms for coherence and relevance" (ICMI Discussion Document, 2018, p. 578). The introduction (Chap. 1) to this book highlights the diverse views and approaches to defining the term 'curriculum'. In that discussion, the notions of 'intended, implemented and attained' as 'curriculum levels' led to the concept of 'curriculum components' such as "goals, content, pedagogy, materials and assessment" (p. 15). These constructs underpin the working definition of the term 'curriculum' that is used in this section's analysis of the coherence and relevance of school curriculum reforms. These elements, along with wider resources and constraints (such as teacher capacity and societal values), are together referred to as the 'curriculum system'. This section highlights that, whatever the curriculum reform's intent, the achieved curriculum is highly contextualised by the entire system, at a variety of levels.

As an important constituent of the curriculum system the professional development of teachers and the conditions and constraints under which they work are critical to the translation of the *intended* curriculum into the *enacted*[1] curriculum. Whilst the preparation and quality of teachers is not directly part of this section, there is some commentary on its importance to faithful enactment of reformed curricula in mathematics, and so, particularly, to the coherence achieved between various curriculum components. This is intended both to illuminate the issues in practice and to emphasise the importance of the quality of teaching.

In relation to the *attained* curriculum, this ICMI study has elicited some quantitative and qualitative data about student attainment, both as a driver for mathematics

[1] The term 'enacted' is preferred to 'implemented'. The latter term suggests that it is faithful to some prior model, such as intended curriculum, whereas there are a number of valid enactments of an intended curriculum.

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curriculum reform and as a measure of success – or otherwise – of reforms. Where appropriate, these data are referenced in the analysis of particular reforms.

Definitions for Coherence and Relevance

At the most general level, we view *coherence* as 'internal' to the curriculum (including materials and technologies designed to support its implementation), with *relevance* seen as 'external' to it, as the interaction between the curriculum and needs and aspirations (of students/young people, the workplace, universities, society, etc.). Both terms require clarification in order to be meaningful – there needs to be some specificity in relation to 'coherence' of what with what, from whose perspective and for what purpose, and, similarly 'relevance' of what to what, or for whom and for what purpose.

A curriculum includes a complex system of components proposed by different agents at different moments of time, under different conditions, and for different purposes. This is why coherence is not always ensured. 'Coherence' might include the vertical and horizontal alignment[2] within the intended mathematics curriculum, as well as its relationship with the parent discipline, with teacher education, with assessment, with the rest of the curriculum. This might (or might not) be a purely mathematical coherence, including an alignment within and across content and processes. It could relate the coherence (or otherwise) of things like priorities, weightings, values, implicit messages across the curriculum system as a whole. We need at least to be clear which of these we are addressing.

There is another practical challenge that confronts identification and analysis of coherence in the context of mathematics curriculum reforms. Instances of lack of coherence between levels and components of mathematics curriculum can be quite apparent and therefore easy to recognise. Further, addressing such incoherence attracts attention as a potential focus for future reforms – we can potentially 'do something about it'. Coherence within the curriculum is, on the other hand, much more unremarkable to us because it is expected as the implicit intended state.

Relevance begs questions of 'relevant to whom, and at what stage? Relevant for what purpose(s)?' These are questions of values or beliefs, and dependent also on context: we need to take care to avoid imposing assumptions or values. We also need to identify and expose underlying assumptions and values adopted. The lexical term 'relevant' is legitimately used in two different ways in the context of mathematics curriculum. The first draws on the mathematical term 'relationship' and has a sense of direct connection that can seem to suggest that a change in one variable causes a change in other(s). The other usage of 'relevant' conveys the sense of 'useful to' and 'fits with'. This is particularly the case for materials designed to support

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[2] 'Vertical alignment' is alignment over time as a student progresses in their experience of the mathematics curriculum. 'Horizontal alignment' is alignment of the different areas of the curriculum at a specific time.

student learning. There are many physical ways of modelling fractions. Some are more 'relevant' (i.e. more useful) than others when planning for the students to develop particular concepts about fractions, or for some particular use or application. This relevance is not 'causal', however, as there are likely to be other models and approaches that are also effective (i.e. relevant) in illuminating the concept(s).

Components of Mathematics Curricula

As indicated in the introduction to this book, "the term 'curriculum' is used with many different meanings and scholars have noted that it seems almost impossible to give a universally acceptable definition of 'curriculum'" (p. xx). However, discussing 'coherence' and 'relevance' in curricula in this section requires a certain consistency of language in describing components of mathematics curricula.

Building on the work of Kilpatrick (1994) and Niss (2016, 2018) proposed "to define a (mathematics) curriculum with respect to a given educational setting as *a vector with six components* [see below]" (p. 70; *italics in original*), which are very commonly evident in mathematics curricula. According to Niss, the components of a curriculum are:

- *goals* (the [declared] overarching purposes, desirable learning outcomes, and specific aims and objectives of the teaching and learning taking place under the auspices of this curriculum);
- *content* (the [names of the] topic areas, concepts, theories, results, methods, techniques, and procedures dealt with in teaching and learning);
- *materials* (the instructional materials and resources, including textbooks, artefacts, manipulatives, and IT systems employed in teaching and learning);
- *forms of teaching* (the tasks, activities and modes of operation of the teacher in this curriculum);
- *student activities* (the activities of, and the tasks and assignments for, the students taught according to this curriculum);
- *assessment* (the goals, modes, formats and instruments adopted for formative and summative assessment, respectively, in this curriculum).

After which he added, "Specifying a curriculum in a given educational setting then amounts to specifying each of these six components. Furthermore, *implementing* a given curriculum amounts to specifying it, as well as to *carrying it out*, i.e. putting all the six components into practice" (p. 70; *italics in original*).

Niss argues that the enactment of the curriculum requires all six components to be in place and evident. Curriculum authorities tend to retain control of the curriculum's goals and content, and often any summative and/or high-stakes assessment. These authorities may, or may not, devolve some or total control of the other components (materials, forms of teaching and student activities), as well as the formative components of assessment, to external agents (textbook writers, assessment developers) and educators at the local level (schools, consultants, teachers). In any of these cases, there are matters of coherence with other curricular components that can and do emerge; even when there is top-down control of a component such as materials through a mandated textbook, there can be some level of choice. Teachers, researchers and developers can exploit this choice in ways that enhance coherence in the enacted curriculum (see, for example, Miyazaki et al., 2018). Alternatively, the choices made can be detrimental to this coherence.

This framework allows for identification and analysis of the coherence between any one of the components of a particular curriculum and, potentially, the five others, to the extent to which they are present or not in the formal documentation. It is the framework used throughout this theme B.

Theme B Analyses

There are two major themes that overlay the analyses in the main chapters (Chaps. 10, 11, 12 and 13) that follow.

Curriculum to Meet Needs (of Students, the Workplace, Higher Education, Society etc.)

The ICMI Study 24 discussion document identified a set of key questions in this territory:

> How are mathematics content and pedagogical approaches in reforms determined for different groups of students (for e.g. in different curriculum levels or tracks) and by whom? How do curriculum reforms establish new structures in content, stakeholders (e.g. students and teachers), and school organisations; and what are their effects? (p. 580)

Perhaps surprisingly, much of the content of mathematics curricula is very similar across the globe, a situation perhaps reflective of common perceived needs in terms of relevance to individual or societal good. The level of commonality evident currently has potentially been reinforced by the comparatively recently-emerging and influential international performance assessments such as TIMSS and PISA, addressed further in theme D of this volume. Because of the influence they have, developments in these assessments have potential to increase relevance – to society and to the individual – of curricula, globally.

For example, PISA 2021 enhances the assessed profile of digital analysis of elementary data sets, surely a key component of data literacy and so centrally relevant to societal needs in a twenty-first-century world. It will be surprising if many curriculum authorities do not adapt accordingly – though the coherence of what is achieved is another matter, and one key message of this section is the challenge in creating and sustaining curriculum coherence at scale. In this theme, we discuss some curriculum reforms that subvert established norms in their attempts to educate young people for appreciation of wider societal challenges (Giménez & Zabala, 2018), or for cross-curricular thinking (Lupiáñez & Ruiz-Hidalgo, 2018), yet fundamental challenges to dominant curriculum norms are few, with Tarp (2018) representing a rare such attempt.

'Meeting needs' is very often a driver and informant of mathematics curriculum reforms. It is one lens on the coherence and relevance of those reforms. Some of the issues include:

- mathematics in compulsory education compared with mathematics when it is no longer compulsory;
- mathematics in different student pathways (science, literature, social sciences etc.), as well as in vocational and general education pathways;
- identification of the goals of specific mathematics reforms (for the citizen, for the future worker, for other disciplines, etc.);
- taking account of and responding to diversity in the classroom, and specific contexts in the classroom including:
  - cognitive diversity (need for diversity of teaching approaches);
  - cultural and social diversity (how to give meaning to mathematics for everybody and drawing on social and cultural aspects);
  - achievement (involving low achievers and extending the more able);
  - the kind of mathematics that gives meaning to the students' world given the geographical and social context of the classroom;
  - intended or possible student pathways for progression within and beyond schooling.

Each of the chapters in this theme address a range of these and other aspects of 'meeting needs' in their analyses of particular reforms.

'Top-Down' as Opposed to 'Bottom-Up' Curriculum Development and Reforms

These can be seen as oppositional, with OECD's Project 2030 and national curriculum reforms in many countries at one end of the spectrum ('top-down'), and reforms where teachers, students and the community play leading roles ('bottom-up') at the other. The reforms discussed in the chapters that follow are drawn from across this spectrum.

However, the level of specification and requirements in the curriculum levels and components of top-down approaches can allow – and even require – schools, teachers, students and communities to make choices and decisions that shape the enacted curriculum. This level of autonomy, where it exists, can lead to opportunities and risks in relation to both the coherence and relevance of mathematics curriculum reforms. These matters are analysed and reported on in what follows.

Theme B Chapters

This section is presented in six components, with the two cross-cutting themes integrated into the analyses and discussion:

Chapter 9 – Introduction
Chapter 10 – Analysis of a range of contemporary mathematics curriculum reforms
Chapter 11 – Reforms that focus on linking mathematics with other disciplines
Chapter 12 – Materials and technologies to support curriculum reforms
Chapter 13 – Theories and methodologies for studying mathematics curriculum reforms
Chapter 14 – Conclusion, achieving coherence and relevance in mathematics curriculum reforms: some guiding principles

The interpretation that coherence is largely ‘internal’ to the curriculum while relevance is ‘external’ as outlined above suggests that the balance of attention to coherence and relevance is different for each of the chapters. The main focus, both for Chap. 10 (general reforms) and for Chap. 11 (interdisciplinary, cross-curricular and STEM-inspired reforms) is largely the coherence and relevance of the *intended* curriculum – the documentation that specifies values and goals, and what is to be taught. Chapter 12 considers both the coherence and relevance of the materials and technologies used to translate the intended curriculum into use in schools and classrooms – the *enacted* curriculum. Chapter 13 analyses theories and methodologies used to study curriculum reforms in ways that identify issues of coherence and relevance that are evident in those reforms. Hence it provides insights into both coherence and relevance through the lenses of theory and methodology.

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Chapter 10
Coherence in a Range of Mathematics Curriculum Reforms

Will Morony

This chapter addresses the series of key questions posed in the ICMI Study 24 discussion document for theme B:

> What is the extent of coherence within and among different aspects of reformed curricula such as values, goals, content, pedagogy, assessment, and resources? How are curriculum ideas organised and sequenced for internal coherence in a curriculum reform? What are the effects of a lack of coherence? For example, regarding relations between high-stakes examinations and curriculum reforms? (2018, p. 580)

The main focus in this chapter is the documentation – written and otherwise – of the curriculum as it sits within the overall 'curriculum system' as outlined in the Introduction (Chap. 9) to this theme. Where possible and appropriate, there is reference to studies and other observations of the curriculum 'in action' as a means of assessing the extent of coherence, how it is achieved (or not) in practice, and its impact and effects.

In line with the definitions adopted for **coherence** and **relevance** in the Introduction to theme B (see Chap. 9), there will be an exclusive emphasis on **coherence** in relation to mathematics curriculum reforms. The response to the key questions posed in ICMI Study 24 – and other issues that emerged – draws on the analysis of a selection of mathematics curriculum reforms. These range in scale from reforms of mandated national curricula to reform initiatives at smaller scale related, for example, to particular aspects of the curriculum such as some particular mathematical content, mathematical process or pedagogical choice(s). The coherence between mathematics curriculum reforms and mathematics itself as the 'parent discipline' is also investigated, as are the interactions between the curriculum and the whole educational context for its implementation. A range of theoretical approaches and practical frameworks have been used in the analyses reported. These are discussed in detail in Chap. 13.

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Our analysis here uses four possible 'lenses' to consider different forms of coherence of mathematics curriculum reforms:

- coherence *between* components of the curriculum;
- coherence *within* components of the curriculum;
- coherence *with* mathematics as the 'parent discipline';
- coherence of the curriculum *with* the wider curriculum system.

The first two of these are related directly to the six components of the framework for describing any mathematics curriculum proposed by Niss (2016). The 'Niss framework' has been outlined in the Introduction (Chap. 9, p. 121) to theme B and is drawn on in various ways in other chapters. The third lens for coherence reflects the need for a mathematics curriculum to facilitate learning of mathematics that is coherent with the logic and structures of the discipline. Whilst the fourth lens is external to the curriculum itself, the level of coherence of curriculum reforms with the existing curriculum system can have a significant impact on the enacted curriculum.

Coherence Between Components of the Niss Framework

This section contains analysis of several national mathematics curriculum reforms. We identify and discuss the coherence between components of those curricula. In each case, the Niss 'goals', taken as the statements of purposes and aspirations for the curriculum, are clearly identified. Coherence of the other curricular components with the goals is needed to support the realisation of those purposes and aspirations – strong alignment of all the components is evident in the success of the example from Portugal, a reform that can justifiably be described as ambitious in the context. The example from Brazil demonstrates some clear coherence between the content and goals, but that this is not uniform, while in Vietnam, although the goals and assessment seem to be well aligned, the current materials, student activity and support for teaching remain locked in the past and out of step with the reform goals.

Portugal

Since 2004, senior secondary education in Portugal has been structured around seven 'tracks', each targeted at a specific student cohort or trajectory. The tracks all contain a combination of compulsory and optional courses. At the commencement of these reforms it was decided to develop and offer a mathematics course (the MACS course) as an option in the social sciences track, in recognition that students on this track benefit from mathematical experiences and learning suited to their particular needs. The goal of the MACS course is, "significant mathematical experiences that allow [students] to appreciate adequately the importance of the mathematical approaches in their future activities" (Carvalho e Silva, 2003, as quoted in

Carvalho e Silva, 2018, p. 310). Rather than focusing on specific concepts, MACS aims to "give students a new perspective on the real world with mathematics, and to change the students view of the importance that mathematical tools will have in their future life" (p. 310). Students engage with real situations, in order to "develop the skills to formulate and solve mathematically problems and develop the skill to communicate mathematical ideas (students should be able to write and read texts with mathematical content describing concrete situations)" (Carvalho e Silva et al., 2001, as quoted in Carvalho e Silva, 2018, p. 310). The approach is therefore interdisciplinary in nature, in order to be relevant for the particular cohort; Chap. 11 contains further discussion of this aspect of the MACS course.

Using the course "For all practical purposes" (COMAP, 2000) as the inspiration, the three topics for grade 10 are decision methods (election methods, apportionment, fair division); mathematical models (financial models, population models); statistics (regression, with graph models, probability models and inference) the areas covered in grade 11 (Carvalho e Silva, 2018, p. 310).

Implementation of MACS from 2004 faced a number of challenges:

- there was no tradition of such approaches to mathematics and, indeed, a previous initiative to implement a quantitative methods course that had similar intentions to those of MACS had failed during the 1990s;
- a lack of teacher knowledge in content areas such as graph theory and the mathematics of elections;
- few relevant teaching materials and no suitable textbooks;
- developing an examination as part of the secondary school diploma.

The last of these was a "very controversial matter [for] the MACS course" (Carvalho e Silva, 2018, p. 311). In Portugal, students need to sit national examinations in four of the subjects in their 'track' in order to achieve their high school certificate. Since MACS is an option in the Social Sciences track, students must have the option of taking the final examination, which makes up 30% of their final grade for the course. However, the MACS syllabus encourages teachers to use a range of means for assessment designed to support students' learning, but that are not generally seen as consistent with examinations. "Group work and individual work is recommended, [with assessment] assuming different forms: essays, personal notes, reports, presentations, debates" (p. 312).

The existence of a national examination was seen by the teachers' association (Associação de Professores de Matemática) as "not compatible with the assessment suggested in the official syllabus (APM, 2007, as reported in Carvalho e Silva, 2018, p. 311). The association complains that teachers lose their freedom and try to 'prepare' students for the examination and this somehow "does not allow the innovation aspects of this program to pass fully into practice". (p. 311) In other words, there was real concern about a lack of coherence between a number of the Niss components of the curriculum and the assessment component, given that at least part of the students' assessment was to be carried out through a national examination.

After some exploration of models and means for achieving coherence between the intent of the MACS course and the external assessment, the current MACS examination "consists of several rather mostly open but simple questions, where some careful interpretation or model construction/analysis is required" (p. 312). Carvalho e Silva (2018, pp. 312–313) provided example items from the 2017 examination. The first asks students to use a graphic calculator to compare exponential and logarithmic models; the second sees them apply a voting method to a particular situation.

Evidence that the approach adopted has been successful in improving coherence of this aspect of the assessment with the intent and form of the enacted MACS curriculum – at least in the eyes of the student cohort – can be found in the current numbers of students opting to take the national examination in MACS. Carvalho e Silva reports:

> As this course is accepted by very few Higher Education degrees, students that take this course can easily opt not to take the national examination. The number of students that take this examination is in fact very high. The total number of students taking exams is around 50, 000, and some 30, 000, take the main Mathematics A examination (p. 313)

In addition to achieving coherence between the goals and content and the assessment, the implementation of the MACS course has also achieved alignment of materials, forms of teaching and student activity through systematic and sustained effort over a number of years. These efforts are discussed later in this chapter as an example of achieving coherence between the curriculum and the curriculum system. The example of the MACS illustrates that an ambitious curriculum can achieve coherence between intentions and enactment.

Brazil

Established under a national legal framework in 2017 (for implementation from 2019), the National Curricular Common Base (BNCC) for elementary school[1] in Brazil is built around five thematic units that guide the formulation of skills to be developed in elementary school. Competence in the BNCC is defined as the "mobilization of knowledge (concepts and procedures), skills (practical, cognitive and social-emotional), attitudes and values to solve complex demands of everyday life, the full exercise of citizenship and the world of work" (Brasil, 2017, p. 8). As a result, "mathematical processes such as problem solving, research, project development and modeling can be cited as main forms of mathematical activity throughout this stage" (p. 8).

Dias and Cerqueira (2018) report on a qualitative and documentary analysis of the final version of Brazil's National Curricular Common Base for the final years of elementary school finding on the one hand that, "The analysis of the BNCC for the Final Annals of Basic Education revealed the presence of social, symbolic and cultural components linked to the objects of knowledge and their respective

[1] In Brazil, elementary school finishes in year 9. It is compulsory for all young people.

[mathematical] abilities” (p. 227). These findings are evidence of broad coherence between the goals and content of the BNCC.

As reported by Dias and Cerqueira (2018, pp. 223–224), he developed a framework for analysing curriculum that combines Bishop’s (1988) identification that the mathematical formation of young people consists of three components (symbolic, social and cultural) with the framework for evaluating mathematics curricula provided by Silva (2009). The latter extends the four criteria proposed by Doll (1993) (wealth, reflection, reality and responsibility) also to include four more criteria (recursion, relationship, rigour and resignification). Dias found that in Brazil’s BNCC for elementary school mathematics the presence of the three components of Bishop and the eight criteria from Silva were not consistent. His analysis showed that while these characteristics are clearly identifiable for the curriculum at years seven and nine, they are not apparent for the content for years 6 and 8. In other words, the coherence was found not to be consistent (pp. 226–227). This type of analysis has the potential to provide curriculum developers with insights into the extent and consistency of a curriculum’s coherence with goals that characterise learning mathematics as a socio-cultural pursuit.

Vietnam

In Vietnam, the university entrance examination is high-stakes and the teaching tradition is one of close adherence to material presented in approved textbooks. There have been several curriculum changes in Vietnam over the last two decades, including in 2019–2020, bringing an intention to move towards a greater valuing of conceptual mastery and engagement with mathematical processes. As is the case elsewhere, these intentions for the goals and content of the curriculum bring with them challenges for teachers in terms of valid enactment of reforms.

Over time, the university entrance examination has become the *de facto* high school graduation examination. Trung and Phat (2018) provide specific examples of assessment items that, despite being constrained to be multiple choice in format, clearly require conceptual understanding rather than recall and reproduction of procedures. Figure 10.1 provides two examples of miultiple-choice assessment items in which identifying the correct response requires substantial conceptual understanding and mathematical reasoning. Hence, the conceptually-oriented questions common in the examination (assessment) since 2017 are coherent with the goals and content of the curriculum.

On the other hand, the tradition of textbooks in Vietnam is highly procedural – it is the textbooks (i.e. the materials in the Niss framework) that are yet to be developed to be coherent with the goals. Chap. 12 contains further discussion of this matter which notes that the “cultural norm, and dominant approach, for mathematics teachers in Vietnam is a focus on procedure and memorization” (p. 181) and that this is strongly represented in the current textbooks.

The first illustration test (October 2016)

Question 4: Let the function y = f (x) be defined and continuous on R and have the following variation chart:

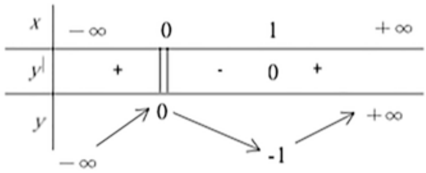

Figure 1: The variation chart for question 4

Which of the following statements is true:

A. The function has exactly one extreme.

B. The function has minimum value equal to 1.

C. The function has absolute maximum value equal to 0 and absolute minimum value equal to -1.

D. Function attains a maximum at x = 0 and attains a minimum at x = 1.

The third illustration test (May 2017)

Question 7: Let the function y = f (x) have the following variation chart:

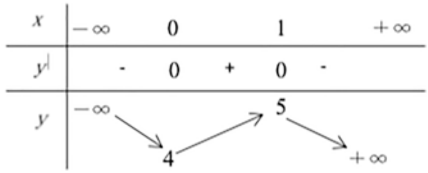

Figure 2: The variation chart for question 7

Which of the following statements is true:

A. $f_{max} = 5$.
B. $f_{min} = 0$.
C. Absolute minimum value equals 4.

D. Absolute maximum value equals 5.

Fig. 10.1 From Trung and Phat (2018, p. 327)

Coherence Within Components of the Niss Framework

The reforms discussed in the previous section are drawn from planning and change initiated at the national level; this section deals with particular aspects of reforms that, whilst they may be situated in a national effort, are considered in terms of a more narrowly defined focus for reform. A number of the examples of mathematics curriculum reform also demonstrate coherence – or lack of it – among the Niss components.

Recent curriculum reforms in both Costa Rica and Vietnam have had an emphasis on problem solving and mathematical modelling as core themes. In both cases, the intention is for student activities to incorporate mathematical modelling; that is, for students to develop, use and refine mathematical models as a means for solving problems and gaining insights that relate to the 'real world'. Japan provides a third example that strives for coherence in the teaching of proof across the curriculum.

Costa Rica

The reform of mathematics education in Costa Rica has involved a significant reorganisation and renewal of many aspects of the curriculum as an "explicit bid to develop Costa Rican society in its full breadth and complexity" through a focus on competence.

The functional focus of the mathematics curriculum advocates knowledge that focuses on development of one's own cognitive strategies, stressing the use of different forms of representation, argumentation abilities, and modelling techniques to pose and solve problems in context. In sum, its purpose is to develop schoolchildren's mathematical competence by improving their thinking and giving them certain autonomy. (Lupiáñez & Ruiz-Hidalgo, 2018, p. 263).

The reform also highlights mathematical processes that are applied across content areas:

- reasoning and argumentation;
- posing and solving problems;
- connecting;
- communicating;
- representing.

Of special importance for the reform are also "disciplinary core ideas" (MEP, 2012) that indicate priorities and that permeate all components of the curriculum including content and topics, advice, suggestions and instructions for teachers.

'Active contextualisation', one of the core ideas for the Costa Rican mathematics curriculum, recognises the importance of posing problems in authentic situations in order to mathematically model situations. The real contexts can have varying origins including the popular media, school and community. Active contextualisation in the Costa Rican mathematics curriculum sees these realistic contexts being explored through questions that are either interesting, authentic, or didactically relevant (after Maaß, 2006, as cited in Lupiáñez & Ruiz-Hidalgo, 2018, p. 265).

A range of initiatives were established to support the implementation of the emphasis on active contextualisation, particularly in relation to building the knowledge and capacities of teachers. Lupiáñez and Ruiz-Hidalgo (2018) report that the teachers involved in one professional development program "showed considerable advance in conceptual clarification of various central notions of the reform" (p. 266). The results were mixed, however:

> [In relation to the core idea of] "active contextualization," although the teachers recognize the role and importance of its application, they have considerable difficulty proposing contextualized tasks […] express[ing] regret and worry that they cannot find phenomena and fields of problems that enable them to propose relevant tasks and authentic questions. (p. 266)

In other words, despite focused professional development on the topic, at the time of reporting many teachers' approaches, and the student activities they motivate, lack coherence with active contextualisation as a major goal and focus of the curriculum reforms in Costa Rica.

Vietnam

From 2015, education in Vietnam has embarked on a major transformation designed to create and develop students' subject-specific core competences, as well as:

> Common competences for all subjects and educational activities that contribute to the formation and development: self-control and self-learning competence, communication and cooperation competence, problem solving and creativity competence. […] (Nguyen, 2018, p. 285)

The 2018 draft curriculum highlights the process of mathematical modelling as a focus. Students are required to develop and use mathematical models to solve problems and describe situations, to relate solutions to the real context being modelled, and to modify models as necessary.

Interdisciplinary application of mathematics is also an emphasis in the curriculum. Comparing the content on trigonometric functions in the mathematics and physics curricula for years 11 and 12, Nguyen (2011):

> find(s) a reasonable arrangement between the contents of the two disciplines. Specifically, circular motions are associated with the trigonometric circle is mentioned in grade 10. Next, the trigonometric function is studied in Mathematics in grade 11 and its applications in Physics like waves, sound, harmonic oscillation, [...] present in grade 12. (p. 289)

Hence the goal of developing mathematical competence in mathematical models is supported by coherence within and between the mathematics and physics curricula.

However, the approach to modelling in mathematics textbooks in Vietnam is not coherent with the emphases intended in the curriculum. Nguyen (2011, as cited in Nguyen, 2018) reports that there are only:

> traces of modeling in the application of mathematical knowledge to some of the problems that arise from reality. In high school mathematics textbooks, these exercises are very rare and are often placed in the readings section or at the beginning of some chapters. (p. 288)

In the case of trigonometric functions, not only do mathematics textbooks contain few real-life examples, but even when these are present, the students are given the model and merely asked to work with it to solve given problems. Whilst this can be seen as experience with applications of mathematics, the students are not engaging with mathematical modelling – certainly not in the spirit of the intended curriculum.

Nguyen (2011, as cited in Nguyen, 2018, p. 289) provided an example (Fig. 10.2).

This exercise requires students to undertake a range of substitutions and computations in a procedural manner. The astronomical context is rich and a different approach would provide students with opportunities to engage with, build and appreciate harmonic models of natural phenomena in ways that are much more coherent with the goal of the mathematics curriculum to promote and provide experience with the process of mathematical modelling and associated processes (reasoning, problem solving, communication, etc.).

In addition to this lack of coherence of the materials for teaching with the goals and content of the reformed curriculum, Vietnam faces issues in ensuring that assessment is coherent with the intentions of the curriculum in terms of evaluating

17. The number of hours of sunlight in the city A in north latitude 40 degrees during the day t is given by:

$$d(t) = 3\sin\left[\frac{\pi}{182}(t-80)\right] + 12, t \in Z \text{ and } 0 < t \leq 365.$$

a) The city A has 12 hours of sunlight on which day of the year?

b) On which day of the year the city A has at least hours of sunlight?

c) On which day of the year the city A has the most hours of sunlight?

Fig. 10.2 A 'modelling' exercise in the algebra and analysis textbook, grade 11

mathematical modelling. As outlined earlier in this chapter, Trung and Phat (2018) note a move to more conceptually-based assessments in the examination system in Vietnam that has significant coherence with the goals of the curriculum. The particular case of mathematical modelling presents additional challenges for assessment. Key among these is to develop assessment strategies for the examinations that test students' capacity to develop and use mathematical models effectively. It is also necessary to provide teachers with means to measure student performance in mathematical modelling if they are to assist their students to develop.

Japan

The 2017 national curriculum in Japan has at its core a general process for 'finding out' and solving problems in mathematics in which phenomena from both the 'real world' and the 'mathematical world' are dealt with through a three-part, common approach:

- problems are represented mathematically;
- problems are 'focused on' (i.e. problem solving);
- results are reflected on and interpreted in terms of the context.

Proof and proving play an important role in this process of 'doing' mathematics in the Japanese curriculum. However, just like their peers in many other countries, it is reported that Japanese students in grades 7–9 (and beyond) experience difficulties in the areas of proof and proving in mathematics. Miyazaki & Fujita (2015, as cited in Miyazaki et al., 2018, p. 275) observed that the course of study that is the statement of the Japanese curriculum only requires proof and proving, but does not propose the way to realise it in the curriculum. Moreover, the course of study requires that students learn various properties of plane and three dimensional figures mainly based on congruency and similarity, and also the meaning of proofs, and how to prove formally.

Although it encourages the gradual introduction of formal proofs until the end of grade 8, the curriculum does not offer a clear plan on how to gradually implement the learning processes of planning and constructing a proof. In other words, students are exposed to *proofs*, but not to the processes involved in *proving*, even though the curriculum emphasises active problem solving, in which *proving* plays a critical role. These authors' response to this lack of coherence has been to propose and develop frameworks for 'exploratory proving' for grades 7–9.

Miyazaki and Fujita "define explorative proving as having the following three components: producing propositions, producing proofs (planning and construction), and looking back (examining, improving, and advancing)" (2015, p. 1399). They subsequently add, "Careful mapping of the transition between these components as part of a teaching and learning sequence allows for systematic development of students' knowledge, understanding and capacity with proofs and proving" (p. 1399). This approach is an example of a way of explicitly teaching students one of the key processes of mathematics – being able to prove a mathematical result, and how to assess the logic of a proof they encounter, "that reflects the nature of mathematics, but also cultivating generic competencies of authentic explorative thinking" (p. 1402).

Miyazaki et al. (2018) provide examples of the framework and transitions of exploratory proving in Geometry, the domain which has traditionally included some treatment of proof, as well as in algebra, functions and data handling – domains in which attention to proof has largely been absent in the Japanese curriculum and, arguably, many other countries. Their analysis shows that the transitions between the components of exploratory proving are different in the different domains. The approach, once translated into actual learning materials by "combining local transitions of our frameworks with units in the Course of Study, the developed curriculum can provide teachers with a realizable plan on how to gradually implement the learning processes, and evaluate students' ability" (p. 275). The approach will enable explicit attention to proving to occur in domain-specific ways. As a result, it is anticipated that students will develop a richer and more robust capacity in proof and proving in mathematics; there will be greater coherence between the goals and the mathematics being taught and learnt through the forms of teaching and student activities that materials motivate, and much more:

> proving activities are flexible, dynamic and productive in nature, and various aspects of proving activities are interrelated and resonant with each other. We can see that proving activities 'breathe life' into mathematics teaching and learning and are intellectually stimulating in numerous ways, for example: producing inductively/deductively/analogically propositions, planning and constructing proofs for these produced propositions, and reflecting on and looking back at producing propositions, including planning and constructing proofs to overcome local and global counter examples and difficulties and then refining propositions and proofs. Mathematics as an activity is continuously developed by these processes which work dynamically together as 'intellectual gears', as if small paddle wheels (various aspects of proving) give power to propel a big paddle steamer (mathematics). (Miyazaki & Fujita, 2015, p. 1397)

Hence, their work can be seen as a means for creating greater coherence between goals and student activities through detailed attention to a key element of the goals (in this case, 'proving'). It is an example that illustrates the challenges in aligning these aspects of mathematics curricula and the significant effort required to achieve coherence.

Coherence with Mathematics as the 'Parent Discipline'

The Niss framework allows analysis of the coherence between and of the six components of a mathematics curriculum it identifies. Another aspect of coherence that is important in mathematics curricula is internal to the mathematics itself. This section considers recent and ongoing reforms to high school geometry curriculum on Israel in some detail as a means for exemplifying the complexity in achieving this form of coherence, followed by shorter comments on some other contemporary reform proposals.

Schmidt et al. (2005, as quoted in McCallum, 2018, p. 4) identify the importance of ordered, logical progression of the mathematics concepts and content:

> We define content standards […] to be coherent if they are articulated over time as a sequence of topics and performances consistent with the logical and, if appropriate, hierarchical nature of the disciplinary content from which the subject-matter derives. (2005, p. 528)

That is not to say that there is only one possible sequence or hierarchy of topics and associated student learning. There is no ordained reason, for example, to introduce fractions before decimals, or negative numbers before elementary algebra – it is possible to create a logical and internally coherent sequence whatever order is chosen for these topics.

As an example of the 'hierarchical nature' of the discipline, the equation of a straight line is learned at school both as a part of analytic geometry as an analytic representation of straight line as a geometric object, and as a graph of a linear function. As a graph of a linear function, it is learned earlier (typically around eighth grade), and generally appears spirally in higher grades, in relevant analytic contexts. The slope as tangent of an angle can only be defined when the trigonometric functions are defined, and at the first stage only for acute angles. Hence this extension of the concept of slope must be delayed until the trigonometric ratios for right angled triangles have been developed.

However, internal mathematical coherence depends not only coherence of 'topics' – what would be traditionally seen as the content – but also on coherence in relation to the 'performances' of doing mathematics with and through that content that are aspects of the student activity. These performances or processes include substantial mathematical capabilities such as reasoning, proving, communicating and formulating and solving problems.

Israel

One feature of reforms of the Israeli mathematics curriculum that commenced implementation in 2014–2015 was a reshaping of the levels of courses to match the abilities and needs of different student cohorts. At the high school level (years 10–12), the intermediate curriculum (called the ‘four point’ curriculum) – which is intended for the middle 50% or so of the students seeking to matriculate – provides students with a basis for study and career trajectories in the life sciences, economics and the social sciences more generally.

The geometry component of the new intermediate (or four point) mathematics curriculum demonstrates the issues and complexities inherent in achieving internal mathematical coherence. Barabash (2018) notes that:

> The guidelines […] include […] integration of analytic geometry, trigonometry, and synthetic geometry; linking mathematical rigour to development of intuition and visualisation-based valid reasoning; the Ministry’s policy (particularly the intended students’ characteristics), technological innovations, possibilities created by [dynamic geometry software], and experimental mathematical ideas that support systematic inductive reasoning. (p. 183)

Based on these guidelines the curriculum is elaborated according to three principles about integration of aspects of geometry, the form of problems that are to be posed to students (number of steps; use of numerical data only) and the use of digital platforms “to enhance inductive conjecturing followed by deductive testing (proof or refutation) of hypotheses thus formulated” (p. 183). The goals of the geometry curriculum emphasise different aspects and forms of reasoning, visualisation and representation, applications and critical evaluation of results. There is a heavy emphasis in the documentation of the curriculum on providing advice on teaching, including teaching sequences and sample problems, such that the list of topics is a relatively small component of the document itself.

The development and implementation of the new four-point geometry curriculum, and especially the extensive advice being given, provided opportunities in relation to ‘internal coherence’ within the geometry curriculum itself. This coherence is between its goals and principles; between the characteristics and possible academic or professional trajectories of the students for whom the curriculum is intended, and the content, the level, the complexity of tasks, etc. (student activity). In addition, there is the matter of coherence between this specific component of the whole high school curriculum and its other parts, i.e. analysis, statistics, algebra – the geometry curriculum’s ‘external coherence’.

The high school geometry curriculum in Israel connects synthetic geometry, analytic geometry and trigonometry in the course of teaching and learning. As noted above, this is explicitly stated as one of the leading features of the geometry part of the curriculum. These connections are pursued consistently through the three-year high school teaching plan. As examples of what can be termed ‘inner *geometric* interdisciplinarity’ Barabash identifies that:

> In addition to the list of topics, clarifications are added to enhance the spirit of the document, such as: "equation of a straight line by slope and a point on it, as an analytic implementation of the axiom of parallels"; "equation of a straight line by two points on it, as an analytic implementation of the axiom claiming the existence and uniqueness of a straight line passing through two given points"; (p. 184)

Thus, the connections between the different ways of thinking geometrically are made explicit and clear.

Continuing this theme of the straight line illustrates some 'inner-*mathematical* interdisciplinary connections' that are central to the coherence of the geometry curriculum with the mathematics curriculum. The straight line, primarily a synthetic-geometric object, appears to have various meanings; in particular, it is of paramount importance as the graph of a linear function. Its central property of constant slope is geometric in nature. The constant slope property is expressed and worked-with using the techniques of analytic geometry and geometric theorems (triangle similarity, angle formed by parallel lines and a secant, etc.). The relationship between the slopes of two perpendicular straight lines also has a geometric basis. Therefore, the straight line in a coordinate system embodies the mathematical inner interdisciplinarity in the sense that it cannot be uniquely attributed to any one mathematical field. This is similarly also the case for points, segments, and many other geometric objects.

Barabash analyses a learning sequence that draws on and ties together analytic geometry, trigonometry and synthetic geometry. The first exercise uses a triangle on the co-ordinate plane to focus on:

> critical testing a wrong supposition that inexperienced students might find correct. … [Subsequent exercises] use another triangle located correspondingly in the coordinate plane for a similarly guided exercise leading to the conclusion that $\tan(a - b) \neq \tan a - \tan b$. The recursive appearance of such questions guides a student toward the habit of doubting and testing "self-evident" beliefs. (pp. 185–186)

Hence, the high school geometry curriculum in Israel is an example of student activities that are coherent with the goals and content and which provide for achieving the goals and students learning the content. The coherence achieved reflects careful attention to the interplay between synthetic geometry, analytic geometry and trigonometry as the mathematical bases of the curriculum and the development of detailed student materials and guidance for teachers.

Japan

Similarly, the approach to proof and proving from Japan as outlined earlier in this chapter is also an example of building components of a curriculum that provide coherence with the mathematical processes of proof and proving as they apply in algebra, geometry, function and data handling (i.e. across the discipline). This is achieved by applying a common theoretical developmental framework that can lead to students having an appreciation of what it means to prove in mathematics in general, as well as the similarities and differences in proof and proving between the domains of the discipline.

An Alternative

The examples above – and indeed all the mathematics curriculum reforms outlined in this chapter – use an orthodox view of the discipline of mathematics, as envisaged by Schmidt and colleagues (above). As a group the examples highlight both the importance and the feasibility of developing curricula that are in some respects coherent with the parent discipline. This is seen to be important if students are to be engaged and inducted into what it means to work in the discipline.

On the other hand, Tarp (2018) proposes a radically different mathematics curriculum that takes as its base a different conception of the discipline that is not obviously coherent with mainstream views of the discipline. His "question guided re-enchantment curriculum in counting [that] could be named 'Mastering Many by counting, recounting and double-counting'" (p. 320) is a creative alternative. A case is made for the internal coherence of the curriculum (pp. 321–324), and this raises the question as to whether basing a mathematics curriculum on an alternative view of the discipline can be sufficiently rigorous, and what might be the challenges and advantages of doing so.

Tarp's proposal brings into focus two issues for mathematics curricula. The first is whether at any particular moment in history there is a tacit assumption that there is a single conception of the discipline, or even one "mainstream" conception of it. Such conceptions evolve with time and, therefore, depends on the educational system considered. Moreover, the ways curricula are formulated and enacted contribute tend to modify or sustain this conception and reject alternative conceptions.

The second issue is that a proposal for a curriculum that is radically different (such as Tarp's) tend to be critiqued in terms of rigour, benefits and the challenges, if the proposal were to be adopted. These factors should also be addressed as carefully in reviews of mathematics curricula that reflect mainstream views of the discipline.

Coherence of the Curriculum with the Curriculum System

The curriculum system is an articulation of many of the factors that are in place in a particular educational setting, including wider resources and constraints, such as policy settings of governments; responses to globalisation; leadership structures; all aspects of teachers' capacity; societal values and so on. There can be many factors that motivate a government or curriculum authority to embark on a reform of the mathematics curriculum. Whether reforms are a response to the society expressing a desire to better equip young people for citizenship; for more young people to undertake STEM careers; findings from national or international assessment programs that aspects of student performance need to be improved; or from other research, reforms both reflect and ultimately have impacts on the curriculum system.

Hence, by definition, mathematics curriculum reforms are designed and have the intention of, to some extent, disrupting the current state of the curriculum system, for example in areas such as the organisation of schooling, teacher beliefs, availability and appropriate use of learning technologies, textbooks, examinations and other assessment structures. This interdependence of curriculum reforms on the one hand, and the curriculum system is therefore an important fourth dimension of coherence of mathematics curriculum reforms. In this section we use examples to explore coherence between the curriculum and the 'curriculum system'. An initial question is what is the impetus for reform? In particular how research on student outcomes, as a measure of the success or otherwise of the existing curriculum and curriculum system, can inform the need for reform of a mathematics curriculum, and, potentially, the directions and emphases of those reforms.

This section begins with a substantial example of research informing reforms in China. We then turn our attention to the issues of coherence between a curriculum and the prevailing curriculum system during implementation. By far the most important issues for the effectiveness of uptake of the reforms outlined in this chapter are those that relate to teacher capacity; some other examples suggest means for building coherence between the curriculum and the curriculum system. This discussion and analysis begins with a reasonably detailed consideration of recent reforms in Mexico. This is followed by observations about the coherence of the curriculum with the curriculum system apparent in examples presented earlier in the chapter.

China

The Mathematics Basic Activity is a key element of the new mathematics curriculum in China designed to change "the Chinese conceptualization of mathematical basics" (Guo & Silver, 2018, p. 245). A new curriculum for grades 1–9 was introduced from 2011; for grades 10–12 the new curriculum commenced in 2017. The focus of this aspect of the new curriculum (i.e. the Mathematics Basic Activity) is experiential learning, rather than learning through instruction.

A key driver for this reform was recognition that whilst Chinese students excel at numerical and algebraic computation, spatial reasoning, and logical reasoning, they do less well with non-routine problems that involve "increased attention to mathematical processes associated with problem solving, invention and creativity" (p. 247) as is seen in the mathematics curricula of other countries such as USA and Japan.

Of the "two main forms of mathematics basic activity experience: practical experience in mathematics and thinking experience in mathematics" (p. 246), Guo & Silver report an investigation of the second of these about thinking. The "new aspect of the Chinese mathematics curriculum is that it is the student's way of mathematics thinking accumulated from experiencing and understanding the processes of mathematics inductive reasoning [initially] and mathematics deductive reasoning [later, in order to verify and prove results arrived at by inductive reasoning]" (p. 246).

They analyse findings from a study by Guo and Shi (2013) of the performance of students from seven middle schools in different parts of China on a set of six problems (with sub-problems) that were "drawn from a variety of sources […] intended to assess students' proficiency in generating a general rule or conclusion through a process that starts from a specific and simple problem" (p. 248). Only about 1% of the students produced responses in the "highest category that involved evidence of proficiency with mathematical reasoning" (p. 250), with the rest of the students either able only to imitate procedures with little or no mathematical reasoning (80%), or showing some capacity with mathematical reasoning.

Given that processes such as mathematical reasoning are highly valued as goals for twenty-first-century mathematics education, these results emphasise the need for the reform embodied in the Mathematics Basic Activity as it is now incorporated in the new Chinese curriculum for grades 1–9. However, much more needs to be done to create coherence between the intended and enacted curricula. The 2013 study shows that the, "instructional practices and curriculum emphases in these Chinese classrooms and schools have not been sufficient to support the majority of students to obtain the kinds of experience envisioned by the curricular reform" (p. 251).

Greater coherence between the curriculum, with its emphasis on the Mathematics Basic Activity and the curriculum system will be an important goal that will be facilitated by developing and making available materials (textbooks, etc.) that are coherent with the goals of the reformed curriculum.

Mexico

The curricular reform of 2011 introduced a competences approach to the Mexican educational system for the first time.

The Integral Reform for Basic Education is a public policy that promotes the comprehensive training of all preschool, primary and secondary students with the aim of favouring the development of life competencies and the achievement of a certain profile at the end of the basic education, [all of this is] based on expected learning and the establishment of Curricular, Teaching Performance and Management Standards. (SEP, 2011, p. 17).

This policy required substantive changes in the approach, goals, and content of the 2011 reformed curriculum:

> The Articulation of Basic Education is the beginning of a transformation that will generate a school focused on educational achievement by addressing specific learning needs for each of its students, so that they acquire the competencies that allow their personal development. (p. 18)

In particular, in contrast with the 1993 curriculum that had separate curricula for elementary and middle school levels and goals that were mostly functional, the 2011 curriculum expected elementary and middle school students to develop the following mathematical competencies:

- solve problems autonomously;
- communicate mathematical information;
- validate procedures and results;
- use techniques efficiently.

These were carried out through content arranged in "three thematic axes" (numerical sense and algebraic thinking; form, space and measure; information handling), rather than the six directly content-based organisers in the previous curriculum.

Importantly, in contrast with the 1993 curriculum which did not deal with assessment, the reforms of 2009–2011 address issues of assessment through guidance about the nature and focus of assessment. Importantly, there is an intention that, "students should be evaluated on their know-how and on the application of the mathematical contents" (SEP, 2011, as quoted in Hoyos et al. 2018, p. 255), as a means for generating coherence between the assessment and the goals and content of the curriculum.

Further questions about coherence between the goals and content of the recent reforms in Mexico and the materials and student activities, as presented in the curriculum documentation and official textbook, are considered in Chap. 12 (materials and technologies), with the overall finding that these components are little changed from the previous curriculum, thus leaving teachers ill-supported in the face of the shifts in the goals and emphases of the curriculum.

The magnitude of the changes has resulted in many challenges to the curriculum system, in particular the teachers' capacities and beliefs. The importance of teacher support – through opportunities for in-service professional development – to help them develop the capacity needed to enact curriculum reforms is well demonstrated in the findings about the effectiveness of successive reforms of the mathematics curriculum in Mexico. Whereas the initial reform of 1993 included coherent classroom and teacher materials in the form of an 'educative' teachers' guide, along with associated in-service professional development, the follow-up reform has been characterised by less coherent materials for teachers and students and no teacher development. In addition, it is not clear that there has been support for coherence of Assessment, beyond the requirements in the curriculum documentation.

As an indication of the potential for these inconsistencies and lack of support for teaching and learning in mathematics classrooms that is coherent with the intentions of the 2011 curriculum, PISA assessments of learner cohorts show that the percentage of Mexican students that were below level 2 (i.e. attaining the level 1 or zero) in PISA 2009 was 51%, with this figure rising to 57% in PISA 2015 for students who have been substantially taught under the 2011 curriculum, indicating that there may be issues for the attained curriculum, with a greater proportion of Mexican students in the poor levels of performance (Hoyos et al. 2018).

Portugal

Issues of teacher preparedness, curriculum materials and support for effective teaching in the implementation of the MACS course in Portugal that was designed to meet the needs of students who are taking a social sciences study and career trajectory – as outlined above (see p. xx) – were substantial. They have largely been addressed through "a carefully designed plan [that has] allowed today's situation where thousands of students opt for this course" (Carvalho e Silva, 2018, p. 314), that was put in place over a decade or more, beginning in 2001.

The approach has had several elements. To support the use of effective teaching strategies and associated student activities, Written teaching materials have been produced by the authors of the MACS program and others. The Ministry of Education edited and made available translations of relevant COMAP publications, and new textbooks have been produced. From 2001, a cadre of specialist MACS teachers were selected and prepared for a role in leading and supporting others through in-service professional development programs that helped them develop the knowledge and skills in teaching the MACS course. Several universities have included content relevant to MACS such as Election Theory, Apportionment and Graph Theory. in their courses for pre-service teachers of mathematics.

An important feature in these and other programs of support has been ongoing engagement with the teaching profession. The Teacher Association APM, in its 2007 report, as quoted in Carvalho e Silva, 2018, p. 315, said that, "APM participated actively with proposals, teacher preparation, discussions, preparation of materials, etc. [… and] the process […] has been exemplary". The authors of the MACS course had a permanent "contact with teachers in the field, asked for contributions from all the teachers, mathematicians and other specialists, integrated in a very satisfactory manner the several suggestions sent to them, and the authors also organized meetings to discuss the work being done in a very open way" (p. 315).

Whilst the extended time provided to generate coherence between the MACS curriculum and the Portuguese curriculum system was necessary, the success of the enterprise is the result of the multi-faceted, systematic and inclusive nature of the initiatives taken. Both elements – sufficient time and targeted initiatives – are necessary; neither is sufficient by itself.

England

Nor are such situations necessarily either static or convergent. Golding (2018), in an extensive suite of longitudinal curriculum enactment studies that focused on learners aged from 5 to 18, shows how recent aspirational curriculum reforms in England, targeting a renewed emphasis on deep conceptual fluency, mathematical problem-solving and reasoning, were initially well-supported in many schools by espoused teacher beliefs, teacher-educative materials (Davis & Krajcik, 2005) and early assessments that were all coherent with curriculum intentions. Over time and given teacher commitment to professional development, good progress was often (though by no means uniformly or universally) made towards classroom practice well aligned with the goals of the curriculum. This underlines that the development and embedding of teacher change is a complex process, so that reasonable stability of curriculum is also valuable – and that there might well be advantages to evolution, rather than revolution, of intended curriculum.

However, England operates with a marketized assessment regime, and teachers commonly talked about choosing assessment providers which are perceived to offer the most accessible routes to good grades in high-stakes assessments, rather than those whose assessments are most coherent with curriculum intentions. Over time, then, and in the context competition between providers, assessments were seen to progressively dilute central intentions. Further, as teachers became more familiar and confident with emerging assessments, they frequently developed alternative classroom practices, often involving mathematically incoherent subsets of the curriculum, which they taught to key groups of students. While curriculum systemic coherence is challenging to establish, then, it would appear even more challenging to sustain, and in this case proved fragile in several respects.

Other Examples

The design of curricular reforms in Costa Rica has the ambitious goal of:

> reorganization of the weight of the main dimensions and elements of the curriculum to give them greater cohesion and depth. As a whole, the changes represent an explicit bid to develop Costa Rican society in its full breadth and complexity. (Lupiáñez & Ruiz-Hidalgo, 2018, pp. 261–262)

However, enactment of that curriculum is compromised by a lack of coherence between its intentions and methods and a key element of the curriculum system – the preparation, training and support of teachers. The authors report on a study that found that many teachers seem to have limited capacity to faithfully enact that curriculum, particularly in relation to “active contextualization of […] one of the disciplinary core ideas” (p. 261) in that curriculum.

Barabash (2018) outlines the challenges to the coherent enactment of reforms of high school mathematics in Israel that are created by the curriculum system in that country. These include factors such as ministerial policy, budget and logistic considerations that have an impact on time allocations, and the fact that many teachers’ beliefs and traditional teaching approaches have become entrenched during several decades of exam-oriented teaching. Work to address these issues and create greater alignment between the curriculum system and the curriculum began in 2014 and is continuing.

Some Possible Responses to Achieve This Coherence

Olsher and Yerulshamy (2018) describe a 'bottom-up' process[2] being used in Israel that brings together mixed groups of teachers, administrators, consultants, researchers, textbook authors and others to curate existing curriculum materials by creating and sharing digital tags that relate materials to aspects of the curriculum. By working together to 'tag' curriculum materials, they build common understandings of what it means for the whole curriculum system to enact the mathematics curriculum in ways that are faithful to the goals and processes of that curriculum. These common understandings have the potential to build coherence in the enactment of the curriculum and the curriculum system through the roles the various educators involved play in that system.

In Vietnam, addressing coherence for the assessment component in relation to mathematical modelling is only part of what is needed for greater coherence within the high school curriculum. Nguyen (2017) has also found that teachers lack the knowledge and skills to effectively implement a modelling approach in their teaching of mathematics and recommends some means for helping teachers build their capacity including:

- training teachers about interdisciplinary teaching and how to incorporate mathematical modelling;
- preparing materials and setting up teaching situations with interdisciplinary themes associated with modelling as a source for teachers to refer and use;
- organising lesson study for mathematics and other subject teachers as ongoing professional development.

It is likely that variations on these types of approaches will be applicable in many settings where, in order for the implementation of reformed curricula to be faithful to the goals, greater alignment between the curriculum and the curriculum system is required.

The examples above provide means for implementation that is faithful to the goals. This is not to say that the goals themselves may not be the source of problems. The goals may have been developed without due regards for the capacity and orientation of the education system that is expected to implement the curriculum. Critical analysis of the goals is also necessary.

Conclusion and Key Messages

In a study such as this, it is only possible to present a snapshot of the field that draws on and analyses some examples of mathematics curriculum reform, in this case in terms of their coherence. We have used four different lenses on coherence and identified ways in which reforms are seen to have coherence when viewed from these different perspectives.

[2] This work is discussed in detail in Chap. 12.

Using the Niss framework to analyse the coherence of the range of curriculum reforms presented in this chapter has highlighted a number of key messages.

- Overall coherence of a reformed mathematics curriculum requires careful and consistent attention to the coherence between all the components of the curriculum.
- The fact that mathematics curricula encompass not just mathematical content, but also mathematical processes and thinking, often with social and cultural overlays, means that it is necessary to give attention to coherence in relation to all these aspects of the curriculum.
- Research and development is needed to support and inform coherence in mathematics curricula. The design of such programs needs to investigate coherence at specific interfaces between Niss' components (e.g. between goals and content or between student activity and assessment, etc.) and provide practical advice that helps promote coherence between components.
- Given that high stakes assessment (examinations) are important in many countries, lack of coherence between assessment and goals of the curriculum can have a significant impact on the enacted curriculum. Many teachers resist change to their existing practice, and textbook writers feel no real urgency to align with the goals of the curriculum when they perceive that examinations have not changed. Burkhardt (1987) coined the term WYTIWYG (What You Test Is What You Get) to make the point that, in mathematics, assessment tends to drive both what is taught and how it is taught. This seems to be as true now as it was more than 30 years ago.

Whilst coherence between and within the components of the curriculum, and between the curriculum and the discipline of mathematics, are all necessary, such coherences are not sufficient for effective enactment of reformed mathematics curricula. It is critical that the curriculum system is also aligned with and supportive of the reforms intended. In example after example the lack of effort to support teachers to develop knowledge and skills that give them the capacity to work with their students (and colleagues) in the spirit of the curriculum has been cited as resulting in inconsistent and inadequate implementation. The best examples of alignment between reformed mathematics curricula and the local curriculum system seem to have three characteristics in common.

- Alignment is achieved over an extended period of time during which the curriculum itself remains a constant.
- There is a comprehensive and targeted program designed to achieve the best possible alignment between the elements of the curriculum system (teacher capacity, values, societal expectations, structure of schooling, place of student voice, etc.) and the curriculum itself.
- The program for alignment is characterised by respect for, and engagement of the wide range of 'players' in the curriculum system (teachers, students, school administrators, officers of education systems, teacher educators, researchers and textbook authors, etc.).

Coherence is likely to be a universal aspiration for mathematics curriculum reforms – truly achieving it presents many challenges and requires commitment to persistent, collaborative work on many fronts.

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Chapter 11
Coherence and Relevance Relating to Mathematics and Other Disciplines

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This chapter focuses on the relations between mathematics and other disciplines through the lens of school curricula. The debates between teaching mathematics as a 'service' discipline and *per se* is old. In mandatory curricula, it is generally mentioned that mathematics is useful and serves valuable purposes in everyday life, other disciplines and for future study and vocational pathways of the students. Despite this, designing a mathematics curriculum that takes into account the relation with other disciplines, with applications and with the future needs of students is a hard task, and the balance between these aspects and the mathematical work itself has to be found.

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This brings a direct focus on relevance in a number of ways:

- relevance of making mathematics interact with other disciplines in school;
- relevance of teaching mathematics that has applications and impacts in the world – physical and social – around us and helps to understand it;
- relevance to provide the mathematics needed for professional specialisation in vocational paths and future careers;
- relevance of preparing future citizens to think critically and emancipating them.

To this end, there is a need for coherence related to the mathematics curriculum:

- coherence when linking mathematics with the external world, both physical and social;
- coherence with curricula of other disciplines;
- coherence with previous and further curricula in the academic journey of the students; and
- coherence with respect to the parent academic discipline.

This brings into sharp focus questions of the 'interdisciplinarity' in mathematics in the sense of connections between different branches of mathematics (algebra and geometry, for example), and the role played by mathematics in interdisciplinarity, the links between mathematics and various situations (everyday life, professional needs, citizenship and understanding of the world).

Fundamental to these links are the inherent links between mathematics and other academic disciplines that are central to 'interdisciplinarity'. Of course, another issue is the coherence of the curriculum system (see Chap. 10) in order to make these interactions exist. Among the important elements of this are the training of teachers of mathematics and other disciplines to be able to work with interdisciplinary learning the available curriculum materials as these have a significant impact on the distance between intended curriculum and enacted curriculum.

In the following, we will start from examples of curricula trying to address these issues and identify the difficulties encountered. We will then elaborate on the relations between disciplines (and mathematics in particular) and between mathematics and the 'external world', showing the central role of mathematical modelling. This will help us to address coherence and relevance in curricula regarding such relations. Based on this, we will finally discuss particular situations where mathematics interacts with other disciplines and the external world in terms of coherence and relevance. This will allow us to discuss the possibility of designing curriculum that integrates links between mathematics and other disciplines or with social and professional needs, as well as the sustainability of such changes.

Emergence of Interdisciplinary Approaches

The division of knowledge into disciplines comes from the human tendency to distinguish the elements that surround us in order to understand and conceptualise them and to propose and solve problems through the benefit of our greater knowledge of the elements. These elements are not, however, isolated, and on most occasions solving the problems proposed requires contemplating relationships, that is, requires a variety of disciplines to solve them (Navarro, 1994).

Interdisciplinarity is a way of describing the process of drawing on several disciplines to solve problems. It has become a central notion in education systems around the world (Samson, 2014; Lenoir & Thompson Klein, 2010). According to Jankvist (2011), the first use of the word interdisciplinary comes from 1937. Jantsch (1972) proposed a spectrum of notions that classify the level of cooperation, coordination and involvement between different disciplines, thus generating a classification in six levels (see Fig. 11.1).

After the isolated consideration of a discipline, *multi-disciplinarity* recognises a grouping of them, but without explicit relationships between them. *Pluri-disciplinarity* also considers several disciplines at the same hierarchical level, but grouped together in such a way that some relationships between them are improved. *Cross-disciplinarity* implies that one discipline is imposed on others, in a way that does not facilitate co-ordination. *Interdisciplinarity* generates an entity at a higher hierarchical level, which is common for a group of related disciplines. This approach offers a novel vision through the fusion of concepts, expectations, methods and theoretical frameworks that come from different disciplines. Finally, *trans-disciplinarity* requires the coordination and interaction of all disciplines based on a widespread axiomatic and an emerging epistemological pattern. In this case, the

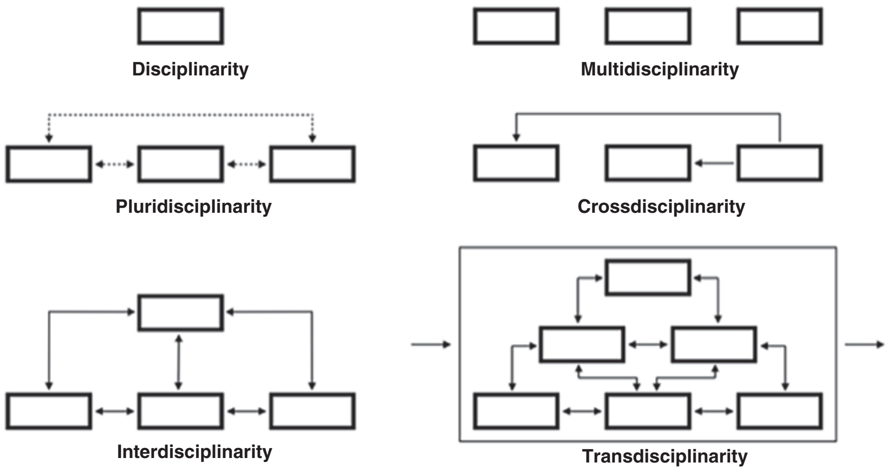

Fig. 11.1 Levels of co-operation and co-ordination between disciplines, as of Jantsch's (1972, p. 410) proposal

education provided by a curriculum transcends the boundaries of conventional academic disciplines. Different disciplines enrich each other and promote the development of a similar set of skills. The trans-disciplinary approach is said to blur frontiers between disciplines (Zuberek, 2007).

Based on these examples, we can draw a picture of issues and questions about curriculum regarding the link between mathematics and other disciplines. From the introduction of the ICME 13 topical survey on “Interdisciplinary Mathematics Education”, Williams et al. (2016) declare:

> it is not only in the mainstream sciences that make up STEM that concerns are raised: in the social sciences too the professional and learned societies are expressing concerns at the lack of adequately numerate recruits. […] Consequently, the task of thinking about mathematics education in this context leads to an increasing concern for how mathematics inter-relates with the other disciplines and contexts involved: for most of the students of concern may only study mathematics for the sake of other ‘leading’ interests and activities, and they may even disidentify with mathematics. On the other hand, if the interdisciplinary significance of mathematics can be understood, there is an opportunity in fact to encourage such students to reconsider and even revisit mathematics. Thus, ‘interdisciplinarity’ should be a major topic for mathematics education in particular, and we can expect it to become much more prominent in educational research and practice. (p. 1–2)

The multiple ways in which disciplines can interact mean that this is complex territory for educators, many of whom find it challenging to work with integration at whatever level is intended due to their training and teaching heritage. The main focus in what follows is on interactions that can best be categorised as interdisciplinary. This is particularly evident in many current STEM initiatives, as well as reforms designed to contribute to ‘citizenship’.

In Education and Curricula

Curriculum reforms that promote interdisciplinarity are widespread in countries across the globe (see, for example, Rocard et al., 2007). Such reforms can promote interdisciplinarity through many of the components of curriculum as developed by Niss (2016), including the goals and content (e.g. in Portugal, Andorra, Vietnam). The enactment of interdisciplinary learning through the Niss component of student activity can be done in a number of different ways such as (UNESCO, 1986):

- establish a link between issues that arise in learning at school in other disciplines;
- research topics that are not limited to a subject or topics that expose problems of real life;
- encourage students’ research into their personal interests and application of mathematics to these interests;
- teaching common concepts and methods implemented in situations involving many different subjects and not belonging to a particular subject;
- teaching and communicating systematically the methods of thinking and acting in a large set of situations.

Whatever the source, interdisciplinary learning involves teaching that engages with a topic, problem or project that is not bound to a single discipline by using the method and language of multiple disciplines, all the while aiming to develop the learning process in each discipline.

The Case of STEM

Notions such as 'social benefit' (Johnson et al., 2015; Felder & Brent, 2016) and 'reducing social gaps (Babaci-Wilhite, 2016; Berube, 2014) have helped drive the recent emphasis on the STEM construct in curricular reforms and educational research, as well as a plethora of activities and tasks available both as hard copy and in digital repositories. This has brought with it a focus on the importance of interdisciplinarity, for the necessary integration of the four disciplines involved: in parallel with an emphasis on 'inquiry-based learning' in recommendations for science education (Rocard et al., 2007).

STEM education eliminates the 'traditional' barriers between the four areas of learning,[1] aiming to integrate them into a cohesive paradigm of teaching and learning. Aguilera et al. (2021) point out that STEM education should be defined, "as the educational approach that promotes the integration of content (concepts, skills and/or attitudes) originating from science, technology, engineering, and mathematics in the resolution of real-world problems." (p. 2). In the STEM context, interdisciplinarity becomes essential if the goal is to develop a combination of skills from the four disciplines in students and the ability to tackle problems not situated entirely within one of the component disciplines. There are a number of practical questions of coherence and relevance that arise as a result of policy directions that promote STEM approaches, including the ease of implementation, given the materials available and the training of teachers, and the coherence between STEM approaches and the learning within each of the disciplines as it is currently established.

The Case of Mathematics and Citizenship

Some recent mathematics curriculum reforms (Portugal [MACS], see Chap. 10, England [Core Maths]) aspire to meet the needs of students taking non-STEM trajectories in study and work through learning that draws on mathematics and the social sciences, in line with the views developed on the *Age of Enlightenment*, and in particular those of Condorcet. These examples can be seen as having 'equipping students for citizenship' as a major goal – though they also serve to develop young

[1] The term 'areas of learning' is used in preference to 'disciplines' to reflect that, while mathematics and science can be seen as singular disciplines, technology and engineering already often draw on multiple disciplines.

people as'users of mathematics' in meaningful ways in a digital world where they are confronted by 'fake news' and 'conspiracy theories'. In a manner similar to STEM, these reforms emphasise interdisciplinary approaches and the role of mathematical modelling of social phenomena, and its specific application that can be different from STEM contexts, through goals, content and student activities designed to meet the needs of particular groups of students.

Issues

The catalysts and potential benefits of such approaches are clear. However, in practice there is evidence of challenges in effective enactment of such approaches. These include:

- different orientations of curricula regarding mathematics and other disciplines;
- different points of view on interdisciplinary approaches;
- different effects in the implemented curricula and difficulties regarding implementing interdisciplinary approaches with mathematics in practice.

Examples of Reforms That Lead to Interdisciplinary Approaches

We turn now to considering issues of relevance and coherence in some examples of recent mathematics curriculum reforms that promote interdisciplinary learning.

Vietnam

Vietnam has undertaken some recent changes in curriculum (2018) in which the discourse on links between mathematics, other disciplines and wider life is reinforced. The discourse in this new curriculum is characteristic of recent trends in many countries.

In Vietnam, in the mathematics general curriculum (published in December 2018 for progressive implementation culminating in full adoption in 2024), mathematics is considered the subject of establishing the connection between mathematics and other subjects (science, economics and other social sciences, etc.), with practical life, and with urgent global issues (such as climate change, sustainable development, financial education etc.). (Nguyen, 2018).

The curriculum provides clear guidance that:

> The content of the Maths program should be strengthened with practical applications, associated with real life or other subjects, associated with the modern development trend of

> economy, science and social life and urgent global issues (such as climate change, sustainable development, financial education …). (p. 4)

In addition, the mathematics program is designed to ensure integration and differentiation. Specifically, the exploitation of integrated interdisciplinary knowledge is emphasised:

> The content of the Maths program should be strengthened with practical applications, associated with real life or other subjects [and] Mathematical knowledge is exploited and used in other subjects such as Physics, Chemistry, Biology, Geography, Informatics, Technology, [...] The exploitation of [cross-disciplinary] integration has contributed to strengthening the knowledge of mathematics, as well as contributing to training students to apply mathematics to practical life. (p. 5)

Hence, the goals of the recently adopted mathematics curriculum in Vietnam emphasise interdisciplinarity. In terms of content, coherent interdisciplinarity requires coordination of content between subjects. This heightened need for coherence between the content of different disciplines is another dimension of coherence in curriculum that is a direct result of taking an interdisciplinary approach. See Chap. 10 for further discussion of this reform for high schools in Vietnam, in particular the example of trigonometric functions.

Spain

Jiménez-Liso, Martínez-Chico and Salmerón-Sánchez (2018) describe a clear example of how the lack of coordination between disciplines can generate issues of coherence in the development of a transdisciplinary student activity. The starting point of this activity is a television advertisement that states that using chewing gum "helps increase oral pH after meals". A sequence of activities was developed with a model-based inquiry teaching approach to engage students in scientific practices, and understand disciplinary core ideas and crosscutting concepts. Learning results showed that students came to differentiate between dilution and neutralisation and their relationship to pH. However, the disconnection with the work in mathematics was evident when it was necessary to model the pH variation using the logarithm function, as the students had not yet studied this topic and did not have the tools to finish the project. Issues such as school organisation, and teachers' interactions to achieve coherence between the curricula of each discipline were not addressed in advance.

This finding highlights a specific issue for connecting learning in mathematics and other disciplines – that of coherence between the content of the different curricula (this is one of the six components of curriculum identified by Niss, 2016). Although the particular example suggests the problem is with the teachers from the different disciplines not coordinating their work, they will be acting on what the different curricula contain – it is these that are not aligned.

The need for coherence between mathematics curricula and the 'parent discipline' is discussed in Chap. 10; this is another dimension of coherence and poses a very interesting question. One line of logic is that if a mathematics curriculum is coherent with mathematics itself and develops the ideas, concepts and content broadly in line with the logic of the subject, then other disciplines cannot expect students to have knowledge of particular content (such as, in this case, the logarithmic nature of the pH scale) until they know about logarithms in mathematics.[2] Alternatively, co-ordination between curriculum developers could make learning about logarithms in mathematics and pH in chemistry coincide, thus enabling the mathematics and one if its applications to be taught and learned concurrently. Mapping of the various curricula can identify inconsistencies and point to more coherent solutions (Segovia et al., 2010).

A third option is for the chemistry curriculum to 'teach' logarithms in the context of their application in the pH scale. The problem with this is that the students' learning about logarithms is not within a coherent development of their mathematics. It may also be that chemistry teachers would take an instrumental approach by simply teaching the techniques, rather than carefully developing the concept that is the intention of the mathematics curriculum.

Epistemological analysis and curricular analysis would be needed to resolve the kinds of issues raised in this example to identify what kind of knowledge of logarithms is needed in chemistry, recognising that mathematics is also about 'decontextualising' concepts (such as logarithms) in order for students to have their knowledge available for many different situations. This latter is one of the great strengths of mathematics.

Andorra

In their description of the implementation of a reformed mathematics curriculum in Andorra, Giménez and Zabala (2018) describe a situation in which interdisciplinarity is almost inevitable. Among the key goals of the curriculum implemented in that country from 2012–13 is:

> the development of specific and [cross-curricular] competences that should allow students to be protagonists and regulators of their learning, and intervene in the different areas of life: personal, interpersonal, social and professional. (p. 231)

In order to achieve this goal, students engage with interdisciplinary spaces:

> Clearly interdisciplinary spaces are called ***global situations***. [… They] act as fields of experience (Boero, 1994) rather than simple contexts, because learners already have such expe-

[2] There is, of course, the question of whether knowledge of logarithms as a function is necessary to understand and use logarithmic scales, with the result that these concepts could be taught and learnt independently.

rienced contexts and from which rich mathematical ideas can emerge. (p. 233; *italics in original*)

It is teachers who create the format of 'global situation' (the student activity component of the curriculum).

Giménez and Zabala provide the example of students aged 12–13 working on modifying a house to achieve greater energy efficiency. They report that the scope of the investigation rapidly expanded from considering the conservation and dissipation of energy to encompass wide ranging practical matters such as improving the natural lighting of a house in a dark location; thermal insulation and distribution of heating in a house; and sustainability. Whilst the students use and further develop mathematical skills such as measuring, proportional reasoning and graphing, they also needed to draw on and develop learning in other disciplines such as physics and design in order to ask and solve problems about energy efficiency in housing; and even language in order to communicate their results and recommendations, given that the final product was a poster.

Working in this way does present some challenges for teachers and students, particularly in relation to the time taken for students to produce meaningful results (a poster is a time-consuming artefact to produce), and in relation to the actual mathematics that the students use. They will tend to use 'lower-level' mathematics with which they are confident. Whilst this will help develop 'deeper' understanding, it is not easily consistent with the expected progression in the content of the mathematics curriculum. Both these factors – time taken and the mathematics used – mean that the coherence of the curriculum is challenged – these Student activities may be valuable and rewarding for students, but a curriculum system that promotes teaching in this way means that the enacted curriculum is unlikely to reflect the intended curriculum, particularly in relation to the Content.

Issues and Questions

Traditionally rigid structures presented by most curricula, do not facilitate these ideal levels of integration between disciplines that arise from interdisciplinary and transdisciplinary approaches. Different disciplines are presented in isolation and rarely make connections between them, in terms of fundamentals, expectations or developments (Martín-Paez et al., 2019).

Connections with other disciplines and the application of mathematics generally have increasingly been included in the mathematics curricula as goals and 'good intentions'. However, having the general orientations about the need to use the mathematics to understand the world is not sufficient to see it in action in meaningful ways. What is missing in many (most) cases is guidance and support on how to do it. Indeed, there are opportunities in making mathematics interact with other disciplines in order to increase relevance of the curricula for the students, relevance to their needs and the specific career intentions, and relevance to the needs for citizenship.

Beyond these reasonable intentions, the examples above show that it is not easy to make it work in a coherent and relevant curriculum, and raise many questions.

- The trigonometric functions (Vietnam) and chewing gum (Spain) examples show application inside the mathematics course by starting from application problems needing specific mathematics.
- The chewing gum example questions the articulation in the introduction of a mathematical concept and its use: should a concept be first taught in order to be available for application in other disciplines, or are interdisciplinary activities good contexts to motivate the introduction and study of concepts of mathematics?
- In the Andorran example, we see how the structure of a curriculum can be designed in order to foster interactions between disciplines, but this example demonstrates issues of how projects can be articulated with disciplinary courses, and questions what curriculum can be implemented and then attained, considering the time spent on such projects. The Andorran approach of the teacher identifying or creating the 'global situation' for the students to work on would not seem to be possible within a rigid structure of the content in the curriculum. The approach has been facilitated by shift the emphasis in the curriculum from content to more general 'competences' as the central goals – the 'content' of the reformed curriculum. That allows teachers to design for student activity (in particular) that is coherent with these goals and content (the competences).
- Another matter is the duality of theoretical/applied mathematics. Mathematics can be learned in specific (interdisciplinary) contexts but the strength of that mathematics for learners comes from it being decontextualised and thereby having the potential to be applied to many diverse contexts. This can be related to the 'institutionalisation process' – extracting concepts from the context in which they have been learned.
- The example also demonstrates the need for a strong analysis of the concepts, in order to better understand how they are used in context: the pH example shows that knowledge of logarithms is needed to deal with logarithmic scales. But which knowledge of logarithms? It seems necessary to draw away from traditional organisation of the knowledge coming from the academic (parent) disciplines. Could logarithmic scales be introduced separately from the classic 'package' of logarithm and exponential functions with their strong focus on analysis content?

These issues all arise from the confrontation between traditional 'rigid' curriculum structures based on disciplines or subjects and integration through inter−/trans−/multi−/cross-disciplinary approaches.

Some structures of the curriculum could be more appropriate to facilitate interactions between mathematics, other disciplines and applications; and we can see that general discourses on making bridges between disciplines are not sufficient to influence the enacted and attained curriculum. Achieving greater alignment also needs a targeted global organisation of the system as a whole (including teacher training, teaching material, and relevant assessments).

In any case, it is not possible to develop interactions between mathematics and other disciplines without questioning the mathematical content of the curriculum. In designing mathematics curriculum, identifying contents and goals, and then building resources and teacher training, accurate analysis of the specific mathematics contents and the potential relation of this content with other disciplines and applications is needed, in order not to encounter the same problems that have created difficulties for previous trends (such as the 'new math' reform), and applying ideas and approaches when these are inappropriate for specific cases (see Boero, 2018).

In particular, the framework in Fig. 11.1 describes the various ways disciplines can interact, but does not take into account any of the specific nature of mathematics, the specific links it has with other disciplines and life, or the power of its abstract nature. This can be done so by considering the notion of mathematical modelling, which enables specific abstract mathematical concepts to be used in various contexts, and how problems in other disciplines can be represented and dealt with using mathematical methods. Mathematical modelling has a central place in the interactions between mathematics and other disciplines and contexts, and permits the integration of the specific epistemology of mathematics and its articulation with other disciplines' epistemologies (social sciences, experimental sciences, engineering needs), and the practical 'rationalities' of the every-day use of mathematics. This is the object of the following section.

Modelling as Central

In this section, we highlight the importance of mathematical modelling in curriculum reforms that promote interactions between disciplines. Pollak's (1979) study (entitled "The impact of mathematics on other subjects at the school") concluded that, mathematics education must be responsible for teaching students how to use mathematics in real life. Since then, teaching and learning mathematics modelling in the school has become a prominent topic on a global scale (Blum et al., 2007). Significant curriculum reforms that emphasise mathematical modelling include in:

- Germany, France, the Netherlands, Australia, the United States and Switzerland, where mathematics modelling is one of the compulsory capabilities of the national education standard in mathematics (Blum et al., 2007; Stillman, 2012);
- Singapore, where mathematics modelling was included in the 2003 mathematics program with the aim of emphasising the importance of mathematics modelling in learning mathematics as well as meeting the challenges of the twenty-first century (Balakrishnan et al., 2010);
- Costa Rica, where its school curriculum includes for all grades an 'active contextualisation' as one of its five disciplinary core ideas (Lupiáñez & Ruiz-Hidalgo,

2018) in which modelling constitutes its central part (*Ministry of Public Education of Costa Rica*, 2012).

According to Nguyen (2018), one of the core competences of mathematics that is defined in the new mathematics curriculum in Vietnam is the ability to create and use mathematical models. The draft of the mathematics program identifies mathematical modelling as important learning for the students by identifying the following student actions:

Mathematical modelling capacity is demonstrated by the implementation of actions:

- use mathematical models (including formulas, equations, tables, graphs …) to describe situations in real-world problems;
- solve mathematics problems in the established model;
- demonstrate and evaluate the solution in the real context and improve the model if the solution is not appropriate (often known as the 'modelling cycle')

The coherence of the content in the topic of trigonometric functions, both within the mathematics curriculum and between the mathematics and physics curricula, was noted earlier in this chapter. However, through her analysis of textbook problems relating to this topic (i.e. materials), Nguyen (2011) discussed examples in which the mathematics model was simply given in the problem. Thus, the work of the student was just to work with the mathematics model:

> Modeling teaching, especially the modeling of recurring cyclical phenomena narrows down in teaching using models. In particular, if the function belongs to the model, it will be presented in the assignment as soon as the actual introduction needs to be modeled. (p. 298)

This example illustrates the issues of coherence and relevance regarding mathematics and other disciplines (and life) in the curriculum. The intended curriculum (goals and content), shows the relevance of interactions between disciplines and of mathematical modelling as an object and means for interdisciplinary learning that is more relevant to the needs of the students. However, the implemented curriculum (through the materials and subsequent student actions) takes a very narrow approach to students' work with mathematical models that compromises the mathematical and interdisciplinary goals of the curriculum: it is not coherent with curriculum intentions.

In Portugal, the *Mathematics Applied to the Social Sciences* (MACS), created in 2001, has as its target student group those who, in contrast with the Vietnamese example, can best be described as non-STEM students. However, the MACS course's emphasis on interdisciplinarity through modelling is similar to that of the Vietnamese high school curriculum, just with content that is different. Topics such as decision methods (election methods, apportionment, fair division), financial models, population models, graph models and probability models are all framed around modelling approaches for solving real problems. Chapter 10 contains a more detailed discussion of the MACS course, including examples of assessment that indicates that the modelling theme is coherently present in the national examination.

Frykholm and Glasson (2005) point out that one of the main difficulties that students face when solving real problems is that they fail to understand the context in which these problems are situated. Hence, promoting modelling problems in the classroom could also result in an advantage for students with particular cultural backgrounds who are more familiar with the context. Hence there is a risk to equitable access and achievement through adopting a modelling approach.

Research on teaching and learning modelling and discussion of its role in the learning of mathematics and its links with other disciplines have been developed for a number of years.

ICTMA (International Community of Teachers of Mathematical Modelling and Applications, affiliated to ICMI) had its first conference in 1983, and has always aimed to influence the curriculum to introduce real problems and applied mathematics in schooling. At ICTMA 7 (in 1995), Blum (1995), defended the teaching of modelling with four main arguments: it permits students to understand and face real situations, preparing them for their future life as citizens and workers; it contributes to the development of students' communication and co-operation, and willingness to engage with new situations; it presents mathematics means for reflection, giving it the image of a science that is part of human culture; it can contribute to students' understanding of mathematical concepts and provide opportunities for students to develop their mathematical reasoning.

Realistic Mathematics Education, based on the work of Freudenthal, according to whom:

> There is no mathematics without mathematizing. [...] This means teaching or even learning mathematics as mathematization [...], it begins with rich, context-laden problems upon which students reflect, gradually progressing from concrete to operational to abstract. (1973, p. 134)

has developed the notions of both horizontal and vertical mathematisations (Treffers, 1978):

> Horizontal mathematization leads from the world of life to the world of symbols. In the world of life one lives, acts (and suffers); in the other one symbols are shaped, reshaped, and manipulated, mechanically, comprehendingly, reflectingly: this is vertical mathematization. The world of life is what is experienced as reality (in the sense I used the word before), as is a symbol world with regard to abstraction. (Freudenthal, 1991, p. 41–42)

The terms of horizontal and vertical mathematisations then appeared in PISA, the international program of student assessment in mathematics conducted by OECD.

These developments led to various ways of describing the mathematical modelling cycle. All of these identify two 'worlds' (reality and mathematics) and at least these four phases: a real situation that has to be treated mathematically; a mathematical problem that has been derived from the real situation (by making choices and simplifying elements); mathematical results from the study of the problem inside mathematics; real results that are related back from mathematics to the reality and can be interpreted in the real problem. They are considered as cycle because the confrontation between the real situation and the results obtained through modelling can lead to a new step of work in order to adapt or change the model to make it fit the real problem better. One of the most used schemes is the one from Blum and Leiss (2005), which has been of influence on curricula through the problems developed in this framework. For more details on modelling in ICTMA, horizontal and vertical mathematisations, and the diversity of modelling cycles, see Yvain-Prébiski (2018).

These considerations show how modelling has been developed and are important in showing the relevance of mathematics in solving ‘real’ problems. They highlight the fact that considering modelling in the interaction of mathematics with other disciplines contributes to coherence. This is especially because modelling makes visible the differences between the mathematical formulation, and the real problems or those that arise in other disciplines to which the mathematical model is applied. Importantly, whilst the problem can be handled within mathematics, a return to the original problem is always needed. Through these means, mathematical modelling contributes to the epistemological coherence of curricula regarding interaction between mathematics and other topics. Of course, introducing mathematical modelling in the curriculum is not easy. Research such the one mentioned above shows that it can be difficult to implement for teachers (who need to be trained for that) with their unfamiliarity with modelling leading to the use of limited and simplistic teaching practices that treat modelling as procedures.

Coherence and Relevance at Different Levels

The use of mathematics in other disciplines is being addressed at multiple levels or scales, from the ‘global’ – such as in the OECD’s PISA – to the national – as in many countries around the world – to the local. Before discussing the issues of coherence and relevance, we introduce few more examples from various countries.

Examples of Ways to Make Mathematics Interact with Other Disciplines

Spanish curriculum (LOMLOE) is an example that emphasizes the need for integration: “The specific mathematical competences, which are related to each other constituting an interconnected whole, are organized in five fundamental axes: problem solving, reasoning and proof, connections, communication and representation, and socio-affective skills. In addition, they provide guidance on the methodological processes and principles that should guide the teaching and learning of mathematics and favor the interdisciplinary approach and innovation” (Ministry of Education and Professional Training, 2022, p. 92). These ‘interdisciplinary’ intentions relate to students at the very start of their schooling, and can be seen as natural, given that young children have not yet experienced the separation of learning into discrete disciplines. It could be argued that any emphasis on interdisciplinarity in the later years of schooling (high school in particular) is a case of returning to the holistic approach that was evident at the start of schooling.

In Australia there are different approaches at the secondary level in the different states, although all are based on the nationally agreed Australian Curriculum: Mathematics (ACARA, 2016). For example, in Queensland (QCAA, 2018), beginning in 2019, five different mathematics pathways are offered. One of them is called general mathematics and "is designed for students who want to extend their mathematical skills beyond year 10 but whose future studies or employment pathways do not require calculus. It incorporates a practical approach that equips learners for their needs as future citizens." (p. 1) So, this is again mathematics offered to non-STEM careers, like social science and the arts. It includes more classical content but also "Statistics, and Networks and matrices". The intention of this syllabus is that students "will experience the relevance of mathematics to their daily lives, communities and cultural background […and also] will develop the ability to understand, analyse and take action regarding social issues in their world" (p. 1).

On the other hand, and at a much more localised scale, a well-co-ordinated and developed project is proposed by Toma and Greca (2018), for elementary education students, for exploring the technologies that in ancient Egypt they could use to raise heavy blocks of stone to the pyramids during its construction. Using an inquiry-based integrative STEM education approach, they developed a module on simple machines through LEGO blocks. In their planning, they propose an "integrative STEM framework" (p. 1385), in which there were a specific methodological design, concepts and learning expectations for science, technology, engineering, and mathematics. This is an example of coherence between materials (the module) and subsequent student actions, and the goals of the curricula in all four disciplines, in the context of a STEM project that implements and interdisciplinary approach to teaching and learning.

The Example of Mathematics and Computer Science Interaction in France

French curriculum has recently and progressively introduced computer science as a subject in particularly in the mandatory lower and upper secondary school years (*collège* and *lycée*) (Modeste, 2018). For practical, historical and scientific reasons, the teaching of computer science has been shared between the mathematics and the technology courses at lower secondary school, whereas in upper secondary school computer science has been progressively developed as stand-alone subject, while algorithmic content has been maintained and strengthened in mathematics curriculum (for details, see Gueudet et al., 2018).

From an epistemological point of view, mathematics and computer science have strong links, but a key question is how these links are taken into account in curricula. To analyse this, Modeste (2015, 2018) has identified four main aspects of the relation between mathematics and computer science: (1) common foundations, logic and proof; (2) continuity and interfaces; (3) computer-assisted mathematics and experimental dimensions; (4) modelling, simulation and relations with other disciplines. Based on these aspects, and the concept of levels of didactic codetermination (see Chap. 13), some features on interaction between mathematics and computer science in the French secondary curricula can be identified.

At lower secondary school level, the 'algorithms and programming' part of the mathematics curriculum is oriented to learning programming through the use of either Scratch software or block programming. We can see here the influence of international trends and the spreading of the idea of developing 'algorithmic thinking'. This leads to two main kinds of activities being suggested: solving mathematical problems using programming and algorithms (relates to Modeste's aspect (3)), and projects to learn (block-)programming where his aspect (4) seems to be underexploited, and often far from mathematics' content and pedagogical uses. Aspects (1) and (2) are almost absent. Besides the intentions, very few interactions exist in the curriculum between technology and mathematics regarding computer science.

At upper secondary school level, algorithms have been introduced into the mathematics curriculum starting in 2009; computer science appeared in 2012 as an optional teaching theme in the final grade. Very recently, it has become an independent subject with a status similar to mathematics or physics in the available choices for students, whilst algorithms remained as part of the mathematics curriculum. However, the connection between mathematics and computer science remains problematic. In the mathematics curriculum, algorithms remain focused on helping illustrate concepts and simulating random experiments; the examples given in the resources concentrate on numerical analysis. This emphasis reflects an orientation to aspect (3) – aspects (1), (2) and (4) are only minorly evident in the curriculum.

Modeste's interpretation of these effects is linked with the mathematics education history in France. One can still interpret the recent orientation toward Python language as a step in the direction of interdisciplinarity (aspects (2) and (4)), as it is favourable for mathematics, computer science, physics and other disciplines. Hence, at both levels of secondary schooling in France, there is a mismatch between the natural connections among the disciplines that are possible and desirable as identified by Modeste, and what is seen at this time in the cycle of curriculum reform in France.

In conclusion, it seems complex to capitalise on the potential for strong and various connections between mathematics and computer sciences in French secondary school in order to achieve coherence between the two subjects. Many factors influenced this situation including the history of the curricula (the evolution of the system depends on its previous states); the structure of the schooling system (until now, there have not been dedicated computer science teachers and, indeed others such as technology and mathematics teachers who protect their territory); and the status and image of mathematics and computer science in society (the general public, among decision-makers and the scientific traditions of the country).

Tensions and Synergies

The examples presented in this chapter illustrate the tensions between different levels: society, culture, school, pedagogy and disciplines, in particular when it comes to relations between disciplines. In terms of coherence and relevance, these tensions show:

- evolution of curricula can be seen as a research for relevance regarding society, culture or discipline in particular, and a new curriculum must keep a certain coherence with the previous one;
- epistemology can be a good tool for questioning the relevance of curricula to the parent disciplines and their coherence between the subject as represented in the curriculum and the discipline itself;
- internal coherence of curricula can be analysed between levels of schooling (middle and high school), between courses and between all levels of co-determination;
- coherence of the curriculum with the curriculum system is very important, particularly regarding interdisciplinary issues.

There is a growing debate on the value and content of mathematics for different kinds of students, namely the ones that are not included in the STEM area of studies (also called sometimes the ‘calculus sequence’), at the secondary school level. For example, in the so-called Villani report (Villani & Torossian, 2018; *authors' translation*) a ‘mathematics for all’ is advocated and it is written that, “mathematics brings tools essential to the exercise of an active citizenship” (p. 30). It is considered that, “mathematics is needed for democracy” (p. 31), and that the educational system should guarantee for everybody “till the end of the compulsory schooling a preparation of mathematics ‘for the citizen’” (p. 31). After that, the mathematics preparation should continue “whichever the preparation path, general, technological or professional, the preparation should be prolonged”, but with different contents depending on the path, the one destined to STEM being more mathematical ‘expert’. For others ‘reconciliation teaching’ is proposed, where the content would be at the same time ‘ambitious’ and would “mesh with the cumulative character of the discipline” (p. 32).

A different kind of line of reasoning, but with a similar goal, was proposed by Timothy Gowers, a Fields Medallist and therefore a very highly regarded mathematician. In a blog entry (Gowers, 2012), following previous discussions, he advocated a different kind of mathematics course for post-16 students. He suggested a curriculum built around interesting questions, so that “the discussions should start from the real-life problem rather than starting from the mathematics”. This curriculum “would be good for a large number of people”, “the (pretty large) cohort of pupils who are intelligent and motivated to learn, but who for one reason or another do not get on well with the traditional mathematics curriculum” – Gowers' hope is that such a course would “help at least some of them to lose their dislike of mathematics”.

In fact, a course now exists, similar to that proposal, called a 'Core Maths' curriculum: "Critical Maths is for study post-16 and is suitable for students intending to enter a wide range of professional careers including social sciences, politics, and others where quantitative reasoning is considered important". Terence Dawson and Stephen Lee (2015) consider that, "Critical Maths is an innovative curriculum, designed to cultivate mathematical reasoning and quantitative analysis through questioning, discussion and solving realistic contextual problems" (p. 1).

In Portugal, the two-year *Mathematics Applied to the Social Sciences* (MACS) course was created in 2001 (this course was discussed in some detail earlier). MACS attracts more and more non-STEM students and has as its main idea that students have, "significant mathematical experiences that allow them to appreciate adequately the importance of the mathematical approaches in their future activities" (Carvalho e Silva, 2003, quoted in Carvalho e Silva, 2018, p. 310).

Creative curriculum designers and teachers are able to develop and implement interdisciplinary learning experiences for their students that link directly to their context and experiences (Toma & Greca, 2018). Such localisation is clearly a strength in relation to relevance and connection with the students' worlds and interests. But the localisation is also a major challenge – a context that connects with one group of students in a particular setting may not be relevant (or even understood) by another group of students.

All these examples (and others that could be quoted) show that it is possible to design curricula that lead to significant mathematics studies for all kinds of students at the secondary level, and not only to (a reduced) number of students on a partially optional level, in order to guarantee a quality mathematics education for all, following some ideas expressed in the UNESCO (2012) document. Different kinds of genuine mathematics can be offered that prepare all students to become full citizens in our century. Of course, the multiple offer of routes for the needs and interests of diverse segments of the student population depends a lot on the socioeconomic and educational conditions that are present in a particular country.

Conclusion and Key Messages

There is clear evidence that curriculum reforms around the world continue to promote learning about the usefulness of mathematics through experiencing and appreciating the connections between mathematics and other disciplines, and with situations experienced in reality. This emphasis in reforms has been strengthened and given more momentum in the context of increasing emphasis on STEM education, and an associated recognition that mathematics – and effective use of mathematical knowledge in particular – is also important for students who are not on a STEM related pathway. All this bring a sharp focus to the challenges inherent in interdisciplinary approaches in curriculum, and the other forms of interaction of mathematics can have with other disciplines, as identified in Fig. 11.1.

These challenges have been identified in the analysis of the examples presented in this chapter. In summary, we see the following.

- Traditional curriculum structures, based on the specific epistemologies, syntax and ways of knowing and doing in the different disciplines (or subjects), are not conducive to identifying the connections between disciplines – the traditional structures keep the disciplines separate. Even where there is a great logic for coherence between the curricula for two closely related disciplines (mathematics and computer science) this is hard to achieve.
- In particular there is a need for new structures that relate mathematics and other disciplines in order to foster interdisciplinarity. A central aspect will be to articulate the rationalities/epistemologies of other disciplines with the rationalities/epistemologies of mathematics.
- Mathematical modelling shows promise as a means for achieving the goals of curriculum reforms that emphasise interdisciplinarity as it is the way in which mathematics is applied to problems in the world. It would seem that coherent and purposeful emphasis on mathematical modelling as an orientation and a set of skills should be explicit and integral to any mathematics curriculum reforms that intend to see students experience and learn the usefulness of mathematics. Indeed, it is likely that mathematical modelling will be inherent in any interdisciplinary curriculum.
- It is important that the goals and content of the curriculum, and the advice for and expectations of teachers are aligned and coherently promote interdisciplinary approaches, such as in the current emphasis on STEM. The curriculum system (teachers' skills, school organisation, materials provided, assessment expectations) needs also to be aligned to support the effective enactment of that curriculum. Even with these two components in place, there remains the challenge of successfully disseminating modelling teaching, and engaging teachers with it, many of whom are committed to more traditional approaches to curriculum and teaching mathematics. It seems that interdisciplinary curriculum reforms need to address all three domains in a co-ordinated manner.

These issues, and others, mean that active research in this field is needed now, and into the future in order to evolve towards greater relevance (external) and coherence (internal) in curriculum reforms.

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Chapter 12
Coherence and Relevance of Materials and Technologies to Support Mathematics Curriculum Reforms

Jennie Golding

In this chapter, we explore the role of the coherence and relevance of curriculum materials and technologies that support mathematics curriculum reforms. Taken together, we conceptualise those as (material) resources for teaching and learning. Although it is in many ways productive to conceive of, for example, student reasoning, or teacher knowledge of mathematics, as a resource for teaching and learning, we restrict ourselves here to physical (including digital) resources. Those include curriculum-related texts, whether digital or printed (and the former can be responsive), physical or virtual manipulatives that can range from plastic teddy bears through 'base ten' representations to mechanical simulations, and generic or subject-specific digital software that allows manipulation of mathematical representations such as graphs or geometric figures. In this sense, digital technologies, although offering distinctive affordances and constraints, can be construed as particular cases of curriculum materials, and here we consider them as such.

We take *coherence* of resources to refer to their internal and mathematical alignment, but also to the alignment of their designed use with the intended curriculum. We argue that curriculum materials and technologies used should also be *relevant* to the needs of the user, whether teacher or student, as well as to the intended curriculum and its valued uses. Otherwise, the user will not fully engage with the materials or technologies in the ways designed, which is likely to result in incoherence with the designer's curriculum-related intentions. McCallum (2018) argues that both coherence and relevance are necessary for learner meaning-making.

Below, we outline some general context in the field. We follow that with analyses of some recent developments around the globe, and use those 'case studies' to identify some of the ways in which curriculum resources, understood as above, can support both the coherence and the relevance of curriculum reform. We consider the

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constraints on that objective, and threats to its effectiveness, and conclude by drawing out some key messages for stakeholders and future research.

Background: Curriculum Resources Supporting Reform

Niss (2016) defines 'curriculum' for an educational setting, as a "vector with six entries – goals, content, materials, forms of teaching, student activities and assessments"(p. 240). These could be regarded as key components of an enacted curriculum, especially within the context of curriculum reforms. Similarly, Schmidt and Prawat (2006) talk about a 'curriculum system' as meaning much more than the intended totality of intended experience and learning within a formal educational environment, and including all major players, artefacts or identifiable capacities that have the potential to impinge on student experience in and related to the classroom: the written intended curriculum, the values and resources prevalent in surrounding communities at a variety of scales, the assessment system, available curriculum materials of whatever sort, teacher capacity – here, for change: their skills, knowledge and affect (Golding, 2017).

Of course, these elements are not independent, so that teacher capacity, for example, can be enhanced by engagement with suitable curriculum resources; in many cases the intended curriculum is built on teacher or other community input and so values, etc. Importantly, Schmidt and Prawat (2006) argue for the need for a deep-seated coherence of all aspects of the curriculum system if curriculum reform aspirations are to be met, since each has the potential to undermine or to support the achievement of that aspiration. Curriculum resources, then, are one critical aspect of the curriculum system.

We also note, though, that curriculum reform enactment is inherently contextually-bound and socially enacted (Gerrard & Farrell, 2013; Ball et al., 2012); and that, further, Supovitz and Weinbaum (2008), in the context of ambitious espoused change in the USA, identify persistent 'iterative refraction' of key messages at successive layers of interpretation from curriculum document writers to students in the classroom. Even if the system as a whole appears coherent, it is naïve to assume that central determination of curriculum intentions effectively leads to an experienced curriculum which exactly implements that which is envisaged, no matter how curriculum agency is framed within different societies and at different scales. We therefore find it helpful to talk in terms of aiming for (a range of) 'valid curriculum enactments', rather than for a single definitive such enactment.

We include in our considerations all textual resources, whether intended for teachers or students: textbooks, workbooks, teacher guides, often communicating curricular intentions, corresponding instructional plans, and support for enacting those. We do know these can influence what and how mathematics is taught, conveying specific views of mathematics and its organisation. Nico and Crespo (2006) show curricular materials can play a significant role in (elementary pre-service) teachers' learning. Remillard (2005) studied the textbook use of practising primary teachers in the US engaging with large-scale professional change towards 'reform' curricula. She showed that the curriculum experienced by pupils showed significant variation, depending on teacher knowledge, beliefs about mathematics, students and about how students learn, and other teacher orientations towards the materials. Not only did many 'reform-oriented' materials place an emphasis on pedagogical guidance, promoting teaching practices that for many teachers required considerable re-orientation, but many teachers then used them in ways that undermined authors' intentions.

Importantly, Stein and Kaufman (2010) found that teachers who engaged with descriptions that articulated the central mathematical ideas of a lesson were more likely to enact tasks in ways that reflected the intentions of the curriculum. Remillard, Harris and Agodini (2014) further showed that different sets of primary age reform curriculum materials developed to align with different theories of learning varied significantly in instructional approach, mathematical emphasis (the mathematics knowledge and practice that are valued and the quality and treatment of mathematics in the curriculum) and support for teachers, and that this led to significantly different learning outcomes even after just one year of use of such materials.

In times of curriculum reform, then, textual materials have potential to deeply inform and influence teacher – and student – practice and thinking, particularly when reforms involve changes in learning approach or mathematical priorities or paradigm: they can directly communicate key fundamental principles intended to be then interpreted, and embedded in classrooms. However, Drake and Sherin (2006) also argue that teachers' narrative identities as learners and teachers of mathematics, which incorporate their past experiences with curriculum and with teaching, fundamentally frame the ways in which they use and adapt a new and challenging mathematics curriculum.

Ideally, then, if curriculum materials are to fully inform classroom enactment that is coherent with intentions, they should contain additional supports, communicating to teachers likely student thinking and misconceptions, key mathematical ideas, and the rationale behind particular design decisions, as well as the range of possible teacher and learner roles within that. Davis and Krajcik (2005) refer to such materials as *educative* because they aim to support teachers in developing practice aligned with curriculum intentions. It is important to note that there is comparatively little evidence around the impact of textual materials on student mathematical functioning or affect.

Of course, textual curriculum materials vary in quality and in appropriateness for a particular context, and so they, in common with all other curriculum materials, are dependent on teacher (and student) choices to realise their potential for supporting curriculum change. Oates (2014) concludes his review of printed textual resources by arguing that the highest quality materials reviewed (judged to be most effective for supporting enactment of curriculum intentions):

- were underpinned by well-grounded learning and subject-specific content theory;
- included coherent learning progressions within and across the subject;
- stimulated and supported learner reflection;

- featured varied application of concepts and principles – 'expansive application'; and
- controlled surface and structural features of texts to ensure consistency with underpinning learning theory.

Such resources cannot be developed overnight, which is one tension in any curriculum reform enacted on a short timescale, such as the English 2014 national curriculum outlined below.

Teacher agency, and the choices and learning opportunities available to teachers through the use of curriculum resources, are analysed by Remillard and colleagues (2009) in terms of their 'structure, look, voice, medium and genre', with corresponding messages for how teachers are positioned in relation to materials and so, how they are likely to interpret texts. Even so, these authors show that teachers working with the same materials might focus on very different 'reading' of the text for activities, for script or for 'big ideas'. For students, teachers mediate curriculum material use both directly and indirectly – but, for example, Rezat (2009) shows student response to, and use of, texts impact also on the choices made by teachers.

Gueudet and Trouche (2009) harness Rabardel's (1995) development of 'instrumentation' to develop a theory of 'documentation work' that encompasses the complex and interactive ways in which teachers, as individuals and groups, come to work with the range of curriculum-related resources, arguing that these are strongly intertwined with teachers' professional development, and therefore, far from static – and also foregrounding the interaction of teachers with resources, that can symbiotically transform. Research on student interaction with curriculum resources, and particularly textbooks, is much less well-developed, although there is a corpus developed around the impact on thinking of student interaction with digital resources, which could equally be conceptualised as instrumentation work.

To date, in general, though, we know less about the particular curriculum reform-supportive potential of digital texts and blended learning. They have potential benefits of easy updating and other editing, and for users, of availability anywhere there is web access. However, we have much less evidence of the potential impact on teachers and learners of their selection, use and shaping in pedagogical discourse. Gould (2011) uses examples from both printed and electronic textbooks to discuss how educational design features can help align the medium of presentation with the content, emphasising that digital texts can provide *different* affordances and constraints in learning mathematics.

The range of curriculum texts, then, have the potential to communicate curriculum in ways that support teacher sense-making of, and adaptation to, reform intentions, especially if they are also educative in nature. Where the communicated 'intended curriculum' does not encompass all aspects of Niss's vector, or of Schmidt and Prawat's 'curriculum system', texts have the potential to offer definition or concretisation of the curriculum – and might be used to do so even where such 'official' interpretation exists elsewhere. The corpus described above, though, shows curriculum work with resources is a complex and highly contextualised process.

The potential of curriculum texts is centrally important to many recent reforms across the globe, where the high-level 'doing mathematics' tasks often valued by twenty-first century intended curricula require deeply informed, selective and creative thinking, frequently prompting student anxiety and opening up classroom discourse in ways that are challenging for many mathematics teachers to manage, especially if they are unfamiliar with such approaches. Related curriculum link-making, designed to deepen conceptual grasp, is perhaps less demanding on teacher skills and subject-specific knowledge, but still more so than the more procedural approaches common in many classrooms historically. Further, curriculum texts necessarily reflect a particular philosophical and/or theoretical approach to enactment, and for coherent messages to learners, it is important both that these are made explicit and that teachers align their understanding and enactment with the espoused approach at a fairly deep level.

Such texts then have the potential to support longitudinal coherence of the experienced curriculum for learners. In early stages of enactment, curriculum materials can carry a considerable share of the instructional load, but after initial engagement with tasks, and for embedded coherence, we know that positive engagement of the teacher with the text, if necessary as learner, become critical (Fullan, 2004). Without that, there is a risk that enactment remains only superficially coherent with curriculum intentions, and in particular with teacher meaning-making of key mathematical concepts and/or processes poorly aligned. Such issues are exemplified below.

Non-textual Resources

There is evidence that the deliberate harnessing of non-textual resources such as concrete manipulatives, has the potential to impact the formation and retention of mathematical concepts and procedures, particularly if careful bridging to symbolic and abstract thinking is supported (Carbonneau et al., 2013). There is a challenge, however, in transferring learning associated with manipulatives to the abstract concepts they represent (Nunes et al., 2009), so that it is helpful to frame such use within curriculum documentation as the Australian curriculum does with technology, in terms of learners engaging with ideas 'both with and without manipulatives': Coles and Sinclair (2019) frame this as engaging with "symbolically structured environments" (p. 470). However, little literature focuses on the role of such materials to support curriculum reform. Exceptions include, for example, that dealing with Singapore's post-1981 primary curriculum, developed with a key focus on a Concrete-Pictorial-Abstract approach to the teaching and learning of mathematics, in which concrete materials play a key role in the intended curriculum throughout the primary phase (Kaur, 2014), and Nigeria's Millennium Development Goal-centred redevelopment of promoted pedagogy (Adeniyi et al., 2013).

Digital manipulatives can go beyond dynamic digital representations of concrete manipulables, whose physical forms are only slowly manipulable, to include for example graphical representations. Suh, Moyer and Heo (2005) suggest that by allowing students to manipulate digital objects to test hypotheses and experiment with ideas, the virtual manipulatives may more closely model the dynamic nature of thinking, which, in turn, may enhance students' thinking and creativity; however, Hunt and colleagues (2011) offer evidence that digital manipulatives may instead be differently, rather than preferentially, supportive of learning. Both sets of authors note that it is important to facilitate connection-making between different modes of representation of mathematical concepts so as to develop students' representational fluency. Some recently-developed curricula, such as that described by Kaur (2014) or the NCTM (2000) 'Principles and standards for school mathematics', explicitly provide for digital manipulatives in the authors' communicated visions of twenty-first century mathematics teaching and learning enriched by informed use of educational technology, and embrace such approaches as key to supporting meaning-making.

The Role of Resources in Recent Curriculum Reforms

We now exemplify the use of materials in recent curriculum reform initiatives to support consideration of how their coherence and relevance might function to enhance, or sometimes undermine, valid enactment of reformed curricula – and what the constraints might be. To do so, we draw on five case studies of recent reforms. The first three focus on top-down national-scale intentions in England, Mexico and Vietnam, respectively. **England** operates under a centralised, politically-controlled curriculum with distributed, market-driven provision of resources and high-stakes assessments. Golding (2018) shows that although serious attempts at systemic curriculum coherence have been made, the coherence achieved was fragile. **Mexico** also has a centrally controlled system, and has in the last 30 years achieved extension of universal education from primary to age 15. For the 1993 national curriculum, materials and teacher development were coherent with curriculum intentions. However, many teachers were not fully equipped to meet the demands of the mathematically more ambitious curriculum introduced from 2011, and did not in 2018 generally have curriculum resource support coherent with intentions; further, it appears learner performance may have dropped (Hoyos et al., 2018). In **Vietnam**, espousal of comparably ambitious curricula at university entrance level was not supported by production of (centrally accredited) textbooks coherent with that, in a culture historically dependent on textbooks and with conservative pedagogic traditions (Trung & Phat, 2018): early outcomes were unsurprisingly incoherent with intentions.

'Curricular reforms' are often generally conceptualised as centralised, large-scale initiatives, but there are also promising approaches and material developments that are bottom-up, capitalising on the intrinsic relevance to participants of such initiatives. Bonissoni and colleagues (2018) focus on local bottom-up development of pedagogy for teaching fractions in **Italy**, tackled using novel disciplinary approaches for which naïve concrete representations are central, offering key relevance to learners. This exposes tensions between relevance and coherence – across the mathematics curriculum and with the parent discipline. In a larger-scale approach to curriculum development, Barabash (2018) focuses on a national curriculum initiative in **Israel**, where early careful approaches to the development of renewed approaches to geometry, together with expansion of school mathematics epistemologies, have led to collaborative design of supportive digital resources, instructive exemplification and draft assessments. Even though there was a measured and deliberate introduction of the reforms, this initiative produced tensions in coherence in the absence of timely planned teacher development and textbooks.

Finally, one challenge associated with more teacher-led bottom-up curriculum material selection, is that of maintaining coherence of the *experienced* curriculum, and Olsher and Yerushalmy (2018) address the development of digital tools to 'tag' and monitor the curricular profile (content and processes) of the selections made, so that for well-informed teachers, the mathematical coherence of the resultant planned curriculum can be monitored and sustained. Visnovska, Cobb and Dean (2012) evidence just how ambitious a task that is.

We now analyse the above situations in greater detail, as 'case studies' of differently coherent resources intended to support reform in five jurisdictions: the categorisations suggested are subjective, but intended to point to the complexity of understanding such relationships.

Case Study 1: England's National Curriculum and Related Post-16 mathematics Provision from 2014 (Top-Down, Time-Pressured Reform with Initial Attempted Coherence)

This centrally-developed, highly aspirational curriculum drew on studies of curricula in high-performing jurisdictions. It features a renewed emphasis on deep conceptual fluency, mathematical reasoning and problem-solving, arguably intrinsically of more relevance to a technology-rich century, than a curriculum focused on facts and procedures. For upper secondary students, changes included attempts to further enhance relevance through mandatory engagement with data handling software to study a 'large data set'. The range of mathematical intentions espoused are widely-valued though there were concerns about speed and scale of introduction, and ambition: a new curriculum for all English 5–16-year-olds was introduced over two academic years, after just 2 years' central planning and preparation, and no time for piloting of curriculum teaching, resource support, or assessment, and very little for teacher professional development.

In England, both assessment and curriculum material provision operate in a market, with assessment heavily constrained by the government-funded body responsible for assessment. Reasonable scale 2–3-year longitudinal studies of the impact of assessments and curriculum materials developed by the major provider, and analysed in Golding (2018), show initial resources were, despite the challenges, coherent with, and supportive of in-classroom progress towards, curriculum intentions. Teachers had to make considerable investment in areas of content and process unfamiliar to them, but were supported in doing so by the materials, which included deeply 'teacher educative' (Davis & Krajcik, 2005) elements such as consideration of necessary prerequisites, of common student questions and misconceptions, and pointers to the related mathematical progression and links. There was mixed reception and use of digital elements of the materials by both teachers and learners.

Over time, many teachers were able to make good progress towards the changes envisaged in teaching and learning, and reported that the focus materials helped them do so in valid ways that economised on preparation time. Learner progress towards confident mathematical engagement post-16 was closely correlated with differential teacher curriculum enactment, though few relatively weak students appeared to thrive in early enactment. Continued progress towards curriculum-aligned teaching and learning at scale, though, appeared fragile, threatened by market-driven assessment, speed of introduction, and high-stakes outcome metrics that mean teachers and students commonly privilege curriculum interpretation in assessment-related resources over that in curriculum-aligned support materials (Golding, 2018). There is a clear threat to sustained coherence of the curriculum system in such marketized and high-stakes assessment contexts.

Case Study 2: Curriculum reforms in Mexico, 1993 to 2011 (Top-Down, Variably Coherent Curriculum System Reform)

Mexico's compulsory education was extended from age 12 to age 15 in 1992; new curricula followed in 1993 and again in 2011. The intended changes over that time have much in common with those described for England, above: a move towards greater emphasis on key mathematical processes of problem-solving and reasoning, with their associated communication, together with flexible, integrated mastery of core knowledge and techniques. For example, in 2011:

It is expected that students develop the following mathematical competencies:

- solving problems autonomously;
- communicating mathematical information;
- validating procedures and results;
- efficient handling of techniques.

The promoted teaching approach is constructivist, building on Brousseau's work and supporting progression from concrete to abstract. Mexican policy is for free distribution of one set of official textbooks for each grade. For the 1993 curriculum, an official mathematics textbook was produced, together with a series of activity books and an 'educative' teacher's guide. Hoyos et al. (2018) argue that in general, in 1993 these materials were coherent with the theoretical approach and content adopted in the written intended curriculum, as was the associated in-service teacher development provided, and that these together supported enactment of the reform intentions. In contrast, for the 2011 curriculum reform, no such teacher development was offered, and the new curriculum materials were not entirely coherent with curriculum intentions, being also less transparent in their support for teachers.

Between the two written curricula there appears little change in the content targeted, and where there are changes made, their alignment with enhanced curriculum intentions is now always clear: for example, under addition and subtraction of fractions there is a move to drop specific mention of manipulatives and of games to underpin meaning-making. 'Informal procedures' are introduced, but their use not followed through. Importantly, for the 2011 curriculum there was no large-scale official textbook series produced, and no national teacher development programme to support practices coherent with curriculum intentions. Teachers consequently had to adapt practice and textbook use to accommodate new emphases. Far from being 'educative' for the new curriculum in Davis and Krajcik's (2005) terms, such work requires sophisticated and subject-knowledgeable instrumentation.

As a consequence, only the best prepared and mathematically-knowledgeable primary teachers, skilled at developing their own materials for the class, could undertake the new approach with clarity, in ways coherent with the approaches intended. Hoyos and colleagues (2018) suggest that the enhanced aspirations of eh 2011 curriculum, consequently resulted in rather poorer quality mathematics curriculum experiences for man children in less privileged (in terms of teacher preparedness) classrooms. It is striking that in PISA assessments of learner cohorts spanning this change, the percentage of Mexican students that in PISA 2009 were *below level 2* (i.e. attaining the level 1 or zero) was 51%, rising to 57% in PISA 2015, perhaps evidencing an early increase in the proportion of Mexican students in the poorest levels of performance, though in comparatively early days of the intended reform. (Hoyos et al., 2018).

Case Study 3: University Entrance Curriculum reform in Vietnam (Top-Down, Not Yet Coherent with Supporting School Curriculum Materials)

As described in Chap. 10, Trung and Phat (2018) present a Vietnamese central intention to move towards, again, a greater valuing of conceptual mastery and engagement with mathematical processes. As elsewhere, such intentions bring with them challenges for teachers in valid enactment. They describe a cultural norm, and dominant approach, for mathematics teachers in Vietnam of a focus on procedure and memorisation, coupled with close adherence to content presented in textbooks – and available textbooks have not yet moved to align well with curriculum intentions. In parallel, the high-stakes university entrance examination has become the *de facto* high school graduation examination, yet the conceptually-oriented questions common in that examination since 2017 are poorly represented in approved textbooks. Specific examples illustrate the dissonance between assessment and curriculum/textbooks. In this case the assessment would appear to be coherent with the intended curriculum – it is the textbooks that are yet to be similarly developed. However, the result for students – and teachers – is that curriculum intentions are not coherent with, and so well supported by, available resources.

Case Study 4: Bottom-Up Italian Development of a Radical Approach to the Fraction-Related Curriculum (Focus on Use of Naïve Concrete Materials to Build Relevance, But with Exposed Tensions for Coherence)

The development of multiple conceptualisations of fractions and related operations is widely recognised as problematic and so is well-represented in the literature, though with few clear pathways to meaning-making at scale. Radical, if yet small-scale, approaches therefore have potential to inform pedagogical approaches that can be taken to scale. The approach of Bonissoni and colleagues (2018) harnesses the familiar natural division of egg boxes of various sizes, so improving relevance and authenticity for grade 3/4 learners via 'intuitive representation' as opposed to 'primitive intuition'. It uses the comparison of 'number of sweets' with 'number of complete egg boxes' to provide an ordered pair identified as a fraction, so privileging the Pythagorean concept of ratio (logos). The approach derives from the historical evolution of the concept of fraction, introducing a mega-concept of fraction from which different sub-constructs are then interwoven. The range of sub-constructs introduced is therefore intrinsically internally coherent. As yet the intervention is only small-scale, and coherence with existing teacher conceptualisations of fractions, and their current didactic practices, have still to be worked through.

This work highlights persistent tensions between relevance and coherence in this context, given also the naïve conceptions of fractions as part-whole that children bring with them to school, and the challenge in relating the promoted representations to later mathematical conceptions. The approach has high relevance, but is not entirely coherent with some of the mathematical structures targeted later in the curriculum.

Case Study 5: Reform of the Israeli Intermediate Geometry Curriculum (Negotiated, Measured Building of Systemic Coherence and Relevance)

This initiative is discussed in more detail in Chap. 10, but we point to it here as an example of the time and co-ordinated effort that is needed to develop a fully coherent curriculum, even for a limited grade and student population target.

A measured and collaborative Israeli curriculum review began in 2014–2015 and continues, led by accountable program committees comprised of mathematicians, mathematics educators, Ministry of Education subject representatives and curriculum specialists, and experienced mathematics teachers. Together they have produced a new geometry curriculum for the second quartile of students. It integrates analytic geometry, trigonometry, and synthetic geometry, linking mathematical rigour with the development of intuition and valid visualisation-based reasoning, embracing possibilities created by dynamic geometry environments (DGEs), and applying ideas of experimental mathematics to high-school geometry. Sets of examination questions coherent with those intentions, together with curriculum enrichment examples, have been produced and exemplified. The committee is now seeking to develop coherent textbook and software, as well as appropriate teacher development opportunities, in the time to first curriculum enactment in 2021, and is confident that the approach adopted will result in a coherent and stable curriculum system (Barabash, 2018).

Case Study 6: Harnessing Technology to Improve Intended/Enacted Curricular Coherence Across Domains and Levels of Teaching, in Bottom-Up Curriculum Development

This case study is rather different as it does not sit within a national reform context. In cultures where teachers commonly supplement any central resources with their own choices of digital or other materials, they need to be able to design curricular sequences skilfully: in particular, teachers need to be sensitive to aspects of curricular coherence, such as continuous mathematical progression, epistemological coherence and alignment with the Goals and Content of the intended national curricula. It should be noted that assumptions about teachers' capacity for such work have been problematised by e.g. Cobb (1999), who argues that the design of a coherent instructional sequence requires specialist support and development, even if teachers work collaboratively.

However, Olsher and Yerushalmy (2018) present tools which enable evaluation of the nature and balance of a collection of learning resources, developed to support teachers with a reasonable grasp of the discipline and its learning: a tagging tool that associates didactic metadata with individual learning resources, and a 'dashboard' representing didactic aspects of the curriculum, for visualising and navigating a tagged collection or textbook. This emerging work respects teachers' professional judgement of resources and promotes connections between teachers, researchers, administrators, authors on an equal footing in the processes of curriculum development; it offers a tool for evaluation and selection of available resources for teachers' identified purposes. It is suggested that the related teacherly judgment could be further developed through collaborative approaches to tagging.

Curriculum Materials: Affordances for Supporting Curriculum Reform Coherence and Relevance

These case studies, set in their wider supporting literature, show how curriculum materials can support relevance, including through specific, sometiomes naïve, manipulatives to support particular pedagogical purpose (for example, modelling concepts of fractions as numbers and fractions as division) (Bonissoni et al., 2018) and support for creation of pathways to mathematically coherent sequencing (Olsher & Yerushalmy, 2018). Curriculum materials are, moreover, widely used to further relevance of curriculum pathways for different groups of students (mathematical 'sense-making'– McCallum, 2018), for example as memorable representations, or by making links with personal or occupational pathways or of the societal, including scientific, world. This relevance might be in support of curricula which are 'nested' so that different students engage with nested subsets of material, though in principle with similar depths and breadths of the content tackled, such as in Singapore (Kaur, 2014) or England (Golding, 2018). Alternatively, resources might support different curricula for groups of students with differing post-school aspirations, as in Portugal (Carvalho e Silva, 2018).

The case studies above exemplify the positive benefits teachers can derive from engaging with curriculum resources that are well-aligned with curriculum intentions and are preferably also teacher-educative – provided the underlying curriculum is internally coherent, and coherent also with the mathematical and wider needs of the target students. The 2019 International Textbook Summit (Royal Society) suggested that such resources can contribute to good use of teacher time – and Golding (2018) found teachers of all phases of ages 5–18 claimed they saved planning time when they moved to working primarily with a single set of trusted resources, compared with selecting their own. They were therefore able to develop a better 'sense' of the intended curriculum and teach more coherent lesson sequences. Teachers did, though, note that making good use of educative resources demands an investment in getting to know the approach, the structure, and the dynamics of the resource. However, that investment supported their own professional development, particularly of subject-specific knowledge and pedagogic knowledge, as well as their confidence – again, supporting their capacity to teach in ways coherent with curriculum intentions.

For knowledgeable teachers, or groups within whom lies sufficient knowledge, there is a valuable teacherly role in involvement in the design of materials to support, or even drive, curriculum change and such development can be empowering, supporting a relevance sometimes harder to achieve in materials brought in from outside (Barabash, 2018; Bonissoni et al., 2018), although bringing with it also a challenge if there is a need to scale up from there, since any small-scale development is necessarily locally contextualised.

The range of such developments, then, potentially have educative purposes for both the teacher and the learner. In particular, recent work suggests curriculum-coherent, and particularly teacher-educative, materials can support curriculum aspirations to educate young people for appreciation of wider societal challenges (Giménez & Zabala, 2018), for cross-curricular thinking (Lupiañez et al., 2018), and for purposeful engagement with twentieth century technologies (Barabash, 2018). A priori reasoning would suggest that resources aimed solely at students should similarly feature coherence with curriculum intentions, and relevance to the young people concerned and the related educational goals. However, as indicated above, there is not yet a well-developed body of work focused on school students' use of resources for learning mathematics.

Given the thrust of much current debate about the future of education in a technology-pervasive world, we give brief additional attention to the potential of digital technologies for supporting curricular coherence, and relevance to students' current and future needs.

Digital Tools Supporting Coherence and Relevance of Enacted Curricula

Purposively-integrated use of digital technologies clearly has the potential to complement traditional approaches and enhance relevance to students of the experienced curriculum for the twenty-first-century. These technologies can support curriculum-relevant computational thinking, and the acquisition, exploration, representation, interrogation and interpretation of a variety of real and realistic data, including large data sets or 'big data'. They offer a variety of modes of communication, teacher to/from student, student to student, or other, including globally, that can again enhance meaning-making and a variety of link-making across representations and conceptualisations. In so doing, digital technologies can bring external expertise into the classroom, and build wider digital literacy, potentially enhancing both curriculum coherence and its relevance to current wider issues.

Golding (2018) evidences the use of text-hyperlinked sources for these purposes, enriching the meaning-making accessible to both teachers and students. In terms of internal mathematical coherence, dynamic software and bespoke digital packages can support inductive exploration and reasoning with curriculum concepts (Barabash, 2018), as well as independent and immediately responsive, non-judgmental self-assessment, and so ownership (and relevance) for learners. Additionally, responsive technologies can support increased sense-making of the experienced curriculum (Golding, 2018). However, a rapidly increasing body of work evidences that the conditions necessary to reliably achieve such desirable outcomes can be quite complex; for example, the TPACK framework (Mishra & Koehler, 2006) identifies the multiple knowledge bases on which effective teachers draw when they teach mathematics with technology.

For teachers, technological affordances have potential to support teacher subject-specific development, and so curriculum-coherent values and approaches, whether through engagement with professional development packages or software principally aimed at students (Golding, 2018). They have a central place in twenty-first century bottom-up mathematics curriculum-making (Barabash, 2018), as well as supporting mathematical coherence of locally-developed curricula (Olsher & Yerushalmy, 2018).

Dynamic digital tools are sometimes included as part of approved curriculum material packages, as described, for example for fractions in Singapore, in Lee and Ferrucci (2012). Such packages appear to engage students in their learning, positively impact progress compared with non-manipulative use (sometimes, including comparison with concrete manipulatives), and can have a positive effect on narrowing the range of students' achievement, as well as supporting both thinking and creativity. However, there may still remain a novelty effect of such use, and large-scale studies often show mixed outcomes. Digital tools, then, *can* support both deductive and inductive approaches, as well as exploratory and experimental work, and bring with them very real benefits for increased student agency, engagement and meaning-making – but our characterisation of those aspects of tools and of teaching which are necessary for such benefits, is not yet well-developed in many instances. Consequently, digital tools often have real, though not always realised, potential for contributing to curriculum coherence, as well as to its relevance.

Curriculum Materials: Constraints for Supporting Curriculum Reform Coherence and Relevance

It is important to note that materials can also be constrained in their impact on experienced coherence – or relevance – if, for example, they are produced in haste, with inadequate investment of money, time or effort, or by resource developers, central or local, whose beliefs, attitudes, knowledge or curriculum-making skills are not fully coherent with curriculum intentions – or with extant teacher or student knowledge resource. Such limitations can lead to superficial, or worse, mathematically incoherent or irrelevant resources (Hoyos et al., 2018; Trung & Phat, 2018), or those which simply lack transparency of objectives or enactment intentions.

If materials are to fully support robust curriculum coherence, developers have to communicate with teachers and students – consistently and in depth – the full range of curriculum intentions, at all levels, in ways which are coherent and relevant to the range of end-users, teachers and students, in the range of target contexts. Given the aspirations of many current curriculum reforms, that is a complex and demanding task. Even then, there are threats from teacher enactment that is faithful to and perhaps unhelpfully reliant on the resource, possibly resulting in lack of flexibility/capacity to adjust to particular students' learning needs, or contributing to teacher de-professionalisation – or equally, from teachers (or local leaders) choosing to ignore or engage only superficially, with challenging messages conveyed therein.

Such responses are often related to educators' beliefs, which are slow to be influenced: curriculum reform without coherent surrounding community beliefs is unlikely to prosper. Even given conscientious teacher investment in coming to know and appreciate the communicated philosophy and values, structure and approaches in a key curriculum-coherent resource, this will not bear fruit if other parts of the curriculum system, such as high-stakes assessment, do not also remain coherent with the intended curriculum (Golding, 2018).

There is an argument that if coherent materials are produced centrally, then teachers do not need to have the skills to develop their own detailed curriculum, but are freed up to develop detailed enactment at lesson and smaller granularity, harnessing their knowledge of individual and classes of learners: development of curriculum vision consistent with that of the curriculum resources, takes time and effort. Further, curriculum trust and curriculum vision are closely related, so that teachers need to have reason to have confidence in the resources they are expected to work with. Even then, more aspirational curricula can be subverted by teachers, e.g. choosing to reduce cognitive demand from that promoted by curriculum-coherent resources. Fundamentally, such approaches will falter if the resources used by teachers are not coherent with curriculum intentions (Trung & Phat, 2018). Others (e.g. Apple, 1990) argue that a fidelity approach may contribute to teacher de-professionalisation, undermining the affirming possibilities of effective teacher ‘curriculum-making’. As above, there is also the view that ‘educative’ resources can constrain, for example by restricting the range of student responses to which teachers are sensitised. In contexts of high stakes assessment, supporting student attainment might involve sacrificing some professional status in relying heavily on texts – but equally, where the system is developed coherently, teachers can also be seen as designer of curriculum, using text as a tool (Golding, 2018), so much depends on the details of the contextualised enactment, and the informed capacity of teachers to move beyond what resources present as possibilities.

In relation to the use of digital technology tools for learning mathematics, we have identified their intrinsic relevance in educating for a digitally-immersed society, as well as a wide range of potentially highly impactful benefits for supporting coherence with curriculum intentions. However, in relation to e.g. dynamic graphing or geometry software, or for developing meaning-making in the use of data, there are demanding implications for teacher learning: of not only newer emphases in the curriculum and their pedagogies, perhaps harnessing technologies for problem solving or for interrogating and so interpreting data, but of the technological pedagogical needs of confidently, effectively, and safely, harnessing technology for such purposes (Mishrak & Koehler, 2006). Without that, benefits might be more about student engagement than mathematics learning that is fully coherent with curriculum intentions.

All curriculum resources then, are likely to have limited impact on coherence or relevance of the experienced curriculum, if there is rushed and/or superficial development, unclear or muted communication of curriculum intentions, or inadequate investment, either financially or in terms of teacher learning; if there are significant limitations to developer beliefs, attitudes, knowledge or skills in relation to curriculum aspirations – or if, for whatever reason, those responsible simply fail to choose to make use of the tools developed. To enhance buy-in more generally, there is a need to balance fidelity of use with attention to the degree of teacher autonomy valued by, and appropriate to, teachers in that context. Finally, curriculum tools are likely to be optimally effective only if the whole system is coherent: for example, in a high-stakes assessment regime, teachers are likely to fully invest, and maintain engagement with, the potential of resources only while those are seen to be coherent with emerging assessments (Golding, 2018).

Conclusion and Key Messages

What, then, are the key messages from this overview of the role of curriculum resources in supporting a curriculum that is both relevant and coherent? First, no curriculum reform exists in a social or contextual vacuum, whatever its scale, so that resources can only be supportive if they are designed to function in the range of target contexts, including that of policy. Materials are part of a larger curriculum system, and the range of evidence we have seems to suggest that systemic coherence is a necessary condition for large-scale sustainability of curriculum enactment coherent with intentions. A key facet of that system is the teacher capacity – their knowledge, skills, and affect (Golding, 2017) – for the intended change.

Central, then, are transparent and detailed exemplification of novel content and/or intended pedagogical approaches and resource-linked messages around those, as well as opportunities framed to support related teacher development. Here, teacher-educative resources might have a central role. The effectiveness with which teachers use well-formulated curriculum-coherent materials will also depend on other macro social educational variables: in addition to the quality of the teachers that is crucial, the presence of appropriate classroom action quality assurance, and an effective control over educational materials, are needed: the presence in the system of inadequate materials can undermine choice and best use of good resources. This is not uncommon in developing countries or with relatively weaker educational systems (Royal Society, 2019). Additionally, we note (e.g. Barabash, 2018; Bonissoni et al., 2018) the potential for bottom-up curriculum reform, and for collaborative efforts – but also their potential constraints.

Once coherent resources are established, their *sustainability* depends not least on continuing and detailed monitoring for systemic coherence, if student experience is to maintain coherence with intentions even in, for example, high-stakes accountability regimes. While recognising the constraints on policymakers, we have seen above the cumulative threats to continued coherence, of tensions within the curriculum system, and of inadequate resourcing or rushed design. We have also seen that textual, manipulative and digital resources can all be harnessed to support increased relevance for students or society. Taken together, high quality curriculum resources have the potential to promote enhanced enactment, supporting teachers in focusing on detailed planning at lesson and smaller granularity, and harnessing their knowledge of individual and classes of learners.

It would seem that the development of high quality, teacher-educative resources coherent with curriculum intentions also has the potential to modify teacher workload while simultaneously enhancing their potential for professionally-affirming, classroom- and wider-scale 'curriculum making'. On a student level, digital materials can, if used in ways coherent with intentions, support a range of mathematical meaning-making, and so relevance, that complements that available by other channels, but there is much that we have yet to understand about the affordances and constraints of digital materials. The knowledge base around school student use of mathematics curriculum resources in general is also under-developed, including in relation to student received coherence with curriculum intentions, and perceptions of relevance to their own current and future needs, warrants further work.

In conclusion, a range of evidence from across the world shows that deep systemic change at scale remains highly challenging, and resource-consuming in all aspects, so that collaborative and measured curriculum *evolution*, rather than *revolution*, has many advantages. The recent research cited then offers some pointers to the development and use of curriculum resources which can effectively support increased and sustained both relevance and coherence within globally aspirational curricula for the twenty-first century.

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Chapter 13
What Theories and Methodologies Are Appropriate for Studying Phenomena Related to Mathematics Curriculum Reforms?

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Curriculum is a contested 'word' and object for schools, society and for our civilisation. Curriculum has been changing its form and status over the years. As Artigue (2018) argues, curricula interact with the conditions and constraints of their functioning to catalyse change in the state of educational systems. Moreover, their design involves a diversity of institutions and agents, and their implementation an even greater number. Institutionally recognised curricula are eventually formalised through the corresponding texts and other resources, which may be understood as products of a complex and dynamic process impacted by many institutions and agents. Where there is some level of freedom available to schools and teachers, the transposition of a curriculum into school affords (and constraints) the range of possible dynamics for the teaching and learning of the discipline.

When curriculum reforms become the objects of study for research in mathematics education, a diversity of theoretical approaches emerges exposing and addressing different research problems linked to curriculum and curriculum reforms. In

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order to gain perspective on the notion of curriculum and to be able to address the variety of phenomena related to their design and implementation, theoretical approaches provide tools to model curriculum and their reforms. The approach to interpreting curriculum reforms, the kinds of entities that are taken into account, and the empirical domain that is considered as the minimal unit of analysis may vary significantly depending on the research framework chosen (Ernest, 2016). Choices about the elements of the unit of analysis can lead to entirely different research problems related to curriculum reforms, diversity of methodologies to address them, and give rise to different or even incommensurable findings.

The objective of this chapter is to give an overview of the current state of the art related to theoretical frameworks and the methodologies used to address phenomena related to school mathematics curriculum reforms. With this aim we begin by presenting an overview of the main theoretical frameworks discussed in the contributions to the ICMI Study 24 and beyond, extending this overview by considering wider research on this topic. This is followed by the analysis of a selection of case studies as representatives of most prominent lines of research about curriculum reforms, depending on which entities they are (or are not) questioning. It aims at identifying theoretical areas related to curriculum reforms, in this extensive territory for research, and at developing new insights that might catalyse further research.

First, we present an overview of the main theoretical approaches that have been used to address research problems of mathematics curriculum reforms, through our analysis of those discussed in the contributions to ICMI Study 24 and beyond. To facilitate this overview, we have opted to organise these frameworks around three main groups depending on their focus on: the conceptualisation of curriculum and its elements, the didactic process of creation and dissemination of curriculum reform, and on the communities involved in curriculum and the factors affecting their success. This is followed by a discussion of the tentative parallels detected among the theoretical approaches, which allow us to delimit certain lines of related research.

The next section presents a set of cases that illustrate the relations between theoretical frameworks and the methodologies offered to approach curriculum reform research. We distinguish between five main lines of research related to curriculum, depending on what is questioned and what is not. These lines are then further exemplified with case studies that we consider as representatives of each line of research. The chapter concludes with some reflections on the main contributions to the research domain of curriculum reforms within mathematics education, and includes some open questions for future research, with respect to the frameworks adopted and the methodological tools proposed for their analysis.

Overview of Theoretical Approaches for Analysing Phenomena Related to Mathematics Curriculum Reforms

Regarding theoretical approaches used in relation to mathematics curriculum reforms, we start by emphasising the diversity of the theoretical frameworks adopted in the papers discussed in the ICMI Study 24 and, in some cases, the difficulties in identifying the adoption of any specific theoretical approaches. This section aims to present the most prominent theoretical approaches discussed in the different themes. These are far from the only theoretical approaches that could be adopted, but we consider our analysis to be useful as it makes explicit the tools and methodologies the different approaches offer to analyse curriculum reforms.

We focus our overview on several such frameworks, each of them approaching curricula and curriculum reforms with different aims and ways of undertaking the analysis. Being conscious of the difficulty in comparing different theoretical approaches in mathematics education, which is not the purpose of this chapter, we have opted to organise them around three main groups according to their main focus. Firstly, we have a group which aims to provide elements to define and to conceptualise curriculum. Here, we consider the TIMSS Curriculum model (Mullis & Martin, 2015; Mullis, 2019) distinguishing different curricula (intended, implemented and attained), and the approach provided by Niss (2016) which adds some particular elements to the 'curriculum' definition.

Secondly, there are further general approaches that aim to analyse the epistemological and didactic process of delimiting the curriculum, including how curriculum reforms are transposed to different institutions for their interpretation, teaching and learning. These tend to include curriculum and curriculum reforms within a wider process of construction and dissemination of the knowledge to be taught and learnt in school institutions. Within this category, we consider the anthropological theory of the didactic (Chevallard, 1992), together with the theory of didactic transposition (Chevallard, 1985) and the didactic analysis curriculum model (Rico, 1997).

The third set of approaches address how cultural, social, contextual factors impinge on the possibilities for that transformation, constraining or supporting curriculum reforms. In this sense, we include more socio-cultural approaches focusing on the analysis of institutional facilitators impacting on the processes that different communities adopt for curriculum reforms, as well as the approaches that focus on identifying and analysing factors affecting the co-creation and implementation of curriculum reforms by the different communities involved.

Approaches Focusing on Curriculum Conceptualisation

Concerning curriculum conceptualisation approaches, the first approach we address is the one presented by Steiner at the Osnabrück meeting (1980), recovered by Travers (1992), and more recently adapted by Mullis and Martin (2015) for the

TIMSS (Trends in International Mathematics and Science Study) assessment framework. In TIMSS, curriculum is broadly defined, as the major organising concept engaged with when considering how educational opportunities are provided to students and the factors that influence how students use these opportunities (Mullis, 2019, p. 4). The TIMSS curriculum model distinguished between the *intended curriculum*, the *implemented curriculum*, and the *attained curriculum*, as three different entities (see Fig. 13.1).

The *intended curriculum* is given by a document that is typically written by staff of national education bodies. Such documents generally identify the expectations of skills, competences and knowledge that students are supposed to reach once the curriculum is developed and being implemented. The *implemented* curriculum is found in school or classroom contexts and refers to the teaching-learning processes that, in fact, occur in them. Finally, the *attained* curriculum focuses on the achievement and attitudes of the students as they are shown in their performances in the tasks and tests.

Secondly, inspired by Kilpatrick's (1994) definition of the term 'curriculum' as, "an amalgam of goals, content, instruction and materials" (p. 7), Niss (2016) extended the framework of Mullis and Martin by proposing a definition of curriculum, with respect to a given educational setting, as a *vector* with six entries: goals; content; materials; forms of teaching; students' activities; assessment. According to the author, analysing an existing curriculum in a given educational setting then amounts to specifying each of these six components. Furthermore, implementing a given curriculum amounts to specifying it, as well as to carrying it out, i.e. putting all the six components into practice (Niss, 2018, p. 70).

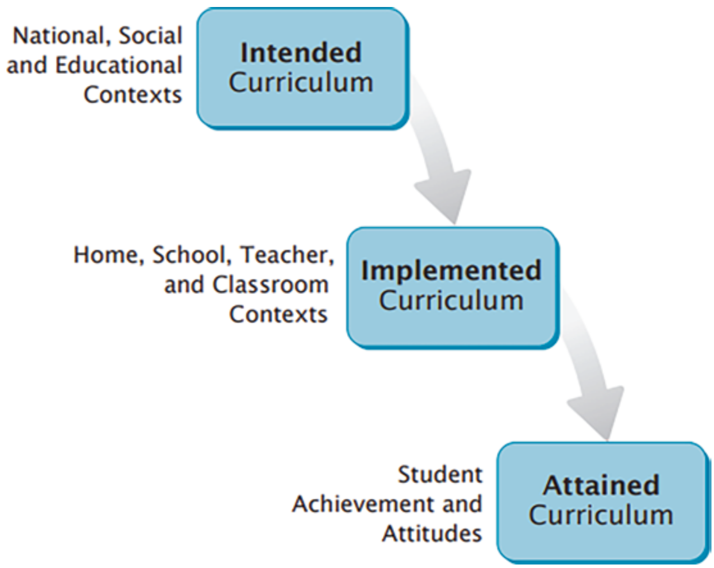

Fig. 13.1 The TIMSS curriculum model (Mullis, 2019, p. 4)

In addition, different agents intervene in the curriculum definition and, in particular, in defining the six entries distinguished. Each agent has more or less impact on some of these entries. For instance, curriculum authorities tend to retain control of the curricular goals, content and the summative components of assessment, at 'intended' level. But sometimes, these authorities may devolve some or total control of the other components (materials, forms of teaching and student activities) as well as the formative components of assessment, to external agents (textbook writers, assessment developers) and educators at the local level (schools, consultants, teachers). The Niss model, therefore, incorporates more aspects of the entire 'curriculum system' than the TIMSS model. Below, it will be seen that much recent work at least implicitly focuses on the latter, at some stage (usually 'intended' or 'enacted': we notice that few recent curriculum studies focus on the *attained* curriculum).

Approaches Focusing on the Didactic Process of Creation and Dissemination of Curriculum Reforms

Related to the second type of approaches, curriculum (as the intended scope of teaching and learning, at each level) is analysed by placing it in a wider didactic process of definition and transposition among different institutions. In this context, the object of study is similar to what defines curriculum in the previous sub-section, though the curricular documents, classroom analyses, etc. are compared and contrasted to mathematics in other institutions, epistemological analyses of content and cultural analyses. Agents and institutions affecting the development of curricular documents are included as objects of study.

In this sub-section, we first have the *anthropological theory of the didactic* (ATD) (Chevallard, 1992) which provides an approach to curriculum reforms by exploring the institutions and their dynamics involved in curriculum definition and dissemination. In particular, this approach refers to the notion of *ecology* (Chevallard, 2002) or *ecological analysis*: the study of the conditions that can facilitate, and the constraints that can limit, the teaching and learning practices. In particular, the conditions and constraints under which curricula are defined, reformed, and transposed are objects of analysis in order to understand the 'perturbations' of the ecological setting for teaching practices. As Artigue (2018) explains in her plenary presentation at ICMI Study 24 (see Conference Proceedings):

> To question the implementation of curriculum reforms, which determines their success or failure, is therefore to try to understand the functioning of these particular dynamic systems in the face of the ecological disruption that is always a curriculum reform, and the means used to regulate these dynamics. […] I consider curriculum reforms as ecological disruptions of education systems and the analysis of their implementation and effects as the study of the responses to these disruptions. (p. 43)

More concrete is the analysis of the *didactic transposition process* of curriculum reforms (see Fig. 13.2). One of the main contributions of the theory of didactic

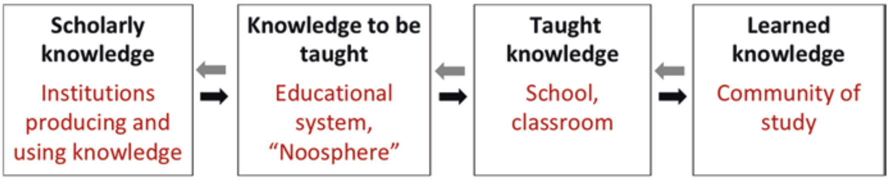

Fig. 13.2 The didactic transposition process. (Chevallard, 1985)

transposition (Chevallard, 1985; Bosch & Gascón, 2006) is taking into account that, in order to analyse what knowledge can be taught and learnt, it is necessary to consider its institutional origin and the conditions and constraints for dissemination. This knowledge undergoes transformations from its production as *scholarly knowledge* to *knowledge to be taught* and, when it is transposed to school institutions and to particular classrooms, as *taught knowledge* and as *learned knowledge* by the community of study involved. Analysing a curriculum reform requires taking into account a diversity of institutions (and agents who occupy different institutional positions) for its (re-)definition and implementation.

As explained more recently by Chevallard (2018), in order to gain perspective on the notion of curriculum, we have to look at the curricular conundrum from the point of view of *society* as a whole. While society is made up of persons and of institutions, "institutional positions are thus the alpha and the omega of the curriculum issue" (p. 214).

The ATD adds another important tool for curriculum analysis: the notion of *praxeology* that appears as the basic unit into which one can analyse human action at large and, in particular, mathematical knowledge and practices. A praxeology is understood as an entity formed by four components: a type of tasks, a set of techniques, a technological discourse, and a theory; it is particularly useful as it provides a unitary vision of different activities. Praxeologies do not emerge suddenly, but are the result of ongoing processes, with complex dynamics, which require analysing what is happening in different institutions setting up the *knowledge to be taught,* and through curriculum and curriculum reforms.

To describe the set of conditions favouring and the constraints hindering the dissemination of certain praxeologies, another important tool in the ATD is the level of didactic co-determinacy (Chevallard, 2002) (see Fig. 13.3). This has been used as a methodological tool for *ecological analysis*, and to illustrate at which level, including those outside school systems, different conditions and constraints appear to support or limit curriculum reforms and their dissemination.

A further approach to curriculum reform is the *didactic analysis curriculum model* (Rico, 1997), which emerged from reviewing and articulating some classical curricular documents (e.g. Stenhouse, 1981; Steiner, 1980; Howson et al., 1981; Romberg, 1992) to elaborate a framework based on both dimensions and levels. This approach distinguishes four levels that expand *from* the particular actions in the *classroom* (first level), the *school system* (second level), *academic disciplines* (third level) and, finally, culminating in a more generic fourth, *teleological level*. Table 13.1

Fig. 13.3 Scale of levels of didactic co-determinacy

Civilizations ↔ Societies ↔ Schools ↔ Pedagogies ↔ Disciplines ↔ Domains ↔ Sectors ↔ Themes ↔ Questions

Table 13.1 The didactic analysis curriculum model (Rico, 1997)

	Cognitive dimension	Cultural dimension	Normative dimension	Social dimension
First level. Action in the classroom. Curriculum is assumed as a work planned by the teacher, based on the influences of the other levels.	Objectives	Contents	Methodology	Evaluation
Second level. School system. Curriculum is a planning instrument for the school system.	Pupils	Knowledge	Teachers	School
Third level. Academic disciplines. Curriculum is devised from disciplinary and erudite reflection, in which different academic disciplines approach and study its theoretical foundations and its technical implementation.	Learning theories	Mathematics, epistemology, history	Pedagogy	Sociology
Fourth level. Teleological. Curriculum is based on the different types of general goals: Cognitive, cultural, ethical, and social.	Training and development goals	Cultural and conceptual goals	Ethical and political goals	Social and utilitarian goals

outlines these four levels and dimensions considered and identifies the curricular elements – organisers – considered in each of these levels.

Socio-Cultural Approaches Focusing on the Conditions for Communities in Curriculum Reforms

The third type of theoretical framework focuses on the socio-cultural approaches to the construction and success of adoption of curriculum reforms. On the one hand, Boero (2018) presents the use of a framework derived from Habermas' elaboration on rationality to deal with the cultural–epistemological orientation of curricular reforms. From this approach, it is proposed to look at the relations between the universal character of mathematics, and the cultures of the contexts where mathematics is taught and of those who are taught. The specific tools proposed aim to analyse the salient characters of different traditions and cultural practices, to identify contact points and differences among them, and to establish relationships between disciplinary culture of mathematics and other cultures, particularly when implementing curricular content (such as teaching and learning modelling, or proof and proving).

On the other hand, other important socio-cultural frameworks may also be highlighted, more focus on the communities collaborating in curriculum reforms and on the conditions facilitating their success. Firstly, the *boundary-crossing approach* (Akkerman & Bakker, 2011) is used as a framework to analyse the collaboration and interaction of different communities, such as the communities of mathematicians, educational researchers and/or school teachers, often part of curriculum reform committees. Sometimes, this first approach is complemented by a framework providing tools for the analysis of discourses emerged in the context of different communities involved.

This is the case of the commognitive theory (Sfard, 2008), a theory based on the notion of commognition, which is premised on the conceptualisation of thinking as one's communication with oneself. The main objects of commognitive research are mathematical discourses, and more specifically the development of mathematical discourses. Within this theory, learning is a form of communication activity that can be conceived as inherently collective, or social, more than an individual phenomenon (Sfard, 2020).

Last but not least, we may mention the framework proposed by Memon (1997) who focuses on identifying the factors that are *enablers* or *inhibitors* of successful curriculum reforms. These factors are classified depending on whether they concern curriculum – primarily the intended curriculum, but also with some attention to other of the Niss components of curriculum that are about implementation – instruction or organisational conditions.

The author presents a number of inhibitors affecting curriculum change. These are divided into three categories. Curriculum factors include mismatch between the official and realised curriculum, not taking the needs of the teachers into consideration, external imposed innovation etc. Instructional factors cover elements such as students' interest, mismatch between a teacher's belief system and curriculum, how to create motivation and engagement, professional development and more. The last category is organisational factors covering influence of political leaders and bureaucracy, resources and physical facilities, communities of participation and other supportive structures. For a full list of factors, see Memon (1997).

Summary of Theoretical Frameworks Used to Address Curriculum Reforms

Above, we have briefly presented some of the most prominent theoretical approaches, organised around three main groups according to their focus. First, we have those which aim to provide definition and conceptualisation of curriculum; second, the more general approaches focused on the analysis of the epistemological and didactic processes used to scope the curriculum, and how curriculum reforms are transposed to the different institutions for their definition, teaching and learning; and third, approaches focusing on how cultural, social, contextual factors impinge on the possibilities for that transformation, constraining or supporting curriculum reforms. As stated, our aim is not to compare the different theoretical approaches but to understand what these approaches aim to question, and what they do not, in curriculum reforms, so as to later be able to select some representative case studies to be described in more detail.

With this purpose, we introduce Fig. 13.4. which shows some tentative parallels among some of the theoretical approaches previously introduced. In particular, this figure presents the parallels among three of them: the TIMSS Curriculum model (Travers, 1992; Mullis, 2019), the theory of didactic transposition (Chevallard, 1985), and the didactic analysis curriculum model (Rico, 1997). There could certainly be other frameworks to include here, when looking for these tentative parallels, such as the one proposed by Niss (2018), but we have focused on these three particular theoretical frameworks as particularly useful to describe and delimit the lines of research on curriculum reforms in the next section.

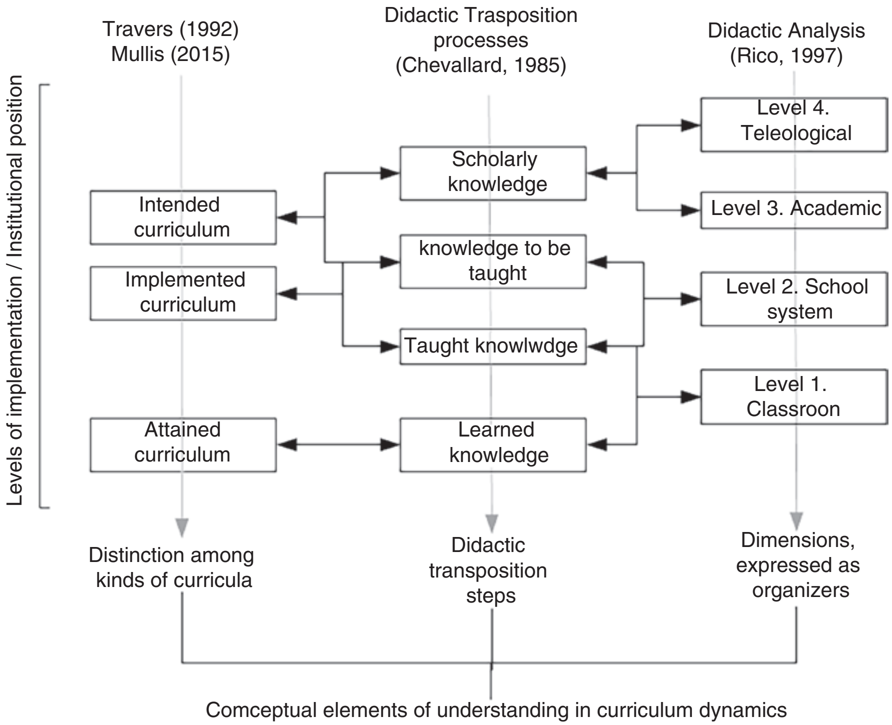

Fig. 13.4 Tentative parallelisms among some of the frameworks used to address curriculum reforms

The figure can be read both horizontally and vertically. A vertical reading allows identification of different levels of curriculum definition and/or implementation or institutional positions for each theoretical approach. A horizontal reading places some of the theoretical approaches previously described with some of their central theoretical constructs used to approach curriculum reforms. This horizontal reading also provides insights about possible relationships about these theoretical constructs. The arrows show relationships among some elements of the linked levels. For instance, the mathematics goals expressed in the curricular documents is an element that is usually included in: the ministerial curriculum (Niss, 2018), the intended curriculum (Mullis, 2019), the knowledge to be taught (Chevallard, 1985), and the school system level (Rico, 1997).

Another more general example could be developed when one refers to the "intended curriculum" (Mullis, 2019) which, in their definition, most of the time becomes the responsibility of the "scholarly institutions" (Chevallard, 1985) and the agents and institutions responsible (the "noosphere") for agreeing the "knowledge to be taught". In this process, the teleological and academic dimensions (Rico, 1997) emerge when the curriculum is analysed in relation to its cultural, social or disciplinary goals. But, its final form (in most of the countries) is the "official curriculum" whose authority lies beyond teachers' community or students. This is the official ministerial (or other authorities') curriculum (Niss, 2018) that is used then to regulate schools', teachers' and students' practice.

This figure has helped us to stress some possible parallelisms that will undoubtedly need further research. But, more importantly for this chapter, this figure is used to delimit certain lines of research about curriculum reforms. More concretely, we distinguish five lines of research, depending on the choices these lines make about: (1) the kind(s) of curriculum taken as object of study: the intended, implemented and/or attained curricula; (2) the institutions considered in the delimitation of curricular knowledge: the scholarly institutions, the 'noosphere', the school institutions (and the classrooms) and/or the particular community of study (teacher/s with student/s); (3) the curricular elements considered at the teleological, academic, school and/or classroom levels. In the next section, not only these lines of research are presented, but also the selection of some case studies has allowed us to look at the particular research questions addressed, the unit of analysis considered, and the methodological choices and tools to problematise curricula reforms.

Research Questions About Curriculum Reforms, Unit of Analysis and Methodologies for Curricular Analysis

When curriculum reforms become the objects of study and research, the diversity of theoretical approaches that can be adopted inevitably delimit the unit of analysis taken into account. Hence there can be significant variation depending on the theoretical framework and foci that are chosen. This delimitation can include different choices concerning the kind of curriculum considered, communities and institutions taken into consideration, and the curricular elements considered.

In this section, we distinguish among the main *lines of research* related to curriculum reforms that we have detected depending on what it is that they question and what they do not. In particular, we have identified the following lines of research, which are then further exemplified with some particular case studies that we consider as representatives of each line of research.

RL1: Research line questioning the *intended curriculum* through the interaction between the *scholarly knowledge* and the *knowledge to be taught.*

RL2: Research line questioning the selection and elaboration of the *knowledge to be taught* and of the resulting *intended curriculum* transposed to school systems.

RL3: Research line questioning the conditions under which *curriculum reforms* are *implemented*, through what means, under which constraints.

RL4: Research line questioning *teachers' actions on curriculum design* and *student attainment*: how the implementation of curriculum is planned and works in classrooms.

RL5: Research line questioning *communities* involved in curriculum reforms.

Table 13.2 summarises the research papers chosen, in correspondence to which research line, making also reference to the research framework(s) used.

Taking these particular cases, we aim to describe several aspects that characterise each particular area of research. In particular, and in order to unify their description, we focus on detecting: (1) the particular research questions addressed; (2) the unit of analysis considered and the particular empirical data taken into account; (3) the methodological choices and tools; (4) results and answers to the research questions.

Table 13.2 Case studies selected in relation to each research line

Research line	Paper(s) considered as case studies	Theoretical framework
RL1	Wijayanti and Bosch (2018)	ATD
RL2	Modeste (2018)	ATD
RL3	Hoyos et al. (2018) Lozano et al. (2018)	TIMSS curriculum model
RL4	Olsher and Yerushalmy (2018)	Didactic metadata
RL4	Ferretti et al. (2018)	TIMSS curriculum model
RL5	O'Meara et al. (2018)	Enablers and inhibitors impacting curricular reform
RL5	Pinto and Cooper (2018)	Commognitive theory Boundary crossing

Research Questioning the Intended Curriculum *Through the Interaction Between the Scholarly Knowledge and the Knowledge to Be Taught*

This first line of research focuses on the higher level of objects to be studied such as the *intended curriculum*, and the interaction between the *scholarly knowledge* and curriculum materials as ministerial documents and textbooks (see Fig. 13.5).

While Wijayanti and Bosch (2018) focused on intended curriculum, scholarly knowledge or the teleological and academic level, their analysis does also point to challenges regarding implementations and teachers' practices. In particular, they develop a didactic transposition analysis to understand why *proportionality* is currently proposed as a particular piece of knowledge to be taught in school. They analyse how this particular mathematical concept has been defined by mathematicians through history, as being part of arithmetic, algebra, geometry or linked to the notion of functions (all representing scholarly knowledge), to analyse which aspects were transposed to be taught, how, and to identify any incoherence that emerges from this process. As the authors describe, they address the following research questions:

> How can the didactic transposition process explain its [proportionality] current form? Where does the current knowledge to be taught about proportionality come from? Why does it have the form it has? How has it been selected, designated, shaped, organized and arranged? What is its role in relation to the other pieces of mathematical knowledge? (Wijayanti & Bosch, 2018, p. 174)

These research questions are explicitly linked to the methodology used, which they denote 'didactic transposition methodology'. They analyse the 'habitats' of the praxeological organisations where the concept of proportionality has existed in scholarly knowledge, as in Euler's *Elements of Algebra*, and how it has been transposed into western mathematics education, the effects of the 'New Math' reform, and through to today's teaching of the concept. The units of analysis include empirical data such as reform documents, textbooks and analyses of former reforms such

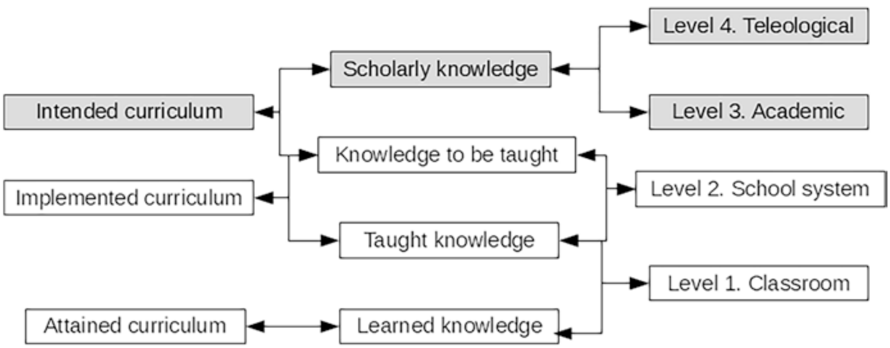

Fig. 13.5 Representation of what it is questioned (in grey) in the first research line

as the New Math movement. In particular, the authors develop an analysis of the kind of mathematical praxeologies that exist concerning proportionality, both as a scholarly knowledge and as a knowledge that is planned to-be-taught. This epistemological analysis allows them to trace what, when and how the migration happened. As we see in the analysis of Artigue (2018), with the case of exponential functions in the French curriculum, the reforms have affected the praxeological organisation in which the curriculum sits, and exponential functions used to be linked with logarithms, but are now taught in relation to physics. The authors Wijayanti & Bosch (2018) conclude:

> We are thus in front of blurred or hybrid organizations made up of pieces taken from different mathematical periods, mixing elements of different praxeologies that maintain redundancies and some incoherence in the kind of tools used. (p. 178)

Thus, the current teaching of proportionality is organised as entities drawing on elements from different former reforms that have quite different mathematical rationales. This results in the approach to teaching proportionality not being entirely mathematically coherent in its own right when compared to scholarly knowledge regarding proportionality. By carefully studying the historic development of the curricula, the authors manage to find the reasons for the current form of the praxeological organisation of proportionality.

Didactic transposition analysis has previously been used to analyse the notion of limit in Spanish upper secondary school. Findings indicated that often the practice block of the praxeological organisation was picked from one mathematical domain, whereas logos belonged to another (Barbé et al., 2005). In this case the incoherence is not historically based as in the work about proportionality by Wijayanti and Bosch, but rather stems from this fundamental disconnection arising from the transposition process.

Together, these findings constitute a genuine challenge for mathematics teachers needing to teach such inherently incoherent curriculum elements so that the mathematics still appears coherent and logical from the students' point of view.

Research Questioning the Selection and Elaboration of the Knowledge to Be Taught and of the Resulting Intended Curriculum Transposed to School Systems

The next line of research is also located at the higher levels of Fig. 13.4, though the main objects of study in this category are documents for implementation in terms of ministerial documents and teaching materials (see Fig. 13.6). The analysis focuses more on the agents and the transformations that the target knowledge undergoes when it is transformed from institutions producing it to the 'noosphere' and subsequently in agents' plans for framing its teaching in school systems.

Modeste (2018) presents one example with a clear research question, theoretical framework and methodology. This paper draws also on ATD when analysing the

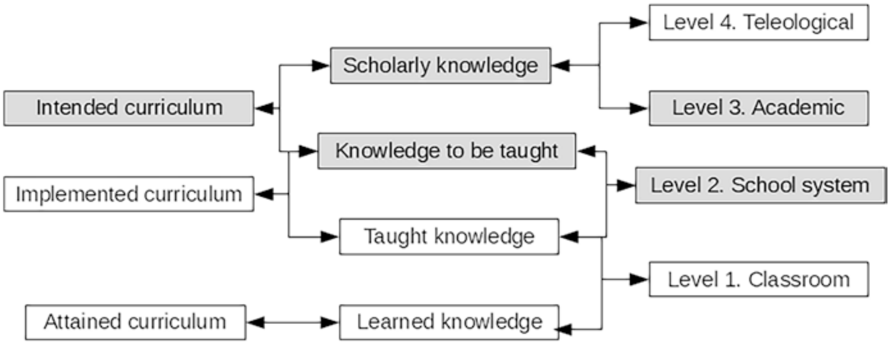

Fig. 13.6 Representation of what it is questioned (in grey) in the second research line

epistemological relations between scholarly knowledge of mathematics and that of computer science - and how that relationship is represented in curriculum documents. In particular, Modeste refers to praxeological organisations: how they are shaped and developed through didactic transposition processes and under which constraints and conditions these take place. He studies their ecology using the scale of levels of co-determinacy, where emphasis is put on the more generic levels beyond the mathematics discipline: that is, according to Chevallard (2002), the levels of the *society, school, pedagogy* and the interacting *disciplines.* The research questions pursued are:

> What do Mathematics and Computer Science share as scientific disciplines and what kind of interactions between them can be developed in secondary school? How do the French curricula deal with this issue and in which direction are they developing? (Modeste, 2018, p. 277)

The methodology adopted is an analysis of (historic and current) documents produced by the 'noosphere' regarding knowledge to be taught from computer science as integrated components of mathematics, e.g. algorithms and programming. The unit of analysis considered includes empirical data (evaluation reports, international reports including the ICMI study (Howson & Wilson, 1986) from the first steps of the didactic transposition to analyse what is (and what is not) finally transposed. Modeste exemplifies this by analysing specific pieces of knowledge such as algorithmic thinking in the intended curricula. The analysis allows him to conclude that:

> Computer Science is still looking for its place in the curriculum, and questions the territories of other scientific disciplines. As we have seen, the interactions with Mathematics are important in scholarly knowledge. […] In the noosphere, many actors influence the didactical transposition of Computer Science which has a direct impact on Mathematics curriculum in the French educational context. In our view, an important issue is the place that a curriculum can lead to the interactions between Mathematics and Computer Science. (p. 283)

Thus, the theoretical framework and methodology allow the author to point out factors and agents affecting the ecology of the teaching and learning of different elements of computer science in mathematics and in more independent course elements of secondary education. Furthermore, the analytic tools point to national, as well as international trends, and how these are related.

Other researchers also analyse intentions for curriculum reform or its development in relation to curricular documents, taking both scholarly knowledge, epistemological aspects and the academic level into consideration when analysing the content of the curriculum. Not all are guided in their analysis by theoretical constructs as scale of levels of co-determinacy and didactic transposition, though their object of study is similar to those addressed by Modeste (2018).

For example, studies that focus on the *intended curriculum* mostly analyse the *knowledge to be taught* and how the 'noosphere' defines what may be taught in a particular school system. For instance, Barquero et al. (2018) turn to an institutional approach using the ATD when analysing how the notion of inquiry has become part of mathematics curriculum across European countries. Lupiáñez & Ruiz-Hidalgo (2018) base their work on the didactic analysis approach to analyse the key notions – specific abilities, processes and active contextualisation – that provide the structure of Costa Rica's most recent curriculum reform. There are also studies that, while not working explicitly with a framework for the analysis of reform efforts, do consider aspects of the construction of mathematics curriculum framed as 'new challenges', as in Nguyen's (2018) approach to analysing the teaching of mathematical modelling.

Research Questioning the Conditions Under Which Curriculum Reforms Are Implemented, Through What Means, Under Which Constraints

We now turn to lines of research in which the units of analysis considered are objects more located in the *implemented curriculum*, in relation to the *intended curriculum*. This third line of research takes into account elements related to the *knowledge to be taught* in school systems and to the *taught knowledge* in particular classroom contexts. The scholarly knowledge and the selections that are represented in the intended curriculum are not questioned or challenged by this research line (Fig. 13.7).

Hoyos et al. (2018), who use the TIMSS curriculum model, provide such a case study, presenting a comparative study of the mathematics curriculum of primary (elementary) school education in Mexico. By considering the distinctions between the intended, implemented and attained curriculum, the paper presents an analysis of empirical data included in official documents relating to two important periods of curriculum reforms, in 1993 and in 2009/2011.

For this analysis, the authors inquired into the characteristics and coherence among these different types of curriculum based on the previous work of Suurtamm et al. (2018), asking, for example, "How is the curriculum in Mexico organized?" and "What is the role of evaluation in the intended curriculum and in the enacted?",

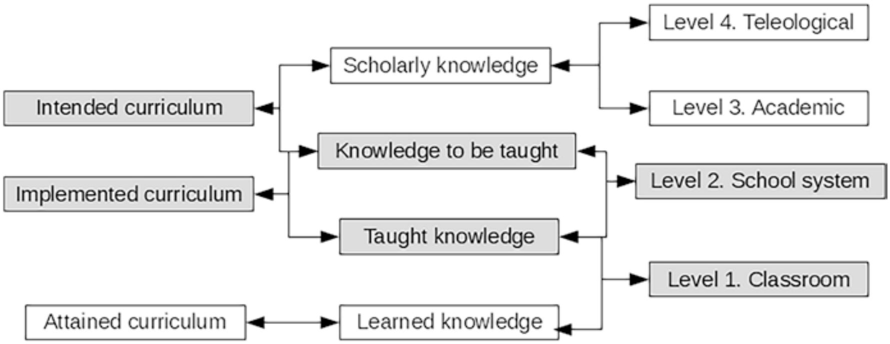

Fig. 13.7 Representation of what it is questioned (in grey) in the third research line

among others questions. They consider a broad unit of analysis including curriculum (general and theme-focused description), textbooks and evaluation results from PISA to analyse the impact of curriculum reforms and the possible connection between PISA results and curriculum reforms.

Specifically, they analyse the official curricula in 1993 and 2011, and focus on the general description of the curriculum and content description provided about adding fractions. They then consider some textbooks in order to analyse the implemented curriculum and contrast this with the approach of the intended curriculum in relation to the topic of adding fractions. Finally, they use selected data and results from the OECD's PISA 2003, PISA 2009 and PISA 2015 as an indication of the attained curriculum for Mexican students. What is noted is that there were minimal changes to the approach to mathematical concepts but an important change in the official discourse that the authors argue caused poorer levels of performance in PISA.

The applications of this broad approach are diverse. Other authors employ more specific frameworks for coherence (e.g. Golding, 2018) that evaluate the alignment between written curriculum, the available resources, the assessment system, and teachers' knowledge to facilitate these reforms, among other things. Giménez and Zabala (2018) combine theoretical approaches by presenting work on the design of a new curriculum from an interdisciplinary perspective, and offer several examples of projects that were consequently implemented in schools. Carvalho e Silva (2018) analyses the origin, rationale and development of the courses of mathematics applied to social sciences (MACS) in the Portuguese secondary school. As part of the analysis of the conditions created by several institutions that have enabled MACS to survive and thrive until the present (nearly two decades), the author discusses the role of the national examinations that may have put at risk the continuity of MACS courses.

A complementary approach is taken by Lozano et al. (2018) who compare and contrast reform initiatives taking place in Mexico and England, particularly paying attention to the resources. Due to the twofold space of research in Mexico and England, as well as the intention of enriching their understanding of the curriculum reforms in both countries, they adopt an approach in which multiple perspectives interact and they assume an enactivist approach to methodology. From this point of view, the authors first analyse innovation in resources both in Mexico and in England independently, obtaining patterns for each case. They then distil meta-themes, such as explicitness of the curriculum, innovative approaches to the teaching concepts, pedagogical aspects, and teachers' autonomy, that allow them to compare and illuminate the changes in both countries.

Research Questioning Teachers' Actions and Engagement with How the Implementation of Curriculum Is Planned and Works in Classrooms

In this sub-section, we discuss the implementation of curriculum in classrooms, considering two specific areas of application of this line of research. The first explores teachers' impact on curriculum design, while the second focuses on students' mathematical activity and includes the attained curriculum as part of its unit of analysis. The theoretical approaches for the first are usually focused on specific mathematical domains (Fig. 13.8).

Regarding teachers, Olsher & Yerushalmy (2018) provide a case study that focuses on teachers' role in shifting from an intended curriculum designed by others to co-designing the intended curriculum to be implemented. The authors analyse aspects of teachers' expertise for designing curricular sequences, such as sequencing that avoids gaps in the mathematical progression, consistent and balanced handling of mathematical objects, and coherence with national curricula. The authors underline three key actions for personalising and managing any curricular sequence and use of interactive textbooks: recognising aspects of affordances of metadata that characterise the resources, developing an awareness of the balance among the learning objects, and developing an awareness of the rationale of the sequencing. Their approach adopts methods based on technological tools: a tagging tool to associate

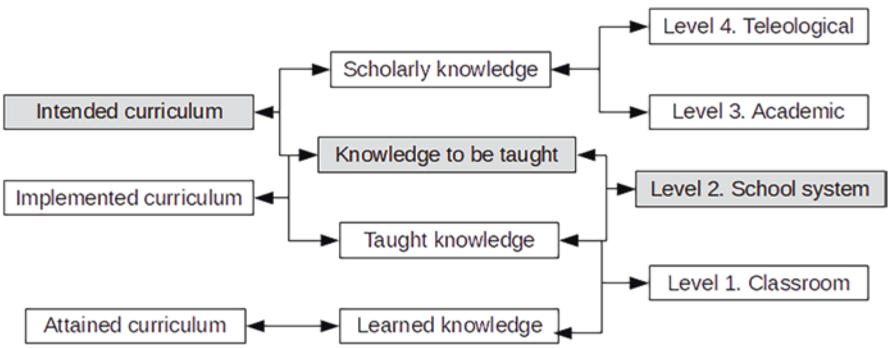

Fig. 13.8 Representation of what it is questioned (in grey) in the fourth research line

metadata with individual learning resources, and a 'dashboard' for providing visual representations of didactic aspects of the intended curriculum, among other things.

In the discussion, the authors underline the role of contributions and methodologies from the domain of data analytics for several stakeholder groups – teachers, textbook authors, and policy makers. On the one hand, for the developers of learning resources there is the opportunity of realising that some tacit intentions should be better explained. On the other hand, teachers gain new insights in the author's intentions. Olsher and Yerushalmy conclude the need for a more symmetrical approach between the variety of designer and practitioner communities that might better serve the evolving state of curriculum design.

Regarding student attainment, Ferretti et al. (2018) argue how the Italian standardised test can be used by teachers to interpret the intended curriculum. Though the authors do not explicitly mention the TIMSS Curriculum model, they locate the paper as linking intended and attained curriculum.

By means of a qualitative and quantitative analysis of the standardised tests, they foster discussion among teachers about the tasks' features and possible student responses, showing that undertaking this kind of analysis can become a means for Italian teachers to engage with and reflect on the Italian Curriculum National Guidelines requirements. As a conclusion, the authors claim that Italian standardised assessment can be both a tool for policy makers for the acquisition of comparative information on students' learning and also a vehicle for teachers to reflect on the goals for development of competencies as stated in the national guidelines.

Research Questioning Communities Involved in Curriculum Reforms[1]

In this sub-section, we discuss Pinto and Cooper (2018) who provide a compelling case study in this line of research through their focus on analysing cases of cross-community interactions and collaboration in reform committees. The research question that guides their contribution is: how can members from different communities in mathematics education collaborate productively in curriculum and reform committees? More concretely, the authors reflect on the potential contributions that members from different communities in mathematics education make when taking part in curriculum reform committees that typically develop the intended curriculum. Their hypothesis is that cross-community collaboration significantly enriches the results of the discussion. However, such committees rarely capitalise on the opportunities of their diversity.

[1] This sub-section analyses work with a particular socio-cultural orientation that does not itself lead to classification in terms of the diagram in Fig. 13.4. Hence, it does not include a diagrammatic representation of the approach.

Pinto and Cooper use the theoretical framework of commognitive theory (Sfard, 2008). They consider curricular discourse as the way in which individuals or communities communicate, think and act with regard to the mathematics curriculum. When committees work, the sociocultural differences of the comprising members are taken as differences in curricular discourse and provide commognitive conflicts. These conflicts force the individuals to make transitions and interactions across different points of view (boundary-crossing), which generate processes of learning by means of mechanisms of reflections and hybridisation. Though the work of reform and policy committees is usually confidential, the authors analyse data from various communities that are stakeholders in mathematics education. The findings show that boundary-crossing should be an explicit aim for committees, and the role of those participants that facilitate this boundary-crossing (named brokers) should be intentionally studied.

O'Meara et al. (2018) employ the framework of Memon (1997) to frame the barriers to a recent national mathematics curriculum reform in Ireland. The barriers were identified in three studies conducted locally to evaluate the implementation of Project Maths. Thus, 'Mind the Gap' sheds light on components that are 'Organisational Factors', the 'Time in Mathematics Education' (TiME) study investigates the 'Curriculum Factors', and the 'Teachers' Perception of Curriculum Reform' study mainly addresses components of the 'Instructional Factors'. All three studies are based on large online questionnaires distributed to 700 primary and 400 post-primary school teachers, exploring their experiences, viewpoints and beliefs. There is therefore a different unit of analysis from for the work of Pinto and Cooper described above, now focused on the primary and post-primary teacher communities.

They investigate teachers' perceptions of the recently reformed mathematics curriculum and identify any misalignments that exist between the beliefs held by teachers and the goals of the reformed curriculum. This study points to a reform effort that is reasonably coherent in terms of, for example, the six components of Niss' (2018) framework. It is noteworthy, however, that the implementation of the reformed curriculum was still challenged by too little attention paid to 'time' as an important component, a variable not explicitly considered in either the Memon (1997) framework or the other frameworks depicted in our Fig. 13.4. Their findings suggest that time is a critical and defining factor in the successful implication of curriculum reform. Teachers in the TiME study clearly indicated that time is impacting on their ability to implement the curriculum as intended, thus adversely impacting on students' opportunities to learn.

Conclusion and Key Messages

This chapter set out to provide an overview of the state of the art of theoretical frameworks and the associated methodologies used to address phenomena related to school mathematics curriculum reforms. Our approach has been to first identify the main theoretical approaches (in the second section), and to organise those by the purposes to which they are typically put. We exemplify such purposes, and approaches, with case studies (in the third section) that show the diversity of lines of research addressing the complex reality of taking curriculum reforms as an object of study for research in mathematics education. We now summarise what we have found and critically analyse those findings to identify the further questions that arise.

The most striking finding from our work to identify theoretical approaches is that, whilst there are clear instances of rigorous and careful use of a theoretical framework to analyse curriculum reforms, such examples are in a distinct minority, both in the work presented to ICMI Study 24 – which has a focus on curriculum reforms – and beyond. In many instances we were unable to find explicit reference to any theory supporting researchers' analysis of curriculum reforms. In others there was what might be called a passing reference to theory without evidence of how it was applied in the analysis and the particular methodology followed.

We have started this chapter, in the second section, by presenting the most prominent theoretical approaches discussed in the different themes identified. We consider our analysis to be useful as it makes explicit the tools and methodologies the different approaches offer to analyse curriculum reforms. We present these theoretical frameworks according to three broad thematic categories, depending on their focus. First, we have those which aim to provide definition and conceptualisation of curriculum; second, the more general approaches focused on the analysis of the epistemological and didactic processes used to scope the curriculum, and how curriculum reforms are transposed to the different institutions for their definition, teaching and learning; and third, approaches focusing on how cultural, social, contextual factors impinge on the possibilities for that transformation, constraining or supporting curriculum reforms. Consideration of these three types of approaches led to the development of a schema (Fig. 13.4 in this chapter) that identifies 'levels of implementation/ institutional position' for each by reading vertically. The schema also identifies some tentative parallels or connections between the three broad categories of theoretical approaches by reading horizontally.

It is acknowledged that this schema represents a tentative means for a visual representation of the components of three quite diverse theoretical approaches and the connections between them, though we acknowledge there are more than three such we could have focused on. We have found it useful to describe the lines of research on curriculum reform in the previous section and, more concretely, to delimit the unit of analysis considered by the different case studies selected, with reference to that schema, validating its use in this chapter. Further work with, and development of, the schema may generate a more robust tool for identifying the components and connections when planning studies to address real research questions about mathematics curriculum reforms.

The current version is included also in the following explanation of the examples, but we have focus on these three particular theoretical frameworks as particularly useful to describe and delimit the lines of research on curriculum reforms we earlier analysed. The previous uses case studies to provide specific examples of research on mathematics curriculum reform. The studies represented are diverse in many respects including geographic location, scale, specific focus and underpinning theoretical approach.

The case studies used are classified according to the line of research they represent: we have distinguished among five broad lines of research related to curriculum reforms, classified according to what it is questioned and what is not. We explore connections and coherence between specific components addressed in the case studies, within those broader research lines. These include the interaction between the scholarly knowledge and the knowledge to be taught in the context of the intended curriculum (RL1); between the knowledge to be taught and the resulting intended curriculum (RL2); in domains including the conditions under which curriculum reforms are implemented, with reference to means and constraints (RL3); teachers' actions on curriculum design and student attainment, with reference to how the implementation of curriculum is planned and works in classrooms (RL4); and the communities involved in curriculum reforms (RL5).

This is an indicative list of lines of research – it is likely that others are able to be identified. For example, the current international interest in comparative student achievement is likely to be generating research line(s) that consider the attained curriculum with others of the Niss components of curriculum. While some of the case studies refer to student attainment as evidence, the lack of representation of studies of student attainment in this collection could be an indication that such studies do not have a strong theoretical base and therefore lack scientific rigour.

The treatment of the main case studies follows the same pattern of identifying the specific research question(s) addressed, identifying the unit of analysis and data considered; outlining the methodological choices and tools used, and providing the key findings and answers to the research question(s). The diversity of the case studies included in the previous section is reflected in the wide range of findings about coherence (or lack of coherence) in mathematics curriculum reforms, aspects such as as challenges that are inherent in reforms, identification of unintended consequences, implications of teachers having an active role in reforms rather than being passive recipients, and lost opportunities, among others. Whilst such findings may well feature in studies that do not have a sound theoretical basis, the theoretical rigour of these studies should make them more credible and more worthy of attention.

In this chapter, we have only been able to scratch the surface of the intersections between mathematics curriculum reforms and the theories and methodologies used for studying them. As a result, the findings cannot be seen as more than indicative of some orientations that can be fruitful in studying reforms. In addition, we have developed a systematic approach that may well appeal to others who want to consider and learn from other analyses of curriculum reforms. Further use of the theoretical approaches will serve to validate, refine and extend the tools available for investigating and understanding phenomena related to mathematics curriculum reforms.

Despite these limitations, one finding that does stand out and is likely to be generally applicable is that – with notable exceptions, some of which are identified in this chapter – many reviews of mathematics curriculum reforms are not well supported by a clear theoretical basis that guides the methodology used. This necessarily limits the robustness of the analyses and, very likely, the impact of the work. The fact that this a-theoretical approach is so common can lead to the conclusion that many of those involved do not perceive having a well-defined theoretical framework as being important to their work – or that the theoretical framing is, unhelpfully, implicit only. The field of research on mathematics curriculum reform would be strengthened by an increased subjection to scrutiny of explicit theoretical underpinnings. The theoretically robust exceptions highlighted in this chapter and elsewhere can be considered examples of 'good practice' that can inform and set the scientific standard for future analyses of mathematics curriculum reforms.

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Chapter 14
Conclusion Achieving Coherence and Relevance in Mathematics Curriculum Reforms: Some Guiding Principles

Will Morony

There is no doubt that the mathematics curriculum will continue to be subject to reforms in response to the changing social and political contexts of education around the globe. The scale of the reforms will range from being global (in response particularly to collaborative international programs), through national and regional levels in different countries to the level of individual schools. We have seen examples at all of these scales from around the world, and analysed these for 'coherence' and 'relevance'. Maximising both these characteristics of any given reform – coherence and relevance – is seen a means for maximising the alignment between the 'intended' curriculum and the 'enacted' curriculum.

As has been demonstrated through the analyses included in this section, the concepts of coherence and relevance are, when applied to mathematics curriculum reforms, multi-faceted and evident within the curriculum itself. The two concepts also pertain to the curriculum in relation to other aspects of education and the society more generally. Achieving deep and sustainable coherence and relevance through contemporary emphases in curriculum reforms that emphasise STEM and interdisciplinarity has been shown to be substantially more complex and challenging (Chap. 11).

A telling finding from Chap. 13 is that there is evidence that there is a lack of conscious and careful application of theory to analyses of mathematics curriculum reforms. As a result, there is no systematic basis in the literature for analysing proposed reforms to identify those likely to be successful, and the conditions required for that success. The comment of Sir Winston Churchill – that, "Those who fail to learn from history are doomed to repeat it" – is clearly relevant to mathematics

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curriculum reform. Until and unless studies of curriculum reforms have a strong theoretical basis and associated methodologies, proposals for reforming mathematics curricula will remain 'good ideas' that are likely to fall well short of their intentions like so many in the past. What is needed is 'good science'.

What then are some key principles that should underpin the design and rollout of mathematics curriculum reform that is coherent and relevant.

- Careful consideration and representation of the mathematics that is the subject of the curriculum – its structure, connections and ways of knowing and doing, both within and outside the discipline.
- The resources that are developed to support implementation of mathematics curriculum reforms in coherent ways need to be carefully designed to be adaptable to different contexts and changing circumstances, in ways that are accessible and sustainable.
- The means and methods used for assessing student attainment, and their coherence with other components of the curriculum, require particular attention in order to support faithful enactment of curriculum reforms.
- A commitment by all stakeholders to consider evidence from relevant scientific studies of curriculum, teaching and student development and attainment, and to initiate such studies as and when these are needed to guide mathematics curriculum reforms. These studies need to be based on the 'good science' outlined above.
- These studies should include continuous evaluation of coherence and relevance in enactment of curriculum reforms with a willingness and capacity to address slippages when these become apparent.
- Respect for the existing knowledge of teachers, and their capacity to adopt new ways of working with their students when provided with appropriate, consistent and sustained support – materials, initial teacher education, ongoing professional development, schooling structures and leadership, along with encouragement in their work, including acknowledgement of their teaching achievements.

A clear theme in our analyses is the importance of alignment between the curriculum and the curriculum system in which it is enacted, and, as we have shown in many cases, the clear negative impacts of misalignment between the two. This misalignment limits the effective and coherent enactment of a curriculum and the reforms it embodies. Just as a curriculum needs to be carefully designed, so too does the curriculum system if the two are to work in harmony and achieve the goals of the curriculum.

Part IV
Theme C – Implementation of Reformed Mathematics Curricula Within and Across Different Contexts and Traditions

Chapter 15
Introduction

Angel Ruiz

In the pages that follow, we include various experiences or examples mainly from papers and discussions about the implementation of mathematics curriculum reforms in the following countries or regions: Australia, China, Costa Rica, Denmark, France, Hong Kong, Israel, Italy, Japan, Lebanon, Luxembourg, Mexico, the Philippines, Quebec (Canada), South Africa, Spain, Thailand, Tunisia, United Kingdom, Vietnam, Wallonia-Brussels Federation. Most of the cases were collected during the *ICMI Study 24 Conference* on the theme 'Implementation of reformed mathematics curricula within and across different contexts and traditions'.

Diversity

The implementation of mathematics curriculum invokes from the beginning what is very important to underline: the enormous diversity of these processes. To begin with, the implementation depends on the nature of the reform: There are reforms that seek to impact fragments of a few grades or dimensions of the curriculum (something that can be very important) but also, we find reforms that affect profoundly all school levels. Some may affect content, aims and certain teaching approaches; others can invoke drastic significant paradigm changes. Diversity among the reforms is one of the first factors that we find. But there are many more, for example:

The general locus There is context diversity that can be about culture (East–West), or socio-economic conditions (developed or developing). And also, the reform impact may affect different geographical or social endeavours (national, regional).

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National political and institutional scenarios There are diverse national institutional features in relation to the education system. For example, curricula can be national, state centralised or not. The structure and influence of government and politics on education can strongly affect the characteristics of a reform (its continuity and support, for example).

General strategies and timeframes The general strategy adopted for the implementation process can generate important differences. For example, if is top-down or bottom-up, or a combination of both. Timeframes (expected) for a reform implementation define different actions. A reform that can be implemented during just a few years is not the same as one that would take one decade, or when the expectation is to have a definitive implementation in a 'generational' time period (during decades).

Influences The reforms can have different combinations of influences that can impact their development: international (twenty-first century abilities, competences, problem solving, STEM, PISA) and local (national constructs, responses).

The situation of educational agents There are different types of teacher preparation, and the characteristics of teacher hiring or professional development systems can be very diverse. The role of education advisers, supervisors and other education officials may be very important for the development of a reform and the role of these agents can be very different. The impact of education and academic associations or even unions within a country may play decisive roles and are different in each context.

These intermingled different conditions play a role in the meanings of a curriculum: 'intended', 'implemented', 'attained. And that multiplicity of scenarios allows us to understand that what in one context may be an insufficient realisation or a failure, in another may be what one can aim for and its realisation represents a success. Every factor can be both an advantage and a constraint when considering the implementation of curriculum reform. For example, on one hand, while a curriculum reform addressing all school levels allows the opportunity for a design with curricular aims developed and connected through many years, it may be too great of a demand on resources to be able to be implemented effectively. On the other hand, a reform addressing only one level of schooling may be the opportunity for teachers to engage in deep professional learning but may be hampered by lack of continuity.

What is the first warning for the reader of this section? Though we try to identify common elements, patterns, local or national models, good practices, or international standards that can provide support to understand the processes of implementation of curricular reforms, we find it prudent not to make many generalisations and extrapolations that could distort the subject. The indications or even lessons that are included in this section and chapters should be covered with that intellectual mantle.

The 'Process' Dimension and the Wider Perspective

Besides diversity, we want to emphasise two more aspects. One is connected to the distinction of curriculum as 'product' or as 'process'. Without a doubt, there is an interplay among 'intended', 'implemented', 'attained' curricula, but in relation to implementation there is a fundamental weight of the agents and strategies involved, and therefore the 'process' dimension is the crucial one. The perceptions and attitudes of these agents, as well as the nature and manner of dealing with implementation strategies, occupy the locus where we seek to investigate what happens according to the diverse experiences that we have. Somehow, this stresses that a curriculum reform should not be considered static, synchronic in all its components or void of historical dynamics: consciously or not, planned or unexpected, many things can change during the 'process' of a reform.

A second aspect: undoubtedly, teachers occupy a privileged place in curriculum implementation as do resources and assessment (we are dedicating one chapter to these), but all these elements, again, play roles that depend a great deal on the nature of the reforms, but essentially on the contexts where they occur. And that raises a crucial issue: It should not be thought that the characteristics and opportunities for the implementation of a reform in the teaching of mathematics depend only within this discipline, it is very common that the timing and fate of the reforms, especially when they are deep transformations, depend on wider social or national variables.

Structure

We have organised our theme through three chapters and a conclusion.

Chapter 16 examines curriculum reforms in Denmark, France, the Philippines, and mainland China in considerable detail. Reforms in Wallonia-Brussels Federation, Tunisia and Quebec (Canada) are discussed in much less detail. In the contributions included, the authors offer elements to answer the first question that guided the theme "Implementation of reformed mathematics curricula within and across different contexts and traditions" of this *ICMI Study*: 'What processes, models, or best/common practices can be identified from the experiences in the implementation of new or reformed school mathematics curricula?' Similarly, the description of the reforms provides a first line of response to the question: 'What are examples of successful or unsuccessful reforms and what are the reasons for their success or failure?' And elements are given on, 'What criteria are used for assessing curriculum reforms and their degree of success or failure?' In this chapter, theoretical or conceptual frameworks are introduced (by M. Artigue and M. Niss) to calibrate the dimensions or general components present in every curricular situation or to understand with more universal categories, the place, interactions, scopes and impacts of the reforms in the educational fabric, social, national or international.

The aim of Chap. 17 is to identify factors that intervene within mathematics curriculum reforms and precisely seek for ‘processes, models, or best/common practices’ that can be relevant for the progress or success of a reform. To collect elements from diverse contexts (cultural, socio-economical and geographical), which can support our analysis, three cases are introduced first with certain detail: Japan, Thailand and Costa Rica. One of our purposes, to fulfil for all curriculum reforms was to identify visions, values and goals, that may condition curricular reforms and their implementation beyond just a few national scenarios.

Chapter 18 first responds to the fourth question of the *ICMI Study* Discussion Document: “What models or processes for professional teacher preparation and continuous development have been carried out in different countries in the implementation of new or reformed curricula; and what are their influences, effectiveness, successes or failures?” It analyses the factors in their initial preparation and in their professional development that act in the curricular implementation, the interrelation between reform and teachers’ action. Secondly, it responds to the fifth question of the cited document: “What are the types of resources and what are their roles (e.g. textbooks, materials, technology) in the implementation of reformed curricula?” Then the participation of the diverse resources is also studied here: material and social, based on technologies or multimedia. The chapter, additionally, points out the role of assessment at the national or international level as a conditioning factor, and at the same time, as a potential instrument in curricular development.

There is no chapter completely dedicated to responding directly to the set of questions, “How is the implementation of new or reformed curricula monitored, evaluated, and acted upon? What are models or mechanisms of continuous improvement in school mathematics curricula? How does the existence of such a mechanism affect the frequency, (dis-)continuity, and perceived challenges and successes of curriculum reforms?” However, some parameters that can serve as a means to gauge the success or progress of a reform are indicated within the first three chapters (especially in Chap. 17).

The conclusion (Chap. 19) seeks to provide a set of ‘laws’ that emerge from the studies carried out in all the previous chapters. It is not, however, a systematic collection of the results associated with each chapter, it is rather a meta-reflection. It also includes a brief insight on the impact of the pandemic provoked by COVID-19 for the implementation of curriculum reforms.

Chapter 16
A First Exploration to Understand Mathematics Curricula Implementation: Results, Limitations and Successes

Angel Ruiz, Mogens Niss, Michèle Artigue, Yiming Cao, and Enriqueta Reston

This chapter is divided into five sections. In the first, Mogens Niss addresses the case of a partially successful curriculum reform in Denmark. It is the KOM Project (that began in 2000) which developed a theoretical proposal that sought to respond to specific difficulties in the transition between various cycles of education including higher education as well as weaknesses in STEM programs. That proposal focused on the concepts of mathematical 'competence' and 'competences', with a perspective that has impacted the international community, especially through its influence on the OECD's PISA tests. To better understand the characteristics of the Danish experience, Niss establishes a much broader conceptual framework that establishes six dimensions in every curricular situation: goals, contents, materials, teaching methods, student activities and assessment. The reform was partially successful as competence-competences were included in the goals of education, in teaching methods and student activities in Denmark, but not, for example, in assessment and content.

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In a second section, Michèle Artigue reports two types of curricular implementation: on the one hand, a successful reform in France at the beginning of the twenty-first century in the senior high school around statistics, the place given to the stochastic domain, the use of computer-simulations, and also the importance of interdisciplinary work. And, on the other hand, it summarises the development of reforms in some Francophone countries or regions: Wallonia-Brussels Federation, Tunisia and Quebec (Canada); some that failed, one crowned with success. In all these cases, strategies, and disciplinary, educational and even social variables are described, which would influence the success or difficulties of these reforms. Artigue analyses these cases by referencing general theoretical concepts of the 'ecological' perspective associated with the Anthropological Theory of the Didactic which is an extension of the Theory of Didactic Transposition.

In a third section, Yiming Cao describes a broad curriculum reform of the teaching of compulsory mathematics in mainland China (which began in 2001 and was finally approved in 2010). Some of the characteristics of the new curriculum are indicated in contrast to the previous one, and its various phases of implementation are discussed.

The fourth section describes various professional development models in the Philippines to try to implement a general curriculum reform approved in 2012. Enriqueta Reston points out the weaknesses of some of them and provides elements of new modalities that, even isolated and sporadic in the Philippines, can support teacher professional development and implementation of this curriculum reform.

In the fifth section, Angel Ruiz synthesises conclusions derived or inspired from the contributions of the preceding sections.

In all these cases, it is possible to observe models of good or inadequate practices, reforms with relatively broad or partial successes, or failures, as well as the role of teacher preparation and support materials for curricular implementation.

National and International Curricular Use of the Competency-Based Danish "KOM" Project – Mogens Niss

This section begins by offering a conceptualisation of 'curriculum' as a vector consisting of six components ('goals', 'content', 'materials', 'forms of teaching', 'student activities' and 'assessment'). So, a curriculum is determined by specifying each of these components and is implemented by enacting them. I present the Danish competence-based KOM Project and discuss the extent to which this project has been implemented in curriculum reforms in Denmark. The answer is 'only partly', primarily because no official measures were instigated to ensure professional development of educational authorities and teachers. Nevertheless, the KOM Project had quite an impact on Danish mathematics education, albeit 'from below', but ~~had~~ perhaps an even greater impact internationally. Finally, I offer some reflections on the conditions for successful implementation of novel curriculum ideas.

Terminological Clarification of Key Concepts

The title of the panel to which this section is a contribution is: "Implementation of reformed mathematics curricula within and across different contexts and traditions". In addition to 'mathematics', this title contains some key words, such as 'curriculum', 'implementation' and 'reform(ed)', that are in common use around the world, yet carry a lot of different meanings. I therefore find it necessary to begin by proposing some clarification (I hope!) of these and some related terms.

The key word 'curriculum' means rather different things in different places (Niss, 2016). Thus, the Collins Cobuild dictionary (1999) offers the following definition: "A curriculum is all the different courses of study that are taught in a school, college or university" (p. 401). Kilpatrick (1994), in contrast, focuses on a single subject rather than on a collection of subjects and writes, "The curriculum can be seen as an amalgam of goals, content, instruction and materials" (p. 7). A somewhat different definition, focusing on the mathematics teacher and on what is actually happening in the classroom, is put forward by Stein, Remillard and Smith (2007): "we use the term curriculum broadly to include mathematics curriculum materials and textbooks, curriculum goals as intended by the teacher, and the curriculum that is enacted in the classroom" (p. 319; *footnote*).

Irrespective of what definition of curriculum we adhere to, any curriculum is situated and lives within an *educational setting*, i.e. the institutional, structural and organisational entity within which the teaching and learning addressed by the curriculum take place. A prime example of an educational setting is the entire public-school system of a given country. As other examples we may think of a particular school or tertiary institution, or a particular course in, say, a university.

In Niss (2016) I proposed, along the lines of Kilpatrick's definition, to define a (mathematics) curriculum with respect to a given educational setting as *a vector with six components*, as follows:

- *goals* (the overarching purposes, desirable learning outcomes, and specific aims and objectives of the teaching and learning taking place under the auspices of this curriculum);
- *content* (the topic areas, concepts, theories, results, methods, techniques, and procedures dealt with in teaching and learning);
- *materials* (the instructional materials and resources, including textbooks, artefacts, manipulatives, and IT systems employed in teaching and learning);
- *forms of teaching* (the tasks, activities and modes of operation of the teacher in this curriculum);
- *student activities* (the activities of, and the tasks and assignments for, the students taught according to this curriculum);

- *assessment* (the goals, modes, formats and instruments adopted for formative and summative assessment, respectively, in this curriculum).

Specifying a curriculum in a given educational setting then amounts to specifying each of these six components. Furthermore, *implementing* a given curriculum amounts to specifying it, as well as to *carrying it out*, i.e. putting all the six components into practice.

The agency that determines a curriculum and has the power to implement it within some educational setting is the *curriculum authority* for that curriculum (Niss, 2016). It may happen that a curriculum authority chooses to leave some of the six components unspecified. Then these components are open for others, e.g. local governments, institutions or teachers, to decide upon and specify, for instance by way of enactment. In some countries national curriculum authorities specify only a few of the components, typically 'goals', 'content' and 'assessment'.

What, then, do we mean by *reformed* mathematics curricula? The term 'reform' suggests some desired changes of a rather fundamental nature, which are likely to affect several or all components of the curriculum. Usually, one wouldn't use the term "reform" unless at least 'goals' and 'content' are explicitly affected. However, the other components are likely to be affected as well, by derivation, even though this may not be explicitly intended.

Competency-Based Mathematics Curricula: The Case of Denmark

In the late 1990s, the Danish Ministry of Education saw a need for reforming the mathematics (and other) curricula in Denmark across all educational levels. This need was spurred by a number of issues and problems that became more and more manifest and visible within and outside the education system. These included that too many students did not benefit enough from the mathematics instruction they were offered, and that there were serious transition problems and severe academic and socio-cultural discontinuities when students moved from one segment of the education system to the next, from primary through to tertiary education.

These transition problems went hand in hand with insufficient progression in students' mathematical learning within and across these segments, which led to 'consumer' complaints about the decrease in students' mathematical capabilities. Moreover, many people thought that not all teachers were adequately prepared for offering high quality mathematics teaching to their students. These problems were seen as (co-)responsible for the fact that students opted away from further education programmes in science, mathematics and technology, which was (and is) considered a serious societal problem.

Against this background, the Ministry, in 2000, established a commission (a task force), composed of mathematicians and mathematics educators (researchers, teachers, and ministerial inspectors) and a few representatives from society at large.

The Commission was chaired by me, while Tomas Højgaard (Jensen) was its academic secretary. The task of the Commission was: (1) to identify, uncover, chart and analyse the entire set of *problématiques* pertaining to mathematics education at all levels of the Danish education system; (2) to propose measures and tools that were likely to be effective in improving the state-of-affairs by counteracting the problems identified and by remedying (some of) the deficiencies observed; these measures were to include drawing up guidelines for the design of new curricula. The Commission worked for 2 years in what became known as the *KOM Project* ("KOM" is a Danish acronym for "Competences and the Learning of Mathematics"), and ended up publishing a report, known as the KOM Report (Niss & Jensen, 2002; Niss & Højgaard, 2011, 2019), which was discussed widely in several places and quarters in Denmark and soon after in a number of other countries as well (e.g. Germany, Norway, Sweden).

The brief for the KOM Project was far from solely focused on proposing new curricula, but had a much wider scope. So, the Project was *not*, first and foremost, meant to be a curriculum project. However, it was assumed by the Ministry, and also by the members of the Commission, that the design of mathematics curricula could be substantially supported by the outcome of the work. I shall return to this issue below.

The KOM Project took its point of departure in the need for creating and adopting a general conceptualisation of mastery of mathematics that goes across and beyond educational levels and institutions. Only then would it be possible to deal with mathematics in a manner that was neither tied to nor dependent on particular levels and types of institutions, which was necessary in tackling the transition problems in the education system. We also wanted to avoid being locked into the specifics of particular mathematical subject matter domains or topics such as algebra, geometry, functions, calculus etc., the place and content of which vary greatly across levels and institutions.

We therefore decided to base our work on an attempt to define and characterise mathematical competence in an overarching sense that would pertain to and make sense in any mathematical context. Focusing (as a consequence of this approach) first and foremost on the *enactment* of mathematics means attributing a secondary role to *specific* mathematical content, which does not mean that mathematical content as such was to be of secondary importance, of course not.

We came up with the following definition of mathematical competence:

> Possessing *mathematical competenc*e – mastering mathematics – is an individual's capability and readiness to act appropriately, and in a knowledge-based manner, in situations and contexts that involve actual or potential mathematical challenges of *any kind*.

In order to identify and characterise the fundamental constituents in mathematical competence, we introduced the notion of mathematical competences:

> A *mathematical competency* is an individual's capability and readiness to act appropriately, and in a knowledge-based manner, in situations and contexts that involve *a certain kind* of mathematical challenge.

A metaphor may illuminate the relationship between competenc*e* and a competency: if we think of mathematical competence as a huge, complex molecule (say a polymer), the competences represent much smaller building blocks (atoms or monomers) in this molecule. *Eight competences* were identified, in the beginning on theoretical and experiential grounds only. Later on, they became corroborated empirically. These are:

- mathematical ***thinking*** competency – mastering mathematical modes of thought;
- ***problem-handling*** competency – being able to pose and solve mathematical problems;
- ***modelling*** competency – being able to analyse and construct mathematical models;
- ***reasoning*** competency – being able to reason mathematically in the context of justification of mathematical claims;
- ***representation*** competency – being able to handle different representations of mathematical entities;
- ***symbols and formalism*** competency – being able to handle symbolic language and formal mathematical systems;
- ***communication*** competency – being able to communicate, in with, and about mathematics;
- ***aids and tools*** competence – being able to relate to the material aids and tools for mathematical activity.

Since the competences are meant to go across all mathematical subject matter domains, in a given educational setting it neither makes sense to consider deriving the competences from such domains, nor to consider deriving domains from the competences. Even though the competences can, of course, only be developed and exercised in dealing with subject matter, the relationship between competences and mathematical domains should be perceived as constituted by two independent, yet interrelated dimensions, of a matrix composed of competency rows and topic columns. Each cell in this matrix represents the relationship between the competency in the corresponding row and the topic in the corresponding column. More specifically, it allows one to specify the ways in which this competency plays out in dealing with a given topic, and the ways in which that topic plays out in exerting the competency at issue.

KOM-Referenced Curriculum Reforms in Denmark in the Twenty-First Century

Even though, as mentioned above, the KOM Project was not primarily established as a curriculum project it was certainly intended and expected that the outcomes of the project, including the eight mathematical competences, would be instrumental in designing new curricula that would help counteracting some of the problems identified prior to and within the project. Although the notion of curriculum introduced at the beginning of this section was not in place at the time of the KOM Project, the project actually adopted a similar notion of curriculum, which was also reflected later in the Danish curriculum reforms of the twenty-first century, in 2009, 2014 and 2017 for primary and lower secondary mathematics (grades K–9) and in 2005, 2013 and 2017 for upper secondary mathematics (grades 10–12). As regards grades K–9, the curriculum reform in 2009 was also much influenced by the report of another committee led by me (*Udvalget til forberedelse af en handlingsplan for matematik i folkeskolen*, 2006), which spelt out ways in which the competence thinking could be implemented in pragmatic terms.

The curriculum documents representing these reforms all included important bits and pieces of the KOM Project but it would be incorrect to say that the reforms were a clear-cut implementation of the Project in its entirety. As to the above-mentioned six curriculum components, these were all addressed in the different curriculum designs, albeit with varying degrees of specification. It follows from what was said above that the 'content' component had to be specified independently from the competences, whereas these contributed to shaping the other components. The 'goals' component, in particular, was typically formulated in competency terms.

In a number of different ways, the KOM Project was a great challenge to traditional conceptualisations of mathematics teaching and learning in Denmark. With the project's primary emphasis on the enactment of mathematics, across education levels and mathematical topics, rather than on mathematical content, curriculum authorities – the official Danish education system, governed by the Ministry of Education, as well as teachers, experienced difficulties in coming to grips with how the outcomes of the KOM Project could in fact guide the design and implementation of new curricula that were not (to be) defined in terms of classical content strands. Furthermore, the issue of assessing competences rather than content knowledge and procedural skills also constituted (and still constitutes) a challenge to the system.

This implied that the new curricula of the first two decades of the century continued to be primarily based on subject matter domains, whereas the competences were presented in the general sections of curriculum documents, stating that the teaching of those domains should pursue competency-oriented goals, whilst paying attention to the competences 'throughout' teaching activities (the vector components 'forms of teaching' and 'student activities').

In Denmark, national exams at the end of grade 9 and again at the end of grades 10, 11 or 12, (depending on which of several possible upper secondary streams the individual student is in) are high stakes exams organised by the Ministry. Without going into details with the somewhat complex exam structure and organisation, the written component of those exams ended up paying almost no attention to the competences. In the oral component, which is mainly dealt with locally within the individual school, there is room for focusing on the mathematical competences, if the teacher so wishes, which is also the case when it comes to formative assessment.

In other words, the crucial curriculum component 'assessment' was never markedly influenced by the competency approach, and since 'what you assess is what you get' this partly jeopardised the competency approach and made it largely rhetorical at the official level. However, other curriculum components, such as materials (including textbooks), forms of teaching, and student activities were oftentimes influenced by the competency thinking of the KOM Project. The same is true of pre-service teacher education and in-service professional development.

So, whilst the competency approach mainly had a rhetorical impact on the official curricula, especially as regards the components that are somewhat tightly controlled by the Ministry of Education (the vector components 'goals', 'content', and 'assessment'), it would not be correct to say that this approach has had no impact on the implementation of these curricula in everyday practice. As a matter of fact, the competency approach and the associated terminology substantially influence the *discourse* amongst mathematics educators in Denmark, who readily express themselves and explain their activities in terms of the KOM competences.

Ironically, then, we may say that what from the point of view of the Ministry should have provided a top-down platform for an entirely new approach to mathematics teaching and learning never became such a platform, primarily due to inertia in the different segments of the official system; whereas, the approach and the thinking of the KOM Project gradually, in a bottom-up process, crept into significant – but certainly not all – aspects of everyday mathematics education in Denmark. This bottom-up process took several different forms, ranging from a large variety of local implementation projects, typically focusing on a few of the competences at a time (many of which were conducted by the KOM Secretary, Tomas Højgaard), over new KOM-inspired textbook systems for primary, lower secondary or upper secondary school, and expository publications by or for practising teachers, numerous articles in national journals or teacher magazines, through to pre- and in-service programmes and resource materials for teachers. Each of these bottom-up activities typically involved a selection of the enactment-oriented components of the curriculum vector.

This development begs an answer to the question: Why did things happen in this way? Well, this is a highly complex issue, which involves a combination of universal as well as national features of curriculum design and implementation. I shall focus on the national ones.

It is clear that the thinking in and behind the KOM Project and the competency approach taken were highly novel, ambitious and demanding for the Danish education system and for teachers to come to grips with. So, it was far too optimistic to expect that the KOM Project ideas could be transposed into curriculum design and implementation without further ado, just by reading the KOM report. Neither the curriculum authorities nor the teachers asked for, or were given, a systematic, thorough introduction to the ideas and their consequences, or were offered professional development activities beyond the written report itself.

This is typical of Denmark, in which political unwillingness to spend public money on human resources, in combination with anti-elitism, has had a strong foothold over the last 50 years. In retrospect it would have been absolutely necessary for a much more forceful and effective implementation of the competency approach in Danish curricula to have had large-scale, systematic in-service activities within all layers of the system. In the absence of such activities, the competency ideas had to enter the system mainly by osmosis, which they certainly did.

Against this background it is remarkable that the KOM Project thinking and the competency approach have in fact influenced mathematics teaching and learning in Denmark as much as they have, especially by way of a multitude of enacted versions of the curriculum vector. This can only be explained by the existence of serious needs amongst educational authorities and mathematics educators for conceptual innovation in mathematics education. The policy lessons that can be learnt from this case are primarily two: (1) you cannot effectively pursue goals and aims unless you are willing to invest and apply material and immaterial means that are conducive to the aims and goals; (2) only very rarely are top-down measures successful. If you really want to achieve change, it is essential that those who are to bring it about have ownership not only of the need for change but also of the means to achieve it. If not, you might be able to see changes on the surface of things, but they will not really affect the substance of what is desired and expected.

The Competency Approach in Other Countries

During the first two decades of this century, many countries and quarters took an interest in the KOM Project and in the competency approach to mathematics education (Niss et al., 2016). This was partly, but not exclusively, stimulated by the fact that competency ideas were involved in shaping all the PISA mathematics frameworks between 2000 and 2012 (Niss, 2014) by underpinning and developing the notion(s) of *mathematical literacy*. However, due to direct personal contacts between mathematics educators in Denmark and in countries such as Germany, Norway and Sweden, these countries early on adopted and adapted aspects of a competency approach as well as some of the related KOM Project ideas in their curriculum development. In particular, the German *Länder*, in the first decade of this century, agreed to take an explicit competency approach when reforming their curricula, leading to the so-called 'Bildungsstandards' (see, for example, Kultusministerkonferenz, 2012). Many countries in Latin America and Spain were also inspired by the competency ideas, primarily via PISA.

It is important to observe, here, that it was never a matter of direct translation and adoption into curriculum design and implementation in other countries of the KOM Project ideas or documents. Rather, it was a matter of modification and adaptation of (some of) these ideas so as to suit national circumstances, needs and traditions. Oftentimes, the eight competences of the KOM Project were modified in various ways, typically into fewer than eight. In some cases, adaptations were not even in accordance with 'the spirit' of the Project, only inspired by some of its features.

Once again, there are lessons to be learnt from these developments. Firstly, one should never aspire to directly translating, transferring and adopting curricula or curricular ideas from one country or setting to another. Such import, even of curricula that were highly successful in their original setting, is almost doomed to failure because the socio-cultural environments and the economic, technological, and institutional boundary conditions vary so much within and across countries. Secondly, the lesson just mentioned should not be taken to suggest that inspiration from others is likely to fail. On the contrary, and this is the second lesson worth highlighting, thoughtful and careful consideration of what others have accomplished, whilst paying attention to the conditions and circumstances under which the accomplishments were achieved, is likely to stimulate positive innovation (and innovation always comes with a "sign" and hence may also be negative) in new places, provided those who are to implement this innovation are genuine shareholders in it.

Implementing Curricular Reforms: A Systemic Challenge – Michèle Artigue

This contribution addresses the challenge of implementing curricular reforms. I first introduce the approach I propose, considering education systems as complex dynamic systems, and the main theoretical elements I rely on, offered by the anthropological theory of the didactic (ATD) and the ecological perspective underlying it. Then I use this approach to discuss the challenge raised by the implementation of curricular reforms using, as case studies, the 2000 high school curricular reform in France, and the implementation of competence-based curricula in three different Francophone countries. I conclude by drawing some lessons from these case studies.

Introduction

As highlighted in the Discussion Document for this ICMI Study, curriculum reforms are transformations that generally affect education systems "as a whole, at a national, state, district or regional level" (2018, p. 572). They modify the conditions and constraints of their functioning to cause changes in the state of these systems. Their *raisons d'être* are situated at different levels: the content of teaching, the balance and relations between school disciplines, pedagogical methods, or more generally the social contract between a society and its schools; more and more these reflect supra-national visions. Their design mobilises a diversity of institutions and agents, and their implementation an even greater number. Design and implementation processes take place over time and their dynamics depend on a multiplicity of factors in interaction.

When a curriculum reform is eventually adopted by authorities, these factors and their possible interactions are only partially identified and even less controlled, if even controllable. The curriculum texts, however constraining they may appear, give some margin of freedom to all those involved in the implementation for expressing their agency, which opens up a range of possible dynamics whose regulation is a crucial issue. In this text, I adopt an *ecological* and *dynamical system* approach. Within this perspective, questioning the implementation of curriculum reforms and what determines their success or failure, is trying to understand the functioning of such dynamical systems in the face of the *ecological perturbation* that a curriculum reform always is, considering the means used to regulate these dynamics.

Such an ecological perspective being at the heart of the theory of didactic transposition (Chevallard, 1985) and of its extension, the anthropological theory of the didactic (ATD) (Chevallard, 2019), I use these theories to approach the dynamics of curriculum reforms. In the next section, I briefly introduce the main elements of these two theories supporting my reflection.

Elements for an Ecological Approach Supported by the ATD

Didactic Transposition

The theory of didactic transposition was developed in the early 1980s to overcome the limitation of the prevalent vision at the time, seeing in taught knowledge a simple elementarisation of scholarly knowledge. Beyond the well-known succession of transformations of knowledge at the basis of this theory, from scholarly knowledge to the knowledge learned by students (see Chap. 13), ecological concepts such as those of *niche*, *habitat* and *trophic chain* (Artaud, 1997) are also essential in it.

The habitat of a species (here a mathematical object, type of task, technique, …) refers to the environment in which it lives, while its niche refers to the function(s) it has in this habitat. This ecological vision invites us to pay attention to the effect of curriculum reforms on habitats and niches. In addition, it invites us to consider the objects at stake as elements of trophic chains, being fed by some objects while feeding others. Even minor curriculum changes can break trophic chains, and be source of learning difficulties impacting the implementation of reforms. As pointed out in Artigue (2011), this phenomenon is linked to the fact that the official teaching time is distinct from the learning time. The teaching of a new mathematical object is an opportunity for consolidating the relationship with old objects; its zone of influence on learning is an area with fuzzy contours, difficult to identify.

Anthropological Theory of the Didactic (ATD)

The ATD enriches this set of conceptual tools. Key concepts here are those of *institution* and *institutional position* (Chevallard, 2019). Indeed, a curriculum reform mobilises a diversity of institutions for its conception and implementation; it also mobilises agents who occupy different positions in these institutions (the position of teacher is neither that of student, nor that of school principal or parent). To these positions are associated different relationships to mathematical knowledge. Curriculum reforms modify positions and relationships intentionally but also unintentionally. Understanding these moves and their possible, actual effects, is important for understanding curricular dynamics.

Another essential tool provided by ATD is the notion of *praxeology* used to model mathematical and didactic practices. At its most elementary level, a praxeology (called pinpoint praxeology) is a quadruplet [T/τ/θ/Θ] where T is a type of task, τ a technique or way of processing this task, θ a technology defined as a discourse making this technique intelligible and justifying it, and Θ a theoretical discourse which in turn makes θ intelligible and justifies it. Types of task and techniques constitute the practical block of praxeologies (praxis), while technology and theory constitute their theoretical block (logos). In a given institution, praxeologies do not live in isolation; they are organised into nested structures. Local praxeologies denote groups of pinpoint praxeologies sharing the same technology, while regional praxeologies denote groups of local praxeologies sharing the same theory or piece of theory. Studying the dynamics of praxeological organisations, both mathematical praxeologies and the didactic praxeologies with which they are in dialectic relationship, is a means of gaining an understanding of curriculum dynamics.

As pointed out in Chap. 13, another conceptual tool provided by the ATD is the *hierarchy of levels of didactic co-determinacy.* This tool helps researchers consider the different conditions and constraints shaping curriculum reforms and their dynamics, those internal to the disciplines at stake with the lower levels of the hierarchy, and more general ones with its higher levels (*Pedagogies – Schools – Societies – Civilisations*). At each level different agents intervene, new power relations, new rules of legitimacy are established. These different conceptual tools support the analyses and reflection developed in the next sections.

A First Case Study: The High School 2000 Reform in France

Main Characteristics of the 2000 Reform

This reform of high school general education from grades 10 to 12 offers an interesting case. Not a curricular revolution, it however introduced substantial changes still in effect today. To make clear the challenges posed by its implementation, I briefly describe these changes. For more detail, the reader may refer to Artigue (2003). At the level of school structures, there were no major changes and the three orientations organising the differentiation of teaching from grade 11 (L for literature, ES for economic and social sciences, S for sciences) were maintained. At the pedagogical level, continuity was also evident. The curriculum discourse remained a constructive discourse and the place to be given to problem solving was reaffirmed. But it was also stated that the school institution was challenged by scientific, technological and cultural developments and should regularly rethink its objectives in the light of these developments.

This consideration led to substantial changes. In mathematics, the main ones were the strengthening of the statistic domain with the ambition to introduce grade 10 students to statistical thinking through the experience of sampling fluctuations with the help of computer simulations, a differentiation according to the L, ES and S orientations more sensitive to their specificities and students' interests with for instance the introduction of graph theory in ES, and an increased emphasis on the interaction between scientific disciplines and more generally on interdisciplinarity, especially with the introduction of interdisciplinary projects called TPE (*Travaux personnels encadrés*) in grade 11.

Due to the change in balance between mathematical domains, trophic chains were cut and new ones had to be created; praxeological organisations had to be built for the new domains introduced as well as their progressive structuring. This was all the more demanding as most teachers had not encountered graph theory or inferential statistics in their academic preparation. Even for those with a university culture in statistics, there was a didactic inversion between the statistics and probability domains, as some introduction to inferential statistics preceded the teaching of probabilities. Moreover, teachers were asked to base the teaching of these domains on study themes selected among those proposed, according to their students' interests, which also required new praxeological reorganisations. Interdisciplinary work, project pedagogy on subjects chosen by students involving the critical use of Internet resources, were also new for most teachers.

The Implementation of the Reform

This reform could have been rejected. Tensions arose between the group in charge of programmes and the General Inspectorate of Mathematics, a key institution for the implementation of curriculum reforms and their evaluation in France. The emphasis on statistics was considered exaggerated by many professionals, especially since it occurred at the expense of other sectors, particularly geometry. Many also wondered about the possibility of making sense of inferential statistics without any probability background, and questioned the sense that students would make of the experimental work based on computer simulations proposed to them. There was also great concern about TPEs, especially among mathematics teachers who wondered whether they would find a role for their discipline in these.

The reform generated vivid and at times hard debates, and the alternation of the political majority in 2002 resulted in some changes. However, globally the reform resisted. The importance given to interdisciplinary projects and modelling, to the stochastic domain, was maintained or even strengthened in the next reform, that of 2010. Several factors undoubtedly made the adaptation of the educational system to this ecological perturbation possible. I list a number of them below, by lowering the levels of didactic co-determinacy. The ambitions of the reform and most of the changes introduced aligned with international perspectives, which contributed to their legitimacy. At the national level, the work carried out by the CREM (*Commission de réflexion sur l'enseignement des mathématiques*), set up at the request of the mathematics community in 1999, contributed to legitimise its global vision (Kahane, 2001).

The reform was carefully prepared by the groups of experts appointed by the CNP (Conseil national des programmes) and bringing together a diversity of expertise. The CNP guidelines ensured coherence at the global level among the disciplines. The expert groups were given 2 years to prepare the programmes, and those of scientific disciplines worked together for instance to ensure that the new introduction of exponential functions as solutions of differential equations would lead to coherent approaches based on the study of radioactivity. This also allowed the creation of new trophic chains involving exponential and logarithmic functions. The groups of experts also produced consistent accompanying documents, covering all new domains and showing how the proposed themes of study could be exploited.

A specific website *Statistix* was created offering teachers the possibility to download dynamic simulations and access statistical data. The IREM network (*Instituts de recherche sur l'enseignement des mathématiques*), an essential actor of in-service teacher education in France, also mobilised, in particular, the inter-IREM Commission on statistics and probability. Locally, IREM groups built situations and progressions, experimented, proposed training sessions and produced a number of paper publications and on-line resources, some in collaboration with the APMEP teacher association. The IREM network and APMEP journals devoted many articles to these innovations. TPE working groups were also created in various IREMs. They supported and analysed the implementation of TPEs in the high schools of their members who were high school teachers, and proposed training sessions based on this experience. French didacticians contributed to these activities. Moreover, which is not frequent in France, a pre-experimentation of TPEs was organised, and when the reform was implemented, its results and a number of tools were made available to teachers by the Ministry of Education. And last, but not least, specific modes of assessment were designed for TPE and the students' marks taken into account at the national examination of Baccalauréat in grade 12.

The collaboration between the different institutional agents acting in different positions at different levels of the hierarchy of co-determinacy that this description shows was certainly crucial for the successful implementation of this reform, making that, retrospectively, it is generally considered as a good reform. Without revolutionising the high school system as is the case for the problematic on-going reform (see Arnoux, 2018), it succeeded in introducing important changes. This story also confirms that the dynamics of a reform is a long-term process, the implementation of a new curriculum being just a step in a process whose stabilisation requires many years.

In the case of this particular reform, after a few years, it was observed that the training demand in the teacher community regarding the teaching of graph theory, the TPE and the new praxeological organisation proposed for calculus, strongly decreased, making clear that the system was reaching some stable state. The process was much longer for the teaching of inferential statistics. Moreover, reaching a stable state does not mean that there are no more problems, that the intended and implemented curricula are fully aligned. For instance, still today finding a niche for mathematics in interdisciplinary projects is challenging for many teachers.

A Second Case Study: Recent Curriculum Reforms in the Francophone Space

In 2012, as part of the EMF conference in Geneva, two round tables were organised on how recent curriculum reforms were designed and implemented in French-speaking countries. Six countries or regions were considered: the Wallonia–Brussels Federation in Belgium, Burkina Faso, Quebec in Canada, France, Romandy in Switzerland and Tunisia. The round tables were prepared by a 2-year collaborative work. The perspective adopted was to conceive curriculum reforms as changes in the social contract between school and society, at a time when the tercentenary of the birth of Jean-Jacques Rousseau was being celebrated in Geneva.

The work carried out considered recent curriculum reforms from their conception to their implementation, specifying the educational and curricular contexts, identifying the institutions involved in the reforms and their respective roles, describing the global curriculum dynamics, before focusing on a dimension particularly important in each case study. In this text, I focus on the implementation of reforms, and due to space limitations, I just briefly contrast three case studies, regarding respectively the Wallonia-Brussels Federation in Belgium, Quebec and Tunisia. Detailed analyses are accessible in the 150 pages of the section of the EMF proceedings devoted to these round tables.[1]

The reason for this selection is that the three case studies share one characteristic of particular interest for this ICMI study (see the Discussion Document): They correspond to curriculum reforms proposing a global reorganisation of the curriculum around the concept of competence. As was the case in Denmark (see the first section in this chapter), in the three cases this move towards competences started nearly two decades ago (in 1997 in Belgium, in 1995 in Quebec and with the 2002 reform in Tunisia). In the three cases also, we observe a proximity of the global aims of these reforms in terms of adaptation to a rapidly changing world and to the technological evolution, of increasing learning opportunities for all students and inclusiveness, development of students' learning autonomy and citizenship. These shared characteristics clearly show the influence on these reforms of conditions situated at the highest levels of the hierarchy of didactic co-determinacy, not specific of a given society.

However, and also one important reason for this selection, the case studies show three different dynamics with very different outcomes. In the case of the Wallonia-Brussels Federation, the analysis provided is rather critical. The co-authors (Baeten & Schneider, 2012) relate the difficulties met to three main factors: first, the fact that the reform went along with a policy of centralisation and increased control of the education system with the creation of assessment tools to serve as external references common to the three education networks existing in the region; second, the emphasis put on transversal competences, valid for all disciplines, expressed in

[1] See round tables in plenary activities at: http://www.emf2012.unige.ch/index.php/actes-emf-2012

very general terms and poorly coordinated with the mathematics content that remained nearly the same; third, limited resources and training for the teachers, not addressing their real needs. Training sessions did not help them to create a new praxeological coherence intertwining competences and mathematical content. Ten years after the implementation of the reform, a very critical report led to substantial revision and a regaining of attention to the specificities of the disciplines while trying to avoid disciplinary compartmentalisation.

In the case of Tunisia (Smida et al., 2012), also, the analysis is critical, but the dynamics is different. The conception of the 2002 reform obeys a new institutional organisation involving three different commissions: a first commission responsible for defining the aims of the education system and preparing specifications for the disciplinary commissions and for setting curriculum structures (something analogous to the French CNP mentioned in the first case study), multidisciplinary commissions (science, languages, humanities, art) in charge of delimiting transversal competences and, finally, disciplinary commissions in charge of writing the programmes, taking into account these competences. Visibly, this structure and also the careful analysis of a selection of foreign programmes by the mathematics commission made it possible to avoid the disconnection between competences and content observed in Belgium.

In the Tunisian case, the difficulties observed mainly situated in the implementation phase, a top-down process under the full responsibility of the Inspectorate, carried out with very limited resources. According to the authors, the negative effect of these conditions situated at the school level were aggravated by two factors: the coincidence of the reform with a policy of decentralisation and the heterogeneity of the body of inspectors whose number had tripled in 5 years. Despite the careful preparation of the reform, these conditions of implementation, the lack of clear training strategies and resources, the importance of the changes expected both in terms of mathematics and didactic praxeologies, led to significant resistance among teachers. The specific study the authors conduct on the algebra curriculum illustrates this very well. Moreover, the analysis of teaching practices carried out in Ben Nejma's (2009) doctoral thesis tends to show that some years after the implementation of the reform, the implemented curriculum in algebra was still a mixture of old and new curricula.

The case of Quebec (Bednarz et al., 2012) contrasts with the two first cases. The authors show a long process of curriculum development beginning with the 'États généraux sur la qualité de l'éducation' in 1995 and ending in 2008. This process co-ordinates the action of a multiplicity of actors, coming from various horizons, and clearly rejects the 'top-down' logic that had prevailed until then. More specifically with regard to implementation, some interesting characteristics can be highlighted:

- large-scale implementation was prepared by previous work in pilot schools with support in context, responding to local needs and ensuring that each school developed its expertise and autonomy;

- implementation was supported during more than a decade by substantial training activities both at national and regional levels. National activities targeted educational advisers, resource persons and managers, and focused on the global elements at the heart of the reform such as the concept of competence. Disciplinary issues were addressed at regional level, targeting teachers and pedagogical advisors. In mathematics, the emphasis was placed on the concrete construction of situations by teachers, with as much as possible experimentation of the situations collectively designed in classrooms and *a posteriori* joint analysis.

In addition, a permanent process of regulation was planned by the Commission des États Généraux. A specific commission to which a mission of continuous regulation was entrusted was officially established by the Minister of Education in 1997 and it worked until the end of its mandate in 2010.

This case shows thus a coherent global process of design, implementation and regulation, conceived as a continuous process obeying a participatory logic, and combining top-down and bottom-up dimensions. The evolution towards a curriculum structured in terms of competences took place in this context. Accompanying and regulation work was required, but the move towards competences was not reconsidered. As the authors point out in the conclusion of their study, what the case of Quebec shows is the case of a curriculum that is constantly developing, a 'living' curriculum that leaves room for teachers and other school stakeholders to make it their own. This is a demanding but visibly productive vision.

More globally, this second case study shows that the move towards curricula organised around the idea of mathematical competence or mathematical competences is a major ecological perturbation. Normally, such a move should lead to reconstructing on other bases the existing praxeological organisations and the associated learning trajectories, and to ensuring the viability of these reconstructions. This can only be a long-term process which, given the uncertainty of its dynamics, must be firmly supported over time and regulated. If it is to succeed, it must also obtain and maintain the adhesion of the various actors, particularly the teachers, organise and support their collaboration. The examples described above, like that of Denmark, show that these conditions are far from always being met for different reasons, and the resulting problems of implementation.

Concluding Comments

In this text, I have adopted an ecological and dynamic perspective to approach curricular reforms, relying on constructs provided by the ATD to support this reflection. What lessons can be drawn from this reflection? First, the case studies briefly reported clearly show that recent curriculum reforms express rather close visions of what our societies expect from mathematics education. Common trends are observed, such as the move towards curricula structured around competences, the increased importance attached to showing the role of mathematics for addressing societal and environmental issues, to the connection between STEM disciplines and to interdisciplinary practices, the increasing space given to the stochastic domain, and the attention paid to students' specific interests, abilities and needs. These confirm that conditions and constraints situated at the highest levels of the hierarchy of didactic co-determinacy influence these reforms.

However, these case studies also show the specificities of each context and the diversity of the resulting curricular dynamics. They make clear that, no matter how carefully a curriculum reform is designed, the dynamics it generates remains partly unpredictable. The vision of curriculum reforms as ecological perturbations and the systemic approach used help understand this unpredictability and also why the information we can gain from pre-experimentations is necessary limited: their experimental status means that they take place in ecologically protected environments. So, the success of a curriculum reform significantly depends on the strategies developed for its implementation, and on the quality of its regulatory mechanisms.

Another clear lesson is that implementation must be conceived as a long-term process, and not something limited to a few years; that long-term support must be provided to all those involved in the implementation and especially to teachers; that the production and accessibility of appropriate resources, the combination of top-down and bottom-up processes, are crucial conditions. The Quebec case study seems a good illustration and the vision of 'living curriculum' a promising one. However, it seems that too often most of the efforts are still focused on the design of reforms, much less on their implementation, monitoring over time and regulation, leading to abrupt changes and ecological disruptions highly damaging for education systems.

In this section, the success and failure of reforms have been discussed. But how are these evaluated, how can they be evaluated? A variety of criteria are undoubtedly to be considered. In the first case study, we mentioned, as criteria of success, the resistance of the reform and that of the main transformations it had brought beyond the reform itself. This is a sort of minimal criterion. It shows that the reform has succeeded in creating a certain level of ownership, which can also be studied from other sources, such as publications and debates generated by the reform. However, as has been pointed out, this does not guarantee the satisfaction of another essential criterion, the alignment of the implemented curriculum with the intended curriculum.

For this, other evaluation instruments are necessary. They may relate to the resources used by teachers and those they produce; the assessments they draw up or the examination papers which are known to strongly condition their practices; they may also be specific surveys, supplemented by observations of real practices. However, the success of a reform can also be appreciated through the way in which the distance between the intended and implemented curriculum, and the feedback from the different actors, are taken into account to regulate it, in the spirit of living curriculum mentioned above. Finally, any reform aims at improving student learning, and a third level of evaluation situates at the level of the achieved curriculum. This requires that the assessment instruments used be aligned with the spirit of the reform and its precise expectations. Unfortunately, the desire to compare the before and after of the reforms too often overlooks this necessary condition.

Chinese Mathematics Curriculum Reform in the Twenty-First Century – Yiming Cao

Curriculum reform is a fundamental factor in pushing forward educational development. In this chapter, I examine the development and implementation of Chinese mathematics curriculum standards. My goal is to present to the world the current situation of mathematics curriculum reform and development in mainland China (i.e. China, excluding Hong Kong, Macao and Taiwan) since 2000.

The Background of New Century Chinese Mathematics Curriculum Reform

Social and economic development in China (especially the development of information technology, digital technology, life-long learning, and democratisation – The Research Group of Mathematics Curriculum Standard, 2002) have raised the bar for mathematics literacy. New demands for modern citizens have required corresponding changes in public schools, especially in mathematics curriculum and instruction (Ma, 2001). From June 1996 to 1997, the division of basic education in the Ministry of Education organised a survey to investigate the status of the implementation of compulsory education in all subjects, including mathematics, across the nation. The data and facts collected from this survey demonstrated that the curriculum used at that time achieved certain goals (e.g. basic knowledge and basic skills training); however, many problems were identified. At the same time, teachers struggled with students having many problems (Liu, 2009). The old curriculum was highly centralised, with little flexibility for local adaption, and it did not meet the different social and economic requirements of a diverse student body. The trends in international and national education that were mentioned above demanded curriculum reform. Similar to the previous education reforms, the current one adopted a top-down approach: however, we cannot negate the fact that it also reflected certain concerns raised from the community.

Mathematics Curriculum for Compulsory Education (Grades 1–9)

The Development of a New Standard for Compulsory Education

The Mathematics Curriculum Standards for Full-time Compulsory Education (draft) (MCSFCE) was completed and put forth for extensive comments from the community in March of 2000. The development of the mathematics curriculum played an important role in this round of curriculum reform in fundamental education, which provides the idea of basic value, the mechanism of implementation, and the way to develop the standard for other subjects in fundamental education. The Ministry of Education formally promulgated and implemented Mathematics Curriculum Standards for Full-time Compulsory Education (Trial version) in June 2001.

The Implementation of Standards for Compulsory Education

Before the release of the MCSFCE a set of textbooks based on the idea of the new curriculum had been designed by a research group for experimental use (the majority of the members were to part in the later development work of the MCSFCE). Since 1994, this group had conducted two rounds of experiments; more than 60,000 students from more than ten provinces (including both well-developed school districts and undeveloped school districts) participated, which provided abundant empirical experience for the later implementation of the MCSFCE.

The Ministry of Education started a national curriculum reform conference to convene the implementation of the new curriculum in July 2001. Several decisions were made at the conference. First, the overall objectives and strategies for the implementation of the new curriculum in public schools were determined. Second, the strategies to spread the curriculum reform to all Chinese public schools were developed. Third, professional development and teacher training programs were set up. The positioning of the trial version of the curriculum standards necessitated a multi-stage process for spreading the new curriculum. The first stage was to set up the goals, then to conduct preliminary experiments before the nationwide implementation, and finally to broaden the experiment gradually.

In the initial round of experimental implementation of the curriculum, school participants were recruited on a county basis, in 2001. First, applications to be volunteer schools were submitted by counties and were examined before being approved by the Ministry of Education. Forty-two regions (3300 elementary schools, 400 secondary schools) participated in the first round of the national curriculum reform with about 270,000 first graders (1% of the population of first graders nationwide) and about 110,000 seventh-grade participants (0.5% of seventh graders) in 2001. Starting in 2002, each province developed a curriculum reform plan at the province level and determined their experimental regions. There was a total of 570 experimental regions with 20% of Chinese first graders and 18% of the seventh graders participating in the new curriculum.

Subsequently, more schools from an additional 1072 counties became experimental regions at the province level, bringing in about 40–50% of the student population of each grade. Including the earlier participants in 2001 and 2002, there were 1642 experimental regions with about 35,000,000 students participating in the new curriculum in 2003. Based on the results of these pilot tests, the new curriculum entered the phase of nationwide promotion. By 2004, 90% of the school districts in

China were using the new curriculum. As of 2005, except for a few places, the new curriculum had been implemented all over mainland China (Ma, 2009).

Epilogue

In the past 10 years of curriculum reforms, including the Mathematics Curriculum Standards for Full-time Compulsory Education (draft) or Mathematics Curriculum Standards for Compulsory Education (2011 Version), the fundamental research was far from enough. In fact, the existing output of research in primary and secondary school education in the Chinese context was too little to allow for the shaping of a persuasive, rational and substantial data-based curriculum standard. But this lack of sufficient research was not a reason to delay. It was an exploratory process which needed to be refined and improved continuously. The curriculum was expected to have different functions. As the curriculum promoter, the government needed to participate in the academic arguments.

The path of reform was an exploratory process. It was necessary to synthesise theory and practice from mathematics, education, psychology and many other disciplines, pooling resources from all areas and levels, from the most academically high-achieving to the rural schools. The success of the curriculum reform demands rigorous academic attitudes, national responsibility and steady work.

Implementing the *K to 12* Mathematics Curriculum in the Philippines: Models and Processes of Teacher Development – Enriqueta Reston

The *K to 12 Basic Education programme* in the Philippines in 2012 is a major reform that posed challenges in closing implementation gaps through more responsive and sustained teacher development programmes. In particular, the intended K to 12 mathematics curriculum adopts a spiral progression approach where five learning domains; namely: numbers and number sense, measurement, geometry, algebra and patterns, and statistics and probability cut across the grade levels with increasing complexity. With the goals of developing students' critical thinking and problem-solving skills, school mathematics teachers are confronted with various challenges as key implementers of the reform. This paper examined the models and processes for professional teacher development that have been carried out in the Philippines to address the needs for school mathematics teachers in expanding their knowledge bases and enhancing their capacities for implementing the reformed mathematics curriculum.

The Philippine Educational System and the Contextual Realities of Curriculum Reform

In 2012, the Department of Education launched the *K to 12* Basic Education program which is a major curriculum reform in the educational landscape of the country aimed at expanding the basic education cycle from 10 to 12 years and, at the same time, enhancing the quality of educational outcomes (DoE, 2012). From a national perspective, this educational reform primarily reflects the shared experience of change of a country's educational system as it adopts to changing contextual realities of the twenty-first century, national priorities and emerging global standards.

The K to 12 Mathematics Curriculum Reform Mathematics Curriculum Reforms in the Philippines

Some salient features of the reformed curriculum which has substantial impact on the teaching of Mathematics and Science include the use of a spiral progression approach to ensure mastery of knowledge and skills at each level and the use of pedagogical approaches that are constructivist, inquiry-based, reflective, collaborative and integrative (DoE, 2012). These features have profound implications on the training of both preservice and in-service mathematics teachers.

The intended *K to 12* mathematics curriculum encompasses five learning domains with the development of problem solving and critical thinking as the twin goals of mathematics teaching, Inspired by Bruner's model of the spiral curriculum, the adoption of the spiral progression approach to curriculum design in the *K to 12* Mathematics curriculum implies that the same concepts are developed and taught from one grade level to the next in increasing complexity and sophistication (Tan, 2012).

Professional Development Models and Processes for K to 12 Mathematics Teachers

The challenge of closing curriculum implementation gaps lies in the hands of the teachers who are the key actors in any curriculum reform Leung (2008). Different stakeholders of Philippine education from both government and private sectors responded to this need for teacher development in response to the reform. The Department of Education (DoE) conducts annual National Training of Trainers (NTOT) among selected teachers by year level and subject areas to build their capacity as teacher-trainers who will conduct the mass trainings by geographical regions and by academic subjects (DoE, 2016). This is an application of the *Cascading Model* where in-service trainings and seminars move from the national, regional, division, then school level with decreasing duration at each lower level (Bentillo et al., cited in Lomibao, 2016). These in-service trainings and seminars usually span for 2–5 days and conducted twice a year, during midyear break and summer break.

Another model of professional development is the *Cluster-based training* which involves teachers from several schools attending the same training program conducted by invited subject specialists as trainers with the content determined by the master teachers and the department co-ordinator of the schools in consultation with the teachers (Ulep, 2006).

More progressive models of teacher development have been explored to address professional development needs of mathematics teachers. The University of the Philippines National Institute for Science and Mathematics Education Development (UPNISMED) has advocated and used the Lesson Study approach for science and mathematics teacher development in various schools within Metro Manila and nearby provinces (UPNISMED, 2017). The Department of Education (DoE, 2016) also institutionalised the Learning Action Cell (LAC) as a school-based continuing professional development strategy where groups of teachers engage in collaborative learning sessions to solve shared challenges encountered in the school facilitated by the school head or a designated LAC leader. The LAC shared some commonalities with lesson study as it promotes teacher collaboration and the growth of professional learning communities or school-based communities of practice, though there are marked differences in focus of the collaborative learning sessions and group structure.

A Needs-Based Professional Development Model for K to 12 Mathematics Teachers

In response to the challenges of the *K to12* reform, the Science and Mathematics Education Department of the University of San Carlos conducted a needs assessment survey in 2015 for Mathematics teachers from public and private schools in Metro Cebu, Philippines. The results revealed that *Probability and Statistics* was ranked 1st by majority of the teachers as the area where they are least confident to teach and in which they need more professional development (Reston & Canizares, 2019).

Based on the need assessment, a 5-year teacher development project entitled *Improving Statistics and Probability among K to 12 Mathematics Teachers in the Philippines* was launched in 2015. The project was implemented in three phases which included: (1) capacity building of workshop facilitators along with the development of activities and learning resources for the workshops; (2) the conduct workshops in parallel sessions for elementary, junior and senior high school mathematics teachers; (3) the development of an online support structure for participating teachers to access additional resources, share best practices and participate in a professional learning community of teachers.

Implications and Future Directions

The evolution of professional teacher development models to address teacher needs in implementing the reformed mathematics curriculum is indicative of the openness and flexibility of various institutions and professional teacher groups to embrace a wide range of options to improve teaching quality and learning outcomes. While the training model may be efficient as it offers a wider reach to the greatest number of teachers in least time, the more progressive and transformative models have provided teachers opportunities for involvement, collaborations and reflections into one's professional development and teaching practice. Evaluation of the impact of these programs are needed to inform both educational practice and research in mathematics teacher development.

Implementations of Reforms Are Diverse, Multifactorial and Non-linear – Angel Ruiz

The implementation of curriculum reforms cannot be viewed as a linear process. There are always ups and downs, and inflexion points. The nature of the reform conditions very much the implementation, but the strategies that are adopted and the educational agents that participate also do. One of the reasons for non-linear developments is that there is always debate and struggle within diverse contexts. Besides, the factors that intervene within reforms implementation are not only multiple, but the weight of each factor is different. Diversity, again, should be emphasised. The aim of this final section is to collect and contrast some aspects that emerge from the reforms introduced so far.

The Nature of the Reforms Studied

In all these experiences, the nature of the reforms is inscribed in strong international trends, such as the cultivation of competence or twenty-first-century skills; here a 'pragmatism' in relation to mathematical preparation usually includes the aim to serve individual and collective progress and a lifelong education perspective. That is why abilities and not only contents are invoked.

These international influences impacted the reforms described on mainland China, although it should also be added that there were some important local inputs: The previous curriculum was questioned as old, complicated, difficult, with an emphasis on memorisation, repetition, rote, and too centralised to allow proper implementation.

The same is true of the Philippines where the so-called *K to 12 Mathematics Curriculum* also insists on achieving mastery of knowledge and skills at each level and the use of pedagogical approaches that are constructivist, inquiry-based, reflective, collaborative and integrative. To support the French reform we have reported, international trends were used that promote the role of statistics (stochastics), the place of technologies and the strengthening of interdisciplinary initiatives in education (especially STEM).

The ‘competence’ and the ‘competences’ and other results elaborated by the KOM Project became one of the crucial nutrients in the theoretical framework of the PISA tests, which has impacted curriculum reforms in many parts of the world. Although these reforms have used or even nurtured general international trends, all responded to local precise needs, different for each one.

Some Particular Aspects

In the implementation of the reform reported in France, one feature we can point out is the building of a crucial convergence between different important educational agents (associated with national authorities, scientific or teaching communities). The case of mainland China shows a carefully managed implementation process in various phases: A trial proposal by a group of experts appointed by the Ministry of Education, which was submitted for consultation in provinces and experimental schools, and that included a very early revision process. The final version was finally published in 2010. We can observe in the Philippines a movement towards the exploration of training models more focused on specific environments, in which there can be greater interaction, collaboration, reflection, what seems to be an international trend. What struck us about the Danish reform it is how educational authorities included the perspective proposed by KOM in some of the curricular components, but not in other key ones (no official implementation processes were created following KOM's ideas, training or elaboration of materials or national assessment).

The Dichotomy Between Top-Down or Bottom-Up Strategies, and Other Factors

Diversity and multifactorial developments can be seen through the prevalence of a general factor of reforms: The role and weight given to top-down strategies or to bottom-up actions.

In the case of the Wallonia-Brussels Federation, a clearly very dominant top-down reform was formulated. The balance after 10 years is not reported as positive. One factor is also indicated: The reform did not take into account the specific disciplines (in this case mathematics) and it tried to force a general framework of competences that had to be applied at all levels and that also included external evaluation mechanisms; with an emphasis placed on a transversal vision. Coordination with mathematics was non-existent. An equilibrium between general and specific constructs was not developed. And another aspect: Very few materials and limited training were provided to support teachers in implementing the reform.

The case of the Tunisia reform was also basically a top-down process, but the same mistakes as described in the Belgian Francophone case were not made here: The 2002 reform avoided the drastic separation between a general competence approach and mathematics. But that was not enough to achieve success. They were unable to provide enough support materials for teachers and there were no clear strategies for their training. This factor was crucial. But another aspect was pointed out: Given general policies of institutional decentralisation (external to mathematics), there was a great heterogeneity of educational inspectors. In this scenario, teachers have resisted implementing this reform.

In Quebec, the reform process studied had an incubation and development phase between 1995 and 2008. Unlike what happened in Francophone Belgium and Tunisia, a process that did not have a top-down dominant orientation was sought. The participation of pilot schools was taken into account, seeking to develop their expertise and autonomy. Then, for more than 10 years, training processes were given at the national and regional level on the competence approach. The properly disciplinary subjects (mathematics) were given at the regional level. The trainings focused on teachers designing problems or problem situations that were later socialised. We have a report on a harmonious combination of factors: An equilibrium between top-down and bottom-up actions and general and specific reform constructs, and teacher's sound participation in the designing of pedagogical resources.

A bottom-up orientation offers important results in terms of appropriation of the reform by educational agents (especially teachers). However, to properly develop this type of strategy requires time, resources and certain quality of these educational agents. And in some contexts, such as in developing countries or regions, it is not easy to have the time to implement a reform, as they often depend on unstable political support and socio-cultural understanding and maturity. It is common that the demands for immediate results (measured for example by better student performance or school promotions) are in these contexts very persistent. Similarly, weaknesses in the quality of teacher preparation (pre-service and in-service) and resources can have significant effects. But additionally, in 'very ample and participative' processes there is a risk that the characteristics of the reforms will be distorted if they are not conducted properly.

On the other hand, top-down strategies can have different levels of success or failure that can be due to other variables. The case of French-speaking Belgium shows us, indeed, a top-down but poorly formulated orientation and mistakes. The reform in China, on the other hand, can be placed as a top-down orientation but where the participation of the provinces and teachers in schools throughout the process is reported. The case of Tunisia shows us another experience better directed from the top but that was not able to articulate resources, training and a positive commitment of teachers. In the French case we see a consistency-convergence between various educational agents (bottom) and politics (top). Promoting bottom-up strategies in reforms is necessary, but not enough. Other factors and the combination of them do impact reform's outcomes.

The reasonable recommendation is what Artigue underlines: To achieve a harmonious combination of top-down and bottom-up strategies. Something that should draw on the international good practices or lessons, but also that requires careful calibration of the specific contexts in which a reform it is to be implemented.

Struggle and Uncertainty

In the French case, there were confrontations of ideas between various professionals and groups, but that did not prevent the reform from continuing. Our reading tells us something obvious: The existence of groups of reformers that are struggling for these changes (and they indeed succeeded). The reforms are processes that have names of individuals or groups, something that is sometimes diluted in institutional anonymity. But here there was another element: There was not always certainty that the reform was going to take place and remain.

In Denmark, reformers (around the KOM Project) show the case of a group of experts that failed to gain national political support in their country to help sustain a new curriculum in all dimensions, but other reasons they had a strong impact outside their country (even in governments, and in other very political international institutions).

The 'ecological' theoretical approach that Artigue uses aids us to better understand this uncertainty in particular by highlighting how a reform impacts many dimensions of education and society, of which we cannot even be aware, and less so in advance. Besides, the characteristics of the local groups within social, cultural, or political scenarios impact the certainty or uncertainty of the implementation of a reform.

Final Comments

The Quebec experience shows us a reform following gradual steps with broad participation. Both this case and that of France occurred in advanced socio-economic, educational and cultural contexts. In China, a controlled and organised reform process within a very stable political context is reported. In all these cases, without a doubt, good materials, extensive training, infrastructure, the supportive participation of professional groups or associations or educational officials, and reasonably good quality teachers have been crucial. In all these cases the process took no less than 10 years. And in the developing world? We believe this would be more difficult to replicate in developing countries or regions; not only because of the weakness in available material or human resources, but also because social and political instabilities and uncertainty tend to outweigh here more than in other contexts.

In the cases studied so far, there are factors that we find relevant in the development of a curriculum implementation, clearly teacher preparation and adhesion to the reform, and resources. But other dimensions also weigh in, such as the role of more general educational agents (such as inspectors in Tunisia) whose participation depends on institutional policies, or the "maturity" of the mathematics and mathematics education communities to help implement the changes; or the legitimacy and consistency of a reform (something that in the French example academic communities were instrumental in achieving). The role of these different factors within each implementation process and the way they get articulated provoke each context to have diverse outcomes.

We have seen success, failure, limitations and positive developments in these reforms. How should such reforms be monitored or evaluated? One first general approach may be using the six categories that Niss offered us: to gauge how successful a curricular reform or proposal has been by calibrating what happens in each of the vectors indicated, and of all of them as a whole. Of course, the "survival" in the time of a reform is a parameter, especially when there is deep struggle in the educational communities. The adhesion to or rejection of the reform by teachers can be another criterion. Artigue points out what can be another criterion: When teachers have reached a level of mastery such that they do not require further training (mostly). Cao points out (and Artigue does too) commonly used mechanisms to gauge understanding or support for reform: Surveys, situation analysis and discussions of special issues.

Reform development assessment is not easy in part due to many variables that intervene, and also because they usually require a long-term where many things can happen in different order. As we said before, curriculum implementation does not follow a linear path, and it is complex to determine, for example, in which point of the path does a reform stand. Within this discussion, it seems wise to underline as a criterion to gauge reform implementation the *alignment* of the reformed curriculum in relation to the different means (resources, teacher training, ICT technology role) developed for implementation, and in particular the quality, adequacy and up to date of these means in connection to the international experience and research. However, for political authorities and the general populations it becomes sometimes difficult to acknowledge the progress of a reform leaning only on such elements, there are a collective demand for 'visible' results in the shortest time. And furthermore, a long-term perspective is more difficult to achieve within developing contexts. This may impact negatively the success of 'good' reforms.

Finally, the curricular vectors that Niss describes in this chapter help us to visualise curriculum implementation not as an indivisible amorphous whole but as a process with various components that can be developed in different ways, and between which there may or may not be consistency. However, one more variable could be added in relation to these vectors, time (or timing). It is possible when evaluating a reform that the goals and content can be stablished in a moment but that forms of teaching and student study develop much later, or even that the assessment component can be postponed even more to allow for other components to advance. The concrete analysis of the specific situation is invoked. This reinforces the recommendation not to extrapolate, since what works in one context may not work in another.

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Chapter 17
Towards a Model for Monitoring and Evaluating Curricula Reforms

Steve Thornton, Maitree Inprasitha, Angel Ruiz, Masami Isoda, Narumon Changsri, and Kristen Tripet

This chapter focuses on general factors that impact on the implementation of curriculum reform programs in the world – some are small scale, targeted interventions while others have national scale and substantial impact; some are located in developing countries, others in more developed countries; some are short-term interventions, others have extended over a long period of time. It is not intended here, therefore, to offer the results of a systematic study on the implementation of curricular reforms around the world, but rather, through some of the experiences or ideas discussed, to identify interesting and relevant dimensions to consider in the processes of implementation of curricular reforms.

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In the first section of this chapter, three national experiences are summarised. These are chosen to provide a variety of elements of curriculum reform programs from which lessons can be learned for the international community. They are chosen with special attention to reform processes in different socioeconomic, geographical and cultural contexts. The criteria for selecting these experiences included: global impact of the reform, diversity between the countries' socioeconomic and cultural development, and relevance to the analytical work we intend to introduce here.

Given the widespread international influence of the Japanese process of Lesson Study (Fernandez & Yoshida, 2012) and the long-established systemic curriculum reform program, Japan was chosen as the first reform experience. In contrast to centralised but often patchily adopted reform efforts in many parts of the world, the Japanese experience provides an example of curriculum reform in a developed country that has become part of the very fabric of the teaching of mathematics. The curriculum intervention in Thailand was chosen as an example of a reform that commenced at a small scale and has rapidly expanded to a much larger scale. A unique feature of the Thailand reform is the use of university agents to implement the reform using a ground-up approach. Costa Rica was chosen as a third reform experience as an example of a wide national-impact process in a developing country. Of particular note is that the reform has so far achieved rare continuity through the support of changing governments.

Together, these three experiences point to some broad dimensions that are considered significant in analysing the impact of curriculum reform implementation internationally. Other reforms will be unpacked in greater detail in the second section of this chapter. Rather than discussing each reform separately, they will be used as exemplars to highlight how particular curriculum reform efforts have taken into account some of the dimensions introduced in the frst section. This will allow discussion of points of convergence or divergence across a larger number of curriculum reform endeavours.

The third section seeks to provide some criteria to assess the development of a curricular implementation. It looks at commonalities and differences across the various reform efforts and at the ways the factors discussed in the previous two sections have affected the impact that each reform has been able to achieve. This section will rely, to a large extent, on anecdotal reflections of the impact of the various reforms, as few have been studied rigorously. We do not, therefore, claim to develop a set of universal criteria to assess the impact of a curriculum reform, but rather seek to identify some pointers arising from the discussions in the first two sections.

Beyond description, we will address the challenge of identifying a model of change, be it explicit or implicit, that underpins the implementation of mathematics curriculum reform. Again, we do not claim universal validity for such a model but offer it as a suggestion that might inform curriculum reform efforts into the future. Together with the discussion in Chaps. 16 and 18, we hope that the lessons learned from discussing the experiences in the implementation of a variety of curriculum reforms will promote more rigorous, systematic and impactful curriculum reform internationally.

Values, Vision, and Goals Within Curriculum-Reform Implementation

The three reform experiences of Japan, Thailand and Costa Rica presented below serve to frame the discussion of factors influencing curriculum reform in the second and third sections. In each case, the reform has meant 'big changes'. However, these changes depended on the realities of those countries; what is to be changed, why it has to be changed, and how it would be changed. Thus, in order to understand the noteworthy success stories of the reform of any country we need to understand the geographical and societal contexts in which the reform is embedded. Japan is a highly developed East Asian country with a strong tradition of centralised curriculum; Thailand is a Southeast Asian country with close links to other ASEAN (Association of South-East Asian Nations) countries; Costa Rica is a rapidly developing Latin-American country.

We draw from these examples three key factors that frame any curriculum reform endeavour: values, vision, and goals. By values, we mean a shared understanding of what is important in the curriculum reform process. By vision we mean the clarity of the intent of the reform. By goals we mean the officially stated goals of the reform. The degree of alignment between the values, vision and goals, the extent to which they match broader societal values and how well they are realised in practice then frame much of the discussion in the remainder of the chapter.

Curriculum Development and Reform in Japan

The Early Years of Curriculum Centralisation and the Beginnings of Lesson Study

Formal education was established in Japan at the university (Daigaku-ryō) in the seventh century CE in order to study written Japanese and arithmetic using Chinese textbooks, including Confucianism. Westernisation of Japanese civilisation[1] and enlightenment began in 1868 after Tokugawa Shogun returned the government to the Emperor, with Japan officially introducing the French public education system up to higher education in 1872. It was an era of educational reform, in which the traditional apprenticeship model moved to whole classroom teaching under a graded curriculum imposed by the government.

[1] Here, the word 'civilisation' does not mean just import Western culture. In the 1860s, the Japanese literacy rate was the highest rate for ordinary people in the world. The International Exposition of Paris (in 1867) became the trigger of Japonism which influenced the European arts, such as Vincent van Gogh and Gustav Klimt, and craftsman industries such as Meissen chinaware. Japanese ethnomathematics was re-developed under the Chinese influence in the sixteenth century and Takakazu Seki and Kanehiro Takebe developed their own original form of calculus in the seventeenth century.

A notable aspect of this early Westernisation was the variety of textbooks adapted from other sources. Despite the introduction of a textbook certification system in 1886 and the introduction of a national curriculum, their content varied. Revising the textbooks through practice then became the custom and gave rise to what has since become known as Japanese Lesson Study. The first theme for lesson study was the improvement of teaching and learning by using Pestalozzi and traditional Zen-Confucian style dialectic methods (Wakabayashi & Shirai, 1883). In 1909, the Elementary School, a laboratory school of the Higher Normal School and the origin of the University of Tsukuba, began to publish the *Journal for Educational Study* to share the themes of Lesson Study for reform. Based on these experiments, the Secondary School proposed a new curriculum for the Ministry in 1910.

In the 1900s, mathematics educators in laboratory schools and the Higher Normal School became aware of and knowledgeable about the Kline movement which aimed to bring different subjects into an integrated mathematics curriculum focused on functional thinking (Isoda, 2019). Despite resistance from some mathematicians, the Ministry promoted the movement by publishing the book *Lehrbuch der Mathematik nach modernen Grundsätzen* (Behrendsen & Götting, 1908) in 1915 and supporting the establishment of the Secondary School Mathematics Society in 1918. In this society, secondary school teachers were able to freely discuss issues of curriculum and pedagogy. In the case of elementary school mathematics, several ideas proposed in the *Journal* and books provided the Lesson Study themes of promoting children as independent learners of mathematics (Isoda, 2007).

Curriculum Development by Teachers and the Evolution of Lesson Study

After World War II, under the government of the United States, the national curriculum was the recommended agenda to enhance school curriculum development. A reform cycle of ten years was established in 1947, with textbooks revised every four years. Curriculum development became the role of every teacher for around ten years, with groups of teachers, educators and mathematicians working to develop curriculum through Lesson Study. One particularly fruitful product of this Lesson Study program was the Japanese didactics of mathematics. This is exemplified in the elementary textbooks developed by the Hiraku Toyama group in the 1960s (Kobayasi, 1989), which have the unique principle of the task sequence moving from the general to the specific. Although these textbooks did not get approval from the government, they were strongly supported by the teachers' union. The union critiqued other approved textbooks.

To address the concerns of the union, educators met the need to systematise terminology in order to more clearly articulate the conceptual sequence in the approved textbooks (Isoda & Nakamura, 2010). The systematised terminology polished theories for: developing mathematical thinking (Isoda & Katagiri, 2012; Katagiri, 1990); designing task sequences (Kobayasi, 1989); representations (Ito, 1971), and; approaches such as open-ended tasks (Shimada, 1977; Becker & Shimada, 1997, re-theorised by Nohda, 1983). These achievements were published as the guidebooks for Lesson Study. Currently, similar ideas can be seen in the world community such as Iszák and Beckmann (2019): however, Japanese educators have used these theories to develop textbooks and to engage in Lesson Study since the 1960s.

Values, Vision and Goals in Japanese Mathematics

In Japan, curriculum authorities and educators have been working to establish coherence between national curriculum, textbooks and assessments tasks, producing better practices and revision in the reform cycle. National Curriculum reform in Japan has synchronised with Lesson Study, promoting both bottom-up and top-down reform. The national reform committee is selected by the government; at the same time various Lesson Study groups enact objectives of mathematics education through carefully designed task sequences.

A consistent vision has been that educators who enrolled as members of the government committee were to establish consistent improvement of curriculum before and after the US occupation. Development of mathematical thinking and attitude have been consistent aims of Japanese education throughout. Before occupation, developing mathematical and scientific thinking and mathematisation were key directions, while fostering activity and appreciation were reform issues under the US occupation. After the occupation, the first reform in 1956 made mathematical thinking and attitude a key under the scientific and technological necessity for societal development, and in the second reform in 1968, extension and integration became a key under the societal modernisation in which creativity was a necessity.

Curriculum reform and Lesson Study are supported by assessment practices. Since 1956, National Curriculum assessment tests have been used to evaluate the implementation of curriculum. Since 1982, because of teachers' reference to the assessment tasks, this has supported curriculum implementation and reform. Assessment tasks have been revised in order to assess mathematical communication, thinking, and attitude as well as children's achievement up to the junior high schools. Currently, common exam tasks for national universities' entrance at the end of high schools have begun to embed dialectic communication into the exam tasks in order to evaluate students' mathematical thinking.

Despite resistance from some quarters, including some mathematicians and the media, Japanese mathematics education has established goals that emphasise mathematical communication and thinking. The goals are underpinned by values that include teacher participation in bottom-up reform and students as independent learners. Together, the goals and values help to realise a vision of mathematics as a creative and inclusive endeavour essential for the scientific and technological development of society. The development of Lesson Study alongside national curriculum reform has led to a coherence that is rare in international mathematics education.

Mathematics Education Reform in Thailand

The educational reform movement in most ASEAN countries gained traction as the new millennium began. Singapore introduced its 'Thinking School, Learning Nation' program in 1997, and followed this with the 'Teach Less, Learn More' initiative in 2005. These programs aimed to enhance the learning experience for students, promote critical thinking, and allow teachers the opportunity to innovate (MoE, 2013). With particular regard to mathematics, the focus was directed to highlight the process of learning rather than just the content, captured in the pentagon model of curriculum describing skills, concepts, processes, attitudes and metacognition, which has been a feature of Singapore mathematics since 1990 (MoE, 2012). Other ASEAN nations such as Thailand (MoE, 2001), Brunei (Khalid, 2007), and Malaysia (Lim, 2006) followed more recently, adopting a similar direction that is part of a global trend.

Although the content that mathematics students are expected to know and be able to put into practice is well known, it is widely recognised (Inprasitha, 2015; Takahashi, 2015) that in many developing countries the approach to teaching is the area where real innovation is needed. However, successfully implementing reform in the mathematics classroom is particularly difficult as is amply demonstrated by the long journey of reform undertaken by the two most developed countries in the region, Singapore (since the 1970s), and post-war Japan (since 1947). This arduous path has not gone unnoticed by the other countries in the region and they have good reason to be cautious when considering learning transformation in mathematics, which is widely accepted as one of the central pillars of education.

In the case of Thailand, major education reform has followed the global trend exemplified in Singapore. In response to the agenda of the first educational act in 1999, which emphasises *'Reforming Learning Process'* (MoE, 2001; Wasi, 2000), a completely new section, *skills and processes*, was added to the 2001 Basic Education Core Curriculum. Policy makers, curriculum developers, other related educational personnel, and teachers were quick to notice the distinguishing features of this new curriculum, which emphasises not only content or subject matter, but also how students learn best and desirable characteristics to be developed in students (Inprasitha, 2018). Unfortunately, the adoption and implementation of an underlying paradigm shift from a product-oriented approach to a product-process oriented approach in this curriculum reform has not been universal in the broader educational community in Thailand.

To begin to address this, the Faculty of Education at Khon Kaen University in Thailand has undertaken an initiative that gives the university a new and central role in curriculum reform implementation. It has instituted and commenced the 30-year Thailand project (see Fig. 17.1), an attempt to create and incorporate a strong research and development cycle as a system of curriculum and instruction (Inprasitha, in press).

At the commencement of the project, a contextual analysis study was conducted with fifteen student teachers during 2000–2002 to introduce the idea of 'open-ended

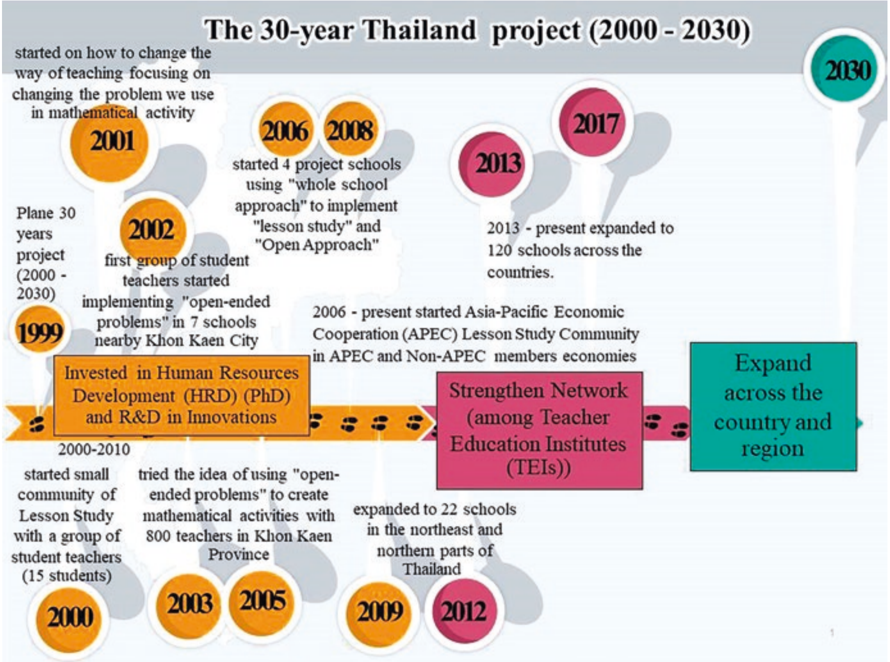

Fig. 17.1 The thirty-year Thailand project

problems' as a part of innovation for teaching mathematics in the collaborative schools in the Khon Kaen city. The Center for Research in Mathematics Education (CRME) was established in 2003 to cultivate a new type of Master's degree program in mathematics education in 2003 and doctoral degree program in 2006. These programs prompted and facilitated professional learning communities among graduate students, teacher educators, mathematics educators, and school principals and teachers.

The role of the graduate students as school co-ordinators, bringing Lesson Study and Open Approach as innovations into schools, is a key initiative aimed at bridging the communication gap between the university and the school. The Open Approach has been adapted by Maitree Inprasitha since 2002 (Inprasitha, 2003) as an innovation for teaching mathematics in Thailand by incorporating three basic steps of Lesson Study (Inprasitha, 2011). The original ideas (Nohda, 2000) are similar to the Open-ended Approach described by Becker and Shimada (1997). Fifth year undergraduate students, trained to use these innovations during the first four years of their teacher initiation program, were sent to schools in 2008.

The first two project schools in 2006 have fully implemented and realised the new section of the 2001 curriculum reform implementation. To institutionalise Lesson Study and Open Approach in the schools, at least three layers of professional learning communities (PLCs) have been created within and among the schools, and in the district (Fig. 17.2). Lesson study teams as members of each PLC

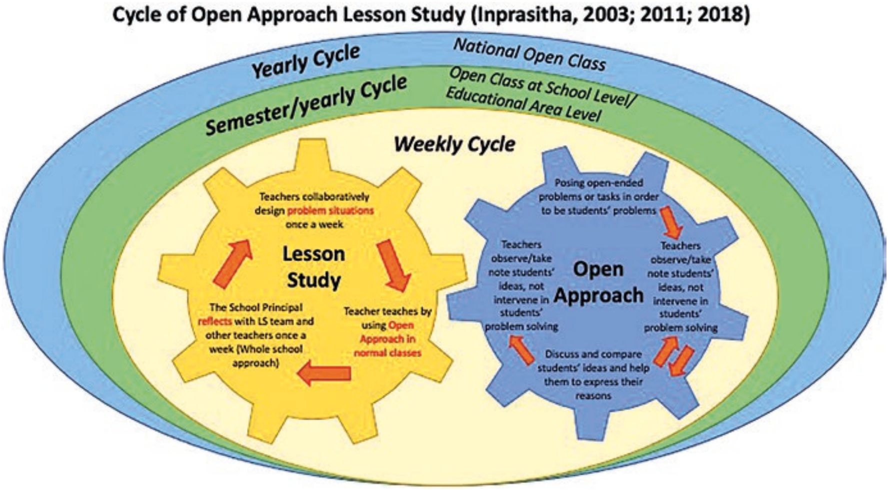

Fig. 17.2 Three layers of professional learning community (PLC)

have been learning together to deeply read the mathematics textbook (translated version) in order to understand new school mathematics; a new teaching approach has been adopted, and; new kinds of assessment have been introduced. This is critical to enable them to faithfully and effectively implement innovations in their schools and in their Lesson Study communities.

During the last twelve years, the Research and Development cycle has been a driving force for curriculum reform implementation with innovations in Thailand and in the region through the Asia Pacific Economic Co-operation Lesson Study[2] project. The first two project schools commenced work in 2006, four schools followed in 2007, twenty-three schools in 2009, which has now increased to nearly two hundred schools in 2018. Approximately fifty Ph.D. candidates and Ph.D. graduates have been working in twenty teacher education institutes across the country.

The Thailand experience shows how a long-term vision, supported at all levels, can grow from small beginnings into a major national reform endeavour. The goals of developing mathematical skills and processes among students are being realised through the agency of PhD candidates and graduates working with teachers to develop a shared vision in Professional Learning Communities. This shared vision is considered essential to the success of this long journey.

[2] See http://www.crme.kku.ac.th/detail_page/Apec2018.html

Mathematics Curriculum Reform in Costa Rica

In Costa Rica, a profound reform of the mathematics curriculum for all primary and secondary education (grades 1–12) began to be gradually implemented in 2013 documented by the Ministry of Public Education (MPE, 2012). A general vision nurtured this reform: It was necessary to respond to decades of curricular backwardness in this school-subject based on up-to-date and appropriate experiences and research from around the world. Global goals were set: to develop higher-order cognitive capabilities across all mathematical areas (to reason and argue, to pose and solve problems, to make connections, etc.), and; to foster a 'mathematical competence' that will enhance understanding and use of mathematics by citizens in diverse contexts. Although there is this a strong emphasis on 'competences', due to local education conditions, the curriculum is based on the mathematical knowledge and abilities that are expected of students (a specific intellectual approach: a curriculum that is neither 'competence-based', nor 'content-based'). Some values were included: an emphasis on real contexts and modelling, as well as the use of technology and mathematics history, are conveyed. Another vision was part of the intellectual foundations: To counterattack 'Mathephobia' (with multiple emphasis or strategies) is a required first aim to achieve learning results, and this nurtures the whole curriculum. To aid these general purposes a specific lesson model for building learning was provided (in other national contexts a model would not be adequate). This model has four steps: problem posing; independent student work; collaborative discussion of strategies, and; closure.

Some international influences can be perceived here: the French Didactique des Mathématiques; the Dutch Realistic Mathematics Education; the NCTM's 'Principles and standards'; the OECD's PISA theoretical framework, and; an *interpretation* of the Japanese Lesson Style. However, as Ruiz (2018) underlines, there are important theoretical roots found in local research developed since the twentieth century. With these visions, values and goals the reformers detached from previous paradigms dominant not only in the teaching of mathematics but in the education establishment itself. It was, using Artigue's (2018) words, a deep "ecological perturbation".

This mathematical reform has had, so far, the support of Ministers of Education of different administrations (2010–2014, 2014–2018, 2018–2022), a political continuity that is rare in Latin American countries. The main means used to design and to guide the implementation of the reform has been the project *Mathematics Education Reform in Costa Rica* (PMERCR), constituted by researchers (specialists in Mathematics Education) from public universities, technology experts and in-service teachers, a team of 12 persons (MPE, 2019a). This combination of professionals has been preserved since the early curricular design. With only the human resources and internal competences within the MPE, it would have been impossible to make progress in the design and implementation of this curriculum.

The researchers and technology experts were funded between 2012 and 2017 by non-governmental organisations; since 2017 the researchers have worked for free and even self-funded diverse activities and technology-related expenses while the teachers have been supported by the MPE. This fusion of expertise and professional trajectories has allowed a balance for relatively successful curricular design and implementation and has created a bridge between theory and practice. Political continuity did not just happen; it has been carefully cultivated by this team.

Implementation was designed assuming a scenario of changing governments where there would be no continuity in the support. The strategy was to have the greatest possible impact in the shortest period. Here Information and Communication Technology (ICT) was decisive. National "blended-courses" (face-to-face meetings, plus online sessions using Moodle) were carried-out between 2012 and 2016, and fully virtual courses following the MOOC modality began in 2014 (using Class2Go, edX). These courses included a combination of mathematical content with specific pedagogy (all associated with the official curriculum), since, as indicated by Hernández-Solís and Scott (2018), the reformers could not assume that the teachers knew well the mathematics they should teach.

To build the human base that would feed the reform throughout the country, the blended courses were developed in two stages: first, executed directly by the Project's team aimed at teachers and officials who could be leaders; then this group replicated the courses in all regions. Thanks to ICT possibilities, the content, methodologies and assessment were the same in both stages.

After 2017, Mini-MOOCs were built. These constituted an innovative modality with compact courses, each to be completed in less than fifteen hours. MOOCs and Mini-MOOCs were designed not only for teachers but, since 2016, also for high-school students who had to prepare for national exit examinations. The large number of videos that these courses require are directly elaborated, edited by members of the Project.

Since 2019, another type of educational support has been developed: *Mathematics Free Resources* (MFR), open virtual materials aimed at secondary school students without any teacher intervention, though the materials can be used by the latter to design lessons, practices, and assessments (see MPE, 2019b). Most content is developed through videos that should not exceed three minutes. These materials can be accessed through computers, tablets and smartphones. Their use is totally free, no registration process is required. Eventually MFR materials may replace textbooks. The rationale is to focus directly on students due to implementation weaknesses in the classroom or to mitigate eventual socio-political unrest that may limit school activities.

This large amount of high-quality free virtual materials (at the end of 2020: over five hundred web sections, five hundred videos, hundreds of fully explained problems for students and teachers) and actions (multiple courses each year) is a unique experience in Costa Rica (Ruiz, 2020), something that has strongly positioned the mathematics reform in the scenario opened by the Covid-19 pandemic, which obliges educators to adopt a radically different perspective for face-to-face and virtual education working together. This leading-edge role could serve as a key support to sustain this curriculum reform in the years to come.

One of the problems reformers dealt with was how to incorporate curricular objects in the task design, classroom actions and assessment, especially higher-order capabilities (processes) and levels of complexity. That is why Ruiz (2018) elaborated a new theoretical framework for task-design that can be used in the preparation of lessons, assessment, and national high-stakes testing. This framework includes a model with sixty-one precise indicators to identify and gauge in three levels the participation of the five higher-order capabilities, or processes, of the curriculum in a mathematical task. This facilitates the determination of the level of complexity of any mathematical task and the conditions for its use in the classroom and in all educational dimensions. This intellectual framework, though not official, goes further than the curriculum approved in 2012.

The Math Reformers in Costa Rica have thus generated a large amount of multiple innovative resources, professional development has been provided for many teachers, and teacher preparation programs at the public universities are synchronised with the new curriculum. However, the curriculum implementation has progressed unevenly. Programa Estado de la Nación (PEN, 2017, 2019) reports feeble use in the classrooms of the four-steps model and the Problem-Solving strategy. Ruiz (2018) points out a weak introduction of higher-order capabilities in the classroom actions, assessment, and national examinations, and also that official documentation and guidelines are not fully consistent with the mathematics curriculum, weakening its implementation.

Reasons for the uneven implementation include:

- An ideological one: in the minds of educational agents and in official documents, behaviouristic paradigms (or curricular views reduced to contents, no abilities, or higher-order capabilities) still dominate;
- The weak preparation of teachers, most of whom come from private universities of dubious quality (a country with just over 5 000 000 inhabitants has more than 50 private universities);
- An inadequate system of teacher recruitment and professional development that is not based on teacher quality performance;
- An inefficient classroom management and teaching system, including:
  - teaching work loads of 30 to 32 hours per week all of which are student contact;
  - overload of administrative tasks assigned to teachers;
  - weak academic use of time in the classroom;
  - feeble advising-supervising-monitoring of classroom action (what happens in a classroom is almost a ‘black-box’), and;
- There has been always resistance from some higher-level and regional officials within the Ministry of Education to implement this curriculum.

These general conditions impact differently on the national regions that have unequal socio-economic and cultural environments, common in most developing countries.

From its inception, it was clear that success in such a wide and deep reform would take 25 to 30 years, depending on factors within mathematics and also on others that would transcend it. It was a bold decision that however would have consequences. As Artigue (2018) has emphasised, a curriculum reform invokes unpredictability; but it will be even more unpredictable if it means a profound 'ecological perturbation' and implies a long-term implementation. For example, here, at least six government transitions will be implied as well as the need for a sustained investment of resources. A 'point of no-return' will never be insured. The situation becomes more uncertain with the general weakening of education processes due to the Covid-19 pandemic.

Some elements to underline include:

- *politics*: taking advantage of a historical 'window' and cultivating support from diverse social-political agents;
- *resources*: with the best international standards, but 'tailored' to the national reality and curriculum implementation; and
- *ICT*: intensive, innovative utilisation.

There was also a central *implementation vision*: Curriculum design should not be done "in vitro" accompanied afterwards by implementation actions; implementation needs to be part of the design from its inception. This vision is what Ruiz (2013) termed a "Perspective of praxis in mathematics education".

One relevant and important feature is the existence and continuity of a team with strong expertise that assumed the mathematical reform as a national and personal commitment. This has secured the permanence of coherent visions, values and goals. This is not easy to replicate, but it may be noteworthy for curricular implementation in developing countries.

What Factors Intervene in the Implementation of Reformed Curricula?

The preceding section presented examples of three different curricular reforms in three different contexts. Together these examples point to some factors that are important to consider when designing or evaluating the implementation of curricular reforms in different contexts. Some of these factors are *external* to the curricular reform, others are *internal*, and others relate to *realisation*. By external we mean those factors that are located beyond the reform itself – these may be international influences, geographic challenges or the political and societal context in which the reform takes place. By internal we mean those factors that are part of the reform itself – these may include the development processes in the reform, the emphases within the reform or the target audience of the reform. By realisation we mean the resources developed as part of the reform, the role of assessment in the reform or professional work with teachers as part of the reform.

For example, Japanese reform is influenced by external factors such as strong cultural traditions of collaboration between teachers and researchers and a school context where education is highly valued, and by internal factors such as clear protocols for planning and implementing curriculum supported by thoroughly documented resources. The curriculum reform in Thailand is strongly influenced by external factors, such as the Southeast Asian context, the influence of Japanese theories and the geographic challenges of implementing the reform in a large developing country, but it is equally influenced by internal factors, such as the role of the university and its post-graduate students in promoting the reform.

In the case of Costa Rica, the ongoing reform is influenced by external factors such as the political context of changing governments and the uneven, often poor preparation and professional development of teachers but a generally agreed need to develop more literate and informed citizens; it is also influenced by internal factors such as the collaboration between researchers from universities, technology experts and teachers and the development of virtual resources (MOOCs, Mini MOOCs, MFR) to implement and support the reform.

This sub-section expands on and elaborates the external, internal and realisation factors considered important in planning and evaluating curriculum reforms. The factors described were identified inductively from the papers and presentation of the various curriculum reform programs during the ICMI Study conference. Critical factors in each reform were identified and summarised, from which key factors in the implementation of mathematics curriculum reform were identified. These are presented below.

Description of General Factors

External Factors

1. International influences
   (a) In what way is the reform implementation influenced by international trends or processes?
   (b) How and to what extent is it influenced by international comparisons of student achievement?
2. Geographical influences and reach
   (a) What is the scale of the reform? Is it localised, regional or national?
   (b) Are there particular geographic challenges that need to be considered?
3. Political influences and ownership
   (a) Does the societal and political context within which the reform takes place impose particular imperatives that need to be addressed in its implementation?

(b) Who has ownership of the reform? Is it centralised or devolved?
(c) To what extent is the reform influenced or supported by the general community?

4. Time-scale

(a) Does the reform have long or short-term goals? How does this influence the implementation of the reform?

Internal Factors

5. Development processes

(a) Who worked on the curriculum development and its implementation? Was it top-down, bottom-up or some combination? How does this impact on ownership of the implementation of the reform?
(b) How much time was invested?

6. Emphases in the curriculum itself

(a) What is the balance between skills and content?
(b) What cognitive competences are emphasised?
(c) What is the role of digital technologies?
(d) How do these factors impact on the implementation of the reform?

7. Target audience

(a) Is the reform for everyone or a particular target group?

Realisation Factors

8. Resources

(a) What resources are provided? What is their role?
(b) Who develops the resources?
(c) Are the resources coherent and in line with the intended curriculum reform?

9. Teachers

(a) What guidance is provided for teachers?
(b) How much autonomy do teachers have in implementing the reform?

10. Assessment

(a) Is the assessment aligned with the reform goals?
(b) What is the role or influence of assessment in the implementation of the reform?

How These Factors Intervene in Implementation of the Reform

External Factors

In addition to the examples provided above, the experiences described in papers relating to curriculum reform in England and Mexico (Lozano et al., 2018) and Luxembourg (Nadimi & Siry, 2018) give somewhat contrasting examples of how international trends have impacted on the implementation of curriculum reform projects.

Lozano et al. (2018) compare and contrast curriculum reform initiatives currently taking place in Mexico and England. In both countries the curriculum reform represents a radical break from existing practice, casting teachers as agents and innovators of curriculum reform rather than as mere implementers. In each case the reforms were at least partly a response to perceived failings of current practice reflected in scores on international assessments of student achievement, and in the case of England the reform was strongly influenced by international practice such as that found in East Asia.

In both cases, resources and texts were produced which challenged existing practice, giving explicit pedagogical guidance to teachers regarding representations and strategies for calculations. The resources emphasise conceptual coherence and understanding, providing innovative approaches to the teaching of concepts. Rather than being concerned with the fidelity of teachers' implementation of curriculum interventions, consistent with the East Asian approach teachers in both countries are offered the opportunity to make decisions based on insights derived from research and practice made explicit within the materials. Although the reforms are works in progress, early indications suggest that the reforms are beginning to transform teaching and learning by re-imagining teachers and curriculum designers as partners in innovation.

Nadimi and Siry (2018) provide a very different example of an historical curriculum in Luxembourg reform strongly influenced by international emphases, but ultimately of limited impact on promoting curriculum reform. Structural reforms addressing the entire school system were proposed in 1958 with the goal of linking all levels of schooling and linking school more closely to active citizenship. Public perceptions were that school was neither preparing Luxembourgian students adequately for further studies in neighbouring countries nor proving useful for developing informed citizens. However, implementation was hampered by external factors including the school system itself and language. A structural reform of secondary schools in 1968, which removed the differentiation between boys' and girls' experiences in school mathematics, provided the impetus for bringing together classic and modern mathematics and emphasising practical applications as well as abstract concepts.

In contrast, attempts to reform primary school mathematics were hampered by challenges such as language. As the language of instruction in Luxembourgian primary schools is German, it was not possible to import Belgian texts that were written in French, and it took several years before Luxembourgian texts including modern mathematics were developed. In short, the reform was not adapted to the existing culture of the school system in Luxembourg and failed to achieve its intended goal of radically reforming mathematics education. Nevertheless, it did provoke discussions about school mathematics in Luxembourg, helping to unify mathematics education for boys and girls and to revise and modernise the applications of school mathematics.

In considering how external factors impact on the implementation of mathematics curriculum reform it is also important to take account of the proposed scale of the reform and whether the reform reaches its target audience. Two contrasting examples are provided by the experience of developing a national curriculum in Australia (Sullivan, 2018) and reforming senior secondary mathematics for non-academic students, i.e. those not intending to study high level mathematics at tertiary level, in Israel (Karsenty, 2018).

After a number of relatively unsuccessful attempts to introduce a more coherent national approach to schooling in Australia, the Australian Curriculum, Assessment and Reporting Authority was constituted in 2008 to develop a national curriculum for Foundation (the year before school) to Year 10. The intent was to improve the quality, equity and transparency of Australia's education system. School education in Australia, however, is constitutionally the responsibility of eight state and territory governments, hence the development of a national curriculum caused a blurring of the lines of responsibility. The result is arguably at best a compromise position in which the national curriculum has been agreed upon in principle yet interpreted and implemented differently across the nation.

In his paper, Sullivan (2018) describes how this differential interpretation has limited the extent to which the underpinning philosophy of the national curriculum is realised in practice. While the scope of the curriculum reform in Australia was national in intent, the political context of eight different states and territories each having ultimate responsibility for curriculum implementation meant that compromises were made and that some of the ideals espoused in the national curriculum have not yet been realised in practice.

In contrast to the centralised national reform described above that was compromised, at least to some degree, by regional interests, Karsenty (2018) describes how the 3 U reform in Israel commenced as a pilot in two schools and is gradually extending. The reform was designed for low-track students in the senior high school years, commencing with an extensive phase of research-based design of new learning materials coupled with an extensive model of teacher support and dissemination. The issue of students' experience of long-term failure in mathematics was tackled head-on through the development of resources that engaged students' common sense and real-life experiences, made extensive and integrated use of a variety of visual and other representations and minimised technical manipulations and notation.

Teachers were introduced to the materials through workshops and summer courses and invited to participate in school-based trials. There was initial reluctance from many teachers based on claims about the limited capacity of students, limited time and the effort required. These concerns informed an extensive program of on-site, ongoing support provided to those teachers who agreed to trial the resources, the success of which has led to the expansion of the program from an initial cohort of two schools, six teachers and a hundred students to thirty-two schools, one hundred and ninety-one teachers and four thousand, seven hundred and fifty students.

Together these contrasting examples show the importance of carefully considering factors such as geographic reach, political influence and ownership. The Australian national curriculum reform impacted significantly on existing state-based curriculum and was effective in stimulating a national debate about priorities in school mathematics. Yet political control in the various states and territories limited the extent to which the lofty intentions of the national reform were implemented in practice. The 3 U curriculum reform in Israel was much more modest in both its target audience and geographic scope, yet the extensive program of school-based support generated a level of ownership among teachers that has led to a significant expansion to, and implementation by, a much wider group of schools.

A final external factor that is important to consider in curriculum implementation is the time-scale of the reform. Short-term acceptance of curriculum reform programs is perhaps the norm; long-term sustainability is rare (Schoenfeld, 2006). Lyle, Cunningham and Gray (2014), for example, in their examination of one school's work in implementing the Australian national curriculum point to the negative impact of "change fatigue" arising from frequent top-down changes in policy. In contrast the contextual and tailored professional learning solutions and respectful support of the 3 U curriculum reform in Israel described above has enabled it to continue for some fifteen years.

The Thailand reform described in the first section is a particularly significant and promising initiative that takes a long-term view of change. Rather than expecting large numbers of teachers to make rapid and dramatic changes in practice, the Thai reform adopts a 30-year implementation timeframe, commencing with post-graduate students as agents of change whose influence will gradually permeate the entire country.

Internal Factors

A key issue in any curriculum reform is the development process. Regardless of whether a reform is top-down or bottom-up, every reform poses its own set of challenges. In particular, the development process impacts strongly on ownership of the reform and hence on its implementation. The Luxembourg and Australian curriculum projects discussed above were very much top-down processes with the inherent challenges of gaining traction among teachers suffering change fatigue. The 3 U curriculum project was much more bottom-up, not seeking to change the existing curriculum but seeking to develop resources and work with teachers to improve outcomes for disinclined students. The challenge here was to achieve reach among a wider group of schools.

An alternative in which top-down and bottom-up processes work together is described in the first section. The project ‘Mathematics education reform in Costa Rica’ was developed through a collaborative effort of researchers and technology experts from public universities funded by non-government organisations and in-service teachers allocated by the Ministry of Public Education. This enabled the project to develop a balanced approach to curriculum design and implementation, bridging theory and practice and cultivating political continuity. While success will take many years, the project is addressing key social, economic and educational issues. The existence and continuity of a team with strong expertise required considerable effort and commitment but serves as a model for the implementation of other national curriculum reform projects, particularly in developing countries.

Every curriculum reform has particular emphases built into it. Many, if not all, involve increased attention to the skills and cognitive competences required for active citizenship in an increasingly technological environment. In some cases, this has meant a corresponding de-emphasis on traditional mathematics content. This raises challenges for the implementation of the reform, particularly when the emphases clash with existing practice.

Tran, Nguyen, Nguyen, Ta and Nguyen (2018) describe a teacher preparation project in Vietnam developed in response to curriculum reform emphasising mathematical modelling as one of five competences including communication, mathematising, reasoning and argument, solving problems and using mathematical tools. Historically the curriculum and texts in Vietnam have made little connection between mathematics and the real world, hence the reform curriculum represents a radical change. The project described by Tran et al. seeks to develop increased mathematical literacy and modelling skills among preservice teachers as agents of change in the Vietnam education system.

The project seeks to investigate effective processes to prepare preservice teachers to teach mathematics contextually and to document the influences, successes and failures of the implementation on preservice teachers’ knowledge and practice. The preservice teachers expanded their knowledge and appreciation of mathematics as much more than a set of isolated skills or concepts. Rather, the skills and competences of mathematical literacy were developed alongside knowledge such as linear programming and regression analysis. Although this is a small-scale project in one university, it holds promise for the implementation of the mathematical modelling reform more widely, as the teachers become agents of change in the Vietnam education system.

Changsri (2018) describes a similar shift in teacher perceptions among preservice teachers in Thailand who engaged in a process of Lesson Study and open approach to problem solving using videos that were part of the APEC Lesson Study project. The emphasis on student thinking challenged preservice teachers’ beliefs about mathematics, moving away from traditional content with right or wrong answers towards valuing processes and students’ ideas through real-world problems. The perceived role of the teacher changed from one of imparter of knowledge and judge of correctness of answers to one of problem poser, listener and prompter of thinking. Again, the small-scale project positions preservice teachers as agents of change in the wider implementation of Lesson Study and the open approach in Thailand.

Every curriculum reform has a specific target audience. In some cases, such as the Costa Rican reform it may be the entire national cohort of students; in others such as the reform in England described by Coles (in Lozano et al., 2018) it may be students from particular year levels; in the 3 U reform in Israel it was a cohort of low achieving students in the senior secondary years. Each approach brings its own implementation challenges and opportunities that need to be addressed in appropriate ways. The experience of a STEM-focused project stimulated by one enthusiastic and knowledgeable teacher in one Hong Kong school (Mok & Sung, 2018) provides an interesting counterpoint to many of the larger scale projects. A three-year enrichment program for talented students, led by the teacher, was progressively introduced by the school to promote communication, analysing and problem-solving skills.

Evaluation of the program indicated that the students in the high ability group developed higher academic achievement, higher order thinking and greater self-esteem. A key to the success of the program was the experimental approach used to design and refine the lessons with careful application of relevant learning theories. Citing Cai and colleagues (2017), Mok and Sung conclude that a key to successful reform implementation is to develop and test learning sequences at a grain size that is useful to teachers. We suggest that many of the issues experienced in the implementation of large-scale curriculum reforms are related to grain size – focusing only on macro-questions of curriculum or textbook design may ignore the day-to-day realities of the teachers responsible for its implementation.

Realisation Factors

The remaining three factors, resources, teachers and assessment, relate to the realisation of the reform. They are discussed in greater detail in Chap. 18. Here we touch briefly on the importance of these realisation factors to help frame the consideration of curriculum reform success discussed in the next section.

Resources play a key role in each of the three case studies discussed in the section "Description of general factors". In the case of Japan, the development of consistent textbooks and assessment practices built on Lesson Study have been instrumental in establishing lasting reform; in the case of Thailand, postgraduates have been key resources as agents of change in the system; in the case of Costa Rica, the development of MOOCs and Mathematics Free Resources has helped circumvent political, teacher quality and geographic issues. Similarly, Lozano and colleagues (2018) highlight the central role played by textbooks that provide pedagogical advice to teachers alongside content, Karsenty (2018) discusses the development of resources aimed at underachieving senior secondary students that present relevant and engaging real life problems, while Changsri (2018) discusses the value of lesson videos as a tool to stimulate preservice teachers' analysis and reflection in a Lesson Study approach.

While resources were instrumental in the implementation of these reform programs, Rodríguez-Muñiz, Díaz and Muñiz-Rodríguez (2018) describe how resources that do not align well with curriculum priorities and emphases can equally limit the impact of a reform. They describe how new secondary curricular learning standards in Spain aim to promote a less formal approach to statistics and probability and focus more on applying mathematics to social science contexts. Statistical literacy, the integration of technology into mathematics and context-based problem-solving involving estimation, simulations and conjectures are key aspects of these standards.

Yet an examination of five full series of textbooks revealed that with one exception every example referred to quantitative rather than qualitative variables, references to variability were extremely rare and more than 95% of exercises were algorithmic in nature. Similarly, probability questions were based on laws of counting with no reference to subjective probability. In this way the textbooks maintained the focus of previous curriculum standards, being an inhibitor rather than a promoter of the changes recommended in the curriculum standards.

An interesting and unusual interpretation of what constitutes a ‘resource’ is discussed in the description of the Australian reSolve: Mathematics by Inquiry project (Thornton et al., 2018). A key aspect of this project is the recruitment and professional development of 300 Champion teachers whom the authors considered to be not only implementers of the reform but part of the project resources. Many of these teachers were involved in the development of the material resources of the project, which, similar to those described in the UK and Mexico resources, are intended to be educative in nature (Davis & Krajcik, 2015). Thornton, Tripet and Patel argue that resources and documentation alone seldom produce sustainable change, even when accompanied by professional learning to promote implementation. In contrast the reSolve project aims to position the three hundred Champions as part of the project resources, integrally intertwined with the material resources through the project philosophy.

Considering teachers as resources in the implementation of curriculum reform is therefore critical in ensuring uptake. In the South African context, Brodie (2018) describes the development of professional learning communities (PLCs) in the Data-Informed Practice Improvement Project (DIPIP). School-based professional learning communities were supported to participate in a sequence of developmental activities analysing learners’ errors in different contexts. They engaged in activities such as test analysis, learner interviews, concept analysis and planning, as well as videoing and reflecting on lessons. The project produced substantial and sustained improvements among teachers in each of the three professional learning communities.

An analysis of the conversations in the PLCs showed an increase in conversations focused on student learning and thinking and highlighted that the focus on pedagogical content knowledge supported teachers to work on their content knowledge. Brodie concludes by arguing that a model of extended inquiry in PLCs, focusing on both knowledge and practice, can be a powerful way of encouraging responsiveness to learners, increasing teachers' professional agency and accountability and hence contributing strongly to the implementation of the reform.

Student errors and learning is also the focus of a curriculum project in Italy that takes advantage of large-scale standardised tests of achievement (Martignone et al., 2018). Rather than seeing the national INVALSI[3] (*Istituto Nazionale per la Valutazione del Sistema Educativo di Istruzione e di Formazione*) tests as a means for comparing schools or groups of students, the researchers worked with groups of teachers to compare statistical data about one's own classes with that of the school or of the population more broadly to identify specific strengths and weaknesses.

This can contribute to the work of curriculum implementation consistent with the goals of the intended curriculum, enabling teachers to reflect on the relationship among the intended, implemented and attained curriculum. Martignone, Ferretti and Lemmo suggest that an analysis of test tasks can thus be used as a tool to modify the system itself and carry key messages about its implementation. However, as discussed in Chap. 18, assessment practices that are not aligned with the curriculum reform may serve to at least partially derail the reform.

This section has identified and synthesised a number of external, internal and realisation factors impacting on the implementation of mathematics curriculum reform. Illustrative examples have been provided as a means of elaborating those factors. This is by no means a complete list of potential factors, nor is it intended to be an in-depth analysis or discussion of the curriculum projects described in the papers and presentations. We hope, rather, that this section sets the scene for the following discussion of the assessment of the success of curricular reform.

The Assessment of Curricular Reform Success

The factors considered in the previous subsection point the way to the possibility of identifying criteria that allow us to evaluate progress or lack of it in an implementation experience. Of course, every curriculum reform has some successes and some failures. It is not our intent, therefore, to attempt to provide a definitive process through which a curriculum reform can be evaluated, but rather to suggest how those responsible for the introduction of a curriculum reform might reflect on the experience. In addition, it is our hope that the discussion might promote systematic evaluation as an integral part of the curriculum reform process rather than an add-on.

Building on the discussion in the previous section, we suggest that three fundamental qualities should be considered in evaluating a curriculum reform: external *cohesion*, internal *coherence* and realisation *fidelity*.

[3] INVALSI is the Italian National Institute for the Evaluation of the Educational System of Education and Training. http://www.invalsi.it/invalsi/istituto.php?page=chisiamo

Relationship and Alignment Between Curriculum Reform Factors

External Cohesion

As discussed above every mathematics curriculum reform takes place within a national, educational and cultural context. Reform that ignores, or worse contradicts the conditions in which it is located is therefore likely to be at best short-lived. The New Math reform in Luxembourg, or indeed in Western society more generally (e.g. Kilpatrick, 2012) provides an example of a reform that failed to take account of at least some external factors. New Math was stimulated by a political context in which strong mathematics education was seen as essential to combating the perceived threat that Western countries such as the USA would fall behind in the international technology race, yet it failed to take into account the educational context in which it was introduced. Teachers were generally unprepared for the radical shifts in emphasis in the curriculum and other important elements of mathematics often described as basic skills were marginalised. In the case of Luxembourg (Nadimi & Siry, 2018) the school system itself was unable to respond to the demands of the new curriculum.

A key element of external cohesion is therefore support at every relevant level. This includes support of the educational authorities involved, support of general academic agents such as mathematical societies, science councils, education boards and universities, support of school related educational agents such as advisors, supervisors, principals, wider support of politicians and the general public, and essentially support of teachers themselves. For large-scale reforms such as those described in Costa Rica, Thailand and the UK and Mexico, it is critical that such support is evident at all levels; for smaller-scale reforms such as those described in Israel or South Africa, gaining the support of those involved is likely to depend at least partly on the extent to which the reform is consistent with external factors such as national priorities and directions.

We therefore suggest two critical implementation questions related to external cohesion:

1. To what degree has the reform been able to gain the support of:
    - teachers and others responsible for its implementation;
    - mathematicians, mathematics educators and mathematical or mathematics education groups, councils or societies;
    - educational and curriculum authorities and unions;
    - politicians, the media and the general public?
2. To what degree has the reform been able to sustain support over time?

Internal Coherence

In the first section of this chapter we identified vision, values and goals as key aspects of any curriculum reform. We described the degree to which vision, values and goals are aligned in practice as internal coherence. Of course, the vision, values and goals are strongly influenced by external factors but once articulated they become a material part of the curriculum reform and its documentation. Successful curriculum reform requires that all elements of, and actors in, the curriculum and its implementation have a shared view of the vision, values and goals. This includes their articulation through tasks or statements of content and proficiencies, assessment and crucially programs of professional learning.

Internal coherence, such as that found in the Japanese approach to Lesson Study and accompanying texts and the Singapore curriculum founded on the pentagon model and implemented in a national system of education in which research, professional learning and preservice teacher education all work in the same direction, is likely to lead to continuity over time. On the other hand, rapid changes in priorities work against the production of a set of shared values, a shared vision and shared goals across the elements of the reform.

In many Western countries, the drivers of recent curriculum reforms have had a political dimension, with an accompanying move away from competences associated with high level mathematical thinking and problem solving and back to facts and content. Most often this has been motivated by perceived poor student performance in international measures of assessment. This has left teachers in an ideological and practical dilemma: on the one hand research and their own experience point to progressive, student-centred and open approaches, on the other hand the political imperative points to more closed, transmissionist and content-focused approaches. Lack of shared vison, values and goals puts any curriculum reform in jeopardy, leading to teacher burn-out and change fatigue and ultimately de-professionalising and disempowering those who are central to the educational endeavour.

We therefore suggest two critical implementation questions relating to internal coherence:

1. To what degree does the reform exhibit coherence and continuity of values, visions and goals associated with:
    - mathematics itself, as a discipline in its own right, as a subject essential for technological and scientific advancement and as a key element of active and informed citizenship;
    - mathematical education, the pedagogical approaches and priorities recommended;
    - assessment of mathematical learning, both at a system level and at an individual school and teacher level?
2. How well are the values, vision and goals communicated in the wider community?

Realisation Fidelity

No matter how well a curriculum reform addresses and is sensitive to the contextual factors in which it is located, nor how consistently the values, vision and goals are documented nor how well they are communicated to key agents of reform, the danger exists that they may not be exhibited in practice. As Schoenfeld (2006) declared, “Indeed, one can imagine curricular materials that, when used in the way intended by the designers, result in significant increases in student performance, but, when used by teachers not invested or trained in the curriculum, result in significant decreases in student performance” (p. 17). That is, the resources developed must not only be faithful to the values, visions and goals of the reform, but as discussed by Mok and Sung (2018) they must also speak to teachers at an appropriate grain-size that enables them to be implemented in practice. Similarly, professional learning and assessment must be both faithful to the values, vision and goals of the reform and have practical impact for teachers and preservice teachers at all levels.

We therefore suggest three critical implementation questions relating to realisation fidelity.

1. To what degree do the resources developed in the reform enable agents to faithfully implement the crucial aspects of the reform?
2. To what extent do professional learning and preservice teacher education programs position teachers as co-designers and agents of reform?
3. Is large and small-scale assessment integral to the reform and aligned with the values, vision and goals of the reform?

A Proposed Model Describing the Relationships Between Factors in Mathematics Curriculum Reform Implementation

Drawing on the lessons from the experiences described in the first section, on the discussion of factors impacting on curriculum reform in in the second section and on the questions regarding evaluation of curriculum reform suggested in this section, we now propose a model describing the relationships between factors in mathematics curriculum reform that might help to inform the planning, implementation and evaluation of reform initiatives. As before, we do not claim completeness nor universal applicability. However, we hope that the model will provide a point of reference for governments, educational systems, universities and for schools as they seek to enhance mathematics education.

Rather than being linear in nature, the model is reflexive and dynamic, recognising that all elements in the curriculum reform interact and influence each other. In this way a curriculum reform is a complex dynamic system in which the factors involved in its design and implementation are far from settled when the reform makes its way into the education system through the official adoption of texts or documents.

The proposed model is presented in Fig. 17.3.

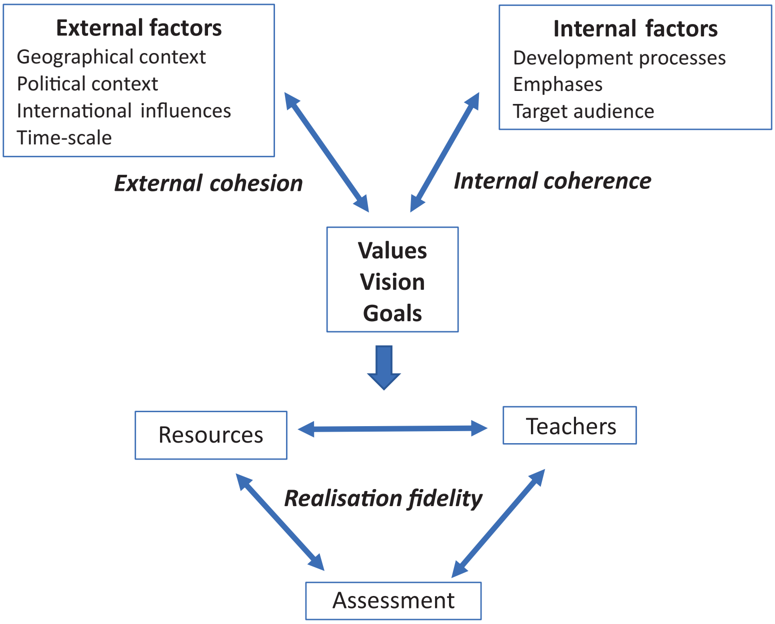

Fig. 17.3 A proposed model describing the relationships between factors in mathematics curriculum reform implementation

Conclusion

This chapter has examined factors associated with the implementation of mathematics curriculum reform. We have identified a number of internal, external and realisation factors. These factors have helped inform questions about internal coherence, external cohesion and realisation fidelity that are important elements to be considered in the evaluation of the implementation of mathematics curriculum reform. Finally, they have helped to suggest a model in which the external, internal and realisation factors interact as parts of a complex dynamic system (Fig. 17.3).

As we have pointed out, the discussion is neither complete nor definitive. Few, if any, of the reforms described in this chapter have been rigorously or systematically evaluated. For this reason, we have avoided labelling them as 'successful' or 'unsuccessful' as every reform has its positive and negative aspects. However, what we can assert from the case studies in the first section and the specific illustrations of the factors discussed in the second, is that unless a curriculum reform works coherently across the external, internal and realisation dimensions, its implementation is likely to be problematic.

We note also that every reform takes place in a particular cultural and political context, among a particular target audience and at a particular scale. Hence the relative importance of the factors identified in this chapter will be specific to the context. For these reasons, we caution against the wholesale importing of a curriculum initiative from one context to another. However, we hope that we have been able to point to some factors that will allow curriculum developers to undertake a systematic and well-considered approach to the planning, implementation and evaluation of a mathematics curriculum reform initiative.

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Chapter 18
Teachers, Resources, Assessment Practices: Role and Impact on the Curricular Implementation Process

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A broad perspective must be taken while discussing teachers, resources and assessment practices in relation to reform and this is what we have attempted in the present chapter. We consider factors in curriculum implementation including physical materials, technologies, but also processes such as classroom and system assessment practices and, in a privileged way, the role of teachers. Curricular change can become just a proposal printed on an official paper if it does not actively involve teachers and their practices, and if it does not secure the needed resources for teachers. In focusing in this chapter on the implementation of curriculum reform,

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we have the classroom firmly in focus. We begin considering issues around the intended curriculum, including the preparation and professional development of teachers and then consider what is attained, which takes us into issues around assessment.

The chapter suggests different roles played by teachers, resources and assessment in helping and/or restraining success in curriculum reforms, so that a reader can identify common elements relating to their own context and, we hope, take forward ideas about how to act and/or not to act when implementing a curriculum reform. The distinction between the *breadth* of reform and its *depth* is particularly significant when it comes to considering teachers, resources and assessments. By *breadth* we mean the number of schools or teachers affected, which could range from all the teachers in a country, or all the teachers in publicly-funded schools in a country, to more regional or local innovations. *Depth* of reform is concerned with the extent to which teachers are brought into the reform process, for instance whether pedagogical innovation is envisaged as part of reform and, if so, whether there is any theory of change driving what takes place and the extent to which changes are harmonious.

To draw on the issue of assessment raised at the start of this chapter, for instance, depth of reform will concern the extent to which assessment practices fit with the new curriculum innovation or not. A deep reform will bring assessment innovation with it, alongside opportunities for teachers to develop and learn in relation to innovation. A shallow reform might be a change in curriculum documentation with little else by way of changes to assessment structures and little consideration given to supporting the envisaged pedagogy.

In this chapter, we have largely drawn from contributions to the ICMI Study 24 Conference that took place in Japan in November 2018. The rationale here is that the Study Conference was the result of a global call and all contributions were peer-reviewed. However, we have supplemented our review with other publications where, as authors, we felt that some important perspective was not represented. We end the chapter by bringing into focus an issue which was not represented at the ICMI Study 24 Conference, which we feel is nonetheless vital to consider, and this is the extent to which curriculum reform pays attention to local or global concerns and challenges.

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Teachers in the Curricular Implementation

Teachers are the most important agents of change during curriculum reforms. The role of teachers in a period of implementation of curriculum reforms is not the same as the one that develops when there is absence of these changes. In this section, we explore the aims and general contexts for teachers' participation in curriculum reform implementation. Teachers' participation can be significant in the process of policymaking and formulation, as well as in the process of design, implementation, reflection, ongoing development and evaluation of pedagogical innovations.

Aims and General Contexts for Teacher Participation in Curriculum Reform Implementation

The basic choices that characterise curriculum reforms emphasise recovery and improvement actions in the school system by attempting to optimise all existing resources, then by sharing and involving all their actors. At the centre of this general interest of the community is the crucial role of the teacher, an aware protagonist, promoter and facilitator of processes of change that are usually involved in curriculum reforms. The renewal process triggered by curriculum reforms often presents different dimensions, among which are the institutional, the organisational and the pedagogical.

Moreover, the role of teacher professionalism in a reform develops, from being within a particular perspective, to opening to potentially new tasks and the feasibility of innovation. We assume that the effectiveness of the laws that introduce novelties in the educational systems depends, to a decisive extent, on the cultural and professional training of those who are called upon to transform these innovations into formative action, an appropriate methodology and a new teaching practice.

Different school systems can involve mathematics teachers in curriculum reform in different ways:

- When teachers are involved before the drafting of the reform, this can be through formally or informally collecting opinions, critical points and good practices. Sometimes teachers get involved through government or other agencies, other times through teacher consultations.
- Teachers could be involved during the preparation of curriculum reform in two ways: by involving teacher-researchers or teachers in the commissions or groups of teachers who produce specific reform documents. In some contexts, this process can actively involve almost all teachers in a region or country.
- Teachers can be contacted immediately before the start-up of reform, by consulting them on a large scale about the possible desired outcomes, or during the implementation of reform; the process could consider trials followed by teachers' opinion or evaluation.

– Finally, teachers can be involved after the implementation of the reform: what are teachers' professional development initiatives? At a global or local level? This phase naturally implies a different involvement, because at the local level there are possibilities for different interactions and comparisons. Does the process foresee the involvement of teachers in evaluation and revision of the reform?

In any case, curriculum reforms are (and must be) important moments, both for teachers' awareness and for the revision and self-analysis of their professionalism. As underlined by Brodie (2018) it is now internationally accepted that curriculum reforms should be accompanied by aligned teacher professional development to support teachers in working with curricula they may not have previously experienced (Borko et al., 2015; Zaccarelli et al., 2018). In fact, as Karsenty (2018) points out, even when most teachers have been positively impressed by new curriculum reforms, few may be willing to implement new practices in their classes. A willingness to implement change depends both on external factors and on the influence of internal beliefs that are often implicit.

An important point to consider when analysing curriculum reform is what kind of discussion between teachers it stimulates. Discussions may take place in institutional contexts, at school level, geographic area, in institutional forms, in teachers' professional training courses, and at an informal level (teacher blogs, chats, etc). Such discussion is important for developing professional awareness that can arise from a new curriculum reform. The aims and role of teachers are affected by national or even regional contexts, which may vary widely across the world. At the same time, teachers' skills become an issue in the progress of curriculum reform implementation.

Finally, what role can teachers take in reforms? Recent approaches (e.g. Lozano et al., 2018) bestow upon teachers a prominent role, since teachers are considered at the same level as the designers of the curriculum. With this in mind, instead of worrying about teachers' fidelity in implementing reformed practices and resources, efforts will be focused on providing support to teachers by guiding them in making informed curricular decisions, based on insights derived from both research and practice.

Certainly, one of the main objectives of a curriculum reform is to ensure that all teachers know the curriculum and have the opportunity to discuss and interpret it. We can, in fact, conclude that one of the fundamental elements for the implementation of a new curriculum is making sure that the teachers feel confident teaching it. Therefore, it is impossible to neglect a considerable commitment to continuous professional development and training inherent to new curricular ideas, with the aim of making teachers aware and competent, in such a way that they feel able and comfortable in the implementation of any new curriculum.

Preparation of Prospective Teachers

A central part of mathematics teaching is the curriculum, which may provide tasks and activities that constitute the instructional core, influencing the sequencing of mathematics topics and the ways mathematical ideas and processes are made available to students. With reformed school curricula, in-service mathematics teachers may need to cope with implementation gaps if their prospective teacher education preparation was based on a past curriculum. Thus, continuing professional development of in-service teachers is an urgent need in the early years of implementation of reform (Reston, 2018). Mathematics teacher preparation and development for implementing a curriculum can be viewed as comprising two stages, the prospective and in-service stages which are generally regarded as a continuum rather than discrete phases (Reston, 2018). In this sub-section, we explore the question: how are prospective teachers connected to a reformed curriculum?

In the scope of the ICMI Study 24 Proceedings, we only find some ideas related to the question above. From the studies of Thailand (Inprasitha, 2018, p. 349; Changsri, 2018, p. 341) and Vietnam (Tran et al., 2018, p. 405), some suggestions can be realised in courses of teacher education programmes at university, such as prospective teachers having opportunities to analyse the new curriculum, new textbooks, or watch and analyse videos of mathematics classrooms. In Thailand, Inprasitha (2018) started an initial teacher education programme with two innovations, the use of Lesson Study and what he labels an Open Approach.

These innovations have been adapted and gradually implemented in all programme and collaboration schools, since 2002 (Changsri, 2018). To meet the demands of the reformed school curriculum following a competence-based learning model, which was implemented from the start of 2020 in Vietnam, Tran et al. (2018) conducted an innovation project, focusing on developing PCK for secondary mathematics prospective teachers, by offering them opportunities to experience mathematical literacy as active learners. The results showed that the prospective teachers began to develop an understanding of mathematical literacy and tried to integrate mathematical literacy into their teaching plans at an increased level of sophistication.

Prospective teachers might be required to connect with the reformed curriculum in their time of field experience. Prospective teachers can be expected to have significant learning from the ways in-service teachers teach in schools. There is evidence that teachers view the internship or field experience as the most valuable and beneficial part of their teaching experience (Behm & Lloyd, 2009). We assume that the learning opportunities arising from the interaction between prospective teachers and university lecturers, or researchers, are also important. Thus, it is desirable to have a variety of approaches or strategies to incorporate a reformed school curriculum across teacher education programmes at universities. Schools and teacher education institutions should ideally cooperate, in order to have coherent and aligned programmes and this has implications for the content and form of teacher preparation programmes. Alternatively, teacher education at university may take a “meta” approach (e.g., developing skills of reflection) which can be applied to a range of curriculum designs.

Along with the challenges of curriculum implementation, Lloyd (2006) suggested that prospective teachers should be experienced with school curriculum materials at university (although we note that not all teacher education, globally, takes place at universities). Through critical analysis of curriculum materials, prospective teachers would have opportunities to develop sophisticated views of the curriculum. These experiences could prepare prospective teachers to make a reasoned match between practices in their classroom and the curriculum that will continue to emerge in the future. Moreover, the teachers' field experience could ideally allow for the involvement of prospective teachers in the organisational aspects of their work, related to any new school programme.

Professional Development and Implementation of Curriculum Reforms

As noted in the previous sub-section, there is a wide consensus about the idea that success of curriculum reforms is linked to professional development (PD) processes that teachers undergo before and during the implementation of these reforms. Kilpatrick related to this idea when interviewed at the ICMI Study 24 conference:

> In a way, it is artificial for us to think of the curriculum as being separate from the teacher's professionalism, because it completely depends on that, and we cannot talk about reforming the curriculum, getting it in a new form, if the teachers are not with us. (interview with Dr. Jeremy Kilpatrick, in Shimizu & Vithal, 2018, p. 41)

We see such a view as being shared among scholars from diverse cultures and contexts. For instance, when describing a curriculum reform in Ireland, O'Meara et al. (2018) maintain that, "without explicit professional development […] any efforts to align revised curricula with existing curricula will result in reform efforts not realizing their full potential" (p. 155). Similarly, Brodie (2018), who works in the South African context, argues that curriculum reforms and ambitious teaching require ambitious professional development if they are to succeed.

Aims of Reform-Oriented Teacher Professional Development

Whilst the overarching goal of reform-oriented teacher professional development is to support teachers in implementing the reformed curricula, different local aims are documented for such programmes in different places internationally. We can identify two main categories: resource-centred aims, and student-centred aims. In the former group, the focus is on helping teachers be acquainted and work with, reformed materials, texts, mathematical concepts at the heart of the reform, and other resources. In this group, we find, for example, the aims of the PD programme held in Luxembourg during the 1960s and 1970s, towards the implementation of the *New Math* reform (Nadimi & Siry, 2018).

In the massive preparation courses conducted for in-service Luxembourgian teachers, the central aim was to introduce teachers to the notions of modern mathematics, so that they would be able to integrate *New Math* in their classrooms. A more recent example is the Champion Programme, a part of the Australian national project *reSolve: Mathematics by Inquiry*, which aims to develop teachers' pedagogical design capacity (Thornton et al., 2018). Champions support teachers in their work with the new curriculum ideas, assist them in understanding the material and human resources as well as using these resources to develop the learners' mathematics.

The second category, student-centred aims, is focused on the work of teachers around student input and student learning. This group is represented, for example, by the South African *Data-Informed Practice Improvement Project (DIPIP)*. The aim of *Professional Learning Communities* in this project was to discuss the reasoning behind learners' errors as a means to better value and understand learner thinking, teachers' own mathematical knowledge, and their teaching practices. It was hoped that teachers' practices will improve in relation to responsiveness to learners' input (Brodie, 2018).

In Lao People's Democratic Republic, the aim of a PD programme was to develop teachers' knowledge and orientations towards a student-centred reformed curriculum, based on task design through Lesson Study (Inthavongsa et al. 2018). Evidently, the aims of a reform-oriented PD programme do not necessarily fall into being just resource-centred or just student-centred. For instance, the *SHLAV* PD programme in Israel had both a resource-centred aim, i.e., to acquaint teachers with the reformed materials for use with students of low prior attainment, but also a student-centred aim – to enhance teachers'confidence in those students' ability to successfully handle matriculation[1] test items (Karsenty, 2018).

Models Suggested for Reform-Oriented Professional Development

Exploring approaches suggested in different countries for supporting in-service teacher learning around the implementation of a reformed curricula, reveals that a wide range of models exist for shaping PD. These models differ in terms of their content; pedagogical orientation (i.e., views about how teachers best learn); and organisational form (school-based, university-based, MOOC-based, etc.). We

[1] Matriculation certificate is a prerequisite examination for entering universities and colleges in Israel.

describe several examples below to demonstrate the spectrum of documented possibilities.

At one end of the spectrum we find individual initiatives for teacher development towards a new curriculum. Mok and Sung (2018) describe a case in Hong Kong, where a lead teacher designed, implemented and disseminated a 3 year enrichment programme for mathematically gifted primary students, around the idea of inquiry-based learning under the umbrella of the school vision and in alignment with the broader context of curriculum reform ideas promoted by the education system. The result was a reformed school-based curriculum that was adopted (with adaptations) by other STEM teachers in the school. This model of reform-oriented teacher professional development thus used a bottom-up method, centering on the experimental design approach, as a powerful way of teacher learning.

Another model that also focuses on individual teacher learning, yet not through a bottom-up approach, is the personalised PD model presented by Karsenty (2018). In this model, professional counsellors provide school-based PD for secondary mathematics teachers of low attaining students, around reformed materials designed to encourage students' learning through doing and understanding. Although the materials are defined by the Matriculation test topics, and not designed by teachers as in the Hong Kong case described above (Mok & Sung, 2018), the orientation behind this model is that the teachers learn best when they feel that the reformed materials are relevant for them. Thus, the endeavour in this model is to tailor the reformed curriculum to the specific context of low-track realities, and furthermore, to adapt it to local practices and constraints, through counselling that reaches out to schools.

Further on in this spectrum we find models that combine university-based and school-based PD. Inprasitha (2018) linked a prospective teacher education programme with an in-service teacher programme. Building up this idea through the implementation of Lesson Study as a professional learning community, experienced school teachers worked collaboratively with prospective teachers and both groups formed habits of 'teachers learning together' and formed a long-term professional learning community.

Inthavongsa et al. (2018) describe another model, implemented in Lao People's Democratic Republic, for the dissemination of Lesson Study. The teachers participating in the project first received instruction from university experts, followed by organised school visits. Then, the next phase of the PD was school-based, and included 3 months of practice within the teachers' own classrooms, directed by Masters' degree students. The implied (although non-explicit) pedagogical orientation here is that teachers learn to implement reformed ideas when they are given institutionalised opportunities to practice these ideas in their classrooms.

As the scale of the PD gets larger, the model offered may include several strategies and modalities. For instance, in the *Project Mathematics Education Reform in Costa Rica* (PMRECR), a large-scale PD for thousands of mathematics teachers was carried out between 2011 to 2017, using both face-to-face sessions and online independent work (Hernández-Solís & Scott, 2018; Ruiz, 2015). The model involved a two-tier PD: one for teacher leaders and the other for large populations of teachers, conducted by the trained leaders (Ruiz, 2015). At a later stage, MOOCs and mini-MOOCs were utilised, serving not only the need to reach a massive number of teachers, but also the hope to bring teachers closer to the use of technology and promote a modern vision of the educator (Hernández-Solís & Scott, 2018).

In the Philippines, the *Enhanced Basic Education Act* reform has shifted from the cascading model (a top-down process moving from the national level to the regional, division, and finally school level) and the cluster-based model (i.e., teachers from several schools attending the same training programme conducted by invited subject specialists), towards more innovative models such as Lesson Study, the Learning Action Cell and the needs-based PD model (Reston, 2018). A Learning Action Cell comprises a group of teachers who engage in collaborative learning sessions to solve shared challenges encountered in the school, facilitated by the school head or a designated Learning Action Cell leader. In the needs-based PD model, the content of the PD is determined following survey research, that identifies and prioritises teachers' needs.

A model similar to the Learning Action Cell, described by Brodie (2018), is the use of Professional Learning Communities as a platform to enhance changes in teacher practices around curriculum and pedagogical reforms. Brodie describes how in the South African project *DIPIP*, a set of activities designed for teachers' collaborative work was applied to support teachers' understanding of the reasoning behind learners' errors. Brodie suggests that Professional Learning Communities can be a useful model for reform-oriented PD, "particularly when curriculum reforms are seen as requiring ongoing interpretation and reinterpretation by teachers in relation to their local contexts, rather than once-off, fragmented inputs by outsiders" (Brodie, 2018, p. 334). This view emphasises once more the pedagogical orientation of *relevance*.

Types of On-Going Support Provided to Teachers

In different cases of curriculum reform implementations around the world, various types of ongoing support offered to teachers are reported, which include digital, textual, and face-to-face modes of support. Digital support may take the form of e-learning systems and communication platforms for participating teachers, as reported in Reston (2018). Through these online systems, teachers can access additional resources and share best practices. Another role that digital platforms can play is to support teachers' interactions with curriculum materials, so that they can become co-designers of the intended curriculum by actions such as tagging (Olsher & Yerushalmy, 2018). One of the advantages of such activities is that they support teachers' ownership of the reformed materials.

Textual support can be provided in the form of Teacher Guides and other materials designed specifically for teachers, as in the Mexican and the UK contexts reported by Lozano et al. (2018). These materials are explicitly designed to support teachers in making informed curriculum decisions, by providing them with guidance on conceptual learning, errors and misconceptions, strategies for differentiation, and more. Face-to-face modes of support are documented in several projects. In Lao People's Democratic Republic, support is given in the form of close collaboration with Masters' students coming regularly to the schools (Inthavongsa et al., 2018).

Similarly, in the Israeli *SHLAV* project, personalised support is provided weekly in schools by specialised counsellors (Karsenty, 2018). In the Australian *reSolve* project, recruited volunteering Champions work with teachers in Professional Learning Communities (Thornton et al., 2018), and the South African *DIPIP* programme is organised around Professional Learning Community weekly meetings with facilitators (Brodie, 2018).

One tool that has been gaining increasing exposure in schools and in research about teachers and PD, is the use of video, i.e., lesson videos for teacher reflection or learning and, in some cases, evaluation (see Gaudin & Chalies, 2015; Major & Watson, 2018). Finally, in relation to ongoing support for teachers, Zehetmeier and Krainer (2011) propose the content-community-context dimensions, which determine the sustainability of PD outcomes into the classroom fitting to context, quality, practicality (content); opportunities for collaborative reflection and discussion, teacher ownership and empowerment, an inquiry stance for teachers (community); administrative support, school-based support and resources (context). The categories provide a further perspective on (and a potential framework for) the planning required for effective on-going PD for mathematics teachers during a time of curriculum change.

Empowering Teachers' Voices in Reform-Oriented PD Programmes

One of the important questions that can be asked, regarding PD programmes designed towards curriculum reforms, is the following: to what degree, if at all, do teachers have opportunities to express their perspectives about the reform and be heard by policy makers? Moreover, do teachers' voices have an effect on the reform?

In some reform-oriented PD programmes, teachers' perspectives are taken into account only for evaluation purposes, e.g. in the Lao People's Democratic Republic Lesson Study reform (Inthavongsa et al., 2018), teachers were interviewed as part of an evaluative research, but this is not reported to have an effect on the programme. Yet, in several documented cases, efforts were made to include teachers' input as a kind of 'formative assessment'for the programme. Osta (2014) pointed out that other models of mathematics curriculum evaluation use more flexible approaches that include the close relationship between teachers and other actors, such as, principals and educational authorities. For instance, an example from Canada (Bednarz et al. 2012, quoted in Osta, 2014) is a hybrid model characterised by its long-term span, formative continuous development, regulated by the roles of the actors with the involvement of teachers and school personnel.

Pegg and Krainer (2008) reported on four examples of large-scale national reform initiatives in mathematics, where teachers were involved in rich professional learning experiences. Teachers were perceived not only as participants but as collaborators and change agents, and their communications with university practitioners formed the basis for fruitful contributions.

Within three of the projects mentioned earlier, opportunities seem to be provided for teachers' voices to be heard. In the South African *DIPIP* programme, teachers communicate their needs in the Professional Learning Communities, and the facilitators bring them back for discussion and consideration. Similarly, in the Israeli *SHLAV* project teachers can define their needs and receive personalised support tailored to these inputs. In the Philippines, teachers' perspectives on the reformed curriculum were documented, in specific studies, within PD initiatives where teachers expressed their opinions in either written or oral forms. We note, however, that these are only sporadic examples; there is an apparent necessity for more empirical evidence on the types and extent of impact that teacher input might have on the design, or the re-design, of reform-oriented PD programmes.

Evaluating the Work with Teachers in Reform-Oriented PD Programmes

A central issue for researchers on reformed curricula dissemination and designers of PD initiatives is the evaluation of the degree to which a reform-oriented PD programme was successful. Two kinds of criteria can be discussed: teacher-related criteria, and student-related criteria.

Teacher-related criteria refer to the evaluations of the PD initiative itself, and to the evaluations pertaining to the teachers' practices. Examples of questions to be posed for PD programmes' evaluations are: were the teachers engaged? did they find the PD initiative relevant? could they point to new things they have learnt? was collaborative work taking place? Whereas examples of questions to be posed for teacher practices' evaluations are: could changes be tracked in the teachers' lessons? which parts of the reformed curriculum were implemented, and how?

Various means are suggested for collecting teacher data: self-reports; surveys; written feedbacks; interviews; questionnaires; analyses of teacher conversations in PD sessions; direct lesson observations; longitudinal tracking of shifts in teachers' choices, and more. For example, in the Israeli *SHLAV* project, teachers' feedback was collected through interviews and questionnaires, showing positive views of the reformed curriculum for low-track students, and high satisfaction of the teachers regarding the personalised support they have received (Karsenty 2012, 2018). Data collection can be used also to better understand difficulties in implementing the PD programme, for instance, the *DIPIP* team (Brodie, 2018) conducted interviews with teachers who left the programme.

Student-related criteria refer to evaluations of student performance under assessments coherent with the curriculum reform. These can be local, national and international tests. In the Philippines, for example, the National Achievement Test (NAT) is used as a basis for ranking school and teacher performance; it is taken by students at the end of each academic year at the end of elementary (Grade 6), Junior High School (Grade 10) and Senior High School (Grade 12). Reston (2018) makes links between the low performance of Filipino students in this test, as well as in international assessments such as TIMSS, and the decision to initiate a *K to 12* basic education reform in the Philippines as of 2012 (as well as to expand the Basic Education cycle from 10 to 12 years). Thus, there is an expectation that the implementation of the reform will be manifested in improvement of student outcomes. However, since complex interrelations exist between student-related and teacher-related criteria, the use of student outcomes to evaluate the success of reform-oriented PD programmes is intricate, to say the least.

In the next section, we move on to consider many kinds of resources that are relevant to the implementation of a curriculum reform, not directly linked to prospective teacher education or in-service professional development. Of course, the separation is in some sense artificial, in that the role of resources is connected with, and in some sense, dependent on, the initial teacher education and the professional development opportunities for teachers. We recognise that adequate training and on-going PD opportunities are critical for teachers, in order to look critically at resources available and make judgments about when to adhere to them and when not.

Resources in the Curriculum Reforms Implementation

Apart from teachers, many resources can intervene in curriculum implementation. We first want to set out what we will and will not be considering in this section. Among the resources of an institutional and social nature, is the role of principals in educational units, the systems of inspection of the classroom action, and the role of parents. Depending on the magnitude of the reform, its educational or social impact, these can play roles of greater or lesser importance. These factors, however, will be touched on here only in an instrumental or tangential way. In what follows, we will focus on ‘traditional’ textbooks and ‘physical’ materials for teachers, as well as resources based on digital technology, but also other resources of a more social nature such as counselling and guidance processes for teachers, and guidance from leaders, experts and educational communities.

When talking about resources, we need to consider the context. In particular, textbooks: in some countries they are mandatory in the sense that the school or the teacher must follow one textbook; sometimes there is only one national textbook or several ones, but they must be certified; in other countries, textbooks are a market product without certification and only dependent on editors, with schools making their own decisions about what to buy. A similar range of structures and considerations apply in the case of other types of resources, but especially for assessments, when they are external to the school.

The resources for curriculum reform considered in this section can all be considered as 'tools' for teachers (textbooks, tasks, digital technology, and advice and guidance around their use) within the assumptions that are shared by the authors. We accept the notion of 'situated abstraction' (Noss et al., 2002, p. 207) in our belief that, in most cases, mathematical abstractions cannot be separated from the context of their creation or application. In this sense, any 'transfer' of knowledge from one context to another, takes place "not as a static entity, but rather through a reconfiguration and re-expression of mathematical relationships as they are reconstituted within different discourses" (p. 207).

Tools, technologies and contexts are, therefore, not 'ready-mades' (Rabardel, 2003, p. 641). Rather users adapt tools and are adapted by tools in a bi-directional relationship (Hollebrands et al., 2010, p. 325). The sensitivity of tool use to context must make us wary of how we read and what we synthesise from reports about reforms in different countries. To re-iterate points made earlier, it is important to keep in mind that the 'same' tool may, in different contexts, afford widely different possibilities and constraints.

Textbooks, Tasks, Teaching Resources

When analysing textbooks as a resource and examining their role in the implementation of curricular changes, it is necessary to consider different facets that vary among countries. These are the process of elaborating the textbooks, the use that textbooks have in the reform, particularly how they are connected to the new curriculum that has to be implemented, and the influence that high-stakes assessments have over textbook content. Moreover, the role of the textbook as a resource varies among countries; the type and the intensity of its use is closely related to other structures and infrastructures, such as the quantity and quality of material resources in the classrooms (including power relationships involved in the process of decision making about the use of resources), how professional development is organised, how the school is organised, what is the role of the principal and/or the teachers in the selection of the materials they use, or what are the working conditions of teachers. For example, in many Latin American countries, for teachers, the number of class hours per week can be more than 30–35, which leaves no room for PD and makes teachers highly dependent on resources such as textbooks.

Learning from different countries, we find that textbooks can be conceived as a public initiative or as a business opportunity. For instance, in Mexico (Lozano et al., 2018) textbooks are created by a core government funded team of mathematics educators and teachers, and the textbooks are distributed by the administration to every student across the country, because they are conceived both as teaching resource and as a tool for general literacy, not only mathematical literacy (especially in the poorer areas, textbooks could be the only books in a family home).

The alignment between curriculum reform and textbooks, when created and delivered in a centralised way, is high in general because of government-driven strategy, despite there being a lack of connections with real life problems or the concerns of local communities. Examples from Tunis (Artigue, 2018) and South Africa (Volmink, 2018) also show how different curriculum reforms were accompanied by the introduction of one textbook.

A certification process for creating textbooks can be found in China (Cao, 2018), where the government decided to move from a former unique national textbook (as it is still in Iran, see Gooya & Gholamazad, 2018) to the possibility of using other textbooks, but only with a prior authorisation. That is a kind of middle point between the government creating a textbook and the government stepping aside and leaving all the process in the hands of publishers. In Thailand (Inprasitha, 2018), Japanese textbooks were translated and introduced to the system within a university and government led process. Any certification process allows the alignment of textbooks with curricular changes, not only regarding contents but also methodologies. Japanese reform (Namikawa, 2018) is a paradigmatic example of this, after the introduction of the methodology of structured problem solving, this way of working in the classroom configured the textbook skeleton.

In other countries, textbooks are created by publishers, without any kind of certification process. This freedom means that, in some countries, books may not reflect the reform, since publishers tend to save elements from previous versions and even, for example in Spain, keep contents that no longer belong to the new curriculum. Moreover, the focus of curricular changes regarding procedures and methodologies can be missed or biased by retaining content from earlier frameworks.

For instance, Statistics in Spanish high school textbooks (Rodríguez-Muñiz et al., 2018) tends to be treated as a summary of rules and calculation procedures, barely insisting on the main ideas about statistical competence, specifically the notion of variability (GAISE project, see Franklin & Garfield, 2006). This fact was also pointed out by McCallum (2018) in his ICMI Study 24 plenary talk, who considered that textbooks are often not faithful to mathematics as it is intended by mathematicians, but they present mathematics as a closed set of concepts, rules and procedures. However, some examples were pointed out (e.g. the Netherlands in van Zanten et al. 2018) in which commercial bookmakers have progressively adopted reform-based approaches.

High-stakes assessments, or curricular-based exit exams, can also have a strong influence in the process of creating textbooks. The case of Lebanon (Osta, 2018) is an instance of a low degree of coherence between assessment practices and curricular objectives, especially regarding reasoning and communication competences, since textbooks are made to be coherent with the standardised high-stakes exams which, themselves, are not aligned with the curriculum reform. This role of external assessments, having more influence on textbooks than curricular changes in their philosophy, goals and objectives, has been also pointed out in other countries like Italy (Martignone et al., 2018) and Spain (Rodríguez-Muñiz et al., 2018).

Thus, we have observed different models about textbooks, from a unique textbook created or endorsed by government, to an authorisation or certification process driven by the government for publishers, up to the complete freedom of distributing and adopting textbooks. In the latter case, the decision variables for many schools not only depend on the quality of the textbooks but also on the commercial offers by publishers. Overall, even assuming the differences, textbooks are still a leading resource in implementing curricular changes, but we have noticed a pattern consisting of: the lower the level of development of continuous training and supporting resources for teachers, the higher the influence of textbook on teachers' practices, and, thus, the higher risk of failure of the curriculum reform if textbooks are not aligned with it.

We now move on to consider, when analysing curriculum reforms, resources in the form of manipulatives, classroom tasks and activities designed for being used by teachers and students, relating to curricular changes. These resources can be more influential than textbooks themselves, depending on the cultural contexts of the countries. That is the case of curriculum reforms in France (Artigue, 2018) and Denmark (Niss, 2018), where banks of resources, classroom activities or methodological guidelines and advice were generated as a support for implementing reform.

As with textbooks, the creation of these resources admits a wide variety of processes that can be built bottom-up, by communities, or top-down, led by authorities, as well as different grey-scales between both extremes.These examples underline that the curriculum is not only a set of contents or an official document. On the contrary, learning from examples, it becomes clear that there is validity in the notion of curriculum as a 'six-dimensional vector' (see, Niss, 2018), consisting of contents, but also of goals, materials, forms of teaching, student activities, and assessment.

Digital Technology

We pointed out the diversity of cultural, social and, therefore, educational contexts in the previous section. If technology is considered, this diversity often turns into inequality, producing huge gaps that become difficult to overcome. Thus, digital technology and its role as an educational resource is an issue to be considered separately. This chapter was conceptualised and mainly written before the Covid-19 pandemic. We want to acknowledge this fact, since we are not able to report on the massive and unexpected shift to online learning which took place during 2020 in many parts of the world.

This shift on-line did not necessarily provoke or entail any change in written curriculum documents, but inevitably resulted in a huge change in teachers' and students' experience of the curriculum. And the inequalities mentioned just now were, it seems clear, exacerbated. Concerns have been expressed in many parts of the world over the differential access of students to technology and support for learning, during times of school closures, as well as over teachers' unequal skills on technology in general and educational technology in particular. It remains to be seen whether the enforced uptake of online learning will have a lasting impact more widely, on technology use in schools.

Literature shows evidence of how and why digital technology improves the learning of mathematics (Drijvers, 2015) under different conditions and by different functionalities, but the focus in this chapter is on how digital technology contributes to curricular change. There is little evidence about this question, further than highlighting the role of inequality and comparing different environments from the technological point of view. No new evidence was found within the *ICMI Study 24 Proceedings* about this question, further than comments remarking that, as Northcote et al. (2010) pointed out, in many cases, technology such as the interactive whiteboard is used as a traditional whiteboard. Hence, the first idea we learn is that the technological resource is not enough, by itself, to produce a change in curriculum or teaching approach. On the contrary, its role strongly depends on its use.

Despite some of the examples presented in the *ICMI Study 24 Conference* mentioning the use of digital textbooks, such as in Serbia where they are going to be mandatory (Milinkovic, 2018, p. 146), no evidence about their role in curricular change was found. Moreover, there are no large-scale analyses about whether digital textbooks significantly change, or not, the format, and more importantly the role, of a traditional paper textbook. So, a key (unsolved) question arises: what are the differences between the use of a paper-based, a digitalised and a digital textbook?

Having answers to this question would lead to posing further ones: what is the role of a digital textbook in changing a curriculum? How can a digital textbook change the methodologies, activities, tasks and ways of learning mathematics? It is not clear at all that textbook makers are taking advantage of the multiple possibilities of integrating technologies in digital textbooks, by embedding different types of software and using technology in supporting learning rather than as using it only as a digital support.

Further than digital-*ised* textbooks, there is an abundance of research on the role of some products (e.g. dynamic geometry software, computer algebra systems) and the ways they are spreading in many classrooms, but they are conceived much more as an innovation resource rather than an instrument for developing and supporting curriculum changes. Developing digital text-book-like resources seems to be a teacher-led movement, with a lack of co-ordination.

In general, we have found there is little top-down guidance about how to connect technology with curricular change, beyond brief comments suggesting the use of technology in the curricular guidelines of some countries, or allowing the use of such technology in high-stake national-wide exams. We could say that the use of digital technology is an example of autonomous organisation of teachers, but its connection with curricular changes remains unstudied.

Nevertheless, some examples can be highlighted. For instance, in Ruiz (2015) we found a relevant experience from Costa Rica of the way technology can support curriculum reform. There, due to the teachers' mobility problems and intensive work days it was difficult for them to attend face-to-face courses, therefore, a PD programme based on e-learning was created in order to overcome these difficulties.

Hence, a set of blended online courses, MOOCs, and also less intensive short-duration Mini-MOOCs, were developed for qualifying teachers, allowing them to follow the courses according to their free time, without commuting, which is an important issue under difficult geographical and transportation conditions.

A further innovation in Costa Rica, with a special focus on independent study by students, is the development of *Mathematics Free Resources*.[2] Ruiz (2020) provides a detailed report on this experience in Costa Rica of technology as a curricular instrument. Such experiences are potentially transferable to other countries in Latin America.

Another highlighted example was found in England (Lozano et al., 2018), where the government supports a National Centre for Excellence in the Teaching of Mathematics (NCETM), which hosts a website with a wide variety of freely available resources, including teachers' materials, guidance, presentations, that can be used by teachers, supporting the implementation of a curriculum, renewed a few years ago, which is being re-interpreted in terms of these resources.

We also note an Australian example, based on the use of technology to overcome huge distances. Under the umbrella of the *reSolve: Mathematics by Inquiry* programme funded by the government (Thornton et al., 2018), selected 'champion' teachers follow a training programme, which is both online and face-to-face, to empower them in the use of different resources and approaches, in order to promote a challenging way of learning mathematics.

We conclude that, at the current time, technology is not a driver of curriculum change, nor are current curricula provoking significant uptake of new technologies for teaching and learning mathematics. And we contrast this conclusion with reports from a decade or so in the past (Sinclair et al., 2009) at which time there were several examples, from across the world, of national implementation of technology. Sinclair and colleagues reported on national projects taking place in Mexico, USA, Italy, Lithuania and Iran, some supporting the existing curriculum and teaching approaches with new technology and, in other cases, pushing the boundaries of curriculum and pedagogy. We have found it instructive to follow up, where possible with some of the instigators of these projects, what was the fate of those with a curriculum innovation element.

The Mexican programme, *Enciclomedia*, was an ambitious project led by the Mexican government to equip a great number of schools with free online textbooks and activities for primary education, provided by internet connection and supported with computers and projectors (Trigueros et al., 2006). The project was not endorsed with a capacitation programme for users and there was lack of technical supplies. Results were consequently not as good as expected. Sadly, a common sight on a recent visit by one of the authors to Mexican schools was the *Enciclomedia* hardware, covered and not being used, in the corner of each classroom.

The USA project, *Sketchpad for Young Learners*, aimed to help primary age students explore and understand key mathematical concepts through the dynamic

[2] See: https://recursoslibres.reformamatematica.net

visualisations possible within *The Geometer's Sketchpad*. The technology, at that time (in 2009), was unfamiliar to many teachers and students and, in common with the Mexican experience, there was a lack of resource for teacher support.

In Italy, the m@t.abel project started in 2006, as a follow up of another project 'La matematica per il cittadino' – Mathematics for the citizen – which developed from 2000 to 2005. M@t.abel lasted until 2012 and had some influence on the national curriculum, which was elaborated in those years (through many political changes). The original project focused around problem situations for which no clear cut or routine procedure is available for solution. A cascading model of training was built into this project, with the aim, within a few years, of reaching almost all teachers in grades 6–10. This extensive training model is interesting to reflect on, in light of the project's longevity as it seems to provide a contrast to the USA and Mexican experiences.

The experience in Lithuania was focused on the implementation of the use of *Geometer's Sketchpad* in grades 9 and 10, as a way to promote dynamic visualisation across the entire curriculum. The project included the development of over 800 sketches, for covering almost all the curricular issues. Apart from specific teacher training, the project highlighted the potential for making an explicit link between curriculum and technology.

In reflecting on the challenges of incorporating digital technology into the school mathematics curriculum, Ruthven (2017) identified *ecological*, *epistemological* and *existential* challenges. This framework seems relevant to considering more broadly the up-take of new resources in a time of curriculum change. In brief, ecological challenges relate to constraints of time, space and infrastructure, in adapting everyday practice in the classroom. Epistemological challenges relate to the requirements for new knowledge and skills in using new resources; and existential challenges relate to the way that values and identities associated with the whole project of school mathematics influence the adoption (or not) and understanding of new tools. There seems evidence, above, that for resources to meet the ecological and epistemological challenges, is a prerequisite for successful implementation (i.e., their absence seems to mean digital projects cannot thrive and grow). We suggest the existential challenge is likely as significant as the other two.

Support Agents for the Use of Resources

There are wide differences across countries in the use made of advisory documents and guidance for teachers, either linked to textbooks or particular resources, or more broadly to curriculum reform. In this section, we offer two examples of country-specific innovations which give a sense of past practice and ways that the two countries are attempting to drive reform through the development and promotion of resources. The two countries are Mexico and England, which were chosen in part because of a similarity in what is taking place across very different contexts but also because they perhaps signalled some unusual imagined relationships between users and creators of new resources.

In 2017, a Mexican curriculum reform was presented through a new 'educational model' which emphasises quality in education for all students. In addition to the new educational model and the National Curriculum, the Secretary of Public Education (SEP) set about developing different materials, including nation-wide textbooks and accompanying teaching guides[3] for each subject. These textbooks are meant to provide, "a common ground for education in the country […], and are conceived as instruments which facilitate diverse and pertinent educational practices" (SEP, 2017, p. 126).

Through the new nation-wide textbooks and teachers' guides, new ways of working are being introduced, with a stronger emphasis than previously on guidance given to teachers. The previous versions of nation-wide textbooks gave general recommendations for each area of mathematics (number, geometry and measurement, data handling), and included brief suggestions specific to the chapters.

In the new materials, specific guidance is provided for each chapter and particular attention is given to: intentions related to conceptual learning; questions that can be asked to promote reflection; common mistakes and misconceptions; strategies for problem solving; strategies for differentiation; manipulatives or models that can be used. Relating this innovation to the previous section, we can see that the teacher guides are being used in an attempt to prompt and provoke classroom innovation, related to curriculum reform. The textbooks in Mexico are mandatory and there is a high degree of consistency in terms of curriculum reform and textbook guidance. Our second example comes from England.

In England, a new curriculum was introduced in 2014. In 2015, there was an explicit government agenda to alter the practice of mathematics teaching, drawing on practices from East Asia, particularly Shanghai, towards what is labelled as a 'mastery' approach. The introduction of a new vision for mathematics teaching took place without a change in curriculum (the 2014 curriculum does not mention mastery). The official government body tasked with promoting and developing the new approach is the National Centre for Excellence in the Teaching of Mathematics (NCETM). The NCETM defines a mastery approach as meaning, "Pupils are taught through whole-class interactive teaching, where the focus is on **all** pupils working together on the same lesson content at the same time" (NCETM, 2016, p. 1; bold in original). In contrast, an organising principle in relation to typical primary school teaching would previously have been that of 'personalisation' a concept at the centre of a past reform of mathematics teaching in England (DfE, 2011, p. 26).

The 2014 National Curriculum in England specifies learning outcomes for each year of study but is deliberately neutral about how these might be achieved. New guidance materials and resources being produced by the NCETM offer an ordering of content alongside themes, such as "equivalence", which are introduced at the start of grade 1, and developed in a systematic manner throughout the primary

[3] See: http://www.snie.sep.gob.mx/descargas/estadistica/SEN_estadistica_historica_nacional.pdf

school years. Students are introduced to the number line and two models for conceptualising part-whole relationships. Having core representations of additive structure that are introduced early in primary schooling, and used consistently and repeatedly, is an innovation in England.

A final distinction to note, compared to previous guidance, is the explicit distinctions made about how “number” is conceptualised, to ensure a balance of cardinal and ordinal or measure-based approaches. Current practice in England would have worked on number in an almost exclusively cardinal manner in grades 1 and 2 (Coles & Sinclair, 2017). According to the new guidance, students’ very first introduction to number work will be in the context of measures (drawing inspiration from the work of, for example, Dougherty, 2008).

We notice a similarity across the Mexico and England examples, which is that resources are explicitly aimed at provoking professional development activity. In other words, in neither country does guidance aim to define precisely what teachers should do. Rather, guidance aims to support teachers in re-thinking their practice and making use of resources in a way that allows for innovation - at least, this is the clear intention of the authors of the guides. The resources, for instance, can be used as part of a collaborative planning process in schools.

We see an interesting development, in these examples, in that teachers are being imagined as co-designers of the curriculum, rather than as implementers of someone else’s design. Of course, whether the resources will be experienced by teachers in a spirit of co-design will depend on a myriad of factors including local leadership and opportunities for continuing professional development related to the new guides and materials.

Final Remarks on the Use of Resources

In looking back across this section, we aim now to distil some of the features of the research cited, which pertain to the question of how and why teachers use resources, in a context of curriculum change. We identify areas in which there is quite significant variation of practice across counties and areas, which could lead to further research.

Control of, and access to, resources ranges from resources mandated for teacher use such as a national textbook, and resources, either freely or commercially available, which teachers can choose to use or not. In-between these extremes there are varying levels and levers of active promotion of resources. For example, where resources are linked closely to high-stakes assessments, there may be a sense of needing resources to avoid disadvantaging students (e.g. in England, Pearson’s Publishing group runs an examination board and also publishes textbooks linked to those examinations).

Government inspection regimes may also have particular areas of focus related to the use of particular resources or pedagogies that are linked to resources. The kinds of resources available range from being freely available online, to being online behind a paywall, to blended resources combining digital and physical versions, to physical artefacts. Again, there is a range of practices in terms of the costs of physical resources. In Mexico, schools are provided textbooks for each child at no cost to the school. In England, schools must meet the full cost of any textbooks they purchase.

We see a range of variation across the creation of resources in terms of the extent of teacher involvement. In some cases, resources are created by 'experts' (who might be teacher educators, mathematicians, publishers), perhaps with trialling and testing in classrooms. In other cases, there is a process of co-creation with teachers and in some cases the resources are created by teachers, perhaps shared via social media. It is beyond the scope of this chapter to offer any evaluation of these different processes and variations but we see such evaluation as a fruitful area of future research.

Similar kinds of variations to those discussed above are apparent in relation to students' use of resources. In some cases, it is mandated by government that students use particular textbooks, in some other cases students access resources decided on by their teacher or school. There are, of course, resources freely available to students online that may be accessed as part of school lessons or independently.

The actual effectiveness of resources in the implementation of a reform is linked to the factors described above. There are examples across the world of innovative resources being made available but not being taken up. As described above, factors at play here include whether particular resource use is mandated or not by local or national government and the extent of training and support provided to teachers. One general question we would like to raise is the extent to which new resources and tools are a good *fit* to new pedagogies implied within curriculum reform. We intuitively feel such a fit, or its absence, must be a significant factor in the use of resources linked to reform, but we are unable to draw on any evidence for this idea. To return to the framework, we suggest that Ruthven's (2017) *ecological*, *epistemological* and *existential* challenges, in the take up of digital resources, can be applied more broadly, to help consider the range of challenges in using resources related to curriculum change.

Role of Assessment in Curriculum Reform Implementation

Assessment has a complex role in teaching-learning processes as well as in the process of curriculum implementation. This section deals with some aspects concerning the role of classroom assessment and external assessment in curriculum implementation. In particular, we focus on how assessment has been used, not only as a source of information about the attained curriculum, but also as a way to perceive the intended curriculum and as a resource for curriculum implementation.

International, National and Classroom Assessment

Depending on its aims, assessment takes many forms and involves students and teachers in different ways. Assessment can be carried out by specific tasks, being part of classroom teaching and learning activities, or of a process of system evaluation. In literature, we find studies in mathematics education on the relationships between the processes of curriculum implementation and assessment that focus on classroom activities carried out by teachers. In particular, the classroom assessment can be identified by, "the activities undertaken by teachers in eliciting and interpreting evidence of the students' learning and using this evidence to inform subsequent action" (Goos, 2014, p. 413). This type of assessment should be distinguished from the external assessment, usually developed for summative or evaluative purposes, which often involves large scale standardised tests.

The complex relations and the interweaving between assessment (classroom and external assessment) with curriculum implementation can be analysed from different perspectives. An issue that has been identified and studied in the last years, concerns the influence of some international surveys (both in terms of framework and in terms of results) on national curricula around the world. The results of these international surveys must be read and interpreted within the different cultural systems. The connection between international assessment and national curricular changes was also pointed out in some papers in ICMI Study 24 Conference focusing on the influence of TIMSS or PISA surveys on design and changes of Mathematics Curriculum (Kadijevich, 2018; Milinkovic, 2018; see Chap. 22).

TIMMS and PISA surveys aim at evaluating education systems worldwide by testing knowledge and skills of students in different school grades. These surveys use standardised tests because they can be administered on a large scale, but standardisation causes some criticisms from the academic world. Moreover, there are also concerns about the contents and the tasks proposed: one of the most commonly criticised aspects is that they focus on calculation skills rather than on mathematical thinking. Unfortunately, many, but not all, standardised tests were constructed following a view of learning (perhaps implicit), according to which learning is most effective when knowledge and skills are broken down into many small steps that can be taken sequentially by learners.

Such a fragmented view of evaluation is not aligned with the more current socio-constructivist and historical-cultural perspectives, which are at the basis of many educational practices suggested by research and new curricula around the world. Although it is certainly true that in a standardised test it is not possible to find certain types of problems, which are however very important for assessing mathematical competences (conjecture, exploration, long and complex problem-solving activities), it must also be said that tests can be made up not only of questions which require the recollection of notions or the production of calculations. It is important to define their aims and what they actually show. External assessment is usually used for summative and evaluative purposes and seen only as related to the analysis of the attained curriculum, despite a range of other possible uses.

We would like to point out some of the other aspects of an external assessment. It can also impact on the complex process of curriculum implementation. For example, changes in national assessments can lead to a change in teaching practices, as has happened in some European countries. In particular in Italy, the National standardised assessment tests, called INVALSI tests, have become a means for teachers and students to deal with tasks that are constructed according to the goals explicitly stated in the National Guidelines. Therefore, national standardised tests can be used by teachers to reflect on curriculum demands and then to perceive the intended curriculum (Martignone et al., 2018). Standardised tests, especially when they are strictly linked to country standards and curricula, can become a resource for teachers who can use them to reflect on curriculum requirements and thus on changes and differences in practices.

Even if the aim of this external assessment is to contribute to a system evaluation, we argue that an analysis of these test tasks can also be a tool to modify the system itself and to carry messages that influence the implementation of the curriculum by leading, for example, the attention on topics rarely developed or on particular types of tasks. In this way, assessment can become a resource influencing the curriculum reform. Still, in the case of Italy, the analysis of national assessment tests is often used in teacher educational programmes because, in order to become agents of the reform, teachers have to be supported by means of educational projects.

During these programmes teachers focus their attention on the understanding of where, how and why students have difficulties in particular problems. This can lead teachers to analyse and reflect on these problems and also to use them in the design of classroom activities. This is not "teaching to the test", but developing formative activities that consider the curricular requirements and that may also have been inspired by problems from standardised tests. Thus, it is possible to build a bridge between external assessment and the activities carried out in the classroom.

In contrast to the case of Italy above, Osta (2007) presents the example of the Lebanese, where the high-stakes national examinations set implicit boundaries for the implementation of the reformed curriculum when they were not aligned with the intended reformed curriculum, and still carried the "assessment culture" rooted in the educational community's understandings and practices, through years of pre-reform stereotyped examinations. Osta highlights the consequent formation of a 'mini-curriculum' consisting of a limited set of stereotyped test items repeatedly included in the national examinations: "This stability and stereotyped structure make teachers and students adopt that mini-curriculum as their set of guidelines instead of the curriculum" (p. 194).

Osta contends that the written, or intended, curriculum is static once a reform is set on paper, while the implemented and the tested curricula are dynamic and variable. In the case where assessments are aligned with a new curriculum, the set of tested curriculum items moves within the intended curriculum by including, each time, different topics, in a way to ultimately cover the intended curriculum. However, in the case where the assessments focus on a small part of the intended curriculum, it is the taught or implemented curriculum that is variable, gradually shrinking closer and closer to the tested curriculum, thus forming a mini-curriculum.

Classroom assessment is usually identified as part of curriculum implementation. It can be different from class to class, from school to school and, of course, from country to country. Therefore, about class practices and implemented curriculum, we can only make general considerations or quote specific examples. Classroom assessment can be carried out for both summative and formative purposes, mainly by the teachers of the classroom. It is often formative because it aims at supporting students' learning and informing teachers' instructional decisions: "Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction" (Black & Wiliam, 2009, p. 9).

The typical activities in formative assessment processes are those through which students have the opportunity to verify their own learning levels, to plan and implement, to interact with the teacher and the classmates, and to develop the strategies necessary to achieve the set learning objectives. Therefore, the formative assessment is carried out during day-to-day practice, in-class exercises, homework, projects, etc. Teachers should develop tasks and methodologies that can give evidence of students' learning by promoting mathematical thinking (abstraction, contextualisation, connection between different concepts, argumentation, problem solving, etc.) and by using different appropriate representations (verbal, tabular, graphic, symbolic, etc.). Like all types of assessment, formative assessment is based on the interpretation of observable variables, which can allow a judgment on the quality of learning and on the effective curriculum implementation.

In this chapter, we have already dealt with teachers in the process of curriculum reform implementation, in addition, in this paragraph, we focus on their fundamental role in the assessment process. This type of assessment may or may not be consistent with curriculum goals. It considers different activities developed at school. Studies on the relationships between curriculum and assessment might take into account components of teacher knowledge of curriculum and assessment methods and how they can be improved (Santos & Cai, 2016).

Teachers use their professional competences to develop teaching strategies that translate the written curriculum into the implemented curriculum. Often, however, teachers are strongly influenced both by textbooks and by external assessment more than by what is written in the official curriculum. For example, if some topics or a specific type of problem are not present in the textbooks or are not assessed, no matter what is written in the curriculum, teachers and students may not focus on those or even, a teacher may decide to skip, or not to assess these topics. There may also be another effect: if a topic or certain tasks are present in the external assessment tests, then the attention of teachers and students will turn to these. Therefore, it is fundamental for effective reform that the assessment, as well as textbooks, be aligned (Schmidt et al., 2005) with curricular goals.

Conclusion

Different roles can be played by teachers as agents of curriculum reform. From getting directly or indirectly involved in drafting the reform, to participating in producing specific documents or materials, or designing or organising professional development programmes associated with the reform. Teachers' engagement with the curriculum reform, and the way professional development is organised and aligned with it, has been pointed out as one of the most important factors in the implementation of a new curriculum.

The acquisition of an adequate mathematical content knowledge during initial training may support teachers in fostering a close connection with future reforms. But, also, specialised knowledge, and specifically, training with school materials, will prepare prospective teachers to match theories underlying the curriculum reform with possible classroom practices.

Reform-oriented PD programmes can focus not only on teachers but also on teachers' resources or even on students. Examples show how helping teachers in working with a new curriculum and supporting them with materials and shared discussions helps in the implementation process. We have seen different implementation experiences from different countries and different cultural contexts, which underline that there are several ways to approach the problem and that just translating one successful case is not enough, because an adaptation to each country's circumstances must be made to transpose any model.

The wide variety of implementation models seems to become even greater when considering the results. We have seen that there are no clear patterns regarding the use of resources during implementation processes. Sources of variation go from the creation to the control of the resources and their availability. New digital technologies are gaining space, not only as supporting resources but also as digital and digitalised books. The way teachers use resources is also a source of great variation. From different examples, it seems that the absence of an authority-driven control of textbooks produces bad effects in their quality and alignment with the reforms.

A similar case occurs with the assessment. Some examples have been pointed out about how external assessment influences teachers' classroom activity as well as PD activities. Particularly, international assessments such as PISA or TIMMS produced changes in the curriculum of some countries. It is difficult to find a prior alignment in such cases, but the most common problem emerges when the assessment is a national one, not aligned with curricular changes, thus weakening its implementation.

From reflections and discussion presented in this chapter, we have summarised, in Table 18.1 some of the key dimensions of variation, as well as questions for the future.

We want, finally, to raise an issue of pressing importance which, however, does not appear in any of the documents presented at the ICMI Study 24 Conference, that is the issue of when and how curriculum reform will pay attention to the global crises of climate change, biodiversity loss, mass migration, access to water and other worldwide issues. In other words, we see daily evidence of the precarious

Table 18.1 Variations and questions identified relating to resources and curriculum reform

Domain	Sub-domain	Range of practices
Teachers	Teacher participation in reform	Drafting; preparation; consultancy; implementation; evaluation; co-designing.
	Teacher preparation	The role of teacher preparation in curriculum reform seems unexplored.
	PD aims	Resource-centred; student-centred.
	Models of PD	Content (the curriculum itself); pedagogical orientation (views on how to teach); organisational form (national; school-led/university-led; face-to-face/online; personalised/collective; cascading/cluster-based/needs-based).
	Teacher voice in PD	Participant; evaluator; collaborator; change agent.
	Evaluation	Teacher-related criteria; student-related criteria.
Resources	Textbooks	Government mandated for all students; limited government certified range of options; commercial marketplace.
	Tasks and teaching resources	Resources as a support for implementing reform; top-down lead; bottom-up lead.
	Digital technology	The connection between take-up of digital technology and curriculum change seems largely unexplored.
	Support agents	Curriculum change; teacher guidance around pedagogy; teacher guidance around content knowledge.
Assessment	International	TIMSS or PISA influence on national curricula; assumption in these tests that learning comes from breaking knowledge into small steps.
	National	Changes in national assessment leading to changes in teaching practice; national assessments perceived as the intended curriculum.
	Classroom	Summative purposes; formative purposes.

nature of life on earth and yet the mathematics curriculum appears to continue, to some extent, as though nothing immediate and different is taking place beyond the school walls.

We have little to say on this point, except to point towards the relative absence of research, within mathematics education, that is considering, for example, a curriculum for an age of climate emergencies. We would, however, like to use this publication to urge scholars to consider *now* what reforms will be needed when catastrophic events force re-thinking the way we live. We want to suggest that there be an urgent need to develop a *curriculum in waiting*, for example, or to find ways of connecting the school curriculum with the concerns of school communities (Coles & Sinclair, 2017). Reflecting on the past implementation of reforms and what we can learn about resources that occasion change is equally vital work, in terms of looking towards an uncertain future.

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Chapter 19
Conclusion 'Laws' of Curriculum Implementation and the Future in Which We Are Living

Angel Ruiz

Several decades ago, the main tendencies in the school curricula of mathematics revolved most of the time around disconnected collections of mathematical topics, with little association to pedagogical approaches, or explicit socio-cultural and pragmatic purposes. In recent times, international trends have established curricula around competences, a pragmatic role of mathematics is fostered to serve societies and their citizens, interdisciplinary perspectives have been promoted and in particular around STEM, larger spaces are incorporated for statistics and probability, and with force: The role of technologies impacts everything. Undoubtedly, the strategies and educational agents to implement curricula within these tendencies are different from those that could be used in the past. And things are moving even faster and toward new perspectives due to the pandemic and a new world scenario.

Is not this expressed in the growing role we give to real contexts in curricular design, textbooks, teacher preparation, or national and supranational assessment? Does not the simple existence of MOOCs, Mini-MOOCs, Tablets, Smartphones, the Internet, remind us that the demands in curricular implementation are no longer the same?

Given the current stage of development of mathematics education as a discipline, it is at least desirable that before proceeding to their implementation the reforms be based on national and international research. Doing so allows us to provide intellectual supports and examples of good practices for the progress of curricular changes. However, the same implementation process can provide elements that lead to an improvement in the relevance and quality of a curriculum. At the end of the day, it must be understood that a curriculum is a historical object that must undergo modifications. A participation of the educational community, within an official process oriented and relatively controlled, can favour this. However, in social or

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national contexts that do not have those levels of ‘control’, resources or a maturity in their education system, the result may be drastically different. This is another example of the diversity of options that we have tried to show in this work.

The issue also has implications for the study or assessment of reforms, and therefore for the research: How has a reform been designed in relation to the results of the international research and experiences? What is special and specific in the implementation of a particular reform?

In curricular reforms, external and internal factors intervene that greatly condition the nature, rhythm and the possibilities of their success or failure. Three variables that generate or sustain the changes are also crucial, and to which we have given a place in this work: their visions, values or goals. And, in the latter, there is a condition that benefits the possibilities of its success: its coherence, which invokes historical and social relevance; but this not only has theoretical dimensions, its materialisation is favoured if the main reforming agents (either within the ministries of education, universities, or teams of individuals) obtain the essential continuity in social processes. Sometimes that continuity occurs as a result of political decisions, sometimes not, chance almost always intervenes and always plays an important role in reforms.

The ‘Law’ of Diversity

The experiences we have analysed and brought to our discussion show important differences both in the nature of the reforms and in the contexts in which they occur.

One example: our review suggests that sometimes reforms can directly touch only some of the components of the curriculum vectors (see Niss in Chap. 16), although inevitably, in one way or another, they affect the other components. This selection of components already makes a difference, but also the way in which these relate to the other ones.

Another dimension: in the development of reform there are different agents and levels of intervention (authorities, scientific or professional organisations, teachers, advisors, inspectors, principals) and the combination of these interventions is crucial. How this occurs can determine the course of the reform. In Tunisia, for example, the role of inspectors played an important role. Other of these agents are politicians and educational authorities of the highest level. Costa Rica and China, although with very different political and socio-economic conditions, are examples that achieved continuity in the support of the reforms by these agents. Denmark provides an example to a certain extend in the opposite direction. The academic associations in the mathematics community and of mathematics education in the French reform -another agent- were very important. They brought legitimacy, coherence and support.

The role of the diversity of contexts can be seen in the ‘time’ or ‘timing’ of reforms. In countries with social, political and cultural stability it is possible to have many years to carry out an appropriate reform (or not), and even to continue advancing it. (Quebec, China and France are examples we described extensively with at least ten years dedicated to implement the reforms.) In other contexts, timing can be provided by political or social circumstances, and times and rhythms may be determined by them. This impacts the chances of success.

The unavoidable diversity, we have insisted on here, allows us to state what we can call a first general 'law':

> *There can be a combination of factors for the implementation of a curriculum reform that is successful in one country and will not be successful in another.*

There is one consequence of this 'law':

> *It is not adequate to try to mechanically translate, import curricula or curricular ideas from one country to another, from one context to another.*

However, this does not mean that it would be inappropriate to affirm that good practices, ideas or strategies, endowed with a serious, critical and responsible assessment, can be a source of inspiration for curricular design and implementation, they can serve as a model in other scenarios.

The 'Law' of Two Directions

Another trend that weighs in the historical scenario has to do with the democratisation of individual and collective efforts, and, in particular, the relationships between institutions or governing bodies and the population. Educational and technological progress enhances that situation. We have described here curricular changes formulated by governments, as well as others raised by universities, by groups of individuals, or by in-service teachers. Different groups provided different scopes for reforms. But everything points to the fact that the success of reforms depends on a harmonious combination between actions that go from the top to the bottom and those that come from the bottom up.

Artigue insisted that, without the individual commitment, internalised, towards a reform, of the teacher, of the director of an educational institution, or of the pedagogical advisor, the reform will not be able to progress. The individual needs to feel part of the reform. And Niss has alerted us that a reform will have less chance of success, projection and continuity if it does not have the national, governmental or institutional support in resources and actions.

This brings us to a second 'law':

> *To achieve success, the existence of appropriate resources, especially for teachers, and implementation strategies must be considered, allowing for both top-down as well as bottom-up developments. What is essential is to create a good synergy between these two processes.*

The harmonious specific point, which integrates efforts and generates progress, depends, of course, on each reality. As we pointed out in the conclusion of Chap. 16. the differences between developed and developing countries or regions are significant.

It should be important to underline that this harmonious point is never easy to determine.

The 'Law' of Alignment

We were able to show multiple roles in the participation of teachers within curricular reforms, from the creation of specific materials, to advising on the design of the curriculum or in the development of professional development programs, and all in varying stages. To facilitate the implementation of curricular reforms nobody doubts that a good preparation in mathematics is required, but it must be emphasised, also that a close contact with the school and pedagogical materials with which educational agents must work (adapted to the historical moments). The way in which this is done varies significantly, but when focusing on the curricular implementation there is a greater and significant value to these last components; something that does not happen in the same way when the focus is only on the design.

It is a consensus that there must be important connections and alignments between the initial preparation and professional development of teachers to sustain curricular reforms, with greater force in the case of profound transformations. But not only the congruence between the reform, its implementation and the conditions of the teachers is raised, it is also invoked for the necessary resources and for the national assessment.

Here emerges a third 'law':

> *For the most adequate implementation of a curriculum reform there must be alignment of all the educational means with the reforming efforts.*

The 'Laws' of the Long-Term and Uncertainty

A fourth 'law' of reform appears to be:

> *Except in the case of curricular changes with very little scope, reforms must be conceived as long-term processes.*

Understanding this is very important, as political actors, educational administrators and societies in general tend to push for results and implementations in the short term. Inappropriate reduction of the time and resources needed for a reform inevitably conspires against its success.

No matter how well designed a reform is or how planned its implementation is, it is inevitable that there are unforeseen events that force adjustments (sometimes substantially) of the implementation actions. The 'ecological' approach introduced by Artigue in Chap. 16 reinforces this perspective.

This leads us to a fifth 'law':

Curricular reforms are not in vitro processes, and therefore inevitably contain a large burden of unpredictability and uncertainty.

A final 'Law' (Within the Scenario of the Pandemic)

The unpredictable has hit us hard in the face with the COVID-19 pandemic. Its implications for the planet are not yet clear. As far as education is concerned, there are some elements that seem to be incorporated in the following years: for instance, the role of 'non-face-to-face' education will play an extraordinary role in all settings. And that implies an intervention of different technologies in ways not seen before. This will impact educational processes at various levels in various ways. The face-to-face and non-face-to-face will be articulated with new perspectives.

Not all educational levels or all school disciplines will be impacted in the same way. Those who demand more face-to-face or individual accompaniment will have to make further adjustments. The learning of mathematics (by the nature of the discipline) has always required greater pedagogical support. Responding to this situation will be a major challenge for the mathematics and mathematics education communities.

Evidently, beyond this sort of issue other educational variables intervene. Aims, values, content, teaching and learning strategies, should be in tune with a changed world where global environmental and general humanity issues demand a stronger place. The new pedagogical mediation will exert pressure on all the actions of the different educational agents (in teaching, advising, supervising, planning, managing). The impact affects the building of learning, assessment, resources. To a certain extend initial teacher preparation and professional development should be renewed.

The impact of the pandemic, however, is not experienced and will not be lived in the same way in all nations and regions. Those with higher levels of poverty and fewer socioeconomic, cultural, educational, and ICT resources will have much greater difficulties to stabilise and progress in the near future. Extraordinary percentages of student dropouts and very serious losses in schooling are foreseen. A recovery will take several years.

The implementation (and design) of curriculum reforms in mathematics cannot evade this complicated scenario. Definitely. This leads us to an awful 'law':

In the scenario opened up by the COVID-19 pandemic, all curricular implementation must include a Factor (RRR) that involves at least three actions in connection to objectives: Rethinking, Reformulating and Reprogramming. This Factor RRR is most likely to be higher in countries or regions within a country with lower socioeconomic, educational and cultural development.

In the new scenario, it seems just reasonable that international understanding and co-operation for education (particularly for mathematics curriculum reforms) should be strengthened.

The Place of Implementation

This may be considered a strong opinion: There has been too much emphasis in the world on curricular design and much less on implementation. Perhaps this is why it is useful to underline the demand for a ‘Perspective of praxis’ (Ruiz, 2013), which helps us to separate ourselves from the visions that see the curriculum as an almost "in vitro" experience that should be implemented later on at some other time. This issue acquires greater dramatic effect when it is found that no matter how well designed a reform is, how good the previous pilot plans are, there will always be large doses of unpredictability.

Implications? To help move future curricular changes more successfully, it will be necessary to intensify research and intellectual constructions on the implementation of the reforms of school mathematics. This would provide elements not only for the implementing action, but for the curricular design itself.

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Part V
Theme D – Globalisation and Internationalisation and Their Impacts on Mathematics Curriculum Reforms

Chapter 20
Introduction

Max Stephens, Djordje M. Kadijevich, Mogens Niss, Nadia Azrou, and Yukihiko Namikawa

Theme *Globalisation and Internationalisation and Their Impacts on Mathematics Curriculum Reforms*: Specific Foci and Questions From the Discussion Document (ICMI, 2017)

1. How have results of international experience and research in the teaching and learning of mathematics influenced curricula changes? To what extent can local curriculum reforms be examined against an emergent 'international' mathematics curriculum?
2. How have particular international studies become drivers for school mathematics curriculum reforms? What new discourses with dominant theoretical and conceptual underpinning have emerged; and how have these been taken up in curriculum reforms in different contexts? For example, how have the OECD's

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PISA notions of mathematical literacy and mathematical competencies been interpreted and expressed in curriculum reforms?

3. How are mathematics curriculum reforms varied (or similar) in different social, cultural, economic and political contexts such as developing versus developed countries or East versus West? How do selected curriculum components such as content, pedagogy, materials technology and teacher preparation vary from one reform, tradition, country or context to another?
4. How can comparative or meta analyses of curriculum reform processes and implementations shed light on what works or does not work in mathematics curriculum reforms in contemporary societies?

This Introduction will address Questions 1, 3 and 4. Questions 2 and 3 will be a focus of Chap. 21 on the impact of international student assessments. Chapters 22 and 23will return to Questions 1 and 3, analysing selected features of the current and anticipated 'international' mathematics curriculum in different national and other contexts. The Conclusion will examine briefly what has been learned and how might we expect globalisation and internationalisation to continue to impact on the curriculum.

Introducing the Major Theme: Globalisation and Internationalisation

Globalisation is not a new phenomenon. The same term can be used to describe the rapid expansion of trade and accompanying colonisations undertaken by the major European Powers during the sixteenth and seventeenth centuries because of dramatic improvements in navigation and in ship-building technology. We can identify who the main players were – Portugal, Spain, France, the Netherlands and England. Their rapid expansion of trade and colonisation into the new world was truly global, contrasting with earlier trading relationships within Europe, such as through the Hanseatic League.

Globalisation in the late twentieth and early twenty-first centuries has taken on new forms through rapid developments in electronic and digital communication that are now characterised as the *Fourth Industrial Revolution*: four specific technological advances are driving these economic and social changes: near universal access to high speed internet, widespread use of data analytics, rapid refinements in artificial intelligence, and availability of cloud internet storage (WEF, 2018). These changes have accelerated the pace and nature of globalisation and internationalisation. Their impact on education and mathematics education remains unclear. The education sector and school systems inevitably move at a much slower pace than industry and the wider society.

Social and economic realities are important in supporting the processes of globalisation and internationalisation. Both terms are constructs that need to be discussed concretely with reference to their specific social and economic drivers.

Definitions and Distinctions

The two terms internationalisation and globalisation have distinct meanings and continue to evolve. Globalisation is more frequently presented as an *outcome* or *consequence* of economic, social and political processes. Its rate of progress is tied increasingly to interconnectivity and speed of communication, especially to evolving global markets which impact on all countries. Some trans-national industries may have a vested interest in globalisation. Some international agencies may be pursuing global goals, but these same agencies rely for their support on national governments and agencies which are not about to lose their identities. In this sense, globalisation may be a *context* in which international actions take place (Larsen, 2016), and the directions of these actions may be modified and shaped by global conditions.

On the other hand, internationalisation, while not a universally endorsed agenda, can be viewed as a *strategic* or *purposeful direction* pursued by individuals, groups and social institutions, national and international agencies. Internationalisation refers to the intentional *actions* of these entities as they actively seek to cross national borders in pursuit of social, economic, political or cultural benefits (Mitchell & Nielsen, 2012). Some writers view Internationalisation as a *driver* or engine facilitating globalisation. Equally, international agendas can be *shaped* by globalisation or global trends globalisation when it refers to conditions influencing various areas of human activities (e.g. trade, education) worldwide. In this sense, internationalisation may be thought of something that institutions *do*, while Globalisation is something that *happens to* institutions (e.g. Larsen, 2016). This distinction is central to this chapter.

According to Cai and Howson (2012), globalisation stands for a process of integration of regional entities (e.g. economies, societies, cultures) through an increasing global network of trade, transportation, communication, and collaboration. As a response to globalisation, apart from focusing on acquiring certain knowledge and skills and developing problem-solving abilities, the mathematics curriculum should also be concerned with fostering cross-cultural communication and collaboration, all supporting the development of creativity and innovation. The same authors refer to internationalisation as denoting a process whereby companies and institutions produce products and services that can be, relatively effortlessly, adapted to the needs of specific local contexts and markets.

Skovsmose (2007) refers to processes of globalisation as an outcome or a result of global processes, drawing attention to the following points which support the positions taken by the above authors and the position taken in this chapter: the processes of globalisation are facilitated by information and communication technologies; globalisation, especially in its current form, is linked with a free-growing capitalism; the processes of globalisation do not follow any simple predictable route (in contrast to Internationalisation which is seen as an institutional response to global trends).

In the ensuing sections of this text, we will take a retrospective view on the New Math movement of the 1950s and 1960s and take some lessons from its significant role as an international curriculum. Next, we will look at the influence of ICMI Studies since 1986 as exemplifying international trends in mathematics curriculum reform. These two sections allow us to look at emerging models of Internationalisation through, for example, new curriculum platforms. The final section of this chapter will introduce key ideas to be examined in ensuing chapters; namely, TIMSS and PISA as vehicles for international curriculum reform; how definitions of numeracy and mathematical literacy continue to evolve internationally and also are subject to global influences; and, finally, the emergence of computational/algorithmic thinking (CT/AT) in the school curriculum as a global phenomenon.

Retrospective on New Math – What Has Been Learned?

Many papers at ICMI 24 referred to the New Mathematics ("New Math") movement in the 1950s and 1960s and focused appropriately on its key role in the development of new ideas for the mathematics curriculum. In the *Globalisation and internationalisation and their impacts on mathematics curriculum reforms* theme, we discuss the New Math movement as an instance of internationalisation. Our brief discussion asks: What form did it take and how was it spread? Was its eventual demise somehow a consequence of internationalisation of the mathematics curriculum? What can be learned from that era to support ongoing curriculum reform? For a more detailed discussion and evaluation of the New Math movement, readers can refer to the various contributions assembled in Theme A *Learning from the past: driving forces and barriers shaping mathematics curriculum reforms*.

The widespread adoption of New Math in the 1950s and 1960s is a good example of internationalisation, as a process by which ideas, programs and textbooks are adopted or adapted for use across different countries. Adopting countries, however, were free to opt into New Math to the extent that they wished (Kilpatrick, 2012). This process was accompanied by the development of texts and other resources which could be taken up or adapted according to local conditions. Each country retained its own specific assessment procedures. In each country, the extent of inclusion of New Math content had to fit with the constraints of the existing curriculum. According to Niss (2018), some countries such as the USA and France were strong adopters. On the other hand, many countries in Eastern Europe chose not to adopt the New Math. In other countries, such as England, the adoption of New Math fell somewhere between these two extremes.

Two related forces appeared to drive the push to introduce the New Mathematics curriculum in the 1950s. Post-World-War-II, high school graduates seemed to be underperforming in mathematics after they left school, and business and industry were calling for reforms. Mathematicians, according to Niss (2018), claimed to have the solution. They argued that students could be made (led) to understand mathematics better, and that current woes were a result of school mathematics courses being developed with little reference to what mathematicians understood mathematics to be.

In 1952, after ICMI was reconstituted as a commission of the International Mathematical Union, ICMI focused its attention during the 1950s and 60s on the reform of mathematics following the New Math movement. The then president of ICMI, Marshall Stone, actively supported the importation of New Math into Latin America with funding provided by the USA. Inter-American Conferences on Mathematics Education (CIAM) were established in 1969 to support this agenda professionally and financially. In its time, the New Math movement enjoyed strong international credentials and the support of national and sub-regional organisations.

The principal drivers of the New Math were professional (pure) mathematicians who supported the 'internationalisation' of the reform. They sought collaborators – including many mathematics educators – who would be associated with this international movement (Nadimi & Siry, 2018) in a supporting role. During its relatively short life New Math was adopted in varying degrees in different countries by mathematicians, educators, and curriculum agencies. However, it is very unfair to compare Internationalisation in the 1950s and 1960s with that of today where communication is immediate and where curriculum materials, resources for teaching and assessment can be supported digitally in ways unimagined by the proponents of New Math.

What can be learned from that experience about what works and does not work in mathematics curriculum reforms? The patterns of distribution and dissemination used at the time were inevitably top-down. Niss (2018) argues that New Math worked well for elite students who could see the connections. But when the collapse came, all the good things were thrown out as well. New Math, as a creation of pure mathematicians, appeared to have few mechanisms of regeneration and review; and was impervious – less receptive might be a better term – to other currents within mathematics and mathematics education, including those emerging from new technologies. As subsequent ICMI Studies show, other areas of mathematical inquiry, such as statistics, modelling and applications, and computer assisted algebra, were easily able to claim a space in the school curriculum.

ICMI Studies as Exemplars of Internationalisation

Clear evidence of responsiveness to international trends and developments can be found in the twenty-five ICMI Studies, demonstrating that the mathematics curriculum is continually developing and open to new questions. ICMI studies have made a strong contribution in the questioning of curricula, that is, in raising questions about what has to be taught at school (in specific domains, levels, etc.) and how, and raising these questions internationally, beyond the specific political, cultural and economic tradition of each country.

Space does not permit an examination of all ICMI Studies, but several are discussed here, starting with ICMI Study 1 starting in 1985 with the theme *The influence of computers and informatics on mathematics teaching*. This important theme was returned to in 2006 by ICMI Study 17, *Technology revisited*. (The ICMI Studies series can be found at: https://www.mathunion.org/icmi/digital-library/icmi-studies/icmi-study-volumes.)

ICMI Study 6 (1992) examined emerging models of assessment in mathematics at a time when many school systems internationally were introducing new forms of assessment to better reflect changing purposes of schooling and a broader appreciation of what it means to know and do mathematics.

ICMI Study 18 (2008) *Teaching statistics in school mathematics* was jointly sponsored by the International Association for Statistics Education. The inclusion of statistics in all years of the mathematics curriculum for basic education is now an almost universal trend, moving the teaching and learning of statistics away from a focus on calculation to a focus on the examination and interpretation of data. The increasing use of technology and the utilisation of real-world 'big data' continue to transform statistics education. Finally, ICMI Study 14 (2007), *Modelling and applications in mathematics education*, is a further instance of how the school curriculum has responded to global changes.

Other ICMI Studies are also driven by changing goals for school education, new demographic patterns for secondary education, and consequent changes in the relationship between schooling and society. For example, ICMI Study 20 (2010), *Educational interfaces between mathematics and industry*, conducted jointly with the international Congress on Industry and Applied Mathematics, represents a clear attempt to examine the relationship between mathematics and the world of work. This study looked outside university settings and extended the scope of ICMI Study 2 (1988), *Mathematics as a service subject*.

Beside reconsidering teaching particular topics (e.g. algebra, geometry, proofs – ICMI Studies 9, 12 and 19, respectively) and improving teacher professional development (ICMI Studies 15, 25), ICMI Studies also have focused on topics relating to gender equity, linguistic diversity and different cultural conditions. The inclusion of these topics is further evidence of the relevance and impact of changing social conditions and priorities on the mathematics curriculum, and illustrate a growing interdependence between regions, states, countries and different cultural areas of the world.

Is There an Emerging International Curriculum or Curricula?

It might be thought that the New Math movement signalled a last attempt towards a truly international curriculum. However, *Cambridge mathematics* (Jameson et al., 2018) can be presented as case study of a local example showing what might be possible in a digital age and what has been learned since the New Math movement. It is presented as one instance of possibly many international curriculums. Essentially, it represents a transformation to a global digital platform for curriculum design by an organisation known internationally for its mathematics textbooks, and examinations conducted internationally by the related *Cambridge Examinations Board*.

Cambridge mathematics is supported by Cambridge University Press, the University's faculties of mathematics and education, and Cambridge Assessment. A flexible and interconnected digital framework supports mathematics curriculum design globally to help local teachers to educate students aged 3–19 years. Its design process is intended to be transparent, collaborative and research- and evidence-informed, and aims to support teachers to develop new mathematics programs and to review their current programs, *without necessarily adopting the texts and assessment systems associated with Cambridge University Press and its related Examinations Board* (our emphasis).

Cambridge mathematics claims *not* to be a top-down international curriculum; but concerned to support local adaptations. Its framework is designed, we are told, to support local teachers and school systems. Seven components or features of its digital platform are designed around a *Mathematics Framework* or 'content spine' to which the other elements are linked. In the summary below, these seven components are *grouped* according to the six *elements* that Niss (2018) considers to be necessary for a successful national or international curriculum reform (*goals*, *content*, *materials*, *forms of teaching*, *student activities*, and *assessment*):

(*goals*)

- to champion and secure access to a quality maths education for all;
- to collaborate to use its position in maths education, to show leadership and to develop an authoritative voice.

(*content*)

- to develop a coherent Cambridge Mathematics Framework for all ages and types of learner with a strong distinctive approach, led by academics and educationalists and supported by a strong research base.

(*materials to support teachers and students*)

- to develop and make available world class teaching and learning materials.

(*enhancing forms of teaching and teacher development*)

- to support an infrastructure to enhance the quality of teacher education and continuing professional development.

(*assessment*)

- to develop forms of assessment that support the development of powerful mathematical reasoning.

(*related values and goals*)

- to develop an approach that is recognised and valued by parents, young people, teachers, institutions and governments.[1]

The first three elements of the Niss (2018) vector were present in the New Math, but the latter three were not so evident. One lesson that has been learned from the New Math movement is that the mathematics curriculum cannot be static. Any international curriculum movement, like *Cambridge mathematics,* needs to have in-built mechanisms for regeneration and review, which permit schools, teachers and school systems to form a connected, coherent, evidence-based program for teaching and learning mathematics. Departing from a top-down model, any international reform should enable teachers to select resources and to engage in their own professional learning. Any digital platform must be designed to promote progressive iterations and multiple solutions to meet different global and local conditions. Any candidates for an internationalised – not necessarily uniform – mathematics curriculum must build on affordances from the new technologies and learn lessons from the past.

Internationalisation or Uniformity? Local Factors, Cultures and Beliefs

The idea that an international mathematics curriculum is emerging may have some traction if one 'zooms out' and looks at commonalities of topics as they might be presented through national curriculum documents. But 'zooming out' has problems because it ignores local cultural factors and conditions. Teaching practices and classroom norms are rarely considered when one 'zooms out', and these present major forms of variation. Likewise, the impact of local and national assessment practices.

How are mathematics curriculum reforms varied (or similar) in different social, cultural, economic and political contexts? The research of Guberman and Abu (2018) shows that it in Israel, a relatively small country, a common national curriculum is implemented quite differently in Bedouin and Israeli schools, despite common teacher training programs. Lessons in the Bedouin sector are more traditional in structure (they end with a summary of class activities and a homework assignment), whereas lessons in the Jewish sector often end with independent work. However, this same research showed that teachers in both sectors insist that students master a specific set of procedures in class and learn how to use them when necessary.

[1] Source: http://www.cambridgemaths.org/images/cambridge-mathematics-symposium-2018-framework-update.pdf

The results point to broad similarities, probably tracing to shared training, curriculum, and materials, and to differences, such as stronger teacher responsibility for learning in the Bedouin sector and more independent thinking and conduct in the Jewish sector. These tendencies in both directions probably trace to stronger adherence to tradition in the Bedouin sector. The results emphasise, among other things, the importance of comparing and contrasting teaching practices within countries as well as among them.

Azrou (2018) identified difficulties in implementing an imported French mathematics curriculum in Algerian primary schools, where local teachers were given a curriculum to implement with very little professional development. In 2009, teachers in Algerian primary schools were informed that every child in the first grade should pass to the second grade, following a similar reform applied in French primary schools. Teachers interpreted this instruction as requiring that children who do not pass their exams and whose scores do not reach some required level should nevertheless pass to the second grade. To comply with a ministry instruction, teachers and administrators agreed that children should all pass to the second grade regardless whether they passed their exams or not at the first grade.

The original intent of this directive was to support all children so that they all reach their learning objectives and that no one would be left behind, allowing all children to proceed to the second grade with complete and strong basis. Even if the correct intention of this change had been explained, teachers needed to learn about the strategies that would make this possible, including how to work effectively with children experiencing difficulties. Teachers needed also to be shown the means, instructions, and assistance to organise their classes so that they can find time to assist children in need.

These studies – by Guberman and Abu, and by Azrou – show that school education and, particularly, teaching and learning mathematics are not free from locally embedded assumptions about teacher education and continuing professional development. There will also be local differences in the support offered to teachers in schools, the local organisation of schools and many other factors that can hold back or re-shape the implementation of any new change.

Looking Ahead

In this final sub-section, there are four areas of focus and issues to be examined in the following chapters of this Part.

1. *TIMSS and PISA as vehicles for curriculum reform*

One may consider TIMSS and PISA studies as examples of educational internationalisation since these studies have, in their different ways, promoted an internationally accepted 'core' of mathematical knowledge and skills to be acquired. It is important, however, to note that TIMSS and PISA are projects to assess and compare the current state of educational systems, not the elaboration of a transformed mathematical curriculum. Since 1995, every 4 years, TIMSS has assessed students' knowledge of mathematics and science in fourth and eighth grades (https://timssandpirls.bc.edu/timss-landing.htm). PISA, which commenced in 2000, has been repeated every 3 years measuring fifteen-year-old school pupils' performance on mathematics, science, and reading (http://www.oecd.org/pisa/).

TIMSS has assessed students' mathematical knowledge for several subject domains and three cognitive domains, mostly by using traditional school tasks. PISA assessed students' mathematical literacy for several content domains and different task contexts, mostly by using non-traditional tasks including real-life questions. Despite these differences, both studies aim to provide participating countries with comparable data to improve their education policies and outcomes. This is usually done through applying components of these studies (e.g. assessment tasks or key ideas) in curriculum reforms undertaken.

Unlike the New Math, TIMSS and PISA enjoy government sponsored participation and a consequent acceptance of regulatory mechanisms co-ordinated internationally to ensure comparability of reporting across participating countries. New countries seeking to join PISA and TIMSS are required to meet the same rigorous conditions for implementation required by their respective international agencies.

In some countries, PISA's framework for mathematical literacy (OECD, 2013) has been a platform for curriculum reform, the development of new national assessment formats and the consequent need for teacher professional learning. For example, in 2005 and 2007, Japan in its national assessment of student performance created a special section consisting of PISA-style questions which may have been previously merged with other test questions. Having a special section on real-life questions allowed the government to report on this section, encouraging schools to appreciate the importance of having students use and apply mathematics in real-life contexts (Namikawa, 2018).

2. *Evolving definitions of numeracy and mathematical literacy*

PISA's definition of mathematical literacy (OECD, 2013) has been influential in the development of national curriculum standards aimed at improving teaching and learning. The subsequent chapter in this book shows how numeracy – sometimes referred to as mathematical literacy – has emerged as a driver for curriculum reform in many international contexts. They trace the emergence and interpretations of numeracy and mathematical literacy and compare their relationship to curriculum reform processes in four countries Australia and Ireland have adopted a cross-curriculum approach. In Japan mathematical literacy is intended to be fostered through the process of reforming the mathematics curriculum.

In a fourth example discussed by Goos and O'Sullivan, South Africa's government in 2006 introduced a new subject called mathematical literacy (Math Lit), centred around real-world problems and not around formal algebra, being an alternative course to the standard pure mathematics course, mathematics (general grades 10–12) in the further education and training (FET) phase of school. Although Math Lit has greatly increased participation in mathematics in the final 3 years of school (about 60% of students take this course), students who have completed this course may be left with the unintended consequence of having a matriculation certificate that does not qualify them undertake university studies in mathematics or science (Jojo, 2019).

Other 'frameworks' not necessarily related to international assessments have influenced curriculum reform, for example, *Common core state standards: Mathematics* (CCSSM, 2019) and NCTM's (2010) *Principles and standards for school mathematics*. There is also a need to compare and connect the numeracy framework of the international *Programme for international assessment of adult competencie*s (PIAAC; OECD, 2019) with that of PISA to look backwards to examine what is happening in schools.

International forums such as the Asia-Pacific Economic Cooperation Agency (APEC, 2016, 2017) and the World Economic Forum (WEF, 2018) have sought to present a global economic perspective on mathematical literacy. To prepare better *students and workers for the yet-to-be-defined jobs of tomorrow* (our emphasis) and for supporting economic growth, APEC (2017), for instance, developed a list of data science and analytics (DSA) competences, including: enhanced skills in data presentation and visualisation; versatile applications of data analytics methods; computational thinking and use of algorithms; all of which are aimed to extend current boundaries and prevailing definitions of mathematical literacy. The World Economic Forum report, *The future of jobs* (WEF, 2018), also links the surging demand for these kinds of competencies to specific technological (ICT) advances. To add some urgency to this perspective, APEC (2018) has predicted that the global shortfall of highly skilled workers in ICT-related fields may be as large as to be 40 million by the early 2020s.

3. *Emergence of Computational/Algorithmic thinking in the mathematics curriculum*

Despite the lack of a widely accepted definition of computational/algorithmic thinking – which may simply be described as thinking involved in applying, modifying, and designing algorithms by using various computational tools – this thinking is an emerging educational notion. There are four reasons why the emergence of CT/AT in education can be viewed as a clear instance of the impact of globalisation and internationalisation. First, there is an increasing reliance on digital technology whose application often combines local and global contexts; in healthcare, biology, manufacturing, education, security, legal processing, the arts and music, to name a few.

Second, there is an increasing use of algorithmic techniques, including artificial intelligence, to deal with various real-world challenges that are not limited to local contexts – none more evident than in the recent *coronavirus* pandemic. Third, there are raised parental and societal expectations concerning a better, technology-assisted education of children that combines local and global contexts. Finally, with computational mathematics in various forms now widely used in undergraduate university courses, there is a mounting case for a better alignment between school courses and those at university.

Raised parental and societal expectations have been stimulated by the involvement of the private and non-government sector in providing online resources for free that are available to students from a young age without the formal mediation of the official school sector. Two examples of such resources, used by millions of young students worldwide, are: *Hour of Code*, promoting computer science (https://code.org/hourofcode/overview), and *Scratch* from MIT Media Lab supporting visual (block) programming language (https://scratch.mit.edu/). *Scratch* also features in many on-line resources to support computational thinking starting in the primary school, such as those provided by the non-government Sadosky Foundation (2018) in Argentina.

One rationale for including CT/AT in school education is based on the importance of developing computational thinking skills in children and young people to enable them to solve (real-world) problems using various computational tools. A second rationale is undeniably a response to changed economic conditions; namely the importance of fostering computational thinking to boost economic growth, fill job vacancies in ICT, and to prepare for future employment (Bocconi et al., 2016). There is no hard and fast separation between these two rationales. The kind of thinking processes advocated in the first rationale are clearly related to life and work contexts of the twenty-first century that are highly influenced by technology use.

Stephens (2018) reported on the expanding number of countries incorporating CT/AT into the curriculum of their elementary and middle school years. These developments, together with those taking place outside formal school hours clearly challenge and expand accepted definitions of mathematical thinking, reasoning and problem solving. CT/AT is not the same a coding; still less is it based on memorisation of procedures. Enlarging the role for CT/AT in the school mathematics curriculum must build upon, connect with, and enhance the way students think about and do mathematics. For a clearer anticipation of the role of CT/AT in the school mathematics curriculum of the twenty-first century, work is required on all six components that Niss (2018) specified for any major curriculum reform. Especially relevant are resources to support student activities, teacher resources, and assessment.

4. *Future visions of the impact of internationalisation and globalisation on school mathematics curriculum*

As twin global drivers, globalisation and internationalisation, remind us that the directions of curriculum reform are complex. Mathematicians and mathematics educators are important players, but reforms are sometimes responses to broader agendas and changing social contexts that have been outlined in this chapter. Failing to respond in an intelligent and timely way risks repeating past mistakes and missing opportunities to make a difference. The last chapter of the theme *globalisation and internationalisation and their impacts on mathematics curriculum reforms* considers how the issues so far considered are likely to play out in the future. In this short chapter we consider what has been learned about the influence of TIMSS and PISA and what might be expected in the near future; we re-examine how our understandings about numeracy and mathematical literacy continue to evolve; and, finally, we present five recommendations regarding the future inclusion of computational (algorithmic) thinking in the curriculum.

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Chapter 21
The Evolution and Uptake of Numeracy and Mathematical Literacy as Drivers for Curriculum Reform

Merrilyn Goos and Kathy O'Sullivan

In many countries the notion of mathematical literacy as a twenty-first century competency has emerged either from international studies, such as the OECD's Programme for International Student Assessment (OECD, 2016), or from national curriculum policy development. In some English-speaking countries, however, it is more common to speak of numeracy rather than mathematical literacy. This chapter traces the emergence and interpretations of numeracy and mathematical literacy as separate but related concepts and examines their role in curriculum reform in four countries: Australia, Ireland, South Africa and Japan. The main question addressed by the chapter is: How have notions of mathematical literacy and numeracy been expressed in curriculum reforms? The analysis aims to shed light on the interpretation and expression of numeracy and its relationship to mathematics.

Conceptualising Numeracy and Mathematical Literacy

Numeracy can be defined in many ways, and sometimes even by using different terms such as mathematical literacy or mathematical competencies. The concept of numeracy evolved from the UK's Crowther Report (MoE, 1959), in which the word 'numerate' was introduced to represent "the mirror image of literacy" (para. 398). In a later UK report, Cockcroft (1982) defined "being numerate" as having two attributes: "The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life" (p. 11). The second attribute is the

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ability to "have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables" (p. 11).

Attempts to operationalise numeracy within school curriculum documents have been made in many English-speaking countries around the world (e.g. Alberta Education, 2019; Australian Curriculum, Assessment and Reporting Authority, n.d.-b; DfE, 2013). Less common are efforts to theorise numeracy in ways that can be used by teachers for curriculum planning and task design. To this end, Goos et al. (2014) developed a multi-dimensional model of numeracy for the twenty-first century. The model consists of four different domains and gives attention to how one can apply *mathematical knowledge* in real life *contexts* by using different representational, physical or digital *tools* while holding positive *dispositions.* These four domains are grounded in a *critical orientation* which involves the ability to make decisions and form opinions based on these four domains.

Compared with numeracy, mathematical literacy is a relatively new term emerging from the OECD's work on PISA. The PISA definition of mathematical literacy has advanced from a basic skills definition of being able to use the mathematics learned in a school setting and apply it to everyday life, to a much broader definition as:

> an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens. (OECD, 2018, p. 7)

Interestingly, the OECD's Programme for International Assessment of Adult Competencies (PIAAC) uses the term numeracy instead of mathematical literacy, along with a focus on literacy and problem solving in technology-rich environments (Tsatsaroni & Evans, 2014). PIAAC defines numeracy as, "the ability to access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life" (Tout et al., 2017, p. 9).

At the ICMI 24 Study Conference, Niss (2018) reminded us that the notion of mathematical competence, developed within the Danish "KOM project" (Competencies and the Learning of Mathematics), shaped the PISA mathematics frameworks from 2000 to 2012 and underpinned the notion of mathematical literacy. In a related paper, Niss and Højgaard (2019) defined mathematical competences in terms of "someone's insightful readiness to act appropriately in response to all kinds of mathematical challenges pertaining to given situations" (p. 4). They revisited the notion of mathematical competences, grouping these into two categories: posing and answering questions in and by means of mathematics; handling the language, constructs and tools of mathematics. As all of the eight competencies in these two categories are specific to mathematics (e.g. mathematical problem-handling competence, mathematical symbolism and formalism competence), it could be argued that the notion of mathematical competence seems to part ways with context-rich definitions of numeracy and mathematical literacy.

Niss and Jablonka (2014) describe mathematical literacy as a concept which is positioned in student and school contexts, whereas numeracy is described as applying mathematics within adult world contexts. On the other hand, Geiger et al. (2015) argued that, while the meaning of numeracy and mathematical literacy varies between countries, being numerate goes beyond using basic arithmetic skills to include the ability to "make sense of non-mathematical contexts through a mathematical lens; exercise critical judgement; and explore and bring to resolution real world problems" (p. 531). Debates surrounding the meanings of numeracy and mathematical literacy need to acknowledge that not only have these terms come into existence at different times, but they are also assumed to operate within somewhat different contexts involving different combinations of school, workplace, and daily life. In his commentary in a journal special issue on numeracy, Askew (2015) claims that much work remains to be done on conceptualising numeracy and mathematical literacy and in realising their role in school curricula.

Conceptualising Curriculum

Remillard and Heck (2014) defined curriculum as, "a *plan for the experiences* that learners will encounter, as well as the *actual experiences* they do encounter, that are designed to help them reach specified mathematics objectives" (p. 707; *emphasis in original*). They presented a visual model of the curriculum policy, design, and enactment system that distinguishes between the official curriculum and the operational curriculum enacted in classrooms. The focus of this chapter is on the official curriculum, as specified by governing authorities, and on curricular aims and objectives as one of its three components proposed by them.

Our comparative analysis is presented via country case studies, each structured around two dimensions: (1) the rationale for including numeracy in the school curriculum; (2) how numeracy is represented in the curriculum through curricular aims and objectives. These countries were chosen for comparison because they highlight contrasting approaches to incorporating numeracy in the school curriculum.

Numeracy as a Cross-Cutting Competency in Australia and Ireland

In both Australia and Ireland, numeracy is identified as one of several general competencies to be developed in all subjects across the school curriculum. This approach has led to curriculum frameworks that attempt to integrate cross-cutting competencies with the disciplinary content of the separate school subjects (Goos & O'Sullivan, 2018).

Rationale for Numeracy in Australia and Ireland

In Australia, the rationale for including numeracy in the curriculum has evolved over 30 years and three national Declarations on the goals of schooling agreed by the State, Territory, and Australian Ministers for Education. In 1989, the Hobart Declaration (Education Council, 2014b) proposed a framework of national collaboration between the States and Commonwealth with ten agreed goals for schooling, including development of skills of numeracy and other mathematical skills. Ten years later, in 1999, the Adelaide Declaration agreed on eight key learning areas for the school curriculum and additionally stated that, "Students should have attained the skills of numeracy and English literacy, such that every student should be numerate, able to read, write, spell and communicate at an appropriate level" (Education Council, 2014a). Whereas the previous declarations were non-binding agreements, in 2008 the Melbourne Declaration foreshadowed action in referring to developing a national curriculum and national assessment program for literacy and numeracy (MCEETYA, 2008), replacing existing state-based curricula and assessments. Having skills in numeracy was seen as essential for creating "successful learners, confident and creative individuals, and active and informed citizens" (p. 8).

In Ireland the rationale for numeracy driving curriculum reform is a more recent phenomenon, in response to the results of the Third International Mathematics and Science Study (TIMSS; Beaton et al., 1996) and Ireland's substantial decline in PISA mathematical literacy performance in 2009 (Shiel et al., 2016). Performance on these international assessments, together with the national economic crisis of 2010, provided impetus for development of a national literacy and numeracy strategy (DES, 2011). The Irish government has agreed that all young people in Ireland should leave school with the appropriate numeracy and literacy skills to live and participate as informed citizens in society. In the strategy document, numeracy is defined as follows:

> Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems. (p. 8)

Representation of Numeracy in the Official Curriculum of Australia and Ireland

In Australia, the relationship between mathematics and numeracy has been explored and contested for many years. The *National Numeracy Review Report* (Council of Australian Governments, 2008), although mixing together research and recommendations regarding both mathematics and numeracy, seemed to set a clear direction for distinguishing between these in its first recommendation:

> That all systems and schools recognise that, while *mathematics* can be taught in the context of mathematics lessons, the development of *numeracy* requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment. (p. 7; *emphasis added*)

The *Australian Curriculum: Mathematics* was developed between 2008 and 2012 and is structured around the three content strands of number and algebra, geometry and measurement, and statistics and probability, and the four proficiency strands of understanding, fluency, problem solving, and reasoning (ACARA, n.d.-a). At the same time, the Australian Curriculum has progressively elaborated the notion of numeracy as a "general capability" alongside literacy, ICT capability, critical and creative thinking, personal and social capability, ethical understanding, and intercultural understanding. General capabilities are meant to be developed in all learning areas, and the curriculum offers advice within each learning area for developing numeracy based on the following general definition:

> In the Australian Curriculum, students become numerate as they develop the knowledge and skills to use mathematics confidently across other learning areas at school and in their lives more broadly. Numeracy encompasses the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations. It involves students recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully. (ACARA, n.d.-b)

The general capabilities section of the Australian Curriculum contains a set of key ideas in numeracy organised into the following elements: estimating and calculating with whole numbers; recognising and using patterns and relationships; using fractions, decimals, percentages, ratios and rates; using spatial reasoning; interpreting statistical information; using measurement. These elements are further represented in a numeracy learning continuum with statements describing what students can typically do by the end of the various years of schooling. However, it is difficult to see how this set of objectives aligns with the curricular aim of helping students "to use mathematics confidently *in other learning areas* at school and *in their lives more broadly*" (emphasis added). The numeracy learning continuum could easily be used to support teachers in implementing the *Australian Curriculum: Mathematics* without the need to engage with other learning areas, or the world outside school, at all.

In Ireland, as in Australia, there is a lack of clarity in curriculum policy about the distinction between numeracy and mathematics. While the Irish document is referred to as the national strategy to improve literacy and numeracy among children and young people, throughout the document there is frequent reference to mathematics rather than numeracy. Nevertheless, a revised curriculum framework for the lower secondary years (known in Ireland as the junior cycle) has introduced a set of key skills that could be interpreted as cross-cutting competencies: being literate, managing myself, staying well, being curious, managing information and thinking, being numerate, being creative, working with others, and communicating (DES, 2015). Teachers are meant to embed these key skills in the learning outcomes of every subject, but there is not yet any explanation within newly developed subject specifications of how this can be done.

Mathematical Literacy as a Stand-Alone Subject in South Africa

In South Africa, post-apartheid mathematical curriculum reform has been driven by political, ideological and social forces striving towards the goal of *mathematics for all* and *mathematics by all* (Volmink, 2018). While the former aspiration refers to equity in curriculum provision, the latter is a statement about quality of mathematical engagement by both learners and teachers. The tension between these twin goals of equity and quality is made visible in recent curriculum reforms that have resulted in a return to differentiated subject offerings at school and the introduction of Mathematical Literacy as a secondary school subject within the field of mathematics.

Rationale for Mathematical Literacy in South Africa

Volmink (2018) argues that curriculum reform has a "contextual ancestry" (p. 101). This means that the expression of mathematical literacy – whether as a general competency or a separate subject in the school curriculum – needs to be understood in the context of choices that have been made during a time of wider political and social reform in South Africa. During the apartheid years, two systems of education co-existed as a means of maintaining severe socio-economic inequalities along racial lines (Graven, 2014). While a People's Education movement was mobilised in the 1980s to oppose educational inequalities, it was not until after the country's first democratic elections in 1994 that education policy-making became the vehicle for transforming society.

A new vision for education as a way of redressing the inequalities perpetuated in the apartheid era was realised through major curriculum change. In 1995 a reformed national curriculum framework was introduced, "premised on a learner-centred, outcomes-based approach to education with an explicit political agenda" (Graven, 2014, p. 1040) emphasising common values and citizenship in the new democratic society. This National Curriculum Statement became known as Curriculum 2005 (C2005), with the intention that it should be implemented in all grades by 2005. Although C2005 was overwhelmingly supported, Volmink (2018) explains that it faced challenges in addressing competing priorities, which he described as follows:

> The post-apartheid challenge: to provide awareness and the conditions for greater social justice, equity and development. This is the challenge of developing new values and attitudes.
>
> The global competitiveness challenge: to provide a platform for developing knowledge, skills and competences to participate in an economy of the twenty first century.
>
> The challenge of developing critical citizens: citizens in a democracy need to be able to examine the many issues facing society and where necessary to challenge the status quo and to provide reasons for proposed changes. (p. 103)

These challenges highlight the multiple expectations of C2005, in particular the expectation that this curriculum should produce critically numerate citizens who can participate actively in society.

Representation of Mathematical Literacy in the Official Curriculum of South Africa

During the apartheid period and up to 2007, students could choose to take mathematics at Higher Grade (HG), Standard Grade (SG), or not at all. At the beginning of the post-apartheid era in 1994, Volmink (2018) reports that only 20% of black students were taking HG mathematics compared with 70% of white students. Of even more concern was the finding that between 2000 and 2005, as many as 40% of students were taking no mathematics at all (Clark, 2012). Following a Department of Education (2003) investigation into the then-current system of curriculum differentiation, the responsible Ministerial Committee recommended that curriculum reform should provide more equitable access to all subjects: as a consequence, it became a requirement that all learners had to take some form of mathematics. In response to this policy, in 2006 a new subject, Mathematical Literacy, was introduced in the post-compulsory phase of schooling (grades 10–12) as an alternative to mathematics. According to the current Curriculum and Assessment Policy Statement for Mathematical Literacy (DoBE, 2011):

> The competencies developed through Mathematical Literacy allow individuals to make sense of, participate in and contribute to the twenty-first century world – a world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of different ways. (p. 8)

Mathematical literacy, as a school subject, has five key elements: it involves the use of elementary mathematical content, authentic real-life contexts, solving familiar and unfamiliar problems, decision making and communication, and the use of integrated content and/or skills in solving problems. The subject is organised into Basic Skills Topics comprising number, calculation, patterns, relationships and representations, and Application Topics including finance, measurement, maps and plans representing the physical world, data handling and probability (DoBE, 2011). It was designed with the intention of providing democratic access to *mathematics for all* rather than as a watered-down subject for mathematically weak students.

Within South Africa concerns have been expressed about the limited capacity of teachers to engage and teach the Mathematical Literacy curriculum, and concurrent criticisms of the curriculum structure as focusing only on achieving minimum standards rather than empowering learners to access a wide range of future careers (Cranfield, 2012). Nevertheless, evidence from small-scale classroom studies, such as that reported by Graven and Buytenhuys (2011), indicates that the subject does have potential for enabling mathematical metamorphosis of learner identities and increasing their access to quality mathematics education.

Questions of quality were highlighted by Volmink (2018) in his explanation of *mathematics by all* as a statement meaning that everyone should be "engaged in a quality mathematical experience" (p. 107), a goal consistent with the intent of the Mathematical Literacy syllabus. However, lack of school-based curriculum leadership in mathematical literacy, lack of teacher understanding of how to teach mathematics in real-life contexts, and disparities in access to resources between private schools and poorer public schools threaten to undermine the potential of the Mathematical Literacy subject to meet its transformational aims (Sidiropoulos, 2008).

Infusing Mathematical Literacy into the Mathematics Curriculum in Japan

In Japan, curriculum reform is undertaken on a regular cycle at approximately ten-year intervals. While neither numeracy nor mathematical literacy are identified as cross-cutting competencies (as in Australia and Ireland) or offered as stand-alone subjects (as in South Africa), the concept of mathematical literacy as operationalised by PISA has had a profound influence on the revision of the secondary mathematics curriculum.

Rationale for Mathematical Literacy in Japan

Namikawa (2018) describes the significant influence of TIMSS and PISA results on government education policy and curriculum in Japan. Although Japan is often regarded as a high-performing country in both assessment programs, this is not necessarily how the results are interpreted within the country by the media and the Ministry of Education, Culture, Sports, Science and Technology. Tasaki's (2017) analysis of the impact of PISA results on educational policy, together with Namikawa's insights into the nature of Japan's TIMSS and PISA "shocks", point to two major concerns. The first is the decline in reading literacy between 2000 and 2003, with a stagnant performance in 2006 that coincided with a decrease in both science literacy and mathematical literacy (where performance was measured in terms of both country ranking and score; see Table 21.1). This apparent decline in academic ability was attributed to so-called "relaxed education" that had led to a reduction of school subject content and increased leisure time for students.

The second concern pointed to students' declining interest and motivation for learning in mathematics and science. Both TIMSS and PISA survey students to ascertain their views in these areas. Namikawa (2018) presented previously unpublished data from the 1995 and 1999 TIMSS, summarising the percentage of Japanese students who responded positively to statements about mathematics or science being important in life and their hopes to be involved in these fields in future professions. In both years, Japanese students were well below the international average in their attitudes towards mathematics and science. Similarly, Tasaki (2017) reported that in PISA 2003 and 2012 (triennial PISA cycles in which the focus was on mathematical literacy), the percentage of Japanese students who responded positively to statements about looking forward to mathematics lessons, doing mathematics because they enjoy it, and being interested in the things they learn in mathematics were below the OECD average. Thus, the rationale for addressing mathematical literacy was influenced not only by international rankings and competition, but also by concerns for motivation and interest in learning and a desire to provide children "with the competencies, including academic ability, to be autonomous in this rapidly changing society" (Tasaki, 2017, p. 152).

Representation of Mathematical Literacy in the Official Curriculum in Japan

Namikawa (2018) argues that, "the importance of PISA lies in not only the result of assessment but also the publication of the framework of assessment with the name of 'literacy'. [… Thus,] a fundamental principle of reform is to foster literacy" (p. 461) in its widest sense. The decline in reading literacy, as assessed by PISA, led to urgent interest in improving reading comprehension – including the ability to understand graphs and other mathematical forms of representation and

Table 21.1 PISA results in Japan (2000–2015)

	Reading literacy		Mathematical literacy		Science literacy	
Year	Score	Rank	Score	Rank	Score	Rank
2000	522 (500)	8 (8)	557 (500)	1 (1)	550 (500)	2 (2)
2003	498 (494)	12 (14)	534 (500)	4 (6)	548 (500)	2 (2)
2006	498 (492)	12 (15)	523 (498)	6 (10)	531 (500)	3 (6)
2009	520 (493)	5 (8)	529 (496)	4 (9)	539 (501)	2 (5)
2012	538 (496)	1 (4)	536 (494)	2 (7)	547 (501)	1 (4)
2015	516 (493)	6 (8)	532 (490)	1 (5)	538 (493)	1 (2)

Source: Adapted from Tasaki (2017)
Score is given as that for Japan followed by the OECD average
Rank is given as Japan's rank amongst OECD countries followed by rank amongst all countries

communication (Tasaki, 2017). Mathematics is thus regarded as a language, and improvement in mathematics is also held to be responsible for improvement in language ability in the form of reading literacy.

A second influence of PISA relates to the way in which the mathematical literacy assessment framework prioritises mathematical activity, that is, not only possessing knowledge but being able to use that knowledge to solve problems. Thus, the new course of study in mathematics for junior high school identifies important mathematical activities, such as using mathematics in daily life and society, that seem to resonate with the notion of mathematical literacy. In addition, a new subject, 'application of mathematics', has been developed for senior high school, and the new topic of statistics has been introduced into the existing mathematics I subject taken by almost all senior high school students. All of these developments suggest that aspects of mathematical literacy emphasising real-life contexts, positive dispositions, and using mathematical knowledge for problem solving in complex social settings are being infused into the regular secondary school mathematics curriculum.

Conclusion

The aim of this chapter was to analyse the emergence of understandings about numeracy and mathematical literacy and to compare their relationship to curriculum reform processes in four countries: Australia, Ireland, South Africa and Japan. In relation to the first aim, we discussed various conceptualisations of numeracy and mathematical literacy, some sitting within the school context and others defining this capability as a critical skill needed by all citizens. As Stephens, Kadijevich, Niss, Azrou and Namikawa argue in the Introduction, definitions of mathematical literacy (and numeracy) are themselves fluid and will continue to evolve as a consequence of globalisation and internationalisation of the mathematics curriculum.

To address our second aim, we focused on what Remillard and Heck (2014) refer to as the official curriculum, examining the rationale for including numeracy or mathematical literacy in the school curriculum and how these concepts are represented in the curriculum of four countries. In all four country case studies, we saw that the official curriculum and related documents espoused transformative goals for numeracy or mathematical literacy, for example, referring to critical citizenship enabling full participation in choices that affect people's lives. However, the representation of these concepts in the curriculum varied. In Australia and Ireland, numeracy was promoted as a cross-cutting competency to be developed in all subjects in the curriculum.

Yet in both countries there was a lack of conceptual clarity concerning the distinction between numeracy and mathematics that threatens to undermine efforts to embed numeracy across the curriculum. In South Africa, a commitment to mathematics for all and mathematics by all frames the offering of a differentiated curriculum, with a stand-alone subject called Mathematical Literacy. Such an approach, while ensuring that all secondary school students will take mathematics in some form, carries a risk of positioning mathematical literacy as a lower-status subject than mathematics. In Japan, concerns over student performance on international assessments have led to aspects of mathematical literacy, as defined in the PISA framework, being infused into the regular mathematics curriculum.

Each of these three forms of curriculum representation has implications for teacher preparation and support, an observation that also raises important questions about who is responsible for developing students' numeracy or mathematical literacy. Whether the responsibility lies with all teachers or only with those who are teaching mathematics, two requirements seem to be essential. First, the official curriculum should provide teachers with a clear conceptualisation of numeracy and its relationship with mathematics. Second, teacher educators and education systems need to provide practical guidance for teachers to implement curricular goals concerning numeracy. These recommendations highlight the importance of all elements of the broad system of curriculum policy, design, and enactment outlined by Remillard and Heck (2014) and point to the central role of teachers in enacting the officially sanctioned curriculum.

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Chapter 22
Impacts of TIMSS and PISA on Mathematics Curriculum Reforms

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The results of international experience and research in the teaching and learning of mathematics, on a global level, have influenced school mathematics curriculum with the emergence of a relatively uniform mathematics curriculum, comprising an internationally accepted core of mathematical knowledge and skills (e.g. Cai & Howson, 2013). From a distance, this perspective has some footing since it regards common topics and notions as they might be presented within national curriculum documents. However, such 'zooming out' may ignore cultural factors and local conditions, including teaching practices, classroom norms, and assessment methods. These forms of variation need closer scrutiny (e.g. Hiebert et al., 2003; Guberman & Abu Amra, 2018).

Considerable variations are also found when specific curriculum issues are considered. For example, within the mathematics curriculum for basic education in many countries, the topic of Statistics focuses on describing, representing, and interpreting data (e.g. Biehler et al., 2018), but there are considerable differences in how statistical content is approached, especially through the use of technology and real-world data (e.g. Ben-Zvi et al., 2018). Furthermore, the ways in which mathematical literacy or computational/algorithmic thinking are defined and included in

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the school curriculum show no consistent pattern of how these notions have been integrated in the school mathematics curriculum (see Goos & O'Sullivan, 2018; Rafiepour, 2018; Stephens, 2018). These curriculum issues are further examined in the accompanying chapters of theme *Globalisation and Internationalisation, and their impacts on mathematics curriculum reforms*.

Among the most important drivers of curriculum reform are probably international assessment studies, which are complex educational products resulting from the processes of globalisation and internationalisation.[1] These studies have contributed to curriculum changes in the following way: they first draw attention to certain aspects of curriculum that need improvement; these aspects are then often improved though the recognition and application of some components of these studies.

By focusing on question "How have international studies driven school mathematics curriculum reforms?", this chapter examines the role of international studies TIMSS (Trends in International Mathematics and Science Study) and PISA (Programme for International Student Assessment) as global drivers for these reforms. Their influences are discussed both worldwide and in individual countries, with an emphasis on the application of particular components in (re)designing and implementing curriculum improvements. Examination of these influences is important in helping educators better understand global trends and their implications for teaching and learning mathematics.

In the remaining part of this chapter, global influences of TIMSS and PISA are examined first. This is followed by four case studies from economically and geographically diverse countries. The chapter ends with a critical summary of the findings presented and outlines directions for further research.

Global Influences of TIMSS and PISA

The section presents the influences of these two international studies worldwide. TIMSS influences are mostly related to a certain curricular convergence regarding topics to cover and skills to foster, whereas PISA influences primarily concern the inclusion of the notion of mathematical literacy in curricula, which is usually expressed in terms of competencies or capabilities.

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[1] The principal characteristics of these processes have been outlined in the Introduction of theme D *Globalisation and internationalisation, and their impacts on mathematics curriculum reforms*. The components of each international assessment study have been influenced by trans-national processes of diverse nature (economic, social, educational, cultural, political) resulting from international actions by various national and international agencies and institutions that strive for concrete economic, social, educational, cultural, and political benefits. Beside such an outward direction, these actions are also directed inwardly through, as Cai and Howson (2013) underlined, adapting globally developed educational products (i.e. assessment packages) to particular countries' needs.

TIMSS Influences

Since 1995, TIMSS has provided data on fourth- and eighth-grade students' achievements in mathematics and science for more than 50 countries worldwide, every 4 years (see https://timss.bc.edu/). Apart from reported achievement data, TIMSS international databases contain the values of many contextual variables, used to explain differences in students' achievements within and among countries, resulting in a great number of secondary analyses. The outcomes of primary and secondary TIMSS research have influenced the development and (re)design of mathematics and science education curricula across the world. The first curricular changes, starting at the end of 1990s, were described in Robitaille et al. (2000). Recent twenty-first-century changes are documented in TIMSS Encyclopedias (e.g. Mullis et al., 2016c).

During the first 20 years of TIMSS studies, most participating countries have implemented reforms to their mathematical curricula, ranging from updates to detailed revisions, by using TIMSS results to review their curricula and improve them (Mullis et al., 2016b). Despite countries differing in many respects (e.g. economical, geographical, religious), their curricula have become increasingly similar, organised around common broad content areas (e.g. number) expressed in terms of their sub-topics (e.g. fractions and decimals). Furthermore, many emphasise the role of problem solving and thinking skills, such as applying mathematics and mathematical reasoning. These researchers therefore argue that TIMSS has brought a certain curricular convergence worldwide regarding topics to cover and skills to foster. This outcome, as put forward by authors participating in the TIMSS project, may be accepted with caution, but a more uniform international mathematics curriculum does seem to have emerged (e.g. Cai & Howson, 2013) although it has not been established what could constitute the details of such a curriculum.

The impact of TIMSS results on curricular and related issues in particular countries is summarised in TIMSS 2015 Encyclopaedia (Mullis et al., 2016c): each country report includes a section "Use and impact of TIMSS", and some describe that impact in a detailed way. An examination of these reports for about 60 countries that participated in TIMSS 2015 revealed that, apart from the influence on curricular reforms in many participating countries, results have also influenced teacher professional development and national assessments. Because some country reports are unclear about all areas of impact, the number of countries implementing substantial change can only be estimated, with at least one-third regarding teacher professional development and one-fifth concerning national assessments.

Having in mind the distinction underlined by Mullis and colleagues (2016c) between intended, implemented and attained curriculum, i.e. between what is expected to be taught, what is actually taught, and what is learned, TIMSS appears to have a dominating influence on intended curriculum. However, its influence on teacher professional development and national assessments clearly relates to implemented and attained curricula. Knowing that teachers' professional development is highly relevant to implemented and attained curricula, it is surprising that the double impact of TIMSS results on curricular reforms *and* teacher professional development, as documented in the 2015 Encyclopaedia, was found in just one-fifth of TIMSS countries represented there. An examination of the section "Use and impact of TIMSS" in the latest TIMSS Encyclopedia for the 2019 project cycle (Kelly et al., 2020), which became available after the first version of this chapter was completed, might reveal similar figures regarding the most recent worldwide impact of TIMSS on curricular reforms, teacher professional development, and national assessments.

Since 2019, an eTIMSS study has been offered in digital format. This change has required the inclusion of innovative problem solving and inquiry tasks that simulate real world situations, whose solutions may be found through the applications of interactive scenarios (Mullis & Martin 2017). These tasks certainty call for new competencies, including computational thinking – a distinctive way of thinking applied while working with problem solutions expressed in representations that could be efficiently processed by technology (e.g. Wing, 2011).

PISA Influences

The Organisation for Economic Co-operation and Development (OECD) launched PISA in 2000 to assess basic competencies in reading, mathematics, and science of 15-year-olds students every 3 years, focusing on mathematics in 2003 and 2012. Apart from these three core subjects, PISA 2018, administered in all OECD member countries and many other countries worldwide (almost 80 countries in total), involved the assessment of two domains, namely: financial literacy and global competence (see: http://www.oecd.org/pisa/).

Like TIMSS, PISA international databases contain the values of many contextual variables, used to explain differences in students' achievements within and among countries, resulting in many secondary analyses. Although PISA research has influenced policy reforms in a number of countries (e.g. Breakspear, 2012), Lingard (2017) and some others claim that PISA primary outcomes in terms of national scores and rankings, rather than its secondary analyses, have been used to initiate and justify curricular reforms. This may also hold true for TIMSS research, but such a conclusion has not been reported so far in the literature, to the authors' knowledge.

Unlike TIMSS, which has assessed students' mathematical knowledge mostly by using traditional school tasks, PISA has assessed students' mathematical literacy (in terms of a matrix of mathematical capabilities by mathematical processes) for several domains and different task contexts (e.g. OECD, 2019), by using tasks mostly related to real-world situations. The use of such tasks – unfamiliar to many students, especially in the first cycles of PISA – has contributed to low or unsatisfactory students' results in many countries. This caused the so-called "PISA shocks" in countries such as Germany and Japan. Consequently, many countries have begun to use PISA-like tasks in their national assessments (Lingard, 2017), which may be considered as one kind of PISA influence on mathematics curriculum. However, over-use of such tasks may not be desirable. While modelling and applications are important components of mathematics education, so are proofs and mathematical structures. Moreover, the difficulty of such tasks may be more affected by the complexity of the contexts used than the mathematics involved in solving them (Stacey et al., 2015).

Whereas the influence of TIMSS on mathematics and science curricula worldwide has been officially documented in TIMSS Encyclopaedias (e.g. Mullis et al. 2016c), few OECD documents have examined the influence of PISA research on national policy reforms. Breakspear (2012), however, using National PISA Reports, showed that PISA had, in varying ways and extents, been embedded in national policies in the majority of countries via performance targets, curriculum standards or assessment practice. However, of the existence of competing policy drivers, such as the recommendations and benchmarks of the European Union (EU), including key competencies of lifelong learning (Michel, 2017) makes it difficult to determine how much these effects can be attributed directly to PISA.

A recent systematic review of research articles on PISA revealed that articles dealing with impact/policy had rarely addressed its impact on the curriculum (Hopfenbeck et al., 2018). An exception is the study of Stacey et al. (2015), which examined PISA influence on mathematics education in ten countries worldwide. This study showed that PISA has influenced mathematics curricula in the majority of these countries. This was done by applying the PISA notion of mathematical literacy, usually expressed in terms of competencies or capabilities. While national assessments have been influenced by PISA framework in Chile and Spain, PISA tasks have been used for formative assessment in Denmark or adapted to meet official standards in France. Stacey et al. (2015) showed that support for PISA-related curricular changes, implemented through teacher professional development, was evident in only a few countries, such as Denmark and Israel. Appropriate teacher professional development promoting such a practice is thus needed, especially when technology is applied in the classroom (e.g. Drijvers et al., 2016).

The framework for mathematical literacy for PISA 2021 study includes Computational thinking (OECD, 2018). Its inclusion into the school mathematics curriculum, is examined in other chapters of theme *Globalisation and Internationalisation, and their impacts on mathematics curriculum reforms*.

Particular Influences of TIMSS and PISA

The particular influences of TIMSS and PISA presented in this section were selected by applying two criteria: countries that are economically and geographically diverse. This selection resulted in four countries from different continents: Australia, Israel, Mexico, and Serbia. Australia and Israel are countries with high-income economies, whereas Mexico and Serbia are countries with middle-income economies, according to the World Bank list of economies, June 2019.

The case study from Mexico examines the influence of PISA, whereas the case studies from Israel and Serbia summarise the influence of TIMSS. The influence of both studies is presented in the case study from Australia.

Australia

The Australian Curriculum: Mathematics (AC:M; ACARA, 2018) is arranged in three content, and four proficiency strands. The content strands – number and algebra; measurement and geometry; statistics and probability – represent, according to Sullivan (2018) a "conventional statement […] of the focus of the curriculums worldwide" (p. 90). This focus is mirrored in the content areas used by TIMSS. The adoption of the AC:M by the all Australian States and Territories from 2014 has provided a framework for Australia's National Assessment Program – Literacy and Numeracy (NAPLAN) and including the use of a calculator-active component of NAPLAN.

The *proficiency* or process strands of the AC:M reflect the language of capabilities and competencies which have become part of PISA and PIAAC. Sullivan (2018) points out that the elaborations of the proficiencies in the AC:M were based on the recommendations of Kilpatrick, Swafford and Findell (2001). The first proficiency is understanding (Kilpatrick et al.'s term was *conceptual understanding*), and the second proficiency is fluency (*procedural fluency*). A third proficiency strand, problem solving (*strategic competence)*, is described as:

> the ability to make choices, interpret, *formulate*, model and investigate problem situations, and communicate solutions effectively. Students *formulate and solve problems* when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify (*evaluate*) their answers (*results*) are reasonable. (quoted from ACARA, 2018, by Sullivan, 2018, p. 90)

The fourth proficiency, reasoning (*adaptive reasoning*) includes:

> analysing, proving, *evaluating*, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from *one context to another*, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices. (quoted from ACARA, 2018, by Sullivan, 2018, p. 90)

Parallels between these two proficiencies, as expressed in the AC:M and in the PISA 2012 Mathematics Framework (OECD, 2013), are shown in Fig. 22.1 by key words in common, such as *formulate*, *evaluate*, and *context* – given above.

One important consequence of the PISA Framework (OECD, 2013) has been the development of several national projects to build teachers' capacity to implement these proficiencies in classrooms and to engage in rigorous professional learning, introducing serious mathematical topics in a spirit of inquiry, embodying high levels of mathematical reasoning and problem solving.

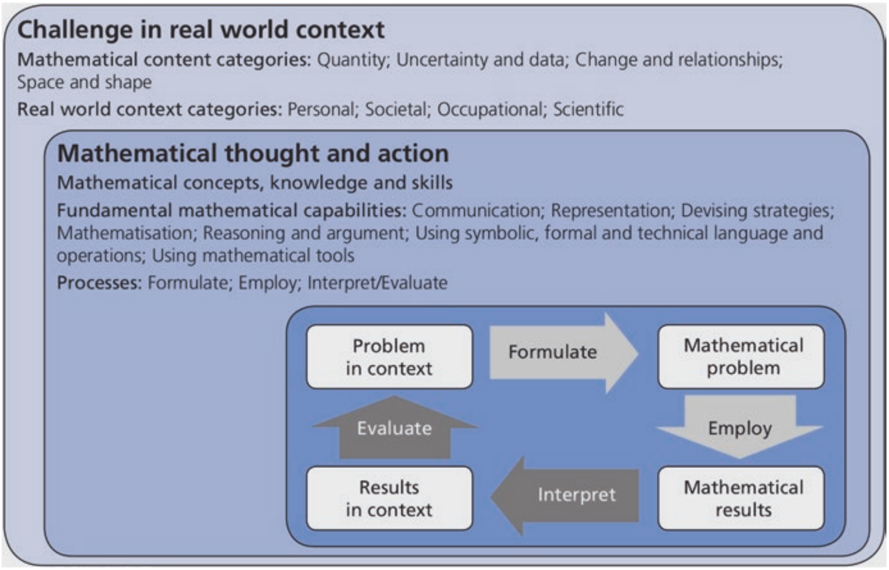

Fig. 22.1 A model of mathematical literacy in practice (OECD, 2013, p. 26)

One such program, *reSolve – Promoting a spirit of inquiry* (https://www.resolve.edu.au) – was funded by the Australian government as a collaboration between Australian Academy of Science and the Australian Association of Mathematics Teachers. The *reSolve* program includes classroom resources for teachers from Foundation Year (K) to Year 10, professional learning modules, and special topics. Many *reSolve* special topics embody resources that address the needs of twenty-first century learners of mathematics following the key elements of the PISA Mathematics Framework, including opportunities for using new technologies in real world contexts.

As stated above, the TIMSS model of diagnostic assessment has been powerful in providing the model for Australia's national assessment program – literacy and numeracy (NAPLAN) which is currently conducted in all schools in Years 3, 5, 7 and 9. However, the need for some reform of NAPLAN has been highlighted in recent debate on whether an exclusive focus on literacy and numeracy might need to be expanded in the future to better serve Australia's schools and young people. This debate is summarised in a 2019 NAPLAN Review Interim Report (McGaw et al., 2019). Unlike TIMSS, NAPLAN occupies considerable school, teacher, and community attention, and so fosters an impression that other areas of thinking (e.g. STEM literacy, critical and computational thinking) may be secondary, and possibly optional.

What can be learned from the case study from Australia? There, TIMSS and PISA occupy an important role as international benchmarks of performance. Their reports now show that Australia's once strong rankings are falling. TIMSS has provided a model for the design Australia's annual assessment of numeracy through

NAPLAN. Growing societal concerns for fostering a wider range of competencies among school students suggest that the TIMSS model may need an overhaul. PISA, on the other hand, appears more responsive to these concerns with its greater emphasis on STEM-literacy and critical thinking.

Israel

The results of the TIMSS-1999 study were a starting point for significant changes in the Israeli mathematics curriculum for all ages. The main change is reflected in the coverage of the mathematical content for secondary schools: in 1999 the coverage rate was 41%; in 2011 it reached 100% (RAMA, 2011). The curriculum change included the addition of mathematical topics and thinking skills, previously not there before (Feniger, 2020). In addition, the study makes it possible to compare the achievements of different groups in the country's education system. The results of the TIMSS-2011 tests showed, for example, that the distribution of student achievements in mathematics in Israel is the largest among countries with high or similar achievements (Mullis et al., 2012).

In order to assist students at 'both ends' in mathematics, thus maximising the ability of those who have difficulties and promoting outstanding students, the "Mitzui–Metzuyanut in Mathematics" program was implemented in Israel. This program included students with difficulties that had potential, whom the schools defined as students who without support might drop out of mathematics studies. The 'Metzuyanut' program included students with interest, motivation and high ability in mathematics (Zaslavsky et al., 2018). The two groups of students studied in separate classes, with appropriate learning methods: the 'Mitzui' groups (grades 7–9) studied the standard 5 h of mathematics per week in a group separated from their regular class. The 'Metzuyanut' group, on the other hand, received an additional one to 2 h of enrichment in mathematics. These programs included a complete layout for their operation: teacher-training courses, unified seminars for teachers from different sectors, tools for locating suitable students, evaluation and monitoring systems of students' progress, and more.

As a result of the implementation of this program, the rate of high-achievers' students increased slightly: from 12% in 2011 to 13% in 2015. This figure places Israel in eighth place in terms of the percentage of outstanding students. Also, compared to countries that are similar in average to their achievements in mathematics, in Israel the percentage of outstanding students is the highest. Unfortunately, despite this program, the rate of low-achiever students increased from 13% in 2011 to 16% in 2015. There are several reasons for this. The first is the division of students into groups according to learning levels from the age of thirteen with almost no possibility of moving from one level to another, especially in the Arabic-speaking sector (Razer et al., 2018).

Another reason, according to findings by the State Comptroller's Office, is that about one-fifth of students experiencing difficulty do not receive additional math study hours in mathematics to which they have been assigned by the Ministry of Education (The State Comptroller of Israel, 2014). Furthermore, teachers who teach in the 'Mitzui' program have less training than the teachers in higher levels (Arcavi & Mandel-Levi, 2014). These changes have led to an even greater difference among Israeli students: The dispersion remains high and there was also a slight increase of eight points in the distribution of scores (Mullis et al., 2016b). The conclusions of the Israeli Ministry of Education following the TIMSS-2015 test indicate that gaps between the sectors must be taken into consideration, and this domain is currently undergoing treatment (The Ministry of Education, 2016).

What can be learned from the Israeli case study? It is important to narrow the gap between low-achieving and high-achieving students, and different approaches should be applied. To this end, the Ministry of Education is working on several projects: new mathematics curricula for lower levels with an emphasis on the relevance of mathematics to daily life; a national Virtual High School where, in addition to their schooling, students from different corners of the country can learn together in a meaningful way; and new professional development of teachers, enabling them to work more effectively with students of different abilities.

Mexico

A comparative analysis of the curricula in Mexico, Chile, South Korea and England was initiated by the National Institute for the Evaluation of Education (INEE) and published in Rojano and Solares-Rojas (2017). It found that, in broad terms, the Mexican curriculum shares with the other countries an important nucleus of mathematical content and common characteristics of its design. The development of competences, problem solving, mathematics in context and the influence of education research reflects a global tendency in line with the PISA assessment program.

PISA exercised an explicit and strong influence on the Mexican curriculum on both previous and current curricular designs (SEP, 2011a, 2017; INEE, 2018). For instance, the 2011 Mexican program of studies asks:

> In its vision towards 2021, the curriculum should lay the foundations of average Mexican society acquisition of the general competences currently shown by level 3 of PISA; eliminating the gap of Mexican children currently located below level 2, and strongly supporting those who are in level 2 and above. The reason for this policy must be understood from the need to drive with determination, from the education sector, the country towards the knowledge society. (SEP, 2011a, p. 85; *translated by Armando Solares-Rojas*)

In Mexico, PISA has produced both positive and undesirable effects (Rojano & Solares-Rojas, 2017, 2018). On the one hand, Mexican curricular design follows an international trend regarding the development of competences, taking into account the impact of local contexts and the conditions where learning takes place. On the other hand, there were some conflicts in the way competences are included in Mexican program of studies; for instance, at the same time the program uses both mathematical competences and content descriptors as leading criteria for the definition and the organisation of the mathematics that students should learn. Moreover, the contents are organised not only by content blocks (defined bimonthly), but also by content themes or 'axes' (applied to each block). The resulting knowledge segmentation can generate discontinuities in the development of mathematical ideas.

To illustrate this discontinuity, we refer to the curriculum of grade 6 which consists of five blocks and three themes, namely: number sense and algebraic thinking; form, space, and measurement; data management. The topic of volume is first studied in Block 3 through comparing the volumes of bodies without using standard units, whereas, 2 months later, in Block 4, the use of cubes as standard units to calculate volumes is introduced (SEP, 2011b). After starting the study of volumes at Block 3, students have to switch to contents of other thematic axes (e.g. to data management at the end of this block). Then, at the beginning of Block 4, they first have to study contents dealing with number sense and algebraic thinking. Only in the middle of Block 4 do students return to study the volume of geometric bodies.

This segmentation by time and thematic axes can not only generate discontinuity in the development of some mathematical notions, it also prescribes rigid teaching times that do not consider the real, diverse and varied needs of the classrooms. Rojano and Solares-Rojas (2017) provide supporting evidence of these issues and propose concrete aspects to be considered in new reforms of Mexican curriculum.

What can be learned from the case study from Mexico? To improve mathematics education, curricular redesign should address not just international trends but also a number of critical issues, such as making clear and explicit the purpose of teaching mathematics to every citizen, explicating the findings of research in mathematics education being applied, and incorporating the teacher's perspective to give flexibility for teachers to adapt the curricular design to their specific classrooms and local contexts. Such an approach would avoid or reduce various content and pedagogical ambiguities present when different trends, advocated by international research and practice, are followed within a reform context without addressing these issues.

Serbia

Prompted by relatively unsatisfactory TIMSS results for Serbia in 2003 and 2007 regarding achievement of students in grade 8 (below 500 points), TIMSS cognitive domains have been incorporated into national educational standards for the end of primary education in grade 4. These cognitive domains, introduced in the TIMSS 2007 assessment cycle (Mullis et al., 2005), were knowing, applying and reasoning. The incorporation was done in the following way: three achievement levels (basic, intermediate and advanced) in the standards were defined in terms of six cognitive categories (knowledge, comprehension, application, analysis, synthesis and evaluation) based on Bloom and Krathwohl (1984).

These cognitive categories were mostly used in a way that corresponds to the TIMSS cognitive domains. The basic level, involving cognitive categories knowledge and comprehension, corresponded to knowing. The intermediate level (with comprehension and application as cognitive categories applied) corresponded to application, whereas the advanced level (requiring analysis, synthesis and evaluation) corresponded to reasoning. However, TIMSS cognitive domains are not mentioned in official documents describing the development of these standards. Instead, the application of Bloom's taxonomy is mentioned (Pejić et al., 2013).

Standards-based curricular changes (IEQE, 2009) were initially supported by comprehensive assessment-related material comprising a CD with one hundred carefully developed, TIMSS-like tasks that assessed mathematical knowledge in grade 4, and detailed documentation including a computer program to enter and analyse achievement data. These tasks were developed for twenty-five learning outcomes, with four similar tasks per outcome. The material was sent to all schools in Serbia in May 2009 with a recommendation to use it to arrange school assessment by the end of the 2008/2009 school year; according to Stanojević (2010) most schools did so.

Because Bloom's respective cognitive and achievement levels were assigned to each learning outcome, i.e. the task assessing it, an empirical evaluation of the incorporation in question was undertaken. This evaluation confirmed that the incorporation occurred to a considerable extent because the cognitive level assigned to particular tasks (e.g. application) was present at the achievement level assigned to it (e.g. intermediate: comprehension and application) for twenty out of twenty-five learning outcomes mentioned above. Seventeen of these twenty were later used as a foundation of the final set of educational standards for the end of primary education in grade 4, and such a contribution was particularly strong for five learning outcomes in the area of measurement & measures (Kadijevich, 2019).

This final set of educational standards (National Educational Council, 2011) was later operationalised in a 20-task-TIMSS-like test that assessed mathematics learned in grade 3. The test was developed by the Institute for Education Quality and Evaluation in 2014, along with a detailed documentation including a computer program to enter and analyse achievement data. This material was sent to all schools to assist them in carrying out an initial assessment in grade 4 at the beginning of the 2014/2015 school year.

Unquestionably, the use of these assessment materials contributed to teachers' and students' familiarity with TIMSS-like contexts and tasks. This probably contributed to above-average TIMSS results in the Mathematics achievement of grade 4 students in 2011 and 2015, with 516 and 518 points, respectively (Kadijevich, 2019). However, the use of these assessment materials has not been monitored in later years to gather evidence about its opportunities, challenges, or need for improvement. In other words, additional research is needed concerning the application of educational standards for mathematics for the end of primary education in grade 4 and assessments based on them. These standards are still in use today.

What can be learned from the case study from Serbia? One important enterprise has been to develop suitable educational standards and proper assessments based on them. Another equally important one is to assist, monitor, and assess their application in practice and to recognise theoretical and practical issues to improve. This latter enterprise has been wanting.

Conclusion

This chapter examined the role of the two international studies, TIMSS and PISA, as drivers for school mathematics curriculum reforms. These drivers were examined both worldwide and in particular countries, with a special emphasis on ways in which particular components of these studies have been applied in curriculum reforms.

Regarding its general, worldwide influence, TIMSS has primarily contributed to a certain curricular convergence regarding topics to cover and skills to foster, and many national curricula have emphasised the role of problem solving and thinking skills, such as applying mathematics and mathematical reasoning (e.g. Mullis et al., 2016a). PISA influences have primarily dealt with the inclusion of the notion of mathematical literacy, usually expressed in terms of competencies or capabilities (e.g. Stacey et al., 2015; Michel, 2017). However, for both TIMSS and PISA, support for related curricular changes through teacher professional development, has been missing in many countries, as evidenced by Mullis and colleagues (2016c) and Stacey and colleagues (2015).

Deficiencies regarding teacher support for curricular changes are reported by four case studies dealing with particular influences of TIMSS and PISA in Australia, Israel, Mexico and Serbia. Although these studies present diverse influences – from critical curricular notions (Australia, Mexico) to instructional programs involving low and high achieving students (Israel) to educational standards (Serbia) – they highlight that curriculum solutions should be justified, flexible, and progressive for the benefits of their implementers, but also be the subject of continuous monitoring.

Integration into national educational policy processes with various stakeholders involved is needed if countries are to benefit from the influence of TIMSS and PISA on mathematics curriculum, through their assessments or national/regional assessments. The study of Lietz et al. (2016), which calls for such integration, evidences that it is unlikely to happen in economically less developed countries, due to*, inter alia*, lack of a continuing and secure line of funding covering the realisation of various assessment stages, including educational policy change. Having in mind the orientation of international assessments studies in recent years to apply e-assessment, many countries would apply this, especially in the time of the Covid-19 pandemic. In Serbia, for example, to help students prepare for their matriculation exam at the end of grade 8 (the so-called *mala matura*), this exam was nationally simulated online in April 2020 and almost all students participated in this kind of knowledge self-evaluation.[2]

Bearing in mind that TIMSS or PISA components have been embedded in national policies in varying ways and to different extents, further research may focus on various interpretations made and implementations applied (Michel, 2017).

[2] Each of the three tests applied was solved by more than 60,000 students (more than 90% of the whole population). Source: http://www.mpn.gov.rs/analiza-onlajn-testova-za-samoprocenu-znanja-za-zavrsni-ispit-i-postavljanje-testova-i-rezultata-sa-prethodnih-zavrsnih-ispita/

This research should not only compare these interpretations and implementations in different countries, but also examine the limitations of these influences, due to the historic and cultural contexts of respective countries. This research would help to us to understand better the development of mathematics curricula worldwide, as well as to identify how better to support teachers' professional development in implementing such curricula for the benefits of all students.

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Chapter 23
Emergence of Computational/Algorithmic Thinking and Its Impact on the Mathematics Curriculum

Djordje M. Kadijevich, Max Stephens, and Abolfazl Rafiepour

The first ever ICMI study, undertaken in 1985, was entitled *The influence of computers and informatics on mathematics and its teaching* (Churchhouse et al., 1986). The contributing authors, mainly mathematicians operated by and large from a European and North American perspective, mostly focused on using computers to model some advanced mathematical ideas. (Papers that focused on teaching and learning were published in a separate supplementary publication.) Despite that, there was the emergence of an international perspective, however limited, on the relevance of computers and informatics to the teaching and learning of mathematics, particularly in the mathematics curriculum of the senior high school that could just make use of spreadsheets and some graphing packages. This was because high powered computer software programs were realised few years later: the first Wolfram Mathematica in 1988; graphing calculators with Computer Algebra Systems (CAS) in the late 1990s.

In 2006, the ICMI Study 17, hosted in Vietnam, returned to the same theme as "Technology Revisited" (Hoyles & Lagrange, 2010). The contributing authors were mostly from education, and, understandably, there was a stronger focus of the papers was on the teaching and learning. This ICMI study adopted a broader

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international perspective, but its conclusions were still cautiously stated. They concluded that some national governments had moved ahead, but generally the position was described as one of limited adoption of technology in the teaching and learning of mathematics, with differences occurring even within different states of the same country. Despite a broader international participation in the study, one can say that the issue had not moved outside the concerns of those mathematics educators who remained its chief protagonists, while other mathematicians and mathematics educators appeared less convinced that the use of digital technologies had an important and indispensable role in school mathematics.

Professional and Societal Perspectives: Connections Between Internationalisation and Globalisation

The so-called fourth industrial revolution, based upon widespread use of data analytics including big data, rapid refinements in artificial intelligence and its applications, and near universal access to high-speed Internet supporting cloud storages, has created economic and social conditions that require an increasing supply of ICT skilled workers. Up to 40 million new positions would need to be filled globally by digitally competent workers, creating an urgent need for young people to leave schools and training institutions digitally literate (WEF, 2018).

As a result, the international debate about curricular issues no longer takes place largely within an educational/academic community. By 2018, major international forums and agencies have taken up these issues. Economic ‘think tanks’, such as the World Economic Forum and inter-governmental agencies such as the Asia-Pacific Economic Cooperation (APEC, 2018), have advocated strongly that educational systems and school mathematics need to respond promptly to the digital revolution. To this end, a prize for promoting the use of digital technology in education is offered by the Islamic Educational, Scientific and Cultural Organisation (ICESCO, 2019), for example.

These interventions are directly relevant to our theme and help us to draw out important connections between internationalisation and globalisation; in particular, showing how global economic and social conditions influence the framing of agendas for internationalisation of the school mathematics curriculum. ICMI Study 1 and ICMI Study 17 are instances of internationalisation, trying to bring together different perspectives from the participating countries and seeking to reach a measure of agreement about what should take place. However, participation in these studies was limited largely to educators and mathematicians, with a still muted role for governments and international agencies.

To understand economic and social conditions that require an increasing supply of ICT skilled workers and to prepare to take part in the fourth industrial revolution in their future work, students need to learn to apply computational thinking (i.e. thinking based on computations) skilfully. The global trends presented above, connecting professional and societal perspectives, are, for example, evident in two directions regarding the rationales for including this thinking in compulsory education: one deals with enabling students to solve problems as an information-processing agent, whereas a second concerns a greatly needed preparation of qualified workforce (Bocconi et al., 2016; Rafiepour, 2018). Clearly, there is no hard and fast separation between these two rationales, each obviously influencing each other.

The emergence of computational thinking is as a clear instance of globalisation and internalisation in education as a consequence of: (1) an increasing reliance on digital technology, whose applications often combines local and global contexts; (2) growing use of algorithmic techniques to deal with various real-world challenges, many of which go beyond local contexts; (3) raised parental and societal expectations concerning a better education of children, involving out-of-school coding and programming activities that are available globally. These issues are considered in the preceding chapter by Stephens, Kadijevich, Niss, Azrou and Namikawa.

Using Technology in Mathematics Education: Computational and Algorithmic Thinking

Technology has been integrated in mathematics curriculum in many countries worldwide. About 90% of countries that participated in TIMSS 2015, for example, reported initiatives for this integration (Mullis et al., 2016). However, there is lack of a solid knowledge of the way in which the integration could affect the content taught and enhance its teaching and learning (Cai & Howson, 2012). Not only have questions (such as, "Would frequent use of computers increase achievement?" and "Is the quality of computers use more important to learning outcomes than the quantity of computers use?") generated inconsistent answers, but also several findings supported just an infrequent use of computers in the classroom (Kadijevich, 2015).

To contribute to the development of this knowledge, research may primarily focus on the way in which the use of computers and other digital tools can mediate the learning of mathematics in a productive way (Drijvers, 2018). Because the mediation in question is based upon problem solving with technology (computers and other digital tools), apart from mathematical reasoning, it involves the above-mentioned computational thinking, i.e. thinking based on computations, often related to the application of tools and techniques from computer science.

The term *computational thinking* (CT) was first used by Papert (1980) in his book, *Mindstorms: Children, computers, and powerful ideas* to describe specific thinking that children applied in learning mathematics (i.e. Turtle geometry) through LOGO programming. CT was later examined by Wing (2006), who viewed it as a fundamental personal ability like reading, writing, and arithmetic. The Royal Society (RS, 2011) described this ability as enabling persons to recognise aspects of computations in various problem situations, and to deal with those aspects, by applying tools and techniques from computer science.

Algorithmic thinking (AT), on the other hand, is one form of mathematical reasoning, required whenever one has to comprehend, test, improve, or design an algorithm – "a precisely described routine procedure that can be applied and systematically followed through to a conclusion" (The Concise Oxford Dictionary of Mathematics, 4th edn, p. 11). This procedure, whereby a mathematical problem is usually solved, processes some numeric, symbolic or geometric data. To deal with algorithms successfully, AT calls for distinct cognitive abilities, including abstraction (making general statements summarising particular examples) and decomposition (breaking a problem down into sub-problems).

CT deals with solutions in representations that could be efficiently processed by information-processing agents (Wing, 2011). As these agents are mostly computers nowadays, we assume that it is precisely the application of automation that separate AT from CT. However, mathematicians may prefer to use term AT even when computer tools are used (see Lockwood et al., 2016, for this preference).

Chapter Outline

In the rest of this chapter – based upon Kadijevich (2019b), Stephens and Kadijevich (2020) and Rafiepour (2018) – CT, as a broader notion, is examined first in detail, by summarising research findings regarding defining, cultivating and assessing it. This examination is followed by a section on CT/AT, discussing different educational priorities and practices regarding CT/AT, its relevance to mathematics education, and emerging implications for this education. The chapter ends with a summary of the findings presented and suggests directions for further research.

Computational Thinking

Despite its widespread use, a widely accepted definition of CT is lacking (Mouza et al., 2017). It has been defined in terms of its main facets, dimensions, concepts, practices, perspectives, etc. For example, core CT facets may be abstraction (data collection and analysis, pattern recognition, modelling), decomposition, algorithms (algorithm design, parallelism, efficiency, automation), iteration, debugging and generalisation (Shute et al., 2017). As regards CT dimensions, there may be three: its concepts (e.g. data, operators, loops), its practices (e.g. abstracting, modularising, debugging), and its perspectives (e.g. questioning, connecting) (Brennan & Resnick, 2012; cf. Kafai & Burke, 2013). In a high school STEM context, CT may comprise four categories of practices, namely: data practices (e.g. collecting, visualising), modelling and simulation practices (e.g. building and using computational models), computational problem-solving practices (e.g. programming, troubleshooting) and system-thinking practices (e.g. defining systems, managing complexity) (Weintrop et al., 2016).

To simplify matters in defining CT, we may just focus on its basic steps or stages used in problem solving, such as decomposition, pattern recognition, abstraction, and algorithmic thinking, recognised by Hoyles and Noss (2015) as main thinking skills required. These stages, as equally important, may be considered as CT cornerstones (see Fig. 23.1).

However, the processes of abstraction and pattern recognition overlap because pattern recognition may be viewed as abstraction and generalisation (Scantamburlo, 2014). In addition, pattern recognition may be an overall goal of CT, like in troubleshooting or managing system complexity. Finally, the use of technology to automate solutions is missing in this four-step model. It may be thus better to assume that basic CT steps (stages) are decomposition, abstraction, algorithmisation and automation, which may be advanced in a complex, nonlinear way by going back and forth between (not only neighbouring) stages (Fig. 23.2). In a preliminary empirical study, Kadijevich (2019a) shows that this cycle is not only relevant to different subject areas, such as mathematics and science, but also relevant to distinctive learning

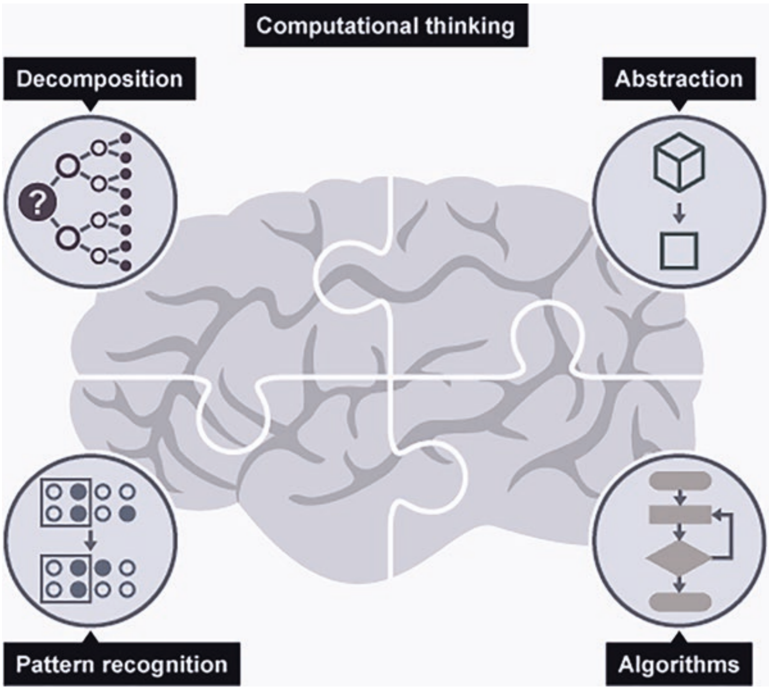

Fig. 23.1 Four cornerstones of CT. (Source: https://www.bbc.com/bitesize/guides/zp92mp3/revision/1)

tasks commonly given in these subject areas, such as data visualisation and spreadsheet modelling.

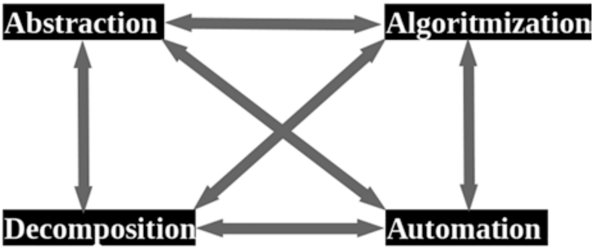

Fig. 23.2 CT cycle. (Kadijevich, 2018a, p. 76)

Cultivating and Assessing CT

Because research is scarce, knowledge about the integration of CT in K–12 education is limited (Voogt et al., 2015). However, to cultivate such thinking, rich computational environments should be used, and students encouraged to develop digital artefacts; in these environments by progressing along a use-modify-create learning path (Lee et al., 2011). Less experienced or novice students should be encouraged to progress along an understand-debug-extend trajectory, i.e. from understanding developed 'artefact' via debugging this 'artefact' to extending it (see Brennan & Resnick, 2012). For an appropriate integration, Mouza and colleagues (2017) discourage narrow use of digital tools (e.g. just concepts mapping tools) promoting just one or two CT components (e.g. problem decomposition).

The learning paths mentioned above may be recognised in CT pedagogy proposed by Kotsopoulos and colleagues (2017), which assumes that the various conceptual or digital objects in mathematical classes make use of four overlapping activities: unplugging (not using computers); tinkering (taking objects apart and changing/modifying their components); making (constructing new objects); remixing (appropriating of objects or their components to produce new objects). As examples of these activities, consider, respectively, sorting mathematical expressions, modifying spreadsheet content, developing interactive geometry presentation, and combining and modifying existing interactive reports to visualise data with dashboard (a set of interactive reports).

A lack of standard CT definition has resulted in diverse measurement of this construct, making comparing results of research studies difficult. Furthermore, CT assessment in classrooms is challenging, requiring real-time assessments that monitor students' progress (Shute et al., 2017). Such assessments could be based on the analysis of students' project portfolios regarding 'artefacts' they develop through progressing along a learning path (e.g. Brennan & Resnick, 2012; Lee et al., 2011), possibly resulted from the application of a suitable pedagogical framework (Kotsopoulos et al., 2017). This analysis should focus on CT features (e.g. stages or components) and their relations aimed to be promoted. To assess CT-based instruction, a technology integration rubric may be used, whose criteria evaluate choosing and applying digital tools and CT components respecting curriculum goals and instructional strategies, simultaneously aligning content, pedagogy, and technology (Mouza et al., 2017).

Implications for Algorithmic Thinking (AT)

By accepting the position that the main stages of a learning cycle describing AT are decomposition, abstraction and algorithmisation, it may be said that, as indicated above, CT occurs whenever AT is supported by automation, i.e. the use of computational tools and environments. This means that approaches to cultivating and assessing CT summarised above may be applied to cultivating and assessing AT. For example, a suitable learning path could use the following trajectory: from understanding developed algorithm via debugging this algorithm to extending or improving it, focusing of AT features (e.g. stages or components) and their relations aimed to be promoted. Furthermore, to assess AT-based instruction, a AT integration rubric may be used, whose criteria evaluate choosing and promoting AT components with respect to curriculum goals and instructional strategies, simultaneously aligning content and pedagogy.

Computational/Algorithmic Thinking

While the above arguments present a case for a clear distinction between CT and AT, their use in practice reflects different interests and priorities. The term 'computational thinking' rightly draws attention to the underlying logical and mathematical processes that are fundamental to computer science and should be contrasted with facility or familiarity in using digital machines. These processes have been introduced to students in the New Zealand program *computer science unplugged* (https://protect-au.mimecast.com/s/SToXCJypvAfqwq6QRhYchXg?domain=csunplugged.org), to give a well-known international example.[1]

Instead of using the term 'computational thinking', recent educational documents in UK (Stephens, 2018) and Argentina (Sadosky Foundation, 2018) mostly use words 'algorithms' and 'algorithmic'. Algorithms and algorithmic thinking are the preferred terms in the *Australian Curriculum: Digital Technologies* (ACARA, 2016) across all years of schooling. A priority to algorithms is given in the French curriculum, *Algorithmique et Programmation* – a domain of both the mathematics and the technology curricula, and thus taught by the teachers of these two disciplines (Ministere de l'Education Nationale, 2016) where *Scratch* is the main

[1] If the reader accepts the definition of CT assumed in this chapter, the *csunplugged* approach, which does not rely on the use of computers i.e. automation, may be viewed as means that primarily promotes AT.

programming language. Later, in high school, algorithmics is also taught in mathematics using *Python*.

In other countries, curriculum documents emphasise coding and programming in basic education. In Finland, for example, there is a clear emphasis on the cross curricular uses of programming, including the use of programming languages, with specific attention to computer-less programming in the early years (PMO, 2019). Similarly, the announcement by the Japanese government to introduce programming in primary and secondary schools from 2020 has a clear focus on programming across the curriculum. In fact, the published materials refer repeatedly to 'programming thinking' as distinct from learning to program a machine (Stephens, 2018).

Relevance to Mathematics Education

The term 'computational thinking' has been used extensively by computer science specialists, who carried out many studies that link CT and computer science topics, mostly programming (e.g. Hickmott et al., 2018). Consequently, CT has become a critical curricular component in computer science (informatics) education (e.g. Webb et al., 2017). It has not had a similar status in mathematics education. The reason may be that studies explicitly linking it and learning mathematics are rather rare (Hickmott et al., 2018), mostly dealing with areas that are traditionally connected to programming (e.g. numbers and operations, algebra, geometry).

In mathematics education, the main task of technology is to mediate the learning of mathematics in productive ways (Drijvers, 2018), including relating procedural and conceptual mathematical knowledge (Kadijevich, 2018b). AT may be critical to developing these knowledge types and relating them. For example, procedural knowledge may be developed through implementing procedures, especially through designing procedures and algorithms, which could result in knowledge that is rich in connections (e.g. Lockwood et al., 2016). On the other hand, AT may be used to develop conceptual knowledge and a deeper conceptual understanding if a special case of a formula, or an algorithm in general, is used as a means for asking advanced questions about the result obtained by applying it (Abramovich, 2015). Research has supported the position that, in digital environments techniques could be used as a means to relate procedures and concepts (e.g. Artigue, 2010).

If CT/AT is to have an enlarged role in the mathematics curriculum, we must continue to ask how these forms of thinking build upon, connect with, and enhance the way students think about and do mathematics. The following examples are intended to make it clear that our emphasis is on *mathematical thinking* – not on following or memorising routines, and still less on equating algorithmic thinking with coding. Productive examples might include: using the language of algorithms to exemplify mathematical concepts and procedures (e.g. starting with multiplication and division); drawing on appropriate mathematical knowledge to construct algorithms (e.g. to model a particular problem and to allow for a solution); using the language of mathematics to explain the key steps of a given algorithm (e.g. a simulation); using the language of mathematics to identify or improve the variables and parameters required to use a given algorithm (e.g. in data practices); critically examining solutions to improve on an existing algorithm; identifying mathematical variables and parameters in order to use a given algorithm (e.g. in data analysis); using an algorithmic application to solve a mathematical problem in order to identify its mathematical structure and to generalise the solutions (e.g. computational problem solving); using an algorithm to deepen understanding of mathematical patterns and relationships.

Emerging Implications for Mathematics Education

In the remainder of this chapter, we tend to use the combined term CT/AT, mindful that some readers may be accustomed to separate uses of these terms. As yet there appears to be no international consensus in these matters. We refer to an increasing trend for CT/AT to be included in the compulsory years (basic education) for all students (Stephens, 2018). CT/AT has the potential to play an important role in problem solving and modelling in the school mathematics curriculum at all stages, where, for example, iteratively developed (deterministic or probabilistic) solutions can be expressed in forms resulted from the application of CT/AT (e.g. a spreadsheet model that determines the profitability of a small business; Kadijevich, 2012). In STEM contexts, CT/AT can develop a synergy between mathematical modelling, computer programming, and engineering design (López-Leiva et al., 2019). The use of CT/AT in STEM contexts should also be considered from the primary school years.

Data analysis, based upon the use of interactive displays for example (Kadijevich, 2019b) is a simple instance of data science, defined as the science of obtaining useful information from data by using various computational methods and tools. Data science reflects the unprecedented growth in the availability of data in most areas of human activity. CT/AT is an essential support for steps in data science learning cycle, such as ask/frame questions, locate/accumulate/evaluate data, analyse data, and interpret data (Gould et al., 2017). Data science latter is an emerging and important area of statistics education, supporting students to acquire data and to use them to make informed decisions in their daily lives (see, for example, *International Data Science in Schools Project* (IDSSP) at: http://www.idssp.org/).

Professional mathematicians apply computation in their disciplinary practice to support various aspects of their work, involving experimentation, approximation, conjecture testing, and visualisation. These areas are now increasingly recognised as important features of mathematical reasoning within school mathematics. Although classroom practice should be different from disciplinary practice, the latter should inform the former and help designing it (Lockwood et al., 2019). Students may use CT/AT to define (construct) objects, identify their possible properties (of algebraic, geometric, or statistical nature) and verify these properties (a number of studies reported in Hoyles & Lagrange, 2010, for example, may be re-examined in that way). The identification and application of geometric properties of shapes, for example, underpins the application of CT/AT in computer design and art, and allows these potentialities to be explored in two and three dimensions much earlier than traditional school geometry has allowed. Like mathematicians who apply computation to find approximate solutions to intractable problems, students may use CT/AT to approximate solutions of mathematical models that cannot (easily) be solved in the context of school mathematics (for examples of such problems, see Kenderov, 2018).

Conclusion

CT originated from learning mathematics with technology. While CT is critical curricular component in computer science (informatics) education (e.g. Webb et al., 2017), CT lacks a similar status in mathematics education. Apart from areas that are traditionally connected to programming (e.g. numbers and operations, algebra, geometry), further research, including curriculum development, is needed to explore other areas of mathematics suitable for technology supported problem solving, such as functions, probability, and statistics explored through modelling, simulations, and data analysis, respectively (Hickmott et al., 2018). Such exemplars of problem-solving utilising CT/AT should aim at developing and interconnecting procedural and conceptual mathematical knowledge.

CT/AT has changed the nature of some contemporary researches in mathematics domains. These are now recognised internationally. For example, computer-based proofs (e.g. four colour theorems) are now accepted in mathematics. New domains of research related to mathematics and computation have become possible, such as Bioinformatics. In this regard, the European Mathematical Society (EMS, 2011) recognised an emerging way of engaging in mathematical research: “Together with theory and experimentation, a third pillar of scientific inquiry of complex systems has emerged in the form of a combination of modelling, simulation, optimization and visualization” (p. 2). Weintrop et al. (2016) try to address CT/AT as a sophisticated and overarching concept through a literature review and interviews with experts who use CT/AT in their professional lives. Accordingly, they have developed a taxonomy of CT/AT which bears a close relation with the third pillar of scientific inquiry mentioned above. Their taxonomy contains four main categories: data practices, modelling and simulation practices, computational problem-solving practices, and systems thinking practices. For each category, they explain how contemporary activities used by mathematicians and scientists are related to CT/AT, arguing that these four areas can be viewed as a future “roadmap for what CT instruction should include in the classroom” (p. 128). For school education, however, we may well need smaller mini road-maps showing how students are introduced to and are led to explore each of these areas. These road maps will be needed to guide the next stages to embed CT/AT in the school mathematics curriculum.

Although AT and CT may in mathematics education denote similar entities, mathematics educators may prefer to use (privilege) the former term to distinguish its place in the school mathematics curriculum from components of the computer science or digital technologies curriculum. The place of algorithms in mathematics has a long history long before the use of computers. Whatever one calls this thinking – computational, algorithmic, programming or even computational algorithmic thinking – the emphasis in mathematics education should be placed on *mathematical thinking* supported by suitable technology. Incorporating CT/AT in the school mathematics curriculum will require important decisions to be taken by national and local curriculum agencies. These will vary from country to country and are outlined in the following chapter. International cooperation and sharing among researchers and educators will be vital, taking special care about defining CT/AT precisely, cultivating this thinking accordingly with a focus on mathematical reasoning, and assessing the contribution of CT/AT to this reasoning appropriately.

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Chapter 24
Conclusion: Future Visions of the Impact of Internationalisation and Globalisation on School Mathematics Curriculum

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The ICMI Study 24 Discussion Document and its research questions for theme D focused our attention on drivers that have influenced changes in the school mathematics curriculum. These were addressed in the previous texts of the theme *Globalisation and internationalisation, and their impacts on mathematics curriculum reforms*. In the Introduction, we identified "international drivers of the curriculum". In Chap. 21, we examined the evolving definitions of numeracy and mathematical literacy. In Chap. 22 we surveyed the role of international assessments, namely PISA and TIMSS, in shaping curriculum reforms internationally and locally; and in Chap. 23, we discuss the inclusion of computational (algorithmic) thinking in mathematics curriculum reforms.

Internationalisation and globalisation are now irreversible phenomena. However, we do not believe that these necessarily lead to a uniform or common international curriculum in mathematics. The *fourth industrial revolution* will certainly allow for immediate sharing of resources for teachers, students, and school systems. But it will also allow for greater variety and local adaptations as more stakeholders and participants are involved in the process of curriculum improvement and curriculum commentary. A wider range of resources will include curriculum materials, resources to support teaching, student activities, and assessment.

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We consider it unlikely that the future will see a return to the *New Math* model of past decades where the range of participants was limited and the boundaries for participation were tightly and somewhat hierarchically defined. A top-down model of promotion and dissemination made sense in a pre-digital age of Internationalisation. Future national and regional priorities should be better informed. In our opinion, globalisation could foster greater innovation and experimentation rather than uniformity.

Internationalisation and globalisation are also intrinsically connected to competition and differentiation between countries. We can expect continued reliance on international assessments by which countries attempt to measure the degree of improvement of their local curriculum and their international rankings. Local cultural factors and historical traditions will not disappear. We are concerned that there will be an ever-widening gap between well-resourced countries who can afford to experiment and choose from the range of available resources and those countries with more limited resources and fewer options.

Influence of TIMSS and PISA

We have seen and compared the impact of TIMSS and PISA in economically and geographically diverse countries, being concerned not simply with reform of the intended curriculum but supporting teacher professional development offered. In some places, our narrative pointed to variations in the curricular solutions applied, as well as to what worked and what did not work in some of these solutions.

Although particular components of TIMSS and PISA studies (e.g. knowledge to cover and skills to foster, mathematical literacy to develop) have influenced curriculum reforms in many countries worldwide, we saw that appropriate policy support for such curricular changes has often been missing or limited. Curriculum solutions should not only be justified, flexible and progressive, but also need continuous monitoring in order to benefit mathematics education.

For TIMSS and PISA, we anticipate several changes: the inclusion of a greater range of abilities in TIMSS and PISA (e.g. computational thinking), and the transformation of traditional paper-and-pencil assessments into e-assessments administered in digital format using some novel assessment tasks. Both changes will influence future curriculum reforms worldwide. We believe that *PISA for development* (OECD, 2016) will show how international assessments can be better tailored to meet the needs of low- and middle-income countries while supporting evidence-based policy making and offering globally applicable tools in monitoring progress towards commonly agreed education sustainable development goals.

Evolving Understandings About Numeracy and Mathematical Literacy

We analysed the emergence of understandings about numeracy and mathematical literacy and compared their relationship to curriculum reform processes in selected countries. Future research could address questions in three areas: (1) how the meanings numeracy and mathematical literacy might continue to evolve; (2) how numeracy and mathematical literacy can be represented in the school curriculum; (3) how to support teachers in developing students' numeracy/mathematical literacy.

We have seen how the PISA definition of mathematical literacy has changed in subtle ways over time in response to "new challenges and opportunities in all areas of life" (OECD, 2018, p. 3). For example, the PISA 2021 mathematical literacy framework retains its emphasis on problem solving, but gives increased emphasis to mathematical reasoning and, for the first time, includes some aspects of computational thinking. These changes present opportunities to investigate their impact on curriculum reform across countries. We also saw different approaches to incorporating numeracy and mathematical literacy into the school curriculum: as a separate subject, a twenty-first-century skill to be developed in all subjects, or a new emphasis on applying mathematics in real-life contexts within the regular mathematics curriculum. Future research needs to investigate the benefits and disadvantages of each of these, and other, curriculum approaches. Finally, we reiterate that teachers are responsible for implementing reforms that integrate numeracy and mathematical literacy into the school curriculum.

For *computational (algorithmic) thinking*, our future vision embraces five key elements:

Realise the importance of CT/AT

Computational thinking is now omnipresent in the sciences, in data analytics and forecasting. Its ever-expanding global applications are defining features of the twenty-first century. We have already seen that PISA 2021 includes computational thinking as a component of mathematical literacy, demonstrating that computational thinking is important for all students not only for those who are interested in computer science or mathematics.

Use CT/AT related resources

We saw that many countries are already launched on this pathway, and while different descriptors may be used, a common focus is on *thinking*. Computational thinking is more than learning to code. We identified the need for research to elucidate the connections between CT/AT and mathematical reasoning and problem solving. Some activities at all stages of schooling will need to be *computer-less* or *unplugged*. We identified resources provided by educational agencies, private foundations, laboratories, and universities to support computational thinking. Internationally available resources will continue to provide different platforms for innovation and experimentation.

Relate CT/AT and mathematical thinking

Computational methods and the use of computer-based algorithms are now established features of undergraduate programs at universities, and in the world of work. Establishing curriculum connections between Computational (Algorithmic) thinking and mathematical thinking is urgently needed for primary and secondary schools. Educational policies are needed to connect curriculum content, teaching approaches, modes of delivery, and assessment – and on the direction and rate of change.

Embedding CT/AT into the mathematics curriculum

This will include at least five dimensions: data *practices*, modelling and simulation *practices*, computational problem-solving *practices*, algorithm design *practices*, and systems thinking *practices*. We expect fluidity among educational policy makers on the degree to which these *practices* are incorporated into the school mathematics curriculum or into other curriculum areas. These practices are all multifaceted and require research on students' computational thinking abilities in parallel with their mathematical and other school studies. International studies are needed to explore these dimensions.

Develop CT/AT related educational policies

As a component of mathematical literacy, computational thinking will have a firm place in the curriculum for compulsory education. A key policy decision is where to split the focus between compulsory education and the later years of schooling where greater opportunities for choice and specialisation can be provided. We recognise that implementing CT/AT in the curriculum will be challenging for teachers with no recent university studies in computational mathematics, computer science, or related areas. A key policy decision, therefore, will be determining the rate of change, and providing for teacher professional learning. In-school models where teachers of mathematics work in partnership with computer science colleagues are likely to improve teacher capacity. The pressure to keep abreast of these changes will weigh heavily on countries and regions that are not resource rich. International leadership including that of ICMI will be essential to prevent disparities widening even more.

Finally, what have we learned from the issues investigated in the theme *Globalisation and internationalisation, and their impacts on mathematics curriculum reforms*? Four *general recommendations* are important for future curriculum reform:

- creating opportunities for collaborative work with different stakeholders (teachers, researchers, curriculum designers, industry partners) to understand complex relations among which components to be included; what national policies to revise; and what reforms to undertake;
- evaluating the opportunities and challenges when adapting components of international studies in national curriculum policies;
- developing and testing practical models of teacher preparation that align with the responsibilities outlined in the official curriculum;
- building teacher capacity to understand these components and integrate them in the classroom for the benefits of all students (extrapolated from Webb et al., 2017).

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Part VI
Theme E – Agents and Processes of Curriculum Design, Development and Reforms in School Mathematics

Chapter 25
Introduction

Ellen Jameson, Peter Sullivan, and Ferdinando Arzarello

Mathematics curriculum reform is as yet a relatively new area of study, and strong theoretical tools for making sense of it have yet to be developed. Moreover, theory alone cannot fully meet the needs of the agents who are directly engaged in development and reform processes – they need examples, concrete actions to take or past experiences to inform them. As a precursor to the development of theoretical tools and in service of building a base of concrete examples, for the sake of both researchers and agents of curriculum reform, we turn to cases of existing practice to begin building insight into the systems of people, institutions and resources involved, with a focus on the following elements, which are based on the questions posed to theme E in the Discussion Document (Shimizu & Vital, 2018):

- processes of curriculum development and reform, and how they are carried out in practice;
- agents involved in designing, developing and implementing reform, and what areas of reform they influence or are influenced by;
- roles played by professional stakeholders – mathematics teachers, teacher educators, researchers, mathematicians and policy makers, how those in each role drive or participate in reform and how they interact with each other;
- roles, if any, given to the public and how the agents of curriculum reform manage this process, whether directly or through media engagement;
- how such roles are formed and influenced;
- the influence of research in curriculum design and development. (p. 583)

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While this list could certainly be expanded, the considerations discussed, encompass many of the characteristics of curriculum reform systems which might prove key to managing complexity in curriculum reform, and which may be observed and compared across cases. The chapters which follow are an examination of these considerations, grounded in the specific contexts of reform efforts discussed in theme E at the ICMI Study 24 conference. While we do not come to generalised conclusions beyond these contexts, we present the ideas which have been suggested by comparison as well as those arising from specific instances of reform.

The authors of these cases described using different tools and approaches to managing complexity in their respective curriculum reform contexts, but in discussion with each other they also identified needs in common. We need communication practices and tools in order to explain, understand and disseminate knowledge, and to know more about achieving consensus, common ground, agreement, reconciliation, shared language and expertise, and autonomy among stakeholders, particularly teachers, who are often cast as receivers of information rather than co-determiners of the curriculum as students receive it.

Having identified these needs, we formed subgroups within the theme to flesh out three topics for the chapters which follow: panel discussions (Chap. 26); exploring how participants in curriculum design discussions negotiate productively (Chaps. 27 and 28); presenting factors which shape the professional identity and agency of teachers in curriculum design, implementation and reform (Chap. 29).

The curriculum reform cases making up the frame of reference for theme E chapters are drawn from different jurisdictions and from different cultural contexts on a macro scale. The elements of curriculum reform systems listed above appear in the cases discussed in each chapter. Additionally, each chapter points to importance of culture on a smaller scale, at the level of different regional communities of practice. Towards the end of our theme E discussions, John Volmink portrayed these cultures as living, growing, responsive things, somewhere on a developmental path. That path stretches from craft culture ("what do I know how to do?") to compliance culture ("what am I expected to do?") to professional culture ("what do I aspire to do?") – at which level creative and constructive ways of surmounting the obstacles to curriculum reform are more likely to be sought and found.

The chapters are summarised below:

Chapter 26: Panel Discussions

Four prominent contributors from different cultures and contexts were invited to present their work as a plenary panel. This chapter present summaries of those plenary papers and presentations, drawing heavily on the words of the authors.

Chapter 27: Agents, Objects and Processes in Curriculum Reform

In this chapter, Jameson and Bobis propose a model of curriculum reform as a system of agents (who is involved), objects (what documents, materials etc. they are working with) and processes (how do agents work with objects and other agents?), characterized in terms of arenas (where reform takes place). This model reflects the papers and discussions in theme E and sets the stage for the two chapters to follow. Subject to validation, it could serve as a framework to structure research in the future.

Chapter 28: Boundary Crossing in Curriculum Reform

In this chapter, Pinto and Cooper examine communication and negotiation between stakeholders in different communities of practice in curriculum reform discussions. They identify factors which support more constructive boundary crossing, according to available evidence, and analyse outcomes of the cases in theme E with respect to these factors.

Chapter 29: Teacher Professional Identity and Curriculum Reform

In this chapter, Lorena Espinoza, Stephen Quirke and Gérard Sensevy address the professional dynamics stemming from the relationship between the stakeholders leading the development or refinement of the official curriculum and stakeholders responsible for translating the official curriculum into the classroom. These dynamics contribute to the degree of agency and intellectual freedom afforded to, and felt by, the teachers responsible for translating curriculum documents into action in classrooms, which in turn has an impact on how effectively an official curriculum can be implemented at a local level.

Conclusion

Based on the chapters of theme E, Ellen Jameson, Peter Sullivan and Ferdinando Arzarello draw some main conclusions and put forward a possible consequent framework, within which future research on agents and processes in curriculum reform can be developed.

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Chapter 26
Case Studies in Agents and Processes of Mathematics Curriculum Development and Reform

Iman Osta, Fidel Oteiza, Peter Sullivan, and John Volmink

Introduction

The four contributors to this chapter represent diverse educational contexts and their case studies emphasise the complex connections between cultural and political factors and aspirations to the learning of mathematics.

Iman Osta discussed the relevance of documentation in the development of curricular reform in Lebanon as a strongly centralised action, its role in guiding the production of textbooks as the main guide for teachers, as well as teachers' role as major agents for the enactment and reshaping of the curriculum, and finally the national examinations as a central focus for curriculum development. She also pointed out the dramatic differences in the whole curricular reform between what happened in private schools with respect to public ones.

Fidel Oteiza discussed how the curricular reform in Chile sought quality education for all, and specifically how this aim was declined for mathematics curriculum, for example adapting it to the international standards within the globalised world, considering also what kind of mathematics should be pursued in an environment

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where technology offers the capabilities to do that properly. He highlighted how the curriculum development in Chile happened through a wide consultation process in the country which involved many agents. However, such in principle useful and democratic process in the end risked to produce too rigid rules, difficult to apply and almost impossible to modify when convenient on the light of application results.

Peter Sullivan, basing on the new mathematics curriculum developed in Australia, pointed out how a curriculum should be seen as an agent of reform with the emphasis being on documentation that both assumes and creates a focus for teachers as active learners about curriculum and pedagogy. He discussed how the Australian commission appointed for this had to face and decide between a series of six basic dichotomies to develop such a curricular philosophy: (1) teacher-proofing or teachers as learners; (2) documenting everything possible versus including just enough information; (3) practitioner versus specialist writers; (4) mathematics as preparation for later study or mathematics as experience; (5) general versus specific descriptions of expected mathematical actions; (6) mathematics for elite or mathematics for all.

John Volmink discussed features of curriculum reforms in South Africa, pointing out the dramatic differences between the racist curriculum before 1994 and the one after that date which was a key project in the transformation of the post-apartheid South African society. He summarises the main challenges that the appointed commissions had to face into three main items: the post-apartheid challenge; the global competitiveness challenge; the challenge of developing critical citizens. The aim was a curriculum where inclusiveness of mathematics curriculum was devised according two concurrent dimensions: the necessity of a mathematics for all and of a mathematics by all. The former means that a curriculum must design the same quality of mathematics for all; the latter that everyone is engaged in a quality mathematical experience.

The Math Curriculum Reform in Lebanon: Achievements, Problems and Challenges – Iman Osta

The educational system in Lebanon is characterised by a high level of centralisation and a national curriculum that is binding to both, public and private schools. Decision making and developments are exclusively under the jurisdiction of the Center for Educational Research and Development (CERD), overseen by the Ministry of Education (MoE). While public schools apply only the national curriculum and textbooks, private schools may apply other programs and may use different series of textbooks, local or foreign. They are, however, bound to cover the national curriculum. A major tool of governmental control is the national examinations, referred to as official exams.

The Lebanese Ministry of Education proposed, in 1994, a project for overhauling the educational sector, as stipulated by the Taïf Agreement (1989), which has put an end to the fifteen-year-long war. In October 1995, the government approved a plan for developing the new curricula. Starting 1995, a reform process of the educational system and national school curricula began, after a stagnation that lasted more than twenty-five years, partly because of the war that hit the country. The older national curriculum initially created in 1946, just after the independence of Lebanon, was partially revised in 1968 and 1971 to include instances of the worldwide, New Math wave, such as the set theory. An extremely abstract, procedural and directive spirit has always characterized the old, long lasting math curriculum, setting up an educational culture guided by, and revolving around stereotypical national examinations (Osta, 2007). In those curricula, conceptualisation was neglected and students were seen as passive receivers of information and executors of algorithms.

Between 1995 and 1999, the reform efforts mobilised politicians, educators, teachers, textbook developers, and other constituents of the Lebanese society. The educational ladder has been organized into two main levels: Basic Education (BE) and Secondary Education (SE). The BE consists of three cycles, three years each – Elementary cycle 1 (grades 1–3), Elementary cycle 2 (grades 4–6) and Intermediate cycle (grades 7–9). Secondary education includes grades 10–12. The main curriculum document, delineating general objectives and objectives of the cycles, as well as the scope-and-sequence and contents to be taught in every grade level, was issued in 1997. The national textbooks were gradually developed and applied over three years thereafter (every year, the new curriculum and textbooks were implemented in one more grade level of each cycle), till the year 2000 that witnessed full implementation at all grade levels, and culminated into the first national exams under the new reformed curriculum.

After a long period of adoption of an old traditional curriculum, the reform of the LMC constituted a revolution. It changed the ways the nature of mathematics and its teaching are perceived by the educational community. The intention was to align the new curricula with the worldwide curricular trends at that time. The methods adopted are defined as constructivist and active, the learner being the ‘centre of the teaching/learning operation’, and the capacities of ‘reasoning and problem solving’ outweighing algorithmic procedures and memorisation of facts. Compared with the old curricula, a real revolution was announced and expected.

The major question remains: has this revolution been maintained throughout the curriculum development and implementation processes? An essential claim of this paper is that, with the marginal role of teachers, absence of internal coherence of the curriculum, and lack of suitable resources, the high-stake national exams determine, to a large extent, the orientations of the curriculum enactment and make it revert back to the deeply rooted old practices.

Reflection on the Lebanese Math Curriculum

In the rest of the text, four of the main components of the LMC will be discussed, namely: (1) the foundational documentation of the curriculum – the role of this documentation was to act as a guiding roadmap for the development of textbooks and an interface between the curriculum philosophical/pedagogical foundations and the educational community; (2) textbooks as the main guiding resource for teachers; (3) teachers as the main agent for the enactment and reshaping of the curriculum; (4) the national examinations as the central focus and determinant factor of the curriculum development, implementation and reorganisation.

Foundational Documentation

The foundational curriculum documentation consists of: (1) the main curriculum document issued by an official governmental decree (CERD, 1997) delineating the aims of the curriculum, its pedagogical recommendations, general objectives (GOs), and objectives of cycles (OCs); (2) the *details of content*, published gradually in three volumes over three years (1997 for the first year of every cycle, 1998 for the second years, and 1999 for the third years). They include the specific objectives (SOs) and detailed information about the contents of the mathematics subject for each grade-level year.

Osta (2003) investigated the internal coherence of the LMC documentation using mapping tables and text analysis of the curriculum documents above. The analysis of the main curriculum document showed a high level of coherence between the general objectives GOs and the philosophical and pedagogical foundations announced in the introduction. They both use a language focused on the development of cognitive abilities, the importance of problem solving, and the appreciation of mathematics as a practical tool related to everyday life. Below are a few examples.

Mathematics is defined in the introduction as, “a fertile field for the development of critical thinking, for the formation of the habit of scientific honesty, for objectivity, for rigor and for precision. It offers to students the necessary knowledge for the social life and efficient means to understand and explore the real world”. As for the recommended teaching methods, “[they] consist of starting from real-life situations, lived or familiar, to show that there is no divorce between mathematics and everyday life”. As described, the recommended teaching methods are clearly constructivist and focus on problem solving. “The stress is mainly on the individual construction of mathematics; it no longer consists of teaching already made mathematics but of making it by oneself. Starting with real-life situations in which the learner raises questions, lays down problems, formulates hypotheses and verifies them, the very spirit of science is implanted and rooted.”

The General Objectives (GOs) are clearly consistent with this approach; they insist on the importance of ‘the construction of arguments’ and on ‘developing critical thinking, and emphasising mathematical reasoning’, the latter being presented as the first GO. Problem solving is presented as the second GO and described as "perhaps the most significant activity in the teaching of mathematics. On the one hand, every new mathematical knowledge must start from a real-life problem. On the other hand, students must learn to use various strategies to tackle difficulties in solving a problem". The student must also "encode and decode messages, formulate, express information orally, in writing and/or with the help of mathematical tools", which makes mathematical communication a third main GO. We will refer to these three objectives by "cognitive objectives", to distinguish them from objectives purely related to the factual and procedural mathematical content.

The curriculum therefore proposes a progressive teaching approach. A constructivist approach, focused on reasoning, problem solving and communication, is reflected in the teaching method and general objectives advocated in the first curriculum declarations. It is to be noted that the three highlighted cognitive objectives are mostly in line with the American "Standards" (NCTM, 1989) which have profoundly affected modern international trends in mathematics education at that time.

However, only partial consistency is found between the cycles' objectives COs and the GOs, with a deviation in the discourse that reflects a beginning of separation from the pedagogical foundations above. Indeed, the COs continue to reiterate the importance of the three cognitive objectives, which systematically appear as the three first objectives for every cycle, followed by content-related, factual and procedural objectives.

One example, where we can touch upon the deviation of discourse, is found in the objectives cited under 'problem solving' for the secondary cycle: "Find the solution of a problem following a given algorithm". Requiring that solving the problem should follow a 'given algorithm' is in opposition to the very meaning of problem solving. It also defeats the purpose stated in the GOs, delineating the traits of the learner as being 'an individual with a critical mind who questions, doubts, proposes solutions', and who 'must learn to use different strategies'.

The deviation from the curriculum's foundations and GOs increases and becomes more serious at the level of the specific objectives in the SOs in the *details of content* volumes. The three cognitive objectives are not maintained in the SOs. Not only have they disappeared as independent objectives, but they are also very rarely reflected in the contents. The analysis of the SOs shows that they mostly represent declarative knowledge and procedural skills related to formal mathematical content, emphasizing the execution of predetermined and automated steps and overlooking conceptual understanding. Very few SOs are linked to the cognitive GOs, which are supposed to perpetuate the link to the constructivist intentions of the curriculum.

The tension is evident in the *Details of content*. The phrase 'to train the student' is frequently used. The learner is seen as a passive receiver of information and executer of algorithms, and the teaching style that is detected from the teaching tips is extremely directive. Consider for example the case of problem solving: Even though the GOs insist on problem solving as a context "from real, lived or familiar situations" for both, learning and applying concepts, we find in the *details of content* clear reluctance to actual situations and mistrust of learners' abilities as problem solvers.

The *details of content* were later used as the main basis for the development of the subsequent documents and tools, including the student textbooks, pedagogical guides and evaluation guides.

Textbooks

School textbooks are the main interface between teachers and the curriculum foundations, as well as the main tool for their educational practices. The question raised here is: considering the fact that the *Details of Content* drifted away from the innovative spirit of the intended curriculum, and the fact that school textbooks are the main tools in the hands of teachers, how can teachers maintain the link between the tools available to them in their professional practice, and the GOs and OCs which ensure the true reflection of the intended curriculum's foundations?

Knowing the fact that the textbooks for the first year of each cycle (grades 1, 4, 7 & 10) were authored just after the development of the foundational documents in 1997 and that the textbooks for the third year of each cycle (grades 3, 6, 9 & 12) were authored two years later in 1999, it may be legitimate to assume that the textbook authors have gradually deviated from the reformed curriculum's foundations and reverted back to the old approaches.

Teachers and the Reform

A radical reform requires involving and preparing the teachers for the enactment of the intended change. It also offers opportunities for teachers' professional development in view of modifying their beliefs about the nature of mathematics they will be teaching and the approaches to its teaching. Educators agree that teachers are main agents for any educational change.

The MoE and CERD have conducted 'training' workshops in the new curriculum, involving a large number of mathematics teachers, especially in the public sector. These workshops proved to be too directive. They mainly revolved around providing information on the new content, as well as the recommended pedagogical approaches.

In a study that solicited teachers' reflections on the reformed Lebanese curriculum and their feedback about the workshops (Osta, 2006), all participants reported that they were not sufficiently prepared to apply the recommended teaching methods. They requested more practice on techniques such as group dynamics, group work management, active methods, design of didactical situations, development of students' autonomy, use of calculator and computer for teaching/learning purposes. Teachers expressed their belief that the educational authorities which 'impose' such methods should provide support to teachers up to the classroom level, such as providing 'model lessons', activity sheets or additional exercises to respond to certain learning problems that may arise.

National Exams

In Lebanon, national (known as official) exams take place every year at two grade levels: the end of the intermediate cycle of study (grade 9), for the 'Brevet' certificate, which gives access to secondary school, and the end of secondary level (grade 12), for the 'Baccalaureate' certificate and graduation from pre-college education. They are high-stakes exams and have an imposing power. In the Lebanese culture, a major goal for schools is to raise their students' test scores in the official exams. It is as well an indicator of school improvement. Teachers whose students pass the official exams gain in reputation and receive good offers with high salaries from private schools. This leads to the observed fact that teachers tend to teach to the test, and that school administrators shape their school policies and focus their academic activities around that goal. As a result, the official tests determine the valued mathematics that should be taught, and the ways it should be taught.

This study showed that the official exams under the old curriculum kept a stable structure and addressed a specific body of mathematical content. It was noticed that many topics in the curriculum were never addressed in the official exams. The topics frequently occurring in test items defined a 'mini-curriculum' that gradually replaced the original one, and was reinforced every year and in every test. This 'mini-curriculum' fosters memorization of answers to stereotyped test items, through drill and practice rather than conceptual understanding.

The study led to a hypothesis expecting that the extremely procedural nature that has always characterised the old math official exams, has established a deeply rooted testing "culture" focused on direct procedural skills. Consequently, the new official exams could not, over the years, reflect the real change intended by new curriculum. This 'culture' was nurtured by the long-lasting old curriculum, and its official exams are still influencing the new official exams. The hypothesis above was confirmed by three studies (Safa, 2013; Shatila, 2014; Sleiman, 2012) that used the framework developed by Osta (2007) to investigate the extent of alignment between the official exams over 10–12 years, with the reformed curriculum. The results of the three studies converged to confirm the hypothesis above. A mini-curriculum was identified, and low levels of alignment are found between the exams and the curriculum guidelines, especially as pertains to the cognitive general objectives. They found, however, that the alignment improved gradually over the years. Global alignment remained, however, lower than enough to reflect actual change in the testing culture.

The nature, scope and structure of the official tests send a clear message to the educational community (teachers, administrators, parents and students) over the years. This implicit "contract" among all involved parties binds, in return, the committees in charge of constructing the tests. Even if they want to include modifications or additions, they find themselves bound to the "mini-curriculum". This closed cycle is sustained by the "doctrine of no surprise" that English and Steffy (2002, p. 46) explain as being the idea that students should not be taken by surprise by any test question.

In Summary: From Design to Implementation

In general, while teachers are in direct contact with the implementation tools, among which the textbooks and the official exams, they are at a distance from the other foundational components, including the pedagogical foundations and general objectives of the curriculum. Those are particularly absent from their direct perception and their day-to-day professional practice, if they are not actively implicated in the reform movement. Even an attentive reading by the teachers and their participation in informational workshops are not enough to guarantee a modification of their professional practices that they developed over many years according to the old curriculum.

Processes and Agents of Curriculum Design, Development and Reforms in Three Decades of School Mathematics in Chile – Fidel Oteiza

Since the beginning of the nineties, Chile has experienced continuous economic and social growth. This process has been slow but sustained. There has been a significant improvement in economic and social development indicators. Reduction of poverty and a substantial improvement in the quality of life are unmistakable signs of a positive change. The continuous clamour for a better education, "*quality education for all*", has forced the above-mentioned period of repeated reform efforts. National and international tests show little progress in learning. These small gains are not compatible or sustainable when compared to the development of the country in other areas. Another driving force is the pervasive and perverse gap between the haves and the have-nots. A single and driving force is inequity as shown by learning results. Evidence shows that learning outcomes in public schools are significantly inferior to the ones in private educational institutions. This gap has shown to be the most difficult barrier to trespass in the Chilean educational system. The search for more equitable educational outcomes may be the most important driving force behind a thirty-year effort to reform the national educational system in the country.

Some Milestones

The reform of school mathematics curriculum is to be understood as embedded in a broader process: the reform of the educational system. The following are some of the milestones in the reform process along the relevant milestones dates, which are major decisions that might impact the educational system as a whole: [as a list] the creation – as a result of a multi-sector consultant committee- of the National Council of Education (CNE), (1996–1998); the extension of compulsory education up to

12 years of schooling (2003); a major reform of the framework defining the education for the country (2009); the creation of the Q*uality Agency* (2011), responsible for the national test as applied in various school levels; a new definition for elementary, secondary and technical education and the creation – in process- of regional entities responsible of the administration of public schools which are accountable for the implementation of the national curricula, a policy that promotes decentralisation of the educational system.

In a minor scale – but significant because they are some of the major results of reform efforts – the following can be mentioned: new infrastructure for schools throughout the country; new standards for teacher selection and teacher preparation; an improvement, although still insufficient, of working conditions and professional development for teachers; the almost universal access to digital technologies; free, newly designed, textbooks for all students in public schools; the extension of school schedules; especially relevant to the subject of this analysis, a renewed and more demanding school curriculum. National tests applied to the entire system, at various school levels, are mentioned separately because, although considered to be a guaranty of quality control, have become, at the same time, the operational definition of school curriculum and the latter competes with the official national curriculum.

Tendencies in the Process of Reform of the National Mathematics Curricula

There has been a remarkable effort to bring the national curriculum closer to international standards. Simultaneously, ideas, themes or content, before reserved for the last two years of schooling or the beginning of university courses, are now included in lower levels. This tendency can be observed in the treatment of functions, previously reserved for grades 11 and 12, now initiated in grade 7 or 8. The same occurs with probability and statistics or patterns and algebra, beginning now in first grade. Geometry includes, now, coordinate geometry and vectors. Another tendency is the emphasis of skills over content. The national curriculum in Chile promotes modelling, problem solving, communication and argumentation, and multiple representation skills. Mathematical reasoning has been of major concern among policy makers of the mathematics curriculum. The new curriculum points to classroom management that encourages the formulation, analysis and verification of conjectures. Modelling skills are emphasised throughout the curriculum. The proposed intense use of digital technologies is another new emphasis.

Agents, Institutions and Driving Forces

The above-mentioned division, which is responsible for school curriculum, has specialized teams in different areas of the curriculum, particularly in mathematics.

What is the role of the mathematics team at the Ministry of Education? When involved in a reform process, the main responsibilities are the analysis of existing curriculum, the compilation and analysis of evidence about curriculum implementation, the search and analysis of the demands and proposals of specific leading actors, the search for significant results of research and, in the field and international experience in mathematical curriculum, the interpretation of general directives as generated by educational authorities within the Ministry of Education.

Moreover, there is the formulation of proposals for the new curricula, the participation in different consultations and validation processes and the incorporation, to proposed curriculum, of the results of the consultation process. Once the new curriculum has been approved, several other tasks are in order: textbook specifications; the search for and the evaluation of different resources including digital ones and digital support; the participation both in the process for the diffusion of new curricula and the implementation of several actions related with diffusion and teacher preparation. Also, there is participation in actions related to the impact of new curricular proposals in teacher preparation and national tests which include the university entrance procedures and their corresponding exams.

The Consultation Process, Its Major Contributors and the Role of the National Education Commission

Several consultations precede the presentation of the curricular proposal to the National Council. The consultation process and the action of the National Council are the mechanism that seek to balance or counterbalance the action of the technical teams of the Curriculum Unit.

Consultation has been shown to be a powerful instrument in the definition of new curricula. Who is addressed in the process of consulting on the new proposals and how consultation instances are organised, are important issues subject to analysis and improvement. Teachers, research centres, researchers, mathematics and mathematics education associations, leaders of private educational organisations and the general public have all been consulted. Consultations have been done, mostly, in the modality of focus-groups, also with small groups of experts and public web questionnaires. Face to face feedback was effective in all the consultation meetings that were organized. Public consultations on the web proved to be more effective in making the proposals be known than in generating a specific contribution. The fact that a reform has been consulted and has received more than 15,000 public reactions is a powerful factor for face validity and acceptance.

A generalised statement can be made for both faces to face and web consultations. Most of the feedback and sometimes the whole of it were about teaching methods or teacher preparation. In a smaller proportion, reactions focused on teachers' abilities needed to put into practice what was proposed and also on the necessary conditions for implementation. A generalized reaction was: "what is proposed is too much; the amount of content exceeds what is possible in the time available to treat it". Those responsible for the proposal often agreed with this evaluation. When authors of this comment where asked about what to remove from the proposal, the most frequent answer was "nothing" and in many opportunities, "nothing, but there are many things missing". It is clear that the entire process of curriculum innovation and the way it has been implemented in the country lead very naturally to a growing curriculum. This is one of the questions to be addressed in the next section.

The role of the *National Council of Education* is now mentioned because it addresses two important needs of a reform that leads to a new formulation of the curriculum: the decision-making regulation and necessary institutional counterweight. The national curriculum in Chile is law enforced. Before a new curricular proposal becomes compulsory, a complex – also a matter of law procedure – needs to be implemented. Proposals are generated in the Unit of Curriculum previously mentioned. Once the design has been approved within the Curriculum Unit, they are subject to approval by de National Council. This process is a guaranty of quality, pertinence and proper formulation.

Two additional consequences of this process are mentioned later as open questions: one is – and this is a statement that reflects only the author′s point of view – the exaggerated weighing that has the opinion of one or very few experts when summoned as reviewers by the Council. This delicate situation has generated distortions or imbalances in the curricula that it has acted on. It is a question to be analysed. The second issue to be considered is the excessive rigidity that the whole reform process gives to the curriculum. Once constituted by law, a change, an improvement, no matter how minor, must go throw the same procedure. The result is unnecessary rigidity that inhibits needed systematic and permanent revisions and makes almost impossible consequent corrections.

Main Social, Cultural and Technical Factors Shaping School Mathematics Curricula, New Questions and Pending Issues

The gap factor shows that there is a significant, odious and until now permanent difference between the learning outcomes of students attending public and private schools. This non-solved situation poses the question of who we are formulating the curriculum for. During decades, national curricular requirements have been growing. Results, in national tests, show that students attending public schools, close to 85% of school population, are not fulfilling those standards. How does mathematical school curriculum contribute to this gap? How might mathematical curriculum be a factor in the reduction of these differences? Topics such as function, systems of inequations or homothetic figures are increasingly lower in the curriculum. Is it advancing topics that make a curriculum be better? Does the maturity of the student matter when deciding these advances? There is tremendous and extremely valuable talent diversity. Can we justify the existence of only one curriculum and only one way to evaluate it through standardised tests?

Testing gives solid information and has impact on the gap between stated and actual curricula. From one point of view, national tests are very much valued as indicators of learning outcomes. Simultaneously, they act as an operational definition of the mathematics curriculum. Teachers, schools, local educational authorities and parents give high value to SIMCE results. In consequence, what is measured ends up being a guide for teachers when making subject matter decisions. As it is very simple to guess, higher-level learning and skills as promoted by reformed mathematics curricula, therefore, are often not covered by classroom teachers.

This is an unsolved dilemma: to test or not to test. Mathematical modelling, argumentation skills, guessing and testing of one's own ideas or those of peers are difficult to measure and, thus, they lose importance for the teachers. What are adequate relations between national curriculum and national tests? How may skills in argumentation, modelling and enquiry be evaluated?

Globalisation has influenced national mathematics curriculum in several ways: media generates access to news, cultural issues, tendencies and frequent expert opinions on educational results; international tests have proved to be very influential. Another factor is the almost universal and instant access to any nation's curriculum, including those of leading countries and economies.

There are strong questions we have not yet addressed in designing the national mathematics curriculum: what is it that mathematics students need know in order to do mathematics in an environment where technology offers the capabilities to do so? What are the skills a person needs to learn to take all the advantages of existing digital technologies when doing mathematics? Information and communication technologies have shaped our culture. The second derivative of this change grows. Is computational thinking a necessary knowledge for everyone? What should a mathematics teacher know about computer science?

Currently there are new social and cultural requirements: gender, the inclusion of those showing physical or learning disabilities and personal and environmental care. All of these pose new questions. How is curriculum worded to promote inclusion? How does one formulation for the curriculum take care of the diversity in talent? How is personal and environmental care included in the school mathematics curricula? How is the mathematics classroom organized and monitored, if handicapped students are to be included?

Another issue to be analysed refers to when and why reforms are initiated. These have begun in a casuistic, not predictable agenda, the opposite to a planned systematic process. Search for long term, periodically evaluated curriculum proposals has been an issue in Chile. A one-year educational committee was appointed (in 2016) to deal with this issue. Nationally recognised educational authorities were asked to generate proposals to create a "*National policy of curriculum development*". The purpose of the Committee was to make reforms of school curricula less vulnerable to political or conjectural factors. These are important questions: What is an appropriate-long- term policy in school mathematics reforms? What are the conditions that make a reform needed? Is there a way to apply significant and defensible school curriculum diagnosis? How is a new reform decided?

There is a fundamental role played by researchers, and research and development centres and institutions. The period of school mathematics curriculum considered in these pages is the first in Chile where researchers – both in mathematics and in mathematics education or didactics – have had significant influences on school mathematics. In another publication, (Rojas & Oteiza, 2014) the authors refer to this as "new actors". However, questions remain:

> How does the knowledge generated by the research reach the classroom?
>
> How do the questions that originate in the classroom reach a research centre or a graduate program?

The Aspirations of the Australian Curriculum in Prompting Reform of School Mathematics – Peter Sullivan

This contribution and the associated presentation provide an opportunity to reflect on the intention and processes for the design and writing of the Australian curriculum: mathematics (AC: M) and to reflect on subsequent developments. The argument is that curriculum reform can be an agent and process for prompting teacher professional learning but whether this happens or not depends on whether the structure of the curriculum documentation and associated support foster such knowledge creation.

In any curriculum reform process there are many dilemmas or dichotomies about which active decisions are taken. One of the meanings of *dichotomy* is that there are two mutually exclusive, opposed, or contradictory positions. This contribution outlines some of the dichotomies in any curriculum reform process and reflects on ways that such dichotomies were and are being resolved in the Australian curriculum.

The Process of Development

Even though there are broader definitions of curriculum, including terms such as intended, planned and enacted (see, for example, Porter, 2004), this discussion focuses on documentation associated with centrally developed curriculums and decisions on the form and substance of that documentation. Of course, the real curriculum results from the ways that such documentation is interpreted, implemented and experienced in schools and classrooms, but the main opportunity for governments to intervene meaningfully is at the level of documentation.

Prior to the creation of a single national curriculum, there were eight Australian jurisdictions that each had their own curriculums and associated supporting resources. The responsibility for such curriculums was jealously guarded. In most cases the curriculums were informed by earlier national profiles so there was substantial overlap in the substance of the content specifications in the various jurisdictions but the extent of collaboration on aligning the documentation was minimal.

The motivation for creating national curriculums in all domains was essentially political. The Australian curriculum started from four domains, mathematics being one. The first step was the development of a discussion paper that set the goals and processes of the curriculum. This was described as the shape paper (ACARA, 2009) and outlines the principles, the aims, the terms used, the focus of the respective levels of schools, various issues such as connectedness and clarity, and a discussion of pedagogy and assessment especially as they related to equity and inclusion. The paper was developed by a broadly-based writing team and sought online and face to face feedback nationally. The following discussion describes some of the dichotomies and is intended to raise some of the considerations in the documentation of curriculums generally.

Dichotomy 1: Teacher Proofing or Teachers as Learners

Curriculum reform and associated teacher learning are integrally connected to views that curriculum developers and system decision makers have of teachers. There is a clear dichotomy of perspectives apparent in the ways that the initial curriculum was designed and has been interpreted.

On one hand, if teachers are seen as unreliable and unable to interpret curriculum documents then the curriculum will be written and supported in a particular way. On the other hand, if teachers are viewed as thinking, flexible and creative agents, then the curriculum documentation and associated support can reflect those perspectives.

The shape paper and the initial curriculum design opted explicitly for the latter position. The underlying assumption is that if systems place trust in teachers, they will come to see the underlying principles of the curriculum. In this process, teachers can become better educators, while increasing the detail of the documentation can be counterproductive to the mathematical intention and also to the learning of teachers.

Another decision taken was to seek to reduce the breadth of the specified content so that the more important aspects were presented. Each time jurisdictions increase the level of detail and breath of expected content, they reduce teacher decision making and the potential for teachers to learn about the broader goals of mathematics learning.

Dichotomy 2: Documenting Everything Possible vs. Including Just Enough Information

One of the initial decisions in the creation of the AC: M was that the curriculum should be described clearly and succinctly. Indeed, the intention was that the content for any one year be presented on a notional single 'page', described parsimoniously and presented flexibly via a dynamic web-based environment to emphasise the need for teachers to make active decisions (ACARA, 2009). The dichotomy is that, on one hand, comprehensive documentation would provide teachers with guidelines of what to teach, while on the other hand it would have the effect of restricting teacher decision making, causing it to be harder for teachers to see the 'big picture'.

The early consensus in the creation of the AC: M was that mathematics is much less a set of isolated micro skills to be learned independently of each other than it is sets of connected concepts and processes and that it is better for teachers to see the connections. An excessive compartmentalisation and documentation can reduce the possibilities of teachers seeing connections. The tendency in some jurisdictions in Australia, subsequent to the initial publication, has been to increase the level of detail in and complexity of curriculum descriptions which has the effect of limiting the extent to which teachers can imagine the bigger picture or even consider seeing the broader perspective as important.

A related aspect is the ways that the curriculum fosters connections between and within strands and sub-strands. A key international perspective which emphasises the importance of connections is Variation Theory (Kullberg et al., 2013). Watson and Mason (2006) outlined the importance of thoughtfully constructed sequences of learning experiences out of which the underlying concepts can be extracted. Similarly, Dibrenza and Shevell (1998) described number strings as an example of the ways that sequences of related exercises can emphasise number properties. Sinitsky and Ilany (2016) explained that considering both change and invariance illustrates not only the nature of the mathematics but also the process of constructing concepts. In other words, finding ways to support teachers in seeing and using connections between and within concepts can support teacher learning and effective teaching. To achieve this, the curriculum needs to be clear and concise.

One of the disadvantages of having the content determined by a student text is that teachers are less required to think about their own broader purposes. The same is true for curricula in which the teachers are 'told' which tasks to teach without having to appreciate the goals, both content and processes, associated with the tasks. One of the critical foci for teacher learning is to enhance their capacity to make their own decisions using the curriculum documents and other resources to which they have access.

A further central aspect that relates to the nature of the documentation is the expectations that teachers will collaborate with colleagues in their planning of sequences of learning. It seems that in some countries the textbook serves as the curriculum and teachers need only to turn to the next page in planning their lessons.

In Australia, it is common for groups of teachers to plan sequences of lessons together. Not only does this allow teachers to learn from each other but also planning together encourages them to anticipate how students might respond, identify potential blockages and misconceptions, share the development of supporting resources, and so on.

Dichotomy 3: Practitioner vs. Specialist Writers

Another early dichotomy relates to whose voice should be heard. One of the initial considerations was whether the curriculum should be written by experts or by practitioners, with the latter option being chosen. The process for creating the curriculum and associated documents was collaborative involving extensive, indeed exhaustive, consultation. Subsequently curriculum writers, predominantly classroom teachers, were employed and an advisory committee formed. There were extensive consultations around successive drafts, piloting in schools across the nation, mapping of the drafts against the various state and international curricula, and many other actions as well. The advantage of this process is that a curriculum was developed which was familiar to many teachers. The disadvantage is that the writing was informed by many and diverse contributions. In other words, there is a tension between seeking consensus and maximising coherence that is not generally acknowledged by commentators.

Dichotomy 4: Mathematics as Preparation for Later Study or Mathematics as Experience

One of the key dichotomies in determining a mathematics curriculum is related to the nature of the mathematics to be described. One perspective refers to the structure and content of many mathematics curricula that create the impression that the main goal of learning mathematics is preparation for study in a subsequent year level. An alternate perspective is that curricula should inform an experience of learning that is like being a mathematician, in which the learning about and using mathematics is the primary goal. Of course, a balanced curriculum will consider both perspectives but the intention in the AC: M was to move away from a curriculum that focused only on the former.

The AC: M took an explicit stance that the mathematics and numeracy that should be experienced by school students is much more than the emphasis on procedures and computational processes that seemed to constitute much of the teaching of mathematics in Australia at the time (Hollingsworth et al., 2003; Stacey, 2010). It is unfortunate that much of the subsequent discussion of the curriculum starts from the perspective that the primary rationale for the inclusion or emphasis on an aspect of content is that it will be used in subsequent study. This tendency is especially evident at senior levels with the pressure from interest groups being to increase the emphasis on procedures and routines and to include additional topics exacerbating the already crowded curriculum.

Dichotomy 5: General vs. Specific Descriptions of Expected Mathematical Actions

The first aspect of the AC: M that teachers gain access to is the descriptions of the concepts or content that form the focus of learning experiences. There are achievement standards available that give advice to teachers of the expected standards of performance. The key device for broadening teacher-focus to encourage them to value specific mathematical actions was described as proficiencies.

ACARA (2009) proposed that the content be arranged in three strands that can be thought of as nouns, and four proficiency strands that can be thought of as verbs. The content strands – number and algebra; measurement and geometry; statistics and probability – represent a conventional statement of the 'nouns' that are the focus of the curricula worldwide.

These four were adapted from the recommendations in Kilpatrick et al. (2001). The first of these, understanding (the Kilpatrick and colleagues' term was 'conceptual understanding'), was described as follows:

> Students build a robust knowledge of adaptable and transferable mathematical concepts, they make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.

A second proficiency strand, fluency (the Kilpatrick and colleagues' term was 'procedural fluency'), was described as:

> choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly used facts, and when they can manipulate expressions and equations to find solutions.

A third such strand, problem solving (the Kilpatrick and colleagues' term was 'strategic competence'), was described as:

> the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify their answers are reasonable.

The fourth proficiency, reasoning (the Kilpatrick and colleagues' term was 'adaptive reasoning'), included:

> analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices.

The proficiencies are represented as intersecting with each of the three sets of content descriptions, illustrating that the proficiencies are not only a focus of learning of all aspects of mathematics, but also can be the vehicle for that learning. There was an explicit intention to support teachers in seeing mathematics learning as incorporating all of these actions.

It is noted that while the first two proficiencies, understanding and fluency, can be prompted by explicit teacher instruction, while the latter two, problem solving and reasoning, require student-centred approaches, further communicating to teachers about the breadth of pedagogies needed and the nature of learning experiences that they can create.

Dichotomy 6: Mathematics for Elite or Mathematics for All

A further key element of the AC: M, which was intended to inform teacher learning is related to the challenge of equity.

ACARA (2018) argued that all students should experience the full range of mathematics in the compulsory years. Mathematics learning creates employment and study opportunities and all students should have access to these opportunities. This is both an equity and a national productivity issue. The curriculum makes the explicit claim that all students should have access to all of the mathematics in the compulsory years.

A fundamental educational principle is that schooling should create opportunities for every student. There are two aspects to this. One is the need to ensure that options for every student are preserved as long as possible, given the obvious critical importance of mathematics achievement in providing access to further study and employment and in developing numerate citizens. The second aspect is the differential achievement among particular groups of students (ACARA, 2009). An explicit goal of education in Australia is the intention to build an inclusive society in which all citizens can participate.

In summary, the claim here is that the initial intentions of the AC: M were that the curriculum should be seen as an agent of reform with the emphasis being on documentation that both assumes and creates a focus for teachers being active learners about curriculum and pedagogy. This intention was also evident in the processes used to communicate to teachers that doing mathematics is as important as skill development, and that not only is it possible to structure classrooms to be inclusive of all students but also that that is an expectation.

School Mathematics Reform in South Africa: A Curriculum for All and By All? – John Volmink

The term *curriculum reform,* like any other concept, always has a contextual *ancestry.* It also has a *career* that needs to be recognised and understood within a particular setting. But while there is general acceptance that curriculum reform grows into its own career and takes shape within a context, we often need to be reminded that this evolution is not bound by some transcendent, universally applicable set of laws which are independent of people. The political aspirations and ideological commitments of the drivers of the reform and the social forces that shaped the reform cannot be ignored and omitted from its ancestral *biography*. I see the purpose of this discussion as an attempt to understand how we can influence the development of these contextual careers of mathematics curriculum reform by understanding how choices were made within the various contexts and to what extent there was a willingness to embrace the complexity and ambiguity for the greater public good.

Curriculum inertia occurs when we choose to ignore the complexity inherent in making educational choices and retreat to the false safety of the universality of mathematics. Behind this wall we see our task as creating access to fixed, universally accepted ways of knowing and learning mathematics, stripped of all the clutter of ideological and cultural expectations.

South Africa is a society in transition. We have moved away from what was a stable but cruel past to a new and dynamic present. The conventional signposts have been swept away and we have been travelling on largely unchartered waters since 1994. One way of describing the new, democratic, educational reality in South Africa is that of celebrating the chaos and turbulence of a new beginning. It has been exciting to be part of this wonderful and dynamic period of our history and for me it has been particularly rewarding to be asked by both the previous and present Ministers of Education to play a key part in educational reform in post-apartheid South Africa.

Challenges Facing Curriculum Reform in South Africa

Over the many years of apartheid two education systems coexisted – one predicated on the goals of a first-world education, the other intended to be merely reproductive. The one was seen to be sufficient to produce enough high-level skills to support the larger economy, the other to reproduce people who were just sufficiently functional to serve the low-level skills demand of the extractive-metals economy. Race was the main determinant of educational access, provision and quality. Throughout the years of apartheid, there was a continuous groundswell of resistance to “Bantu education” culminating finally in the 1976 Soweto uprising. Since that time the Mass Democratic Movement (MDM) and the politics of confrontation in education, became increasingly organised until it established the National Education Crisis Committee (NECC) in 1980.

The failure of the then government to respond to the crisis in education led the MDM to resolve to strive for *People’s Education for People’s Power* at its first Education Crisis Conference, in December 1985. People’s Education (PE) would lead to educational practices that would enable the oppressed to understand and resist exploitation in the workplace, school and any other institution in society. It would also encourage collective input and active participation by all in educational issues and policies, by facilitating appropriate organisational structures. These ideals found expression in the work of three commissions, one each in the fields of History, English and Mathematics. When it became clear that PE would be introduced in schools by mid-1986, the apartheid government moved in very quickly to restrict its impact. The momentum for PE, during the years after the restrictive measures, was sustained for a while in large part, by the work of the Mathematics Commission, but this momentum also finally ground to a halt for a variety of reasons.

An underlying assumption in educational policy in South Africa is that the achievement of democracy requires a (national) curriculum to realise its goals. Curriculum change in post-apartheid South Africa thus started immediately after the election in 1994. So, the genesis of new curriculum thought in South Africa finds its roots in the debate within the Mass Democratic Movement over previous decades. The first major curriculum statement of a democratic South Africa was known as Curriculum 2005 launched in 1997. It signalled a dramatic break from the past with its narrow visions and concerns for the interests of limited groupings at the expense of others. But it was also bold and innovative in its educational vision and conception. It introduced new skills, knowledge, values and attitudes for all South Africans and stands as the most significant educational transformation framework in South African education.

At the dawn of democracy in 1994, South Africa had nineteen different educational departments separated by race, geography and ideology. While these were merged into nine provincial departments, there was also a need for a single core syllabus. It did not touch the core content since a part of its brief was not to necessitate new textbooks. So, beyond the rationalisation and consolidation of the existing syllabi, the process could at best sanitise the syllabus by removing overtly racist and other insensitive and offensive content forms from the syllabi.

After the completion of the syllabus revision process in late 1994 the national Department of Education (DoE) set in place a new vision for education through a series of policy initiatives in 1995. This included a vision for curriculum development and design. At the same time South Africa adopted **a** National Qualifications Framework (NQF) as the focus for systematic transformation of the *education and training* system. Some of the objectives of the NQF are to create an integrated national framework for learning achievements and to accelerate the redress of past unfair discrimination in education, training and employment opportunities.

Furthermore, an outcomes-based education approach was chosen as the vehicle to implement the objectives of the NQF at all levels and sectors of education and training in the country. When the Minister of Education announced the introduction of a new curriculum framework in 1995, there were plans to introduce it into all grades by 2005. In line with this timetable, the new National Curriculum Statement (NCS) became known as Curriculum 2005 (C2005). At a broader level, eight critical outcomes have been chosen to ensure that learners would be prepared for life in a global society. These generic, cross-curriculum outcomes also reflect the aims of the Constitution.

C2005 was inspired, not so much by the theories of others, nor on experiences elsewhere, but was an attempt to respond in an authentic manner to the realities facing the South African classroom. But it was also flawed in several ways. Some of these were design flaws while others were directly attributable to the rate and scope of implementation. None of these however, outweigh the significance or detract from the impact of C2005 as the curriculum policy that would forever change the landscape of education in South Africa.

The development of an NCS was seen as a key project in the transformation of South African society. The thrust of the project is towards achieving, in the words of the DoE, *a prosperous truly united, democratic and internationally competitive country with literate, creative and critical citizens leading productive, self-fulfilled lives in a country free of violence, discrimination and prejudice* (DoE, 1997, p. 4).

Curriculum reform since 1994 faced several challenges**.** These include:

- The post-apartheid challenge: to provide awareness and the conditions for greater social justice, equity and development. This is the challenge of developing new values and attitudes.
- The global competitiveness challenge: to provide a platform for developing knowledge, skills and competences to participate in an economy of the twenty first century.
- The challenge of developing critical citizens: citizens in a democracy need to be able to examine the many issues facing society and where necessary to challenge the status quo and to provide reasons for proposed changes.

The view taken by the curriculum designers was that the best route to greater social justice and development is through a high-knowledge and high skills curriculum and that mathematics education can play a vital role in the realisation of this vision. The general expectation was that the NCS would result *in learners who are literate, numerate and multi-skilled, but who are also confident and independent, compassionate, environmentally respectful and able to participate in society as critical and active citizens.*

Review Committee on Curriculum 2005 recommended major changes to the NCS (C2005) in May 2000 and the Revised National Curriculum Statement (RNCS) was implemented immediately thereafter. The vision adopted by the Review Committee in 2000 keeps in focus the dual challenge for C2005 of addressing the legacies of apartheid on the one hand and preparing learners to participate in the global village on the other – these two are taken as indivisible. The RNCS has been further refined in 2011 through a new statement called the Curriculum and Assessment Policy Statement (CAPS) (DoBE, 2011) that specifies content and assessment criteria in a more integrated manner.

Mathematics Curriculum Pre-1994

During the apartheid period the canonical syllabus for mathematics, although compartmentalised by race, had remained roughly invariant for everyone over decades. In a sense, the content was almost immaterial and by itself, made very little difference to the way mathematics, as a school subject, was used as a means of control and social stratification. Some attempt was made to revise the mathematics syllabus every eight years or so, but this rarely made any substantive change to the core content. Even in the current South African curriculum parlance, mathematics is referred to as a 'gateway subject' precisely because it provides access as a gatekeeper. More than any other subject, mathematics will decide who will stay behind and who will go ahead. Although some may feel that mathematics has only been able to assume this central position in the curriculum because it is over-admired and over- privileged, very few will question the need for all learners to be 'mathematically literate'.

In fairness it must be acknowledged that a feature of school mathematics during the late 1980s and early 1990s was a concerted effort by some mathematics educators to adopt a different approach to the teaching and learning of mathematics at school. The impetus for this change came largely from the world-wide swing towards a *constructivist perspective* that was implemented mainly in white primary schools in South Africa. Euphemistically called the '*problem-centred approach*', this perspective came across in the South African context as a prescriptive methodology, a new orthodoxy, which dismissed and replaced any set of ideas mathematics teachers may have had about the teaching of the subject. Nevertheless, few will deny that where this constructivist approach was piloted, it made a significant change to the classroom culture. Pupils at these schools developed very positive attitudes to mathematics and there is strong evidence that they also developed powerful ways of learning mathematics. It would therefore be unfair to say that this "socio-constructivist" approach to mathematics did not have a beneficial effect on classroom practice. It is however the case that the classroom of majority population in South Africa, where the teacher typically has to cope with a large class and poor resources, was left virtually unreached and therefore unaffected by this approach.

During the pre-1994 period People's Mathematics developed independently and indigenously rather than an attempt to embrace the "loudest fad from the West". In addition to facilitating discourses around mathematics in the communities, People's Mathematics also developed a unique emphasis and character. Cyril Julie (1991) argues that the four major distinguishing features of People's Mathematics were:

- its ability to reveal how school mathematics can be used to reproduce social inequalities;
- its rejection of absolutism in school mathematics and its contribution towards seeing mathematics as a human activity and therefore necessarily a fallibilist one;
- its incorporation of the social history of mathematics into mathematics curricula and its belief in the primacy of applications of mathematics.

Julie acknowledges that People's Mathematics did not have the desired effect on the development of a mathematics culture at the time. This he claims, is partly due to the preoccupation of the advocates of People's Mathematics to design mathematical activities that had a direct bearing on the day-to-day political struggles of the people. Another reason for its lack of efficacy was the sense of scepticism and even distrust about the notion of People's Mathematics as a poor substitute for the 'real mathematics'. People's Education (PE) failed to re-direct its focus away from *a struggle in the streets to a struggle within the classroom*. While it may be the case that it was too overtly political or even woolly at times, the People's Mathematics Movement did provide a focus for mathematics curriculum debate and indeed for PE itself and it was encouraging to how the spirit and core ideas of PE became mainstreamed in the National Curriculum Statement.

Mathematics Curriculum Reform Post-1994

In the post-apartheid era, mathematics curriculum reform continues to be influenced by two main considerations namely, a call for mathematics **for all** and the need to ensure mathematics **by all.** The first deals with the legacy of the past and considerations of equity, while the second is response to a renewed focus on quality of provision and global economic challenge of participating in a global village.

Mathematics for All

In a country where there has been a neglect of provision for decades, the need for massification of provision remains a major challenge for the future of education in general, and of mathematics in particular. The legacies of gross discrimination of the past meant that blacks were actively discouraged from taking mathematics as a subject. Historically between 30% and 40% of secondary schools in the country simply did not offer any mathematics beyond grade nine. We now have a policy that requires that everyone must take some form of mathematics. "*Mathematics for all*" is fundamentally a statement of policy, and as such it is a statement of provision. Of course it is a statement about curriculum, but essentially it signals that every learner should have the opportunity to learn mathematics.

But mathematics for all does not necessarily mean the same content for all. It is a truism that what content is used must be tied to purpose. It is therefore perfectly reasonable to assume that while all learners need mathematics, not all need the same mathematics. Mathematics for all however, must mean the same **quality** of mathematics for all. Although this seems to be an educationally defensible position, the idea of a differentiated approach to subject offerings at school (including mathematics) was rejected in favour of a single undifferentiated approach to mathematics. This decision should be seen within its historical and political context. During the pre-democratic era and up until 2007, more than 10 years into the new democracy, mathematics, like all other subjects was offered at two levels namely Higher Grade (HG) or Standard Grade (SG). At the dawn of democracy only twenty percent of blacks were taking HG mathematics while seventy percent of whites took mathematics at the same level. A Ministerial Committee on Differentiation (DoE, 2003a) recommended that curriculum reform in South Africa move away from differentiation at subject level.

In order to comply with the new policy that all learners to take some form of mathematics, mathematics literacy was introduced as a high-school subject from grade ten level in 2006 as part of the field of mathematics. Although seen as part of the 'field of Mathematics', it had a very different purpose from that of mathematics. While mathematics is important as a foundation for those with an interest to pursue work and further study in fields that require mathematics (such as business, science and engineering), mathematical literacy is about helping people to participate more fully in the choices that affect their lives. Mathematical literacy may help individuals to engage in discussion with employers over what constitutes fair wages and conditions of service, make sense of even participate in national debates on issues such as health, crime etc., particularly where quantitative arguments are used.

Generally, mathematical literacy was intended to assist learners to take charge of their own experiences as self-managing individuals and critical citizens in a democracy, crucial for nation-building and the strengthening of the new democracy. However, it was never meant to be a dead-end low-level subject that represents a kind of watered-down mathematics in the same way that SG mathematics differed from HG mathematics. In short, the difference between mathematics and mathematical literacy is a difference in kind rather than level or degree. Initially, many more learners opted for mathematical literacy, but in recent years there has been a more even split with 56% of the 617, 982 grade 12 candidates enrolled for mathematics literacy in 2018.

One of the points of departure is that the South African school curriculum is composed of 'learning areas' rather than subject disciplines. Integration within and across learning areas is another important building stone of the curriculum.

In the learning area of mathematics there are five learning outcomes (DoE, 2003b). They are:

1. **Numbers, operations and relationships**: The learner is able to recognise, describe and represent numbers and their relationships and can count, estimate, calculate and check with competence and confidence in solving problems.

2. **Patterns, functions and algebra**: The learner is able to recognise, describe and represent patterns and relationships, and solves problems using algebraic language skills.
3. **Space and shape**: The learner is able to describe and represent characteristics and relationships between 2-D shapes and 3-D objects in a variety of orientations and positions.
4. **Measurement:** the learner is able to use appropriate measuring units, instruments and formulae in a variety of contexts.
5. **Data handling**: The learner is able to collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, as well as interpret and determine chance variation.

As in the case of the other learning areas, the mathematics learning area is based on the principles of high knowledge, high skills and integrates within mathematics and with other learning areas. It infuses concerns of human rights and inclusivity throughout the assessment standards.

There is however always a danger that there would be a lack of fit between the intended curriculum and the actual or implemented curriculum. This danger is of course very great in South Africa where the biggest challenges for implementation are the lack of resources and adequate teacher training, infrastructure and leadership capacity. Teachers implementing C2005 indicated that although they believed it to be beneficial to their learners and were eager to implement it, they were undermined in their efforts to do so in the absence of the necessary support.

Mathematics By All

While mathematics for all is a statement of provision, *mathematics by all* is a statement of participation and a statement of mathematical engagement. If we are concerned only with provision of opportunity and the construction of mathematics curricula, without considering who is engaged in mathematics and how they are engaged, we will be giving ourselves a false sense of comfort. There is very little point in laying a table with the best food without inviting those around the table to participate in the eating and enjoyment that goes with it. There is a recognition that if we are going to effect change in South Africa, we have to accept that both "mathematics for all" and 'mathematics by all' are essential ingredients of a transformation agenda. The focus in education generally has been shifting from provision and access to quality.

At the same time the educational measurement industry both locally and internationally has, with its narrow focus, taken the attention away from the things that matter and has led to a traditional approach of raising the knowledge level. South Africa performs very poorly on the TIMSS study. In the 2015 study South Africa was ranked 38th out of 39 countries at grade 9 level for mathematics and 47th out of 48 countries for grade 5 level numeracy. Also, in the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), South Africa was placed 9th out of the 15 countries participating in Mathematics and Science – and these are countries which spend less on education and are not as wealthy as we are. South Africa has now developed its own Annual National Assessment (ANA) tests for grades 3, 6 and 9. In the ANA of 2011, grade 3 learners scored an average of 35% for literacy and 28% for numeracy while grade 6 learners averaged 28% for literacy and 30% for numeracy.

Although these performances are pertinent in assessing educational quality of mathematics in the country, we have become pre-occupied with the political pressure to 'do better' and to improve our relative standing in relation to other countries using the comparative construct provided by these studies. In this process our focus has been fixed on the 'knowing of mathematics' instead of the 'doing of mathematics'. In our attempt to get teachers and learners to demonstrate knowledge we forget sometimes that teaching and learning are actions and that people rather than knowledge must be at the center. Mathematics by all is about changing the focus away from provision and compliance towards engagement and taking charge of our own mathematical experiences. This is not being reckless about the importance of knowledge but to see the key challenge facing mathematics teachers and learners as that to engage with the subject and to get them to believe that mathematical engagement could be part of their 'possible selves'.

Mathematics by all means that everyone is engaged in a quality mathematical experience. Quality of mathematical teaching and learning depends on whether the teacher can select cognitively demanding tasks and plan the learning experiences by encouraging learners to go beyond the "answer" to seek elaborations and generalisations whenever appropriate to do so through these tasks. This will require learners and teachers alike to commit to extra time on task and be engaged cognitively, socially and mathematically.

Allocating sufficient time for the learners to engage in and spend time on mathematical tasks in an already overcrowded curriculum presents a significant challenge. To address this challenge policy makers are currently in engaged in developing a new *Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding* (DoBE, 2018). It is not intended to be a new curriculum but supports the implementation of the existing CAPS curriculum by introducing a model to help teachers change the way they teach. Taking its bearing from the work of Kilpatrick et al. (2001), the model of teaching mathematics has four dimensions: *conceptual understanding*, *mathematics procedures*, *learners' own strategies* and *reasoning* while each of these takes place in a *dynamic classroom culture*. In addition, the topics in the existing mathematics curriculum will be re-sequenced and even where necessary, removed to make space and time for deeper mathematics engagement.

While it is recognised that one of the major problems in mathematics education in South Africa is the level of teacher knowledge, it is felt that there has been too much emphasis on "teacher blame" when trying to explain the poor level of learner proficiency in mathematics. While teachers with strong content knowledge are more likely benefit from high level interventions and they therefore are more likely to lead their learners into richer mathematical experiences, strength in content knowledge does not always transfer to pedagogical knowledge. However, we need now to go beyond this and ask what we can do within the current reality. To wait until teachers' knowledge has all radically improved would drive us into paralysis. Transformation of the classroom practice must begin with an enabling framework. Teachers' re-socialization into the new mathematics landscape envisaged in the new framework would have to start with unfreezing and deconstructing existing notions of working mathematically. The work of Leone Burton (1999, 2004) and Jo Boaler (1998, 2002) illustrate how important it is for teachers to themselves be immersed in mathematical experiences that will give them an insight into the practice of mathematicians.

In summary, South Africa has a new set of values: *democracy, social justice and equity, equality, non-racism and non-sexism, ubuntu (human dignity), an open society, accountability (responsibility), the rule of law, respect, and reconciliation* are the ten fundamental values of our Constitution. The promotion of these values is seen as important, not only for the sake of personal development, but also for the evolution of a national South African character. These values have been infused in all learning areas and school mathematics in particular is expected to respect these values. The need is to develop a mathematics curriculum that will not only recognise the global competitiveness challenge by providing a platform for developing the knowledge, skills and competences to participate in an economy of the twenty first century, but also to show how our fundamental values can be lived out in our everyday experience while at the same time illuminating and exposing violations of these values. The mathematics curriculum reform in South Africa holds in tension the need to provide mathematics for all on the one hand, while creating opportunities to ensure that mathematics achievement is seen and experienced as part of the 'possible self' of every learner.

Final Conclusions

The four experiences illustrated above show some commonalities as well some crucial differences insofar the same issue is sometimes faced from unlike and even opposite standpoint because of context historical and social differences.

Among the commonalities is the theme of documentation as an agent of reform: however, also this is approached in different ways. For example, the report from Lebanon underlines this as a centralised action that can guide the production of textbooks as a main guide for teachers, who in their turn can so become agents of reform. On a different stream, the report from Australia argues that documentation can assume and create a focus for teachers as active learners about curriculum and pedagogy. Also, the rationale of documentation can be different, as pointed out in curricular dichotomies, which can determine the philosophy of curricular reforms: for example, documentation of everything is possible vs. including just enough information.

A theme which is faced in very different ways is that of inclusion, which may have opposite or at least different results with respect to the waited ones. For example, in Chile this issue was interpreted as the aim of having quality education for all, in compliance with the international standards of the globalised world, and with a wide consultation process in the country, which involved many agents; however, the result consisted in having too rigid rules, which made it difficult to apply them. Another issue concerning the possibility of having an effective inclusion was also linked to socio-economic features of the countries, which can strongly influence the way different types of agents can realize the curricular reforms.

This aspect was underlined in the report from Lebanon a discussing the differences between public and private schools as agents of reforms, and from South Africa, in the crucial difference in the realisation of curricular reforms and in the concrete actions of their agents, namely that between a mathematics for all and a mathematics by all: the former meaning that a curriculum must design the same quality of mathematics for all, while the latter that everyone is engaged in a quality mathematical experience. This last aspect highlights a subtle but important aspect in the way agents can be really effective in the implementation of curricular reforms.

In a sense, this issue is present also in another feature of the complex landscape produced by the variety of curriculum reforms agents, that is in what one of the curricula dichotomies in the Australian report formulates as the difference between mathematics for elite or mathematics for all. This conflict is present in different forms in almost all the contributions to the panel, and assumes interesting connotations in other curricular conflicts, represented in different curricular dichotomies: from the possible differences between the writers of the curriculum, practitioners vs. specialists, to the aims of the mathematics curriculum, as preparation for later study vs. mathematics as experience, to the ways, general versus specific, which describe the concepts or content that form the focus of learning experiences.

In general, what appears as a main question from the different contributions, and is explicitly pointed with reference to the curricular reform in Lebanon, is the way teachers are involved in the processes of curricular reforms. In fact, the success of a curricular reform heavily depends on the way they concretely interpret and apply it in their classrooms, as well as how their students react to it. Theme C expresses this issue as the 'law of alignment' (Chap. 19): the effective implementation of a curricular reform in the classrooms depends on the way its mathematical content and pedagogical assumptions, materially written in the official documents, are effectively interpreted by the 'terminal' chain of the curriculum agents, that is the teachers, and how the interpretation determine/change their beliefs and practices.

All this happens in concrete historical and social contexts, which produce different, possibly opposite effects, to apparently similar actions: this poses delicate and very difficult questions for researchers. In the following chapters of theme E, this point emerges as a relevant stream of discussion and analysis and, as it will pointed out in the final comments to the theme-related chapters, the issue of resilience of a curriculum plan remains as one of the main problems to be solved in the face of possible disruptions to the planning and enactment of curriculum reforms.

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Chapter 27
Modelling Curriculum Reform: A System of Agents, Processes and Objects

Ellen Jameson and Janette Bobis

Defining successful processes and outcomes in curriculum reform can make important contributions to meeting the goals of reform. Although definitions of success might be specific to each instance, a generalised model of curriculum reform can help to identify the factors involved. In this chapter we propose a model of curriculum reform that incorporates a system of agents, processes and objects derived from the foci and content of theme E papers. This chapter starts with a discussion surrounding definitions of curriculum. We then introduce the precedents for proposing the processes and flow of knowledge in the model and support our proposal with reference to and analysis of individual theme E papers. Our purpose is not to validate the model or to comprehensively cover every aspect of all papers but to use evidence from these papers to illustrate the model's applicability and plausibility as a framework for future investigations of agents and processes in curriculum reform. It is envisaged that the validation and elaboration of the model be the agenda of future researchers and readers of this chapter.

Definitions of Curriculum

It is well noted that the term curriculum takes on different meanings according to the context in which the authors are writing (Remillard & Heck, 2014) and too often is used in educational contexts without any explicit clarification (Way et al., 2016).

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The latter was the case for the majority of papers presented as part of theme E. The lack of clarification made it challenging at times to precisely determine what was being referred to, since authors sometimes used the same terms to refer to slightly different aspects of curriculum planning and enactment processes. At other times, authors used different terms to mean almost the same thing. We found that authors introduced curriculum-related terminology by situating their work in their local context of curriculum reform and enactment. For the majority of papers, we needed to infer a definition of curriculum based on the context in which it was used by the author(s).

Remillard and Heck (2014) broadly define the mathematics curriculum “as a plan for the experiences that learners will encounter, as well as the actual experiences they do encounter, that are designed to help them reach specified mathematics objectives” (p. 707). They conceptualised various curricular elements (e.g., policy, assessments, textbooks, student outcomes etc) as existing within a curriculum policy, design and enactment system. Their proposed system of curriculum focuses on two major components – the official curriculum and the operational curriculum. According to Remillard and Heck, the official curriculum specifies what should be taught and the operational curriculum specifies what actually occurs during the enactment process; some of which exists outside the official, sanctioned curriculum. The operational curriculum comprises the enacted curriculum, which includes aspects of curriculum leadership, teacher development in terms of their pedagogy and knowledge, interactions between students and teachers during instruction, the tools and resources used by teachers, and the actual mathematics presented to students.

When used by theme E authors, the term curriculum was generally synonymous with the official curriculum as defined by Remillard and Heck (2014). For instance, the term was used by Quirke (2018) to refer to the official curriculum in his exploration of factors affecting the reshaping of a new curriculum in Ireland. While Montecillo et al. (2018) used it to refer to the detailed specification of content to be taught in a range of southeast Asian countries, Jameson et al. (2018) referred to the “intended” curriculum as relevant to their framework for linking research to mathematics learning at all stages of the curriculum process – from the “intended” to the “enacted” to that which is “received by students” (p. 531).

Meanwhile, Bobis, Downton, Hughes, Livy, McCormick, Russo and Sullivan (2018) used the term “documented curriculum” (p. 499) to refer to the official Australian mathematics curriculum from which they developed planned sequences of learning experiences that would initially challenge students and eventually consolidate their learning. Solis and Scott (2018) explained how the term curriculum in Costa Rica has a dual meaning. The first meaning refers to the official curriculum, ordered according to grade-appropriate content and the second refers to a four-staged curriculum process for its implementation in the classroom. In France, Arnoux (2018) explains that the term ‘syllabus’ is preferred over curriculum, and more specifically refers to “the programme” (p. 491) which incorporates both content and its organisation.

The central message we derived from our review of theme E papers in terms of defining curriculum, was that to provide an international perspective on this topic, we need to be aware that 'curriculum' can mean different things to different people from different (or even the same) context. We do not advocate for consistency of terminology because the variation in contexts may necessitate differences. However, if curriculum research is to progress on an international scale, it is critical that authors clearly present what they mean by curriculum at the outset.

A Model of Curriculum Reform

All authors of theme E papers drew upon one or more theoretical perspectives to inform their research and interpret the processes of curriculum reform at play in their contexts. These perspectives varied enormously and included commognitive theory (Pinto & Cooper, 2018), variation theory (Bobis et al., 2018), socio-political and historical perspectives (Arnoux, 2018; Bonilla & Huamán, 2018), cultural and cognitive perspectives on curriculum coherence (Jameson et al., 2018), narrative (Quirke, 2018) and the theory of didactical situations (Espinoza & Barbé, 2018). The different theoretical perspectives undoubtedly had implications for the variety of approaches researchers adopted in their studies and expositions.

Taken together, the papers reveal the richness and complexity surrounding the design, enactment and outcomes of mathematics curriculum from an international perspective. Despite the differences, there were also similarities in terms of the agents and processes involved in curriculum development and reform. To help identify these differences and similarities across each context, we used Remillard and Heck's (2014) model of curriculum policy, design and enactment system as a starting point to visually represent the curriculum aspects explored in each paper. Two members from theme E collaboratively analysed each paper to identify the agents reported as influencing curriculum and mapped the processes described.

Based on this analysis, we modified specific components of Remillard and Heck's original model to reflect the agents, processes and objects described by theme E authors. With a proto-type of our model, we revisited each paper and mapped the agents, objects and processes described onto it, adjusting model components when necessary to ensure the agents and flow of knowledge reflected in individual papers were represented as closely as possible to that which we perceived the authors intended. The resultant models are presented in Figs. 27.1 (objects) and 27.2 (processes) and briefly explained with respect to agents and contexts. We provide support for various components and connections in the proposed model with reference to individual theme E chapters.

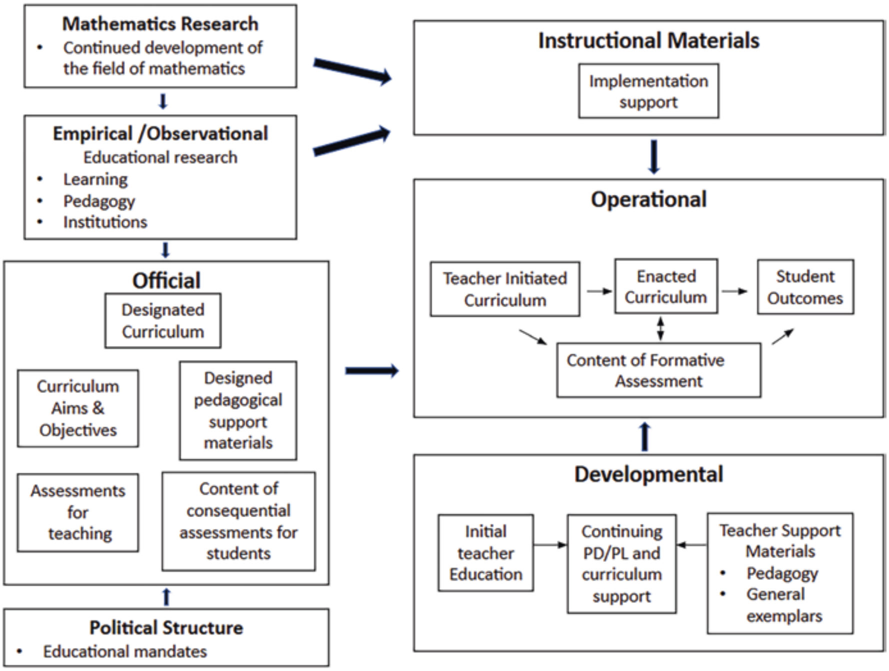

Fig. 27.1 Model of curriculum reform, part 1: objects in curriculum reform arenas. (Adapted from Remillard and Heck (2014) – agents not shown; see Table 27.1)

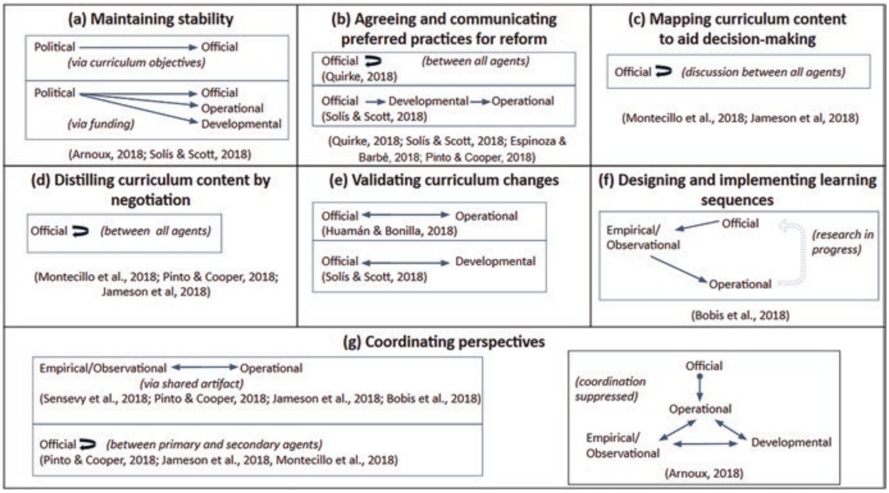

Fig. 27.2 Model of curriculum reform, part 2 – processes in curriculum reform

Agents, Objects and Arenas

In our model, objects (shown in boxes in Fig. 27.1) are things which might be designed, taught or experienced such as documentation, teacher education programs, or the designated, teacher-initiated, and enacted versions of a curriculum. Processes (summarised as arrows in Fig. 27.1 and shown in more detail in Fig. 27.2) are the various ways that curriculum reform agents can act on objects. They determine the overall direction and path of impact on teaching and learning in the Operational arena. Arenas (adapted from McGinnis, 2011) are bounded contexts in which specific agents act in specific ways on specific objects. Each type of agent has an arena in which they primarily act (e.g. teachers in the Operational arena), but they may also cross over as secondary actors in some other arena (e.g. teachers consulted in the development of a curriculum or of instructional materials).

Figure 27.1 shows the seven arenas in our model: of these, official, operational and instructional materials are adapted from Remillard and Heck's original model. We have added the others based on discussions among theme E authors and analysis of the papers in theme E. Table 27.1 lists the primary agents shaping the objects and processes in each arena and the secondary agents involved.

In many cases, the main institution within the Official arena might be the curriculum committee or equivalent body, which creates the designated curriculum,

Table 27.1 The primary and secondary agents shaping the objects and processes in each arena

Arena	Primary agents	Secondary agents
Official	Education administrators Bureaucrats	Curriculum designers Teachers Assessment designers Educational researchers
Operational	Teachers School/regional administrators	Educational researchers Teacher educators
Developmental	Teacher educators	Teachers Educational researchers Curriculum designers Pedagogical advisors
Instructional materials	Publishers and independent authors or developers	Teachers Teacher educators Educational designers (may also be teachers) Educational researchers
Educational research	Educational researchers (social scientists)	Teachers Teacher educators
Mathematics research	Mathematicians	Educational researchers
Political	Politicians Political appointees	Curriculum design committees Constituents inside and outside the education system

including aims and objectives, possibly some form of instructional guidance for teachers, and assessment frameworks for students and sometimes teachers.

Such committees may consist of representatives from several groups, including curriculum designers, teachers, assessment designers, educational researchers, and regional and/or national education administrators. Politicians and bureaucrats may be part of a committee or above it. The administrators structuring the process and the bureaucrats managing accountability for it are primary actors in the official arena. Members of the other groups are occasional actors here but primarily act elsewhere (see Table 27.1). Researchers act between the educational research and instructional materials arenas, teachers act among the operational, instructional materials and research arenas, and so on.

In the operational arena, the primary agents are teachers and school or regional administrators. In this arena, teachers are interpreting (teacher-initiated curriculum) and operationalising (enacted curriculum) the intended curriculum from the official arena as they work with students in classrooms. They gather their own information about students' experiences either informally or through their choice of formative assessment activities. Teacher educators and educational researchers may play secondary roles here when teachers draw on their work, either through direct interaction during student teaching or research participation, or as a result of professional development efforts.

In the developmental arena, teacher educators are the primary agents. They develop curricula, practices and materials for the education and training of new teachers and develop continuing professional development opportunities for existing teachers. Educational researchers may contribute data to inform this process, and curriculum designers may provide additional guidance for teachers. Pedagogical advisors may also provide guidance and training which is specific to the official curriculum.

In the instructional materials arena, publishers commission, produce and sell instructional materials. Independent authors or developers may do the same. Some of the people involved in designing or consulting on these materials may be primarily educational designers, teachers, teacher educators or educational researchers. The objects produced in this arena are in some way aligned to objects in the official arena and are used by teacher in the instructional arena.

In the empirical/observational arena, social scientists do educational research in a variety of fields (e.g. educational psychology, sociology of education, curriculum and instruction, educational philosophy, learning sciences, etc.) on issues relevant for mathematics curriculum design and enactment. Some of this work may be translated into the operational arena directly by teachers who are also researchers, but much of it reaches this arena indirectly via the instructional materials which are designed for teachers to use and teacher education and professional development.

In the research in mathematics arena, professional mathematicians contribute to the further development of fields in mathematics. Some educational researchers, teacher educators and teachers have experience in this arena, and some mathematicians have experience in educational outreach, but there is less direct influence on the curriculum here than in the other arenas.

In the political arena, politicians can appropriate funding for curriculum reform and potentially set high-level agendas or mandates for curriculum reform, which then may create direction or constraints for those acting in the operational arena. This political action may be at a national or regional level, depending on the scale and locus of curriculum reform efforts. It may or may not involve the influence of existing or former curriculum design committees, or other constituencies with agendas for curriculum reform.

Processes

Looking across the papers which shaped our discussion in theme E, it became apparent that we could describe what happens within and between arenas in terms of several processes. We present these processes in Fig. 27.2. They involve unidirectional or bidirectional actions between arenas, or cyclical actions within an arena, shown as arrows in Fig. 27.2.

Maintaining Stability (Fig. 27.2a)

Both Arnoux (2018) and Solís and Scott (2018), when analysing curriculum reform efforts in France and Costa Rica, respectively, identified policy makers as key to the overall stability and continuity of reform efforts. In the Political arena, policy makers set objectives for curriculum reform which provide structure for the approaches taken in the Official arena. Policy makers also determine the level of funding available for reform, which affects several arenas. In the Official arena, this determines how often curriculum committees can meet and how much change they feel can be supported. In the Operational and Developmental arenas, this plays a role in determining what resources schools can purchase and how much professional development teachers can undertake.

Agreeing and Communicating Preferred Practices for Reform (Fig. 27.2b)

Theme E authors analysing curriculum reform efforts in Ireland (Quirke, 2018), Chile (Espinoza & Barbé, 2018) and Costa Rica (Solís & Scott, 2018), as well as research conducted in Israel (Pinto & Cooper, 2018), all described the importance of a process of deciding on preferred practices for conducting and implementing reforms, and for communicating these practices to the actors involved. This can take place within the official arena, where those serving on curriculum committees can negotiate and determine how to carry out their mandate for reform. Past reform efforts and data from current efforts (if available) can inform the practices which curriculum committees adopt. Quirke (2018) gave an example of curriculum documentation in Ireland serving to communicate preferred practices to teachers by providing public narratives which could influence their professional identities, helping them to relate to practices necessary to carry out the desired reforms.

Preferred practices for reform can also be coordinated between the official, developmental and operational arenas. In the case of recent curriculum reform efforts in Costa Rica, Solís and Scott noted that it had been useful to have the same team in charge of both reform and implementation and alliances between the public and private education sectors. They also noted steps taken to provide teacher support which greatly improved communication of preferred practices and are being emulated in further reforms: MOOCS (Massive Open On-line Courses) were created to serve all secondary teachers and most primary teachers, and support for teachers was included within the curriculum documentation itself (Solís & Scott, 2018).

Mapping Curriculum Content to Aid Decision-Making (Fig. 27.2c)

Two of the projects described by theme E authors featured the process of mapping mathematics curriculum content for the purpose of comparison and decision-making in curriculum reform. As part of an effort to develop regional mathematics standards across several countries, the Southeast Asia Ministers of Education Organization (SEAMEO) convened a team of specialists and collaborating teachers through the Regional Education Centre for Science and Mathematics (RECSAM) to examine variation in the countries' mathematics curriculum structure and content (Montecillo et al., 2018). They mapped the countries' existing curricula to each other to find the intersection of content held in common and compared this content in turn to existing international benchmarks. They found this mapping-based comparison to be an important step, but one requiring further collaboration between agents in the Official arena at local and regional levels.

Jameson et al. (2018) present a theoretical case for the use of the Cambridge Mathematics Framework as a reference curriculum framework in mapping processes to contribute to decision-making in curriculum design. They suggest that mapping to a reference framework can contribute to increased domain coherence and system coherence: helping curriculum designers improve the coherent alignment of content within the curriculum while also improving alignment with classroom resources and professional development. Mapping processes have been developed through pilot implementations of this framework and will continue to be refined (Jameson & Horsman, 2019).

Distilling Curriculum Content by Negotiation (Fig. 27.2d)

Three of the papers in theme E mention the importance of the process of negotiation among stakeholders in the Official and Political arenas. Pinto and Cooper (2018) closely examined boundary-crossing between stakeholders engaging in curriculum reform discussions, where incommensurate views on teaching and learning between professions can make collaboration difficult. They observed that the negotiation process was more effective when goals, norms, values, perspectives and key language were made explicit. Negotiation was also more successful when some participants were members of at least two of the communities involved and so were able to act as brokers, facilitating development of shared understanding between them. Based on these interactions, they described processes of reflection, perspective-making and perspective-taking occurring at boundary-encounters which were facilitated by brokers (Pinto & Cooper, 2018).

In the SEAMEO–RECSAM case, negotiation occurred around mappings between curricula in the pursuit of a shared set of standards. This helped to make assumptions about differences and commonalities between curricula explicit and allowed the collaborating groups engaged in creating shared standards to move past those assumptions to nuanced interpretation involving different curriculum contexts (Montecillo et al., 2018). Similarly, the Cambridge Mathematics Framework was presented as a tool designed to make implications from research for conceptual connections in mathematics more explicit to contribute to curriculum content discussions (Jameson et al., 2018).

Validating Curriculum Changes (Fig. 27.2e)

Validation of curriculum reform efforts occurs between the official and operational arenas when it involves assessing and comparing the performance of students or teachers in the classroom. Bonilla and Huamán (2018) described the development of authentic tasks to assess student achievement with respect to newly defined mathematical competencies in Peru; this process was carried out by teams of specialists in mathematics education. On the basis of this work, Bonilla is currently developing a framework for characterising levels of teacher co-determination in curriculum reform.

Inclusion of teachers, teacher educators and professional development designers in curriculum reform efforts can serve as a form of ecological validation of the structure and pacing of reforms as they are implemented, and of concurrent support through professional development. In his discussion of recent curriculum reform efforts in Costa Rica, Solís and Scott (2018) noted that the participation of teacher representatives and NGOs engaged in professional development was instrumental to implementation. The pace of reform in this case was gradual and deliberate, which gave these collaborators to develop and implement more effective support for curriculum changes.

Designing and Implementing Learning sequences (Fig. 27.2f)

Learning sequences or structured activities which are based on an intended curriculum help to translate it into the classroom and, along with teacher implementation, help to shape the enacted curriculum (Remillard & Heck, 2014). Bobis et al. (2018)

suggested that the process of designing learning sequences and implementation guidance should span the official, empirical/observational and operational arenas. They reported on an on-going project featuring the iterative design of learning sequences, incorporating data from implementations and feedback from teachers.

Co-ordinating Perspectives (Fig. 27.2g)

The importance of the process of coordinating perspectives between curriculum reform agents was highlighted in six theme E papers. The authors' examples fell into three categories. Researchers may co-ordinate their perspectives with teachers using shared artefacts to spark and focus of discussion (Bobis et al., 2018; Jameson et al., 2018; Pinto & Cooper, 2018; Sensevy et al., 2018); this spans the empirical/observational and operational arenas. Members of curriculum committees of various roles may co-ordinate perspectives between each other when discussing proposed curriculum changes (Jameson et al., 2018; Montecillo et al., 2018; Pinto & Cooper, 2018), with knowledge brokers potentially playing a special role (Pinto & Cooper). Arnoux (2018) painted a more complex picture of curriculum reform in France, in which coordination was occurring between researchers, teachers and teacher educators, but was suppressed between those agents and government officials. This suppression, between agents in the political arena and primary agents in the official arena on the one hand, and secondary agents in the official arena on the other, was flagged as an obstacle to curriculum reform.

Other Processes

Additional, more granular processes were raised in the theme E discussion at the ICMI Study 24 conference which set the stage for this chapter. While these processes were not the direct focus of the theme E papers, they are integral parts of curriculum reform. These processes are listed below along with the arena(s) in which they take place.

Setting principles which influence curriculum writing (*official*)
Selection of the writing team (*official*)
Writing the reformed curriculum, including decisions about what to include or exclude (*official*)
Setting scope and sequence (*official*, *operational*)
Consultancy, in which draft documents are distributed for feedback to expert groups and educators (*official*)
Refinement of draft documents following feedback from consultancy (*official*)
Gaining the support of teachers and of the public (*official*, *developmental*)
Gaining official endorsements (*official*, *political*)

Trialling of curriculum with teachers/students before full/mandatory implementation (*operational*, *empirical/observational*)
Implementation, either gradual phasing-in or sudden 'punctuated' equilibrium (*operational, sometimes empirical/observational*)
Planning related professional development (*official*, *developmental*)
Providing related professional development (*developmental*)
Preparation of teaching/learning materials (*official*)
Alignment of national documents and teaching (*official*, *developmental*, *operational*)

Summary

Using Remillard and Heck's (2014) model of curriculum expression in the education system as a starting point, we have proposed a model for curriculum reform which reflects the agents, objects, processes and perspectives on the nature of curricula reported by theme E authors. Many of the components of this model were identified in multiple curriculum reform cases, and evidence from these cases suggests that, while this model has not been validated, it could be a reasonable framework for structuring future research. We also hope that this model will help those involved in future curriculum reform efforts to translate evidence from other contexts into their own and give due consideration to factors which have been identified as important across jurisdictions. Finally, theme E participants concluded from the panel presentations described in Chap. 26 that there are gaps between local and global levels of agents and processes in curriculum reform. We hope this can be a step towards a more explicitly multi-level framework which may help to characterise and navigate across these gaps.

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Chapter 28
Boundary Crossing in Curriculum Reform

Alon Pinto and Jason Cooper

Processes of curriculum reform in mathematics involve a broad range of stakeholders, including teachers, politicians, mathematicians, education researchers, parents, industrialists, and possibly others. These stakeholders – representatives of diverse communities – may have very different notions of what it means to learn, to know and to practice mathematics. Many of these stakeholders attain some degree of influence – direct or indirect – on mathematics curricula and reform. For example, ministerial curriculum committees will often comprise some combination of mathematicians, education researchers, teachers, superintendents and other ministry officials.

However, such representation does not guarantee that the communities represented on such committees will endorse their conclusions. A recent international survey among 310 university mathematics instructors from thirty countries has found that more than a third of the respondents (105) explicitly claimed that students are arriving at tertiary education less prepared or less inclined to learn mathematics at a high level because of reforms in school mathematics in their respective countries (Koichu & Pinto, 2019). In fact, 'math wars' are often enacted in criticism of curriculum reforms, which may have the political power to significantly influence mathematics curricula, as in cases of open letters from mathematicians to ministers of education (Klein et al., 1999; Israeli, 2010).

We choose to frame this situation of conflicting views among stakeholders in two complementary theoretical frameworks. First, we draw on the theory of

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commognition (Sfard, 2008) in viewing stakeholders as communities that are distinguishable in their Discourse – patterns in their communication about mathematics and its teaching and learning. While all communities involved may profess similar goals for mathematics education, the keywords that feature in these goals (e.g. mathematical competence) may have different meanings for different stakeholders, possibly related to scores on national or international comparative exams, to productive and responsible contribution to society, or to a smooth transition across various academic or professional paths. Furthermore, different communities may have different ways of evaluating and endorsing narratives about how mathematics is best taught, relying on educational research (education researchers), on a deep understanding of the discipline (mathematicians), or on accumulated teaching experience – be it teaching mathematics at school (teachers) or didactics of mathematics in college (teacher educators). This state of affairs can create commognitive conflict – a situation where incommensurabilities in the discourse of different communities (i.e. using the same keywords with different meanings) lead to conflicting narratives. Often it is the case that such conflicts cannot be resolved to the satisfaction of all parties involved.

The second framework we draw on uses the metaphor of 'boundary' to describe discursive conflict across communities. Akkerman and Bakker (2011) use this term to describe "sociocultural differences leading to discontinuity in action or interaction" (p. 133). Research on 'boundary crossing' has shown how communities can collaborate across a boundary (i.e. in the face of commognitive conflict), without resolving the conflict and without achieving consensus. Such collaboration can be supported by a carefully designed boundary object, which can mediate the work of the different communities involved in the collaboration. The nature of such mediation can vary, from supporting practices that minimise conflictual interaction to encouraging communities to be explicit about their differences in order to learn from and with each other.

Theoretically, a diversity of perspectives on mathematics education could contribute to a well-informed and balanced approach to curriculum design, yet there is reason to believe that productive collaboration among stakeholders in curriculum design and reform is the exception and not the norm. Descriptions of reform processes in the literature, and testimonies of former reform committee members suggest that conflicting perspectives on mathematics education tend to hinder productive reform, and that as a result issues that could instigate tension often end up as the responsibility of homogeneous sub-committees. However, this reality is by no means a necessity.

In this chapter, we present some examples of conflicting perspectives between stakeholders in mathematics curriculum reforms, drawing on the diverse work that took place in theme E of ICMI Study 24. We examine these examples from the theoretical perspective of boundary crossing. On the basis of these examples, and within this particular framing, we propose a model of collaboration, where the diversity in stakeholders' perspectives on mathematics education may act as a resource for achieving a deeper and more comprehensive understanding of the issues at stake. While conflicts between stakeholders may not be resolvable, this model suggests how it may be possible for diverse communities to collaborate without achieving consensus regarding their points of dispute, achieving an appreciation of the rationality of other perspectives, and seeking out hybrid practices that draw on diverse discourses.

Cases of Stakeholders in Interaction

Curricular Reform in France

Arnoux (2018) provides an insider view of reform processes that took place in France over the last 20 years. Focusing on three main stakeholders – politicians, administration of the Ministry of Education, and the educational community – Arnoux draws attention to conflicting stances of these stakeholders regarding the evaluation of curriculum, and to the power relations between stakeholders as a significant force driving and shaping reform. While the professional educational community (researchers and teacher educators) is concerned primarily with the content of curricular reform (mathematical content and the nature of mathematical activity in schools), politicians and ministry administrations, who are held accountable for rankings on international tests such as PISA and TIMSS, do not encourage critical evaluation of educational programs.

An interesting consequence of the power relations in France is the emergence of an independent 'mathematical education community' in which secondary and higher education teachers, associations of mathematicians and teachers of mathematics work together. It would appear that the 'common enemy' in the form of instability created by political powers has brought together stakeholders from communities who in some other contexts do not have a good record of productive collaboration.

Developing a Pan-Asian Curriculum

Montecillo, Teh and Isoda (2018) describe efforts to develop a mathematics curriculum that will be shared by the countries that belong to ASEAN (Association of Southeast Asian Nations). Tensions in this case were not only between the various professional stakeholders (teachers, teacher educators, education researchers, curriculum experts), but also across multiple nationalities, where different traditions and understandings regarding mathematics education have emerged. Eventually, an international perspective, not favouring any of the participating nations, helped overcome tensions.

Curriculum Documentation in Australia

Bobis et al. (2018) demonstrate a curriculum documentation process designed to inform the construction and implementation of innovative teaching materials. In this process, curriculum preparation and implementation are conducted in iterative cycles (design-test-redesign-retest) with ongoing feedback from teachers and student work samples. Accordingly, teachers have more influence on the curriculum design and documentation, which is led by education researchers and teacher educators. This documentation process is reported to have helped in reducing tensions between the designers of the curriculum and the teachers responsible for implementing it.

Teacher Identity as Designated in Curriculum Documents in Ireland

Quirke (2018) draws attention to the ways by which public policy documents in mathematics education frame and promote certain understandings regarding teachers and teaching. Tensions in this case may be found between teachers' self-identities – the way they define themselves to themselves and to others as they account for their practices and their positionings within official and unofficial discourses of teaching – and teachers' current and designated identities as narrated by policymakers in public documents.

Boundary Crossing in Israel

While reform committees in Israel typically comprise representatives of various stakeholders in mathematics education, Pinto and Cooper (2018) provide circumstantial evidence suggesting that interactions between the communities is minimised by carefully dividing responsibilities among them. However, some local initiatives (e.g. a professional development course for primary school teachers run by mathematicians and the 'Math-Ed Crossings club' organised at the Weizmann Institute of Science) have brought together mathematicians, teachers and education researchers in ways that encourage members of different professional communities to interact and learn from and with each other.

Coherence in Curriculum Design in the UK

Jameson et al. (2018) draw attention to the importance of coherence across curriculum design as a way to increase effectiveness of teaching and learning, by coordinating policies, resources, and actions. The Cambridge mathematics framework (CMF) is being developed to help achieve such coherence by helping actors base their curricular decisions on published research in mathematics education. The framework provides a single database of educational resources, educational practices and supporting research findings, yet provides different views for the different stakeholders. The single database can support a coherent approach to curricular design, while the multiplicity of ways to work with the database allows diverse stakeholders to collaborate in joint projects of curricular design without the need to achieve consensus regarding points of conflict.

Agents and Tensions

Based on the cases we have examined, the roles of stakeholders in curriculum reform vary considerably across contexts. Nevertheless, there do appear to be some common patterns. Mathematicians were involved in planning and preparing curriculum documents in five of the six contexts we have examined, with Australia being the only exception. However, mathematicians' involvement in designing curricular activities, appears to have occurred in practice only in France.

Teachers were involved in curriculum planning and in the design of curricular activities in all six cases, but in different roles and to varying degrees.

In France, a selected group of teachers was drafted at the early stages of reform by the '*conseil supérieur des programmes*' to write a first version of the proposed curricular program. In UK, the core writing team of CMF consists of people who have mathematics teaching experience as well as undergraduate degrees in mathematics. In the ASEAN case, teachers, because of their familiarity with their country's curriculum, were in charge of interpreting the different mathematics curricula with their national biases and assumptions. In Australia, teachers were not part of curriculum design teams, but individual teachers were selected to provide feedback on curricular activities in the redesign stage.

In all the reported contexts, both education researchers and teacher educators had a central role at the stages of planning, design and teacher preparation. However, while the professional orientations and expertise prevalent in these two communities are quite distinct, it appears that in most contexts they have similar roles and responsibilities. Hence, the distinction between education researchers and teacher educators tends to be blurred.

In some contexts, Ministries of Education had a role in planning and in design (France, Ireland and Israel). In some countries there are unique communities that are responsible for specific expertise, such as curriculum specialists (ASEAN) or didacticians (France) – a community quite unique to France specialising both in mathematics (often at university level) and in its teaching and learning.

The CMF project's point of departure is the assumption that tensions between the communities are grounded in different priorities and obligations (often framed in terms of teachers' obligation to students, mathematicians' obligation to the discipline and researchers' obligation to theory-building). Yet not all these tensions play out in all contexts. In France and in Australia, the main tensions are between the reform committees and the general body of teachers who need to implement the reform, whereas in the ASEAN project, where the aim is to produce a pan-Asian curriculum, tensions are mainly across educational systems. Quirke's study of public policy documents in Ireland (2018) demonstrates that tension may also arise between policy makers and teachers around the narration of ‘good practice’.

It seems that tensions between stakeholders are often minimised through the division of labour across communities, assigning specific tasks in the planning and the design of curricula to individual communities. In the ASEAN context, for example, the initial draft of a curriculum document, which required an academic comparison of the curricula of different countries, was prepared by curriculum specialists, researchers in mathematics education were responsible for attending to twenty-first-century skills, Ministry of Education officials were required to approve the design documents, and teacher educators then provided assessment tasks as a practical elaboration of the design document.

As a result, much of the interface between communities is mediated by documents. When and how such documents are shared affects the nature of interactions. In the Australian context, curriculum design and documentation – including teacher guidelines and references to research – were led by education researchers and teacher educators, with teachers providing feedback, clearly delineating which communities have more agency over curriculum design. In France, in Australia and in Israel, drafts of design documents were shared with the public, and comments of some communities did influence the final version. Yet at least in one case in Israel, in spite of the invitation to comment on drafts, the community of mathematicians severely criticized the final version of the middle school reform in an open letter to the minister of education, and as a consequence the reform was halted, and eventually significantly revised (Israeli, 2010).

While the sharing of documents in such contexts may help minimise or resolve tensions, it does not support deep or meaningful communication across communities. In an attempt to support such communication, CMF provides a multi-linked database of curriculum ‘components’ – activities, the rationale of their design, and supporting research. The goal of this framework is to support informed discussion and negotiation of the curriculum and its implementation.

Boundary Crossing in Curriculum Reform: Challenges and Opportunities

Research on the challenges and opportunities that cross-community cultural diversity brings to mathematics education is scarce, and in the context of the work of mathematics education reform committees it is virtually non-existent. Yet, Pinto and Cooper (2018) surmised on the basis of circumstantial evidence that cross-community tensions are prevalent in the planning and execution of reform in mathematics education, and are often considered an obstacle that should be avoided, rather than a resource that could be tapped. Common techniques for avoiding cross-community tensions include what we call compartmentalisation – assigning potentially controversial tasks in the planning and the design of curricula to culturally uniform sub-communities, and detoxification – diluting or completely removing elements of reform that trigger tension across communities. Analysis of the cross-community tensions that arose between agents of curricular reform in the six cases corroborates Pinto and Cooper's conjecture.

There are clear advantages for applying various degrees of compartmentalization at different stages of the work of reform committees. Culturally uniform subcommunities generally work more efficiently, the work of reform committees can progress in parallel, and members of different communities can bring forth their diverse expertise where it is most relevant. However, there is also a cost to consider. First, compartmentalisation means reform committees function at best as 'the sum of their parts', drawing on individual fields of expertise separately, rather than drawing on multi-faceted expertise to develop novel ideas and insights regarding issues of mathematics-education policy, which are rarely single-faceted.

Secondly, compartmentalisation may result in fragmented reform documents, in which different aspects of reform may be closely aligned with the (often tacit) goals, norms, values and perspectives of one particular community, and end up rejected by the other communities. Thirdly, compartmentalisation may result in substantial discursive inconsistencies or incommensurabilities throughout policy documents. For example, if different parts of curricular documents, authored by members of different communities, speak of 'student understanding' of mathematics, it seems more than likely that they will not have the same notion of understanding in mind. This is likely to impede the coherence of the reform documents, and to increase the risk of misinterpretation across different communities.

The theory of boundary crossing suggests how cultural diversity and multi-faceted expertise can be leveraged towards achieving productive cross-community collaboration (Akkerman & Bakker, 2011). To his end, several distinct processes are pertinent. Reflection is the process of explicating aspects of one's discourse with respect to the discourse of others, thus coming to learn something new about one's own perspective, while possibly changing it in the process. Hybridization is a process of transforming discourse or practice, drawing on the discourse of two or more communities to create something new and unfamiliar. After analysing two cases of cross-community collaboration, Pinto and Cooper (2018) concluded that reflection and hybridisation processes in these cases, based on incommensurabilities between the mathematical and pedagogical discourses of teachers and mathematicians, supported the emergence of insights that were not available for the different communities on their own.

We recognise in the six cases examined here opportunities for boundary crossing that could support the work of reform committees. The spontaneous emergence of cross-community organisations in France, bringing together teachers, mathematicians and representatives of teacher unions, provides a promising context for boundary-crossing. The ASEAN reform initiative and the curriculum documentation initiative in Australia seem to have involved different communities at different stages. It would be interesting to see what insights emerge from a cross-community evaluation of the outcomes. In Israel, some of the reported cases of boundary-crossing involved a mathematician who currently serves as chair of secondary-school mathematics national committee; it would be interesting to see how his interactions with teachers, with teacher educators and with mathematics education researchers have influenced his approach to learning and teaching mathematics at various curricular levels.

In the case of Ireland, Quirke (2018) describes how current and designated identities of teachers are narrated by policy makers in policy documents to encourage reform by "the endorsement of a new discourse for teachers to account for their practice" (p. 561). However, different communities – mathematicians, teachers, education researchers – are likely to have different, possibly conflicting narratives on teachers and on 'good practice'. Narration that is grounded in a nuanced, hybrid discourse is more likely to be endorsed by the various stakeholders of reform. The CMF, along with its accompanying technological interface, was developed with the express intention of achieving coherence across curriculum design. While it appears to privilege the community of mathematics education researchers, it too can serve as a mediation for cross-community discussion and boundary crossing.

We conclude by noting that capitalising upon cultural diversity is far from straightforward. Research indicates that more often than not, cultural diversity hinders cross-community collaboration rather than enhancing it. However, literature on boundary crossing highlights the role of certain individuals – brokers – as instrumental in facilitating boundary crossing in cross-community collaboration. Broker are typically members of more than one community, or are at least 'conversant' in these communities' discourse. Pinto and Cooper (2018) suggested that education researchers are natural candidates for brokering in reform committees, since their profession requires them to take an explorative approach to the discourses of multiple communities, in an attempt to see the sense in them.

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Chapter 29
Teacher Professional Identity and Curriculum Reform

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In many countries, centralised educational systems introduce mathematics education reforms in a top-down manner (Potari et al., 2018). These reforms often fail to gain traction for a multitude of reasons, not least the failure to appreciate the central role of the teacher in classroom practices and their agency in misinterpreting, subverting, and even disregarding reformed curricula (Remillard, 2005). Teachers through their teaching practices interact with the curriculum in a number of ways. For instance, teachers may act as curriculum-transmitters embracing a fidelity approach by prioritising content transmission; they may act as curriculum-developers implementing an adaptation approach by adjusting the curriculum or they may act as curriculum-makers adopting an enactment approach by designing the curriculum in action drawing on student experience (Shawer et al., 2009; Shawer, 2010). Inherently, these approaches describe distinctive ways that teachers may act and interact with the curriculum incorporating their values, feelings, thinking and beliefs. Therefore, by engaging in these certain kinds of practices, a teacher is negotiating a way of being in that context - their identity (Wenger, 1998). As such, identity and practice mutually shape who a teacher is.

The teacher–curriculum relationship is interwoven with teaching practices and teachers' identities. This relationship is dependent upon the individual teacher and

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curriculum, and is located in a particular context (Remillard, 2005). Remillard explains that by virtue of this relationship, a distinction emerges between the written curriculum and the enacted curriculum. Drawing on the work of Clandinin & Connelly (1998), she argues that teachers are not just conduits or implementers of curriculum, rather they act as agents by working with students to construct the enacted curriculum. She also refers to the work of Ben-Peretz (1990) to indicate that curriculum development occurs in two phases. The first phase concerns the work of curriculum writers where the official curriculum reform is materialised through the creation of curricular plans, guides and resources for teachers. The second phase elicits the work of teachers as they read, evaluate, adapt, alter or translate these curricular materials into events in their mathematics classrooms. Remillard (2005) opts to describe teachers as designers in the second phase of curriculum development owing to the creative and improvisational work of mathematics teaching.

In a top-down approach to curriculum reform, teachers have little or no involvement in phase one of curriculum development. In such cases, teachers may be positioned as receivers of curriculum knowledge (Cochran-Smith & Lytle, 1999). They are expected to implement reforms determined by an external body, without being actively involved in their design or organization (Vähäsantanen & Eteläpelto, 2009). This has been the case in many countries where curricular reforms have adopted performative, managerialist agendas through the tighter coupling of teaching practices to standardised assessment of students (Buchanan, 2015; Day, 2002; Sachs, 2003). These kinds of reforms which consist of the implementation of national curricula, national tests, criteria for assessing the quality of schools and the publication of schools' results in these assessments in the public domain erode teachers' autonomy and challenge teachers' individual and collective professional and personal identities (Day, 2002). This new discourse of accountability has implications for teacher professionalism as it has changed what it means to be a teacher and how teachers understand themselves (Buchanan, 2015; Day, 2002).

Teachers draw on their pre-existing identities which have been continuously formed and reformed over the course of their careers to interpret, learn from, evaluate, and appropriate new mandates for their teaching in their schools and classrooms (Buchanan, 2015). Even in such cases where teachers are excluded from phase one of curriculum development, they do not submissively accept the curriculum reform and its associated goals nor the intended changes it requests in their work and identities (Vähäsantanen & Eteläpelto, 2009). How teachers negotiate their identities in such circumstances is an issue of agency. It is based on the professional actions that teachers take within contexts that mediate that action (Buchanan, 2015). Consequently, teachers' agency is significant in the implementation of mathematics curriculum reform and in the negotiations of teachers' professional identities (Vähäsantanen & Eteläpelto, 2009).

In other approaches to curriculum reform, teachers may be afforded the opportunity to act as co-constructors of the curriculum through increased participation in phase one of curriculum development. Teachers acting as co-constructors during phase one may affect teacher professionalism and the identities that teachers enact in their mathematics classrooms. It is rare, however, for teachers to act as co-constructors during phase one of curriculum development.

This chapter, therefore, seeks to investigate the constructs of 'teacher as co-constructor' and 'teacher as receiver' by examining issues surrounding teacher professional identity, teacher professionalism, reform agendas and classroom practices.

Identity

Teacher professional identity stands at the core of the teaching profession; it provides a framework for teachers to construct their own ideas of 'how to be', 'how to act' and 'how to understand' their work and their place in society (Sachs, 2005). We will now offer a brief elaboration on existing identity theories that led to our perspective on the concept and how this can be applied in the context of mathematics education reform. We will start by addressing the fundamental dualisms which are encountered when theorising about identity.

Essentialism vs. constructionism

Identity theories diverge between essentialist and constructionist paradigms (Benwell & Stokoe, 2006). Essentialist theories view identity as a property of the person - that is, identity is a product of minds, cognition, or socialisation practices (Bamberg et al., 2011; Benwell & Stokoe, 2006). Conversely, constructionist theories view identity itself as a socially constructed category in discursive activities (Bamberg et al., 2011).

Agency vs. structure

Identity theories differ between granting primacy to agency or structure (Benwell & Stokoe, 2006; Penuel & Wertsch, 1995). The 'agency' view posits that people are free to construct their identity as they wish. The 'structure' perspective positions individuals as subjects whose identity construction is constrained by various forces (Benwell & Stokoe, 2006). Penuel and Wertsch (1995) argue that both components should be brought together to form a sociocultural approach to identity formation that overcomes this dualism.

Continuity vs. discontinuity

The final dualism in identity theories which Benwell and Stokoe (2006) address is between people generating a stable identity and yet, simultaneously contending with identity being fluid, fragmentary and contingent on the sociocultural context. This dualism is concerned with the degree of continuity that is required to maintain a unitary identity and the degree of development that is required to change one's identity (Bamberg et al., 2011). To overcome these entrenched dualisms in identity theories, we look towards discourse and identity.

Discourse and Identity

Benwell and Stokoe (2006) argue that discursive approaches to identity, from a broadly socio-constructivist perspective, address and advance identity theorising beyond these dualisms. These authors refer to the work of Butler (1990) who conceptualises identity as a discursive practice and performance that is interpreted by other people. As such, a discursive view theorizes and operationalizes how language and other communicative means in text and context enable the enactment of socially situated and recognizable identities (Bamberg et al., 2011; Gee, 2011a).

Socially Situated and Recognisable Identities

Individuals talk and act as members of various social and cultural groups. Socially situated identities incorporate the particular ways members of various social and cultural groups speak and act (Gee, 2011b). More specifically, socially situated and recognisable identities involve the enactment and recognition of big 'D' Discourses. Gee (2011a) uses the term big 'D' Discourse when referring to "distinctive ways of speaking/listening and/or reading/writing […] coupled with distinctive ways of acting, interacting, valuing, feeling, dressing, thinking, and believing […] coupled with ways of coordinating oneself with […] other people and with various objects, tools and technologies" (p. 177).

Members of each group also draw on a suite of typical stories, or *figured worlds*, to go about the business of communicating, acting, and living as recognisable group members (Gee, 2011b). These figured worlds consist of images, metaphors, and narratives, and are populated with identifiable persons and practiced identities (Holland et al., 1998), which in turn shape members' identities and relative positions and relationships within the group. Through the lens of figured worlds, Holland, Lachicotte, Skinner and Cain (1998) assert that the agency of individuals is manifested through their improvisation in response to the voices of others in group discourse and thus, provides the opportunity for the making of new activities, new worlds and new ways of being.

Identity and Learning

From a narrative discursive perspective, Sfard and Prusak (2005) delineate between current and designated identities. Current identities refer to stories which are about the current state of affairs. Designated identities refer to narratives which are expected to be the case in the future. These stories and narratives can be told by the person themselves and others. The storytellers who hold powerful and authoritative positions are referred to as significant narrators.

Policymakers act as significant narrators for teachers' identifying stories. Through mathematics curriculum reform materials, policymakers circulate discourse on teachers' socially expected identities. These can become a dominant frame for the teachers' own designated identities. At a time of curriculum reform, teachers are expected to bridge the gap between their current identities and the expected identities circulated through curriculum reform discourse. In effect, they must enact different Discourses leading to changes in their socially recognisable identities; however, teachers can use their agency to improvise in different ways to form their responses.

Teacher Agency and Curriculum Reform

Buchanan (2015) describes teacher's agency as the capacity of a teacher to take actions to be the kind of teacher that they want to be. These actions are mediated by the discourse of reform policies and the school's commitments to implementing the reform policies. The teacher's professional identity underpins their response to reform policies and their implementation of these policies in the school context. Buchanan found that in cases where the teacher's professional identity aligned with the school culture, commitments and practices, the teacher exhibited agency by *stepping up* and mentoring other teachers and leading professional development sessions. Buchanan noted that in a school that was committed to the accountability regime of new policies, a teacher who viewed herself as fitting with these commitments engaged in *stepping up* by supporting the implementation of the new policies. This demonstrates that in the reform context, professional identity negotiations are easiest for teachers whose existing professional identity is in alignment to the socially expected identity emerging in the reform discourse (Vähäsantanen & Eteläpelto, 2009).

Buchanan (2015) also found that in cases where a teacher's professional identity does not fit with the school's commitment to reform policies, the teacher can exhibit their agency by *pushing back*. Thus, professional identity negotiations are more difficult for teachers whose existing professional identities are in conflict with the socially expected identity circulated through the curriculum reform discourse (Vähäsantanen & Eteläpelto, 2009). Buchanan (2015) concludes that teachers use their identities to interpret and engage with new social practices, and their agency to find ways to be the kind of teacher that they want to be. In doing so, they improvise to form their own pathway through the curriculum reform process.

The Teacher's Trajectory Through Curriculum Reform

Wenger's (1998) notion of trajectories can be used to understand teachers' improvisations and pathways through curriculum reform. The notion of trajectory indicates that each teacher's pathway is a process of becoming that is a continuous motion connecting past, present and future with a field of other influencing voices.

Vähäsantanen and Eteläpelto (2009) examined the trajectories of teachers through the curriculum reform process and found some distinctive pathways. Teachers may employ an empowerment pathway whereby the teacher experiences continuity and a strengthened sense of professional identity, owing to a positive and approving disposition prior to and during the curriculum reform. Teachers who are critical of reform may adopt a critical but adaptive pathway. This incorporates both continuity and conflict in one's professional identity as one is resistant to change but adjusts to the situation as a matter of deliberate strategy.

Teachers who approach a curriculum with an open mind, and opt to wait to interpret the experiences during the reform before making a decision, are said to follow the open and expectant pathway. Teachers may experience a successful transformation pathway, resulting from initially being resistant to reform before becoming positively disposed towards the change. Teachers who are initially positive towards reform before being disappointed by it, as it does not meet their expectations, are described as following a struggling pathway. Vähäsantanen and Eteläpelto (2009) conclude that these positions and negotiations are ultimately based on the teachers' emotion towards and interpretations of their experiences during reform.

We suggest that these pathways illustrate some trajectories which teachers may take as they are expected to move from their current identity to the expected identity in the context of mathematics curriculum reform. Next, we explore several theories and concepts which are useful for characterizing what takes place along these pathways.

Theories and Concepts Which Help to Frame the Transformation of Teachers' Identities

It is important to recognise the power of social institutions relative to individuals, while at the same time recognising the potential of individuals to change the environments that condition their lives. Understanding the means by which these bidirectional influences occur may have bearing on understanding and improving the role of teachers in mathematics curriculum reform. Activity theory (Roth & Lee, 2007) and the concepts of figured worlds and Discourses can help us to examine the interplay of identity and agency which contributes to the success and expansion of teachers' ability to act productively in reform contexts.

Potari et al. (2018) examined the teaching, research and policy components of the curriculum reform process by characterising them as interacting activity systems (Engeström, 2001), in which peoples' actions are constrained by specific rules and mediated by specific artefacts. They observed that conflict arose when members of the teaching, research and policy communities relied on different artefacts and rules when trying to act sensibly within the system. This conflict was sometimes resolved by brokers, described by Pinto and Cooper in Chap. 28, as participants who belong to more than one of the interacting communities.

The division of labour in an activity system creates different positions for participants to inhabit (Daniels, 2007). These different positions are likened to practiced identities in figured worlds which are taken up, constructed and resisted through continued participation. Figured worlds provide the opportunities to develop new activities from within the larger activity system (Rainio, 2008). Social practices can be reshaped by their participants, so that they can reposition themselves and reshape their identities (Edwards, 2008). Figured worlds provide an elaboration on the subject and agency within activity systems, while activity systems account for the regulation of practices, positions and identities within such social worlds.

A teacher's socially situated identity is then the nexus of figured worlds, position and voice in the configuration of the activity of teaching at specific moments in the history of persons and collectives (Ottesen, 2006). For teachers, artefacts become real to the activity of teaching through their use in the processes of production and meaning making. In the context of curriculum reform new artefacts, such as curriculum materials, do not simply move in and occupy empty slots in ongoing activities; instead, the tools, and the activities in which they are used, are re-constructed and given meaning through the actions of the teachers and other stakeholders (Ottesen, 2006). For example, curriculum reform policies incorporating strict teacher accountability can establish a dominant discourse which positions teachers, through their actions in that particular figured world, in such a way that they must align their practices with those which are defined as legitimate, and thus reshape their professional identities (Buchanan, 2015). For such curricula reforms, the ways in which the supporting curriculum artefacts that the teachers work with are reconstructed seldom incorporates the teachers' voices and consequently, these artefacts may not be used productively or as intended. In contrast, policies which make space for teachers' voices and actions in curriculum reform and implementation may afford teachers greater agency and efficacy, and lead to more effective and aligned implementation.

Mathematics Curriculum Reform Materials

In the context of school mathematics, curriculum reform materials act as mediating artefacts (Brown, 2009). They co-ordinate the community with one another through their material and conceptual form. The possibilities for engagement with these artefacts is determined by the community's participants' practiced identities (Daniels, 2007). The types of engagement teachers experience with these artefacts reflects their socially situated identities and has implications for how they put these artefacts into use. In some contexts, the socially situated identities of teachers results in their engagement with curriculum reform artefacts being that of a receiver. For example, in the context of the Literacy and Mathematics Strategy (LEM) in Chile, once the support that was provided to the teachers, which showed them how to teach in manner befitting the curriculum, was removed, the teachers abandoned the reformed practices. In other cases, for example, as part of the Arithmetic and Comprehension at Elementary School (ACE) curriculum in France, teachers enacted different Discourses and produced alternative socially situated identities through engaging with curriculum reform materials as co-constructors. In ACE, teachers, mathematics lecturers and mathematics education lecturers worked as a team to develop, implement and refine a mathematics curriculum. The resultant ACE curriculum, which is evidence- and research-based has shown to positively impact student learning. What the LEM and ACE curricula reforms have demonstrated is that what curriculum reform materials, such as review documents, curriculum guidelines, teacher guidelines, syllabi, textbooks and other teaching resources, become to teachers is determined by how they are *used* in practice. For LEM, without the opportunities to co-create curriculum reform materials, the teachers failed to sustain the reformed teaching approaches as part of their practice. For ACE, the teachers' role in the co-construction of the materials resulted in sustained implementation, review and refinement. Thus, the production, form and use of curriculum materials requires consideration.

Designated Curriculum and Instructional Materials

Remillard and Heck (2014) demarcate between the designated curriculum and instructional materials. The designated curriculum, which is part of the official curriculum, provides teachers with instructional directions to guide them towards addressing the curricular aims and objectives. Instructional materials, such as textbooks and mathematics tasks, are the resources designed to support and supplement the teacher's instruction. In some cases, the designated curriculum comprises a range of materials to shape the content, pacing, processes and tools of mathematics teaching. In other education systems, the official curriculum is instead communicated through aims, objectives and assessments.

These approaches range in terms of the agency and expected identities they afford teachers. An official curriculum incorporating a designated curriculum and instructional materials provides detailed structures for teachers to operate within. It authors comprehensively the socially expected identities of teachers and negates their sense of agency. Other official curricula with lower degrees of specificity may offer the teacher greater affordances in their instructional practices and designated identities; however, even in these cases, the content and form of consequential assessments may constrain teacher's enacted identities and negate their sense of agency.

Remillard (2005) reports that traditionally the authors of the official curriculum have sought to speak *through* teachers. This top-down perspective views the teacher as an conduit for the curriculum (Kilpatrick, 2009; Remillard, 2005). They are seen as receivers who have no role in co-constructing the curriculum (Cochran-Smith & Lytle, 1999; Kilpatrick, 2009). Instead, their job is to adopt a fidelity approach and enact curriculum transmission strategies, such as using the teachers' guide as a single-source of pedagogic instruction (Shawer, 2010).

According to Remillard (2005), in more recent times, official curriculum writers have explored ways to speak *to* teachers. This perspective follows a curriculum adaptation approach whereby teachers apply the actions of macro strategies through their use of micro strategies to act as curriculum developers in their classroom (Shawer, 2010). Shawer describes that the teacher as developer engages in curriculum planning and experimentation through flexibly drawing on curriculum materials to adapt lessons and tasks.

Choppin et al. (2018) describe another approach that may be adopted to reduce that the distance between the authors of the official curriculum and the teacher. This approach involves the official curriculum writer speaking *with* teachers. In this way, teachers are co-constructors of the curriculum materials in use as they *collaborate* with curriculum developers to act as designers beyond their own classroom. Each of these approaches to developing the curriculum reform materials has implications for teacher professionalism.

Teacher Professionalism

Historically, there has been much scholarly debate regarding whether teaching can be classified as a professional or semi-professional job (Demirkasımoğlu, 2010). Krejsler (2005) explains that the functionalist approach within the sociology of professions characterises full-scale professions based on meeting requisite criteria. This criterion is premised on the development and upkeep of social values, and is largely based on law and medicine. From this perspective, a profession is characterised by the fact that:

> Its knowledge and practice are based on systematised theory; the professional has authority in the sense that she/he knows best about his/her field; the professionals exercise formal as well as informal control over the development of knowledge within their field and over education of future professionals; the profession is guided by an ethical codex that regulates relations between colleagues and with clients; its members understand themselves within a comprehensive professional culture of common norms, symbols, and language. (Hall, 1969, cited in Krejsler, 2005, p. 342)

Based on this definition, teachers are designated as semi or quasi professionals given their restricted professional autonomy as a result of being directed and shaped by administrators to achieve organisational goals (Demirkasımoğlu, 2010). As such, teachers operate within a school framework that is subjected to a largely bureaucratically regulated administration upon which they have little or no influence (Krejsler, 2005).

Sachs (2016) explains that there have been several attempts to classify teacher professionalism. These classifications differ between conceptualising professionalism as an occupational value based on trust, competence, occupational identity, and cooperation; as an ideology premised on occupational dominance and monopoly control of work; and as a discourse of occupational change and managerial value (Evetts, 2011).

Hargreaves (2000) demarcates between teachers being professional and teachers being a professional. For teachers, being professional refers to the quality of their practice, including their conduct, demeanour and standards; this is defined as professionalism. For teachers, being a professional concerns them with how they are seen by others and in particular, the status and regard in which they are held; this is defined as professionalisation. Hargreaves explains that although professionalism and professionalisation are conveyed as complementary projects, this is not always the case in teaching.

Drawing on anglophone culture, Hargreaves distinguishes between four historical ages of teacher professionalism, namely, the pre-professional age, the age of the autonomous professional, the age of the collegial professional, and the post-professional or postmodern professional. Teaching in the pre-professional age was seen as a managerially demanding, but technically simple job. Teachers learn to teach by watching others teach and after they serve their practical apprenticeship, they no longer collaborate and can only improve through trial and error. Effectively, as teachers carry out the directives of others, they are seen as virtually amateurs in this age.

Teachers in the age of the autonomous professional had greater status than those in the pre-professional age. The autonomous age was marked by teachers having the authority to choose the teaching methods they believed were best for their students. This age was not unproblematic as it led to individualism, with teachers being isolated and unable to make lasting changes in their teaching practices. The age of the collegial professional followed, emanating from the heightened complexities of schoolings brought about by the proliferation of teaching methods and curricular reforms.

In response to imposed changes and associated uncertainties, there are increasing efforts to develop strong professional cultures of common purpose. Hargreaves indicates that, since 2000, teacher professionalism may be moving into the new era with contrasting possible outcomes. One potential outcome is the age of the post-professional whereby teachers' professionalism will be eroded and discarded. The de-professionalisation of teaching would return teachers to the pre-professional age.

An alternative outcome is the age of the post-modern professional which would see teachers working collaboratively with colleagues and other communities to augment the idea of the collegial professionalism. For this to occur, teachers must draw on the ages of the autonomous and collegial professional to ensure that they receive competitive salaries, restore public faith in the profession, obtain the requisite time to adequately plan and collaborate with colleagues and, set and meet professional standards of practice. Consequently, the role of the teacher in curriculum reform and the discourse on teacher professionalism circulated through the curriculum reform process significantly impacts who teachers are and how they are regarded by others.

Teacher Professionalism and Teacher Professional Identity

Sachs (2001) argues that democratic and managerialist discourses on teacher professionalism are shaping the professional identities of teachers. Reform initiatives within managerialist discourse promote competition through the allocation of funds based on teachers' and schools' performance on externally defined measures. Emerging from this discourse is the entrepreneurial teacher professional identity which is characterised by being individualistic, competitive and complying with externally set performance indicators of high-quality teaching.

By contrast, democratic professionalism is underpinned by collaboration and co-operation between teachers and other educational stakeholders. The activist teacher professional identity emerging from this discourse is underpinned by equity and social justice. The activist identity is built on co-operative and collaborative action with the effective communication of aims and recognition of each individual's and collectives' expertise in an environment of trust and respect (Sachs, 2000). This approach was evident in the development of the ACE curriculum, with teachers positioned as co-constructors of the curriculum.

By contrast, according to Buchanan (2015), the discourse of accountability circulated in many curricula reforms has positioned teachers as technicians and moved teacher professionalism toward the age of the post-professional. This is an example of where a reform agenda targeting teacher professionalism does not result in heightened teacher professionalisation. Thus, externally determined reforms have the capacity to diminish teachers' ability to raise standards and challenge their professional identities, rather than improve the quality of teaching and learning (Day, 2002).

Summary

For school reform to occur, there must be changes in one's sense of the way things should be done - that is, changes in how the various stakeholders in curriculum reform talk and act as members of their social and cultural groups. Thus, professional Discourses must be altered to implement reform initiatives in schools (Toll, 2001). This is complex as there are competing Discourses of change among and between teachers and policymakers. It is almost unrealistic to call for policymakers and teachers to change their ways of being powerful within their communities; instead, it may be beneficial to develop a meta-Discourse in school settings (Toll, 2001). This meta-Discourse would provide an awareness of the competing

Discourses in educational change. With this awareness, the emphasis would not be on superiority of one's own Discourse, but rather, on the differences between the Discourses.

Curriculum reform in this manner requires intricate work that aims to create a middle ground to afford space for negotiation (Clandinin & Connelly, 1998; Toll, 2001). The purpose of this middle ground would be to move away from reform as a war-zone with buy-ins and buy-outs, and towards negotiation, improvisation, imagination and possibility (Clandinin & Connelly, 1998). For reform to succeed long term, teachers' professionalism and identities must be transformed through sustained, critical dialogue, mutual trust and respect (Day, 2002). Reform, then, would no longer be about urgent problem solving and control determined by external bodies distanced from the classroom, but instead, used as a research instrument premised on a willingness to listen, negotiate and change (Clandinin & Connelly, 1998).

The goal of such an approach would be to make a new figured world of mathematics curriculum reform that produces new Discourses for teachers and policymakers alike, and provides teachers with opportunities to transform their professional and socially situated identities (Clandinin & Connelly, 1998; Holland et al., 1998). This would result in a shift in reform ownership to a middle ground between those situated outside of the school and those working in the classroom (Coburn, 2003). A move from teachers as receivers to teachers as co-constructors.

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Chapter 30
Conclusion Implications for Active Curriculum Reform Work and Future Research

Ellen Jameson, Peter Sullivan, and Ferdinando Arzarello

Each of the previous chapters in this section proposed research tools – methods, questions, theoretical lenses – that might help those engaged in mathematics curriculum reform to learn from other reform efforts through examination of different aspects of the systems and contexts in which they take place. The authors and theme E participants made these explorations because the study of curriculum reform is still in its infancy, and its foundations are in the early stages of being built. The papers should be read individually in order for the reader to make full use of their nuanced implications. However, several key considerations stand out across these chapters that we suggest should shape approaches to future curriculum reform and reform research efforts.

Considering Curriculum Reform Contexts as Systems Can Make Important Choices Stand Out

Curriculum reform is not carried out in the abstract but is always situated in a particular context at the jurisdiction level, and further instantiated within multiple local contexts like schools and training programmes. While these contexts are all unique,

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there are features of the institutions, communities and individuals involved and the interactions between participants at the broad and local levels which may be particularly productive for reformers to pay attention to when deciding what to do in their own contexts, although some specific feature might be more tractable or crucial for success in one context than another.

Therefore, when planning or analysing a reform effort, in accordance with the 'law of diversity' proposed in theme C (Chap. 19) it is important to remember that a systematic look at the forces influencing reform in another context can contribute to designing and carrying out reforms in a new context, even though direct replication would typically not be appropriate or successful. Tools like the model proposed in Chap. 27 can help reform agents to critically examine the dynamics at play in their own contexts and find novel solutions suited to their particular goals and constraints.

The Integration of Teachers Into Reform Efforts Is a Key Factor in the Success of Those Efforts

Teachers translate and transform a reformed curriculum as they and their students enact it in their classrooms. This gives them a critical role in determining how and whether students will experience the curriculum as its designers intended. Moreover, when teachers are part of the design process itself, they help to determine that what the designers intend can make sense to the teachers who must implement it (as in theme C's 'law of alignment', in Chap. 19).

Different curriculum reform contexts might afford teachers a greater or lesser degree of professional agency, investment and confidence in the processes of curriculum design and implementation. This then feeds back into teachers' responses to the reformed curriculum (as in theme C's 'law of two directions' in Chap. 19). Teachers can be positioned in ways which increase their agency in both determining curriculum reforms and implementing them in their local contexts. Theme E authors in Chap. 29 and panelist Osta (in Chap. 26), discussing reform in Lebanon, highlighted that closer teacher proximity to reform processes would give teachers greater familiarity and facility with the underlying structure of their curriculum and its pedagogical assumptions, enabling reform efforts to change not only the designed curriculum but their practices as well.

Enhancing Communication Across Community Boundaries Benefits Reform

Of the features of curriculum reform systems modelled in Chap. 27, theme E authors paid particular attention to the significance of interaction between professional communities, within and across levels, by direct dialogue and mediated through documents and materials. Even as these interactions can be a source of friction or misunderstanding, a focus on improving them can make a substantial improvement in how effectively the design and implementation of reforms can benefit from multiple forms of professional expertise, practice, and engagement.

In Chap. 28, Pinto and Cooper identified processes of cross-community collaboration suggested by the theory of boundary crossing, and noted aspects of the cases of curriculum reform discussed by other participants in theme E to which they might be applied: explicit reflection on individual practice in terms of the wider discourse and hybridisation of the discourses of multiple communities. Past research indicates that these processes might not spontaneously benefit curriculum reform, but that the involvement of key agents who are members of more than one community, brokers, would be important for helping the group to make sense of its multiple discourses.

Currently, most individual teachers are not direct participants in the structuring and design of curriculum reform, and in some cases even representatives of teachers as a group may not be substantively involved. Without a voice in the design aspects of curriculum reform, teachers are left with a choice between adopting or resisting the reforms which are handed down to them. Involving teachers more effectively in the discourse around the design of reforms might widen these options and contribute to the development of teachers' identities as change agents, and subsequently their investment and success in working with the resulting reforms in ways which allow them to be more responsive to their specific circumstances.

Some Challenges to Applied Curriculum Reform Research

There are numerous practical challenges for researchers in this area to overcome, but theme E authors focused on two types of challenges which are even more basic. The ability to develop more informed reform practices depends in part on the ability to target research and observations productively. This in turn depends both on the theoretical frameworks guiding research approaches and on the practicalities of collecting the necessary data.

Data availability is affected by two important factors: the long time-scale of reform and the political sensitivity of reform processes at each level. For example, data from close observation of the inner workings and dynamics of curriculum committees would contribute greatly to strategies for improving communication between professional communities engaged in reform. However, individuals are reluctant to agree to participate in such research out of concern that it might open the process up to forces which could overwhelm their voices and take the dialogue out of participants' control. This is a legitimate concern, and while methods might be established which would help to mitigate it, this also suggests that researchers' relationship-building will be crucial for making progress in this area.

Choices of research questions, research designs and interpretation of results all depend on the development of a research community sharing some common theoretical foundations and employing compatible methodological approaches. While this entire research community is in the early stages of building these foundations, we hope that this section and this volume as a whole will contribute to the process.

Outstanding Questions

Of the initial guiding questions for theme E listed in the Introduction, there were some which did not end up being central to our discussions because for whatever reason they were not central to the papers and experiences we were discussing.

What are the roles, if any, given to the public and how do the agents of curriculum reform manage this?

Although we focused on the roles and communities of professional stakeholders in curriculum reform (teachers, policy makers, researchers, etc.) we did not spend time talking about the role of the public specifically, even though it was sometimes implicit in discussing the pressures on policy makers and teachers.

How are the roles of agents in curriculum reform formed and influenced?

We touched on how these roles might be influenced in the case of teachers and those participating in curriculum discussions with brokers to assist boundary crossing. However, we didn't spend as much time talking about how the roles in our contexts had been formed. This issue did not rise to the surface as being as universal a topic in our particular discussion, not because it is unimportant but because the roles were viewed as being inherited by the reform processes we discussed rather than being formed for them, and in our contexts it was not known directly how the roles formed. The closest we came was discussion of differences in roles between contexts and whether they seemed entrenched. This topic would be very useful to explore in cases where there is greater latitude to form or reshape roles.

What is the influence of research in curriculum design and development?

While we discussed the role of research in our specific contexts, and there was notable variation between contexts, it did not become a primary theme of Part V in this volume. We agreed that research should often have more of a role in curriculum reform than it does due to constraints and lack of consensus in some relevant research areas. Some incorporation of research may be superficial in some contexts as a result, while in others it may be adopted but difficult to implement, while in still others research-inspired solutions may succeed greatly. More comparison across contexts would be warranted. Approaches which streamline the resources which must be devoted to finding, incorporating and discussing research in the design of curriculum reforms would create more opportunities to study the impact of research on mathematics curriculum reform.

Curriculum Reform, Resilience and the Unexpected

Some tools and practices are more useful for responding to unexpected events or circumstances than others – but which? As we have all seen in recent months when responding to COVID-19, and as some jurisdictions have experienced previously due to regional circumstances, disruptions to the planning and enactment of curriculum reform can arise, transforming strategies and placing more weight on reactive solutions than we might prefer they bear. A curriculum might need to be enacted through means not anticipated by its designers, which means that the design itself should ideally be structured in a resilient way.

Resilience is a property that allows a system to experience some degree of shock and disruption while still fulfilling its basic characteristics and functions. How might the design of a curriculum and its designers conception of the systems in which it must operate contribute to resilience? There are a variety of characteristics contributing to resilient systems which could be relevant to curriculum design (having to do with how hard it is to disrupt, how prepared participants are to respond and how well the system can stretch to adapt) but we will highlight in particular the ability of such systems to make available multiple possible means to an end (Kerner & Thomas, 2014).

Some curriculum reform efforts seek to provide this through multiple course pathways, some through teacher professional development, but regardless, many systems recently found themselves stretching further than they had expected they'd need to. In theme E, we discussed specific cases presented by panelists Volmink and Otieza which centred around reforming systems which had been significantly disrupted, but we will now further emphasise the need to develop reform practices which can make curricula more resilient in the face of change.

Acknowledgements The participants in theme E contributed to a highly collaborative and productive discussion based collectively upon our mathematics curriculum reform contexts across the world. The chapters in this section express ideas that we all agreed to bring forward into this volume. Thanks to all theme E participants: Pierre Arnoux, Ferdinando Arzarello, Janette Bobis, Maria del Carmen Bonilla, Jason Cooper, Lorena Espinoza, Teh Kim Hong, Masami Isoda, Ellen Jameson, Iman Osta, Fidel Oteiza, Alon Pinto, Stephen Quirke, Patrick Scott, Gérard Sensevy, Peter Sullivan and John Volmink. Our thanks as well to the reviewers and editors with whose help this section was shaped to complement other sections, and to the organisers of ICMI Study 24 conference for making these discussions and collaborations possible.

Reference

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Part VII
International Perspectives and Influences on School Mathematics Curriculum Reforms

Chapter 31
Introduction

Yoshinori Shimizu and Renuka Vithal

School mathematics curriculum reforms that take place in one country or region of the world sometimes impact well beyond the boundaries of the contexts for which those reforms were intended or enacted. In this section, two such contrasting curriculum reform perspectives are presented in the first two chapters, from two major influences – that of the Organisation for Economic Co-operation and Development (OECD) and the United States of America (USA) – which have had impact on school mathematics reforms not only in their own country or particular region but internationally and have been taken up in many other countries. These two major curriculum reform efforts are reflected upon and insights drawn from a third country perspective in the third chapter, that of Singapore, which has itself been influenced but has also influenced reforms in other contexts in the interconnected world of today.

As discussed in theme D, globalisation and internationalisation and their impacts on mathematics curriculum reforms was one of the major issues to be addressed in this volume. Large-scale international surveys of students' achievements in mathematics, such as TIMSS and PISA, have served as vehicles for curriculum reform. In particular, OECD's PISA has advanced notions of mathematical literacy and mathematical competences that have been interpreted and found expression in curriculum reforms in many countries outside the OECD and participating countries. In Chap. 32, Miho Taguma, who presented a keynote address at the ICMI Study 24 conference and is leading the OECD Learning Compass 2030 framework, shows the significance of the framework for future curriculum reforms. With her colleagues

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Kelly Makowiecki and Florence Gabriel, the implications for mathematics curricula are drawn out.

In Chap. 33, William McCallum, who also presented a keynote presentation at the ICMI Study 24 conference and was involved in the Common Core State Standards in Mathematics (CCSSM) in the USA, from a very different context, presents his perspective on mathematics curriculum reforms by juxtaposing two different stances – the sense-making stance on one hand, and the making-sense one as a complementary stance on the other. Based on his experience of writing the CCSSM, he argues for the importance of co-ordinating these two stances, as well as identifying the difficulties of their co-ordination due to the political aspects in the divisions as illustrated by the 'Math Wars' in the United States. For him, the CCSSM is an existence proof of the possibility of overcoming such political differences.

Following these two chapters, in Chap. 34 Berinderjeet Kaur presents her reaction to these two chapters and adds reflections and insights based on her experience and involvement in the Singapore school mathematics curriculum reforms. These experiences offer an interesting counterpoint, in that they have been influenced by reforms from some contexts and, in turn, have also themselves influenced reforms in other countries due to consistent high achievements in mathematics of their students in international studies, such as TIMSS and PISA.

These three chapters together demonstrate the international nature of school mathematics curriculum reforms that are taking place nowadays, by looking forward through the OECD framework as it charts future curricula perspectives, but equally by looking back and learning through the CCSSM experience, as well as showing how countries are both influenced and, in turn, influence mathematics curriculum reforms. This volume, as an ICMI Study, is a part of how such influences are mediated internationally as school mathematics curricula are designed and implemented in any one country or a region of the world.

Chapter 32
OECD Learning Compass 2030: Implications for Mathematics Curricula

Miho Taguma, Kelly Makowiecki, and Florence Gabriel

We are facing unprecedented challenges – social, economic and environmental – driven by accelerating globalisation and an increasing rate of technological developments. At the same time, those forces are providing us with myriad new opportunities for human advancement. The children entering education in 2021 will be young adults in 2030. Education policy makers must consider how to prepare students for jobs that have not yet been created, to tackle societal challenges that we cannot yet imagine, and to use technologies that have not yet been invented. Education can equip students to thrive in an interconnected world where they understand and appreciate different perspectives and worldviews, interact respectfully with others, and take responsible action toward sustainability and collective well-being.

The future is often unpredictable, but by being attuned to some of the trends now sweeping across the world (OECD, 2019a) we can learn – and help our children learn – to adapt to, thrive in and even shape whatever the future holds. Students need support in developing not only knowledge and skills but also attitudes and values, which can guide them towards ethical and responsible actions. At the same time, they need opportunities to develop their creative ingenuity to help propel humanity towards a bright future.

The opinions expressed and arguments employed herein do not necessarily reflect the official views of the Member countries of the OECD.

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OECD Future of Education and Skills 2030 Project and the OECD Learning Compass 2030

In 2015, the OECD launched the Future of Education and Skills 2030 (E2030) project with the aim of helping countries prepare their education systems for the future. The project looks at the bigger picture – the longer-term challenges facing education – even as policy makers are busy with more immediate policy concerns (OECD, 2018).

The long-term implications have emerged from how economic, societal and environmental changes are making the world more volatile, uncertain, complex and ambiguous (VUCA). For example, one of the biggest social concerns today is the environmental challenge. Brought on by rapidly and profoundly changing societies, it calls for urgent action and adaptation to address the issue of climate change and the depletion of natural resources. The OECD's recent look at major environmental trends includes analysis of climate change with a focus on global emissions of greenhouse gases (GHG), which continue to grow: they have increased by 50% since 1990, and by 35% since 2000, driven by economic growth and fossil energy use (OECD, 2020a). While the growth rate of GHG emissions has been slowing down in OECD countries since 2007 and emissions decreased (Fig. 32.1), overall progress is insufficient. This is one of many factors putting the environment, its natural resources and ecosystems under high pressure.

While younger people tend to have increased consciousness about climate change (Reinhart, 2018), they are also more likely to have misunderstandings related to scientific knowledge and often live lifestyles similar to older generations (Ojala & Lakew, 2017). A change in the course and speed of environmental threats requires a shift in people's mindsets: from consumer to steward of nature, from utilitarianism to sustainability, from predatory behaviour to nurturing, restoring and rebuilding for a better future (OECD, 2020a). Education can play a critical role in making these shifts in people's mindsets.

Another long-term challenge includes demographic changes, such as migration, urbanisation, ageing and global population growth. These changes raise questions on how limited resources can stretch to meet growing social demands. While

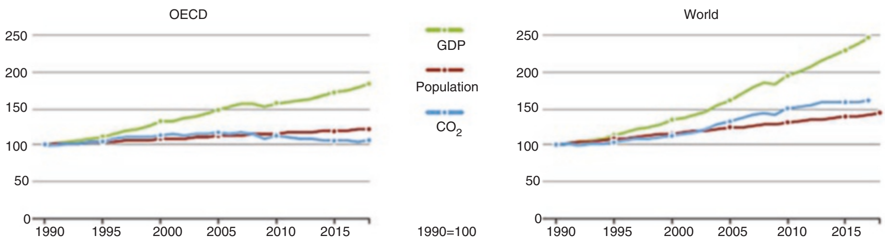

Fig. 32.1 Carbon emissions growth in OECD countries and globally. (*Source*: OECD, 2020a)

science and technology develop at an unprecedented pace, the well-being of societies and individuals in many parts of the globe is threatened by growing inequalities in living standards, access to health care and social inclusion in diverse societies, as well as by violent conflict. For example, income inequality among OECD countries today is at its highest level since the 1980s (OECD, 2015b).

In recent decades, as much as 40% of the population at the lower end of the distribution has benefited little from economic growth in many countries (Fig. 32.2). When such a large group in the population gains so little from economic growth, the social fabric frays and trust in institutions is weakened (OECD, 2015b). Here too, education is key to learning to build and rebuild trust in oneself, others and institutions.

Another example of long-term macro-trends is the dramatic shift in types of skills required for work over the past fifty years, driven by automation with technological developments. Routine manual and cognitive tasks were once the norm, but today's jobs require more non-routine analytic and interpersonal skills (Autor and Price (2013), in Bialik and Fadel (2018, p. 7)). As trends continue to evolve, the set of competencies required for some new and emerging jobs have been described as 'fusion skills' – a combination of creative, entrepreneurial and technical skills – allowing workers to shift into new occupations as they emerge (OECD, 2015a). As a result, education is increasingly expected to foster whole-person development, including such 'fusion skills' as well as 'soft skills', in addition to fundamental literacy, numeracy and other non-routine cognitive skills.

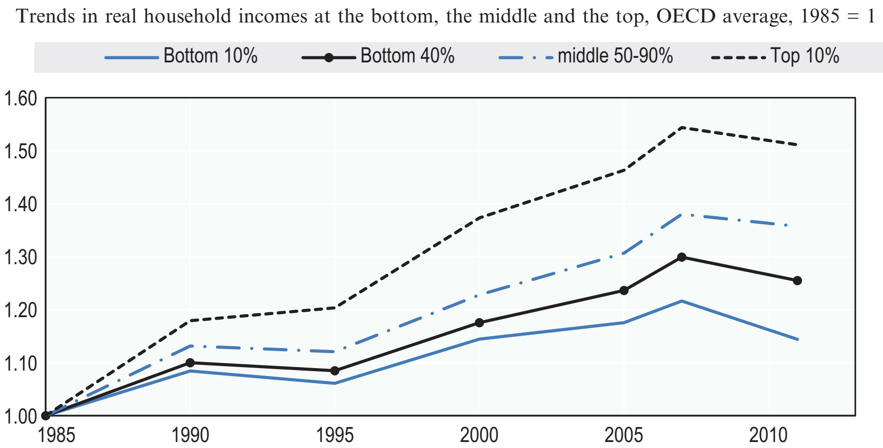

Fig. 32.2 Lower and lowest incomes increasingly left behind (Income refers to disposable household income, corrected for household size. OECD is the unweighted average of 17 countries (Canada, Germany, Denmark, Finland, France, United Kingdom, Greece, Israel, Italy, Japan, Luxembourg, Mexico, Netherlands, Norway, New Zealand, Sweden and United States). *Source*: OECD Income Distribution Database (IDD) (www.oecd.org/social/income-distribution-database.htm))

These rapidly evolving global trends put pressure on schools and school systems to modernise their curricula so that students can develop a broader set of competencies to help them cope with changing realities and new demands. However, education systems can be relatively slow to adapt. For example, whereas education was a driver for technological advances following the industrial revolution, technology has outgrown and outpaced changes in education in the digital revolution (see Goldin & Katz, 2009). When the demands and development of technology exceed the education and skills of children and adults, there can be a gap in productivity and prosperity.

The march of technological advancement has been driven by opportunities to tackle challenges brought on by our rapidly changing world. These trends, which are changing the dominant paradigms for work, life and schooling, are explored further in the next section.

To help keep schools up to pace with these changes, the first phase of the project (2015–2019) focused on 'what' questions: what kinds of competencies – knowledge, skills, attitudes and values – do today's students need to thrive in and shape the future for better lives and for individual and societal well-being? The second phase (2019 and beyond) focuses on 'how' questions: how do we design learning environments that can nurture such competencies, and how do we implement curricula effectively?

Through the E2030 project, policy makers, researchers, school leaders, teachers, students and social partners from around the world worked together with the OECD from 2016 to 2018 to co-develop a vision of education and a learning framework, known as the OECD Learning Compass 2030 (see Fig. 32.3), that sets out the types of competences today's students need to thrive in and shape their future.

Although the project focuses on secondary education as a starting point, it recognises the importance of all levels of formal and informal education, and of life-long learning, and the applicability of project principles to all levels of learning. The framework can thus serve as a common language to build a shared understanding from the local to the global level that every learner, no matter her or his age or background, can develop as a whole person, fulfil his or her potential and participate in shaping a future that improves the well-being of individuals, communities and the planet. Such a shared language can also facilitate comparisons and learning across a wide range of education systems. With a shared learning framework, stakeholders can communicate with each other and learn about and compare best practices. The E2030 project, in other words, stimulates a discussion we need to have now (Schleicher, 2018).

What Is the OECD Learning Compass 2030?

The OECD Learning Compass 2030 sets out an aspirational vision of education in 2030 and provides points of orientation towards the future we want: individual and collective well-being. It aims to articulate core goals and elements of a shared future

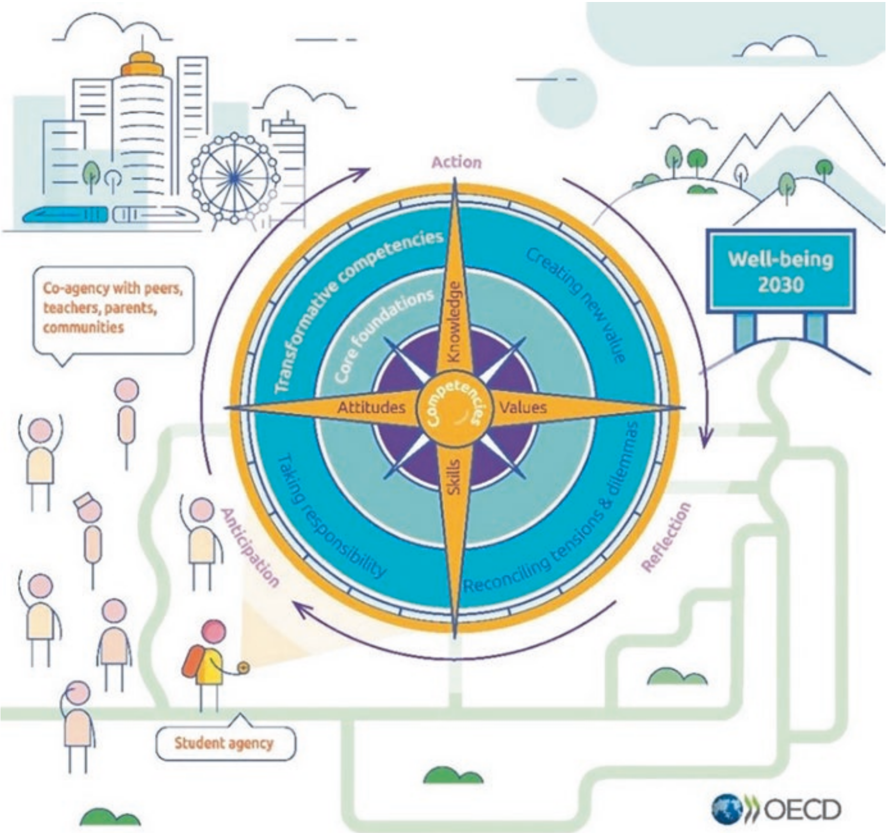

Fig. 32.3 OECD learning compass 2030. (*Source*: www.oecd.org/education/2030-project/teaching-and-learning/learning/)

in a way that can be used at multiple levels (by individual learners, education practitioners, system leaders, policy designers and institutional decision makers) to clarify, connect and guide their efforts (OECD, 2019b).

Just as a compass orients a traveller, the OECD Learning Compass 2030 orients the learner by indicating the knowledge, skills, attitudes and values they need to weather the changes in their environment and to shape their future. The compass is composed of seven elements.

Student agency/co-agency
Student agency is defined as the capacity to set a goal, to reflect and to act responsibly to effect change. It is about acting rather than being acted upon; shaping rather than being shaped; and making responsible decisions and choices rather than accepting those determined by others. In education systems that encourage student

agency, learning involves not only instruction and evaluation but also co-construction. The concept of co-agency recognises that students, teachers, parents and communities work together to help students progress towards their shared goals.

Core foundations
The OECD Learning Compass 2030 defines core foundations as the fundamental conditions and core skills, knowledge, attitudes and values that are prerequisites for further learning across the entire curriculum. The core foundations provide a basis for developing student agency and transformative competencies. All students need this solid grounding in order to fulfil their potential to become responsible contributors to and healthy members of society.

Transformative competencies
To meet the challenges of the twenty-first century, students need to be empowered and feel that they can help shape a world where well-being and sustainability are achievable for themselves, for others and for the planet. The OECD Learning Compass 2030 identifies three "transformative competencies" that students need in order to contribute to and thrive in our world, and shape a better future: creating new value; reconciling tensions and dilemmas; taking responsibility.

Knowledge
As part of the OECD Learning Compass 2030, knowledge includes theoretical concepts and ideas in addition to practical understanding based on the experience of having performed certain tasks. The 2030 project recognises four different types of knowledge: disciplinary, interdisciplinary, epistemic and procedural.

Skills
Skills are the ability and capacity to carry out processes and be able to use one's knowledge in a responsible way to achieve a goal. The OECD Learning Compass 2030 distinguishes three different types of skills: cognitive and metacognitive, social and emotional, and practical and physical.

Attitudes and values
Attitudes and values refer to the principles and beliefs that influence one's choices, judgements, behaviours and actions on the path towards individual, societal and environmental well-being. Strengthening and renewing trust in institutions and among communities require greater efforts to develop core, shared values of citizenship in order to build more inclusive, fair, and sustainable economies and societies.

Anticipation–action–reflection cycle
The Anticipation–Action–Reflection (AAR) cycle is an iterative learning process whereby learners continuously improve their thinking and act intentionally and responsibly. In the anticipation phase, learners become informed by considering how actions taken today might have consequences for the future. In the action phase, learners have the will and capacity to take action towards well-being. In the reflection phase, learners improve their thinking, which leads to better actions towards individual, societal and environmental well-being.

The OECD Learning Compass 2030 sets out a 'learning framework', not an 'assessment framework'. The framework offers a broad vision of the types of competences students need to thrive in 2030, as opposed to what kind of competences should be measured or can be measured. The OECD Learning Compass 2030 recognises the intrinsic value of learning by elaborating a wide range and types of learning within a broad structure. At the same time, assessment initiatives can use the learning framework to help focus discussions on what kinds of learning could be prioritised in particular contexts, for example for the purpose of monitoring and supporting student progress.

The OECD Learning Compass 2030 is not a 'curriculum framework' either. It acknowledges the importance of formal, non-formal and informal learning alongside education that is bounded by formal curricula and instructional strategies. Moving towards 2030, it is increasingly important to recognise the multiple layers and directions of learning in which students participate, including at school, at home and in the communities to which they belong.

As mathematics is one of the E2030 project's subject-specific analyses, a mathematics-specific learning compass – the OECD 2030 Mathematics Learning Framework – has been developed in the same vein as the OECD Learning Compass 2030 and is to be published in 2022 (OECD, forthcoming-a).

How Was This Framework Developed?

The OECD Learning Compass 2030 is an evolving framework in that it will be refined over time by the wider community of interested stakeholders. It is the product of a collaboration among government representatives, academic experts, school leaders, teachers, students and social partners who have a genuine interest in supporting positive change in education systems. These stakeholders come from a wide variety of countries.[1] The framework was designed through iterative, continuous discussions among all stakeholders, and thematic working groups were established for each of the underlying key concepts that comprise the OECD Learning Compass 2030.

[1] OECD Future of Education and Skills 2030 stakeholders come from the following countries and economies: Argentina, Australia, Belgium, Brazil, Canada (the provinces of British Columbia, Ontario, Quebec and Saskatchewan), Chile, China (People's Republic of), Costa Rica, the Czech Republic, Denmark, Estonia, France, Finland, Germany, Greece, Hong Kong (China), Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, Kazakhstan, Korea, Latvia, Lebanon, Lithuania, Luxembourg, Malaysia, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Romania, Russia, Saudi Arabia, Singapore, Slovenia, South Africa, Sweden, Switzerland, Turkey, United Arab Emirates, United Kingdom (England, Northern Ireland, Scotland and Wales), United States and Viet Nam. OECD Future of Education and Skills 2030 stakeholders also come from the following international organisations: Council of Europe, European Union, UNESCO and UNESCO IBE.

The E2030 project began by revising the OECD Definition and Selection of Competencies: Theoretical and Conceptual Foundations (DeSeCo) project (OECD, 2005; Rychen & Salganik, 2003). This latter project was developed by the OECD between 1997 and 2003 with an aim to provide theoretical and conceptual foundations for identifying the competencies needed for a successful life and a well-functioning society. DeSeCo identified three categories of competencies as OECD Key Competencies:

- *Use tools interactively (e.g. language, technology)*
  - The ability to use language, symbols and text interactively;
  - The ability to use knowledge and information interactively;
  - The ability to use technology interactively.
- *Interact in heterogeneous groups*
  - The ability to relate well to others;
  - The ability to co-operate;
  - The ability to manage and resolve conflicts.
- *Act autonomously*
  - The ability to act within the 'big picture';
  - The ability to form and conduct life plans and personal projects;
  - The ability to assert rights, interests, limits and needs.

Building on the DeSeCo framework, the OECD Learning Compass 2030 includes new insights and emerging concepts from thought leaders that may not be fully researched yet, e.g. student agency and co-agency, the interconnected nature of knowledge, skills, attitudes and values, and transformative competencies like creating new value. It aims to increase its relevance to policy makers by linking the framework to curriculum design issues. The framework was constructed, and is understood by stakeholders, as actionable and multi-directional.

Student Agency and Co-agency at the Centre of the Learning Framework

The concept of student agency is central to the OECD Learning Compass 2030, as the compass is a tool that students can use to orient themselves as they exercise their sense of purpose and responsibility. There is no global consensus on the definition of student agency. In the context of the OECD Learning Compass 2030, student agency implies a sense of responsibility as students participate in society and aim to influence people, events and circumstances for the better. Agency requires the ability to frame a guiding purpose and identify actions to achieve a goal (OECD, 2019c).

Student agency is not a personality trait; it is something malleable and learnable. The term “student agency” is often mistakenly used as a synonym for “student autonomy”, “student voice” and “student choice”; but it is much more than these concepts. Acting autonomously does not mean functioning in social isolation, nor does it mean acting solely in self-interest. Similarly, student agency does not mean that students can voice whatever they want or can choose whatever subjects they wish to learn. Students need support from adults in order to exercise their agency and realise their potential.

Competences: Importance of Knowledge, Skills, Attitudes and Values

The E2030 project defines competences as a holistic concept that includes knowledge, skills, attitudes and values. To be ready and competent for 2030, students need to be able to use their competencies to act in coherent and responsible ways that change the future for the better.

Knowledge, a key component of the OECD Learning Compass 2030, encompasses the established facts, concepts, ideas and theories about certain aspects of the world. Knowledge (disciplinary, interdisciplinary, epistemic and procedural) usually includes theoretical concepts and ideas as well as practical understanding based on the experience of having performed certain tasks (OECD, 2019d).

Knowledge and skills are interconnected and mutually reinforcing. Researchers have emphasised the growing importance of being able to understand, interpret and apply knowledge and skills in various situations. As defined by the OECD Learning Compass 2030, skills are the ability and capacity to carry out processes and to be able to use one’s knowledge in a responsible way to achieve a goal. The types of skills include:

- cognitive and meta-cognitive skills, which include critical thinking, creative thinking, learning-to-learn and self-regulation;
- social and emotional skills, which include empathy, self-efficacy, responsibility and collaboration;
- practical and physical skills, which include using new information and communication technology devices. (OECD, 2019e)

In wide acknowledgement that competencies go beyond knowledge and skills, attitudes and values are increasingly integrated into curriculum frameworks. The OECD Learning Compass 2030 defines attitudes and values as the principles and beliefs that influence one’s choices, judgements, behaviours and actions on the path towards individual, societal and environmental well-being. To acknowledge local differences, values are classified into four categories: personal, social, societal and human. Approaches to developing attitudes and values often draw on cultural and societal traditions while addressing global challenges (OECD, 2019f).

How Can the OECD Learning Compass 2030 Influence Mathematics Learning and Teaching?

Underpinning the work of the E2030 project is the idea that education needs to aim to do more than prepare young people for the world of work; it needs to equip students with the knowledge, skills, attitudes and values they need to become active, responsible and engaged citizens. In this broader context, mathematical thinking is increasingly relevant to modern life in terms of the economic, political and social aspects of life. For example, as artificial intelligence and computer simulations are expected to address quantitative questions using large amounts of data, basic mathematics and critical thinking are expected to be required more than ever in daily lives in 2030; mathematical reasoning will be used in more occupations in one form or another; and mathematical thinking will become more important in seizing new opportunities for human advancement.

The modern, digital, tech-focused world is different to the pre-digital age. The global infiltration of computers, automation and other technologies into our professional and personal lives has had – and will continue to have – a deep, broad and lasting impact. Societies today require a fundamental shift from the twentieth-century manufacturing economy to a twenty-first-century digital knowledge and service economy that challenges the dominant paradigms for *work*, *life* and *schooling* (Ananiadou & Claro, 2009; Trilling & Fadel, 2009).

- The changing paradigm for *work* pressures education systems to prepare students for more rapid social, demographic and economic changes, for technologies that have not been created yet and for jobs that do not yet exist. A study estimated that 47% of current occupations were at high risk of becoming automated in the near to medium term (Frey & Osborne, 2017). Many of the jobs most resilient to automation will require workers to have the mathematical and computational competencies to work within highly technological environments.
- The changing paradigm for *life* is marked by technological advancement driven by the promise of opportunities to solve big problems, such as demographic challenges, environmental challenges and growing inequality. Mathematical literacy is crucial to resolving such problems. For example, illustrating environmental issues – population growth, wastefulness, resource scarcity, air and water pollution, and electrical energy demand – requires mathematical competencies, such as knowing how to solve mathematics problems involving basic computations, percentages, ratios, tables, circle charts and graphs (Schwartz, 1985).
- The changing paradigm for *school* gives way to a broadened perspective on mathematical literacy, which emphasises disciplinary knowledge, interdisciplinary knowledge, more contemporary aspects of mathematics (e.g. computer simulations), and twenty-first century competencies needed for civic life (e.g. critical thinking). Thus, a mathematics curriculum that can deliver these competencies is

essential. Mathematics curricula are key to improve mathematics teaching and learning as they are set up to structure students' learning experiences in school education (Schmidt et al., 2013).

The OECD Learning Compass 2030 can help shape the development of mathematics curricula as it identifies the knowledge, skills, attitudes and values students will need for the future. In this context, a mathematics-specific learning compass is to be published in due course (OECD, forthcoming-a). Below are the intermediary outputs from the extensive discussions on how the key concepts of the learning compass be contextualised specific to mathematics curriculum, teaching and learning.

Student Agency

In the OECD Learning Compass 2030, the metaphor of the learning compass is used to emphasise the need for students to learn to navigate by themselves through unfamiliar contexts and find their direction in a meaningful and responsible way, instead of simply receiving fixed instructions or directions from their teachers. This is referred to in the learning framework as the concept of student agency and co-agency, which students can develop with their teachers, peers, family and community. Deep learning over time requires students to be active agents, and they must come to understand that learning is a continuous, ongoing process (Confrey, 2019).

As students become actors of their own learning and are given the tools to become life-long learners, they have the potential to develop positive dispositions towards mathematics. Such dispositions have been shown to have a direct influence on students' mathematics achievement (Newcombe et al., 2009; Gabriel et al., 2018). For example, PISA results show a positive correlation between students' self-efficacy and their mathematical literacy (OECD, 2015c). Taken together, this suggests mathematics learning may be improved by supporting student agency and co-agency and promoting positive attitudes towards mathematics.

Core Foundations and Knowledge

While core foundations in numeracy, data literacy and digital literacy, and disciplinary knowledge (e.g. number systems, geometry and operations) will continue to be important, mathematics curricula should also include greater focus on more contemporary topics, such as statistics, data analysis and computational thinking. Epistemic knowledge, or knowledge about a discipline, such as knowing how to think like a mathematician, statistician or an engineer, for example, will also play a significant role. This will enable students to extend their disciplinary knowledge by studying how practitioners have discovered or invented mathematical concepts and methods.

With globalisation, technological progress and demographic change, the demand for workers with competence in science, technology, engineering and mathematics (STEM) is increasing in many countries (Australian Department of Employment, Skills, Small and Family Business, 2019; EU Skills Panorama, 2014; Yamada, 2017). Therefore, the importance of interdisciplinary knowledge (i.e. the capacity to think across the boundaries of disciplines and make connections between disciplines) is also increasing.

With regard to curriculum design, however, experts participating in the E2030 mathematics analysis have noted that STEM presents both challenges and opportunities, such as:

1. the lack of common language across STEM fields to ensure interdisciplinary dialogues;
2. differing 'thinking processes' (e.g. scientific thinking, mathematical thinking, design thinking) in a STEM activity as a specific thinking process;
3. differing degrees of 'compatibility' (e.g. science and technology seem to integrate better, and mathematics seems to be more of a standalone subject);
4. interdisciplinary STEM knowledge remains a challenge in many schools where discipline knowledge is still considered in silos;
5. the lack of a consistent understanding of what integrated curriculum such as STEM is in lower secondary school remains a challenge. (O'Keefe et al., 2019)

Skills, Attitudes and Values

Students will also need to develop a broad range of skills, including cognitive and meta-cognitive skills; social and emotional skills; and practical and physical skills. Skills that automation cannot easily replace are particularly important. These are skills that let people act and react properly under uncertain, ambiguous and nuanced situations and that can be very powerful when working together with technology. Many of these fall under the umbrella term of twenty-first-century skills, for example: problem solving, critical thinking, creativity, communication, self-efficacy and learning to learn.

The OECD Learning Compass 2030 also emphasises the importance of students' attitudes and values. The capacity to combine and apply knowledge with skills, attitudes and values in unfamiliar circumstances is uniquely human. Take, for example, artificial intelligence (AI): its unprecedented range of applications can only be maximised through the creativity and imagination of the users and designers of AI. When Luckin and Issroff (2018) identify a number of things that people should know and be able to do with AI, they mention a combination of knowledge (basic

AI concepts, digital literacy, data literacy, online safety protocols), skills (basic AI programming, AI systems building), attitudes and values (ethics of AI). A mathematics curriculum for 2030 must have the space to promote knowledge in addition to skills, attitudes and values.

Transformative Competences

Transformative competences include creating new value, taking responsibility, and reconciling tensions and dilemmas. They can be used across a wide range of contexts and situations – and they are uniquely human. All three are higher-level competencies that help learners navigate across a range of different situations and experiences. In that sense, they are highly transferable: these competencies can be used throughout a lifetime (OECD, 2019g). Each one is relevant to developing mathematics literacy and can be taught in schools by incorporating them into existing curricula and pedagogy, for example, by embedding them into mathematics using an inter-disciplinary approach.

Anticipation–Action–Reflection (AAR) Cycle

The AAR cycle component of the OECD Learning Compass 2030 is defined as an iterative learning process whereby learners continuously improve their thinking and act intentionally and responsibly, moving towards long-term goals that contribute to collective well-being (OECD, 2019h). Through planning, experience and reflection, learners deepen their understanding and broaden their perspective. Each step of the AAR cycle is critical to developing mathematics literacy, and each step informs, complements and strengthens the others. For example, if action is taken without anticipation, the learner is not taking into account the possible consequences of the action. And while skills such critical thinking and decision making are developed through reflection, they are also skills that are required for effective anticipation.

How Can the OECD Education 2030 Project Influence Mathematics Curriculum?

The stakeholders of the E2030 project identified six major challenges linked to curriculum redesign commonly found in their respective countries.

Curriculum Overload[2]

Curriculum overload has become a widely discussed policy issue. Societal, technological and economic changes have placed pressure on school systems to adapt their curriculum by including various competencies (e.g. digital and data literacies, financial literacy, coding and programming, and health literacy). However, teaching time over the last decade has not changed much, creating tensions and competing demands for students to stretch themselves too thinly and not having time for deeper learning; for teachers to embed these competencies within limited instruction time; and for policy makers to resist accommodating all these demands by adding more hours to curriculum (OECD, 2020b).

More hours of instruction do not necessarily lead to effective learning and higher academic achievement. Results from the PISA 2012 study show that simply increasing the number of hours students spend in mathematics lessons will not automatically improve their performance (Fig. 32.4; OECD, 2015d).

Curriculum overload affects the quality of the intended curriculum in terms of rigour, coherence and focus. These three terms gained prominence in the field of mathematics education following publication of the first results of the Trends in International Mathematics and Science Study (TIMSS) Curriculum Analysis in the

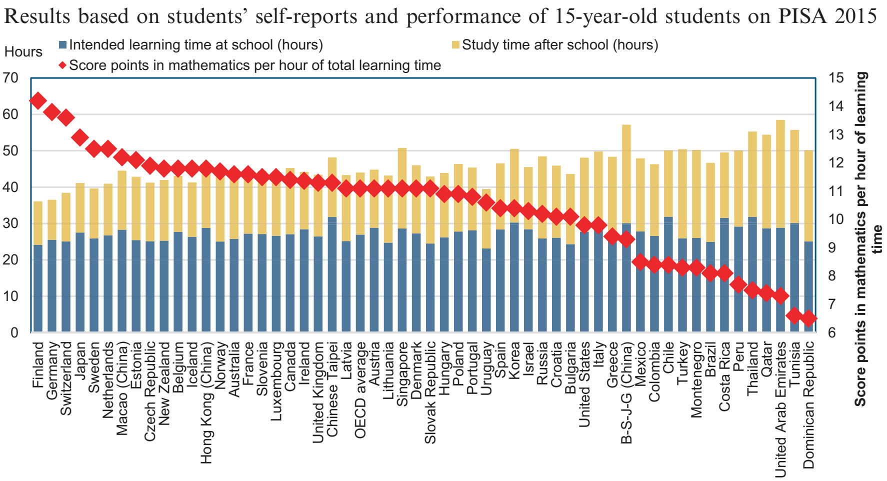

Fig. 32.4 Learning time and the ratio between learning time and score points in mathematics per hour of total learning time, PISA 2015 (Countries and economies are ranked in descending order of the score points in mathematics per hour of total learning time. *Source*: OECD, PISA 2015 Database, Tables I.2.3, I.4.3, I.5.3, II.6.32 and II.6.41)

[2] See www.oecd-ilibrary.org/education/curriculum-overload_3081ceca-en

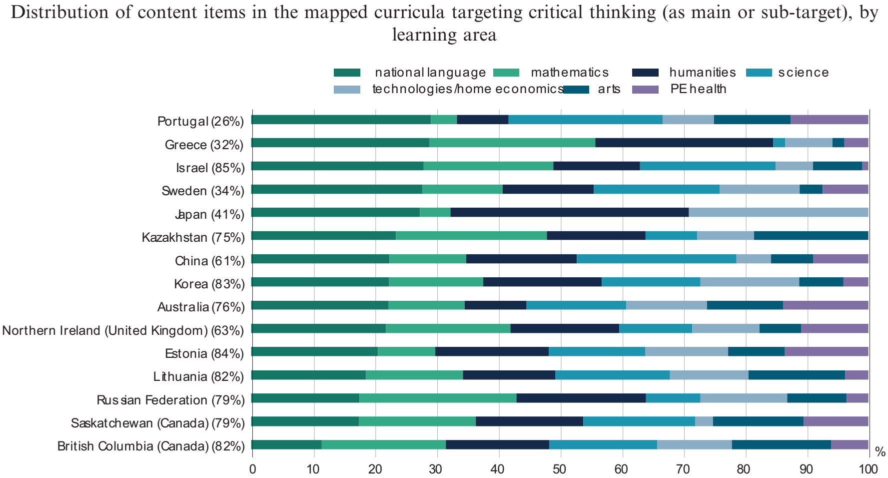

Fig. 32.5 Critical thinking in curricula (The percentage next to the name of the country/jurisdiction refers to the total percentage of the mapped curriculum that embeds the competency. (*Source*: Data from the Education 2030 Curriculum Content Mapping exercise (OECD, 2020c). StatLink 2 https://doi.org/10.1787/888934195511)

late 1990s. Rigour refers to educators pursuing conceptual understanding, procedural skill and fluency, and application with equal intensity. Coherence refers to progressions of learning from one grade to another that help students build knowledge and understanding within and across grades. Focus refers to the fact that students will learn fewer concepts, and focusing on them in greater depth and detail.

To address the challenges of curriculum overload, countries and schools are called to rethink what to change on the scope and structure, what to prioritise/remove among topics without compromising rigour, how to manage change process, etc. They are making changes such as regulating the quantity and ensuring the quality of learning time; translating emerging societal needs into connecting topics/themes or developing competencies across learning areas; focusing on conceptual understanding or 'big ideas' to avoid an excessive number of subjects and/or topics per subject – often described as 'mile-wide, inch-deep"'; carefully defining the pitch of what is included in curriculum; building in coherent learning progressions across grades; and managing perceptions by adjusting the size and/or format of curriculum documents (OECD, 2020b). For example, in Australia, general competences and cross-curriculum priorities are embedded within learning objectives, and general capabilities such as critical thinking, creative thinking, and ethical understanding are defined in mathematics and other subject areas.

The E2030 project conducted a curriculum content mapping exercise to gain insight on how OECD Learning Compass 2030 competences and concepts are embedded in curricula. Figure 32.5 depicts how critical thinking, in particular, is taught across numerous subjects, including mathematics, in 15 countries/jurisdictions.

An increasing number of countries/jurisdictions have made a clear distinction in curriculum between 'key concepts' and 'facts and procedural knowledge' to facilitate deeper learning. As such, the concept of 'big ideas (similar to 'key concepts') commonly appears in curricula as a way to highlight essential ideas that, approached from different angles, are crucial to multiple learning areas. The simplicity of indicating clearly what are the 'big ideas' in a learning area can help teachers remain focused when deciding what to prioritise from the more exhaustive curriculum without being overly prescriptive at the level of content items (OECD, 2020b).

Additionally, in mathematics, some countries/jurisdictions are shifting away from disconnected factual knowledge towards more holistic conceptual understanding to make mathematical learning more meaningful to students. For example, word problems have long been used to convey real-world situations, which help students understand how mathematical concepts can be used outside of school (OECD, 2014), e.g. problems on purchasing furniture with a discount and determining someone's age based on a relationship to the age of others.

Emerging twenty-first-century challenges are also reinforcing the need to foster a deeper conceptual understanding of mathematical content as opposed to rote learning. Addressing these challenges requires equipping students to think mathematically (OECD, 2014). In already crowded curricula, it is difficult to make sufficient room for increased mathematical reasoning and developing students' ability to apply problems in the real world.

The E2030 project launched the Mathematics Curriculum Document Analysis (MCDA) in 2018 to take a close look at mathematics curricula in over twenty countries and investigate the extent to which they are equipping students with the necessary mathematical skills for the twenty-first century. MCDA results (Schmidt et al., 2022), provide insight on how individual countries compare to contemporary international benchmarks. The results will help inform ongoing reform efforts towards a twenty-first-century vision of mathematics curriculum.

Managing the Time Lag Between Future Needs and Current Curriculum[3]

Time lag in curriculum occurs when the curriculum content that children are learning in school today lags behind what they will be expected to know and do with that knowledge, and how they will engage in the world, when they grow up. Globalisation and rapid changes in technology are accelerating social, economic, and environmental challenges worldwide. Many of these changes are also opportunities for human advancement, but citizens must be equipped to handle them via a high

[3] See www.oecd-ilibrary.org/education/what-students-learn-matters_d86d4d9a-en

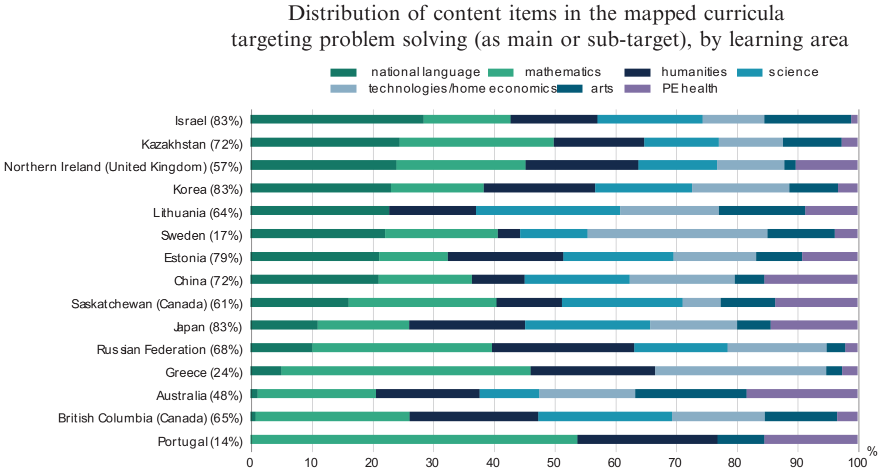

Fig. 32.6 Problem solving in curricula (The percentage next to the name of the country/jurisdiction refers to the total percentage of the mapped curriculum that embeds the competency. *Source*: Data from the Education 2030 Curriculum Content Mapping exercise (OECD, 2020c). StatLink 2 https://doi.org/10.1787/888934195549)

quality and appropriately designed education. Current predictions around novel industries due to changes in technology and demands from a changing environment will certainly shift the skills required by future graduates. Curriculum can be refined and improved to prepare students for a world of challenges and opportunities (OECD, 2020c).

There are four types of time lag: (1) recognition; (2) decision-making; (3) implementation; (4) impact. Most delays arise in the implementation phase due to a lack of stakeholder buy-in, insufficient teacher preparation or teacher capacity to implement reforms, and variations in the pace of change across regions, localities or schools in decentralised education systems (OECD, 2020c). Articulating a shared vision for the kinds of students needed for the future can help address time lag in curriculum by creating a common language for desired outcomes and setting the stage for movement towards a shared goal.

The E2030 project's Curriculum Content Mapping exercise sheds light on strategies to minimise time lags. Analyses suggest that there are common twenty-first-century competencies that countries tend to embrace in their curricula to close time lags, for example, problem solving. Figure 32.6 demonstrates the prominent place problem-solving skills have across the curricula of fifteen countries/jurisdictions, emphasising the need to prepare students to enter an increasingly complex and volatile world. The inclusion of them in curriculum may also suggest ways to mitigate time lags.

Professionals working in sectors using mathematics (e.g. health, marketing, data science, finance) highlight a need for mathematics curricula to promote better coding skills, better understanding of probability, enhanced analytical skills, critical thinking skills and problem-solving skills. These trends are reflected in the PISA 2021 assessment framework through the inclusion of mathematical reasoning (i.e. the ability to reason logically and critically in mathematics) and computational thinking.

Ensuring that curriculum remains responsive to societal needs is an ongoing challenge for countries/jurisdictions around the world. The Ministry of Education in Singapore, for example, commissions and produces reports on global trends and future demands, developments in the global and local economy and international syllabuses. This process guides the decisions that will be made by curriculum designers and policy-makers when reviewing their mathematics curriculum.

Equity Through Curriculum Innovations[4]

The types of curriculum innovation that may promote equity include personalised curriculum, digital curriculum, cross-curricular or competency-based curriculum, and flexible curriculum. While there is a risk that curriculum design can lead to or compound inequities, there is also much potential for curriculum to help increase fairness, justice and inclusion for all students. Research on individual differences, particularly on disparities in learning and access related to students with special education needs and students of lower socio-economic backgrounds, suggests that curriculum design approaches can be leveraged to respond to the needs of diverse students (OECD, 2021a).

Equity in education can be seen through two dimensions: fairness and inclusion (Field et al., 2007). Equity as inclusion means ensuring that all students reach at least a basic level of skills. Equity as fairness implies that personal or socio-economic and cultural circumstances should not be obstacles to educational success.

Using innovative teaching strategies is important when teaching mathematics to students who have different abilities, motivation and interests. There are many innovative cognitive-activation teaching strategies that have been shown to be strongly associated with student success in mathematics. These strategies are designed to give students the opportunity to think deeply about problems, discuss methods and mistakes with their peers, and reflect on their own learning (Echazarra et al., 2016). However, this benefit only appears to hold true in schools with high socio-economic

[4] See www.oecd-ilibrary.org/education/adapting-curriculum-to-bridge-equity-gaps_6b49e118-en

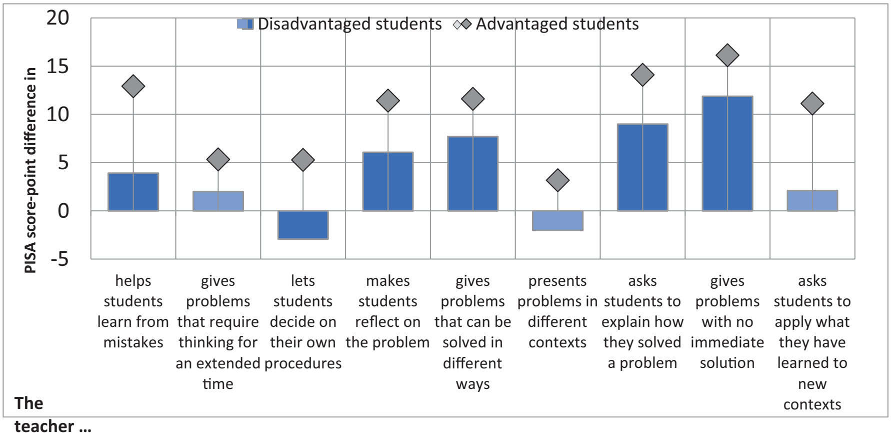

Fig. 32.7 Mathematics teachers' teaching strategies and student performance in mathematics, by socio-economic status (Disadvantaged (advantaged) schools are those whose mean PISA index of economic, social and cultural status is statistically lower (higher) than the mean index across all schools in the country/economy. *Source*: OECD, PISA 2012 database.)

status, where students are more exposed to these teaching strategies than students in disadvantaged schools (see Fig. 32.7; OECD, 2016). If the reasons behind this divergence are not resolved, the introduction of these kinds of pedagogical tools risks entrenching already existing inequity.

Successful strategies have been deployed to overcome equity issues. For example, equity and equality have been leading values of the Finnish education system and they are embedded in many ways in their National Core Curriculum. The idea of "school for all" means that every child has access to high quality, free education regardless of their background. Denmark put in place comprehensive strategies that include creating an inclusive learning environment varied teaching approaches, a focus on individual student well-being at school. Every student has a learning plan with individual learning goals and the use of teaching differentiation allows teachers to ensure that each student has the opportunity to reach their own learning goals in different ways and at a different pace.

Curriculum Flexibility and Autonomy

Curriculum flexibility is conceptualised as adaptability and accessibility of the curriculum for schools and teachers to respond to students' needs and capabilities, and it assumes autonomy of schools and teachers with regard to the curriculum or parts of it (Saarivirta & Kumpulainen, 2016; Newton & da Costa, 2016). Finding the right balance between curriculum flexibility and school autonomy can be difficult to achieve in practice (Ko et al., 2016; OECD, forthcoming-b). Giving teachers more autonomy can empower them to set their own priorities, but on the other hand, too much autonomy might overburden them, in particular when they lack opportunities for further professional development. Recommended approaches to tackle this issue include the development of a common, comprehensive and cohesive curriculum framework with room for school-based variety, and adequate continuous professional development for teachers.

Research shows that teachers who feel supported and autonomous deliver more adaptive and more adequate education for their students, who increasingly have different cultural background and individualised educational needs (Paradis et al., 2018). School autonomy over learning is important with regards to mathematics curricula as it has been linked with higher levels of mathematical literacy in high school students. This is supported by results from PISA 2012 showing a positive correlation between school autonomy and students' performance in mathematics (see Fig. 32.8).

Embedding Values in Curriculum[5]

The inclusion of values in the curriculum is not a new issue, but there is a greater need for teaching values as our world is becoming more complex and pluralistic. In the OECD Learning Compass 2030, values play an important role and serve as the ethical basis for the development of key competencies. Evidence from research also highlights the importance of values in the context of mathematics education. Schukajlow's (2017) study with year 9 and 10 German students showed a positive correlation between students' values and their mathematics performance. The links between values and mathematics performance is significant as it affects the cognitive processes and affective states of the students, which in turn will influence the quality of their learning (Seah, 2018). At the policy level, more and more countries are embedding values in their curricula. For example, the curriculum in Singapore highlights that competencies should be learned alongside core values, and these values are expected to be embedded in every subject, including mathematics.

Including values in curriculum requires a clear decision-making process to identify and select shared values that support the overall mission and goals of the curriculum. This obviously raises questions about which values – and whose values – to

[5] See www.oecd-ilibrary.org/education/embedding-values-and-attitudes-in-curriculum_aee2adcd-en

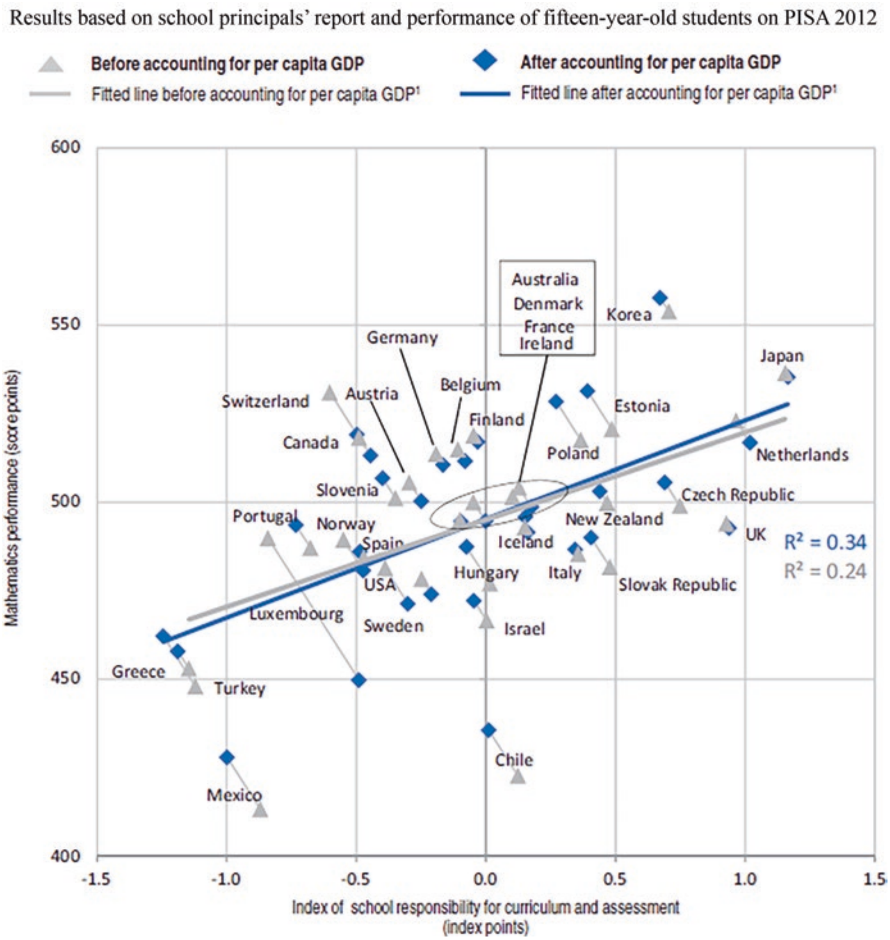

Fig. 32.8 School autonomy over curriculum and assessment and mathematics performance across OECD countries, PISA 2012 (A significant relationship ($p < 0.10$) is shown by the solid line. *Source*: OECD, PISA 2012 Database, Data Figure IV.1.15)

include and or exclude from curriculum and how to balance such these choices in the context of multicultural societies with evolving value systems (OECD, 2021b). Curricula should reflect the complexity represented in the community to make a positive impact with a diverse student population (Gecan, 1993; Cline & Necochea, 1996), and schools should be able to choose from those core values that are more pertinent to the mission, goals and objectives of their educational system (Cline & Necochea, 1996).

Ecosystem Approach to Curriculum Redesign and Implementation

Decades of research on the efficacy of curricular reforms has found that implementation dictates outcomes (McLaughlin, 1990). Moreover, assessments, particularly high-stakes assessments, have an impact on what is taught and, ultimately, on what and how students learn. Curriculum implementation is a complex process at the intersection of multiple policy dimensions, a range of people and diversity of places – ideally linked in an eco-systemic way, building on a co-agency approach (OECD, forthcoming-c).

The key factors influencing curriculum implementation can be divided into three levels (Fullan, 2008, 2016): local (characteristics of the schools, principals, teachers and communities); external (government and other agencies); and intrinsic (the characteristics of the reform itself). Examples of local factors include inadequate support for end-users (e.g. lack of funding for local practitioners to effectively implement the new curriculum), a lack of coherence between the intended curriculum and teacher training, and a lack of appropriate continuous professional development for teachers. The major external factor is insufficient cooperation between various actors in curriculum change, in particular in terms of alignment of assessment to curriculum content. Intrinsic factors include unclear and overly complex goals and a lack of internal consistency.

An example of a positive strategy to address the challenge of variable implementation of curriculum was established in British Columbia, Canada. Instead of solely looking to the Ministry of Education for advice, teachers from British Columbia have been encouraged to connect with their peers in professional mathematics associations, empowering them to take leadership in their communities and ensure a more coherent implementation of their mathematics curriculum.

Implications for Action

The OECD Learning Compass 2030 is a globally informed, locally contextualised learning framework and can be used as a tool to guide curriculum reform, including the development of mathematics curricula. The E2030 project also looks into curriculum design, and the project stakeholders have identified the main issues experienced in curriculum reform and ways of addressing the issues, providing a rich source of ideas and examples for countries to consider and learn from when preparing for their own reform.

While mathematics-specific rigorous analysis is being conducted in the E2030 project, two areas of implications for action in designing a mathematics curriculum can be already set out: (1) embracing diversity in individual students' learning trajectories while recognising commonalities in the science of mathematics teaching;

and (2) twelve design principles that have been consolidated as those that can endure across time, across countries, and across different subjects by the E2030 participating country delegates, school leaders, teachers and students.

Learning Trajectories in Mathematics

All students learn at different rates. When there is a discrepancy between a student's current state of understanding and the expectations set out for them in their mathematics curriculum, this creates a dilemma for teachers; teachers need to address the learning needs of all of their students while maintaining school year-level expectations. One way of resolving this dilemma is to include paths of learning and learning trajectories in curriculum documents (Confrey, 2019; Groff, 2017).

Learning trajectories[6] can be defined as a representation of the expected paths a student may follow as they gain successively more sophisticated ways of thinking about an idea, concept or topic (Simon, 1995; Sztajn et al., 2012; Groff, 2017). If a student is considered to be 'lagging' behind the standards set out in the curriculum, there is the risk of the system losing track of them and that they fall behind even further. A way to mitigate this may be to assess both a student's level compared to the curriculum and also their rate of progress along their learning trajectories (Clements et al., 2011; Confrey, 2019). This practice may help support equity in education by making sure the needs of individual students are met.

Learning trajectories are a valuable resource for ongoing teacher self-assessment and dynamic feedback for students (Heritage, 2008; Confrey, 2019). Teachers can use learning trajectories to assess and update their decisions regarding the use of curriculum materials, instructional approaches and assessment practices. For students, learning trajectories can provide a big picture view of what they need to learn. This can serve as a baseline for formative assessment, and help teachers formulate dynamic feedback about the gap between their students' current and desired performance and provide suggestions on how to improve (Heritage, 2008).

There is a degree of hierarchy and structure in the field of mathematics, which means that it matters how topics are arranged and ordered within a curriculum (Schmidt, 2017; Nasuno, 2017). Curriculum experts and learning scientists should therefore work hand in hand. Experts in mathematics and mathematics education would develop curriculum documents that provide organisational guidance about

[6] Note that learning trajectories is the term preferred in mathematics education research, whereas learning progression is more commonly used in science education research.

what to teach and when to teach it, while learning scientists in mathematics would develop evidence-based learning trajectories. Confrey (2019) argues that these are two important educational tools that should support each other.

Twelve Design Principles in Curriculum Redesign

Based on a number of curriculum reform experiences and research findings across the OECD countries/jurisdictions and partner countries, twelve design principles have been consolidated that can endure across time, across countries and across disciplines. Curriculum designers, teachers, academics and teacher educators in the field of mathematics can use these either as a checklist or as a list of food for thought.

Design Principles

Design principles within a discipline

1. **Focus.** Focus refers to the introduction of a relatively small number of topics in each grade in order to ensure depth and quality of students' learning. For example, many countries/jurisdictions foster focus in the curriculum by incorporating cross-curricular or interdisciplinary themes. Thus, instead of including additional courses or subjects, important themes and concepts are taught across the curriculum.
2. **Rigor.** A rigorous curriculum should include topics that are challenging and enable deep thinking and reflection. Regardless of historical presence, influential voices, tradition and bias, curriculum content should be justified for the evidence-based contribution it makes to the development of students, ensuring high and relevant standards, with appropriate breadth and depth of topics. A rigorous curriculum incorporates content that develops and strengthens students' capacity to utilise knowledge and to apply skills in new and different contexts.
3. **Coherence.** Coherence in the context of curriculum design refers to the extent to which there is a meaningful sequential structure of topics that reflect the logic of the academic discipline(s) on which they draw, from which the relationships between the different elements of curriculum become clear. A coherent curriculum enables progression from basic to

(continued)

more advanced concepts, is pitched at developmentally appropriate levels (grade and age), and supports teachers to respond to learners' needs where student learning progress is framed by broader purposes.

Design principles across disciplines

4. **Transferability.** In curriculum design, transferability entails structuring curriculum to allow students to understand fundamental concepts or big ideas that underpin a particular discipline and see how they apply across different disciplines. A transferable curriculum should also recognise how students can develop skills, attitudes and values in particular disciplinary contexts, while also applying them across different disciplines and contexts.
5. **Interdisciplinarity.** A curriculum that favours interdisciplinarity and interrelatedness should provide students with opportunities to discover how a topic or concept can link and connect to other topics or concepts within and across disciplines and further into their life outside of school.
6. **Choice.** A curriculum built in line with the principle of choice should offer a wide range of topics, project options and opportunities for students to suggest their own topics and projects of interest, with support to help them make well-informed choices, especially for disadvantaged students. Such a curriculum allows for flexibility in terms of opening up subject areas to new topics, new resources, innovative and alternative approaches to planning, teaching and assessing, and enabling teachers to engage their students in meaningful and relevant learning experiences.

Design principles beyond school

7. **Authenticity.** An authentic curriculum should provide space and links to the real world where appropriate. It is a measure of quality of the extent to which the content is current, relevant and applicable to contemporary times. Therefore, it requires interdisciplinary and collaborative experiences outside school, alongside a mastery of discipline-based knowledge in school. When curriculum content is authentic, it engages students in learning experiences that involve exploration of real and relevant issues that speak to them, their environment and their needs. Such a curriculum explores how subject matter relates to students' future lives and work options, as well as enabling them to access topics and undertake project tasks that have a clear purpose, thus equipping them for further lifelong learning.

(continued)

8. **Flexibility.** A flexible curriculum grants schools and teachers the possibility to update, adapt and align the curriculum to reflect evolving societal issues, as well as individual learning needs. Such a curriculum is dynamic and responsive to different and changing circumstances and allows for the incorporation of new content and priorities. This helps the curriculum to be currently relevant and future-focused at the same time. A flexible curriculum also allows teachers to make decisions on when to spend more or less time on subject areas, adding more or less context when needed, in line with local priorities and individual student needs.
9. **Alignment.** When thinking about the principle of alignment, there are various dimensions within and across curriculum to take into account. First, pedagogies, and assessment practices should be well aligned with the curriculum. Second, initial teacher education and professional development should be aligned with the curriculum. Third, in order to ensure continuity of lifelong learning, it is crucial to ensure alignment and conceptual coherence between curricula across different levels of education. While the technologies to assess many of the desired outcomes may not yet exist, new teaching and assessment methods should be developed that value holistic student outcomes, including both learning and well-being.

Design principles for processes

10. **Engagement.** Strong engagement from teachers, students and other relevant stakeholders is of critical importance in the development phase of the curriculum, to ensure their ownership and buy-in during the implementation phase. Engagement is essential if students are to fully immerse themselves in learning experiences, develop positive attitudes towards learning and better understand themselves as learners. It is also crucial in order to receive buy-in from stakeholders and avoid time lag at the recognition, decision-making and implementation phases, as well as to make teachers feel at ease with the changes by engaging them from the onset of the reform process
11. **Student Agency.** A curriculum that grants students agency offers them a carefully designed space to participate in the curriculum design and implementation processes to ensure the relevance of the curriculum for learners. By motivating students and building on their prior knowledge, skills, attitudes and values, such a curriculum ensures that they feel a sense of ownership of their own learning. When students are empowered and granted agency, they are able to influence and determine what, when and how they are learning, meaningfully equipping them for their future.

(continued)

12. **Teacher Agency**. Teacher agency refers to empowerment granted to teachers to use their professional knowledge, skills and expertise to co-design and deliver the curriculum effectively. Granting teacher agency in the development and delivery of curricula has emerged as an important design principle in relation to the issues of overload, flexibility and autonomy, as well as effective implementation.[7]

For further information about project research, findings and outputs, visit: www.oecd.org/education/2030-project/. The OECD will publish a mathematics curriculum analysis report, including the Mathematics Learning Framework 2030, in 2023 (OECD, forthcoming-a). Results from the Mathematics Curriculum Document Analysis (MCDA) study have been published in OECD Education Working Paper No. 286 (Schmidt et al., 2022), available at www.oecd-ilibrary.org/education/when-practice-meets-policy-in-mathematics-education_07d0eb7d-en.

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[7] *Source:* OECD (2020d), *Curriculum (re)design: A series of thematic reports from the OECD Education 2030 project – Overview brochure.* (www.oecd.org/education/2030-project/contact/brochure-thematic-reports-on-curriculum-redesign.pdf).

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Chapter 33
Making Sense of Mathematics and Making Mathematics Make Sense

William McCallum

From 1989, the year of the publication by the National Council of Teachers of Mathematics in the United States (NCTM) of the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), until 2010, the year of the publication of the Common Core State Standards in Mathematics (CCSSM) (see McCallum, 2015), a debate about mathematics education raged in the United States. The debate, known as the math wars, was between a reform movement inspired by the NCTM standards, and a reaction to that movement which brought together under one umbrella a number of different groups, including parents puzzled by changes in their children's curriculum, traditionalists worried about a decreased emphasis on basic skills, and mathematicians concerned about the correctness of school mathematics. The debate was political and not always informed by much evidence (Schoenfeld, 2004).

Neither camp was as homogeneous as their apparent unity at the height of the wars might suggest, as became apparent with the publication of the CCSSM, which drew support from both sides (and, to a lesser extent, opposition from both sides). My own career as a mathematician has spanned this period, from my involvement in the early 1990s in reforming the undergraduate calculus curriculum as a member of the Harvard Calculus Consortium to my role as one of the lead writers of the CCSSM. During that period, I have often wondered whether, underneath the irrationality and politics, there was a coherent duality which at least in part explained the difference between the two sides, and which also shows how they can come together, as they did with the writing of the CCSSM. This paper is an attempt to describe such a duality.

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Views of Mathematics

> I [want to] emphasize the practices, because from my point of view that's where the content lives.
> (Alan Schoenfeld, 3rd April, 2013)

> at first, I thought no, that's wrong, the practices live in the content standards, and then I realized we were both saying the same thing, namely that having this separate free-floating set of practices that are independent of the content is a bad idea. (William McCallum, 4th April, 2013)

The exchange above is from a meeting that Alan Schoenfeld and I attended at the Mathematical Sciences Research Institute in Berkeley, CA in 2013 (MSRI, 2013). The content and practices referred to are the Content Standards and Practice Standards in the Common Core State Standards in Mathematics (CCSSM "Common Core State Standards in Mathematics (CCSSM)", 2010, a collaborative effort of the fifty US states to write common standards. I will return to a discussion of this document later in this paper, but first I would like to use the exchange to lay out a dichotomy in views of mathematics.

Schoenfeld described a spectrum of views of mathematics:

> At one end of the spectrum, mathematical knowledge is seen as a body of facts and procedures dealing with quantities, magnitudes, and forms, and relationships among them; knowing mathematics is seen as having "mastered" these facts and procedures. At the other end of the spectrum, mathematics is conceptualized as the "science of patterns," an (almost) empirical discipline closely akin to the sciences in its emphasis on pattern-seeking on the basis of empirical evidence. (1992, p. 335)

A casual internet search on "mathematics as facts and procedures" does not find anybody advocating it as a complete definition, but finds many saying that mathematics is more than that. It is true, however, that this view of mathematics seems embedded in the culture of US classrooms. Stigler and Hiebert wrote:

> In the United States, […] the level *is* less advanced and requires much less mathematical reasoning than in the other two countries [Germany and Japan]. Teachers present definitions of terms and demonstrate procedures for solving specific problems. Students are then asked to memorize the definitions and practice the procedures. (1999, p. 27; *italics in original*)

Despite efforts to reform this state of affairs going back to the 1989 NCTM standards, this culture remains prevalent today.

Schoenfeld associates one end of his spectrum, the view of mathematics as facts and procedures, with what he calls the content perspective:

> A consequence of this perspective is that instruction has traditionally focused on the content aspect of knowledge. Traditionally one defines what students ought to know in terms of chunks of subject matter, and characterizes what a student knows in terms of the amount of content that has been "mastered." […] From this perspective, "learning mathematics" is defined as mastering, in some coherent order, the set of facts and procedures that comprise the body of mathematics. The route to learning consists of delineating the desired subject matter content as clearly as possible, carving it into bite-sized pieces, and providing explicit instruction and practice on each of those pieces so that students master them. (1992, p. 342)

Note there are really two perspectives here, one on what mathematics is, and another on how it is learned. One could in principle hold the first and not the second. In contrast to the content perspective, and by preference, Schoenfeld proposed the process perspective. In writing about *Everybody Counts*, a report of the National Research Council (NRC, 1989), he claimed:

> there is a major shift from the traditional focus on the content aspect of mathematics [...] to the process aspects of mathematics—to what Everybody Counts calls doing mathematics. Indeed, content is mentioned only in passing, while modes of thought are specifically highlighted in the first page of the section. (1992, p. 343)

The process perspective has taken various forms over the years: the NCTM (2000) process standards, the focus on problem-solving as a core activity in reform curricula, and the practice standards of CCSSM "Common Core State Standards in Mathematics (CCSSM)" (2010). Again, one might hold a content perspective on what mathematics is and a process perspective on how it is learned; for example, problem-solving could be a way of learning facts and procedures. Schoenfeld (1992) described his own version of the process perspective as a view of mathematics as pattern-seeking.

The last sentence in the second quotation above captures a danger of the process perspective: “content is mentioned only in passing”. The danger is that mathematics content is a backdrop to the action, a backdrop that can be inaccurate or forgotten.[1] For example, curricula written from the process perspective might be organized around large projects that pull different mathematical tools in at different times. Without careful planning there is the danger that mathematical dependencies get mixed up. Some curricula are organized around “big ideas,” lists of overarching themes that recur throughout the curriculum. This can work well if done judiciously; but some ideas in mathematics are not well-described as “big”: rather they are small but consequential. Completing the square is an example of such an idea (see McCallum, 2018).

Approaches from the process perspective—mathematics as pattern seeking, mathematics as problem-solving, big ideas—have in common what I call the sense-making stance. In this stance, mathematics is a source of material for important processes such as problem solving and communication. It is an important stance, but it carries risks. If mathematics is about sense-making, the stuff being made sense of can be viewed as some sort of inert material lying around in the mathematical universe. Even when it is structured into “big ideas” between which connections are made, the whole thing can have the skeleton of a jellyfish.

I would like to propose a complementary stance, which carries its own benefits and risks.

[1] To be clear, that is not what Schoenfeld is advocating; indeed, at the same conference mentioned above he explicitly said that he intends neither to ignore nor downplay mathematics.

The Making-Sense Stance

Where the sense-making stance sees a process of people making-sense of mathematics (or not), the making sense stance sees mathematics making sense to people (or not). These are not mutually exclusive stances; rather they are dual stances jointly observing the same thing. The making-sense stance is related to the content perspective described by Schoenfeld, without the unappetizing “carving it [content] into bite-sized pieces” (p. 342). It views content as something to be actively structured in such a way that it makes sense.

That structuring is constrained by the logic of mathematics. But logic by itself does not tell you how to make mathematics make sense, for various reasons. First, because time is one-dimensional, and sense-making happens over time, structuring mathematics to make sense involves arranging mathematical ideas into a coherent mathematical progression, and that can usually be done in more than one way. Second, there are genuine disagreements about the definition of key ideas in school mathematics (ratios, for example), and so there are different choices of internally consistent systems of definition. Third, attending to logical structure alone can lead to overly formal and elaborate structuring of mathematical ideas. Just as it is a risk of the sense-making stance that the mathematics gets ignored, it is a risk of the making-sense stance that the sense-maker gets ignored.

Student struggle is the nexus of debate between the two stances. It is possible for those who exclusively take the sense-making stance to confuse productive struggle with struggle resulting from an underlying illogical or contradictory presentation of ideas, the consequence of inattention to the making-sense stance. And it possible for those who exclusively take the making-sense stance to think that struggle can be avoided by ever clearer and ever more elaborate presentations of ideas.

The work entailed in the making-sense stance is mathematical work, so it is not surprising that much of the work of mathematicians in mathematics education falls under this heading. Wu (2015) has written about “textbook school mathematics” as a degraded subject that is not faithful to mathematics as it is understood by mathematicians. Howe and Epp (2008) have written about the mathematical ideas behind place value. Baldridge (2018) has constructed a vast edifice of grade-level-appropriate, internally consistent definitions of ideas that arise in school mathematics.

An important strand of research in mathematics education is composed of work where the two stances are taken simultaneously, often by pairs of mathematicians and education researchers. For example, Ball and Bass argue that:

> Making mathematics reasonable is more than individual sense making […] making-sense refers to making mathematical ideas sensible, or perceptible, and allows for understanding based only on personal conviction. Reasoning, as we use it, comprises a set of practices and norms that are collective, not merely individual or idiosyncratic, and rooted in the discipline. (2003, p. 29)

Another example is the work of Izsák and Beckmann (2019), who propose a unified definition of multiplication that applies to the many situations modeled by multiplication. In their definition, a product is measured simultaneously by a base unit and by a group, which is itself measured by base units. Their work provides a nice example of co-ordinating the making-sense stance with the sense-making stance. On the one hand, their work is an attempt to make the diverse array of multiplication situations make sense through a unified definition. On the other, it recognizes the role of the sense-maker, the person who must make the choice of base unit and group in order to make sense of a multiplication situation.

> We think that mathematics education as a field should seek more completely worked out coherent approaches to the [multiplicative conceptual field] based on consistency and logical interconnection. The absence of such articulation may be constraining our capacity to help students and teachers use prior knowledge and experience to effectively relate topics and construct interconnected bodies of knowledge. It is one thing to know that multiplication can be used to model a variety of situations and another to perceive a common underlying structure. (Iszák & Beckmann, 2019).

Coherence

Coherence is the *sine qua non* of the making-sense stance. Schmidt et al. (2005) talk about coherence of standards:

> We define content standards […] to be coherent if they are articulated over time as a sequence of topics and performances consistent with the logical and, if appropriate, hierarchical nature of the disciplinary content from which the subject-matter derives. […] This implies that, for a set of content standards to 'to be coherent', they must evolve from particulars […] to deeper structures. (p. 528)

This definition was elaborated by Cuoco and McCallum (2017) to include coherence of curriculum and coherence of practice. Izsák and Beckmann (2019) argue for a coherent view of multiplication in mathematics education research. Attempts to bring coherence to school topics also underlie the work of mathematicians mentioned above.

Coherence was a guiding principle in the writing of the Common Core State Standards in Mathematics (CCSSM) (McCallum, 2015) in 2009–2010. An important precursor was the report in 2008 of the National Mathematics Advisory Panel, which laid out the following principles:

> A focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics, should become the norm in elementary and middle school mathematics curricula. (NMAP, p. xvi)

By the term 'coherent', the Panel means that the curriculum is marked by effective, logical progressions from earlier, less sophisticated topics into later, more sophisticated ones.

Standards have an inherent tendency to interfere with focus and coherence, in that they attempt to reduce a subject to a list, Schoenfeld's "bite-sized pieces". The pieces can lose connection with each other, breaking coherence, and there is a danger that everybody's favorite pieces get added to the list, breaking focus. Maintaining focus in CCSSM was a matter of resisting temptation. Maintaining coherence was a matter of building structures that transcended the bulleted list. See Daro et al. (2012), Zimba (2014) and McCallum (2015) for more detail on the process.

One important way of maintaining coherence was to build the standards on progressions: narrative descriptions of how the mathematical ideas in a particular domain evolve over a sequence of grades (CCSSWT, 2018). These were the first documents produced in the writing of the standards. For example, there was a progression for Number and Operations in Base Ten (NBT) in grades K–5, which told the story of that domain over the grades. Different progressions were tied together by cross-domain connections. For example, it makes sense that the place in the NBT progression where students learn about multiplication should come in the same grade where the geometry progression talks about area of rectangles. These connections tied the different stories together into a coherent whole.

A particularly knotty area in mathematics curriculum is the progression from fractions to ratios to proportional relationships. Part of the problem is the result of a confusion in everyday usage, at least in the English language. In common language, the fraction, the quotient $a \div b$, and the ratio a: b, seem to be different manifestations of a single fused notion. Here, for example are the mathematical definitions of fraction, quotient, and ratio from Merriam-Webster on-line:[2]

> fraction: a numerical representation (such as, or 3.234) indicating the quotient of two numbers;
>
> quotient: (1) the number resulting from the division of one number by another (2) the numerical ratio usually multiplied by 100 between a test score and a standard value;
>
> ratio: (1) the indicated quotient of two mathematical expression (2) the relationship in quantity, amount, or size between two or more things.

The first definition says that a fraction is a quotient; the second says that a quotient is a ratio; the third one says that a ratio is a quotient. Thus, it would appear that these words all mean the same thing. The definitions are not wrong as descriptions of how people use the words. For example, people say things like, "mix the flour and the water in a ratio of 3/4," confusing ratios with fractions.

From the point of view of the sense-making stance, this fusion of language is out there in the mathematical world, and we must help students make sense of it. From the point of view of the making-sense stance, we might make some choices about separating and defining terms and ordering them in a coherent progression. In CCSSM, the following choices were made.

1. A fraction a/b is the number on the number line that you get to by dividing the interval from 0 to 1 into b equal parts and putting a of those parts together end-to-end. It is a single number, even though you need a pair of numbers to locate it.
2. It can be shown using the definition that a/b is the quotient $a \div b$, the number that gives a when multiplied by b. (This is what Iszák & Beckmann, 2019, call the Fundamental Theorem of Fractions.)
3. A ratio is a pair of quantities; equivalent ratios are obtained by multiplying each quantity by the same scale factor.

[2] www.merriam-webster.com

4. A proportional relationship is a set of equivalent ratios. One quantity y is proportional to another quantity x if there is a constant of proportionality k such that $y = kx$.

   Note that there is a clear distinction between fractions (single numbers) and ratios (pairs of numbers).

   This is not the only way of developing a coherent progression of ideas in this domain. Zalman Usiskin (private communication) prefers to start with (2) and define a/b as the quotient $a \div b$, which is assumed to exist. One could then use the Fundamental Theorem of Fractions to show (1).

There is no *a priori* mathematical way of deciding between these approaches. Each depends on certain assumptions and primitive notions. But each approach is an example of the structuring and pruning required to make the mathematical ideas make sense; an example of the making-sense stance.

Fidelity

Another principle of the making-sense stance is fidelity. I define fidelity as, "the extent to which a curriculum, or a collection of curriculum materials, faithfully presents the underlying mathematical concept as it is situated in the discipline of mathematics" (McCallum, 2019, p. 80). I go on to say that, "mathematical fidelity is not the same as mathematical formality; a mathematical concept can be presented in a way that is appropriate for the age of the students, while still being presented with fidelity" (p. 80).

Examples of lack of fidelity abound on the internet. Consider, for example, this representation found at Lumen[3] (Fig. 33.1)

The image would seem to violate the condition that a function have one output for each input, since an apple has two halves. Or, if we take the caption to mean that the machine is throwing away one of the halves, there is still the question of which half. A function does not randomly choose outputs from two possible choices.

Fidelity is to some degree a matter of taste. Consider, for example, the distinction between order of operations – the set of rules for how to read arithmetic expressions, such as giving precedence to multiplication over addition – and the properties

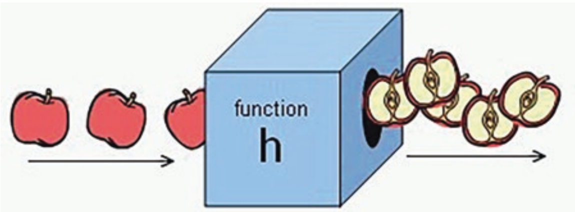

Fig. 33.1 Fruit-halving function (this shows a function that takes a fruit as input and releases half the fruit as output)

[3] https://courses.lumenlearning.com/boundless-algebra/chapter/introduction-to-functions/

of operations – the set of rules governing how operations work, such as the distributive property. In school mathematics these topics are often given equal salience. However, most mathematicians would regard the first as merely convention and the second as fundamental law. The order of operations could be changed; there is nothing mathematically wrong with saying that addition takes precedence over multiplication, in which case the distributive property would be written $a \cdot b + c = (a \cdot b) + (a \cdot c)$. But the distributive property itself is fundamental, and has the same meaning no matter how it is notated. Although it would not be mathematically incorrect in a curriculum to present order of operations and properties of operations in a flat list with the same degree of emphasis, it would be a little tone-deaf.

This subjective aspect of fidelity means that there can be reasonable disagreements about it. A making-sense stance takes seriously the task of discussing those disagreements with evidence from the professional norms of the discipline.

Concluding Thoughts

I have spent most of this paper describing properties and examples of the making-sense stance: the properties of coherence and fidelity, the example of ratios and fractions. However, a complete view of mathematics and learning takes both stances at the same time, with a sort of binocular vision that sees the full dimensionality of the domain. An example of this is Arcavi's (Arcavi, 1994) article on symbol sense, which shifts beautifully back and forth between the two stances.

In wondering about how the duality between the two stances relates to the math wars, I am drawn to the observation that some participants in that debate may have made unwarranted assumptions about what each stance implied on the other side. It was sometimes assumed that a proponent of mathematical correctness – the making-sense stance – would also be in favor of instructional practices that come under the heading of "stand and deliver": the teacher standing at the front of the room, explaining a concept or demonstrating the procedure for solving a certain type of problem, and then asking the students to mimic the procedure with a set of similar problems. It was also sometimes assumed that a proponent of the sense-making stance would embrace an arrangement of mathematics by extra-mathematical organizing principles, such as large real-world projects, or nebulous big ideas.

However, we have known better for a long time. For example, in Hiebert et al. (1996), they describe a reform approach to instruction based on the principle that, "students should be allowed and encouraged to problematize what they study, to define problems that elicit their curiosities and sense-making skills" (p. 12), a principle which falls squarely in the sense-making camp. However, they illustrate this approach with an account of a second-grade class where students work working on the most traditional of word problems: "find the difference in the height of two children, Jorge and Paulo, who were 62 inches tall and 37 inches tall, respectively" (p. 13). Thus, the sense-making stance is applied to an ostensibly traditional organization of material. In the other direction, the freely available curriculum (Illustrative Mathematics, 2017)[4] takes the making-sense approach to ratios and proportional relationships prescribed by CCSSM, organized into lesson plans that support problem-based instruction as described in Hiebert and colleagues (1996).

However, this co-ordination of the two stances does not always happen. This is partly because there is a political aspect to the division, as illustrated by the math wars in the United States (Schoenfeld, 2004). The Common Core is an existence proof of the possibility of overcoming these political differences. Part of that success was the result of the usual grind of diplomatic work; a lot of listening and trying to find third ways, while at the same time insisting on principles of mathematical coherence and pedagogical appropriateness. It is difficult to draw a general lesson there, apart from the lesson that if you keep trying at something you occasionally succeed.

But there is one lesson worth pointing out, about the power of the word "common", meaning shared. In the end, the fact that almost fifty states agreed on the same set of standards was at least as powerful as the quality of those standards. Having a shared set of standards means being able to share curriculum, teaching strategies, and resources across state lines. And, although a country with a centralized education system has already solved that problem, I think the idea of shared understanding also has the power to bring together the two sides in whatever version of the math wars might be happening in that country, or indeed in the international community.

I hope that spelling out the two stances will contribute to productive dialog in mathematics education, such as the one that started this article, allowing for conscious recognition of the stance one or one's interlocutor is taking and for acknowledgement of the value of adding the dual stance.

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[4] Produced by an organization of which the author is a co-founder.

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Chapter 34
School Mathematics Curriculum Reforms: Insights and Reflections

Berinderjeet Kaur

> Mathematics education does not take place in a vacuum. It is greatly influenced by and must reflect or even anticipate changes in the educational and social system. (Howson, 1978, p. 183)

A critical part of mathematics education in any system of schooling is the curriculum, which includes the prescribed mathematics content in the form of syllabuses and or standards and pedagogical approaches. Over the span of the last two centuries although the content of the school curriculum has expanded to meet the changing needs of society, there has been stability in the structure even as waves of reform have swept across the surface (Kilpatrick, 1996). The fact that more than fifty countries participating in the Trends in Mathematics and Science Studies from 1995 till the present concur on the core mathematical knowledge that is tested at the fourth and eighth grade levels suggest that most countries appear to have the same basic mathematical content knowledge taught by certain grade levels (Linquist et al., 2017). This is not unexpected as the canonical school mathematics curriculum, a result of curriculum developers in one educational system simply copying what is being done in another, was developed in Western Europe in the aftermath of the Industrial Revolution and adopted practically in every country during the twentieth century (Howson & Wilson, 1986). The adoption was either voluntary or via colonisation.

Despite its seemingly structural stability, school mathematics curriculum has made significant shifts in emphasis periodically. These shifts have been consequences of national or international initiatives resulting from the evolution of mathematics content knowledge, developments in learning theories and needs of societies (Howson et al., 1981). Chapters 32 and 33 illustrate two such cases that provide food for thought about underlying reasons that result in reforms in the teaching and

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learning of mathematics in a nation or in a wider context. The purpose of this chapter is to augment Chaps. 32 and 33 and provide some background of reforms in mathematics education as to glean knowledge about the why and what of these reforms.

School Mathematics Reforms

The Era of "New Math"

The "New Math" reform, sparked by the successful launch of an unexpected first satellite into orbit around the earth by the Soviet Union in 1957, began in the US. With mathematicians taking a deeper interest in what to teach in the schools a revamp of the content was carried out and new topics like modular arithmetic, set theory, abstract algebra, etc. were introduced (Hayden, 1981). Groups of teachers guided by mathematicians wrote the "New Math" textbooks, and by 1960, New Math curriculum materials were available for use in schools in the US. In a rush to put the books in the classrooms, many other aspects of implementing a curricular change were lacking.

As such in the US, the reform was short lived and by the early 1970s New Math was dead (Klein, 2003). This was so as social issues overtook curricular ones. It is apparent from the book *Why Johnny can't add: The failure of the new math* (Kline, 1973) that, in haste to implement New Math, teachers had to enact it without knowledge and understanding whilst the wider society was ignorant of the need for change. Schoenfeld (2004), in his reflection on the New Math, aptly notes that:

> Specifically, it provides a cautionary tale for reform. One of the morals of the experience with the New Math is that for a curriculum to succeed it needs to be made accessible to various constituencies and stakeholders. If teachers feel uncomfortable with a curriculum they have not been prepared to implement, they will either shy away from it or bastardise it. If parents feel disenfranchised because they do not feel competent to help their children, and they do not recognize what is in the curriculum as being of significant value (and what value is someone trained in standard arithmetic to see studying 'clock arithmetic' or set theory?) they will ultimately demand change? (p. 257)

Elsewhere the reform made its way over time. In some places it did not take root (for example Hungary) while in others it did influence the intended school mathematics curriculum (for example in Singapore). In Hungary, "news of the sputnik shock and the subsequent boom of the New Math with its fresh wind and dust storms did not reach us [them] before some years later" (Halmos & Varga, 1978, p. 225). When it did, the local (Hungarian) reform in school mathematics was already well underway with mathematicians, teachers and the wider public working together towards "efficient mass education in mathematics" (p. 227), that included "working on miscellaneous problems" (p. 227) and "mathematical literary – reading formulae, graphs, statistical tables, understanding what is behind them" (p. 227). The approach adopted by the local reform and the content focus was affirmed when news began circulating about the shortcomings of New Math – typified by the claim “why Johnny can’t add” (p. 228).

In Singapore, up to the late 1950s, several mathematics syllabuses were in use as schools were vernacular in nature with the Chinese, Indian, Malay and English schools (where the medium of instruction was Chinese, Indian, Malay and English respectively) adopting their curricula from China, India, the Malay Archipelago and Britain respectively. The first local set of syllabuses for mathematics for use in all schools, both primary and secondary schools, was drafted in 1957 and published in 1959 (Lee, 2008). A revision of this set of syllabuses took place in the early 1970s in response to the New Math reform that was traversing the world.

While the primary mathematics curriculum was added an outcomes-based approach, the secondary school mathematics curriculum included ‘modern mathematics’ topics such as modular arithmetic, set theory, transformations, data representations and analysis. However, by the end of the decade it was replaced as globally the curriculum was no longer in tandem with the University of Cambridge Examinations Syllabuses (Lee, 2008) that was adopted by the Ministry of Education (MoE) for mathematics. It may be said that this was perhaps due to the colonisation of Singapore by the British from 1819 till 1963 that impacted many adoptions by Singapore’s Education System, a significant one being the Cambridge Examinations. As mentioned in the introduction, school mathematics curriculum is canonical and therefore world trends que curriculum adoptions in many countries, including Singapore.

Realistic Mathematics Education (RME)

As the “New Math” reform was making its way into countries around the world, in the Netherlands the emergence of an initiative, Realistic Mathematics Education (RME), led by mathematics didacticians in 1968 ensured that the Dutch mathematics education was not influenced by the formal approach of the ‘New Math’ movement. Freudenthal (1983), a mathematician with deep interest in mathematics education, introduced the method of didactical phenomenology that made RME a domain-specific instruction theory for mathematics.

Based on the teaching principles of RME, a number of local instruction theories and paradigmatic teaching sequences focusing on specific mathematical topics have been developed over time (see van den Brink, 1989; Streefland, 1991; de Lange, 1987). Design research guided the local instruction theories (Gravemeijer, 1994). With the availability of technological tools for mathematics instruction, the development of local instruction theories included technology (for details, see Drijvers, 2003; Bakker, 2004; Doorman, 2005). Almost five decades on, RME is still work in progress (van den Heuvel-Panhuizen, 2020). It may be considered as a research and development venture that directly inputs into mathematics classroom instruction. The RME approach has completely influenced the intended school mathematics curriculum in the Netherlands through the textbooks for both primary and secondary schools (van den Heuvel-Panhuizen & Drijvers, 2014).

During the 13th International Congress on Mathematical Education (ICME) held in 2016 during the Thematic Afternoon on "European Didactic Traditions", RME was illuminated, through presentations by the traditional owners of RME and others who have adopted or adapted aspects of RME in their countries, states, or classrooms. An outcome of the afternoon is a 366 pages publication that has documented the adoptions and adaptations. This is a testimony to the impact of RME in many countries beyond the Dutch mathematics classrooms (van den Heuvel-Panhuizen, 2020). The publication notes that making acquaintance with RME was in most cases the result of a personal encounter at a gathering of mathematicians or mathematics educators somewhere in the world. This suggests that the spread of RME has been an educational endeavor and not a politicised one. International collaborations have also led to RME-based textbooks in the US, "Mathematics in Context" (Wisconsin Centre for Educational Research and Freudenthal Institute (2010) and a mathematics education reform, known as Pendidikan Matematika Realistik Indonesia (PMRI), based on RME in Indonesia for more than two decades, that began in 1994 (Zulkardi et al., 2020).

The 'Model' Method: A Pedagogical Reform in Primary School Mathematics

In Singapore, school mathematics curriculum reforms have been driven by both global trends and national needs. A reform that arose out of a national need to improve the learning of mathematics was a pedagogical reform in primary school mathematics that has had outreach internationally. A study carried out in 1975, revealed that 25% of students, after six years of primary school, failed to meet the minimum numeracy level by the standards of the MoE (MoE, 2009a). This and similar findings for other school subjects prompted the Prime Minister in August 1978 to call for a review of the education system that resulted in formulation of a New Education System (NES) (MoE, 1979). The NES was implemented in 1981. The goal of the NES was to provide improved education for every child in the system.

As part of the NES, the establishment of the Curriculum Development Institute of Singapore (CDIS) in June 1980 was an important milestone. Its main function was development of curriculum and teaching materials. It was directly involved in the implementation of syllabuses and systematic collection of feedback at each stage of implementation so that subsequent revisions and refinements would be strategic (Ang & Yeoh, 1990). Among the various project teams, at the CDIS was the Primary Mathematics Project (PMP) team. The task for this team was to produce instructional materials for the teaching and learning of primary mathematics with effective teaching approaches and professional development of teachers (MoE, 2009a). In 1981, the team administered diagnostic tests of basic skills of mathematics to a sample of 17000 Primary 1 to Primary 4 students. The findings were dismal with more than half of Primary 3 and 4 students doing poorly on items that tested division, 87% of Primary 2 to 4 students could solve word problems when key words like 'altogether' and 'left' were provided but only 46% of them could solve word problems without key words (MoE, 1981).

The PMP team comprising experienced educators from schools and the MoE, together with expertise of international consultants, produced the new primary mathematics curriculum in 1981. The curriculum adopted a concrete–pictorial–abstract approach for the teaching and learning of mathematics. This approach provided students with the necessary learning experiences and meaningful contexts, using concrete manipulatives and pictorial representations to construct abstract mathematical knowledge (CDIS, 1987). In the new primary mathematics curriculum, the 'model' method, a heuristic to solve word problems, was included. Theories that underline the method and the method, are discussed and described respectively, elsewhere (see Kaur, 2019a).

The concrete–pictorial–abstract approach pervaded the design of the materials and pedagogy of teaching primary mathematics henceforth. The 'model' method, helps students visualise the abstract mathematical relationships and the varying problem structures through pictorial representations (Kho, 1987). The method, initially meant for upper primary school grades (4–6) in 1983, is now an essential feature of the Singapore primary mathematics curriculum and introduced to students in primary 1 (MoE, 2009a).

The stellar performance of Singapore students in International Benchmark Studies (Kaur et al., 2019), Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA), has drawn a lot of attention to the method that is often referred to 'Singapore Math'. Several countries, including Brunei, Thailand, South Africa, and various states in North America have attempted to adopt the method by customising Singapore textbooks for their use and initiate reform in their mathematics classrooms (see Sim, 2014; Teng, 2014; Tham, 2014). However, successful adoption would need teachers to have a good grasp of the pedagogy and sound mathematical knowledge. This is crucial, as when the PMP team set forth to develop the materials there was equal emphasis placed on the professional development of teachers alongside them.

Era of Problem Solving

In the US, the aftermath of the New Math era, was a 'Back to Basics' turn, i.e. the pre-New Math curricula was re-adopted. The outcome of this turn, as noted by Schoenfeld (2004) was:

> By 1980, the results of a decade of such instruction were in. Not surprising, students showed little ability at problem solving—after all, curricular had not emphasized aspects of mathematics beyond mastery of core mathematical procedures. But performance on the "basics" had not improved either. (p. 258)

In response to the poor performance, the National Council of Teachers of Mathematics (NCTM) in the US published *An agenda for action* in 1980. This agenda called for problem solving to be the focus of school mathematics (NCTM, 1980). The agenda was timely as more evidence appeared about the dismal performance of US students in the Second International Mathematics Study (SIMS) in the mid-1980s:

> There is one consistent message. Students from the United States, regardless of grade level, generally lag behind many of their counterparts from other developed countries in both mathematics and science achievement. That, perhaps, is the only consistent message. (Medrich & Griffith, 1992, p. 29)

However, as to what 'problem solving' entailed was again a contentious issue. It is apparent that the agenda for action, followed by the curriculum and evaluation standards (NCTM, 1989) and principles and standards for school mathematics (NCTM, 2000) were sources of continued debate on mathematics education in the US (Schoenfeld, 2004). The present common core state standards in mathematics appear to be where the focus is at in the US at present (CCSSM, 2010).

As noted in the preceding sections, in Hungary and the Netherlands, 'problem solving' was already part of their school mathematics curricula much earlier than the intent of the US. Nevertheless, the agenda was timely as globally there was an emerging interest in problem solving as it essentially emphasised the acquisition of mathematical knowledge to solve non-routine problems or doing realistic mathematics through applications, modelling and mathematisation (de Lange, 1996). This emphasis also stemmed from the need to prepare students to be competent citizens for democratic life as opposed to qualifying for the future work force (Keitel, 1993).

In Singapore, in the early 1980s, there was also the 'Back to basics' turn as basic numeracy skills of students across the school system continued to decline following adoption of Modern Maths which was done in haste to keep abreast of global trends. However, the 'Back to basics' turn is best known as the 'Mathematics for every child' reform in Singapore (Kaur, 2019b). This reform was in sync with the New Education System (MoE, 1979) that was implemented in 1981. The 1980 NCTM's agenda for action drew attention of educators in Singapore and a decade later in 1990, after careful deliberations by educators at the Institute of Education, MoE and classroom practitioners the framework for school mathematics curriculum was detailed with mathematical problem solving as its primary goal. This framework, shown in Fig. 34.1, has been steadfast for the last three decades (MoE, 2018).

The framework makes apparent that for students to be mathematical problem solvers they must acquire conceptual knowledge, mathematical skills, mathematical processes, have good attitudes for learning and be metacognitive. This framework is robust as it also aligns with the student outcomes of twenty-first-century competences, shown in Table 34.1, which are: a confident person, a self-directed learner, an active contributor and a concerned citizen (Wong, 2016).

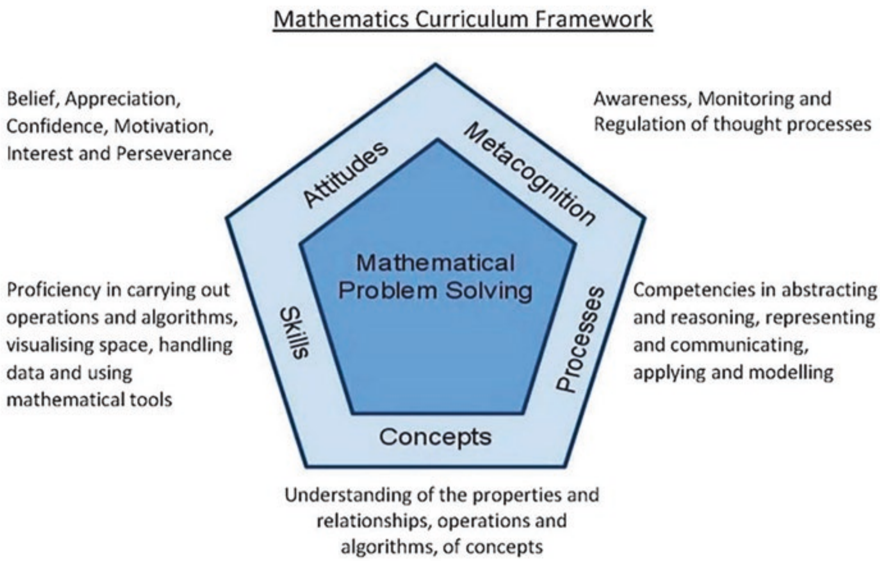

Fig. 34.1 Singapore school mathematics curriculum framework. (MoE, 2018, p. 15)

Table 34.1 Student outcomes (twenty-first-century competences) and components of the Singapore school mathematics curriculum framework

Student outcomes	Components of the school mathematics curriculum
Confident person	Attitudes, concepts, skills, processes
Self-directed learner	Metacognition self-regulation of learning
Active contributor	Attitudes, processes
Concerned citizen	Processes, problem solving

'Looking Forward' and 'Looking Back

In this section, we examine the key issues related to reform in school mathematics curriculum illuminated in Chaps. 32 and 33.

The OECD appears to have envisioned and mapped the future direction for school mathematics curriculum 2030. As noted in Chap. 32 by Taguma et al., the future is unpredictable. However, by being cognisant of the current trends "we can learn – and help our children learn – to adapt to, thrive in and even shape whatever the future holds" (Taguma et al., Chap. 32 Abstract). The OECD Learning Compass 2030, formulated after extensive research carried out by the OECD, has seven elements. The elements are: 1. student agency/co-agency; 2. core foundations; 3. transformative competences; 4. knowledge; 5. skills; 6. attitudes and values; 7. Anticipation–Action–Reflection cycle. Details of these elements are in Chap. 32.

Implications of the Learning Compass 2030 for mathematics curricula stem from future workplace demands in an environment that is highly automated. In a nutshell, we note that content and practices in mathematics lessons must:

- go beyond core foundations in numeracy and disciplinary knowledge (e.g. number systems, geometry and operations) and place a greater focus on contemporary topics such as statistics, data analysis and computational thinking;
- engage students with epistemic knowledge of maths as part and parcel of the work students do in their mathematics lessons;
- engage students in interdisciplinary tasks so that they apply mathematical knowledge in authentic settings;
- facilitate the development of skills needed for life-long and self-regulated learning.

This is so that our students are future-ready for the decade ahead of us! In incorporating the above in the school mathematics curriculum, a unique challenge confronting the custodians of the curriculum would be about what to keep or replace in the present curriculum in view of the time allotted for mathematics instruction.

As every leaner must reach his or her potential in mathematics there is a need to cater to individual learning needs. For this, curriculum experts and learning scientists need to work hand-in-hand and prepare curriculum guides that illuminate learning trajectories in mathematics. Lastly, for the vision 2030 to be carried out, teachers must be developed and society be kept abreast of the rationale for change in school mathematics content and practices.

Some elements of the OECD Learning Compass 2030 are already present in the Singapore school mathematics curriculum as of the "*Values-based, student-centric phase (2012-present)*" of mathematics education of Singapore (Kaur, 2019b). The twenty-first-century competences framework 2010 (MoE, 2009b) was put forth by the Curriculum 2015 committee set up in 2008 to study twenty-first-century skills and mind-sets needed to prepare future generations in Singapore for a globalised world framed this phase.

The framework led to a review of all mathematics syllabuses for Singapore schools in 2010. The revised syllabuses of 2012 (MoE, 2012) implemented in 2013, made explicit that learning mathematics is a twenty-first century necessity and it is a key fundamental in every education system that aims to prepare its citizens for a productive life in the twenty-first century. It also noted that for Singapore as a nation the development of a highly skilled and well-educated manpower was critical to support an innovation- and technology-driven economy. The goal of the national mathematics curriculum was to ensure that all students achieve a level of mastery of mathematics that will serve them well in their lives, and for those who have the interest and ability, to pursue mathematics at the highest possible level.

The syllabuses placed heightened emphasis on the role of learning experiences for mathematics learning. They stated that:

> Learning mathematics is more than just learning concepts and skills. Equally important are the cognitive and metacognitive process skills. These processes are learned through carefully constructed learning experiences. For example, to encourage students to be inquisitive, the learning experiences must include opportunities where students discover mathematical results on their own. To support the development of collaborative and communication skills, students must be given opportunities to work together on a problem and present their ideas using appropriate mathematical language and methods. To develop habits of self-directed learning, students must be given opportunities to set learning goals and work towards them purposefully. A classroom rich with these opportunities, will provide the platform for students to develop 21st century competencies. (MoE, 2012, p. 22)

In 2011, nation-wide professional development of mathematics teachers was carried out to prepare them for the implementation of the 2012 revised mathematics curriculum. The implementation of these syllabuses began in 2013. Following a six-year cycle of review and revision of mathematics curriculum, in 2018 the next revision of the syllabuses has taken place. The 2018 revised school mathematics curriculum has included emphasis on "epistemic knowledge of maths as part and parcel of the work students do in their mathematics lessons". This again affirms the impact of global initiatives on the school mathematics curriculum in Singapore.

McCallum, in Chap. 33 presents the product of the 'math wars', the Common Core State Standards in Mathematics in the US. The math wars began with the publication of NCTM's Curriculum and Evaluation Standards for School Mathematics in 1989 (NCTM, 1989). Through the lens of mathematicians and mathematician educators, what appeared to be a resolved product, the Common Core State Standards in Mathematics (CCSSM, 2010), have raised challenges for the robust enactment of it. Views of mathematics amongst the curriculum writers and enactors have led to two significant perspectives – one on what mathematics is, and another on how it is learned.

Aptly, McCallum has surfaced a duality that merits attention of mathematician educators and researchers. The sense-making stance, a process of people making sense of mathematics, and the making-sense stance where the mathematics content is structured in ways that it makes sense. This duality calls for both the stances to be examined simultaneously as in the works of Ball and Bass (2003) and Iszák and Beckmann (2019) elaborated in the chapter.

On one hand, a pitfall of standards, as in the CCSSM, is that the subject may be reduced to a list of items, what Schoenfeld (1992) refers to as 'bite-sized' pieces, which, when enacted without coherence and fidelity may lead a learner to view "mathematics as facts and procedures". On the other hand, McCallum also cautions that sense-making and making-sense could be futile if learning materials are inappropriately organised as in the two examples Hiebert et al. (1996) and Illustrative Maths (2017) he cited in the chapter. It looks like there is much work ahead for all, mathematicians, mathematician educators, researchers, curriculum designers, and teachers to curtail desired outcomes of the standards. Just like the RME in the Netherlands, this could be work in progress for the next few decades!

In Singapore, the sense-making stance has dominated instruction for a long while. However, in the revised school mathematics curriculum (MoE, 2018) for implementation in 2020, the making-sense stance has been initiated through big ideas, not meant to be authoritative or comprehensive, namely notations, diagrams, proportionality, models, equivalence, measures, invariance and functions for secondary schools, and notations, diagrams, proportionality, models, equivalence and measures for primary schools.

The curriculum notes that there are two orientations to mathematics learning that are relevant to the design of the syllabuses. They are: (i) learning mathematics as a tool that places emphasis on using mathematics to solve problems; and (ii) learning mathematics as a discipline that places emphasis on understanding the nature of mathematics illuminating the practices of mathematicians. As every review of the curriculum in Singapore is guided both by internal needs and by global trends, one may speculate that the 'math wars' in the US has initiated the making-sense stance in the curriculum in Singapore. For the initiative to take root, development of teachers, curriculum materials and research on the what and how of BISM (Big Ideas in School Mathematics) have started and in due course there will be lessons to share with the mathematics education fraternity.

Concluding Remarks

Curricula reforms are inevitable as systems must continuously strive for improvements. The stimulus for reform can be external or internal to a system. The examples in this chapter together with Chaps. 32 and 33 provide us insights of a few reforms in school mathematics. Chapter 33 illuminates tensions in intentions of the curriculum at the micro-level that may lead to dire consequences of the outcomes of intentions. Such micro-level deliberations are important as not all such deliberations are in the purview of many curriculum policy initiators or others involved in the process of enacting a reform. What is significant is the research aspect warranted for inputs into the efficacy of a reform.

Reforms like the RME and the 'model' method are each examples of reforms that grew out of the needs of an educational system. Their positive impact on the learning of mathematics merited adoptions in other countries. Global trends and societal needs have also fuelled reforms and "the era of problem solving" is one that though may be said to have a formal launch in the US, was a much needed one world-wide. The theme helped many educational systems to chart directions of their school mathematics programs. These directions have not been unique. In Singapore "problem solving" is still the primary core while in the USA, other developments have over shadowed it. In a similar vein the OECD Learning Compass 2030 has ignited global reform for stakeholders to adopt or adapt in responsible ways.

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Part VIII
Commentaries on ICMI Study 24

Chapter 35
Introduction

Renuka Vithal and Yoshinori Shimizu

In this penultimate section, two commentaries on the ICMI Study 24 volume as a whole are presented. These reflections are included from two leading scholars in mathematics education research with a keen interest in school mathematics curriculum reforms, Anjum Halai and Paola Valero, who did not participate in the ICMI Study 24 conference. These prominent researchers from different backgrounds, were invited to react to the study volume by providing critical commentaries and to consider gaps between, the findings and discussions raised by ICMI Study 24, and the reality of school mathematics curriculum reforms in the broad diversity of contexts.

Anjum Halai and Paola Valero bring two very different perspectives and reflections on the volume given the contexts of their own research and experiences, which spans different continents of Asia and Africa, and South America and Europe, respectively. They draw attention to particular issues and aspects that are arguably, under-represented in this volume on school mathematics curriculum reforms.

Anjum Halai, in her reaction, foregrounds the critical issue of language in mathematics curriculum reforms and draws attention to the fact that many learners and teachers of mathematics participate in teaching and learning in schools in languages other than their home or first languages. She presents the case of Pakistan but this is the case for many countries, especially in the former colonies such as is found in most regions in Africa (a context that is under-represented in this volume). She points to the issue of equitable access for all learners to the mathematics content and processes in the classroom, and argues that the language of implementation of

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mathematics curriculum reforms serves as a gatekeeper in accessing the curriculum content, which potentially marginalises students from diverse language backgrounds.

Paola Valero offers a cultural–political reading of school mathematics curriculum reforms in her commentary on the volume. Her reflections take different vantage points on mathematics education research in this area, highlighting both insider and outsider perspectives and thereby demonstrating the value of deeper and extended theoretical analyses of school mathematics curriculum reforms (an aspect identified as needing development in this volume). She emphasises that the study of the transformations of the mathematics curriculum needs to embrace the inseparability of the curriculum from its context and that exploring the cultural politics of the mathematics curriculum reforms requires engaging in interdisciplinary research with scholars from other educational disciplines or social sciences.

These reflections and commentaries signal in a real and concrete way that this volume has opened an important and overdue conversation in mathematics education scholarship and has only made the tip of the proverbial iceberg visible. Much more research and focused work is needed in the study of all aspects of school mathematics curricula reforms, especially at macro system levels.

Chapter 36
Language of Teaching and Learning and School Mathematics Curriculum Reform: Tensions in Equity and Access

Anjum Halai

This Chapter provides reflections and commentary on this ICMI Study 24 volume entitled *Mathematics Curriculum Reforms Around the World*. The book looks at school mathematics curriculum reforms where curriculum is conceptualised from two dimensions, *curriculum levels* (intended, implemented and attained) and *curriculum components* (goals, content, teaching approaches, materials and assessment). Curriculum reforms are then studied under these two dimensions and in five thematic areas namely: (a) learning from the past: driving forces and barriers shaping mathematics curriculum reforms; (b) analysing school mathematics curriculum reforms for coherence and relevance; (c) implementation of reformed mathematics curricula within and across different contexts and traditions; (d) globalisation and internationalisation and their impacts on mathematics curriculum reforms; (e) agents and processes of curriculum design, development and reforms in school mathematics.

The above thematic framework is comprehensive and covers the field historically bringing it up to date to issues of globalisation and internationalisation. However, the issues and questions are largely framed within the western tradition of mathematics and its manifestation in school mathematics curricula reforms.

The rationale for a comprehensive and in-depth piece of work on curriculum reform in school mathematics education is well justified. The editors state that, "while there is considerable scholarly developments in general education on curriculum reform studies, these have not crossed boundaries to the same extent into mathematics education as other areas or have not been developed by mathematics educators themselves to the same extent" (Shimizu and Vithal, Chap. 1, p. 4). This issue reflects a broader concern i.e. the need to ensure that the field of mathematics education research is not insular to other developments in the field of education.

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There are several threads that could be expanded upon in this commentary. For focus and depth, I look at the role of language of instruction or the language of teaching and learning and its impact on curriculum as implemented and attained.

The language in which the intended curriculum is implemented significantly impacts the curriculum components of teaching, learning and assessment. However, decisions about the language of instruction are not necessarily taken from a perspective of cognition and learning. For a variety of cultural, historical and political reasons ministries of education and policy makers employ national or global languages as the language of instruction, which are often not the languages children speak at home or their proximate language. This decision of introducing a global or a national language of instruction in mathematics classrooms is often guided by a strongly prevalent view that mathematics has a universal language of abstract symbols and signs and therefore mathematics transcends culture (Parker Waller & Flood, 2016). Hence, from this perspective it is seen as immaterial which language of instruction is employed to implement the mathematics curriculum in schools.

However, as is noted in this volume that the ‘social and cultural turn’ in curricular reform is based on sustained and compelling evidence that learning mathematics is socio-culturally embedded (Kilpatrick, Chap. 2). Language is a strong cultural tool that mediates the implementation of curriculum in the classroom this is especially the case in problem solving and the application of components of the curriculum. Students who learn in their first or proximate language are able to engage deeply with the curriculum process as compared to students who are learning mathematics in a second or third language.

Consequently, “a government or educational authority, in very unequal societies, can claim to offer the same curriculum when in fact they are referring only to the intended official curriculum, the implemented and/or the attained curriculum can reveal deep inequalities given by different resources etc” (Shimizu and Vithal, Chap. 1, p. 15). Halai and Muzaffar (2016) drew on a large-scale study in Pakistan that involved qualitative observations of teaching and learning processes in one hundred and twenty-six primary classrooms in Punjab, the largest province in the country. In the province, new policy had introduced English as a medium of instruction in primary school classrooms where English was the third language for almost all the teachers and students. Classroom interaction patterns showed that students were mostly silent, and seldom uttered a full sentence in English, and were not engaged in meaningful mathematical communication.

It stands to reason that acquiring academic knowledge and higher order thinking is not just a cognitive function, it is also dependent on the tools of thinking that are provided by culture, significant among them being the language. This limited nature of students’ participation in the classroom potentially defeated the goals of the intended mathematics curriculum that aimed to promote problem solving and critical thinking.

Curriculum as attained is often reflected in students' performance in tests and is also mediated by the language of instruction and testing. It is very common to find that children from linguistic minority (or low socio-economic status) to be among the low performers in mathematics. For example, in reviewing Pakistan's performance in the country's first ever participation in TIMSS 2019,[1] Halai (2020) holds that the percentage of students in the sample of fourth graders from Pakistan who reported speaking the language of the test at home was: always (22%); almost always (10%); sometimes (32%); never (36%). However, overall results of TIMSS 2019 showed that there were few students (5%) at fourth grade, on average who 'never' spoke the language of the test at home and had much lower average achievement in mathematics as compared to those students at fourth grade, who, on average, reported 'always' (63%) or 'almost always' (32%) speaking the language of the test at home.

Elsewhere, in a study that investigated the role of language in students' performance in examinations in science and mathematics, Rea-Dickins et al. (2009) concluded that students are normally disadvantaged when they are assessed in a language other than the language they speak at home. The students in their study demonstrated difficulties in the interpretation and understanding of examination questions especially word problems. A challenge in assessing the curriculum attained by the students is ensuring that all are provided the same opportunity to demonstrate their skills and understanding. However, when mathematics tests are written in language that may not be the home or proximate language of all students test items are skewed in favour of some (Halai et al., 2015).

To conclude, this volume *Mathematics Curriculum Reforms Around the World* is a much needed and comprehensive work that documents the curriculum reforms in school mathematics in all its complexities and identifies the challenges and opportunities in the process. Among the challenges in mathematics curriculum reform is the issue of equitable access for all learners to the mathematics content and process in the classroom. Language of implementation of mathematics curriculum is a gatekeeper in accessing the curriculum content and potentially marginalises students from diverse language backgrounds. However, several evidence-based approaches and strategies have emerged in the field of education that provide a way forward for curriculum, teaching, learning and assessment in settings where language of instruction is not the first at times not even the second language of the learners (e.g. Barwell et al., 2015; Street et al., 2005). It is important for school mathematics education community to be outward looking in its gaze on curriculum reforms to ensure that it benefits from cross fertilisation of ideas, frameworks and approaches from the broader curricular reforms in school education.

[1] https://timssandpirls.bc.edu/timss2019/

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Chapter 37
A Cultural–Political Reading of School Mathematics Curriculum Reform

Paola Valero

The call of ICMI Study 24 to develop research that provides understanding of the changes in the school mathematics curriculum has resulted in this valuable and comprehensive collection of systematic and nuanced reflections on its multiple dimensions and its constant transformations. In this commentary, I point to a central issue that needs further attention to advance the area of research that the Study 24 intends to contribute to.

I have engaged in the reading of the chapters from a theoretical and analytical perspective which can be called the study of the cultural politics of mathematics education (e.g. Kollosche, 2016; Valero, 2018). Such perspective studies mathematics education as a wide network of cultural and political practices. It is interested in tracing how mathematics education emerges in the relations between people, institutions and materialities, where different mathematical practices are assembled as teaching and learning are performed. Mathematics education is constantly contested and negotiated given the value and importance that is attributed to mathematical knowledge and competence in a contemporary techno-scientific, dominantly capitalist world.

A special sensibility to the Foucaultian notion of discourse (e.g. Foucault, 1970, 1972) had been helpful in identifying recurring enunciations that appear across the chapters and that configure statements on what is the mathematics curriculum, what is its reform and how these two have become an object of research. The examination of elements of the discourse in this volume is important since it will have a role in the further making of curricular change as an object of scientific examination in mathematics education. As such, the notions in this book actively – and not neutrally – shape the very same phenomena that it intends to study. And since, as all authors here seem to agree, the mathematics curriculum has increasingly become a

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central area of schooling, the ways in which we conceive of and study it sets part of the direction for its further transformations in the future.

A statement in the book is that the mathematics curriculum is influenced by several *contextual factors* across time (different periods) and space (different countries and national cultures). In a way, this is an evident observation since it is impossible to deny that the mathematics curriculum makes part of the curricular technologies that, since the nineteenth century, organise state-controlled mass education. But it is also salient insight that sets the object of study 'mathematics curriculum' in relation to a 'context'. The question emerges of how such relationship is conceptualised in the chapters.

One first discursive recurrence is the way this relationship is expressed in the use of formulations such as 'the factors that influence', the 'vested interests of stakeholders on' or 'the values that permeate' the mathematics curriculum. The recurrences and the types of constructions in the expressions in different chapters seem to convey the idea that the mathematics curriculum is constructed as an *object* that is separated from the *context,* although obviously linked to it. Such relationship is similar to that of a liquid contained in a cup: the nature and character of the liquid does not necessarily change by the form or characteristics of the cup; the former are only circumstantially shaped by the latter. As an object of study of mathematics education research, the way the mathematics curriculum is referred to puts forward the idea that researchers mainly conceive of it as an object in itself, with a core and nature rooted in mathematics, and that we can study the external, non-mathematical influences that affect it. For most, the context is conceived as a series of forces that shape the contours of the curriculum, but seems not to alter its core nature of the mathematics that should be mobilised in education.

Such a statement on the curriculum as a distinct object of mathematics education research may be articulated with respect to the inevitable disciplinary framing that defines the interest of mathematics education research with respect to a focus on mathematics and its related processes of teaching and learning. For example, the idea of a curriculum as a didactic transposition from mathematical scholarly knowledge to a group of students' learnt knowledge is a particular way of narrowing the broad anthropological enterprise of knowledge transformation in cultural and institutionally framed processes in a pedagogical organisation (e.g., Chap. 13).

The effect of such theoretical framing foregrounds the mathematical knowledge and backgrounds its cultural assemblage as it circulates through different institutional norms and practices. Even if in some chapters there is an interest in exploring the contextual setting on which the mathematics curriculum appears (e.g. Chap. 6), such exploration is carried out to pinpointing the interest, values or influences that may explain the changes in the mathematical contents or orientations in the curriculum. This is of course an important endeavour that casts light into the "very complex relations between [mathematics,] the mathematics curriculum and different cultures (Chap. 6, p. 89) [… and offers insight into] major driving forces behind mathematics curriculum changes that are not necessarily mathematical in nature" (Chap. 6, p. 97). The latter, of course, should be carefully considered.

Such an understanding of the relationship between the curriculum and its context leaves unexplored the possibility that those non-mathematical driving forces may indeed be equally or on occasions even more determinant or constitutive of the school mathematics curriculum than mathematics itself. What would happen in that case? Would we still have an object of study of mathematics education research called the mathematics curriculum and its transformation? From most chapters one can draw the statement that the school mathematics curriculum is political in that it has become increasingly valued and privileged; there is an increasing interest in governments and supranational agencies to steer it; and its enactment has consequential effects on the population, groups and individuals, to the point that in our recent history it has acquired a strategic cultural and economic importance. In other words, there is power at stake in the mathematics curriculum.

As a field of study, we posit the importance of the curriculum on the salience and power of mathematics as a central form of knowledge in the making of the modern world. With this, we foreground the mathematics and background the cultural project that has configured a modern rationality in different spheres of life, also in education. This is the backgrounded context in which mathematics, education and the mathematics curriculum as an important power device of such culture has formed.

In contrast to this reasoning, it is possible to present a different, still complementary and quite productive alternative. What would happen if we foregrounded the understanding of modern education and subordinated the making of the school mathematics curriculum as part of it? In taking this turn we need to explore the conditions on which modern mathematics curricula emerged. Modern school systems started configuring as part of the political process of consolidation of nation-States. Following historians of education such as Tröhler (2016), in most of the Western world new political constitutions were followed by laws of education that became operationalised in official curricula. Education in modernity is political since it has been one of the most effective technologies to bind people into an invented community called *nation* and into a political body with the rational organisation of the *state*.

Education can be thought as a political technology for "making types of people" (Popkewitz et al., 2017) with particular cognitive, behavioural and moral characteristics. In a rational, knowledge-based political regime, such aspirations are made operational through plans of study. Thus, the curricula of school subjects amalgamate a series of political aspirations for who the desired, virtuous citizen of a nation should be, with transformed disciplinary knowledge and pedagogical practices, all of which together offer the frame to form the mind, the body and the soul of children and people.

Following this perspective, the study of the mathematics curriculum and its transformation is undetachable from both the larger political project of governing the population through education, and from the micro-organisation of the pedagogical practices and what they should do for people, for the nation and for the state. This means that understanding changes in the mathematics curriculum requires asking questions and investigating the particular contribution of school mathematics to the making of modern subjectivities in general and desirable citizen in particular. It is digging into how the changes in contents and orientations bring forward clear political aspirations for people (students as citizens) and which subjectivities and sensibilities mathematical contents and curricular orientations may potentially fabricate.

In other words, the study of the transformations of the mathematics curriculum needs to embrace the inseparability of the curriculum from its context. Phrased in other terms, further research on the curriculum can further embrace one of the major contributions of socio-cultural-political studies in mathematics education, namely, the recognition that the objectivation of mathematics is inseparable from the subjectivation that is effected through learning and education (e.g. Radford, 2018).

A final point is that the exploration of the cultural politics of the mathematics curriculum requires engaging in interdisciplinary research with scholars from other educational disciplines or social sciences to examine together the generalities and specificities of the changes in the mathematics curriculum. Such collective work would result, with no doubt, in the advancement on the productive line of research that this volume aims to advance.

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Part IX
Conclusion to ICMI Study 24

Chapter 38
Key Messages and Lessons from Mathematics Curriculum Reforms Around the World

Renuka Vithal and Yoshinori Shimizu

Changing Contexts of ICMI Study 24

In this concluding chapter, the key messages from the themes represented in the different parts and their contributing chapters are pulled together to distil some major learning points for understanding and informing school mathematics curriculum reforms. The five themes that were identified and formed the basis for the organisation of the ICMI Study conference have been retained in the structure of this volume (see Shimizu & Vithal, 2018). These themes are represented in Parts II to VI of this volume – a historical perspective, a focus on coherence and relevance of curriculum reforms, on implementation aspects across contexts, on the impact of internationalisation and globalisation and on agents and processes of school mathematics curriculum reforms.

Each of the five themes were explored by addressing a number of key questions (see Chap. 1). Just as the conference attracted more papers in some themes than others, some questions were addressed in greater detail and others much less. This is reflected in this volume as authors in each part engage on the questions addressed in the respective chapters. In addition, these concluding remarks also draw on the chapters from keynotes and the plenary panels, which sought to focus on more macro perspectives in school mathematics curriculum reforms.

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This study volume seeks to build on and extend earlier works in this area. When Howson, Keitel and Kilpatrick released their seminal book *Curriculum Development in Mathematics* in 1981, which captured the state of the art of mathematics curriculum reforms in the preceding decades, the world was a very different place. Four decades later this ICMI Study 24, attempts to similarly, provide an understanding of school mathematics reforms taking place around the world. The last ICMI Study that focused on mathematics curricular was ICMI Study 2 on *School Mathematics in the 1990s,* edited by Howson and Wilson and published in 1986, shows how much this area has grown and how complex it has become (ICMI Studies may be accessed from the ICMI website).

This ICMI Study 24 was initiated in 2016 and conceptualised in a Discussion Document which was disseminated internationally in 2017. The Study Conference took place in November 2018 in Japan, and work on developing the volume was underway in 2019, well before the major global catastrophic event in 2020 in the form of the COVID-19 pandemic. This means that the volume has been finalised while major changes in the functioning of all levels of education systems and societies have been taking place, which were not part of the study and were not deliberated on in the study conference. This significant unpredicted COVID-19 pandemic, has impacted schooling fundamentally and changed life for learners and teachers in the vast majority of societies generally, and specifically, in mathematics education. Hence, all authors were requested to consider including their reflections on the impact of the pandemic on school mathematics curriculum reforms in their respective chapters and themes; and these will be similarly drawn on in this concluding chapter.

This study itself offers an approach to the study of mathematics curriculum reforms across multiple countries or regions of the world. The methodology adopted in the themes of the study has been to present rich descriptions of school mathematics curriculum reforms in diverse contexts and to cluster these in a variety of ways for analysis and to learn lessons about what happened, what worked or did not work, in a variety of different aspects of curriculum reforms.

The volume as a whole draws on curriculum reforms from some thirty-two countries and regions, including: the Netherlands, Hungary, France, Brazil, Japan, Ireland, Italy, Serbia, South Africa, Iran, Vietnam, Portugal, Denmark, Unites States of America, Costa Rica, Israel, China/Hong Kong, Mexico, England, Spain, Andorra, Australia, Lebanon, Luxembourg, Philippines, Canada, Tunisia, Wallonia-Brussels Federation, Chile, Peoples' Democratic Republic Lithuania and Singapore. However, many more countries or regions are referred to in the chapters, which draw on the related literature in this area.

It also includes discussion on curriculum frameworks that have had more global or international impacts in the mathematics curricula of many countries beyond what may have been intended. For example, implications for mathematics curricula of recent curriculum frameworks developed by OECD (Organisation for Economic Co-operation and Development) for their member countries (Taguma et al. Chap. 32); as well as the Pan-Asian curriculum efforts to develop a mathematics curriculum shared by countries in ASEAN (Association of Southeast Asia Nations)

(Pinto and Cooper, Chap. 28). Despite this wide diversity, what is also acknowledged, is the lack of representation from several parts of the world in this volume, such as Africa, especially sub-Saharan countries (excluding South Africa). Nevertheless, this broad inclusion of such diverse countries and regions is a unique strength of this ICMI Study upon which many more studies have the potential to build.

The approach in this concluding chapter to distilling key messages or lessons from the chapters and the themes, is not to summarise what has been set out in the conclusions of the various sections and chapters, but rather to step back and review these from across the volume and to extract elements that have stood out by virtue of being repeated or significantly emphasised and that may be of relevance and value to school mathematics curriculum policy makers, analysts, practitioners and researchers. Authors of chapters were encouraged to draw out key conclusions from their respective chapters and similarly theme leaders extracted the main learning points from their theme. The aspects discussed below are therefore a selection of the key messages and main conclusions from those identified by authors and theme leaders. Readers are referred to the main texts for the extensive rich detailed and varied deeper nuances of the points highlighted below.

Challenges in Defining School Mathematics Curriculum Reforms

What is meant by curriculum, by reform or even by school mathematics within curriculum reforms? These were abiding questions throughout the study.

Chapter 1 describes the challenges of defining the key concepts of the study and how the study drew on different definitions of '*curriculum*'. An evolution can be observed from the definitions of 'intended', 'implemented' and 'attained' curriculum as set out in the Discussion Document; to the definition presented by Niss (2016) comprising six curriculum 'vectors' in the study conference that was taken up across a number of themes and chapters; to further definitions drawn on by authors in their chapters in this volume.

It is possible to observe that the difficulty of defining the key concepts increases with greater diversity in the cases being considered in the analysis. While international studies have developed definitions that are applicable across contexts, two major driving forces identified as significant, that of cultural values and political movements (Bosch et al., Chap. 8), point to how sensitive definitions need to be to particular contexts in order to adequately grasp the working of a particular reform effort in any analysis.

Difficulties in defining school mathematics curriculum reforms also come from its complex and multi-facetted nature as an endeavour with a wide variety of stakeholders and a general public (parents and students) who receive and experience the impact of reforms. As was discussed in theme E on agents and processes of curriculum design, development, and reforms in school mathematics, curriculum reforms are shaped by actors from a wide variety of sectors - business, industry, media, teacher unions, and parents and so on.

The very use of the word ‘reform’ in school mathematics curriculum reforms has also come under critique especially in cases were the intention is to completely overhaul a curriculum – where a ‘transformation’ or ‘reimagined’ curriculum is being argued. With new areas being introduced in mathematics curriculum reforms (e.g. statistics, modelling, computational/ algorithmic thinking, mathematical literacy) foundational questions about what constitutes mathematics in the school mathematics curriculum have equally been raised. The challenges and difficulties of defining mathematics curriculum reforms may therefore be linked to the sheer complexity and magnitude of school mathematics curriculum reforms as well as their deep context boundedness. We will return to this point.

Lack of Research, Gaps in Theories and Methodologies in the Study of Reforms

It became evident early in the study, as Kilpatrick points out in Chap. 2,

> Despite enormous amount of curriculum development, we do not have an enormous amount of curriculum development research. (p. 29)

Across the themes of this volume, authors raise this lack of scholarly work in mathematics curriculum reforms as a major challenge and give different explanations. It is identified as one of the barriers that have hindered reform efforts historically because as Bosch and O’Meara(Chap. 8) point out research in mathematics education “tend to focus on smaller units of analysis like the teaching of a given topic or recurrent student difficulties in learning. Analysing a whole curriculum and its evolution over time […] requires specific research tools” (p. 114) and those which will allow a more neutral stance by researchers who are variously involved in the reform.

Authors focusing on the agents and processes in curriculums reforms expand the research challenge, pointing to the long time-scale of reforms, the political sensitivity of reform processes at each level, and difficulties of obtaining data on the inner workings and dynamics of curriculum committees which could contribute to greater understanding of reforms and hence to improving different reform processes and communication (Ellen et al., Chap. 30). Hence, it appears that there is wide-spread practice of mathematics curriculum reforms nationally and regionally across the world but with limited in-depth studies at macro or policy levels to inform and drive evidence-based improvements over time and across contexts.

While there is research published on mathematics curriculum reforms, especially of pilots conducted before a reform is rolled out or on smaller aspects of a reform, and especially on the outcomes of a reform in the form of students’ achievements, the point being made is that much less is known, for example, about the inside workings at a curriculum policy level, and on how particular decisions get made, implemented and resourced. Furthermore, much more research is needed about the broad scaling up of the implementation after the piloting stage of a mathematics curriculum reform (where piloting does take place). It is evident that developing, implementing and evaluating a mathematics curriculum reform, say, at a country, state or district level, constitute different domains of practice and, therefore, of research.

Another significant related point is the dearth in the development and use of theories. A direct question was posed in the theme on curriculum reform coherence and relevance regarding this issue. Morony (Chap. 14, p. 219) concludes that "there is a lack of conscious and careful application of theory to analyses of mathematics curriculum reforms." This is not to discount development and applications of some theories (e.g. on didactic transposition – see Bosch et al., Chap. 7; Artigue, in Chap. 16; Barquero et al., Chap. 13) or models for curriculum reforms (e.g. Jameson and Bobis, Chap. 27; Thornton et al., Chap. 17). Barquero et al. (Chap. 13) provide a useful exposition on this situation in mathematics curriculum reform research and state that the most striking finding from their work to identify theoretical approaches in analyses of curriculum reforms is that such examples are in a distinct minority and many reviews of curriculum reforms are not supported by a clear theoretical basis that guides the methodology used. As a relatively under-developed area of research in mathematics education, it is evident that "Choices of research questions, research designs and interpretation of results all depend on the development of a research community sharing some common theoretical foundations and employing compatible methodological approaches." (Jameson et al., Chap. 30, p. 471).

A main conclusion is that mathematics curriculum reforms, currently, therefore do not appear in the main, to be strongly evidence-based or guided by research. The challenge is double-edged in that, on the one hand, curriculum policy makers may not use research or use only research that supports the stance they seek to promote, while on the other hand, mathematics education researchers are not engaging the kind of research that can appropriately speak to and inform large-scale curriculum reform efforts. Morony (Chap. 14) highlights the need for a commitment from all stakeholders to consider evidence from relevant scientific studies of curriculum reforms, and to initiate such studies to guide reforms. In addition, several themes acknowledge the importance of continuous monitoring and evaluation of curriculum reforms. It is hoped, therefore, that this study will galvanise such efforts in the mathematics education community.

With the lack of theory and theorising, there appears to be a concomitant lack of development of appropriate methodologies in mathematics education literature for studying large reforms. The primary methodology in this volume, across themes, is one that draws on selected groups of national or regional case studies of mathematics curriculum reforms to extract key messages and learning points. Although the volume is driven by authors who were delegates (whose papers were accepted) in the ICMI Study 24 conference, the selected invited keynotes and plenary panellists as well as members of the International Programme Committee (IPC), who were variously leading or involved in major reforms of their countries or regions, provided a unique rich data source and reflections on the inner working and challenges of macro curriculum reforms across a wide variety of diverse contexts. These include: Angel Ruiz (Costa Rica); Peter Sullivan and Will Morony (Australia); Bill McCallum and Jeremy Kilpatrick (USA); Yoshinori Shimizu (Japan); Mogens Niss (Denmark); Fidel J. Oteiza (Chile); John Volmink and Renuka Vithal (South Africa); Berinderjeet Kaur (Singapore) and Miho Taguma (OECD, for member countries), among others. This, in itself, represents a significant strength of this study volume as it brings together the reflections of key figures in school mathematics curriculum reforms from diverse countries and contexts.

This ICMI Study volume itself therefore, offers a methodology for studying curriculum reforms while presenting an emergent approach to international reviews and collaborations, to jointly learn about and from each other about school mathematics curriculum reforms.

Shifts in the Content of School Mathematics Curriculum Reforms

A central and major element in all school mathematics curriculum reforms is the concern about mathematical content. What is the appropriate content in school mathematics, how is it represented at all levels and for different groups of students, what is its purpose, how does it compare with other countries, and so on, which take centre stage in any mathematics curriculum reform effort. A few key messages are highlighted on this aspect from a complex, multi-faceted and substantive discussion in the volume.

From a historical perspective, Kilpatrick (Chap. 2) points to a major movement from a pure mathematics focus, especially in the secondary grades in the period before the eighties, to a much more applied mathematics focus since the nineties, which is explained, in part, by the advancements and ubiquitous availability of technology and shifts in student interests. The evidence for this is in the introduction of new areas such as mathematical modelling, statistics, financial mathematics and programming in curriculum reforms observed across many countries. Mathematics curriculum reform content and pedagogies are also being influenced by particular mathematics education movements such as Realistic Mathematics Education, Critical Mathematics Education and Ethnomathematics as well as discourses on social justice and equity, which have become established in the past few decades and may be observed explicitly or implicitly in some reforms.

Another major shift has been in the introduction of mathematical literacy or numeracy or similar content in multiple ways. This shift may be explained, in part, from the wide media coverage and public interest in international studies like PISA which focus on mathematics literacy but also from concerns about ensuring student learning outcomes impart necessary mathematics knowledge and skills appropriate for functioning in contemporary society. It is underpinned by wider concerns about how economic and social inequalities may be entrenched through mathematics education provisions. This debate is also extended into how mathematics curriculum reforms build in different pathways for different groups of learners which serve as gateway or gatekeeping functions into further study of mathematics and ultimately into different career opportunities. This aspect is arguably under-explored in this volume.

The very question of the relevance of content matters in mathematics curriculum reforms is raised with contemporary and future curriculum reforms pointing to a major shift from content-based to a competency-based curriculum. The OECD frameworks, as presented in the chapter by Taguma et al. (Chap. 32) shows how these frameworks are taken up across countries and the shape of possible future curriculum reforms. The widespread adoption of the USA Common Core State Standards in Mathematics to inform content and pedagogical choices in mathematics curriculum reforms across many other countries, is another case in point.

The chapters in the theme on globalisation and internationalisation describe and analyse in detail how these curriculum reform processes are unfolding and their impacts. In particular, this theme explores how TIMSS and PISA are serving as vehicles for curriculum reforms across countries, how numeracy and mathematical literacy are evolving and finding representation in curriculum reforms in multiples ways; and the most recent emergence, that of computational thinking or algorithmic thinking is finding expression in some of the most recent national curriculum reforms. The inclusion of some aspects of computational thinking in the PISA 2021 Mathematics Literacy Framework will no doubt, open questions about the relation between the disciplines of computer science and mathematics in future curriculum reforms.

From a coherence perspective, Morony (Chap. 14) points out that a key principle underpinning the design and rollout of a mathematics curriculum is the careful consideration of the subject of the curriculum reform – the mathematics - its structure and ways of knowing and doing, not only within mathematics but also in relation to other disciplines. Arguably, an area not adequately addressed in the volume is that of mathematics curriculum content and its relation to curriculum pathways that open or close for student progression in schooling and beyond (see Kilpatrick, Chap. 2).

Key Role of Teachers, Teacher Education and Professional Development

There is no doubt that there is wide recognition of the role of multiple agents in school mathematics curriculum reforms. However, the critically important role of teachers stands out across themes in this ICMI Study. Even in themes that did not specifically pose research questions related to teachers, it is evident that teachers have the potential to make or break a mathematics curriculum reform.

This lesson was learnt very early as one key explanation for the failure of “New Maths” reforms of the mid twentieth century. There is a recognition of the importance of teacher education and professional development and of different models of professional development that can be scaled up so that they reach a great majority of teachers in any system of mathematics curriculum reform. From a historical perspective O'Meara and Milinkovic (Chap. 5) conclude that in any reform movement, teachers are key agents in the effective delivery of the new curriculum, and so the potential success of the reform is heavily dependent on them. Respect for the existing knowledge of teachers, building their capacity to adopt new ways of working with their students when provided with appropriate and sustained support in the form of materials, induction, initial and continuing professional development and acknowledgement of their work is spelt out as a key principle in the focus on coherence and relevance of curriculum reforms (Morony, Chap. 14).

Teacher involvement, ownership and commitment to the reform are regarded as important for successful implementation of a curriculum reform, and both bottom up and top-down strategies are needed (Ruiz, Chap. 19). Several themes emphasise that appropriate forms of resources need to be provided to teachers that are coherent with the curriculum and main objectives of the reform. Central to this is developing and testing practical models of teacher preparation that align with their responsibilities in the intended curriculum (Stephens et al., Chap. 24). Stakeholder communication and negotiation (Pinto and Cooper, Chap. 28) are important at all stages of a curriculum reform. However, finding ways to meaningfully involve all teachers who will eventually have to implement a reform is difficult but also critical in a mathematics curriculum reform.

The Rise of the Importance of Resources and Technology

The question of the role of resources, including teaching and learning materials and technology in mathematics curriculum reforms was expressly asked in all themes except for the one focusing on agents and processes of curriculum reforms. With the emergence of the Covid-19 pandemic and the resultant shift to online, blended or hybrid teaching and learning, this issue of resources and technology has been brought into much sharper relief. However, the study identified the importance of adequate and appropriate resources before the pandemic struck.

One of the key principles identified for the design and rollout of a curriculum reform that is coherent and relevant is that resources developed to support the implementation need to be "adaptable to different contexts and changing circumstances, accessible and sustainable" (Morony, Chap. 14, p. 220), and Golding (Chap. 12) describes the conditions and some of the pitfalls for this in large scale reforms. Success in implementation of mathematics curriculum reforms requires not only the existence of appropriate resources for teachers and learners but a close relation between the school and any pedagogical materials, in a bi-directional process that is both top-down and bottom-up, noting that the way this is done varies significantly (Ruiz, Chap. 19).

The range of issues related to the development and use of a broad range of resources including digital technologies and the diversity across countries and regions as well as invariant aspects in curriculum reform implementation processes is explored in detail by Coles et al. (Chap. 18). In school mathematics curriculum reforms, as they point out, "textbooks arise as an important resource to promote good alignment with the curricular reform, considering not only their content but also their methodological approaches. They are strongly influenced by external high stakes assessments, which can be coherent or not with curricular reforms" (p. 291).

While warning of the dangers of inadequate resourcing or rushed design of resources in school mathematics curriculum reforms and how these can create tensions within the curriculum system, Golding (Chap. 12) concludes, that textual, manipulative and digital resources can be harnessed to support increased relevance for students and promote enactment of a curriculum reform by teachers; and notes the lack of research about the affordances and constraints of digital materials, and on student use of resources in relation to curriculum reform intentions. The need for research assumes much more significance in the Covid-19 and most likely post-Covid era where new digital technologies as well as digital and digitalised books and other teaching and learning resources will gain considerable traction within education systems.

Alignment as a Key Feature Within School Mathematics Curriculum Reforms

While the theme on coherence and relevance in mathematics curriculum reforms is defined and explored in detail, the issue of alignment has arisen consistently alongside it. Morony (Chap. 14) concludes that their analyses demonstrate "the importance of alignment between the curriculum and the curriculum system in which it is enacted, and […] the negative impacts of the mis-alignment between the two. This mis-alignment limits the effective and coherent enactment of a curriculum and the reform it embodies" (p. 220). Alignment is, in fact, identified as one of the 'laws' for successful implementation of reforms in that "all the educational means (and) the reforming efforts" must be aligned (Ruiz, Chap. 19, p. 326).

Within any curriculum reform, a central key message for its success is the alignment of the aspects and components of the curriculum, however, it is defined. That is, any curriculum reform effort must seek to work towards and sustain the alignment of the intended, implemented and attained curriculum as well as the alignment of curriculum vectors as defined by Niss (see Chap. 1) – goals, content, materials, teaching, student activities and assessment.

Given that a curriculum reform by its nature involves multiple stakeholders, each with different vested interests, roles and responsibilities at different levels of an educational system, sustaining alignment will always be a challenge. To this end the 'law' of alignment may be associated with the 'law' of two directions in that "implementation strategies must be considered allowing for both top-down and bottom-up developments. What is essential is to create a good synergy between these two processes" (Ruiz, Chap. 19, p. 325).

In addition, to maintain this alignment would require recognising the importance of communication across boundaries of different communities (Pinto and Cooper, Chap. 28). Attention is drawn to the range of stakeholders in any curriculum reform and "the significance of interaction between different professional communities, within and across levels by direct dialogue and mediated by documents and materials" (Jameson et al., Chap. 30, p. 471–2), pointing to how the design and implementation of reforms can benefit from multiple forms of professional expertise, practice and engagement. In particular, the involvement of key agents who are members of multiple communities can serve as brokers mediating different discourses to facilitate alignment in its different forms and facets.

School Mathematics Curriculum Reforms as Context-Bounded and Invariant

The notion that school mathematics reforms are deeply bounded to their context and yet also demonstrate invariant aspects across contexts, is well captured in the final comment by Kilpatrick (Chap. 2)

> I think the idea of curriculum as a process, and one that needs to be shaped by the situation in the school, the situation in the country, the situation in the classroom – all of that has changed from what it was in the 1980s. Today, I would say we are moving much more toward recognising that the goals for school mathematics may be different across different school systems, countries, and situations. Each country has to figure out what its goals are, and in what directions it wants to go. (…)
>
> The bipolar nature of school mathematics (pure and applied mathematics), in contrast, shines through regardless of the curricular context or level. We have learned since the new math era that school mathematics is complicated, contextualized, not easily changed, and not easily studied. (p. 33–4)

In all the themes, there is a recognition of how context-bound mathematics curriculum reforms are by virtue of different cultural values, social, political and economic systems and living conditions. In the implementation of curriculum reforms, this is a key message presented as one of the 'laws' of the enormous *diversity* in different dimensions observed in experiences of a variety of countries and regions. This diversity can refer to: the many different agents involved in the various levels of intervention in reforms, the cycles or timing of reforms which may be linked to social, cultural and political stability or some change event or condition; or to the scale and breadth of a reform which could involve some or all of the curriculum vectors identified in the definition by Niss. A consequence of this diversity is that a curriculum reform that may be successful in one country may not be so in another country and the importing/exporting of mathematics curriculum reforms from one context to another cannot therefore, be translated in a mechanical or an instrumental approach.

Despite this diversity, there are aspects and components of mathematics curriculum reforms that are invariant across contexts, given in part, by the internationalisation and globalisation processes. The impact of international studies like TIMSS and PISA on mathematics curriculum reforms being undertaken in individual countries or regions has been recognised across the themes. Stephens et al. (Chap. 24) in the theme on internationalisation and globalisation, document in detail the influence of TIMSS and PISA in diverse countries on intended curricula, support for teacher professional development, coverage of content knowledge and skills, and inclusion of concepts of mathematical literacy or numeracy in mathematics curriculum reforms. They caution however, on the importance of evaluating these when adapting components of international studies in national curriculum reforms.

It is noted that the activities of ICMI itself may influence and impact on particular globalisation and internationalisation processes in mathematics curriculum reforms. A wide range of ICMI activities bring diverse participants from many different countries together and this sharing leads to similar aspects being taken up across very different contexts. The point however, is not against such activities but rather that great consideration should be given to the unique features of a particular context in adopting any aspect of a curriculum reform from it into another context.

The role and impact of technology in mathematics curriculum reforms is already observed and has been highlighted as likely to increase in future, which will take different forms and emphasis in different contexts but is invariant in that it will influence school mathematics curriculum reforms in multiple ways across the world. Some reasons advance are the increased reliance on digital technology by their application to all facets of life in contemporary society, the expectations of parents and students for a better technology-assisted education that combines the peculiarities of local contexts with the global in opportunities for further education and jobs, the increasing use of algorithmic techniques including artificial intelligence and the general unfolding of what is being referred to as the fourth industrial revolution (4IR) across diverse societies. These have been accelerated by the impact of the Covid-19 pandemic.

In the theme on agents and processes of curriculum design, development and reforms, curriculum reform contexts are considered as systems that can make important choices stand out. That is, within a particular country or region, Jameson et al. (Chap. 30) conclude that curriculum reform is, "always situated in a particular context at the jurisdictional level, and further instantiated within multiple contexts like schools and training programmes" (p. 469) in which agents may make different choices in what is focused on and emphasised in a curriculum reform. Hence, the lessons learned from importing curricula at a global level to a national context also apply within countries in recognising diversity within a country or region in the design and implementation of any curriculum reform and its impacts and outcomes.

School Mathematics Curriculum Reforms as Long-Term and Unpredictable Endeavours

Large macro-level mathematics curriculum reforms are typically conceived of as long-term. These type of curriculum reforms are often conducted in cycles of five to ten years or when there is a change event, which triggers a mathematics curriculum reform. Depending on the extent of the reform in terms of its departure from the status quo, the actual curriculum redesign and development might take place over a shorter period of a year or more but the implementation could be rolled out over many years through an education system (e.g. primary and/or secondary grades). This long-term timeframe has also been identified as one of the ‘laws’ for successful implementation with experience showing “inappropriate reduction of time and resources needed for a reform inevitably conspires against its success” (Ruiz, Chap. 19, p. 326).

The long-term timeframe together with diverse agents and stakeholders at multiple levels in any reform contributes to several unknowns that are invariably part of any curriculum reform design and implementation plan. Those involved in or leading large-scale mathematics curriculum reforms point to how unpredictable the implementation of a reform can be and hence, the ‘law’ of uncertainty has been proposed to recognise that reforms are not linear processes as debates and challenges arises in diverse contexts (Ruiz, Chap. 19). The theme exploring agents and process of curriculum reforms refer in their concluding key message to unexpected events and circumstances that may arise since a reform is by its nature a disruption and some degree of shock to the system. Jameson et al. (Chap. 30) connect this with the notion of resilience in the design and implementation of a curriculum reform and outline characteristics that contribute to resilient systems.

The impact of the COVID-19 pandemic has demonstrated in very real terms how curriculum reform design, implementation and outcomes will need to account for the unexpected, the uncertain and the unpredictable by being open to continual “rethinking, reforming and reprogramming” (Ruiz, Chap. 19, p. 327). This difficulty is exponentially increased in education systems already characterised by deep inequities, instability and disruption. In this context, Jameson et al. (Chap. 30) similarly cite “the importance of ‘resilience’ as a property that allows a system to experience some degree of shock and disruption while fulfilling its basic characteristics and functions” (p. 473) and argue for how a mathematics curriculum reform may be designed to withstand unexpected changes.

Further Issues in the Study of School Mathematics Curriculum Reforms

This ICMI Study demonstrates the magnitude and complexity of school mathematics curriculum reforms as well as the challenges of studying these. This complexity and difficulty increases many-fold when school mathematics reforms are but one part of a broader curriculum reform that involves many other school subjects and involves one or several levels of an education system. In this ICMI Study, the unit of analysis has been *school mathematics* curriculum reforms and to that extent the study has not focussed on these reforms as part of a broader school curriculum reform.

The challenges of studying mathematics curriculum reforms are many-fold. Not only are there theoretical and methodological challenges requiring particular knowledge and skill sets, large scale studies require large funding grants which may be difficult to obtain. Nevertheless, the need for undertaking these studies is abundantly demonstrated in this study and it is hoped it will be a catalyst for this area of research and development within mathematics education.

As it is observed in the study, and for which there is a long tradition in mathematics education research and practice, boundary crossings and borrowings from other disciplines such as from general education studies, policy studies and curriculum studies may greatly advance and benefit studies in mathematics curriculum reforms (Halai, Chap. 36; Valero, Chap. 37). Furthermore, while it may be difficult to mount large studies in mathematics curriculum reforms, approaches adopted in other disciplines (such as the medical sciences) that actively undertake synthesis studies, meta-analysis and reviews across different countries or contexts need to be more deliberately planned and conducted so that what is context specific and what is invariant becomes better understood and able to be implemented with greater confidence in and through mathematics curriculum reforms.

One of the areas which did not generate papers to the study conference and remains under-explored is the role of media in macro school mathematics curriculum reforms. Experience across countries show that major changes in school mathematics curriculum generate sharp interest across many different sectors and stakeholders. This is explained, in part, by the role of school mathematics curriculum outcomes as either, an obstacle to or facilitating of further educational and job opportunities for students. Yet this area on the role of media is not understood even though it can and does directly impact the eventual curriculum reforms that get accepted and implemented in real terms in any given context. Carefully crafted and co-ordinated media releases across many countries simultaneously by major international studies like TIMSS and PISA provide lessons in demonstrating their influence in curriculum reforms and which need to be studied.

Curriculum reform processes are as much an educational matter as they are political. They involve a broad range of stakeholders and agents with vested interests. Actors from outside education such as politicians, business, industry, various professional associations and trade unions together with and through the traditional and newer forms of (social) media influence and shape curriculum reforms as much as (if not more than) those with different expertise such as curriculum policy makers, educators, mathematicians, and researchers.

An area that requires further exploration and study is the relation between mathematics education/educators in the broadest sense and other stakeholders, especially those in government with the power to initiate, develop, implement, review and evaluate school mathematics curriculum reforms. School mathematics curriculum reforms are no longer the preserve of mathematicians primarily driving such processes as they once were such as in the "New Math" era. In countries around the world the processes by which curriculum reforms are undertaken are as complex as the reforms themselves. In today's world mathematics educators, are but one (among many) of the key agents in any reform, hence, their understanding and ability to communicate and negotiate with multiple stakeholders, agents and the general public is critically important for their expertise to find expression in the reform itself.

In conclusion, this ICMI Study 24 points to an emerging scholarship in school mathematics curriculum reforms (especially large macro-reforms) that is still in its infancy and requires much more deliberate and focused attention to develop it into a fully-fledged area of study in mathematics education. This is critically important if mathematics education researchers and practitioners seek to significantly impact the day to day functioning and outcomes of the vast majority of mathematics classrooms on the ground, in whichever context they find themselves. It is by influencing school mathematics curriculum policy at a macro level that will create the necessary openings for the inclusion of the considerable research and knowledge that exists about mathematics education to find expression and impact the largest numbers of mathematics learners and teachers, and eventually any society as a whole.

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Afterword

Jill Adler

The launch of ICMI Study 24 in late 2016 by the 2013–2016 Executive Committee of ICMI was welcomed by the mathematics education community. The focus on school mathematics curriculum reform from an international perspective was timely. A previous ICMI study on curriculum dated back to the early 1990s, and in the intervening three decades, mathematics curriculum reforms had taken place and continue to take place across the globe. It was indeed time to bring together collective understanding and insight, and offer the uniqueness of the ICMI study process to inquire again into this important phenomenon that impacts the experiences of mathematics curriculum developers, teacher educators, teachers and school learners everywhere.

As I write the afterword in March 2022, some five years later, the ICMI Study 24 volume is reaching conclusion and will shortly be published. So much has already been written about the contents of the volume – in the introductory and concluding chapters, and in the foreward – about its breadth and how it has managed to also provide in-depth views into diverse reform processes and products across country contexts. This is a remarkable achievement, particularly given that the major work of writing the chapters for the volume was undertaking during the height of the Covid-19 pandemic, a period of dramatic upheaval and uncertainty. I am sure we can all attest to little following 'normal' paths in our work these past two years, with many publication deadlines being extended. The authors of the working group chapters in this volume had the task not only of re-presenting the work at the conference, but further locating and relating these stories to the significant unfolding of the impact of school closures, rapid development of digital platforms for communication and simultaneous deepening of inequitable effects around them. And they have succeeded in capturing not only the themes and engagements during the study conference in 2019, what was elicited, what remained obscured, but also reflection on this all in the light of a changing educational landscape. ICMI Study 24 of mathematics curriculum reform was an extraordinarily complex process, and the study volume offers a significant contribution to this important, and as we have read, under-developed domain of our work.

Y. Shimizu, R. Vithal (eds.), *Mathematics Curriculum Reforms Around the World*, New ICMI Study Series, https://doi.org/10.1007/978-3-031-13548-4

I was ICMI President from 2017 to 2020, and thus privileged to participate, ex-officio, in the whole process of Study 24. It is from this insider perspective that I write the afterword and offer a few reflective insights into the complex journey travelled these past five years by the co-editors and chapter authors who together formed the International Program Committee (IPC) for the study.

In the introductory chapter, the volume co-editors describe the challenges faced in the first IPC meeting where the discussion document was being consolidated, and plans made for the study conference. With some consternation, I would say, the committee came to realize what we probably all knew but did not fully appreciate. Curriculum experiences are ubiquitous in school education: all concerned know about and experience curriculum reform. But who and where were the leading scholars in this field? What was the accumulating scholarship? Some of the members of the IPC themselves had had leading roles in curriculum reform in their countries, and while this brought extraordinary expertise into the design and development of the study, their research, and others who had participated with them, was directed elsewhere in the field. We could not easily identify leading and influential theories and methodologies. The issue of the basis on which the study could proceed, on anticipated building blocks that were not evident, confronted the first IPC meeting and have permeated the study. It is this ICMI study volume that now provides a structured and insightfully co-ordinated platform from which further research can proceed.

This was a fundamentally different situation from most previous ICMI studies where there was an established research field, or an identifiable emerging field. In the first IPCs for previous studies, discussion and debate was over who and what to select to shape the conference; and on how to ensure that the breadth of and diversity in the field was engaged, and so the inclusion of ranging ideas, theoretical perspectives and methods. ICMI consistently reminds our community that our work extends beyond research to include interests in practice, policy and curriculum development. Yet, when it came to identifying persons of stature who could be plenary speakers at the Study 24 conference, our eyes turned to research profiles. The consequences of this contradiction for the study has been reflected on in the first chapters of the volume, and form the important backdrop to what follows. Study 24 stands now as an exemplar of what is possible for ICMI studies in critical areas of our work where there is perhaps limited or limiting self-evident research, but abundant expertise, and where systematic reflection can advance ICMIs goals.

A second issue for the study related to TIMSS and PISA and their organizations. All in the IPC could agree that both these international assessment bodies continue to have an influence on mathematics education globally, and even moreso within particular countries. In south Africa, and no doubt equally elsewhere, a disappointing ranking in the results is used for reflection on the system as a whole, but at the same time creates space for media sensationalism, and frequently with this, a blame game typically for teachers. It is worthwhile noting here that the role of the media in curriculum reform processes is an important area for further research. It was thus clear that these international studies be part of a theme in the Discussion Document, the conference and the volume. In the IPC, the suggestion for a plenary dedicated to this, and including the voice of the studies was fervently debated. We discussed different views on the role and position as ICMI in relation to the influence of the work of these organizations on our field. How does/can ICMI interact with these policy shapers? Here too the IPC took up this challenge and the stories related to both TIMSS and PISA in the volume speak to this.

Third, and as for all ICMI activities, ICMI Study 24 desired and aimed to be inclusive – in participation and voice. Both the introductory and concluding chapters discuss the constraints faced. Notwithstanding the desires and goals, inclusion of participants and thus their voices related to curriculum reform did not spread across all regions of the world. The one impact of this – the concerns of how language shapes curriculum enactment everywhere – is the focus of Anjum Halai's commentary, and the relative under-discussion of this crucial aspect of curriculum everywhere. The co-chairs of the study were and remain painfully aware of the challenge they faced and what was and was not possible to accomplish. Their challenge is a wider challenge for ICMI as a whole, and then of course for future studies.

Finally, I suggest that it is precisely the uniqueness of the ICMI study processes that have enabled this significant contribution to our field. This volume depicts and indeed enhances the ICMI vision for its studies. The able leadership of the co-chairs, Renuka Vithal and Yoshinori Shimizu, together with their dedicated IPC members, and all who contributed to and participated in the study conference, co-produced a coherent, reflective and forward gazing account of school curriculum reform through the ICMI spirit of international, collaborative scholarship. Our recent survey of previous studies reflected most clearly that this was its value, and while challenges were evident in relation to stated goals for the ICMI Studies program, it was certainly worth pursuing. ICMI studies provide a special perspective on an aspect of our field – a perspective developed through a palpable critical and collaborative spirit evident in any ICMI study conference. As study 24 got underway, and we understood the different basis on which it would need to develop, our hope was that through this special process, a path would be formed. And indeed, it has!

South Africa
March 2022

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