38 Schriften aus der Fakultät Sozial- und Wirtschaftswissenschaften der Otto-Friedrich-Universität Bamberg

Redistributive Taxation in Dynamic General Equilibrium with Heterogeneous Agents

Christian Putz

University of Bamberg Press

38 Schriften aus der Fakultät Sozial- und Wirtschaftswissenschaften der Otto-Friedrich-Universität Bamberg

Schriften aus der Fakultät Sozial- und Wirtschaftswissenschaften der Otto-Friedrich-Universität Bamberg

Band 38

University
of Bamberg
Press
2019

Redistributive Taxation in Dynamic General Equilibrium with Heterogeneous Agents

Christian Putz

University of Bamberg Press
2019

Bibliographische Information der Deutschen Nationalbibliothek
Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliographie; detaillierte bibliographische Informationen sind im Internet über http://dnb.d-nb.de/ abrufbar.

Diese Arbeit hat der Fakultät Sozial- und Wirtschaftswissenschaften der Otto-Friedrich-Universität Bamberg als Dissertation vorgelegen.
1. Gutachter: Prof. Dr. Matthias Muck
2. Gutachter: Prof. Dr. Thomas Egner
Tag der mündlichen Prüfung: 18.06.2019

Herstellung und Druck: Digital Print Group, Nürnberg
Umschlaggestaltung: University of Bamberg Press

http://www.uni-bamberg.de/ubp/

ISSN: 1867-6197
ISBN: 978-3-86309-702-8 (Druckausgabe)
eISBN: 978-3-86309-703-5 (Online-Ausgabe)
URN: urn:nbn:de:bvb:473-irb-466512
DOI: http://dx.doi.org/10.20378/irb-46651

To my family.

Acknowledgements

This thesis is the result of my time as a doctoral candidate at the Chair of Banking and Financial Control at the University of Bamberg. Everyone who has ever taken up such a task knows that it certainly takes its own kind of time and effort to be completed, but yet it is hardly possible without the support from many others. With this in mind, I would like to thank those who generously gave their time and support to contribute to its success.

In particular, I want to thank my supervisor Prof. Dr. Matthias Muck for his invaluable support. I am especially grateful to him for giving me the academic freedom to create and develop my own ideas, while providing me with the right guidance whenever necessary. Along with the very friendly atmosphere that he created, it certainly has been a great time for me as a doctoral candidate at the Chair of Banking and Financial Control. Further, I would like to thank Prof. Dr. Thomas Egner and Prof. Dr. Andreas Oehler who kindly agreed to join the doctoral committee.

I would like to cordially thank my colleagues and friends at the Chair of Banking and Financial Control, especially, Dr. Michael Herold, Dr. Tobias Kaufmann, Tobias Meyer, Dominik Staniewski, and Dr. Stefan Weisheit for making it a most enjoyable environment. I thank them for stimulating discussions and helpful comments that greatly improved my work. It was a wonderful experience working with them.

Furthermore, I am indebted to Dr. Niko Pomakis for proofreading and providing valuable comments on the pre-final version. I am grateful to him and Alexander Schwarz for many encouraging discussions.

Finally, I am more than thankful to my family. I cannot overestimate the support and motivation they provided me with and the impact they took on my personal development way beyond my academic career. Thanks to my brothers Dr. Tobias and Dr. Florian Putz - their personal and academic careers are sources of great inspiration to me. I am deeply grateful to Elisabeth Sixt for her ongoing love and to her and her parents Maria and Heinz Sixt for their unconditional support. Lastly, my greatest appreciation goes to my parents Maria and Dieter Putz for their continuing encouragement and absolute support throughout my whole life.

June 2019 *Christian Putz*

Contents

List of Figures . . . xiv
List of Tables . . . xvi
List of Acronyms . . . xvii
List of Symbols . . . xviii

1 Introduction . . . **1**

2 Empirical and Theoretical Background . . . **7**
2.1 Motivation . . . 8
2.1.1 Income, Wealth and Consumption Disparity . . . 8
2.1.2 Redistributive Taxation and Transfers . . . 13
2.1.3 Individual Heterogeneity . . . 17
2.1.4 Conclusion and Objective . . . 22
2.2 Methodical Foundation . . . 24
2.2.1 Neoclassical Foundation . . . 25
2.2.1.1 Fisher Separation . . . 27
2.2.1.2 Expected Utility and Life-cycle Consumption . 31
2.2.1.3 Neoclassical Production and Growth Theory . 35
2.2.2 Consumption-based Asset Pricing . . . 41
2.2.2.1 Consumption-based Asset Pricing in an Exchange Economy . . . 46
2.2.2.2 Consumption-based Asset Pricing with Linear Production Technology . . . 48
2.2.2.3 Asset Pricing with Heterogeneous Agents . . 50
2.2.3 Overlapping Generations Model . . . 54

2.3 Literature Review and Conclusion 58
2.3.1 Literature Review 58
2.3.2 Conclusion and Survey Approach 63

3 Redistributive Taxation in an Exchange Economy 67
3.1 The Model 70
3.1.1 Economic Setup 70
3.1.2 Tax System and Redistribution Mechanism 73
3.1.3 The Agents' Optimization Problem 75
3.1.3.1 Heterogeneous Patience and Expected Lifetime Utility 75
3.1.3.2 Evolution of Wealth 77
3.1.3.3 Maximization Problem 79
3.1.4 Market Equilibrium 80
3.2 General Equilibrium Solution 82
3.2.1 First Order Conditions, Equilibrium Consumption and the SDF 83
3.2.1.1 The First Order Conditions 84
3.2.1.2 The Consumption Share Function 86
3.2.1.3 Alignment of Marginal Rates of Substitution and the SDF 90
3.2.2 Asset Prices 91
3.2.2.1 The Risk-free Return 91
3.2.2.2 The Price-Dividend Ratio 93
3.2.3 Individual Investment and Net Transfers 95
3.2.3.1 Total Wealth Budget Constraint and Transfer Capital 95
3.2.3.2 Marginal Propensity to Consume out of Total Wealth 98
3.2.3.3 Investment Policies 100
3.2.3.4 Net Transfers 103
3.2.3.5 Optimal Expected Lifetime Utility 105
3.2.4 Equilibrium Condition 106
3.3 Quantitative Analysis 107
3.3.1 Baseline Results 109

3.3.2 Impact of the Model Horizon 112
3.3.3 Impact of the Tax Rate 115
3.3.4 Impact of Heterogeneous Patience 117
3.4 Conclusion 120

4 Redistributive Taxation in a Production Economy with Overlapping Generations 123
4.1 The Model 127
4.1.1 Population 127
4.1.2 Economic Setup 130
4.1.2.1 Production Technology 130
4.1.2.2 Economic Output 131
4.1.2.3 Firms' Optimization 132
4.1.2.4 Linear Technology and Asset Markets 135
4.1.3 Tax System and Redistribution Mechanism 136
4.1.4 The Agents' Optimization Problem 140
4.1.4.1 Heterogeneous Patience and Expected Lifetime Utility 140
4.1.4.2 Evolution of Wealth 142
4.1.4.3 Maximization Problem 144
4.1.5 Market Equilibrium 146
4.2 General Equilibrium Solution 148
4.2.1 Aggregate Economic Behavior 150
4.2.1.1 The First Order Conditions 150
4.2.1.2 Aggregate Production, Consumption and Investment 154
4.2.1.3 The Stochastic Discount Factor and the Risk-free Return 159
4.2.2 Life-cycle Consumption and Investment Behavior 162
4.2.2.1 Total Wealth Budget Constraint, Human and Transfer Capital 163
4.2.2.2 Consumption Policy 166
4.2.2.3 Investment Policies 172
4.2.2.4 Net Transfers 178
4.2.3 Equilibrium Condition 180

4.2.4 Welfare Measures 183
4.2.4.1 Individual Welfare Measure 184
4.2.4.2 Social Welfare Measure 186
4.2.5 The Impact of Government Debt 191
4.3 Quantitative Analysis 195
4.3.1 Baseline Parameterization 196
4.3.1.1 Earnings Profiles 199
4.3.1.2 Subjective Time Discount Factors 202
4.3.2 Baseline Results 205
4.3.2.1 Aggregate Economic Statistics 206
4.3.2.2 Life-cycle Profiles 208
4.3.2.3 Consumption and Wealth Disparity 213
4.3.3 The Impact of Redistributive Taxation 214
4.3.3.1 Macroeconomic Effects 215
4.3.3.2 Individual Life-cycle Effects 219
4.3.3.3 Consumption and Wealth Disparity 224
4.3.3.4 Welfare Effects 228
4.3.3.5 Summary 235
4.3.4 Robustness Analysis 237
4.3.4.1 Impact Earnings Disparity 237
4.3.4.2 Heterogeneous Subjective Time Discount Factors 240
4.3.4.3 Impact Level of Risk Aversion 251
4.4 Conclusion 253

5 Conclusion and Outlook **259**

A Figures **263**

B Proofs **269**
B.1 Redistributive Taxation in an Exchange Economy 270
B.1.1 The Government's Optimization Problem 270
B.1.2 Derivation of First Order Conditions 271
B.1.3 Derivation of the Pre-tax Stochastic Discount Factor . . . 273
B.1.4 Derivation of the Consumption Share Function 274
B.1.5 Properties of the Risk-free Return 277

B.1.6 Derivation Total Wealth Budget Constraint 278
B.1.7 Derivation Price-Dividend Ratio Transfer Capital 280
B.1.8 Derivation Marginal Propensity to Consume out of Total Wealth 282
B.1.9 Optimal Expected Lifetime Utility 284
B.1.10 Clearing on the Stock Market 284
B.2 Redistributive Taxation in a Production Economy with Overlapping Generations 286
B.2.1 Clearing on the Market for Wealth and Consumption Goods 286
B.2.2 The Government's Optimization Problem 288
B.2.3 Derivation of First Order Conditions 289
B.2.4 Derivation of the Pre-tax Stochastic Discount Factor . . . 292
B.2.5 Derivation of Condition (4.59) 293
B.2.6 Derivation Total Wealth Budget Constraint 294
B.2.7 Derivation Human Capital 295
B.2.8 Derivation Transfer Capital 297
B.2.9 Derivation Marginal Propensity to Consume out of Total Wealth 299
B.2.10 Clearing on the Bond Market 300
B.2.11 Derivation of Individual Welfare Measure 301
B.2.12 Derivation of Social Welfare Measure 304
B.2.13 Impact of Government Debt 305

Bibliography 312

List of Figures

2.1 Income and wealth shares of top percentiles in selected OECD countries 10
2.2 Evolution of top income shares across the world and global wealth shares (1980 - 2016) 13
2.3 Tax revenues from and cash benefits to households as well as implied redistribution effects in selected OECD countries 15
2.4 Neoclassical general equilibrium model 29
2.5 Demographic structure overlapping generations model 56

3.1 Baseline parameterization: consumption, wealth, equity investment and net transfer profiles 111
3.2 Impact model horizon: initial and terminal consumption share 114
3.3 Impact tax rate: consumption, wealth, equity investment and net transfer profiles 117
3.4 Impact heterogeneous patience: consumption, wealth, equity investment and net transfer profiles 118

4.1 Individual life-cycle earnings profiles for baseline parameterization 201
4.2 Life-cycle profile subjective time discount factor for baseline parameterization 205
4.3 Baseline parameterization: life-cycle profiles of consumption, wealth, income, and consumption growth, as well as total wealth, human and transfer capital for the average agent 209

4.4 Baseline parameterization: life-cycle profiles of total, equity and bond investment as well as net transfers for the average agent . . 211
4.5 Impact tax rates: macroeconomic quantities 216
4.6 Impact tax rates: life-cycle profiles of consumption 220
4.7 Impact tax rates: life-cycle profiles of wealth 222
4.8 Impact tax rates: life-cycle profiles of investment and net transfers for the average agent 224
4.9 Impact tax rates: consumption and wealth disparity 226
4.10 Impact tax rates: individual welfare 230
4.11 Impact tax rates: aggregate welfare 234
4.12 Impact earnings disparity: individual welfare 239
4.13 Impact earnings disparity: social welfare 240
4.14 Impact heterogeneous patience: macroeconomic quantities . . . 243
4.15 Impact heterogeneous patience: life-cycle profiles of investment and net transfers 244
4.16 Impact heterogeneous patience: life-cycle profiles of consumption 245
4.17 Impact heterogeneous patience: consumption Gini coefficient over the life-cycle 247
4.18 Impact heterogeneous patience: individual welfare 249
4.19 Impact heterogeneous patience: social welfare 250
4.20 Impact relative risk aversion: individual consumption growth and macroeconomic quantities 252
4.21 Impact relative risk aversion: individual and social welfare 253

A.1 Baseline parameterization: life-cycle profiles of total, equity and bond investment and net transfers for the low income agent . . . 264
A.2 Baseline parameterization: life-cycle profiles of total, equity and bond investment and net transfers for the high income agent . . 264
A.3 Impact of tax rates: life-cycle profiles of investment and net transfers for high income agent 265
A.4 Impact of tax rates: life-cycle profiles of investment and net transfers for low income agent 266

List of Tables

2.1 Consumption, income and wealth distribution 12

3.1 Baseline parameterization for the model featuring an exchange economy 108

4.1 Baseline parameterization for the model featuring production and overlapping generations 197
4.2 Coefficients for the normalized cohort labor income profile 200
4.3 Coefficients for the subjective time discount factor polynomial . . 203
4.4 Aggregate statistics for the model economy and historical data . . 206
4.5 Asset pricing moments for the model economy and historical data 207
4.6 Percentage share of tax revenues in GDP by source 208
4.7 Consumption distribution under baseline parameterization . . . 214

List of Acronyms

CAPM	Capital asset pricing model
CARA	Constant absolute risk aversion
CRRA	Constant relative risk aversion
EIS	Elasticity of intertemporal substitution
EZ	Epstein-Zin
GDP	Gross domestic product
i.i.d.	Independent and identically distributed
IMF	International Monetary Fund
MPCTW	Marginal propensity to consume out of total wealth
OECD	Organisation for Economic Co-operation and Development
OLG	Overlapping generations
RA	Representative agent
RBC	Real business cycle
SDF	Stochastic discount factor
U.S.	United States
WIL	World Inequality Lab

List of Symbols

The following list contains symbols used in this book and essential for the reader's understanding. They are sorted in order of appearance and congruence. Throughout the course of this thesis subscripts are used to denote time t, and in some places to indicate a certain economic state ω (or the realization of a possible uncertain outcome z). In this context, further special notational conventions apply in Chapters 3 and 4. When necessary, and unless otherwise stated, tildes indicate after-tax values and bars denote variables in the presence of government debt. Finally, subscripts and superscripts in brackets are optional.

Common Symbols

t	Time (period)
$\mathbb{E}_{(t)}\left[\cdot\right]$	(Conditional) expectation operator
$u\left(\cdot\right)$	Instantaneous utility function
$\mathcal{F}_t$	Information filtration
γ	Coefficient of relative risk aversion
Ω	State space
$\omega \in \Omega$	Economic state
$\mathcal{L}$	Lagrangian function

Symbols Chapter 2

U_t	Expected lifetime utility function
N	Individual time horizon
δ	Subjective time discount factor
c_t	Individual consumption
l_t	Individual non-financial endowment/income
a_t	Individual financial income
q_t	Individual financial investment
v_t	Individual wealth
$R_{v,t+1}$	Individual gross return on financial investments from t to $t+1$
$F_t(\cdot)$	Neoclassical production technology
K_t	Stock of physical capital
L_t	Amount of human labor
A_t	State of technology/knowledge
Y_t	Aggregate consumable production output
λ	Positive scaling factor
I_t	Aggregate gross investment
d_k	Capital depreciation rate
C_t	Aggregate consumption
Π_t	Profit representative firm
ω_t	Wage rate
$R_{K,t+1}$	Capital rental rate / return on capital investment from t to $t+1$
X_t	Gross payoff financial asset at date t
P_t	Market price financial asset at date t
α_t	Individual holdings in financial asset from t to $t+1$
V_0	Indirect utility
M_{t+1}	Stochastic discount factor from t to $t+1$
i	Date of birth generation i

Symbols Chapter 3

In this chapter, different agent types are characterized by a superscript m.

T	Economic and individual time horizon
$m = 1, 2$	Index indicating agent type $m = 1, 2$
c_t^m	Consumption of agent type m in period t
C_t	Aggregate consumption
D_t	Exogenous production output / aggregate dividend payment
$G_{(t+1)}$, G_z	Random i.i.d. gross production growth (from t to $t + 1$) and gross production growth if outcome z is realized
$z = 1, 2$	Index indicating the realization of possible outcome $z = 1, 2$
$P_{k,t}$	Ex-dividend stock price / value of production means
α_{-1}^m	Initial stock endowment of agent type m
$R_{f,t(,t+n)}$, $\tilde{R}_{f,t(,t+n)}$	Gross risk-free return before and after tax from t to $t + 1$ (t to $t + n$)
k_t^m, $\tilde{k}_t^m$	Net capital income at date t of agent type m before and after redistribution
κ	Parameter measuring the strength of friction costs associated with the redistribution of capital income
τ	Tax rate on net capital income
S_t	Aggregate tax revenues
s_t	Individual transfer payment
δ^m	Subjective time discount factor of agent type m
U_0^m	Expected lifetime utility function of agent type m
α_t^m, β_t^m	Stock and bond holdings of agent type m from t to $t + 1$
v_t^m	Wealth of agent type m in period t
V_0^m	Indirect utility of agent type m
$P_{s,t}$	Price of transfer capital
μ_t^m	Lagrangian multipliers associated with the agent m's constraints

$2\mu_{t+1}^{(m)}/\mu_t^{(m)}$	Stochastic discount factor after tax from t to $t+1$ (induced by agent m)
$2\lambda_{t+1}^{(m)}/\lambda_t^{(m)}$	Stochastic discount factor before tax from t to $t+1$ (induced by agent m)
g_t^m	Consumption share of agent type m in period t
$PD_{k,t}$	Stock price to dividend ratio
w_t^m	Total wealth of agent type m in period t
b_t^m	Marginal propensity to consume out of total wealth of agent type m in t

Symbols Chapter 4

In this chapter, different agent types m and their date of birth i are denoted by superscripts, whereas individual's age $t-i$ is characterized by a subscript.

i	Date of birth generation i
M	Number of different agent types
$m = 1, ..., M$	Index indicating agent type $m = 1, ..., M$
$c_t^{i,m}$	Consumption in period t of agent type m born in period i
g_{t-i}^m	Consumption share of agent type m aged $t-i$
$N+1$	Individual initial life expectancy
$h_t^{i,m}$	Earnings income in period t of agent type m born in period i
f_{t-i}^m	Share in aggregate earnings of agent type m aged $t-i$
H_t	Aggregate earnings
$O+1$	Retirement age
Y_t	Aggregate consumable production output
K_t	Stock of physical capital
L_t	Amount of human labor
A_t, A_z	Random i.i.d. state of technology in period t and state of technology if outcome z is realized

Z	Number of possible realizations of i.i.d. random variable A
$z = 1, ..., Z$	Index indicating the realization of possible outcome $z = 1, ..., Z$
$\theta, 1 - \theta$	Output elasticity of capital and labor
I_t	Aggregate private investment
C_t	Aggregate private consumption
X_t	Aggregate investment share / private saving rate
D_t	Profit representative firm / aggregate capital income
ω	Exogenous and constant wage rate
Ξ_1	Auxiliary constant linear production technology
$R_{E,t+1}$, $\tilde{R}_{E,t+1}$	Gross return on equity investment before and after tax from t to $t + 1$
$R_{f,t}$, $\tilde{R}_{f,t}$	Gross risk-free return before and after tax from t to $t + 1$
$k_t^{i,m}$, $\tilde{k}_t^{i,m}$	Net capital income at date t of agent type m born in period i before and after redistribution
$h_t^{i,m}$, $\tilde{h}_t^{i,m}$	Earnings at date t of agent type m born in period i before and after redistribution
d_{t-i}	Share of aggregate tax revenues distributed to an agent of age $t - i$
κ_c, κ_l	Parameter measuring the strength of friction costs associated with the redistribution of capital and earnings income
τ_c, τ_l	Tax rate on net capital and labor income (incl. retirement income)
S_t	Aggregate tax revenues
s_t^i	Transfer payment to an agent born in period i
δ_{t-i}^m	Subjective time discount factor of agent type m aged $t - i$
$U_t^{i,m}$	Expected remaining lifetime utility function at date t of agent type m born in period i
$\alpha_t^{i,m}$, $\beta_t^{i,m}$	Stock and bond holdings of agent type m born in period i from t to $t + 1$
$v_t^{i,m}$	Wealth in period t of agent type m born in period i

$V_t^{i,m}$	Indirect utility at date t of agent type m born in period i
$p_{h,t}^{i,m}$, $p_{s,t}^{i}$	Price of human and transfer capital at date t of agent type m born in period i
b_{t-i}^{m}	Marginal propensity to consume out of total wealth of agent type m aged $t-i$
ν	Endogenous parameter equilibrium condition
$\mu_{t+n}^{i,m}$	Lagrangian multipliers associated with the constraints of agent type m born in period i
$Z\mu_{t+1}^{(i,m)}/\mu_t^{(i,m)}$	Stochastic discount factor after tax from t to $t+1$ (induced by agent i, m)
$Z\lambda_{t+1}^{(i,m)}/\lambda_t^{(i,m)}$	Stochastic discount factor before tax from t to $t+1$ (induced by agent i, m)
G_1, G_2	Auxiliary constants aggregate investment share and SDF
$w_t^{i,m}$	Total wealth in period t of agent type m born in period i
$a_t^{i,m}$	Financial wealth after tax in period t of agent type m born in period i
ρ, Ξ_2, Ξ_3	Auxiliary constants human and transfer capital
$\eta_{h,t-i}^{m}$, $\eta_{s,t-i}$	Price-to-production ratio of human and transfer capital of agent type m aged $t-i$
$\mathcal{V}_{t-i}^{m}$, $\mathcal{V}$	Individual and aggregate welfare measure
ε_{t-i}^{m}	Auxiliary function aggregate welfare measure
$\beta^G Y_t$, I_t^G, α_t^G	Government debt, investment and equity holdings from t to $t+1$
λ	Replacement ratio
F_{t-i}	Share in aggregate earnings of a generation aged $t-i$
χ^m	Income group-specific share in aggregate earnings
$\bar{\phi}$, ϕ_1, ϕ_2, ϕ_3	Constant and coefficients for cohort labor income polynomial
$\bar{\varphi}^{(m)}$, φ_1, φ_2	Constant and coefficients for subjective time discount factor polynomial

1

Introduction

The consideration of the distribution of income, wealth and consumption within countries around the globe reveals one basic and consistent picture: inequality in the allocation of resources is a prevailing global phenomenon and subject to an ongoing negative trend since, at least, the last three decades. Although economic inequality may be inevitable to some extent, the observed levels and trends in the distribution of resources have provoked a widespread debate and the call for policy makers to tackle rising inequality. In general, governments are equipped with a range of instruments and tools to influence distribution, with redistributive tax systems being considered the most direct, powerful and popular instrument in this context. Besides its influence on resource allocation, however, tax and transfer policies also possess substantial effects on many economic areas, for example, economic growth, financial markets and individual as well as aggregate welfare.

Undoubtedly, the knowledge about the true relationships of all these aspects and the understanding of the underlying causes is decisive for any policy maker. In particular, being aware of the interdependencies is crucial when designing public policy instruments, like redistributive tax systems, with the aim to affect macroeconomic growth, distribution of resources, and the formation of asset prices. Nevertheless, although widely discussed, the effects of taxation and redistribution as well as the underlying causes are not yet fully understood (see Fischer and Jensen (2015) and Pastor and Veronesi (2016)). This is the fundamental motivation of the present work, which aims to help fill this gap in order to provide a better understanding of the described relations. In particular, the objective is to simultaneously study the impact of redistributive taxation on the behavior of a heterogeneous population, macroeconomic development, welfare and asset prices.

The present thesis addresses this task quantitatively by the means of economic equilibrium models. In order to be of use in this context, these models need to capture several real-life properties that will be identified in the course of Chapter 2. Notably in this respect are the two dimensions of heterogeneity across individuals focused on in the present work: First of all, differences in the attitude concerning the intertemporal allocation of consumption, i.e., time preferences, are found to be of major importance in the context of inequality and redistribution. Beyond that, in order to capture the natural source of heterogeneity across individuals that is induced by differences in age, life-cycle characteristics are captured as a second important source of heterogeneity.

The thesis provides the following main contributions to the existing literature: First, in the course of the thesis two related models of different complexity are proposed that allow to study the simultaneous impact of redistributive taxation and heterogeneous time preferences in dynamic general equilibrium. Second, tractable solution methods are established to solve these frameworks. In this vein, analytical and partly closed-form solutions are obtained that facilitate the analysis of the model results and help to derive key relations. Third, the present work is the first to investigate the meaning of heterogeneity in time preferences across individuals' life-cycle within a dynamic general equilibrium framework. The resulting consumption function allows to reproduce and, thereby, helps to explain the empirically well-established hump-shaped pattern of consumption in the absence of any borrowing or short-sales constraint. Fourth, individual and aggregate welfare measures are derived that allow to distinguish between the short- and long-run effects of redistributive taxation on individual and aggregate well-being. To this end, this thesis is structured as follows:

Chapter 2 starts out by amplifying the motivation of the present work. Building on recent statistics the global phenomena of inequality and redistributive taxation are characterized, while the meaning of individual heterogeneity within this context is emphasized. Then, to be in accordance with these observations, the required model dimensions, i.e., real-life properties to be captured by an analytical approach to the above problem, are identified. Afterwards, the theoretical foundations needed to study the research objective within a consistent analytical equilibrium framework are outlined.[1] Finally, the chapter concludes with reviewing the relevant literature and presenting a brief overview of the survey approach that will be followed in the subsequent chapters.

Chapter 3 abstracts some of the dimensions and presents a simplified modeling approach. To be precise, the proposed framework builds on an exchange economy with classical demographic structure populated by different agent types heterogeneous with respect to their time preferences and initial financial endowment. Beyond that, the approach concentrates on taxation of capital income and ignores labor income taxation. The chapter is meant to provide first insights with regard to the complex equilibrium impact of a tax-based reallocation mechanism and agent heterogeneity on households' consumption and investment behavior as well as on

[1] Despite this review of relevant theories from finance and economics, the reader should be familiar with basic principles of these fields - especially in the area of asset pricing. Useful textbook treatments with a focus on finance (or asset pricing) are Cochrane (2005), Back (2010), Munk (2013) and Danthine and Donaldson (2015), whereas Ljungqvist and Sargent (2004), Lengwiler (2006) and Miao (2014) serve as references with a stronger focus on economics. Finally, some basic knowledge of mathematics, especially in the filed of optimization and probability theory, is required.

asset prices. Furthermore, the approach is used to further motivate the concentration on differences in time preferences as source of preference heterogeneity within the present thesis. As production output is exogenous in this setting, the framework is apparently unsuitable to investigate the effect of redistributive taxation on economic growth and any related feedback mechanism. The solution method in this chapter builds on the assumption of complete markets. Although a closed-form solution is not available in the context of heterogeneous time preferences, a tractable analytical solution is derived, where all equilibrium processes are given as functions of the initial distribution of consumption shares. Due to the time-dependent but deterministic nature of this function, a particularly simple deterministic equilibrium condition is determined that can readily be solved for by using numerical methods. Quantitative implications of redistributive taxation along with agent heterogeneity are provided by means of numerical examples.

Chapter 4 builds on the model of Chapter 3 but adds the missing dimensions. In particular, the framework is based on an economy with endogenous linear production technology populated by a finite number of overlapping generations, where every cohort is composed of different types of agents that are heterogeneous with respect to their stream of permanent life-cycle labor income and time preferences. Beyond that, and in line with the empirical evidence, time preferences are explicitly assumed to vary over the individual's life-cycle. Lastly, the approach considers both capital and labor income taxation. Due to the endogenous production decision, agents' individual investment and consumption behavior becomes decisive in determining aggregate production output. Consequently, there will be real effects on economic growth by redistributive taxation. The solution method in this chapter builds on a "guess and verify" approach, while the derivation of a stationary equilibrium solution is facilitated by the restriction on independent and identically distributed (i.i.d.) aggregate production risk. Age-dependent but again deterministic consumption shares are the result. In this vein, agents strive for a linear sharing rule, align their marginal rates of substitution in equilibrium and every equilibrium process can again be derived as a function of the consumption distribution, i.e., one endogenous parameter. Along the lines of Chapter 3, the result is a deterministic equilibrium condition that can be solved numerically. In order to study the quantitative implications of redistributive taxation and agent heterogeneity on macroeconomic and individual behavior as well as its welfare effects, numerical examples are used. For this analysis, an empirically plausible parameterization is chosen that is calibrated to match macroeconomic moments, tax revenues and life-cycle earnings profiles, while allowing for a hump-shaped life-cycle pattern of consumption.

Both Chapters *3* and *4* concentrate on heterogeneity in time preferences. They do, however, differ to some extent in where they locate their respective origins of heterogeneity due to the described differences in the model frameworks. To be precise, in the course of Chapter *3*, heterogeneity emanates per assumption by supposing different time preferences across all individuals that simultaneously live through the finite lifespan of the economy. In contrast, in the framework considered in Chapter *4* individuals may also hold identical preferences across their finite life-cycles, but agent heterogeneity would still arise due to the fact that various age groups coexist in every time step throughout the infinite lifespan of the economy.

Chapter 5 summarizes the ideas and findings of the preceding chapters and deduces several key issues important for policy makers. Finally, an outlook on future research is provided by addressing various ways in which to extended and enrich the present work.

2

Empirical and Theoretical Background

Over the last thirty years the distribution of income, wealth and consumption has become increasingly unequal across many countries around the world. This trend has provoked a widespread debate and the call for policy makers to tackle rising inequality. In general, governments are equipped with a range of instruments and tools to influence distribution, where redistributive tax systems are to be considered the most direct, powerful and popular instrument in this context. Besides a large body of studies within this area, the actual (simultaneous) effects of redistributive taxation systems on individual behavior, macroeconomic development and asset prices are still largely unclear.

Economic equilibrium models can help to better understand these relationships by addressing the task quantitatively. In order to be of use in this context, these models have to capture several real-life properties (model dimensions). To this end, established theories from finance and economics have to be combined to obtain a solid theoretical foundation. Past research has been fruitful in producing an enormous body of literature and has achieved a profound understanding with respect to many economic phenomena. Nevertheless, for several important financial and economic questions - like the one above - no or no satisfying answer has been found yet.

These considerations form the basis of the present work and will be further elaborated within this chapter. The remainder is organized as follows: Section 2.1 is used to motivate the present work. It starts by giving detailed information about the global phenomena of inequality and tax-based redistribution systems. Subsequently, it highlights the meaning of individual heterogeneity within this context. It closes by summarizing these observations and deriving the present research objective. Next, Section 2.2 establishes the theoretical requirements needed to study this objective within an analytical model framework. This methodical foundation deals with the concepts of neoclassical theory, asset pricing and overlapping generations. Section 2.3 starts out by reviewing the relevant literature and, finally, concludes the present chapter with a brief overview of the theoretical survey approach followed in Chapters 3 and 4.

2.1 Motivation

2.1.1 Income, Wealth and Consumption Disparity

Considering the distribution of income, wealth and consumption within countries around the globe reveals the basic and consistent picture that inequality in the allocation of resources is a prevailing global phenomenon. While economic inequality is inevitable to some extent, excessive inequality may be harmful for a sound economic development. In many countries and regions of the world such a degree of inequality may have been reached, as indicated by the increasing attention to and awareness of this subject that manifests itself in the enormous amount of recent reports and articles in this area. In this regard, reports have been published by numerous global organization, like for example the Organisation for Economic Co-operation and Development (OECD), the International Monetary Fund (IMF) and most recently the World Inequality Lab (WIL).[2] These reports take global and multi-country perspectives and document a high level of inequality that is accompanied by an increasing trend. In contrast to these reports, most academic research in this area has its main focus on the developments in the United States (U.S.).[3] Exceptions include Fuchs-Schündeln et al. (2010), Atkinson et al. (2011) and Alvaredo et al. (2013). Besides these geographical differences the findings, however, are basically identical.

When looking at distributional differences, income inequality is typically the first reference point and, hence, certainly the one that is best documented. In this context, the left panel of Figure 2.1 gives an overview of the distribution of income in some developed countries. In particular, it depicts the income shares of the top 10% (white bars) and top 1% (gray bars) of the pre-tax national income distribution in selected OECD countries, using the latest data from the World and Income Database. While inequality does vary by this measure between the different countries, it is at a remarkably high level in all developed countries considered. The lowest degree of income inequality is documented for Denmark, where the top one and ten percent of the population receive about 6.4% and 26.9% of the total pre-tax national income, respectively. The United States are located on the other side of this spectrum, where the top one and ten percent own a share of about 20.2% and 47.0% of the total pre-tax national income, respectively.

As pointed out by Krueger and Perri (2006), however, studying (exclusively) the distribution of current income may not be sufficient, especially if one aims to study welfare aspects. One way to expand this view is to addition-

[2] See OECD (2015), IMF (2017) and Alvaredo et al. (2017).

[3] See Krueger and Perri (2006), Heathcote et al. (2010), Favilukis (2013), Attanasio and Pistaferri (2016), Krueger et al. (2017), Wolff (2017) and Fisher et al. (2018).

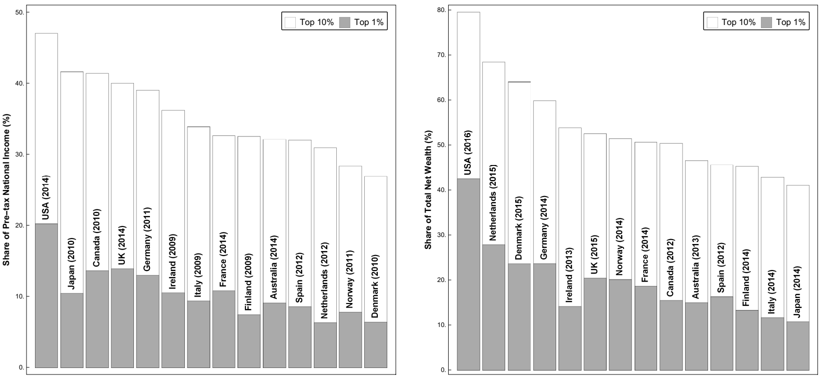

Figure 2.1 – This figure shows the income (left panel) and wealth (right panel) shares of the top 10% and top 1% of the respective income and net wealth distribution in selected OECD countries. (Source: World and Income Database (income data), OECD Income Distribution Database (wealth data))

ally consider inequality in wealth across the population. In these terms inequality tends to be even more pronounced than in income terms. One reason for this is that wealth can itself generate income and, thereby, widen income inequalities, which in return widens wealth inequalities again (Keeley (2015)). According to the OECD (2015), half of total wealth is held by the wealthiest top 10%, nearly the other half is owned by the next 50%, while solely about 3% is held by the remaining bottom 40%. The right panel of Figure 2.1 confirms this picture for various selected OECD countries by depicting the wealth shares of the top 10% (white bars) and top 1% (gray bars) of the total net wealth distribution, using the latest data from the OECD Income Distribution Database. Compared to the documented income inequality, illustrated in the left panel of Figure 2.1, wealth is considerably more concentrated in all of these countries. Again, figures vary across countries, but are at high levels throughout. Wealth inequality is tremendously pronounced in the United States, where the top one and ten percent of the population hold about 42.5% and 79.5% of total net wealth. In Japan, where the top one and ten percent own about 10.8% and 41.0% of total net wealth, the distribution is more moderate.

Large parts of the inequality debate are actually based on the distribution of income and wealth. Nevertheless, in order to study welfare implications, inequality in individual consumption levels seems to be a more direct and suitable measure of well-being, as pointed out by Heathcote et al. (2010). This becomes apparent when considering the classical approach taken in most theoretical economic models, where individual behavior is assumed to be exclusively driven by the households' ultimate goal to optimize lifetime well-being over consumption (Attanasio and Pistaferri (2016)). In this context, income and wealth are just means to an end, while only consumption is associated with utility.

One reason for the concentration on income and wealth within the ongoing discussion is the fact that appropriate and consistent data on household consumption is usually rare, especially compared to income data (Attanasio and Pistaferri (2016)). As is often the case, most data in this regard exists for the United States and, therefore, large parts of the academic research is limited to this region. One exemption is the experimental statistics on the distribution of consumption in selected European countries provided by Eurostat. Building on this data, Table 2.1 gives an overview of the consumption distribution in various European countries as well as the United States and compares these figures to the distribution of national income and wealth. In particular, it reports the consumption, income and wealth distributions of these countries as measured by their Gini coefficient.[4] The general picture is as follows: inequality in consumption is basically a little less pronounced compared to income inequality and considerably lower than wealth disparity. A considerable level of inequality, however, is also prevailing in the consumption distributions of these developed countries.

Beyond that, distributional inequality is not an exclusive phenomenon in western countries but widespread and can be observed in a more or less

[4] The Gini coefficient is a common measure of inequality. Its range is between zero and one, where zero expresses perfect equality (all households of a population hold the same share) and one corresponds to maximal inequality (only one household owns all).

Table 2.1 – This table reports the consumption, income and wealth distributions for various European countries (2010) and the USA (2006) as measured by their Gini coefficient. (Source: Eurostat, "Gini coefficient on household population - experimental statistics", 2010; U.S. data is for 2006 and taken from Krueger et al. (2017) based on data from the Panel Study of Income Dynamics)

	Gini coefficients (2010)		
Country	Consumption	Income	Net Wealth
Denmark	0.30	0.37	n/a
Germany	0.33	0.37	0.75
Ireland	0.33	0.35	n/a
Spain	0.35	0.38	0.58
France	0.34	0.36	0.68
Italy	0.37	0.37	0.61
Finland	0.36	0.35	0.65
UK	0.36	0.39	n/a.
USA (2006)	0.40	0.42	0.77

pronounced manner in any major geographical region as well as from an aggregate global perspective. Moreover, it is not a static phenomenon. Quiet the contrary, an ongoing negative trend in the allocation of resources within countries can be documented for, at least, the last three decades (see, for example, OECD (2015), Alvaredo et al. (2017), IMF (2017)). Figure 2.2 illustrates this development. In particular, the left panel depicts the persistent increase of the shares in national income received by the top 10% in India, USA and Canada, Russia, China as well as Europe from 1980 to 2016. During this period, income inequality rose in all of these world regions, although at a different pace. While Europe experienced only a moderate increase, the development in the other regions has been much more pronounced, especially in India and Russia. Moreover, in almost all world regions income inequality has reached its highest value since the last thirty years.

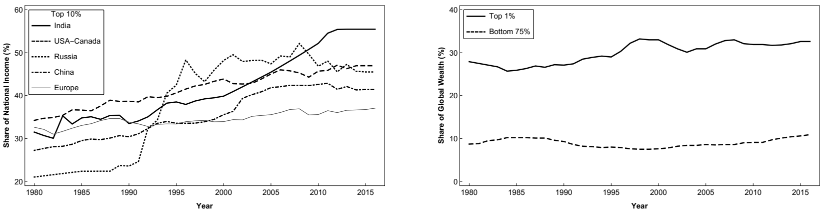

Figure 2.2 – This figure shows the evolution of the top 10% income shares for major geographic regions across the world (left panel) as well as the evolution of the top 1% and the bottom 75% global (represented by China, Europe and the United States) wealth shares (right panel). (Source: adapted from Alvaredo et al. (2017, Figures E2a and 4.1.1) based on data from the World and Income Database)

This trend also carries over to the distribution of wealth from a global perspective, as shown in the right panel of Figure 2.2. It illustrates the evolution of the wealth shares held by the top 1% and the bottom 75% in total global wealth from 1980 to 2016.[5] While the share in global wealth of the wealthiest top 1% was at about 28% in 1980, it went up to about 33% in 2016. In contrast, the share in global wealth owned by the bottom 75% of the population oscillates around 10% at the same time.

Finally, aggregate global statistics for the development of the distribution of consumption shares are not available. Data for the United States, however, indicate a similar trend, although consumption inequality seems to grow at a more moderate speed (see Krueger and Perri (2006)).

2.1.2 Redistributive Taxation and Transfers

Although economic inequality may be inevitable to some extent, the documented levels and observed trends in the distribution of resources have provoked a widespread debate and the call for policy makers to tackle rising inequality (see Bloch et al. (2013), Piketty and Goldhammer (2014), OECD (2015), IMF (2017) and OECD (2017a)). National policies and insti-

[5] Following Alvaredo et al. (2017), the global level is represented by China, Europe and the United States.

tutions are actually decisive in this context, as evidenced by the differences in inequality levels across countries of the same development state and over time (Alvaredo et al. (2017)). In general, governments are equipped with a range of instruments and tools to influence the distribution of (especially) income and wealth across households. In this regard, a tax-based redistribution system constitutes a direct, powerful and popular instrument to tackle inequality. As pointed out by Fischer and Jensen (2014, 2015), in large parts of the industrialized world individual income taxation along with social insurance and income support programs were introduced within the last century. Transfer payments to households by means of a redistributive taxation system is, therefore, found to be a globally observed phenomenon.

Besides being widespread, income taxation and transfer payments account for a considerable share in the national accounts of most developed countries, as illustrated by the left panel of Figure 2.3. It shows the total tax revenues raised (white bars) from households and the total public cash benefits paid to households (gray bars) for selected OECD countries in percent of the country's gross domestic product (GDP) in 2013. On OECD average, total tax revenues raised from households constitute 24.1%, while public spending on cash benefits aggregate to 12.4% of GDP. The size of tax revenues and cash transfers, however, varies significantly across countries. In the United States, for example, tax revenues amount to 20.1% and cash benefits to only 9.3% of the GDP in 2013, whereas in Denmark these instruments measure 41.6% and 13.8% in the same period, respectively.

The right panel of Figure 2.3 highlights the implied redistribution effect due to taxes and transfers for the selected OECD countries in 2010 (gray rectangles) and 2013 (white bars). In particular, redistribution is measured as the difference between market income and disposable income inequality (Gini coefficient), expressed as a percentage of market income inequality. In 2013 (2010), redistribution by means of taxation and transfers reduced income inequality by about 26.2% (27.7%) on OECD average.

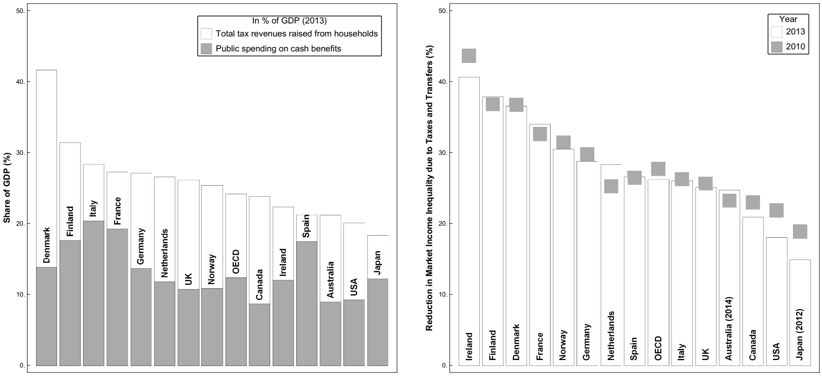

Figure 2.3 – This figure plots tax revenues from and cash benefits to households as well as redistribution effects due to taxes and transfers for selected OECD countries. The left panel shows total tax revenues raised from households and total public cash benefits paid to households in percent of the country's GDP in 2013. Total tax revenues raised from households comprise taxes on income, profits and capital gains of individuals, taxes on property, taxes on goods and services as well as employees' social security contributions (Source: OECD Tax Statistics Database). Public cash benefits refer to old age and survivor pensions, incapacity benefits, family cash benefits, unemployment and other social policy areas categories (Source: OECD Social Expenditure Database). The right panel presents the income redistribution effect due to taxes and transfers for 2010 and 2013. Redistribution is defined as the difference between market income and disposable income inequality (measured by the Gini coefficient), expressed as a percentage of market income inequality (Source: adapted from OECD (2016, Figure 5) based on data from the OECD Income Distribution Database).

Again, country specific figures differ significantly. Taking the same example as above, income inequality in the United States is reduced by about 18.0% (21.8%), whereas it is decreased by about 36.5% (36.7%) in Denmark due to taxes and cash benefits in 2013 (2010).

While the observed effects are considerable throughout, redistribution weakened on average between 2010 and 2013. Furthermore, inequality figures rose at the same time, which fueled the public debate about more equality and the call for more redistribution. With regards to the instrument of redistributive taxation, more redistribution would apparently imply increasing transfers by raising tax rates. Nevertheless, and unfortunately, the effects of tax and transfer policies do not exclusively affect equality measures but have substantial effects on many economic areas, especially on growth. Both in the public as well as in the academic debate there is no consensus about the macroeconomic effects of taxes and transfers (Cloyne (2013)). The arguments range from the conviction that increases in tax rates may support growth, and the conviction that they do not necessarily harm growth, to the insistence that they hinder economic growth. Most academic evidence seems to support the latter argument, as, for example, documented in Blanchard and Perotti (2002), Romer and Romer (2010), Barro and Redlick (2011) and Cloyne (2013). However, as argued in IMF (2017), there is no systematic adverse trade-off between increasing growth and decreasing inequality, for instance, due to increased taxes and transfers. To the contrary, excessive inequality, i.e., not redistributing, is associated with lower economic growth. This view finds (at least) some support in the results by Romer and Romer (2016), who report an increase in aggregate consumer spending following a permanent increase in social security benefits.

In this context, one may clearly ask for the meaning of economic growth for a society. There appears to be, at least in this regard, a broader consensus in that growth (at least to some extent) is an elementary aspect for a positive development of social well-being. Nevertheless, as often argued, it may not be equated with social well-being, which might be higher considering lower growth and greater equality in the distribution of income, wealth and/or consumption. If redistribution, however, is associated with negative effects on growth, income, wealth and consumption levels will be affected negatively at the aggregate level, too. In this vein, redistributive taxation might lead to a higher degree of equality but lower economic welfare and, moreover, even reduce individual welfare levels of net recipients of the transfer system (see Fischer and Jensen (2014)). Hence, not only the direction but also the size of the effects might be important.

Besides the effect on economic growth, the empirically observed influence of tax changes on asset prices should be pointed out (see Dai et al.

(2008) and Sialm (2009)). Well operating capital markets are the backbone of modern economies and considered a key factor of economic growth. Moreover, they are important from a household perspective, as they provide them with a broad asset universe and the possibility to participate in the economic development. Hence, the effects of taxes and transfers on investment possibilities, i.e., asset prices, might play a decisive role in the context of economic equality and welfare. Unfortunately (from an equality perspective), however, stock market participation rates are low, especially within low income households. While the participation rate (considering direct or indirect stock holdings) of the top 10% of households with highest income lies around 90.6%, it is only about 12.5% for the bottom 20% (Fischer and Jensen (2015), based on data from the 2010 Survey of Consumer Finances).[6] Herein lies an additional fact that might foster distributional inequality.

The knowledge about the true relationships of all of these aspects and the understanding of the underlying causes is clearly decisive for any policy maker. In particular, being aware of the interdependencies is crucial when designing public policy instruments, like redistributive tax systems, with the aim to influence macroeconomic growth, distributional effects, and the formation of asset prices.

2.1.3 Individual Heterogeneity

One aspect in the context of inequality and redistribution has not been considered yet: the fact that individuals are not all identical. On the contrary, real-life economies are characterized by a large amount of individuals that are heterogeneous with respect to a number of different char-

[6]There is a large body of literature studying stock market participation rates. See, for example, Mankiw and Zeldes (1991), Campbell (2006), Calvet et al. (2007), Christiansen et al. (2008), Calvet et al. (2009a), Calvet et al. (2009b) and Giannetti and Koskinen (2010). The impact of participation rates on asset prices is studied by, for instance, Brav et al. (2002), Gomes and Michaelides (2008), Favilukis (2013),Gomes et al. (2013), Fischer and Jensen (2015) and Pastor and Veronesi (2016).

acteristics. From an economist's point of view, differences in households' preferences, especially over consumption as well as the distribution of consumption across time and economic states, are most important. These preferences determine households' consumption-saving as well as investment decisions and, thereby, link individual behavior to individual welfare and, on an aggregate level, to the distribution of income, wealth and welfare as well as to macroeconomic development. In the light of heterogeneity and different tastes, a total equalization through redistribution may not only be economically harmful, but also counterproductive in the attempt to achieve higher levels of welfare (equality). When studying inequality and the effects of redistributive taxation, it is most important to capture and model the different preferences of individuals as well as the heterogeneous behavior that results from them. To that end, an overview of the most important preference characteristics and the empirically observed socioeconomic and age-related differences is needed.[7]

The existence of preference heterogeneity is supported by numerous studies, which mostly rely on household survey data. The results indicate heterogeneity in time preferences (or patience), risk aversion and the elasticity of intertemporal substitution (EIS) in consumption (Hendricks (2007)). Although there is a broad consensus about the existence of differences across household preferences, the concrete results are often inconsistent. To maintain clarity, the present overview will, therefore, concentrate on the relationship between preference heterogeneity and the key household characteristics income and age.[8]

[7] It is important to note that in the present work it is assumed that (age-unrelated) preference heterogeneity arises from the socioeconomic characteristics individuals are born into, and not the other way around. In particular, this implies that preferences of households remain unchanged regardless of any policy instrument that may influence their actual situation. Undoubtedly, this is the prevailing approach also chosen in many other studies. Nevertheless, the direction of the causality between heterogeneous behavior and socioeconomic factors is generally unclear (Lawrance (1987)).

[8] It is noteworthy that in many studies income is used as proxy for wealth.

Starting with the differences in households' risk aversion, i.e., the attitude towards atemporal risks, it is important to stress that it expresses an individual's aversion to a consumption stream that varies across different states of nature. In this sense, higher levels of risk aversion imply that agents tend to increasingly prefer certain claims to future consumption to uncertain claims. When inspecting the existing evidence of the link between risk aversion and household income, the direction of causality remains uncertain. Guiso and Paiella (2008) find a negative relation by empirically documenting that richer agents exhibit smaller levels of risk aversion than poorer households. In sharp contrast, the studies by Booij and van Praag (2009) and Bosch-Domènech and Silvestre (1999) estimate higher degrees of relative risk aversion with increasing income. This implies that gambles proportional to wealth become less attractive with increasing levels of wealth. As pointed out by Booij and van Praag (2009), however, there seems to be no a priori reason for neither direction of causality. In support of this view, and one possible reason for the contradictory findings, is a parabolic relationship between risk aversion and wealth, as found by Halek and Eisenhauer (2001). Nevertheless, the true relationship remains vague. With regard to the link between age and risk aversion the documented relationship is clearer. In general, the empirical evidence finds the degree of risk aversion to be either increasing or U-shaped in age (see Pålsson (1996), Donkers and van Soest (1999), Halek and Eisenhauer (2001), Hartog et al. (2002)). Yet the findings in Booij and van Praag (2009) indicate a negative correlation with age, although the effect is not significant.[9]

The elasticity of intertemporal substitution in consumption describes the attitude towards consumption shifts over time.[10] Intuitively, it measures

---

[9]With regard to other socioeconomic factors there exists conformity that males are less risk-averse than females (see Booij and van Praag (2009)). Considering education, results are ambiguous. Alan and Browning (2010), for instance, find that less educated households are less risk averse, whereas Outreville (2015) documents a negative correlation.

[10]In economic models with preferences defined by standard power utility risk aversion

the individual's willingness to pre- or postpone consumption across time in response to changes in the expected real interest rate. Consumers with a high EIS, for example, save more if interest rates are high. Despite a large body of literature trying to determine the size of the EIS, there is no consensus about its real value (see, for example, the discussions in Vissing-Jørgensen (2002), Havranek et al. (2015) and Thimme (2017)). Fortunately, the evidence regarding the link between EIS and income is more definite. In particular, several studies directly deduce a positive correlation between the level of income and the individuals' willingness to shift consumption intertemporally (see Lawrance (1991), Blundell et al. (1994) and Guvenen (2006)). This finding is supported indirectly by a number of related studies. Attanasio and Browning (1995), for example, find a positive link between the EIS and household consumption, which can be interpreted as a proxy for income. Furthermore, parts of the relevant literature concentrate on the differences in the EIS between asset holders and non–asset holders. In this regard, Attanasio et al. (2002) and Vissing-Jørgensen (2002) find higher EIS for asset holders than for non-asset holders. As the former are on average wealthier than the latter, this also supports the view that the EIS may be increasing in income (Guvenen (2006)). Finally, Havranek et al. (2015) compare the elasticity of intertemporal substitution across countries and find the EIS to be higher for households in rich countries.[11] Unfortunately, there are no studies that document the impact of age on the elasticity intertemporal substitution.

Time preferences characterize the attitude concerning the intertemporal allocation of consumption. There is unambiguous evidence that patience alters with age and across income levels. In the latter's case, there also ex-

and EIS are not independent but directly linked. In particular, the elasticity of intertemporal substitution is given as just the inverse of the risk aversion coefficient, in this case. Since some of the studies mentioned here rely on this type of preference specification, results found for EIS partly carry over to results found for risk aversion, and vice versa.

[11] Besides these relationships, Alan and Browning (2010) find a negative correlation between the EIS and the level of household education. That is, the more educated are found to be less willing to shift consumption intertemporally than the less educated.

ists a high level of conformity within the empirical literature regarding the relationship. To be precise, patience is consistently found to be higher for high income households than for low income households across several studies (see Samwick (1998), Lawrance (1991) and Booij and van Praag (2009)). With regard to the former, however, the documented relationship remains ambiguous. Although some studies document an opposite relation (see Rogers (1994) and Samwick (1998)), most studies find that patience is higher for young adults than for elderly individuals. This relationship is explained by declining mental and physical abilities, decreasing fertility and increasing mortality or generally shortened remaining lifetime with age. Nevertheless, there exists inconsistency regarding the relation between discounting of middle-aged individuals and discounting of young adults. On the one hand, relying on the above mentioned factors, a number of studies predict that discounting is a monotonic function of age, implying patience to be highest for young adults and constantly declining over the life-cycle (see Trostel and Taylor (2001) and Booij and van Praag (2009)). On the other hand, numerous studies find a hump-shaped pattern of subjective time discount rates over the life-cycle. That is, patience increases until middle age and declines afterwards, so that elderly individuals display the highest degree of impatience, whereas middle-aged individuals are the most patient (see Harrison et al. (2002), Sozou and Seymour (2003), Read and Read (2004), Chu et al. (2010) and Kageyama (2013)).[12]

Finally, although the described evidence is often vague, allowing for preference heterogeneity is essential in order to rationalize and replicate several observed properties of the wealth and consumption distribution. With respect to the inequality patterns documented above, and as pointed out by Alan and Browning (2010), the most important of these properties is provided by the observed heterogeneity in households' lifetime wealth accu-

[12]Again, other socioeconomic characteristics have equally been studied. For example, Coller and Williams (1999), Donkers and van Soest (1999), Read and Read (2004) as well as Booij and van Praag (2009) find women to be more patient than men. Furthermore, Harrison et al. (2002) as well as Kapteyn and Teppa (2003) report a positive correlation between education and patience.

mulation (even when earnings profiles are identical). In order to capture this fact within an economic model, heterogeneity in time preferences is required to be included in the analysis (see Samwick (1998), Krusell and Smith (1998), Hendricks (2007) and Gomes et al. (2013)). Motivated by this fact, and the presumably clearest evidence on the income- and age-related relationship, the present work will focus on this type of preference heterogeneity.

2.1.4 Conclusion and Objective

The preceding exposition documented the basic and consistent picture that inequality in the allocation of resources is a prevailing global phenomenon. Moreover, an ongoing negative trend in the distribution of income, wealth and consumption within countries can been found for, at least, the last three decades. Although economic inequality may be inevitable to some extent, the observed levels and trends in the distribution of resources have provoked a widespread debate and the call for policy makers to tackle rising inequality. In general, governments are equipped with a range of instruments and tools to influence distribution, where redistributive tax systems are to be considered the most direct, powerful and popular instrument in this context. Besides its influence on resource allocation, tax and transfer policies also possess substantial effects on many economic areas (for example growth, asset prices and welfare). Both in the public as well as the academic debate, however, there is no consensus about the actual macroeconomic effects of these policies.

Knowing about the true relationships of all of these aspects and understanding the underlying causes is undoubtedly decisive for any policy maker. In particular, being aware of the interdependencies is crucial when designing public policy instruments, like redistributive tax systems, with the aim to influence macroeconomic growth, distributional effects, and the formation of asset prices. Economic equilibrium models address this task quantitatively and can help to understand these relationships as well as the impact on individual consumption and investment behavior. In this context, it is, however, essential to capture the fact that populations are not uniform but composed of a large number of individuals that, depending on their diverse characteristics (income and age), differ in their preferences. With respect to the reported inequality patterns, heterogeneity in individual time preferences is found to be most important.

Although well documented empirically, the effects of taxation and redistribution in general equilibrium as well as the underlying causes are not yet fully understood (Fischer and Jensen (2015) and Pastor and Veronesi (2016)). Herein lies the fundamental motivation of the present work, that aims to help fill this gap in order to gain a better understanding of the described relations. In particular, the objective is to simultaneously study the impact of redistributive taxation on the behavior (consumption-savings and portfolio decision) of a heterogeneous population, macroeconomic development, welfare and asset prices.

To that end, and to be in line with the documented facts, the study approach ought to comprise six dimensions: Obviously, a suitable economic model must allow for the reallocation of resources through a taxation system. Hence, *redistributive taxation* is assumed to be the first dimension. Next, as explained above, reallocation is assumed to be necessary because of the documented inequalities in the distribution of resources. In order to account for this property within an economic model, it requires heterogeneity in income, preferences and age. The first two aspects are taken together into the second dimension - *agent heterogeneity*. The latter aspect is considered separately and forms the third dimension - *life-cycle characteristics*. Beyond that, in order to investigate the influence on individual consumption-saving and portfolio decisions, the objective requires a household behavior that follows endogenously from the model. *Endogenous individual behavior* is, therefore, the fourth dimension. Finally, taxation and transfer systems possess substantial macroeconomic effects.

Most important are the impacts on macroeconomic growth and the formation of asset prices. In order to capture the feedback effects between growth and redistributive taxation, macroeconomic development must follow endogenously from the individual behavior of the model's agents. This requires the fifth dimension of an *endogenous* (macroeconomic) *production* decision. Lastly, the feedback mechanism between redistribution and capital markets is captured by the *asset pricing* dimension.

The remainder of this chapter is structured as follows: Section 2.2 establishes the theoretical requirements by presenting the methodical foundation for the analytical model framework. The first part of Section 2.3 then gives a comprehensive review of the relevant literature, while the second part closes the present chapter by presenting a brief overview of the theoretical survey approach followed in Chapters 3 and 4.

2.2 Methodical Foundation

In order to address the research objective described above, a theoretical foundation is needed that formalizes the problem and allows to treat it mathematically. In particular, it must allow to address all six dimensions of the problem described above and, in this sense, settle it within a consistent economic framework.

The fundamental theoretical approach that allows to establish the formal basis can be found by the assumptions underlying what is called "neoclassical economics".[13] A metatheory or set of rules that has become the center piece of mainstream finance and economic theories developed over the past century. Its fundamental assumptions help to capture the dimensions *endogenous individual behavior*, *agent heterogeneity*, *endogenous production* as well as partly *life-cycle characteristics* and are outlined in Section 2.2.1.

[13]The term "neoclassic" in the context of economics seems to be mainly due to Veblen (1900), who is considered to be the first to have used the expression "neoclassical economics".

Although neoclassical theory provides a consistent formal framework, it needs some more concretization in order to address the present research question. This specific model framework is provided by general asset pricing theory and its stochastic discount factor or "SDF"-concept that developed from the assumptions of neoclassical economics. In the context of the present work, this theory establishes the structure for the dimensions of *redistributive taxation* as well as *asset pricing*. The relevant aspects are described in Section 2.2.2.

Finally, in order to capture the natural source of heterogeneity across individuals that is mainly due to differences in age and to complete the dimension of *life-cycle characteristics*, the concept of an overlapping generations or "OLG"-framework is introduced in Section 2.2.3. It implies a nontrivial population structure that is not accounted for in the standard asset pricing framework.

2.2.1 Neoclassical Foundation

The neoclassical paradigm provides a consistent mathematization of economic theory and, hence, represents a methodical framework suitable to formally address the problem outlined above. Although there is not one general definition of neoclassical economics, especially because it is subject to ongoing development itself, there are some assumptions that can be considered fundamental in neoclassical theory. In this regard, for instance, Jäkel (2006) presents a comprehensive overview of neoclassical theory, while Aspromourgos (1986) and Colander et al. (2004) as well as the textbook treatments of Henry (2012) provide detailed examinations of its historical development. In line with these authors, the present exposition gives an overview of the relevant building blocks of the neoclassical paradigm and, thereby, establishes the methodical foundation required for the analytical assessment of the economic problem outlined above.[14]

[14]Since this implies that the present section does not give an extensive treatment of the history of neoclassical economics, the interested reader shall be referred to the references

In the sense of Colander et al. (2004), the corner stones of neoclassical economics can be summarized by the "holy trinity" - *rationality*, *selfishness*, and *equilibrium* - which will be outlined in the following.

The first neoclassical pillar, *rationality*, proposes that all relevant economic agents follow a certain kind of behavior. In particular, they are considered to possess equal access to identical and full information and they use this information to form rational expectations about the future, which is supposed to culminate in independent and rational decisions.

Related to this is the second pillar, *selfishness*, that establishes the objectives driving rational decision making and the way they are pursued. Thereby, the neoclassical approach distinguishes two types of agents, households and firms, that follow selfish interest. On the one hand, buyers or households seek to obtain goods in order to maximize the gains obtained from consuming them. This gain is measured by expected utility, a concept which builds on the axiomatization of preferences established by Bernoulli (1738) and was further developed by von Neumann and Morgenstern (1944). On the other hand, producers or firms generate the consumption goods by implementing productive factors like labor and capital into a neoclassical production technology that, like utility, builds on the axiomatization of Bernoulli (1738) and was extended to the production side by Walras (1874) and Marshall (1890). The selfish objective followed by firms is the maximization of profits obtained by minimizing costs and selling the output to households. In this very context, the center piece of neoclassical economic behavior dates back to the marginalist revolution that was developed independently by Jevons (1871) in England, Menger (1871) in Austria and Walras (1874) in Switzerland (Clarke (1991)). It implies that both utility and production technology are characterized by marginal diminishing utility or productivity.

Closely connected to this marginalism is the last neoclassical pillar, the concept of *equilibrium*. It implies that all relevant prices and quantities are

mentioned in the text above.

eventually found through the supply and demand forces of the market. In particular, it is the desire of households to maximize utility and the associated insatiable appetite for consuming goods that constitute the demand force in the economic market. Households, however, are constrained in the sense that the amount of goods available or affordable to them is limited. The limitations result from the firms' ultimate goal to maximize profits and the constraints they face, which are mainly due to scarcity of resources or production factors. Firms decide upon production quantities in order to achieve their objective. In this vein, goods become costly and its supply is restricted. Referring to marginalism and equilibrium, on the demand side, the concepts imply that households will adapt their demand for goods until the marginal gain corresponds to the marginal costs associated with an extra unit of the good. Similarly, on the supply side, firms will adapt their supply until the cost of producing a marginal unit of the good is just balanced by the revenue it generates. Following this adjustment process, an equilibrium is finally achieved in which supply equals demand and market prices result accordingly. In other words, an economy is in equilibrium if, given preferences, technologies and limitations, each agent buys or sells the optimal quantities of goods at a certain price, for which aggregate supply equals aggregate demand.

These ideas of neoclassical theory can be clarified using the seminal model developed by Fisher (1930). This will be done in the subsequent Section 2.2.1.1. After that, Sections 2.2.1.2 and 2.2.1.3 will deepen the understanding of the demand and supply side of the economy by formalizing them. To this end, the concepts of neoclassical utility and production will be introduced.

2.2.1.1 Fisher Separation

The model introduced by Fisher (1930) is one of the most important contributions that builds on the neoclassical rationale. It develops a consistent economic framework that combines the independent production and consumption decisions of firms and households mentioned above in a general equilibrium model with a perfect capital market. It is thought to be the foundation of modern financial economics and asset pricing theory.[15]

In a stylized deterministic two-period world, rational households maximize their utility from consumption by trading off the distribution of an initial endowment between present and future consumption. In order to substitute consumption units between today and tomorrow, households possess two investment opportunities. On the one hand, they can buy equity shares (or stocks) of a firm and, thereby, invest into the production technology of the economy. According to neoclassical marginalism, this technology is characterized by marginal productivity, which implies that the marginal rate of return earned by the households falls as investments rise. On the other hand, they have access to a perfect capital market that provides them with the possibility to intertemporally exchange consumption units with each other at a fixed interest rate. Finally, the firm's manager (installed by the households) maximizes profits by adjusting the demand for investment inputs.

Figure 2.4 illustrates the elements of the model graphically. The firm's investment opportunity set is characterized by a concave production (or intertemporal transformation) technology. The slope of this curve in one point reflects the marginal rate of transformation (or return) for a certain investment, and its general shape, just in line with neoclassical marginalism, reflects that marginal productivity is diminishing. Households' utility maximizing behavior is characterized by convex indifference curves that display different consumption plans which provide them with the same amount of utility. Its slope corresponds to an individual's marginal rate of substitution of consumption, i.e., it displays his willingness to shift consumption between the two time steps. The shape of the indifference curves

[15] See, for example, Copeland et al. (2014) for a textbook treatment of the Fisher (1930) model and the "Fisher Separation Theorem".

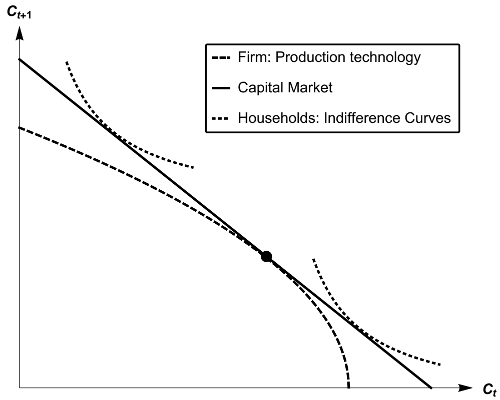

Figure 2.4 – This figure illustrates the neoclassical general equilibrium model featuring firm's production, households' utility maximizing consumption and a perfect capital market.

follows directly from the assumptions about the individuals' intertemporal preferences over consumption and, hence, reflects the property of diminishing marginal utility indirectly. Finally, the capital market is depicted by the straight line that is tangent to the firm's production technology and the households' indifference curves in equilibrium. Its slope represents the return on the capital market.

The existence of a perfect capital market implies that the investment decision of households will consist of two steps. First, all rational households, independent of their preferences, will want the firm's manager to invest into the production technology until the marginal rate of return earned from this investment opportunity equals the fixed interest rate paid on the capital market. In this point, all investments that provide a higher return than what is earned on the capital market are realized. In other words, the manager is forced to implement the profit-maximizing strategy by the firm's rational shareholders. This solution constitutes the firm's point of optimal production and, thereby, determines the available amount of aggregate consumption in the economy. From this point on, in a second step, households choose to borrow or lend on the capital market in order to maximize utility from consumption according to their intertemporal preferences over consumption. Their preference-dependent individual point of optimal consumption, therefore, lies on the capital market line and is ultimately achieved when the individual's marginal rate of substitution of consumption equals the interest rate. Note, however, that since aggregate consumption is specified by optimal production, capital market activities require market participants that take complementary positions.

The fundamental results of this model are known as the "Fisher Separation Theorem": First, the existence of a perfect capital market implies that the equilibrium investment into the production technology and, hence, firm's profit maximization is independent of the preferences of its owners. And second, households achieve their individual utility maximizing consumption plan independent of this investment decision by borrowing or lending on the capital market.

The Fisher Separation Theorem and its neoclassical general equilibrium model are the origin of the development of many modern finance and economic theories. In particular, it constitutes the starting point of the split into the two sub-categories of general equilibrium modeling known as the *consumption-based* and *production-based* approach. As indicated by its name, the former focuses mainly on the consumption side of the economy. It links the utility maximizing consumption decision of the rational households, more precisely their marginal rates of substitution, to the evolution of asset prices and returns. In contrast, the latter approach concentrates on the production side of the economy and substitutes firms and production technologies with consumers and utility functions. In this vein, asset prices and returns are linked to the firm's marginal rate of transformation.[16]

Since the objective of the present work is to study the general equilibrium effect of a tax-based mechanism of redistribution between house-

[16] This interpretation, i.e., the separation of general equilibrium theory into two sub-categories, builds on Cochrane (2005) and Jäkel (2006).

holds, in particular its impact on the consumption-savings and portfolio decision of heterogeneous individuals as well as the associated macroeconomic, welfare and asset pricing implications, it necessarily focuses on the consumption-based approach. To be more precise, the consumption-based asset pricing model will serve as concrete theoretical foundation. Details about the relevant aspects of modern consumption-based asset pricing theory will be presented in Section 2.2.2. Leading up to it, the following sections will concentrate on the formal specification of the neoclassical consumption and production decision and, thereby, relax some of the major simplifications of the model presented above.

2.2.1.2 Expected Utility and Life-cycle Consumption

To formalize the demand side of the economy, the decision problem faced by households has to be further specified. For that purpose, it is important to note that neoclassical consumption theory has developed compared to the model of Fisher (1930). On the one hand, modern economic models, without exception, allow for uncertainty in the consumption process. The foundations for this were laid by the advancement of the axiomatization of preferences established by Bernoulli (1738) to expected utility by von Neumann and Morgenstern (1944). On the other hand, economic research has removed the limitation of the intertemporal consumption problem to only two periods and replaced it by multidate or life-cycle consumption and income theories. In this context, the models developed by Friedman (1957) and Modigliani (1966) are considered to be the seminal contributions. In order to capture all the dimensions of the research question brought up above, this progress is crucial. In particular, expected utility and life-cycle consumption theory especially provide the basis for the dimensions *endogenous individual behavior, agent heterogeneity* and partly *life-cycle characteristics.*[17]

[17] The exposition in the present section is mainly inspired by Jäkel (2006) and Munk (2013).

Consider individuals that care for the consumption of goods only, where the number of different goods shall be restricted to one in the following.[18] In that sense, individuals possess preferences over consumption and need to decide between different (uncertain) consumption plans that represent distinct consumption distributions over time and states of the world. This means that individual consumption takes place at different points in time over the individual life-cycle and that, although a certain consumption plan is chosen, the realization of future consumption remains uncertain due to some sources of risk that influence the availability and/or affordability of the consumption good.

As outlined above, the neoclassical objective followed by the households is to maximize the (expected) gains obtained from consumption. In order to find the optimal consumption plan that maximizes lifetime well-being, preferences are formalized by a von Neumann-Morgenstern expected utility function over consumption (von Neumann and Morgenstern (1944)). Formally, at a certain point in time t a distinct individual maximizes his expected lifetime utility given by the following time-separable additive utility function

$$U_t = \sum_{n=0}^{N} \delta^n \mathbb{E}_t \left[u(c_{t+n}) \right], \tag{2.1}$$

where $u(\cdot)$ is the time- and state-independent, instantaneous utility function common to all individuals of the economy. It is assumed to be monotonic increasing and concave in individual consumption c_{t+n}. That means that a higher level of consumption is always considered with a higher level of utility, but that, in line with neoclassical marginalism, the utility gain from consuming an additional unit of the consumption good decreases in the level of individual consumption. The fact that expected utility is time-separable implies that the utility obtained from consumption in a certain

[18]This rules out the existence of any bequest motive and abstracts from any labor-leisure decision faced by households.

period is not directly dependent on the consumption in other periods. Formally, this is modeled in (2.1) by considering that lifetime utility is given by the sum of expected instantaneous utilities from consumption at different points in time.[19]

The parameter δ is the subjective time discount factor. It weights the instantaneous utility from consumption at different points in time, i.e., the summands of lifetime utility. In this way, differences in preferences for consumption at different dates are formalized. Put differently, the parameter δ reflects the time preferences or patience of an individual. As in the above Equation (2.1), the subjective time discount factor is typically assumed to be constant and the same for all economic agents. Moreover, individuals are usually considered to be impatient, i.e., to prefer early consumption to late consumption, which implies $0 < \delta < 1$. Since the above motivation (see Section 2.1.3) has shown that *agent heterogeneity* and *life-cycle characteristics*, especially in terms of differences in time preferences, should be dimensions to consider when studying the impact of redistribution systems, the present work will deviate from this common practice and allow for heterogeneity in subjective time discount factors. Nevertheless, in order to ease the representation of the methodical foundation, the common simplification will be maintained within this chapter.

Beyond that, $\mathbb{E}_t[\cdot]$ is the conditional expectation operator. It says that agents form their expectations about the future outcome of any random event conditional on the set of information currently available to them at date t. According to neoclassical theory, agents possess full and identical access to this set of information and the expectations resulting from them are rational.

The individual's maximization of expected lifetime utility (2.1) is constrained by the availability and/or affordability of the consumption good. On the one hand, individuals are subject to endowment streams that equip

[19] For a more detailed exposition of utility theory in the context of economics and asset pricing see the excellent textbook treatment in Munk (2013).

them with a certain or uncertain quantity of the consumption good at different time steps. Common examples are the permanent streams of labor or retirement income. On the other hand, in order to allow the individual consumption decision to deviate from the distribution of endowments over time and across states, households need to be able to reallocate consumption over these two dimensions. A financial market that allows for the exchange of different financial assets constitutes such a trading place.[20] Current endowment l_t and the income from past financial market activities a_t in conjunction with the current investment on financial markets q_t constitute the constraint of the maximization of Equation (2.1) that is formalized by the individual's consumption budget constraint

$$\begin{aligned} c_t =& a_t + l_t - q_t \\ =& v_t - q_t, \end{aligned} \tag{2.2}$$

where $v_t = a_t + l_t$ is wealth currently available to him. It can alternatively be stated in wealth-return form, which highlights the dynamic evolution of wealth over time,

$$v_{t+1} = (v_t - c_t)\, R_{v,t+1} + l_{t+1}, \tag{2.3}$$

where $R_{v,t+1}$ is the one-period return on financial savings. According to constraint (2.3), wealth is the link between consumption and any source of income. Moreover, it implies that individual's consumption at a certain date is not just determined by current income but also dependent on future wealth and, thus, future income. This is the essence of Friedman's (1957) "Permanent Income Hypothesis".

[20] Especially when the consumption good is perishable in the sense that it cannot be stored by individuals across periods, financial assets are the only vehicle for the reallocation of consumption. In this vein, the demand for securities becomes interconnected with the individual consumption decision. This is the basis of the consumption-based asset pricing theory that will be addressed below.

Beyond that, the constraint maximization given by Equations (2.1)-(2.3) allows to capture the changing consumption behavior of individuals over their lifespan. First, agents' lifetime is finite and will end at some terminal date. Objective (2.1) reflects this property when considering a finite number of future consumption periods N. Second, the stream of permanent income l_t considered in constraints (2.2)-(2.3) may end or change before the terminal date, for example, at some retirement age, representing differences in labor and retirement income. In the sense of Modigliani's (1966) "Life-cycle Hypothesis", these aspects influence individual consumption and lead to a certain life-cycle behavior, in which agents will save during their working years and spend these savings during retirement to finance consumption.

Finally, note that when there is more than one asset traded on the financial market, the outcome of a_t in constraint (2.2), or alternatively of $R_{v,t+1}$ in (2.3), is not just dependent on the evolution of asset prices or returns but also on the portfolio decision made by the individual in the past, i.e., the allocation of the unconsumed part of wealth between the different tradable assets. In this sense, the optimization of expected lifetime utility (2.1) subject to constraint (2.2) or (2.3) comprises two dimensions of decision taking faced by each household in each time step: the consumption-savings decision and the portfolio choice.

2.2.1.3 Neoclassical Production and Growth Theory

Like the demand side, the supply side of the economy can be formalized by specifying the investment decision faced by firms. This constitutes the basis of the dimension of *endogenous production*. Again, and in contrast to Fisher (1930), the following considerations are not limited to a deterministic and two period decision problem.[21]

[21]The present exposition is based on Miao (2014), Barro and Sala-i Martin (2004), and Sardadvar (2011).

Neoclassical production theory is closely connected to growth theory, which, from a chronological viewpoint, dates back to the seminal contribution of Ramsey (1928) and was independently developed by Solow (1956) and Swan (1956). While accounting for production, capital accumulation, population growth, as well as technological progress, its main focus is to explain long-run economic growth. Integrating Ramsey's analysis of consumer optimization, Cass (1965) and Koopmans (1965) advanced the neoclassical growth model of Solow (1956) and Swan (1956) by specifying the saving rate endogenously. Kydland and Prescott (1982) as well as Long and Plosser (1983) introduced uncertainty in terms of technology shocks into the deterministic growth model and, thereby, pioneered what is known as real business cycle (RBC) theory. It basically considers technological shocks to be the main source driving the evolution of business cycles.

Neoclassical Production

The key aspect of growth and RBC theory is the neoclassical production technology $F_t(\cdot)$ that combines the input factors capital K_t, labor L_t and technology (or knowledge) A_t to produce aggregate consumable output according to

$$Y_t = F_t\left(K_t, L_t, A_t\right), \tag{2.4}$$

at date t. Equation (2.4) constitutes the most general form of a production function that incorporates technological progress besides the other two input factors (Barro and Sala-i Martin (2004)). In particular, there are three distinct ways of introducing exogenous technological progress into the model. First, technological progress may be labor-augmenting, i.e., $Y_t = F_t\left(K_t, A_t L_t\right)$, implying that it magnifies the effective amount of labor. Second, technological progress may be capital-augmenting, $Y_t = F_t\left(A_t K_t, L_t\right)$, implying that it increases the effective amount of capital.

Third, it may enter as neutral scale factor $Y_t = A_t F_t(K_t, L_t)$.[22] For neoclassical growth models, technological progress A_t is deterministic, while it is subject to exogenous random shocks in the case of RBC theory.

Following Barro and Sala-i Martin (2004), the production technology (2.4) is considered to be "neoclassical" if it holds the following three properties. First, the production function $F_t(\cdot)$ must be homogeneous of degree one in the two rival inputs K_t and L_t. That is, scaling capital and labor input by the positive factor λ, output is scaled by the same factor:

$$\lambda Y_t = F_t(\lambda K_t, \lambda L_t, A_t) . \tag{2.5}$$

Production functions that exhibit this property are also said to have constant return to scales. Second, and once more in line with marginalism, neoclassical production technology is characterized by positive and diminishing marginal products with respect to the two rival inputs K_t and L_t

$$\begin{aligned} \frac{\partial F_t}{\partial K_t} > 0, \frac{\partial^2 F_t}{\partial K_t^2} < 0, \\ \frac{\partial F_t}{\partial L_t} > 0, \frac{\partial^2 F_t}{\partial L_t^2} < 0, \end{aligned} \tag{2.6}$$

for all t. That is, holding all other input factors constant, each additional unit of capital or labor employed in production is always associated with an increase in the level of output. As in the case of utility, however, the increase in output from using an additional unit of the input factor decreases in its level of input. Third, and finally, neoclassical production must exhibit the "Inada conditions", named after Inada (1963).

[22] The assumption of labor-augmenting technological progress is prevailing in neoclassical growth theory, since it is the only form that is consistent with balanced growth when considering general production functions (see, for example, Solow (1999), Barro and Sala-i Martin (2004) or Miao (2014)).

It implies that the marginal product of the rival inputs approaches infinity as its input level goes to 0 and approaches 0 as its input level goes to infinity:

$$\begin{aligned} \lim_{K_t \to 0} \frac{\partial F_t}{\partial K_t} = \lim_{L_t \to 0} \frac{\partial F_t}{\partial L_t} &= \infty, \\ \lim_{K_t \to \infty} \frac{\partial F_t}{\partial K_t} = \lim_{L_t \to \infty} \frac{\partial F_t}{\partial L_t} &= 0. \end{aligned} \tag{2.7}$$

In addition to these three properties of neoclassical production, it must hold that each input factor is also an essential ingredient in production. This assumption is called essentiality and means that a positive amount of each input is needed in order to generate a positive aggregate output. However, as shown by Barro and Sala-i Martin (2004), this characteristic is already implied by the three neoclassical properties given in Equations (2.5)-(2.7).

One simple production function that fulfills all of these neoclassical properties is the famous Cobb-Douglas function (Cobb and Douglas (1928)). It is named after the labor economist Paul H. Douglas and the mathematician Charles W. Cobb. It has become famous and frequently used, since it provides a reasonable but simple description of actual economies. Its key characteristic is that, in a competitive economy, capital and labor are both paid their marginal products.

Given the basic structure of neoclassical production technology as in Equation (2.4), and abstracting from growth in the technological level, the only forces that can drive economic growth are changes in labor or capital input. The former may vary, for instance, because of changes in the population size, participation rates, average working hours or improvements in the skill of workers.[23] The latter is subject to variations due to changes in the stock of capital through time that depends on aggregate economic investment and the depreciation of capital.

[23]The present work, however, generally abstracts from such explicit modeling approaches of labor dynamics and will rely on a simplified implicit approach when necessary.

Formally, the evolution of capital through time is typically assumed to follow a process like

$$K_{t+1} = I_t + (1 - d_k)\, K_t, \tag{2.8}$$

where I_t represents aggregate gross investment and d_k is the depreciation rate of capital. When $d_k = 1$, there is full capital depreciation in every period and the future capital stock will just depend on the current gross investment. Furthermore, considering a closed economy with no net government purchases or spending, production Y_t will be composed of aggregate private consumption C_t and aggregate gross investment I_t only. In such an economy, aggregate output equals aggregate private income and, thus, private savings will correspond to gross investment. In that case, the evolution of the capital stock and, hence, the development of economic growth will depend on the fraction of aggregate output reinvested into production, which is given by the aggregate saving rate of private households. The endogenous determination of this saving rate is generally non-trivial, which is why Solow (1956) and Swan (1956) assumed it to be constant and given exogenously. One of the major tasks in Chapter 4 will be to specify this aggregate saving rate in the context of heterogeneous agents as result of their endogenous individual behavior arising from the fundamental assumptions on expected utility and life-cycle consumption defined above.

The Firm's Optimization Problem

The economy's firms that make up the supply force are assumed to be identical and in perfect competition. As defined above, they use capital and labor inputs to produce consumption output according to a neoclassical production technology. Following Miao (2014), the assumption of constant returns to scale implies that it is sufficient to consider only a single aggregate firm that combines capital and labor to generate aggregate output according to Equation (2.4). In line with the neoclassical rationale, the selfish objective followed by the representative firm is the maximization of its profits Π_t. For that very purpose, the firm hires labor at the wage rate ω_t and rents capital at the rental rate $R_{K,t}$ from the households.[24] The firm takes these factor prices as given. Since the firm does not face any adjustment costs, there are no intertemporal elements and the optimization problem it solves is given by the following static maximization:

$$\max_{L_t,K_t} \Pi_t = Y_t - \omega_t L_t - R_{K,t} K_t, \tag{2.9}$$

for every time step t. Put differently, Equation (2.9) states the classical relationship that profits are given by revenues (output) minus costs (labor and capital input). The identity of revenues and aggregate output implies that the price for the consumption good is normalized to one. This is without loss of generality, since the consumption good serves as numeraire in the economy.

The first-order conditions to this optimization problem with respect to capital and labor are given by

$$R_{K,t} = \frac{\partial F_t}{\partial K_t}, \tag{2.10}$$

$$\omega_t = \frac{\partial F_t}{\partial L_t}, \tag{2.11}$$

respectively. According to neoclassical marginalism, optimization implies that the amount of each input factor used in production should be increased until the marginal product of the last unit equals its factor price.

[24] In this setting, households are assumed to own the entire capital of the economy.

2.2.2 Consumption-based Asset Pricing

The theoretical foundation presented so far has mainly focused on the link between agents' (firms and households) optimal rational behavior and the determination of aggregate economic quantities as well as their evolution. An equilibrium, in this context, was characterized by the fundamental property that demand needs to equal supply. The asset prices or returns that resulted from or supported this equilibrium, however, are either assumed as given or, at least, mainly neglected within the area of research considered so far. This missing dimension and apparent gap is filled by *asset pricing* theory. Unlike the previous theories, its major task is to explain the prices of claims to uncertain payments and the implied asset returns. The starting point of modern asset pricing theory is certainly the development of the famous capital asset pricing model (CAPM) by Sharpe (1964), Lintner (1965), and Mossin (1966). Asset pricing models, since then, have become one of the major tools used in theoretical and empirical finance research. In this context, the concept of the stochastic discount factor and the related simple representation of the basic asset pricing equation is the fundamental building block of asset pricing models, which led to an enormous body of literature. A comprehensive overview of the asset pricing field is provided by Campbell (2000) as well as by the textbook treatments in Cochrane (2005), Back (2010) and Munk (2013). The present section draws heavily on these contributions.

According to Cochrane (2005, Preface), the whole field of asset pricing theory can be broken down to the simple concept that "price equals expected discounted payoff", where it is the so-called *stochastic discount factor* that is used to discount future (uncertain) payoffs, i.e., to price an asset. In fact, once determined, the SDF relates (uncertain) payoffs to market prices for all assets in an economy. The SDF-based asset pricing representation is, therefore, both simple and universal. It generalizes the standard discount factor idea to a world of uncertainty. Its roots are presumed to lie in the famous Arrow-Debreu general equilibrium model (Arrow (1951), Debreu

(1951) and Arrow and Debreu (1954)) and its applications by Cox and Ross (1976) and Ross (1978) to option pricing as well as in the Arbitrage Pricing Theory of Ross (1976). The representation in continuous time was refined by Harrison and Kreps (1979), while Rubinstein (1976), Lucas (1978), Grossman and Shiller (1981), Hansen and Richard (1987) and Hansen and Jagannathan (1991) have further developed the SDF-approach in discrete time.

In line with the separation mentioned in the context of the Fisher Separation Theorem, general equilibrium asset pricing theory can be subdivided into the field of consumption-based and production-based asset pricing. The latter links asset prices and returns to firms' profit maximizing investment decisions and, thereby, identifies the marginal rate of transformation as the relevant stochastic discount factor. In contrast to its consumption-based counterpart, however, production-based models are to be considered an exception within the field of asset pricing research. Important contributions are, for instance, the seminal works of Cochrane (1991, 1996) and the more recent article by Croce (2014) featuring long-run productivity risk. The major number of asset pricing models that have been developed so far build on the consumption-based approach. It uses the utility maximizing consumption behavior of rational households to explain the evolution of asset prices and returns. Core to this approach, therefore, is the definition of the stochastic discount factor as the households' marginal rates of substitution of consumption. In discrete time, consumption-based asset pricing was mainly pioneered by Rubinstein (1976) and Lucas (1978), while its continuous time counterpart is due to Breeden (1979).[25]

In line with the general trend, and as explained above, the present work focuses on the consumption-based approach. Formally, the basic pricing equation results from the utility maximizing rationale given in Equation (2.1) subject to constraint (2.2). Suppose an agent that pursues this objec-

[25] Reviews of the consumption-based asset pricing theory are provided by, for instance, Campbell (2003) and Mehra (2012) as well as the textbook treatments in Cochrane (2005), Constantinides (2005), Back (2010), Munk (2013) and Danthine and Donaldson (2015).

tive and that this agent can freely trade his wealth by buying or selling a financial asset with future gross payoff X_{t+1} and current market price P_t. Denote by α_t the units of the asset the agent chooses to hold from period t to $t+1$. Then, the optimization problem faced by this agent at date $t = 0$ can be redefined to:

$$\max_{\{c_t,\alpha_t\}} V_0 = \sum_{t=0}^{N} \delta^n \mathbb{E}_t \left[u(c_t) \right], \tag{2.12}$$

$$s.t.$$

$$c_t = \alpha_{t-1} X_t + l_t - \alpha_t P_t, \qquad and \tag{2.13}$$

$$c_0 = l_0, \tag{2.14}$$

where (2.12) is denoted the agent's indirect utility, which is the maximum expected lifetime utility of current and future consumption (Munk (2013)), and Equations (2.13)-(2.14) are the agent's budget constraint as well as his initial endowment. The first order condition of this intertemporal optimization problem with respect to α_t reads

$$P_t \frac{\partial u(c_t)}{\partial c_t} = \mathbb{E}_t \left[\delta \frac{\partial u(c_{t+1})}{\partial c_{t+1}} X_{t+1} \right], \tag{2.15}$$

or, alternatively,

$$P_t = \mathbb{E}_t \left[\delta \frac{\frac{\partial u(c_{t+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t)}{\partial c_t}} X_{t+1} \right]. \tag{2.16}$$

Equation (2.15) represents a stochastic version of the famous consumption Euler equation, which equates the marginal costs associated with the purchase of an extra unit of the asset today, $P_t(\partial u(c_t)/\partial c_t)$, to the expected marginal benefit of the additional payoff received tomorrow, $\mathbb{E}_t \left[\delta(\partial u(c_{t+1})/\partial c_{t+1}) X_{t+1} \right]$. That is, the agent adjusts his asset holdings until the marginal loss equals the marginal gain.

Equation (2.16) is the basic equation of asset pricing.[26] As indicated above, this representation simply states that the market price of the asset is given by its expected discounted payoff. The factor used for discounting is a stochastic discount factor, which is based on the agent's optimal consumption plan. In the present consumption-based approach, it is specified by the agent's intertemporal marginal rate of substitution of consumption:

$$M_{t+1} = \delta \frac{\frac{\partial u(c_{t+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t)}{\partial c_t}}. \tag{2.17}$$

This is the rate at which the agent is willing to forgo current (at date t) consumption in exchange for extra consumption tomorrow (at date $t+1$). The existence of a positive stochastic discount factor, as defined by Equation (2.17), is guaranteed if asset prices do not admit arbitrage (see, for example, Munk (2013, Theorem 4.5)).[27] Beyond that, the stochastic discount factor is uniquely determined for the sole case the market is complete (see, for example, Munk (2013, Theorem 4.6)).[28] If the market is incomplete, however, multiple stochastic discount factors exist. In the present context, for instance, there might be a number of different agents with various marginal utilities and, hence, different SDF processes. Nevertheless, and most importantly, each of these stochastic discount factors must satisfy Equation (2.16). The fact that there might be multiple agents in the economy considered, however, does not necessarily imply that the market is incomplete. This is just a matter of the number of different non-

[26] If there is more than one asset traded, the utility maximizing individual will have to decide about what amount to buy or sell of all of these assets, while satisfying his budget constraint. In this case, there will be pricing relations, similar to (2.16), for each of these assets.

[27] According to Mehra (2012, Footnote 5), "A securities market is arbitrage free if no security is a free lottery and any portfolio of securities with a zero payoff has a zero price."

[28] A very informal definition of market completeness is given by Munk (2013, Chapter 3), who also provides a formal definition for the discrete as well as the continuous time case. According to him, a market is said to be complete, if all dividends (net payoffs) one can think of are spanned by traded assets (i.e., can be created by trading the basic assets).

redundant assets traded and the possible realizations of future economic states (see, for instance, Munk (2013, Theorem 3.3)). In complete markets with multiple individuals, the SDF is unique because agents have the possibility to align their marginal rates of substitution in equilibrium. Put differently, the market structure allows agents to trade with each other in order to eliminate any idiosyncratic component in their marginal utilities (Campbell (2003)).

As indicated above, the representation given by the basic pricing Equation (2.16) is very general. To be more precise, and as pointed out by Cochrane (2005), Equation (2.16) is not yet a complete solution to the model, where exogenous inputs on the right hand side are used to explain the endogenous results on the left hand side. It, apparently, needs some more concretization to gain usefulness. For the present purpose, specifications with respect to two fundamental aspects of the economy are necessary in order to further complete the model.

First, it requires a characterization of the nature of the consumers for a definite representation of the stochastic discount factor. Real-life economies are populated by a large number of agents heterogeneous with respect to various characteristics (see Section 2.1.3). To capture all facets of agent heterogeneity, however, is almost impossible and complicates the analytical problem substantially. For modeling reasons it is, therefore, inevitable to group agents according to their individual behavior and, in so doing, to focus only on a limited number of characteristics of heterogeneity. One extreme variant of a simplification in this respect is the concept of a representative agent (RA). The idea is to use one single individual with one representative utility function who holds all the wealth of the economy and consumes aggregate consumption. Such an individual is said to be a representative agent of the economy, if the equilibrium asset prices in the stylized one-agent-economy are identical to the equilibrium asset prices of the represented economy with many different agents. If the aggregate behavior of the different individuals of an economy can be represented by a representative agent in this way, then the SDF can be defined by using the utility function of this representative agent. Hence, the SDF will be related to the marginal utility of aggregate consumption. Although the RA-concept has its advantages, especially since it is much easier to analyze, it is hardly useful for addressing the present research problem, as will be further pointed out in Section 2.2.2.3.

Second, the origin of the aggregate amount of consumption goods available at a certain date, i.e., the assumption regarding the underlying consumption good producing technology of the economy, has to be determined. In the context of consumption-based asset pricing, the explicit modeling of a neoclassical production technology with diminishing marginal productivity, as in the Fisher Separation, however, is not very common. Quiet the contrary, by taking the stream of consumption goods as exogenously given the prevalent approach generally abstracts from specifying a certain production technology. This extreme concept of an "exchange economy" will be outlined in Section 2.2.2.1. Beyond that, a simplified approach featuring a genuine production side, based on a linear production technology, has gained some importance within the asset pricing literature. This concept will be treated in Section 2.2.2.2.

2.2.2.1 Consumption-based Asset Pricing in an Exchange Economy

The concept of an exchange economy is certainly the most reduced possibility of modeling the production technology of an economy. Perishable consumption is assumed to just appear in every period. That is, production is characterized by an exogenously specified stream of random aggregate and nondurable consumption. In this vein, there exists no possibility for agents to transform consumption goods from one period to another by, for example, saving, storing or investing. Graphically the production opportunity set implied by such an exchange economy is depicted by the central point in Figure 2.4. For an equilibrium, asset returns, or equivalently prices, need to adjust until the utility maximizing individuals are just happy consuming the aggregate endowment (or consumption) stream. This approach was developed by Lucas (1978) and is also referred to as the "Lucas fruit-tree model", since the exogenous aggregate endowment can be interpreted as the perishable fruits produced periodically by a tree. In this analogy the tree represents the production means of an economy.

Following this approach, an enormous bunch of literature emerged that abstracted from the explicit modeling of a production side. Like Lucas (1978) himself, most of these studies also rely on the assumption that the economy can be described by a representative agent, with standard power or constant relative risk aversion (CRRA) utility function, who consumes the exogenous aggregate endowment distributed to him as the dividend of the aggregate stock market. That way, exogenous aggregate consumption data is linked to asset prices, reflecting the market portfolio of all wealth in an economy.

By empirically testing standard asset pricing models of this kind, however, it has been shown that some severe discrepancies between model predictions and empirical data exist. The three most important of these empirical challenges are known as asset pricing puzzles. First of all, it has been shown by Mehra and Prescott (1985) that, for reasonable parameter values, the model implied risk premium, i.e., the excess return on equity over the risk-free return, is much smaller (more than an order of magnitude) than the empirically observed historical average. Although originally derived using data for the United States, this so-called "equity premium puzzle" has proven to be a robust phenomenon using data for other countries with well-developed capital markets and for other data periods (see, for instance, Mehra (2012)). Second, due to the low consumption volatility that drives the volatility of stock returns within the standard model, the model implied variation in returns is significantly smaller than its empirical counterpart. According to Campbell (1999, 2000), this is referred to as the "stock market volatility puzzle". Third, for reasonable parameter values, the model implied risk-free interest rate is counterfactually high. This is the "risk-free rate puzzle" found by Weil (1989a). In order to address these misspecifications of the consumption-based asset pricing model, the last decades have produced various alternative specifications of the standard model. These alternative approaches, for example, consider different utility specifications, alternative aggregate consumption dynamics, and/or abandon the RA-concept and assume heterogeneous agents. Important contributions are, for instance, the habit formation model by Campbell and Cochrane (1999), the overlapping generations model by Constantinides et al. (2002), the long-run consumption risk model by Bansal and Yaron (2004), and the heterogeneous agent model by Gârleanu and Panageas (2015).[29]

Finally, considering an exchange economy clearly eliminates any feedback effect between the households' consumption-savings decision and aggregate production output. That is, it ignores the dimension of *endogenous production*. As a result, this model approach is unsuitable for considering the equilibrium impact of a redistributive taxation system on macroeconomic development and is of limited use with respect to welfare considerations. When abstracting from a representative agent, however, such a model can provide first insights with regard to the equilibrium impact of a tax-based reallocation mechanism and agent heterogeneity on households' consumption and investment behavior as well as on asset prices. This is the modeling approach followed in Fischer and Jensen (2015) and the basis of the equilibrium model developed in Chapter 3.

2.2.2.2 Consumption-based Asset Pricing with Linear Production Technology

As pointed out above, the exchange economy approach is of little use when studying the impact of a redistributive taxation system on macroeconomic development and social welfare. To capture the feedback effects between

[29] Although the explanation of these puzzles is a very interesting subject on its own, it clearly exceeds the scope of the models considered in the present work, which are designed to address the objective outlined above.

resource allocation, consumption-savings decision and macroeconomic production, it rather requires a truly explicit modeling of the production side. Abstracting from the neoclassical rationale of diminishing marginal productivity, in the context of asset pricing models, a simplified approach, based on a linear production technology, has garnered some importance within the asset pricing literature. These models have been pioneered by Sundaresan (1984), Cox et al. (1985a,b), and Constantinides (1992).

When considering linearity of the production technology, the marginal rate of return on physical capital (i.e., the marginal rate of transformation) becomes constant and will, thus, be unaffected by the amount that is invested. Graphically the production opportunity set implied by such a production economy coincides with the straight line of the capital market depicted in Figure 2.4. Since the physical rate of return is fixed by the assumption regarding the linear production technology, consumption has to adjust to this given rate. Beyond that, however, the production output and, therefore, aggregate consumption possibilities will depend on the amount invested. In this sense, the approach captures the dimension of *endogenous production*.

The advantage of a linear production technology lies in the simplification of the analytical complexity of the general equilibrium model due to the technologically given physical return, while allowing for the described feedback mechanism. For that reason, Fischer and Jensen (2014, 2017) applied this approach to discrete time consumption-based asset pricing and used it to study the equilibrium impact of redistributive taxation in the context of a simple dynamic general equilibrium model. The linear production model developed in Chapter 4 builds on this approach and these contributions.

2.2.2.3 Asset Pricing with Heterogeneous Agents

When considering asset pricing models, the use of a single representative individual is certainly the prevailing modeling approach to specify the consumption side. This is because of the apparent advantages of the RA-concept in the context of classical asset pricing. First, it is easier to analyze a single-agent-economy than a model with numerous heterogeneous agents. Second, it is favorable from an empirical point of view, since aggregate consumption is an observable quantity for which sufficient and appropriate data is available. Nevertheless, there are also some severe disadvantages of the RA-concept to be considered.

First, and in most cases, the aggregation from many agents to a representative of an entire economy is non-trivial. In general, the preferences of a RA are given by a complex average of the preferences of all agents of the economy, where the weights are dependent on the wealth distribution across them. Simple preferences of a representative agent, i.e., preferences independent of the wealth distribution, require strong assumptions on individual preferences (Munk (2013)). As pointed out by Danthine and Donaldson (2015), the easiest way of constructing a RA exists, when agents are perfectly homogeneous, i.e., identical with respect to their preferences, endowments and future income. Apart from this very simplistic approach, some less restrictive assumptions under which a representative agent can be constructed have been developed by Wilson (1968), Rubinstein (1974) and Constantinides (1982).

Second, and more importantly in the context of the present work, the "loss of distributional information" implied by the application of the RA-concept needs to be considered (Lengwiler (2006, Box 2.10)). That is, when assuming that the economy can be described by a representative agent, all information regarding the equilibrium distribution of consumption and wealth holdings is lost. In the present work that studies the equilibrium impact of tax-based consumption reallocation, however, it will be elementary to capture this distributional information on individual consumption and wealth.

Moreover, in order to draw a more realistic picture of a real-life economy (see Section 2.1.3), it will be essential that the models developed in Chapters 3 and 4 take some characteristics of *agent heterogeneity* explicitly into account. The RA-concept, thus, is not suitable for the present purpose.

Since real-life economies are characterized by many different individuals, there are various different ways to capture agent heterogeneity within an asset pricing model. The most important approaches consider heterogeneous constraints (see, for example, Campbell and Mankiw (1989), Mankiw and Zeldes (1991), Brav et al. (2002), Heaton and Lucas (1999), Vissing-Jørgensen and Attanasio (2003), Constantinides et al. (2002), Gomes and Michaelides (2008) and Gomes et al. (2013)), heterogeneous (irrational) information (see, for instance, Wang (1993), Wang (1994), Hong and Stein (1999), Chen et al. (2012), Branger et al. (2013), Piatti (2014) and Chabakauri (2015)), heterogeneous uninsurable (idiosyncratic) income processes (see, for example, Zeldes (1989), Krusell and Smith (1998), Brav et al. (2002), Constantinides and Duffie (1996) and Athanasoulis (2005)) and heterogeneity regarding the specification of agents' preferences. While Campbell (2000) as well as the textbook treatment in Guvenen (2011) provide a comprehensive overview of this subject, the present work will focus on heterogeneity regarding preferences and, moreover, assume heterogeneous income without idiosyncratic risk throughout. Beyond that, life-cycle effects that provide a natural source of heterogeneity across individuals will be considered exclusively in the overlapping generations model of Chapter 4. Since the OLG-framework implies a nontrivial population structure that is typically not accounted for in heterogeneous asset pricing models, its concept will be presented in Section 2.2.3.

A large fraction of asset pricing studies featuring heterogeneous agents considers differences in individual preferences as source of heterogeneity. In line with the empirical observations presented in Section 2.1.3 and depending on the precise form of the instantaneous utility function, several distinct dimensions of preference heterogeneity might exist. The most important types of utility functions used in the context of asset pricing are certainly the famous standard power or CRRA preferences and the so-called recursive or Epstein-Zin (EZ) preferences, which are getting increasingly popular. The major difference between both specifications is that in the former case the coefficient of relative risk aversion, i.e., the attitude towards atemporal risks, is directly linked to the elasticity of intertemporal substitution (EIS) in consumption, i.e., the attitude towards consumption shifts over time, while in the latter the coefficient of relative risk aversion is separated from the EIS (Munk (2013)).[30]

As an immediate consequence, studies featuring heterogeneous CRRA preferences (mainly) focus on the differences in risk aversion across individuals. Compared to the results derived in the standard homogeneous agents equilibrium, individual consumption shares as well as the market price of risk are not constant but may become state-dependent in the presence of multiple CRRA individuals with heterogeneous risk aversion. The intuition works as follows. Since agents with a higher risk-tolerance hold more risky assets, they control a greater fraction of wealth in good states than in bad states. As a result, the risk aversion of the economy falls in good states and rises in bad states, implying a state-dependent aggregate risk aversion. This dependency incorporates additional time-variety into the model (Campbell (2000)). Within this area of research, the contribution by Dumas (1989) is to be considered a seminal work. It provides the foundation for a number of subsequent papers, like for example Wang (1996), Dieckmann and Gallmeyer (2005), Vasicek (2005), Cvitanić and Malamud (2010), Longstaff and Wang (2012), Chabakauri (2013) and Bhamra and Uppal (2014), of which not all concentrate exclusively on

[30] As pointed out by Campbell (2000), the Epstein-Zin utility model is situated outside the classical von Neumann–Morgenstern framework of expected utility. It builds on the work of Kreps and Porteus (1978) and was proposed by Epstein and Zin (1989, 1991) as well as Weil (1989a).

heterogeneity in individual risk aversion. Moreover, Fischer and Jensen (2015) assume two agents with CRRA preferences heterogeneous with respect to their coefficient of risk aversion in order to study the impact of redistributive taxation in the context of agent heterogeneity.

In contrast, studies using EZ preferences are often concerned with both sources of heterogeneity risk aversion and EIS. The reason for this is that considering agents with different EIS but identical coefficients of risk aversion does not affect the risk premium (see Gârleanu and Panageas (2015) and Chabakauri (2015)). When agents are heterogeneous with respect to both characteristics, however, it may help to replicate important empirical asset pricing properties. In this vein, Gomes and Michaelides (2008) consider two types of Epstein-Zin investors settled within a general equilibrium life-cycle model featuring an endogenous neoclassical production technology. Agent heterogeneity in conjunction with idiosyncratic labor income shocks and borrowing constraints generates a large equity premium while matching individual asset holdings and stock market participation rates. Chabakauri (2015) combines heterogeneity in risk aversion and EIS with rare disaster risk in order to explain excess stock return volatility, procyclical price-dividend ratios and interest rates, as well as counter-cyclical market prices of risk. Gârleanu and Panageas (2015) use an overlapping generations model with two types of Epstein-Zin investors to explain the risk premium, interest rates, and the volatility of stock returns.

Beyond that, and independent of the choice of the instantaneous utility function, individual preferences also depend on the weight individuals put on the instantaneous utility from consumption at different points in time, i.e., patience.[31] As pointed out by Lengwiler (2005, p. 890), "There is no reason to believe that all members of an economy have the same preferences with respect to the timing of consumption. After all, some people seem to be more patient than others." In this sense, preference

[31] This has been shown in the context of the neoclassical utility maximizing rational in Equation (2.1).

heterogeneity may also be characterized by differences in subjective time discount factors. As explained above, this is the approach taken in the present work.

In contrast to heterogeneity in risk aversion, where individual consumption shares become time- and state-dependent in equilibrium, differences in the patience of agents lead to time-dependency only.[32] In this sense, it does not add additional time-variability to the model and, hence, is less suitable to explain the high variations in, for example, returns and asset prices. Nevertheless, and as pointed out above, this is not the focus of the present study. More importantly, a simple and tractable analytical solution can be found under the assumption of heterogeneous levels of patience. In addition, Chapter 3 shows that, when setting up the model along the lines of Fischer and Jensen (2015) while allowing for heterogeneous patience instead of risk aversion, both sources of heterogeneity actually have the same effects with regards to the impact of a redistributive taxation system.

Finally, equilibrium models featuring heterogeneous subjective time discount factors are rare, especially in the context of asset pricing. Exemptions are Lengwiler et al. (2005), Beaumont et al. (2013) and Gomes et al. (2013). Other studies featuring heterogeneous patience are presented by Becker (1980), Lawrance (1987), Gollier and Zeckhauser (2005), Lengwiler (2005) and Vasicek (2005). A more detailed review of the relevant literature will be provided in Section 2.3.

2.2.3 Overlapping Generations Model

As pointed out earlier, real-life economies are populated by a large number of agents that are heterogeneous with respect to various characteristics. Besides the (more or less) artificial approaches discussed above, another way

[32] This holds for classical asset pricing models with standard population structure. In the context of an overlapping generations structure, dependencies may be different, as shown in Chapter 4.

of adding heterogeneity to the model is to use a population structure that induces heterogeneity across individuals by considering the natural differences caused by the distinct phases of agents' life-cycle. An equilibrium model that provides such a life-cycle perspective and, thereby, contributes to the dimension of *life-cycle characteristics* is provided by the overlapping generations model. While the origins of this approach are found in Allais (1947), the development of the model is often attributed to the famous contributions of Samuelson (1958) and Diamond (1965), which made this approach extremely popular (Malinvaud (1987)). Because of its properties, the framework has become a workhorse in the area of public finance. Comprehensive treatments of the classical OLG models are, for instance, presented by La Croix, David de and Michel (2002), Ljungqvist and Sargent (2004), Weil (2008) and Miao (2014).

In contrast to the OLG approach, the standard population structure mainly used in economic as well as asset pricing models assumes that multiple agents (or one single individual in the case of a RA model) enter the economy at just the same point in time and, from then on, simultaneously populate the economy until its very end. That is, they live for exact the same time span, where their existence also determines the lifespan of the economy. This implies that, when agents are considered to face a realistic and, thus, finite horizon, the existence of the economy is limited to an (from a macroeconomic point of view) extremely short time span. In comparison, the overlapping generations framework constitutes a type of model with a typically infinite economic horizon in which agents live for a finite length of time. In each time step a new generation enters (is born) while an old generation (dies) leaves the economy. In this vein, at any point in time different generations (cohorts), i.e., individuals of different age, coexist simultaneously. This structure produces a natural heterogeneity within the current population as well as nontrivial life-cycle effects for each individual over his lifespan.

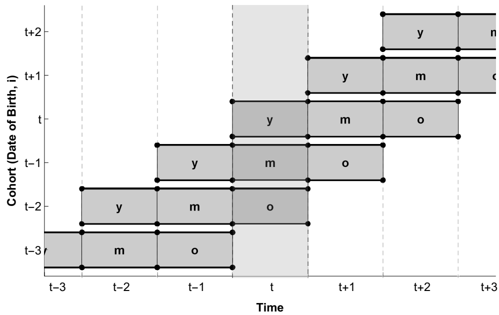

Figure 2.5 – This figure illustrates the demographic structure implied by a classical overlapping generations model with three generations (Source: adapted from Jäkel (2006, Fig. 3.1.)).

Figure 2.5 depicts the demographic structure implied by the classical overlapping generations model with three cohorts in discrete time, as in Samuelson (1958).[33] That is, every agent in the economy is assumed to live for just three periods, while facing a very artificial life-cycle: individuals enter the economy at date $i = t$ (age $t - i = 0$) as part of the generation of young workers ("y"); in the following period $t + 1$ (at age 1), they constitute the cohort of middle-aged ("m") workers, and retire in the subsequent period $t + 2$ (at age 2), where they form the generation of the old ("o"); finally, they vanish from the economy. The fact that new generations are always coming along implies that there will be three generations, one from each kind (young, middle-aged and old), alive and overlapping in every time step t. This is why the demographic structure is called overlapping generations. In this classical version of the OLG model life expectancy is certain and all individuals live through the designated phases of their life-cycle. Moreover, all members of a given generation are assumed to be perfectly identical (or homogeneous).

One disadvantage of the classical overlapping generations framework is its analytical intractability. Compared to the standard population structure,

[33]The most basic form of an OLG model requires only two generations, as used in Diamond (1965).

an aggregate consumption function in an economy populated by overlapping and finitely lived individuals is hard to derive. As pointed out by Blanchard (1985), the reason for this can be found in two sources of agent heterogeneity introduced by the OLG structure. First, differences in age cause the level as well as composition of wealth to vary across individuals. Second, and more importantly, facing a different remaining life expectancy, agents hold different consumption-wealth ratios. To overcome this aggregation problem, Blanchard (1985) proposed a different modeling approach. He assumed that all individuals, starting from birth, face a constant and age-independent survival probability. As an immediate consequence, life expectancies become constant and, therefore, identical across individuals of all generations. Since all agents face the same horizon in this setting, they also have identical consumption-wealth ratios. This solves the aggregation problem and has become known as the "perpetual youth" model.[34] Although it considers the aspect of finite lives, it does not capture life-cycle effects, i.e., the changes in individual behavior over the lifespan. In this respect, it is closer to the infinite horizon representative agent model and, hence, less suitable for the present purpose.

Although it has generally become very popular in the area of public finance (see, for example, Atkinson and Sandmo (1980) and Auerbach and Kotlikoff (1983)), the OLG approach was mainly ignored in the context of financial economics for quiet a long time. Constantinides et al. (2002) were the first to use the classical demographic structure of overlapping generations, as depicted in Figure 2.5, to address asset pricing issues. Other important contributions in this filed are, for instance, Gomes and Michaelides (2008) and Gomes et al. (2013). Considerable asset pricing models that are based on the perpetual youth approach have been developed by Athanasoulis (2006), Campbell and Nosbusch (2007), Farmer et al. (2011) and Gârleanu and Panageas (2015).

[34] Weil (1989b) proposed a similar approach. While classical OLG models typically assume a discrete time framework, the models developed by Blanchard (1985) and Weil (1989b) are in continuous time.

The life-cycle model studied in Chapter 4 builds on the classical overlapping generations approach in combination with preference heterogeneity. In this vein, a framework is set up that allows to mutually capture inter- and intra-generational distributional as well as life-cycle effects.

2.3 Literature Review and Conclusion

2.3.1 Literature Review

So far, the present chapter has developed the motivation and given an overview of the methodical basis of the research question raised above. Before turning to the development and analysis of the economic models in Chapters 3 and 4, the present section provides an overview of the most relevant theoretical contributions. According to the six dimensions identified earlier, these studies deal with *redistributive taxation*, *agent heterogeneity* (especially heterogeneous patience), *life-cycle characteristics* (especially OLG), *endogenous production* and/or *asset pricing*, while allowing for *endogenous individual behavior*. In order to structure the survey, the relevant theoretical literature is subdivided into three strands: first, theoretical studies that focus on the equilibrium impact of heterogeneous patience; second, contributions that explicitly study the equilibrium impact of taxation on welfare, individual behavior and/or asset prices; and third, contributions that focus on asset pricing and overlapping generations but (mostly) ignore or only supplementally consider heterogeneous patience or taxation.

Starting with the first strand, it is worth reiterating that in the context of asset pricing there exists only a small number of contributions that is concerned with heterogeneous patience. The few exemptions can be found in Lengwiler et al. (2005), Beaumont et al. (2013) and Gomes et al. (2013). However, only the first two of these studies explicitly focus on the equilibrium impact of heterogeneity in agents' subjective time discount factors.

Lengwiler et al. (2005) concentrate on the asset pricing puzzles and extend the classical Lucas (1978) exchange economy by multiple agents with standard preferences heterogeneous in endowment risk, the coefficient of risk aversion and patience. Their main finding is that the joint heterogeneity in risk aversion and patience increases the equity premium, while it decreases the risk-free return. In contrast, Beaumont et al. (2013) investigate the market dynamics of asset holdings and the pricing function over time in a standard Lucas (1978) economy, in which individuals have heterogeneous subjective time discount factors. They show that only the most patient agent remains in equilibrium, and that the resulting pricing function is identical to the one obtained in a homogeneous agent economy populated exclusively by the most patient individual. The topic of the asset pricing field aside, a similar result has previously been documented by Becker (1980).

Other notable studies considering differences in patience are the contributions by Lawrance (1987), Lengwiler (2005), Gollier and Zeckhauser (2005) and Vasicek (2005). While Lengwiler (2005) and Vasicek (2005) build economic models with heterogeneous agents to study the term structure of interest, Gollier and Zeckhauser (2005) examine the aggregation of heterogeneous time preferences considering multiple agents with different discount factors for utility, possibly not exponential. In the context of this strand of literature, the study that is closest to the focus of the present work, however, is Lawrance (1987). Using a deterministic and neoclassical production economy, she examines the impact of different rates of time preferences on long-run savings of two types of consumers, patient rich and impatient poor, as well as the associated equilibrium effect of transfers from rich to poor. Her results indicate that the long-run capital accumulation of individuals is rather insensitive to lump sum transfers. Finally, neither of these contributions investigates the meaning of heterogeneous patience in the context of redistributive taxation or within an overlapping generations framework.

The second strand of literature considers contributions that explicitly study the equilibrium impact of taxation on welfare, individual behavior and/or asset prices. This research field originated with Ramsey (1927) and the early macroeconomic studies of the 1960s until 1980s that were mainly concerned with an optimal, i.e., welfare maximizing, tax analysis and the transition path between steady states. Prominent representatives are the perfect foresight equilibrium models building on the overlapping generations approach of Diamond (1965): while Diamond (1973), Pestieau (1974), Auerbach (1979) as well as Atkinson and Sandmo (1980) rely on the original framework with only two coexisting cohorts, Auerbach and Kotlikoff (1983) consider a larger number of overlapping generations. Modern macroeconomic studies in this field have further developed this approach and mostly feature large scale overlapping generations models and various sources of uncertainty, such as Conesa et al. (2009) and Benhabib et al. (2011). However, these contributions typically lack the asset pricing dimension. In general, as noted by Fischer and Jensen (2015), dynamic general equilibrium models that study the impact of redistributive taxation on individual welfare, asset prices, and/or optimal investment strategies are surprisingly rare. However, there are some notable exemptions. Sialm (2006) analyzes the general equilibrium effects of stochastic tax rates on asset prices in a Lucas (1978) exchange economy with a representative agent. He finds that the risk introduced by tax changes increases the term and equity premia. Campbell and Nosbusch (2007) construct a perpetual youth overlapping generations model with two Lucas trees to study the impact of a tax-based social security system on both asset prices and intergenerational risk sharing. They conclude that a social security system that optimally shares risks between cohorts counterintuitively requires a net transfer of physical capital from the old to the young, while human capital risk is already shared optimally in the absence of transfers. Furthermore, they show that such a system increases the risk-free rate but lowers the risk premium. The articles that are closest to the present study in the field of asset pricing and taxation, however, are Fischer and Jensen (2014, 2015).

Fischer and Jensen (2015) investigate the impact of redistributive taxation in a standard exchange economy assuming agents heterogeneous with respect to their initial endowment and/or degree of risk aversion. In their article, they mainly focus on the explanation of stock market participation rates. They show that the reallocation system, besides redistributing consumption, also reallocates market risk. As a result, poorer agents, which are net recipients of the system, lower their equity and increase their bond holdings. The study abstracts from an endogenous production technology as well as a life-cycle perspective. It is used as the basis of the simplified model developed in Chapter 3. In a related article, the authors slightly modify this framework (see Fischer and Jensen (2014)). With the objective to examine the impact of a redistributive taxation system on individual and macroeconomic welfare, they add an endogenous linear production technology but abstain from including elements of heterogeneity except individual initial endowment. In this setting, they show that redistribution from rich to poor individuals may result in Pareto inefficient production and, as a result, even net recipients of the reallocation system can be better off in the absence of transfers. Although the approach ignores preference heterogeneity and life-cycle considerations, the focus on welfare and endogenous production provides the basis of the model developed in Chapter 4.

Finally, the last strand of literature to be considered are articles that concentrate on asset pricing and overlapping generations, but (mostly) ignore or only supplementally consider heterogeneous patience or taxation. As described above, Constantinides et al. (2002) were the first to use the classical demographic structure of overlapping generations to explicitly address the asset pricing puzzles. They construct an OLG model with three generations and exogenous production in which the members of the young generation face high future labor income risk but are excluded from trading in the stock due to borrowing constraints. As a result, equity is exclusively priced by the members of the middle-aged generation. This increases the equity premium and lowers the risk-free rate. Athanasoulis

(2006) and Gârleanu and Panageas (2015) use the perpetual youth overlapping generations structure in combination with an exchange economy to study asset prices. Following Constantinides et al. (2002), Athanasoulis (2006) restricts young workers from trading in the equity market and examines the equilibrium impact of demographic parameters, like the age individuals begin to work, retirement age and survival probability. According to his findings, notable effects on asset prices result only from changing parameters that have a large impact on the percentage of the constrained young population. Gârleanu and Panageas (2015) assume two types of Epstein-Zin investors heterogeneous in their risk aversion and EIS to study the implications of preference heterogeneity for asset pricing. Because of the overlapping generations structure, both types of agents exist in the long run, where the process of optimal consumption-allocation derived in the stationary equilibrium causes persistence in individual consumption growth, even though aggregate consumption growth is independent and identically distributed. Agents, hence, demand a higher compensation for risk, which results in a large equity premium.

With respect to the life-cycle framework studied in Chapter 4, Gomes and Michaelides (2008) as well as Gomes et al. (2013) are to be considered the most relevant contributions, since they capture all the essential dimensions identified above (incl. agents heterogeneous in patience in the case of the latter). Besides these similarities, however, the specific framework as well as the scope of theses studies differ considerably from the present work. In particular, the authors assume a neoclassical production economy populated by households with heterogeneous Epstein-Zin preferences, uninsurable labor income shocks as well as borrowing constraints. Gomes and Michaelides (2008) apply their model to simultaneously match asset pricing moments, stock market participation rates and individual asset holdings, whereas Gomes et al. (2013) focus on the impact of fiscal policy decisions on behavior of macroeconomic quantities and asset prices, while capturing wealth and consumption distributions.[35] The impact of tax-based reallocation effects or transfers on individual behavior and welfare, however, is not studied. Moreover, because of the model complexity, numerical solution techniques with large scale simulations are required. Analytical solutions do not exist.

2.3.2 Conclusion and Survey Approach

Motivated by the explanations in Section 2.1, the objective of the present work is to study the impact of a redistributive taxation system, particularly on the consumption-savings and portfolio behavior of a heterogeneous population, macroeconomic development, welfare and asset prices. In this context, six dimensions have been identified that should be captured (ideally simultaneously) by economic models developed to address this problem: *redistributive taxation*, *endogenous individual behavior*, *agent heterogeneity*, *life-cycle characteristics*, *endogenous production* and *asset pricing*. Looking at the relevant literature, however, the comprehensive overview given above has shown that at least one of the dimensions is disregarded or at least not explicitly taken into account by existing research - although the economic and finance theories which the mentioned literature developed and applied are capable to do so. These theories have been reviewed in Section 2.2 and establish the methodical foundation to address the research objective within the present work. In order to enhance comprehensibility and to further motivate the concentration on heterogeneous patience, a two step approach based on the successive development of two economic models is taken:

First, Chapter 3 will abstract from some of the dimensions and present a simplified approach. By assuming an exchange economy with classical demographic structure populated by agents heterogeneous in their patience

[35] Among other things, Gomes et al. (2013) also examine the equilibrium impact of changes in the capital income tax rate. Hence, the study may equally be considered within the second strand of literature classified here.

and endowments, a dynamic general equilibrium asset pricing model is developed that captures the dimensions of *redistributive taxation*, *endogenous individual behavior*, *agent heterogeneity* and *asset pricing* but ignores the aspects of *endogenous production* and partly *life-cycle characteristics* (especially OLG). This approach helps to clarify the equilibrium impact of redistributive taxation on asset prices, individual consumption and investment behavior as well as the implied complex relationships within a simplified model setting. Beyond that, the meaning of heterogeneous patience for the relevant model results is shown to be identical to the results that are derived from considering heterogeneous risk aversion, as given in Fischer and Jensen (2015).

Second, Chapter 4 will add the two missing dimensions - *endogenous production* and (partly) *life-cycle characteristics* - to the model developed in the preceding chapter. Considering an endogenous linear production technology agents' individual investment and consumption behavior will become decisive in determining aggregate production output. A redistribution mechanism that influences agents' decisions will then have real effects on economic development. Assuming overlapping generations explicitly takes into account the finiteness of human life and the individual behavior over the life cycle, while allowing for a long-lived nature of the economy. On the one hand, this also influences production output itself. On the other hand, redistribution between different classes within a generation (intra-generational) might be considered as well as a reallocation of resources across generations (inter-generational), which are features of redistribution systems that can be observed throughout many countries in the world.

3

Redistributive Taxation in an Exchange Economy

Elaborating on the previous considerations, this chapter develops and analyzes a dynamic general equilibrium asset pricing model featuring the dimensions of *redistributive taxation*, *endogenous individual behavior*, *agent heterogeneity* and *asset pricing*. In the context of the above mentioned required properties, it represents a simplified approach, since it abstracts from the identified dimensions of *endogenous production* and partly *life-cycle characteristics* (especially OLG). As explained above, however, such an approach can be useful in providing a first insight with regard to the equilibrium impact of a tax-based reallocation mechanism and agent heterogeneity on households' consumption and investment behavior as well as on asset prices. Beyond that, the model can be used to further motivate the concentration on differences in time preferences as a source of preference heterogeneity.

The present chapter builds upon the dynamic general equilibrium asset pricing model by Fischer and Jensen (2015). They consider an exchange economy with classical demographic structure populated by agents heterogeneous with respect to their preferences and initial financial endowment. Motivated by the discussion on inequality and individual heterogeneity in Section 2.1, the present model specification deviates from this approach in one important respect. While the referred article investigates the impact of heterogeneity in agents' risk aversion, the present chapter will concentrate on the implications of heterogeneous time preferences.[36] The general model specification is as follows: time is assumed to evolve in discrete steps towards a finite economic horizon. The population structure is standard, which means that agents enter the economy at the same point in time and, from then on, simultaneously populate the economy until its very end. Moreover, the economy provides a financial market in which two types of non-redundant assets are traded, a risky stock and risk-free bonds. For the sake of a simple model solution, two restrictions apply. First, financial markets are complete given the two non-redundant assets. And second, the number of heterogeneous households is restricted to two individuals, which may differ in their initial financial endowment and their attitude concerning the intertemporal allocation of consumption possibilities.[37]

The contribution of this chapter is threefold: First, it presents the first study to investigate the simultaneous impact of heterogeneous time preferences and redistributive taxation on households' consumption and investment decision and on asset prices within a dynamic general equilibrium

---

[36] In the expositions in Section 2.1 it was shown that both time preferences as well as risk aversion coefficients vary across households with different income levels. Nevertheless, heterogeneous patience was found to be of special importance in order to capture certain properties of the observed wealth and consumption distribution. Beyond that, the evidence on the relationship between time preferences and income as well as age is presumably clearer than for risk aversion.

[37] The assumption of complete markets and the limitation to two individuals is not necessary but simplifies the model solution. A more general solution would require a more extensive approach, following an equivalent solution method as presented in Chapter 4. Since the results are qualitatively robust with respect to these assumptions, the procedure seems reasonable.

model. Second, and in contrast to large parts of the related literature, it establishes a tractable framework and solution method in the context of asset pricing with heterogeneous agents, for which analytical results are obtained. This provides the possibility to examine the underlying relationships and, thereby, to enhance the understanding of the derived model results. Third, by comparing the results to the findings in Fischer and Jensen (2015), the effects of heterogeneous patience and risk aversion are found to be comparable in the context of redistributive taxation. While the general effects of both sources of preference heterogeneity are comparable, a simple and tractable analytical solution is not available under heterogeneous risk aversion.[38] This motivates the focus on heterogeneous subjective time preferences in the context of a tax-based reallocation mechanism.

The solution method presented in this chapter builds on the aforementioned assumption of complete markets. It is a well known fact that, when markets are complete, agents align their marginal rates of substitution, strive for a linear sharing rule and the stochastic discount factor underlying the economy is unique (see, for example, Munk (2013)). The present analysis shows that, when individuals are heterogeneous regarding their subjective time discount factors (time preferences), the share of individual consumption in aggregate consumption is not constant but time-dependent, unlike in the case of homogeneous preferences. Contrary to the setting with heterogeneous risk aversion, however, the share of individual consumption in aggregate consumption will be state-independent. This is decisive with respect to the model solution, as the analytical results will depend on the households' consumption shares, which are to be determined endogenously within the model. Given the deterministic nature of the recursive consumption share function in the presence of heterogeneous time preferences, it follows that once the consumption distribution at one point in time is found, the consumption distribution for any other period is determined as well. In order to solve for equilibrium the analyti-

[38] A closed-form solution exists for the case of homogeneous preferences, as shown by Fischer and Jensen (2015). It is given as special case in the present setting.

cal solutions regarding individual consumption behavior and asset prices can be used to derive a nonlinear deterministic equation in the initial distribution of consumption. It constitutes a single equilibrium condition that can be solved for the initial consumption shares using numerical methods.

The chapter is arranged as follows: Section 3.1 introduces the detailed model specification defining the economic setup, the redistributive tax system, the individuals' optimization problem and conditions for market clearing. Section 3.2 derives the general equilibrium model solution and presents the analytical results. In Section 3.3, the quantitative implications of redistributive taxation along with agent heterogeneity on individual consumption and investment behavior are illustrated. Section 3.4 concludes the present chapter.

3.1 The Model

In the present section, the stylized model setup is specified. The representation follows Fischer and Jensen (2015), but deviates especially with respect to the assumption of heterogeneity in agents' patience. The section starts by developing the economic framework before presenting the redistributive tax system. Subsequently, the agents' optimization problem is described. The section is closes by formally defining market equilibrium.

3.1.1 Economic Setup

Within the stylized model economy considered in the present chapter, time is discrete, starts at date 0, faces a finite horizon T, and is denoted by $t = 0, 1, \ldots, T$. The economy is populated by two heterogeneous and finitely lived agents, whose lifespan coincides with the existence of the economy. That is, they are born at date $t = 0$ and dissolve economically at the horizon T. These individuals are indexed by $m = 1{,}2$ and are heterogeneous with respect to two important characteristics. First, and in line with

Fischer and Jensen (2015), agents may differ in their initial financial endowment, which generates persistent heterogeneity in consumption and wealth levels across agents. Second, and closely connected, individuals' attitude concerning the intertemporal allocation of consumption possibilities may also alter across income levels, which constitutes heterogeneity in preferences. Particularly, patience is found to be higher for relatively rich households than for relatively poor households (see the discussion in Section 2.1.3). In the present setting, this is captured by considering heterogeneity within the agents' subjective time discount factors as, for instance, in Lawrance (1987).

For clarity, the following notational convention shall apply. Lowercase letters denote per capita and capital letters aggregate variables. The current period t is denoted by a subscript, whereas the agent type m is denoted by a superscript. Then, for instance, the date t consumption of agent type m is given by c_t^m, whereas aggregate consumption is C_t.

Economic production is exogenous to the model and generates an uncertain but positive output (or dividend) D_t of a single non-storable consumption good in every time step. This implies that economic development is independent of both the agents' behavior as well as any government policy. As outlined in Section 2.2.2.1, this concept is called an exchange economy, where production output is often interpreted as the perishable fruit from a so-called "Lucas tree" (see Lucas (1978)). Since the fruits (consumption goods) cannot be stored, output is not only exogenous but will also be consistent with aggregate consumption C_t in equilibrium.

The exogenous and stochastic process of production output $D_{t\in\{0,1,\dots,T\}}$ is specified by determining an initial value as well as the properties of the random per period growth rate in production. Without loss of generality, the former is achieved by normalizing production output at date $t = 0$ to $D_0 = 1$. For tractability, the latter is specified by assuming that gross production growth $G_{t+1} = D_{t+1}/D_t$ from period t to $t+1$ is given by an independent and identically distributed (i.i.d.) binomial random variable with possible growth rates denoted by G_z, where $z = 1, 2$ are the two possible realizations. Each realization occurs with equal probability $1/2$.[39]

The economy provides two non-redundant assets that are traded in a financial market. First, agents can trade a single stock, which is the claim to the exogenous stream of the consumption good (or dividends) D_t produced by the Lucas tree. Since the realization of the growth rates and, with it, production output are uncertain, the development of the stock value and the resulting asset return are considered risky. The ex-dividend stock price (value of production means) at time t is denoted by $P_{k,t}$. The initial holdings of the agents in this risky security are denoted by $\alpha^m_{-1} > 0$, for $m = 1, 2$, and its net supply is normalized to one, which implies $\alpha^1_{-1} + \alpha^2_{-1} = 1$. Furthermore, in the following agent type 1 is assumed to be endowed with a smaller fraction of initial wealth than agent type 2; that is, for the agents' initial stock holdings it holds that $\alpha^1_{-1} < \alpha^2_{-1}$.

Second, agents can trade a risk-free one-period bond. The payout of such a security is said to be certain, as it is known by the agents one period in advance. The gross return on the risk-free asset before tax between time t and $t+1$ is denoted by $R_{f,t}$. The subscript displays the period at which the return is known to the individuals, not at which it is realized. There is no technology that generates the risk-free payout and, hence, the net supply is zero. In other words, trading in the risk-free bond requires a market equilibrium that yields a risk-free rate at which agents are willing to take complementary positions in such an asset.

Finally, assets can be traded without transaction costs, do not admit arbitrage and agents do not face any borrowing or short-sales constraints. Since in every time step the number of possible subsequent economic states is equal to the number of traded non-redundant assets, the financial market is complete (see, for example, Munk (2013, pp. 91-92)).

39 Binomial or multinomial processes for consumption and asset prices are widely used within the discrete-time asset pricing literature (see, for example, Mehra and Prescott (1985), Rietz (1988), He (1991), Dumas and Lyasoff (2012), and Chabakauri (2015)).

3.1.2 Tax System and Redistribution Mechanism

The assumptions regarding the stylized redistributive tax system follow the specifications of Fischer and Jensen (2015). These postulate that there exists a government that pursues the objective to reduce the disparity in consumption opportunities across agents. In order to achieve its objective, government taxes the agents' income and redistributes the collected revenues, a process which can be observed in real-life tax systems throughout many countries in the world (see the discussion in Section 2.1.2). The taxable income of individuals populating the economy is defined to consist of their net capital income, which comprises gains from equity and bond investments in every time step. Since capital income is the only source of income received by the agents and the risky stock is the only asset in positive net supply, it follows that aggregate net capital income in the economy at date t is given by $P_{k,t} + D_t - P_{k,t-1}$. In other words, the tax basis in period t is given by the current value of the production means, plus current production output, minus the value of the production means in the previous period.

While the government attempts to reduce the disparity in consumption opportunities by taxing and redistributing net capital income, it is confronted with friction costs that limit harmonization. This trade-off faced by the government is formally modeled by a quadratic objective function. Denote by k_t^m the net capital income at date t of an agent of type m before redistribution and by $\tilde{k}_t^m$ the net capital income after redistribution, then the government's optimization problem at time t is given by:

$$\min_{\{\tilde{k}_t^m\}_{m=1}^M} \sum_{m=1}^{2} \left\{ \left(\tilde{k}_t^m - \frac{1}{2} \left(P_{k,t} + D_t - P_{k,t-1} \right) \right)^2 + \kappa \left(\tilde{k}_t^m - k_t^m \right)^2 \right\} \quad (3.1)$$

subject to

$$\sum_{m=1}^{2} \tilde{k}_t^m = P_{k,t} + D_t - P_{k,t-1}, \tag{3.2}$$

where the first term in Equation (3.1) indicates the prevailing disparity in the distribution of net capital incomes before transfers and the second term the costs associated with redistribution. The strength of the frictions is measured by the parameter $\kappa \geq 0$. The solution to the government's optimization is given by the following linear feedback rule:[40]

$$\tilde{k}_t^m = \frac{\kappa}{1+\kappa} k_t^m + \frac{1}{1+\kappa} \frac{1}{2} \left(P_{k,t} + D_t - P_{k,t-1} \right). \tag{3.3}$$

The redistribution system implied by Equation (3.3) can be realized by imposing a flat tax of $\tau = \frac{1}{1+\kappa}$ on net capital income and redistributing aggregate tax revenues equally among the two individuals. This implies that the agent that earns an income below the average becomes net recipient of transfers, whereas the agent that earns an income above the average becomes net contributor. Furthermore, assuming an equal allocation of tax proceeds ensures that the order of agents according to their pre-tax income level remains unchanged by the redistributive tax system. No friction costs, i.e., $\kappa = 0$, correspond to tax rates of 100%. Contrary, in the case of friction costs that tend to infinity, i.e., $\kappa \rightarrow \infty$, the tax rates become zero.

Aggregate current tax revenues are immediately redistributed to the agents and, hence, equal aggregate current transfer payments. This simplifying assumption ensures that the tax system is balanced each period. Since the risk-free security is in zero net supply, disposable tax revenues depend on the current dividend payment and the change in the stock price only.[41]

---

[40] The formal derivation is shown in Appendix B.1.1.

[41] When considering "gross" tax revenues, government also collects a positive amount of revenues by taxing bond market gains. The taxation scheme presented here, however, implies immediate tax credits of equal size granted to individuals that hold short positions in the risk-free security. As trading in the bond requires agents that take offsetting positions in that asset,

Overall, this implies that government collects a total of

$$S_t = \tau \left(P_{k,t} + D_t - P_{k,t-1} \right) \tag{3.4}$$

in disposable tax revenues at time t. From the redistribution mechanism and Equation (3.4) follows that the transfer payment at date t received by any agent is $s_t = S_t/2$.

3.1.3 The Agents' Optimization Problem

In line with neoclassical theory, all agents are expected utility maximizers with time-additive preferences over a single consumption good, where they form rational expectations about the uncertain future development of the economy based on a full set of information equally available to all of them. Formally, there is an increasing sequence of information sets (a filtration) $\{\mathcal{F}_t : \ t = 0, 1, ..., T\}$ underlying the economy and available to all of them in period t. It contains all information regarding the past and current values of the random growth rate G_t up to time t as well as the consumption and investment histories of all individuals up to period $t-1$. For the agents populating the economy this means that the realization of G_t and, hence, the stochastic production output D_t become known to them at the beginning of period t. Furthermore, since individuals have access to $\mathcal{F}_t$, they know about the whole history of realizations up to date t. Based on this information set, agents form their (conditional) expectations and make their economic decisions.

3.1.3.1 Heterogeneous Patience and Expected Lifetime Utility

As outlined in Section 3.1.1, the present setting features agent heterogeneity with respect to two dimensions. On the one hand, agents may differ

there are no disposable revenues from bond market activities in the aggregate.

in their initial financial endowment. On the other hand, individuals are assumed to differ in their patience regarding the intertemporal allocation of consumption possibilities. As a consequence of these heterogeneous characteristics their optimization problems diverge as well.

Formally, agents' heterogeneity in patience is modeled by assuming that they hold varying subjective time discount factors $0 < \delta^m < 1$ as, for instance, in Lawrance (1987) and Gomes et al. (2013). As explained in Section 2.2.1.2, considering time additive utility, the subjective time discount factor weights the instantaneous utility from consumption at different points in time, i.e., the summands of lifetime utility. In this vein, it describes the preferences of individuals regarding consumption between two time steps t and $t+1$. For smaller values of δ^m, agents tend to have increasing preferences for early over late consumption, i.e., for consumption at t over consumption at $t+1$. In other words, they become less patient. When δ^m becomes larger patience increases, as agents decrease their preferences for early over late consumption. In the present model the subjective time discount factor is exogenously specified and constant over time but may differ across agents as indicated by the superscript m.[42]

Then, assuming that both agents have standard, time separable, instantaneous utility functions $u(c_t^m)$ over consumption c_t^m, the expected present discounted utility, or lifetime utility, U_t^m at date $t = 0$ of agent type m is given by

$$U_0^m = \sum_{t=0}^{T} (\delta^m)^t \, \mathbb{E}_0 \left[u(c_t^m) \right], \tag{3.5}$$

[42]The assumption that the subjective time discount factor is exogenous implies that patience is unaffected by changes in the income or wealth level due to the redistribution mechanism. Considering subjective time discount factors that endogenously depend on the agent's income, consumption or wealth level, therefore, seems to be reasonable and would certainly enrich the model and its implications. However, this would complicate the model and foil its analytical solution. Because of the complexity it is common practice within the asset pricing and relevant taxation literature to assume constant and exogenous preference parameters. Studies featuring endogenous subjective discount factors are, for instance, Mendoza (1991) and Jahan-Parvar et al. (2013).

where $\mathbb{E}_t$ is the conditional expectation operator, forming expectations conditional on the agent's current set of information, i.e., $\mathcal{F}_t$. Equation (3.5) displays the impact of the finite horizon on the individual's expected lifetime utility. When agents get older, the number of summands decreases until their decisions will only depend on the current instantaneous utility at time T. As long as the economic horizon T is finite the agents' lifetime will be finite, too.

3.1.3.2 Evolution of Wealth

The agent's consumption decision and, hence, his lifetime utility is constrained by the resources available to him. In the present setting the evolution of individual wealth (cash on hand) after taxes comprises three components. First, at date t an agent receives the net payout from his stock investment made at period $t-1$. Denote by α_t^m the share of the single risky stock held by agent type m from time t to time $t+1$. Then, his time t after-tax income from stock market activities follows by

$$\alpha_{t-1}^m \left((1-\tau)\left(P_{k,t} + D_t\right) + \tau P_{k,t-1}\right). \tag{3.6}$$

Second, agents receive the net payout from their investment activities in the market for the risk-free bond. Let the number of the risk-free security held by an agent of type m from time t to $t+1$ be given by β_t^m, then the associated after-tax income at date t is

$$\beta_{t-1}^m \tilde{R}_{f,t-1}, \tag{3.7}$$

where

$$\tilde{R}_{f,t-1} = (1-\tau) R_{f,t-1} + \tau \tag{3.8}$$

is the gross risk-free return (accumulation factor) after tax from time $t-1$ to t. Together, Equations (3.6) and (3.7) define the individual's net income from financial market activities. Beyond that, and third, agents receive non-financial or permanent income in terms of transfer payments as defined above (see Section 3.1.2) and restated below

$$\begin{aligned} s_t =& \frac{1}{2} S_t \\ =& \frac{\tau}{2} \left(P_{k,t} + D_t - P_{k,t-1} \right). \end{aligned} \tag{3.9}$$

Overall, comprising all income sources stated above, the evolution of wealth of an agent of type m at date t follows

$$v_t^m = \alpha_{t-1}^m \left((1-\tau) \left(P_{k,t} + D_t \right) + \tau P_{k,t-1} \right) + \beta_{t-1}^m \tilde{R}_{f,t-1} + s_t, \tag{3.10}$$

for $0 < t \leq T$. When agents enter the economy at time 0 no trading activity has taken place yet and they are solely endowed with the exogenously specified initial stock holding. That is, for $t = 0$,

$$v_0^m = \alpha_{-1}^m \left(P_{k,0} + D_0 \right) \tag{3.11}$$

is the entering level of wealth of an agent type m. Equation (3.10) can be rewritten to display the mode of operation of the redistributive tax system (see Fischer and Jensen (2014)). Using Equations (3.9) and (3.4) in constraint (3.10) and rearranging one obtains

$$\begin{aligned} v_t^m =& \alpha_{t-1}^m \left(P_{k,t} + D_t \right) + \tau \left(\frac{1}{2} - \alpha_{t-1}^m \right) \left(P_{k,t} + D_t - P_{k,t-1} \right) + \\ & \beta_{t-1}^m \tilde{R}_{f,t-1} \end{aligned} \tag{3.12}$$

for the evolution of wealth of an agent of type m. The second term of Equation (3.12) represents the net transfers received by agent type m. It turns out that the tax rate τ is multiplied by the term $(1/2 - \alpha_{t-1}^m)$, which represents the deviation in agent m's stock holdings from an equal distribution. This shows that the effective tax rate on the risky stock depends on the agent's share in aggregate production means. As a result, the relative rich (poor) individual that holds a fraction of stock market wealth that is larger (smaller) than the average, i.e., $\alpha_{t-1}^m > 1/2$ ($\alpha_{t-1}^m < 1/2$), faces positive (negative) effective tax rates and, hence, is subject to negative (positive) net transfers. He is, therefore, the net contributor to (recipient of) the redistribution system. With raising inequality, effective tax rates increase (decrease) for the relative rich (poor) agent, which implies that effective tax rates are progressive, although the tax system implemented by the government is based on a flat tax rate.

3.1.3.3 Maximization Problem

With the objective to maximize expected lifetime utility over consumption, agents have to take two decisions in every time step t. On the one hand, this is the consumption-savings decision that determines the share of available resources to be consumed or saved in every period. The relevant decision variable is given by the agent's consumption c_t^m. On the other hand, since two financial assets are traded, individuals face a portfolio choice. That is, the decision of how to allocate the savings fraction of available resources between the different securities available in every time step. The associated decision variables are the share of the stock α_t^m and the number of risk-free bonds β_t^m to be held from one period to the next.

Agents are assumed to hold homogeneous instantaneous preferences of the commonly used constant relative risk aversion (CRRA) type with common risk aversion parameter $\gamma > 0$.[43] That is, the date t utility obtained by agent type m from consumption c_t^m in this period is formally described by

$$u\left(c_t^m\right) = \begin{cases} \frac{\left(c_t^m\right)^{1-\gamma}}{1-\gamma} & if\ \gamma > 0, \gamma \neq 1, \\ \ln\left(c_t^m\right) & if\ \gamma = 1. \end{cases} \tag{3.13}$$

[43] It is noteworthy that the properties of CRRA preferences imply that optimal consumption will be strictly positive, i.e., $c_t^m > 0$. Hence, a non-negativity constraint on consumption is redundant, since it will never be binding (see, for instance, Munk (2013, p. 171)).

Finally, the agent's optimization problem is defined by bringing together Equations (3.5)-(3.13). The objective function of agent m is given by

$$\max_{\{\{c_t^m\}_{t=0}^{T},\{\alpha_t^m,\beta_t^m\}_{t=0}^{T-1}\}} V_0^m = \sum_{t=0}^{T} (\delta^m)^t \, \mathbb{E}_0 \left[\frac{(c_t^m)^{1-\gamma}}{1-\gamma} \right], \tag{3.14}$$

subject to

$$c_t^m = v_t^m - \alpha_t^m P_{k,t} - \beta_t^m, \tag{3.15}$$

$$\alpha_T^m = \beta_T^m = 0, \tag{3.16}$$

in combination with constraints (3.10) and (3.11). Equation (3.14) is denoted the agent's indirect utility, which is the maximum expected lifetime utility of current and future consumption (Munk (2013)). Equation (3.15) is the agent's budget constraint and (3.16) is the agent's terminal portfolio condition. It describes the fact that at the horizon T agents do not enter into any asset position and consume all their remaining wealth, i.e., $c_T^m = v_T^m$. On the one hand, this is a direct consequence of the assumption that consumption is the only source of utility. Any positive investment at the horizon would result in leaving units of the consumption good without future consumption possibility and, therefore, contradicts utility maximizing behavior. On the other hand, condition (3.16) constrains an agent from leaving debt at the cost of the other agent. That is, it states that any private debt must be settled within the model horizon.

3.1.4 Market Equilibrium

Before turning to the characterization of equilibrium the concept of transfer capital has to be introduced first. In the presence of a stream of permanent income, the expected remaining lifetime consumption (hereinafter referred to as total wealth) of an agent will include the present value of his expected future payments received from this source of income. In the present model, such an income is given by the permanent stream of transfer payments s_t to the individuals, if $\tau > 0$. Accordingly, transfer capital $P_{s,t}$ is defined as the present value (or price) of expected future transfers.[44] Like the stock price, the price of transfer capital is determined in equilibrium.

An equilibrium for the economy described above consists of a set of individual consumption decisions $\{c_t^m\}_{t=0}^T$ and investment policies $\{\alpha_t^m, \beta_t^m\}_{t=0}^{T-1}$ for each type $m = 1, 2$, endogenously determined prices $\{P_{k,t}, P_{s,t}\}_{t=0}^T$ as well as the endogenously determined risk-free returns $\{\tilde{R}_{f,t-1}\}_{t=0}^T$, such that for every period t (i) each agent maximizes his expected lifetime utility (3.14) subject to constraints (3.15)-(3.16) in combination with (3.10)-(3.11) and (ii) markets clear. Market clearing (ii) comprises clearing on the market for consumption goods, i.e.,

$$\sum_{m=1}^{2} c_t^m = C_t = D_t, \quad t = 0, 1, ..., T, \tag{3.17}$$

and clearing on both asset markets, i.e.,

$$\sum_{m=1}^{2} \alpha_t^m = 1, \quad t = -1, 0, ..., T, \tag{3.18}$$

$$\sum_{m=1}^{2} \beta_t^m = 0, \quad t = 0, 1, ..., T. \tag{3.19}$$

Condition (3.17) implies that the sum of the individual consumption of both agents must equal aggregate production, while conditions (3.18)-(3.19) indicate that agents hold all outstanding shares in the risky stock and that the market for the risk-free bond is in zero net supply, respectively. The definition of market equilibrium formally closes the model and the following section can now turn to the model solution.

[44] A formal definition of transfer capital is given below in Section 3.2.3.1.

3.2 General Equilibrium Solution

Given the assumptions regarding the economy in Section 3.1.1, there exists a finite and discrete state space of possible outcomes Ω underlying the economy. Given one current economic state $\omega \in \Omega$, the assumptions regarding the growth rates G_z imply the realization of only two possible direct successive future states $\omega'(z)$, with $z = 1, 2$, for every time step $t = 0, 1, \ldots, T$. Since the number of possible successive future economic states corresponds to the number of non-redundant assets traded on the financial markets between two time steps for every $t = 0, 1, \ldots, T$, markets are complete in the present setting (see, for example, Munk (2013, pp. 91-92)).

The solution of the given dynamic general equilibrium model builds on the properties of complete markets. In particular, under market completeness agents share risk efficiently and align their marginal rates of substitution in every state and time step (see Munk (2013, Theorem 7.4 and 7.2)). Put differently, the market structure allows agents to trade with each other in order to eliminate any idiosyncratic component in their marginal utilities (Campbell (2003)). Furthermore, individual optimal consumption is strictly increasing and affine in aggregate consumption. That is, agents strive for a linear sharing rule (see Munk (2013, Theorem 7.4 and 7.5)).

The current section shows that these properties hold for the model setting presented above. Moreover, it is proven that, when individuals are heterogeneous regarding their subjective time discount factors (patience), it further holds true that the share of individual consumption in aggregate consumption is state-independent. This is contrary to the case of heterogeneous risk aversion, where consumption shares are state-dependent. In contrast to the setting that presumes homogeneous preferences, the distribution of consumption, however, is not constant but time-dependent. Particularly, the consumption share function is found to be a non-linear recurrence equation.

Since the distribution of consumption is only time-dependent, but state-independent, the consumption shares at any time step t can be expressed in terms of the consumption shares at date $t = 0$. Building on this result, all equilibrium processes of the analytical model solution are characterized as functions of these consumption shares. For them, a closed-form solution does not exist. To solve for equilibrium, the analytical model solution is used to determine a nonlinear deterministic equation in the initial distribution of consumption shares. It constitutes a single equilibrium condition and can be solved for the endogenous initial consumption shares using numerical methods.

The remainder of this section is organized as follows: Section 3.2.1 presents the first order conditions as well as the special equilibrium model properties. Section 3.2.2 determines the equilibrium process of the stochastic discount factor and asset prices. Next, Section 3.2.3 derives the analytical results for agents' consumption and investment behavior. Finally, in Section 3.2.4 the equilibrium condition is established.

3.2.1 First Order Conditions, Equilibrium Consumption and the SDF

The present section focuses on the derivation of the special equilibrium properties of the model economy featuring complete markets. Initially, the first order conditions implied by the individuals' optimization problem described above will be presented. Based on these, it is shown that in the present setting agents' consumption shares are state-independent and specified by a non-linear recurrence equation. The section is closed by demonstrating that agents align their marginal rates of substitution in equilibrium and that the stochastic discount factor is uniquely determined.

3.2.1.1 The First Order Conditions

For the optimization problem given by Equation (3.14) and constraints (3.15)-(3.16) in combination with (3.10)-(3.11) the first order conditions are derived. Since the instantaneous utility (3.13) is strictly concave in consumption, second order conditions are satisfied and, hence, first order conditions provide the optimal choice (see Munk (2013, p. 256)).

After-tax Representation

From the individuals' optimization problem described above follows that the first order condition for agent type m with respect to consumption is given by[45]

$$\mu_t^m = \left(\frac{\delta^m}{2}\right)^t (c_t^m)^{-\gamma}, \quad t = 0, 1, ..., T, \tag{3.20}$$

where $\{\mu_t^m\}_{t=0}^T$ are the Lagrangian multipliers associated with the agent's constraints and $\{c_t^m\}_{t=0}^T$ is the individual's optimal equilibrium consumption sequence. Furthermore, the first order conditions at time t for agent type m with respect to his stock and bond investment strategy are, respectively,

$$P_{k,t} = \mathbb{E}_t \left[\frac{2\mu_{t+1}^m}{\mu_t^m}\left((1-\tau)\left(P_{k,t+1} + D_{t+1}\right) + \tau P_{k,t}\right)\right], \tag{3.21}$$
$$t = 0, 1, ..., T-1,$$

$$1 = \mathbb{E}_t \left[\frac{2\mu_{t+1}^m}{\mu_t^m}\right] \tilde{R}_{f,t}, \quad t = 0, 1, ..., T-1, \tag{3.22}$$

with $P_{k,T} = 0$. As explained in Section 2.2.2, Equations (3.21) and (3.22) are the Euler equations that must hold equally for all individuals. They simply state that the market price of an asset is given by the expectation

[45] Appendix B.1.2 provides the details of the derivation.

of its discounted future payoff. In conjunction with condition (3.20) these equations link prices (or returns) to the optimal consumption plans of the agents populating the economy and participating in the financial markets. They contain the definition of the stochastic discount factor in terms of the agent's marginal rate of substitution of consumption over time. Accordingly, the one-period SDF from time t to $t+1$ after taxes induced by agent m is given by

$$\frac{2\mu_{t+1}^{m}}{\mu_{t}^{m}} = \delta^{m}\left(\frac{c_{t+1}^{m}}{c_{t}^{m}}\right)^{-\gamma} . \tag{3.23}$$

Equation (3.23) demonstrates the trade-off an individual faces between consumption in period t and period $t+1$, when making his investment decision (Munk (2013)). As indicated above, it is the rate at which the agent is willing to forgo current (at date t) consumption in exchange for extra consumption tomorrow (at date $t+1$). Since the heterogeneity in subjective time discount factors δ^m, this willingness may vary across agent types m in the present setting. The SDF is further specified, as soon as the consumption share function is determined.

Pre-tax Representation

Given the results just presented, it can be shown that in the presence of taxation on financial investments an alternative representation of the basic pricing equations can be found in terms of a pre-tax stochastic discount factor.

According to pricing relation (3.21) the pre-tax stock price is defined as the conditional expectation over the discounted next period capital payoff after tax using the after-tax stochastic discount factor (3.23). The pre-tax version of this condition can be found by solving condition (3.21) for the current stock price $P_{k,t}$ (see Fischer and Jensen (2015)):[46]

[46] Details are given in Appendix B.2.4.

$$
\begin{aligned}
P_{k,t} =& \mathbb{E}_t \left[\frac{2\mu^m_{t+1}}{\mu^m_t} \frac{\tilde{R}_{f,t}}{R_{f,t}} \left(P_{k,t+1} + D_{t+1}\right) \right] \\
=& \mathbb{E}_t \left[\frac{2\lambda^m_{t+1}}{\lambda^m_t} \left(P_{k,t+1} + D_{t+1}\right) \right],
\end{aligned} \tag{3.24}
$$

where

$$
\frac{2\lambda^m_{t+1}}{\lambda^m_t} = \frac{2\mu^m_{t+1}}{\mu^m_t} \frac{\tilde{R}_{f,t}}{R_{f,t}} \tag{3.25}
$$

is a pre-tax stochastic discount factor. It is defined by the after-tax version of the stochastic discount factor multiplied by the ratio of after-tax risk-free return to its pre-tax counterpart. Expression (3.24) indicates that the pre-tax equity price can alternatively be derived by discounting the future pre-tax capital payout using the pre-tax version of the stochastic discount factor. This will be useful, when determining the stock price below.

Similarly, expression (3.25) can be used to derive a pre-tax version of condition (3.22):

$$
1 = \mathbb{E}_t \left[\frac{2\lambda^m_{t+1}}{\lambda^m_t} \right] R_{f,t}. \tag{3.26}
$$

This is a direct link between the pre-tax risk-free return and the pre-tax SDF. Pricing relation (3.26), therefore, indicates why (3.25) is considered to be before taxation.

3.2.1.2 The Consumption Share Function

Based on the first order conditions just presented one can now turn to the specification of the equilibrium distribution of consumption shares. In particular, the present section gives a brief sketch of the derivation while concentrating on the presentation of the results. Details are provided in Appendix B.1.4.

Denote by $g_t^1 = c_t^1/D_t$ the consumption share of agent type $m = 1$ at date t, then it follows from clearing on the market for consumption goods, Equation (3.17), that the consumption share of the other agent $m = 2$ is given by $g_t^2 = 1 - g_t^1$. That is, in the present setting with only two agent types, the consumption distribution at date t is fully defined as soon as the consumption share of one of the agents is determined for this time step. Using this result along with the first order conditions derived above, a system of two equations in two unknowns, namely the possible future consumption shares $g^1_{t+1,\omega'(1)}$ and $g^1_{t+1,\omega'(2)}$ conditional on state ω at date t, can be deduced for all time steps t. Then, given the assumptions on financial markets, only one feasible solution results for every time step t. In particular, it must hold that the possible future consumption shares at $t+1$ conditional on state ω at date t are identical, i.e., $g^1_{t+1,\omega'(1)} = g^1_{t+1,\omega'(2)}$. From this equality and the system of equations it follows immediately that the consumption share at time $t+1$ is just given as a function of the consumption share in state ω at time t. Since this holds equally for all periods, the consumption distribution is found to be generally state-independent.

To be precise, the non-linear recurrence equation determining the consumption share of agent type $m = 1$ in period t reads

$$g_t^1 = \frac{g_{t-1}^1}{\left(1 - \left(\frac{\delta_2}{\delta_1}\right)^{\frac{1}{\gamma}}\right) g_{t-1}^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{1}{\gamma}}}, \quad t = 0, 1, ..., T. \tag{3.27}$$

It can be solved backwards until date $t = 0$ in order to state the consumption share of agent type $m = 1$ at date t in terms of his initial consumption share:

$$g_t^1 = \frac{g_0^1}{\left(1 - \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right) g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}, \quad t = 0, 1, ..., T. \tag{3.28}$$

As indicated above, this is a time-dependent, but state-independent, function. Since $g_t^2 = 1 - g_t^1$, the consumption share of agent type $m = 2$

in period t follows immediately. Beyond that, Equation (3.28) implies that the whole sequence of consumption shares $\{g_t^m\}_{t=0}^T$ for both agents $m = 1, 2$ is determined as soon as the initial consumption share g_0^m of one of the agents is found. Furthermore, as individual consumption is given by $c_t^m = g_t^m D_t$, Equations (3.27) and (3.28) imply that agents' optimal consumption is a strictly increasing linear function of aggregate consumption, i.e., a linear sharing rule (see, for instance, Munk (2013, p. 264)).

Although g_0^m cannot be derived in closed form, the analytical solution found above allows to study the impact of heterogeneity in subjective time discount factors on the evolution of consumption shares over time. Taking the partial derivative of g_t^1 with respect to t and considering the assumptions about preferences and the model parameters, it follows that

$$\frac{\partial g_t^1}{\partial t} = \begin{cases} > 0 & if\; \delta^1 > \delta^2, \\ < 0 & if\; \delta^1 < \delta^2. \end{cases} \tag{3.29}$$

Equation (3.29) states that the share in aggregate consumption of agent $m = 1$ is a strictly monotonically increasing (decreasing) function in time, if he holds a higher (smaller) subjective time discount factor, i.e., $\delta^1 > \delta^2$ ($\delta^1 < \delta^2$), than the other agent $m = 2$. Because of the direct link between the consumption shares of both agents $g_t^2 = 1 - g_t^1$ it immediately follows that $\partial g_t^2 / \partial t = -\partial g_t^1 / \partial t$. Hence, the relation described in (3.29) is just the opposite for the other agent $m = 2$. In general, this implies that the share in aggregate consumption is a strictly monotonically increasing (decreasing) function in time for the individual that is relatively more (less) patient. Since this property holds independent of the initial consumption distribution, it is also independent of the initial endowments and the tax rate.

Considering an infinite model horizon and period t that tends to infinity, i.e., $t \to \infty$, the observed properties further imply that the consumption share of agent m has a limiting value. In particular, it follows that the limiting value is finite and given by

$$\lim_{t\to\infty} g_t^1 = \lim_{t\to\infty} \frac{g_0^1}{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right) g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}} = \begin{cases} 1 & if\ \delta^1 > \delta^2, \\ 0 & if\ \delta^1 < \delta^2, \end{cases} \tag{3.30}$$

for agent type $m = 1$. Again, this relation is just the opposite for the other agent $m = 2$. Equations (3.29) and (3.30) demonstrate that, as time evolves, the relatively more patient individual successively drives out the relatively less patient individual and in the long run remains the only economically relevant agent. In other words, at an infinite horizon the model solution coincides with the homogeneous agent case, where the highest subjective time discount factor of the heterogeneous agent model corresponds to the unanimous subjective time discount factor of the homogeneous agent case. This result is in line with the theoretical observations found in, for instance, Lengwiler (2005) and Beaumont et al. (2013).

In the case of homogeneous subjective time discount factors, $\delta^1 = \delta^2$, Equations (3.27) and (3.28) simplify and individual consumption shares become constants. This is the classical result derived in standard asset pricing models with homogeneous agents and identical to the baseline solution in Fischer and Jensen (2015). Only for this special case a closed-form solution is available.

Finally, one point remains noteworthy. Although the tax rate τ does not enter Equations (3.27) and (3.28) directly, consumption shares are affected by τ indirectly through the initial consumption distribution. This relation results from the equilibrium condition (3.62), which will be derived in Section 3.2.4.

3.2.1.3 Alignment of Marginal Rates of Substitution and the SDF

Having established that the optimal solution has the property that agents' consumption shares are time-dependent, but state-independent, it immediately follows that individuals align their marginal rates of substitution in equilibrium. This fact can be shown by equating the basic pricing equation for the risk-free return (3.22) for both agent types $m = 1, 2$, substituting the definition of the SDF as the agent's marginal rate of substitution of consumption over time, Equation (3.23), and using the result of state-independent consumption shares $g_t^m = c_t^m / D_t$ just derived:

$$\begin{aligned} \delta^1 \left(\frac{g_{t+1}^1}{g_t^1} \right)^{-\gamma} &= \delta^2 \left(\frac{g_{t+1}^2}{g_t^2} \right)^{-\gamma} \\ \Leftrightarrow \delta^1 \left(\frac{g_{t+1}^1}{g_t^1} \right)^{-\gamma} \left(\frac{D_{t+1}}{D_t} \right)^{-\gamma} &= \delta^2 \left(\frac{g_{t+1}^2}{g_t^2} \right)^{-\gamma} \left(\frac{D_{t+1}}{D_t} \right)^{-\gamma} \\ \Leftrightarrow \delta^1 \left(\frac{c_{t+1}^1}{c_t^1} \right)^{-\gamma} &= \delta^2 \left(\frac{c_{t+1}^2}{c_t^2} \right)^{-\gamma} . \end{aligned} \tag{3.31}$$

As intended, Equation (3.31) indicates that the marginal rates of substitution of both agents are identical in every economic state and time step. That is, in equilibrium, aggregate consumption risk will be distributed in a way that all individuals possess the same marginal desire to allocate consumption across time and states (Munk (2013)). This property is referred to as efficient risk sharing, since it implies perfect insurance against everything except fluctuations in aggregate consumption (see Back (2010)).

Furthermore, since the marginal rates of substitution are identical for both agents, it conversely follows from Equation (3.31) in conjunction with condition (3.23) that the stochastic discount factors induced by both agents are identical in every economic state and time step:

$$\frac{2\mu_{t+1}}{\mu_t} \equiv \frac{2\mu_{t+1}^1}{\mu_t^1} = \frac{2\mu_{t+1}^2}{\mu_t^2} . \tag{3.32}$$

This implies that the SDF is uniquely determined and, consequently, all properties of complete markets indicated above hold in the present model setting. Finally, from the second line of Equation (3.31) and the properties of equilibrium consumption shares it results that the unique stochastic discount factor follows a time- and state-dependent binomial process:

$$\frac{2\mu_{t+1}}{\mu_t} = \delta^m \left(\frac{g_{t+1}^m}{g_t^m}\right)^{-\gamma} G_{t+1}^{-\gamma}. \tag{3.33}$$

Since it depends on the (initial) distribution of consumption, there is no closed-form solution for the stochastic discount factor. As outlined above, this further implies that, although the tax rate τ does not enter Equation (3.33) directly, the stochastic discount factor is affected by the tax rate indirectly through this channel.

3.2.2 Asset Prices

Building on the equilibrium model properties, the present section focuses on the characterization of the equilibrium results with respect to financial asset prices. At the outset, the analytical solution for the risk-free rate is presented, while the section closes with the presentation of the analytical solution to the equilibrium stock price process. All results are given as functions of the initial consumption share of agent $m = 1$.

3.2.2.1 The Risk-free Return

Given the results derived so far, the determination of the risk-free return is straightforward. As above, substitute the definition of the SDF, Equation (3.33), into the basic pricing relation, Equation (3.22), and apply the property of state-independent consumption shares, in order to derive the following representation:

$$
\begin{aligned}
\tilde{R}_{f,t} &= \left(\mathbb{E}_t \left[\frac{2\mu_{t+1}}{\mu_t} \right] \right)^{-1} \\
&= \left(\delta^m \left(\frac{g_{t+1}^m}{g_t^m} \right)^{-\gamma} \mathbb{E}_t \left[\left(\frac{D_{t+1}}{D_t} \right)^{-\gamma} \right] \right)^{-1} . \qquad (3.34)
\end{aligned}
$$

Equation (3.34) reveals a standard result in asset pricing theory, the condition that the (gross) risk-free return is the inverse of the expected value of the stochastic discount factor (see, for example, Cochrane (2005), Back (2010) or Munk (2013)). Then, using the i.i.d. property of aggregate production growth, i.e., $G = G_{t+1} = D_{t+1}/D_t$, and the solution (3.28) determining agent 1's consumption share, in order to specify the one-period risk-free rate after tax by

$$
\tilde{R}_{f,t} = \left(\frac{\left(1 - \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right) g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}{\left(1 - \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+1}{\gamma}}\right) g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+1}{\gamma}}} \right)^{\gamma} \left(\delta^1 \mathbb{E}\left[G^{-\gamma} \right] \right)^{-1} , \qquad (3.35)
$$

as a function of agent 1's consumption share at date $t = 0$. The one-period risk-free rate after tax is time-dependent, but state-independent. In particular, given the restrictions on the parameters and heterogeneity in the subjective time discount factors of the two agents $\delta^1 \neq \delta^2$, the risk-free return is a strictly monotonically decreasing function in time.[47] As in the case of the consumption share function, the tax rate τ does not enter Equation (3.35) directly. However, since it depends on the initial distribution of consumption, the risk-free rate is affected by τ indirectly through this channel.[48]

[47] It is shown in Appendix B.1.5 that this property holds.

[48] Again, this relation results from the equilibrium condition (Equation (3.62)), which will be specified in Section 3.2.4.

When preferences are homogeneous, i.e., $\delta^1 = \delta^2$, the first factor in Equation (3.35) vanishes and the risk-free return after tax becomes constant and independent of the tax rate. The solution is then identical to the one found for the standard asset pricing model with i.i.d. aggregate risk, one representative CRRA agent and without taxation (Fischer and Jensen (2015)).

In any case, the one-period risk-free return before tax follows immediately from Equation (3.8) by $R_{f,t-1} = (\tilde{R}_{f,t-1} - \tau)/(1-\tau)$. It depends directly on the tax rate.

Finally, the risk-free return can also be stated in a multi-period version. In so doing, the following simple notational convention shall apply. As before, with only one time subscript attached to it, $\tilde{R}_{f,t}$ denotes the one-period risk-free return that is realized from period t to $t+1$. Henceforth, when adding a second time subscript, $\tilde{R}_{f,t,t+n}$ shall denote the n-period risk-free return that is realized from period t to $t+n$. Formally, the multi-period risk-free return after tax follows immediately from its one-period counterpart given in Equation (3.35). That is, the n-period risk-free return after tax from period t to $t+n$ reads

$$\tilde{R}_{f,t,t+n} = \left(\frac{\left(1 - \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right) g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}{\left(1 - \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}\right) g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}} \right)^{\gamma} \left(\delta^1 \mathbb{E}\left[G^{-\gamma}\right]\right)^{-n}. \quad (3.36)$$

This notation will simplify the representation in the following.

3.2.2.2 The Price-Dividend Ratio

In order to determine the equilibrium stock price process, it is convenient to define the price-dividend ratio, $PD_{k,t} \equiv P_{k,t}/D_t$, by dividing both sides of the pre-tax pricing Equation (3.24) by D_t:

$$PD_{k,t} = \frac{\tilde{R}_{f,t}}{R_{f,t}} \mathbb{E}_t \left[\frac{2\mu_{t+1}}{\mu_t} \left(PD_{k,t+1} + 1\right) G_{t+1} \right]. \quad (3.37)$$

Substituting the equilibrium stochastic discount factor (3.33), the price-dividend ratio at date t reads

$$PD_{k,t} = \frac{\tilde{R}_{f,t}}{R_{f,t}} \delta^m \left(\frac{g^m_{t+1}}{g^m_t} \right)^{-\gamma} \mathbb{E}_t \left[\left(PD_{k,t+1} + 1 \right) G^{1-\gamma}_{t+1} \right]. \tag{3.38}$$

Having established Equation (3.38), the price-dividend ratio can be solved by backward induction starting from the terminal date (Fischer and Jensen (2015)). Due to the fact that $P_{k,T} = PD_{k,T} = 0$ at the horizon T, the price-dividend ratio at date $T-1$ is given by

$$\begin{aligned} PD_{k,T-1} &= \frac{\tilde{R}_{f,T-1}}{R_{f,T-1}} \delta^m \left(\frac{g^m_T}{g^m_{T-1}} \right)^{-\gamma} \mathbb{E}_{T-1} \left[G^{1-\gamma}_T \right] \\ &= \frac{\tilde{R}_{f,T-1}}{R_{f,T-1}} \delta^m \left(\frac{g^m_T}{g^m_{T-1}} \right)^{-\gamma} \mathbb{E} \left[G^{1-\gamma} \right], \end{aligned} \tag{3.39}$$

where the assumption of i.i.d. consumption growth has been used again. Equation (3.39) shows that the price-dividend ratio at date $T-1$ is state-independent. Applying this result to Equation (3.38) it follows that the price-dividend ratio at date $T-2$,

$$PD_{k,T-2} = \frac{\tilde{R}_{f,T-2}}{R_{f,T-2}} \delta^m \left(\frac{g^m_{T-1}}{g^m_{T-2}} \right)^{-\gamma} \left(PD_{k,T-1} + 1 \right) \mathbb{E} \left[G^{1-\gamma} \right], \tag{3.40}$$

is state-independent, too. Proceeding this way and iterating backwards through time, the price-dividend ratio is generally state-independent for all time steps t and given by the following recurrence equation:

$$\begin{aligned} PD_{k,t} &= \frac{\tilde{R}_{f,t}}{R_{f,t}} \delta^m \left(\frac{g^m_{t+1}}{g^m_t} \right)^{-\gamma} \left(PD_{k,t+1} + 1 \right) \mathbb{E} \left[G^{1-\gamma} \right] \\ &= R^{-1}_{f,t} \left(PD_{k,t+1} + 1 \right) \frac{\mathbb{E} \left[G^{1-\gamma} \right]}{\mathbb{E} \left[G^{-\gamma} \right]}, \end{aligned} \tag{3.41}$$

where the second line is due to the definition of the risk-free rate given in Equation (3.34). Alternatively, starting at the horizon (using the fact that $PD_{k,T} = 0$) and solving the recursive definition (3.41) by backward induction, the price-dividend ratio at date t can be stated explicitly by

$$PD_{k,t} = \sum_{n=0}^{(T-t)-1} \left(\frac{\mathbb{E}\left[G^{1-\gamma}\right]}{\mathbb{E}\left[G^{-\gamma}\right]} \right)^{(T-t)-n} R^{-1}_{f,t,T-n}, \tag{3.42}$$

which is a time-dependent, but state-independent, function.[49] Beyond that, it depends on the initial consumption distribution through the risk-free rate before tax and, hence, is not a closed-form solution. Finally, it depends directly and indirectly on the tax rate τ through the risk-free rate before tax.

3.2.3 Individual Investment and Net Transfers

Having established the equilibrium model properties and the implied explicit analytical asset pricing solutions, the present section concentrates on the determination of equilibrium results on an individual level. In a first step, agents' total wealth budget constraint is derived. From there, the formal definition of transfer capital and subsequently its explicit analytical equilibrium solution follow. Second, agents' marginal propensity to consume out of total wealth (MPCTW) is specified and, in conjunction with the previous results, used to determine individual investment policies. Finally, the section closes by determining net transfer income and optimal expected lifetime utility.

3.2.3.1 Total Wealth Budget Constraint and Transfer Capital

As outlined above, in the presence of a stream of permanent income, the expected remaining lifetime consumption of an agent will include the present value of his expected future payments received from this source of income.

[49] Note that in Equation (3.42), $R^{-1}_{f,t,T-n}$ follows by applying the definition of the multi-period risk-free return after tax (3.36) to the sequences of one-period risk-free rates $\{R^{-1}_{f,i}\}_{i=t}^{T-n}$ for all $0 \leq n < (T-t)$ that are found by solving Equation (3.41) recursively.

As a result, the concept of total wealth in the present setting incorporates transfer capital along with financial wealth. This is of importance, since the present value of future transfers influences the initial consumption distribution and, therefore, the equilibrium solution. The concept of total wealth is used, for instance, by Epstein and Zin (1991) and Campbell (1993) in order to incorporate human capital, i.e., the present value of future earnings, into the analysis of an intertemporal asset pricing model.

In the following, it will be shown that transfer capital can be considered a nontraded asset, the market value of which can be calculated in equilibrium. Nevertheless, markets are found to be complete in the present setting with only two non-redundant assets traded on the financial market, as outlined above. These assets are given by the risky stock and the risk-free bond. As an immediate consequence it follows that any additional asset must be redundant. Hence, although there is no active market for a transfer capital security, it must be tradable through a portfolio composed of the risky and risk-free asset (see, for instance, Munk (2013, pp. 84-86)). Then, being similar to capital income, transfer income should be valued the same way. That is, its present value should be derived by discounting future income streams using the equilibrium stochastic discount factor.

The formal pricing relation for transfer capital follows naturally, while specifying the total wealth budget constraint. The latter can be obtained by using the SDF and solving forward the dynamic budget constraint (3.15) (Miao (2014)). In so doing, total wealth of agent m at date t eventually reads[50]

$$w_t^m \equiv \sum_{n=0}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} c_{t+n}^m \right] \tag{3.43}$$

$$= \alpha_{t-1}^m \left((1-\tau) \left(P_{k,t} + D_t \right) + \tau P_{k,t-1} \right) + \beta_{t-1}^m \tilde{R}_{f,t-1} + s_t + P_{s,t}. \tag{3.44}$$

[50] Details of the derivation are given in Appendix B.1.6.

Equation (3.44) defines the current total wealth of agent m as the present value of his expected life-time consumption. In this expression, non-financial total wealth is separated into two components: current transfers s_t and transfer capital $P_{s,t}$. This representation illustrates the fact indicated above, namely that non-financial total wealth can also be interpreted as a nontraded asset. In particular, while current transfers s_t can be interpreted as its stochastic dividend, $P_{s,t}$ constitutes the shadow price of the nontraded asset (see Epstein and Zin (1991, Footnote 3)). Like for liquid wealth v_t^m, the agent that holds the higher fraction in aggregate total wealth might be considered to be more wealthy since he owns the larger proportion in the expected present value of the future aggregate consumption stream.

The derivation of the total wealth budget constraint (3.44) implies the following formal pricing relation for transfer capital:

$$P_{s,t} \equiv \sum_{n=1}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} s_{t+n} \right] \tag{3.45}$$

$$= \mathbb{E}_t \left[\frac{2\mu_{t+1}}{\mu_t} \left(P_{s,t+1} + s_{t+1} \right) \right]. \tag{3.46}$$

According to Equation (3.45), the current transfer capital of agent m is the sum over his expected future transfers discounted by the equilibrium stochastic discount factor. By substituting the definition of the SDF (3.33), current transfers (3.9) as well as the price-dividend ratio for the stock (3.41), an equilibrium solution for the transfer capital security can be determined. As for the stock, its price-dividend ratio is given by a recurrence equation[51]

$$PD_{s,t} = \tilde{R}_{f,t}^{-1} \left(PD_{s,t+1} \frac{\mathbb{E}\left[G^{1-\gamma}\right]}{\mathbb{E}\left[G^{-\gamma}\right]} + \frac{\tau}{2} \left(R_{f,t} - 1 \right) PD_{k,t} \right), \tag{3.47}$$

[51] The derivation is shown in Appendix B.1.7.

and can, alternatively, be stated in explicit form

$$PD_{s,t} = \frac{\tau}{2} \sum_{n=0}^{(T-t)-1} \left(\frac{\mathbb{E}\left[G^{1-\gamma}\right]}{\mathbb{E}\left[G^{-\gamma}\right]} \right)^{(T-t)-n-1} \left(\frac{R_{f,T-n-1} - 1}{\tilde{R}_{f,t,T-n}^{-1}} \right) \cdot PD_{k,T-n-1}. \tag{3.48}$$

Equations (3.47) and (3.48) show that the price-dividend ratio for transfer capital is time-dependent, but state-independent. Moreover, it is a function of the initial consumption distribution and directly and indirectly dependent on the tax rate τ.

3.2.3.2 Marginal Propensity to Consume out of Total Wealth

The marginal propensity to consume out of total wealth is defined as the agent-specific function that determines the fraction consumed out of total wealth currently available to him. In order to determine agent m's MPCTW, it is helpful to start by defining individual consumption grows as a function of the initial consumption share of agent $m = 1$. As shown above, Equation (3.31) implies that agents align their marginal rates of substitution of consumption in equilibrium. Rearranging this equation and substituting the equilibrium consumption share function (3.28), it follows that agent m's consumption grows from time t to $t+1$ is given by

$$\frac{c_{t+1}^m}{c_t^m} = \left(\frac{\delta^m}{\delta^1} \right)^{\frac{1}{\gamma}} \frac{g_{t+1}^1}{g_t^1} G_{t+1} \tag{3.49}$$

$$= \left(\frac{\delta^m}{\delta^1} \right)^{\frac{1}{\gamma}} \frac{\left(1 - \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right) g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}{\left(1 - \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+1}{\gamma}}\right) g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+1}{\gamma}}} G_{t+1}. \tag{3.50}$$

This is a function of agent 1's initial consumption share and follows a time-dependent i.i.d. binomial process. Consumption grows rates are identical for both agents only if their subjective time discount factors are identical, i.e., $\delta^1 = \delta^2$. Again, the tax rate τ does not enter Equation (3.50) directly. However, due to its dependency on g_0^1, it is affected by τ indirectly through this channel.

Then, in order to determine agent m's marginal propensity to consume out of total wealth, divide both sides of the total wealth budget constraint (3.43) by current consumption c_t^m and substitute the SDF expression (3.23):

$$\begin{aligned}\frac{w_t^m}{c_t^m} =&\sum_{n=0}^{T-t}\mathbb{E}_t\left[\frac{2^n\mu_{t+n}}{\mu_t}\frac{c_{t+n}^m}{c_t^m}\right]\\ =&\sum_{n=0}^{T-t}\mathbb{E}_t\left[(\delta^m)^n\left(\frac{c_{t+n}^m}{c_t^m}\right)^{1-\gamma}\right].\end{aligned} \tag{3.51}$$

Taking the reciprocal of this expression and substituting individual consumption growth (3.50), the equilibrium solution for the marginal propensity to consume out of total wealth of agent m at date t follows by[52]

$$\begin{aligned}b_t^m \equiv&\frac{c_t^m}{w_t^m}\\ =&\left\{\sum_{n=0}^{T-t}\left(\frac{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right)g_0^1+\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}\right)g_0^1+\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}}\right)^{1-\gamma}\right.\\ &\left.\left((\delta^m)^{\frac{1}{\gamma}}\left(\delta^1\right)^{\frac{\gamma-1}{\gamma}}\mathbb{E}\left[G^{1-\gamma}\right]\right)^n\right\}^{-1},\end{aligned} \tag{3.52}$$

where the i.i.d. property of random production growth has been used again. The consumption to total wealth ratio of agent m is a time-dependent, but state-independent, function. With growing (declining) subjective time discount factors, the MPCTW decreases (increases), which implies a lower (higher) proportion of total wealth being consumed today. In return,

[52] Details of the derivation are given in Appendix B.1.8.

the fraction postponed for future consumption rises (drops), reflecting increasing (decreasing) patience. With a coefficient of relative risk aversion of $\gamma = 1$, Equation (3.52) incorporates the special case of logarithmic utility. It implies the well-known and simplified consumption propensity, which only depends on the agent's personal subjective time discount factor (see Merton (1969) and Samuelson (1969)). The behavior of such an individual is independent of any other factor, as, for instance, the risk and return properties on the financial market. With risk aversion coefficients larger than one the marginal propensity to consume out of total wealth is still a deterministic function, which is a consequence of the i.i.d. property of G. Nevertheless, it depends on the expectation about the realization of the aggregate shock. Hence, such an individual, takes the risk characteristic underlying the economy into account when making his consumption decision.

3.2.3.3 Investment Policies

In order to maximize lifetime utility every agent has to take two decisions in every time step. On the one hand, this entails the consumption-savings decision, which has already been determined by the consumption share function and the MPCTW. On the other hand, this comprises the portfolio choice that determines the allocation of savings between the different financial assets available. In the present model two types of securities are traded, the risky stock and risk-free one-period bonds. Building on the results derived so far, the optimal individual investment policies for this opportunity set are specified in the following.

Bond Investment

Using the total wealth budget constraint in conjunction with the marginal propensity to consume out of total wealth just derived, consumption at date t of an agent m can be stated by

$$
\begin{aligned}
c_t^m =& b_t^m \Big(\alpha_{t-1}^m \left(\left(1-\tau\right) \left(P_{k,t}+D_t\right) + \tau P_{k,t-1} \right) + \beta_{t-1}^m \tilde{R}_{f,t-1} + s_t + P_{s,t} \Big) \\
=& b_t^m \left(\left(\alpha_{t-1}^m \left(1-\tau\right) + \frac{\tau}{2} \right) \left(PD_{k,t}+1\right) D_t + PD_{s,t} D_t + \right. \\
& \left. \left(\alpha_{t-1}^m - \frac{1}{2} \right) \tau PD_{k,t-1} D_{t-1} + \beta_{t-1}^m \tilde{R}_{f,t-1} \right), \qquad (3.53)
\end{aligned}
$$

where the second line follows by applying the definitions of the price-dividend ratios for the stock and transfer capital. Beyond that, consumption is given by $c_t^m = g_t^m D_t$, in which g_t^m was found to be state-independent. That is, in equilibrium, optimal consumption was found to be a function linear in aggregate production. As an immediate consequence, the consumption expression given on the right-hand side of Equation (3.53) must also be linear in current output. Therefore, and in order to be in line with optimal behavior, all terms that do not involve D_t must vanish from the right-hand side of expression (3.53). Formally, this implies the following equilibrium condition:

$$
b_t^m \left(\left(\alpha_{t-1}^m - \frac{1}{2} \right) \tau PD_{k,t-1} D_{t-1} + \beta_{t-1}^m \tilde{R}_{f,t-1} \right) = 0. \qquad (3.54)
$$

Equation (3.54) is similar to the condition found by Fischer and Jensen (2015, Equation (A.13)) and can be rearranged in order to determine agent m's bond market position from time $t-1$ to t in terms of his equity investment:

$$
\beta_{t-1}^m = \frac{\tau}{\tilde{R}_{f,t-1}} \left(\frac{1}{2} - \alpha_{t-1}^m \right) PD_{k,t-1} D_{t-1}, \qquad (3.55)
$$

which is a time- and state-dependent function. In line with the findings in Fischer and Jensen (2014, 2015, 2017), in the presence of taxation on net capital income trading on the bond market plays a decisive role for agents in order to establish their optimal consumption policy. Particularly, the investment strategy for the risk-free security is induced by the fact that transfers financed by capital taxation are subject to additional market risk. This is due to the fact that transfers from capital gains are large (small) when stock returns were high (low) in the recent past, which is the case when production growth was strong (weak) over the last period.[53] In addition, the taxation system is designed in a way that the individual with financial income below (above) the average is the net recipient (contributor) of transfer payments. As a result, this individual is endowed with a proportion of market risk that is higher (lower) than the risk exposure faced by the other agent. Attempting to compensate this unequal distribution of risk, agents enter into bond contracts with each other.

In this vein, Equation (3.55) implies positive bond holdings for the individual that holds a proportion of the stock, α^m_{t-1}, that is smaller than the average. In contrast, the agent with stock holdings larger than the average enters into short positions on the bond market. This result is surprising, since it implies dynamic trading in the risk-free security, although there is no positive net supply in the bond market. As outlined above, this means that a market equilibrium is established that produces a risk-free return at which agents are willing to take complementary positions in such an asset.

Stock Investment

The investment strategy for the stock market follows from the total wealth budget constraint defined in Equation (3.53) by substituting the solution for the bond market position (3.55) and rearranging for α^m_{t-1}. That is, agent m's stock holdings from time $t-1$ to t are given by

$$\alpha^m_{t-1} = \frac{1}{(1-\tau)\left(PD_{k,t}+1\right)}\left(\frac{g^m_t}{b^m_t} - \frac{\tau}{2}\left(PD_{k,t}+1\right) - PD_{s,t}\right), \quad (3.56)$$

[53] The fact that transfers carry additional market risk will be discussed in more detail below.

which is a time-dependent, but state-independent, function. Equation (3.56) implies that the share of equity holdings increases in the individual's consumption share g_t^m and decreases in his marginal propensity to consume out of total wealth b_t^m. Consequently, it is the more wealthy agent, i.e., the one that owns the higher fraction in aggregate total wealth, that also holds a larger fraction in the stock. This relation might explain low stock market participation rates for poorer individuals, as outlined by Fischer and Jensen (2015).

Overall, in order to establish optimal consumption in the presence of a redistributive capital gains tax, agents have to dynamically trade in both the bond and the equity markets. This result was first found by Fischer and Jensen (2015). Accordingly, the linear sharing rule would not be achieved without dynamic trading in both securities. This becomes apparent by considering Equation (3.53). In the absence of trading in the risk-free security, the term from Equation (3.54) would not vanish and, therefore, prevent agents from establishing their optimal linear consumption policy. At the time the investment decision is made, however, this term is not uncertain but known to the agents. As a matter of fact, they can find the bond market investment strategy given in (3.55) and eventually establish the linear sharing rule.

3.2.3.4 Net Transfers

Having established the equilibrium investment policies, one can now refine the presentation for net transfers.[54] In so doing, first note that the taxation system implies that the total amount of tax payments of agent m at date t is given by

$$\tau\alpha_{t-1}^m \left(P_{k,t} + D_t - P_{k,t-1}\right) + \tau\beta_{t-1}^m \left(R_{f,t-1} - 1\right). \tag{3.57}$$

[54]The present section draws on the derivations in Fischer and Jensen (2015, Theorem 1, Item 7).

That is, individual tax payments contain payments from taxes on stock (first term) as well as on bond (second term) investments. Beyond that, the same agent receives transfer payments (see Equation (3.9)) at date t according to

$$\frac{\tau}{2}\left(P_{k,t}+D_t-P_{k,t-1}\right), \tag{3.58}$$

which does not contain tax revenues from positions in the risk-free security, since there is no aggregate net supply. Then, subtracting Equation (3.57) from (3.58) and substituting the bond market investment policy (3.55), agent m's net transfer at date t reads

$$\tau\left(\frac{1}{2}-\alpha_{t-1}^{m}\right)\left(P_{k,t}+D_t-P_{k,t-1}\frac{R_{f,t-1}}{\tilde{R}_{f,t-1}}\right). \tag{3.59}$$

Equation (3.59) implies that the agent holding a smaller (larger) fraction in the stock than the other individual is subject to positive (negative) net transfer payments. That is, the agent receiving lower capital income than the average ($\alpha_{t-1}^{m}<\frac{1}{2}$) is the net recipient of the transfer system, while the agent with higher capital income than the average ($\alpha_{t-1}^{m}>\frac{1}{2}$) is its net contributor.[55]

Rewriting Equation (3.59) and applying the recursive equilibrium solution to the stock price-dividend ratio (3.41), it can be shown that transfers carry additional market risk:

$$\tau\left(1-\frac{\mathbb{E}\left[G^{1-\gamma}\right]}{\mathbb{E}\left[G^{-\gamma}\right]}\tilde{R}_{f,t-1}^{-1}G_t^{-1}\right)\left(\frac{1}{2}-\alpha_{t-1}^{m}\right)\left(PD_{k,t}+1\right)D_t, \tag{3.60}$$

First of all, net transfer payments at date t depend on the uncertain market development through the current realization of production D_t. In this vein, they carry the same amount of market risk as implied by regular stock market payoffs. In addition, however, the efficient tax rate implied by Equation (3.60) carries additional market risk, as it is given by the statutory tax

[55] This result is similar to the one found in Fischer and Jensen (2015, Theorem 1, Item 7).

rate τ multiplied by $(1-(\mathbb{E}\left[G^{1-\gamma}\right]/\mathbb{E}\left[G^{-\gamma}\right])\tilde{R}_{f,t-1}^{-1}G_t^{-1})$. That is, it includes current production growth G_t itself and is, thus, also dependent on the recent market development. In particular, when the realization of G_z is high and the economy is booming, production is large and the effective tax rate on capital income is high as well. In contrast, when the economy is in a bust, with the realized growth rate G_z being small, capital output is small and the effective tax rate on capital income is low as well. In other words, the effective tax system is pro-cyclical.

Since this implies that market risk is redistributed from the net contributor to the net recipient of transfers, it follows that the latter enters into long positions on the bond market, whereas the former takes short positions in the risk-free security. This way, equity investments are reduced for the net recipient, while they are increased for the net contributor of transfers.

3.2.3.5 Optimal Expected Lifetime Utility

Agents' equilibrium consumption and investment decision are driven by the objective to maximize expected lifetime utility over consumption. Given the definition of individual consumption shares $g_t^m = c_t^m D_t$ and the optimal equilibrium solution found above, particularly the MPCTW (3.52), one can specify the indirect utility (3.14) as a function of the initial consumption distribution. In so doing, agent m's maximum expected lifetime utility reads[56]

$$V_0^m = (b_0^m)^{-1}\,\frac{(g_0^m)^{1-\gamma}}{1-\gamma}D_0^{1-\gamma}. \tag{3.61}$$

This solution is a standard result that arises in the context of CRRA utility and i.i.d. risk (Back (2010)). In the present representation, it depends directly on the agent's initial marginal propensity to consume out of total wealth, his share in aggregate consumption as well as on aggregate produc-

[56] Details of the derivation are given in Appendix (B.1.9).

tion. Furthermore, Equation (3.61) implies that agent m's indirect utility is a decreasing function in his initial consumption share.[57]

3.2.4 Equilibrium Condition

With the dynamic general equilibrium model presented in Section 3.1, followed the analytical equilibrium solution of the previous section. In particular, it was found that optimal consumption plans require the consumption share function (3.28) to be a time-dependent, but state-independent, function of agent 1's initial consumption share g_0^1. Building on this result, the remaining equilibrium solution, i.e., the SDF as well as asset prices and individual investment behavior, were also specified as functions of the initial consumption distribution.

Although markets are complete, heterogeneity in subjective time discount factors implies that a closed-form solution to g_0^1 does not exist. Hence, in order to specify equilibrium, the initial consumption share has to be determined endogenously from the analytical model solution. In this vein, recall that clearing on the market for consumption goods (3.17) implies $c_0^1 = 1 - c_0^2$ and substitute agent 2's initial consumption in terms of his MPCTW and total wealth at date $t = 0$, as in Equation (3.53), to obtain the following equilibrium condition:

$$g_0^1 = 1 - b_0^2 \bigg(\alpha_{-1}^2 \left(PD_{k,0} + 1 \right) + PD_{s,0} \bigg). \tag{3.62}$$

Equation (3.62), in combination with the analytical equilibrium results (3.35), (3.36), (3.42), (3.48) and (3.52) derived above, form a non-linear equation in agent 1's initial consumption share. In order to eventually solve this condition for g_0^1, standard numerical methods can be applied.

[57] The impact of the redistributive taxation system on consumption shares and, thus, indirect utility will be analyzed using numerical examples in the quantitative analysis below.

It can be noted that the solution implied by Equation (3.62) is optimal, since it builds on the utility maximizing equilibrium results determined above. In addition, however, the solution must also be in line with market clearing, i.e., feasible given current economic resources. Clearing on the market for consumption goods follows immediately, as the equilibrium condition is based on this restriction. Clearing on the asset markets follows in return from consumption clearing. To demonstrate this, substitute budget constraint (3.15) along with optimal bond investment (3.55) into the condition for consumption clearing (3.17). Then, starting at the horizon and working backwards through time, clearing on the stock market, first, follows for date $T - 1$ and, subsequently, for any time step t.[58] Finally, with clearing on the consumption goods as well as on the stock market, clearing on the bond market follows automatically by Walras's law. Therefore, the solution implied by equilibrium condition (3.62) is both optimal and feasible.

Lastly, since the price-dividend ratios for both the stock and transfer capital are affected by the tax rate τ, the equilibrium condition and consequently the initial consumption distribution are also tax-dependent. Being dependent on the initial consumption distribution, all equilibrium results derived above are affected by the redistributive taxation system.

3.3 Quantitative Analysis

Having established the analytical solution, the present section studies the impact of redistributive taxation along with heterogeneity in the agents' subjective time discount factors using numerical examples. In order to ensure comparability, the parameterization is chosen in line with the base case parameter setting presented in Fischer and Jensen (2015). In particular, each model period is assumed to correspond to one year, so individual

[58] Appendix B.1.10 details this derivation.

Table 3.1 – This table reports the baseline parameterization for the model featuring an exchange economy.

Description	Parameter	Value
Economic horizon	T	60
Growth rates	$\{G_1, G_2\}$	$\{1.0315, 1.0087\}$
Initial wealth shares	$\{\alpha^1_{-1}, \alpha^2_{-1}\}$	$\{10\%, 90\%\}$
Tax rate	τ	20%
Degree of risk aversion	γ	5
Homog./Heterog. time discount factors	$\delta^{1,2}/\{\delta^1, \delta^2\}$	$0.96/\{0.94, 0.96\}$

decisions are made at an annual frequency. The time horizon determining the lifespan of both the economy and its agents is limited to $T = 60$ years. In accordance with the empirical evidence for the U.S. reported in Lettau and Ludvigson (2005), the two possible realizations of the production growth rate are given by $G_1 = 1.0315$ and $G_2 = 1.0087$, which are also referred to as the boom and bust state, respectively. Agents' common relative risk aversion coefficient is $\gamma = 5$, which lies well within the range of reasonable values usually considered in the asset pricing literature (see Mehra and Prescott (1985)). The share in the initial wealth endowment held by the less wealthy agent 1 is set to $\alpha^1_{-1} = 10\%$, which implies a corresponding value of $\alpha^2_{-1} = 90\%$ for the relatively wealthy agent 2. The tax rate is fixed at $\tau = 20\%$.

Beyond that, the baseline parameterization distinguishes two settings for the agents' subjective time discount factors in the present analysis. In the first case, homogeneous patience is considered by assuming $\delta^1 = \delta^2 = 0.96$.[59] The second case captures heterogeneity in subjective time discount factors. To be in line with the empirical evidence, the relatively poor agent 1 is assumed to be less patient than the relatively wealthy agent

[59]This case is identical with the base case parameter setting used in Fischer and Jensen (2015).

2 (see Samwick (1998), Lawrance (1991), Booij and van Praag (2009) and the discussion in Section 2.1.3). More precisely, the subjective time discount factor of the former is set to $\delta^1 = 0.94$, while it is set to $\delta^2 = 0.96$ for the latter in the heterogeneous case. The parameter setting is summarized in Table 3.1.

The remainder of this section is organized as follows: while Section 3.3.1 illustrates the evolution of the dynamic consumption and investment behavior over time, Section 3.3.2 studies the equilibrium impact of the length of the model horizon on the consumption distribution. In Section 3.3.3, the implications of different levels of the tax rate on the quantitative model results is investigated. Finally, Section 3.3.4 closes the numerical analysis by examining the impact of varying levels of heterogeneity in the subjective time discount factor.

3.3.1 Baseline Results

This section illustrates the impact of the remaining model horizon on the equilibrium profiles of agent 1 under both cases of the baseline parameter setting featuring homogeneous ($\delta^{1,2} = 0.96$) and heterogeneous ($\delta^1 = 0.94$, $\delta^2 = 0.96$) subjective time discount factors. In this vein, Figure 3.1 plots the evolution of agent 1's consumption share (top left panel), his share in aggregate wealth (top right panel), his equity share (bottom left panel), as well as his net transfer income in percent of aggregate production for both the pure boom and the pure bust scenario (bottom right panel) over time.

In line with the results derived in the analytical section, the top left panel depicts the fact that in the case of heterogeneous patience the share in aggregate consumption is a strictly monotonically decreasing function in time for the individual that is relatively less patient. To be precise, in the present case, agent 1's consumption share starts at around 18.5% and declines over the model horizon until it accounts for 15% of aggregate consumption at the terminal date. When it comes to homogeneous time preferences, however, agent 1's consumption share is constant over time at about 18%. Comparing both consumption profiles illustrates the increased impatience of agent 1 in the heterogeneous case, as the lower subjective time discount factor increases his share in aggregate consumption slightly in early time steps at the cost of decreasing it in later periods. In general, being endowed with an initial wealth share of only $\alpha^1_{-1} = 10\%$ means that the redistributive taxation system has a positive effect on the consumption share of the less wealthy agent 1 in both cases featuring heterogeneous and homogeneous subjective time discount factors.

The upper right panel of Figure 3.1 depicts the evolution of agent 1's share in aggregate economic wealth ($v^1_t/(P_{t,k} + D_t)$) under both homogeneous and heterogeneous patience. In both cases the proportion starts out at the initial wealth share of $\alpha^1_{-1} = 10\%$, but increases monotonically in the former case, while it is first subject to a decline in the latter case before it starts to increase. Considering increased impatience of agent 1, his wealth share remains below the level achieved in case of equal patience over the whole model horizon. This is a direct consequence of the increase in early consumption in the baseline setting with heterogeneous subjective time discount factors. It comes at the expense of decreasing the share of early savings. In return, a negative effect on agent 1's future wealth levels results.

Beyond that, in both cases agent 1's wealth share stays below his consumption share over the whole model horizon, while approaching it towards the terminal date. This illustrates that agent 1 uses large parts of his current and future permanent transfer income to finance current consumption. This is the case because the less wealthy agent 1 is subject to positive net transfer income for the whole investment horizon, as displayed in the bottom right panel of Figure 3.1. In the same vein, the value of his net transfer capital is positive, too. When the remaining model horizon is still long, agent 1's current and future permanent transfer income are large and the

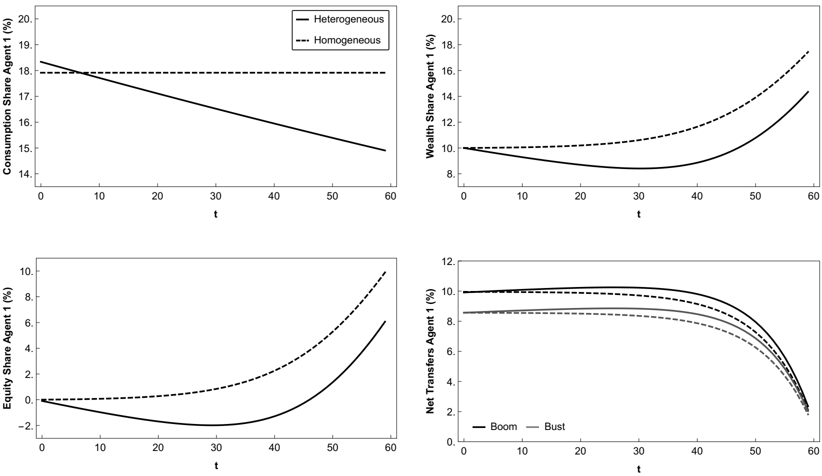

Figure 3.1 – This figure shows the equilibrium time profiles for agent 1 in both cases of the baseline parameter setting featuring heterogeneous (solid line) and homogeneous (dashed line) subjective time discount factors. The top left panel depicts the evolution of agent 1's consumption share, while the top right panel illustrates the evolution of his share in aggregate wealth over time. In the bottom left panel the development of agent 1's equity share is shown, and the bottom right panel plots his net transfer income in percent of aggregate production for the pure boom (black lines) and bust (gray lines) scenario over time.

desired current consumption level can be attained holding a relatively low wealth level. In contrast, when the remaining model horizon shortens, current transfer income and the value of net transfer capital decline. Then, in order to obtain the desired consumption levels, he needs to hold increasing wealth levels.

The bottom left panel of Figure 3.1 shows the evolution of agent 1's equity share over time under both homogeneous and heterogeneous patience. As for the wealth share, the proportion starts at the same low investment share $\alpha_0^1 = 0\%$ in both cases, but increases monotonically in the homogeneous setting, while it first declines before it recovers to increase in the heterogeneous case. Again, this pattern is driven by the net transfer income, as can be deduced from the analytical solution (see Equation (3.60)). In particular, the additional market risk carried by the transfers drives the net recipient of transfers out of the stock market. Since net transfers are higher when the less wealthy agent 1 is also less patient than agent 2, his equity share is particularly low in this case. This comprises even short asset holdings in the equity market.

The bottom right panel of Figure 3.1 depicts the evolution of agent 1's net transfer income in percent of aggregate production for the pure boom and the pure bust scenario under both homogeneous and heterogeneous patience. As indicated above, net transfers are generally positive for the less wealthy agent 1. Beyond that, his net transfer income is higher compared to the homogeneous case, when he holds a smaller subjective time discount factor than the wealthy agent 2. This is the case, because being less patient, agent 1 increases his consumption share around the initial periods. As a result, he has to reduce savings, which in return lowers his wealth share over the whole economic lifespan. Then, holding an even smaller wealth share, the design of the redistributive taxation system implies increased net transfer payments to him.

Finally, in neither case is an equal distribution for the consumption or the wealth share attained. Moreover, and in line with the results in, for instance, Krusell and Smith (1998) or Hendricks (2007), considering heterogeneous patience (according to the baseline parameter setting) induces even further consumption and wealth inequality. Beyond that, it also aggravates the low stock market participation rates for poor individuals. Heterogeneity in time preferences, therefore, might be an important factor in understanding low participation rates and crucial when studying the equilibrium impact of policy instruments, like in Gomes et al. (2013).

3.3.2 Impact of the Model Horizon

In this section, the impact of the length of the model horizon on the equilibrium consumption distribution is illustrated by varying the model horizon between $T = 0$ and $T = 400$ under both cases of the baseline parameter setting featuring homogeneous ($\delta^{1,2} = 0.96$) and heterogeneous ($\delta^1 = 0.94$, $\delta^2 = 0.96$) subjective time discount factors. Figure 3.2, therefore, depicts agent 1's consumption share at the initial ($t = 0$) and the terminal date ($t = T$) for different model horizons T and for the cases featuring heterogeneous and homogeneous patience.

In both baseline parameter cases, the initial consumption share increases with the length of the model horizon. Since consumption shares are constant in the homogeneous setting, the terminal and initial consumption shares are, in this case, identical. To the contrary, when the less wealthy agent 1 is also less patient, his initial and terminal consumption share deviate. In particular, agent 1's terminal consumption share starts out to increase with growing model horizon, peaks at around $T = 25$, and continuously falls afterwards. Two opposing effects lead to this pattern. First, being subject to exclusively positive net transfer payments, the initial value of agent 1's net transfer capital increases with the length of the model horizon T. As a result, his initial consumption share and, in return, his whole sequence of consumption shares, including the one at the terminal date, rise. Since future transfers are discounted, however, the initial consumption share does not grow linearly, but approaches a limiting value (about 18% in the present setting) as the length of the time horizon increases. This positive effect holds for both the homogeneous and the heterogeneous setting. Second, and as outlined above, in the heterogeneous case the consumption share is a strictly monotonically decreasing function in time for the less patient agent 1. Consequently, when the length of the model horizon rises, the time span over which the consumption share may shrink increases. Accordingly, for longer time horizons, other things being equal, this leads to lower terminal consumption shares. Taken together, when T is small, the first effect dominates, as there is just not enough time for agent 1's consumption share to decrease as much as to compensate the positive effect it has on his initial consumption share. When T gets larger, however, the initial consumption share approaches its limiting value and the time span increases over which the negative ef-

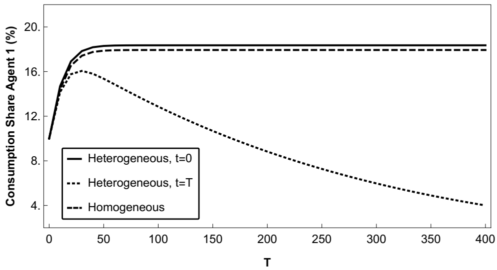

Figure 3.2 – This figure shows the impact of varying model horizons T on the initial ($t = 0$) and terminal ($t = T$) equilibrium consumption share of agent 1 in both cases of the baseline parameter setting featuring heterogeneous and homogeneous subjective time discount factors.

fect is effective. That is, the second effect gradually gains influence until it ultimately starts to dominate.

In this regard, the analytical results (see Section 3.2.1.2) have shown that the terminal consumption share of the less patient agent tends to zero as the length of the model horizon approaches infinity. Figure 3.2 indicates this result graphically. As outlined above, this implies that, although transfer payments are made in each time step, the less patient agent is successively driven out of the economy, which is then exclusively dominated by the patient agent 2. Similar results have been documented in models without redistribution systems by, for example, Becker (1980), Lengwiler (2005) and Beaumont et al. (2013). It is, however, surprising at first sight that this result holds similarly true in the presence of permanent transfers. Nevertheless, this is due to the absence of any short-sales or borrowing constraints in the present model setting. As illustrated in the top panels of Figure 3.1, and described above, agent 1's wealth share stays below his consumption share over the whole model horizon, while approaching it towards the terminal date. In the presence of permanent (transfer) income, however, wealth shares need not to be positive throughout in order to attain exclusively positive consumption shares. On the contrary, when the remaining length of the model horizon is large and the present value of future net transfers is large as well, agent 1, being unconstrained and impatient, incurs debts by borrowing against his future permanent (transfer) income. When the remaining length of the model horizon gets smaller, however, he has to run down consumption gradually in order to steadily reduce debts towards the terminal date. Put differently, agent 1's consumption shares become marginal at the long horizon.

3.3.3 Impact of the Tax Rate

This section visualizes the effect of different levels of the tax rate τ on the equilibrium profiles of agent 1 considering the heterogeneous baseline parameter setting ($\delta^1 = 0.94$, $\delta^2 = 0.96$). In line with Figure 3.1, Figure 3.3 plots the evolution of agent 1's consumption share (top left panel), his share in aggregate wealth (top right panel), his equity share (bottom left panel), as well as his net transfer income in percent of aggregate production for the pure boom scenario (bottom right panel) over time for different tax rates, varying between $\tau = 0\%$ and $\tau = 50\%$.

The top left panel of Figure 3.3 demonstrates that the redistributive taxation system has a positive effect on the consumption share of the less wealthy agent 1 over the whole economic time span. That is, it increases monotonically in the tax rate for any time step. To the contrary, the consumption share of the relatively wealthy agent 2 must be subject to decreasing consumption shares, when τ increases. In that sense, the redistributive taxation system reaches its objective to reduce consumption disparity. Accordingly, as implied by Equation (3.61), the transfer mechanism increases the expected lifetime utility of the less wealthy agent, while reducing it for his relatively wealthy counterpart.

The impact of varying tax rates on agent 1's wealth share is more complex, as can be seen from the top right panel of Figure 3.3. For low tax rates it is also a monotonically decreasing function in time, like the consumption share. In particular, when $\tau = 0\%$, the time profiles for the consumption and wealth share are identical, i.e., at $t = 0$ they both start at the initial wealth endowment $\alpha^1_{-1} = 10\%$ and similarly decrease over time. In contrast, when tax rates are high, agent 1's wealth share first decreases over time before it recovers to rise even more aggressively. For growing τ, this pattern becomes more and more extreme. The reason for this effect is the increase in the present value of net transfers with increasing τ in combination with its depreciation over time. In particular, when the tax rate is high and the remaining time horizon is still large, agent 1's present value of future net transfers is high as well. Being endowed with a higher level of total wealth, agent 1 finances higher current consumption levels by borrowing against future transfers and running down current wealth levels. When the remaining time horizon shortens, however, the present value of future net transfer payments declines and agent 1, in order to achieve his desired future consumption levels, needs to save and build up wealth. In sum, the impact of redistributive taxation on wealth inequality is ambiguous, because it dependents on the remaining investment horizon. Compared to the no-tax case, inequality increases in early time steps, while it decreases in later periods with the tax rate.

The bottom left panel of Figure 3.3 illustrates the impact of varying tax rates on agent 1's equilibrium stock investment over time. Similar to the wealth share, in the presence of taxation, equity holdings first decline, reach a minimum, and increase afterwards. Again, this pattern is caused by the impact of the tax rate and the remaining length of the model horizon on the present value of net transfers. Beyond that, the redistribution mechanism also transfers stock market risk from the relatively wealthy agent 2 to agent 1. For growing tax rates, net transfers to agent 1 increase, as shown in the bottom right panel of Figure 3.3. In this vein, the amount of risk that is reallocated to agent 1 rises and drives him out of the stock market. Put differently, the stock investment of the less wealthy agent declines with increasing tax rates.

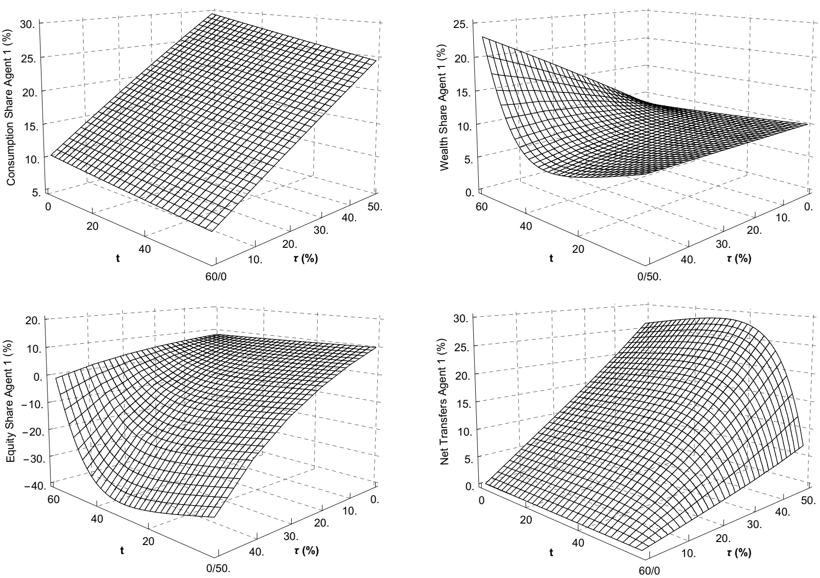

Figure 3.3 – This figure shows the impact of varying levels of the tax rate τ on the equilibrium profiles for agent 1's consumption share (top left panel), his wealth share (top right panel), his equity share (bottom left panel), as well as his net transfer income in percent of aggregate production for the pure boom scenario (bottom right panel) over time under the baseline parameter setting featuring heterogeneous subjective time discount factors.

3.3.4 Impact of Heterogeneous Patience

The subjective time discount factor determines which agent is the more or less patient individual populating the economy and, as shown in Section 3.2.1.2, who will face a decreasing or increasing sequence of consumption shares over his lifespan. In order to illustrate its quantitative impact on the equilibrium life-cycle profiles, Figure 3.4 depicts the impact of varying subjective time discount factors of the less wealthy agent 1 in the baseline parameter setting. To be precise, Figure 3.4 plots the evolution of agent 1's consumption share (top left panel), his share in aggregate wealth (top right panel), his equity share (bottom left panel), as well as his net transfer income in percent of aggregate production for the pure boom scenario

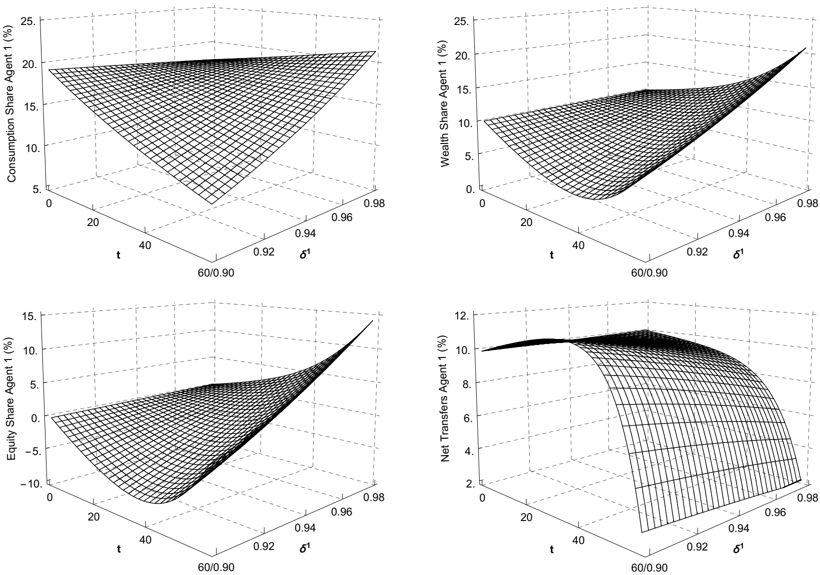

Figure 3.4 – This figure shows the impact of varying levels of agent 1's subjective time discount factor δ^1 on the equilibrium profiles for his consumption share (top left panel), his wealth share (top right panel), his equity share (bottom left panel), as well as his net transfer income in percent of aggregate production for the pure boom scenario (bottom right panel) over time.

(bottom right panel) over time for different subjective time discount factors of agent 1, varying between $\delta^1 = 0.90$ and $\delta^1 = 0.98$. Since in the baseline parameter setting $\delta^2 = 0.96$, this scenario comprises all three possible cases: as long as $\delta^1 < 0.96$, and in line with the empirical evidence, the less wealthy agent 1 is also less patient (see Samwick (1998), Lawrance (1991) and Booij and van Praag (2009)); for $\delta^1 = 0.96$, both agents are identical with respect to their preferences and only differ in their initial endowment; finally, when $\delta^1 > 0.96$, the less wealthy agent 1 is relatively patient.

The top left panel of Figure 3.4 illustrates the impact of varying patience on consumption. Reflecting patience, agent 1's share in aggregate consumption at the beginning of his lifespan is decreasing in his subjective time discount factor, while it is increasing in δ^1 at later stages. Moreover, and in line with the analytical results, being relatively impatient (patient) means that his sequence of consumption shares decreases (increases) over time. Being equally patient, consumption shares are constant throughout.

These results translate into agent 1's wealth share, as shown in the top right panel of Figure 3.4. Since his early consumption decreases in the subjective time discount factor δ^1, savings at the beginning of his lifespan increase. A direct consequence is that, when the less wealthy agent 1 becomes more patient, his fraction in aggregate wealth increases for all time steps $t > 0$. In return, the design of the redistributive taxation system implies that in this case net transfer payments to him decrease, as illustrated in the bottom right panel of Figure 3.4. Moreover, since this reduces the amount of stock market risk that is transferred to the less wealthy agent, his equilibrium equity holdings climb. This is visualized in the bottom left panel of Figure 3.4.

Qualitatively, these results are in accordance with the findings in Fischer and Jensen (2015) for the case of heterogeneous coefficients of relative risk aversion. In particular, they show that, when the less (more) wealthy agent becomes relatively less (more) risk averse, his level of net transfer income decreases (increases), while his equity and wealth shares increase (decrease) throughout. This is just in line with the effects of increasing (decreasing) time preferences observed above.

3.4 Conclusion

In this chapter, the simultaneous impact of redistributive taxation and household heterogeneity is analyzed using a dynamic general equilibrium asset pricing model. The framework features an exchange economy with classical demographic structure populated by two agents heterogeneous with respect to their time preferences and initial financial endowment. The model solution is based on the assumption of complete financial markets, where the developed solution method provides tractable analytical results for the equilibrium processes - like, for example, optimal consumption, investment, and asset prices.

In the presence of heterogeneous patience, the share in aggregate consumption becomes a deterministic and strictly monotonically decreasing function in time for the individual that is relatively less patient. This carries over to the SDF and risk-free return that become time-dependent, too. In the long run, however, the relatively more patient individual remains the only economically relevant agent.

In line with the findings in Fischer and Jensen (2015), the design of the redistributive tax system implies that transfers are pro-cyclical and, hence, carry additional market risk. Trying to compensate the resulting unequal distribution of risk across agents, the poorer individual (net recipient of transfers) reduces his stock investments and increases his bond holdings. Considering heterogeneous time preferences (as observed empirically) induces even further consumption and wealth inequality. As a result, transfer payments increase and, thereby, aggravate the low stock market participation rate for the poor individual. Heterogeneity in time preferences, therefore, might be an important factor in understanding low participation rates and crucial when studying the equilibrium impact of policy instruments, like in Gomes et al. (2013). The impact of redistributive taxation on wealth inequality is ambiguous. Still, it reduces consumption disparity and redistributes welfare from high to low income households in the present setting.

The meaning of heterogeneous time preferences on an individual level is as follows: as the less (more) wealthy agent becomes relatively less (more) patient, his level of net transfer income increases (decreases), while his equity and wealth shares decline (grow). This is neatly in line with the effects of increasing (decreasing) coefficients of risk aversion, as documented in Fischer and Jensen (2015). While the general effects of both sources of preference heterogeneity are comparable, a simple and tractable analytical solution is not available under heterogeneous risk aversion. This motivates the focus on heterogeneous subjective time preferences in the context of a tax-based reallocation mechanism.

The model presented in this chapter is intentionally simple with respect to a variety of aspects. To increase realism it can be enriched in various ways. For example, agents are not constrained with respect to their financial market activities within the present setting. This results in unrealistically large fractions of negative stock holdings for the poor and impatient household, that borrows against future transfers - which serves not or at the most only as a very limited collateralization within real-life economies. Therefore, one way of extending the model would be to consider borrowing or short-sales constraints. Beyond that, the present framework abstracts generally from human capital as source of income. Introducing unspanned labor income would, hence, be another way of expanding the model. Nevertheless, both approaches would foil the tractability of the solution method.

Finally, with respect to the required dimensions identified in Chapter 2, the present chapter disregards the dimensions *endogenous production* and partly *life-cycle characteristics* (especially OLG). This weakening will be addressed in the following chapter.

4

Redistributive Taxation in a Production Economy with Overlapping Generations

The previous chapter provided first insights with regard to the equilibrium impact of a tax-based reallocation mechanism and agent heterogeneity on households' consumption and investment behavior as well as on asset prices. Moreover, the concentration on differences in time preferences as source of preference heterogeneity was further motivated. By assuming an exchange economy with classical demographic structure populated by agents heterogeneous in their time preferences and initial financial endowment, a dynamic general equilibrium asset pricing model was developed that captured the dimensions of *redistributive taxation, endogenous individual behavior, agent heterogeneity* and *asset pricing*. In such a setting, production output is exogenous and consistent with aggregate consumption. Consequently, economic development is unaffected by households' consumption and investment decisions and the reallocation mechanism only alters the distribution of a fixed amount of resources across agents. Furthermore, in order to capture the finiteness of individuals' lifetime, the demographic structure requires the economy to face an unrealistically short time horizon.

As motivated in Chapter 2, in order to study the general equilibrium impact of redistributive taxation within a realistic setting, two additional dimensions ought to be considered: *endogenous production* and *life-cycle characteristics* (especially OLG). Therefore, the present chapter builds upon the dynamic general equilibrium asset pricing model of the previous chapter but adds the two missing dimensions. Considering an explicit production side, agents' individual investment and consumption behavior will become decisive in determining aggregate production output. A redistribution mechanism that influences agents' decisions will then have real effects on economic development. Assuming overlapping generations explicitly takes into account the finiteness of human life and the individual behavior over the life cycle, while allowing for a long-lived nature of the economy. On the one hand, this again influences production output itself. On the other hand, redistribution between different groups within a generation (intra-generational) might be considered as well as a reallocation of resources across generations (inter-generational). These are features of redistribution systems that can be observed throughout many countries in the world.

The general model specification is as follows: time evolves in discrete steps and the economic model horizon is infinite. The population is composed of finitely lived overlapping generations, where in each time step a new cohort enters, while an old cohort leaves the economy. In this vein, at any point in time individuals of different age coexist simultaneously. Every cohort is composed of different types of agents that are heterogeneous with respect to their stream of permanent life-cycle labor income and time preferences. Furthermore, and in accordance with the empirically observed evidence (see Section 2.1.3), patience levels may also alter with individiduals' age. Beyond that, aggregate perishable and consumable output is endogenous to the model. It is produced by a constant returns to scale Cobb-Douglas technology that employs capital, labor and technology. For simplicity, the firm hires labor at a fixed wage rate, as in Abel (1983) and Nakamura (1999), which implies a production technology linear in capital.[60] Accordingly, agents do not face a labor-leisure decision but supply labor at the exogenously given constant wage rate. Finally, the taxation and redistribution mechanism applied in the previous section is extended to account for taxation on both income types labor and capital.

The contribution of this chapter is fourfold: First, it presents the first study to investigate the simultaneous impact of heterogeneous age-dependent time preferences and redistributive taxation on individuals' life-cycle consumption and investment behavior, macroeconomic development and asset prices within a dynamic general equilibrium model. Second, and again in contrast to large parts of the related literature, it establishes a tractable framework and innovative solution method in the context of asset pricing with heterogeneous agents, overlapping generations and endogenous production, for which analytical and partly closed-form solutions are obtained - like, for example, the aggregate endogenous saving rate. Third, the combination of overlapping generations and age-dependent time preferences allows to reproduce and, thereby, helps to explain the empirically well-established hump-shaped pattern of consumption (see, for instance, Gourinchas and Parker (2002) and Fernández-Villaverde and Krueger (2007)) in the absence of any borrowing or short-sales constraint. Fourth, this chapter derives individual and aggregate welfare measures that allow to distinguish between the short- and long-run effects of redistributive taxation on well-being.

The derivation of a simple stationary equilibrium solution is facilitated by the restriction on i.i.d. aggregate production risk. The solution method in

[60] In the redistribution context, such linear production functions have already been used by, for example, Ball and Mankiw (2007) or Fischer and Jensen (2014).

this chapter builds on a "guess and verify" approach, like, for instance, in Athanasoulis and Shiller (2001), Viceira (2001), Athanasoulis (2005, 2006) and Gârleanu and Panageas (2015). It starts out by conjecturing the form of the agents' optimal consumption policy and subsequently derives the analytical equilibrium solutions based on this guess. Finally, given the entire equilibrium solution, the conjecture can be verified by showing that it is consistent with general equilibrium. In particular, the share of individual consumption in aggregate production turns out to be age-dependent, but state- and time-independent, in the present setting. As in the previous chapter, this is decisive with respect to the model solution, since the analytical results will partly depend on the individuals' consumption shares, which are to be determined endogenously within the model. To be more precise, parts of the model solution are found in closed from. Interestingly, these are individual consumption growth, the stochastic discount factor as well as the risk-free return. Due to the overlapping generations structure and heterogeneous preferences, the solution to aggregate production, consumption and investment as well as to the individual policy functions will depend on the distribution of consumption among agents. For them, closed-form solutions are not available. Given age-dependent but deterministic consumption shares, however, implies that agents strive for a linear sharing rule and align their marginal rates of substitution in equilibrium. In this vein, all analytical results can again be found dependent on only one endogenous parameter. In the present setting, this parameter is defined by the ratio of consecutive consumption shares of an agent of one specific type and age. In order to solve for equilibrium, the analytical solutions to the individual policies and asset prices in combination with the clearing conditions can be used to derive a nonlinear deterministic equation in this parameter. It constitutes a single equilibrium condition that can be solved for using numerical methods.

The chapter is structured as follows: Section 4.1 introduces the detailed model specification defining the population structure, the economic setup, the redistributive tax system, the individuals' optimization problem and the conditions for market clearing. Section 4.2 starts by discussing the solution method and, thereafter, presents the general equilibrium model solution along with the analytical results. It closes by deriving individual as well as aggregate welfare measures and discussing the impact of government debt in the present setting. In Section 4.3, the quantitative implications of redistributive taxation and agent heterogeneity on macroeconomic and individual behavior as well as its welfare effects are illustrated. Finally, Section 4.4 concludes the present chapter.

4.1 The Model

The present section presents the stylized model setup. The dynamic general equilibrium asset pricing model builds upon the specifications of the previous chapter but adds the two missing dimensions: *endogenous production* and *life-cycle characteristics* (especially OLG). The section starts by elaborating the overlapping generations population structure and presents the economic setup afterwards. Subsequently, the redistributive tax system and the individuals' optimization problem is described. The section closes by formally defining market equilibrium.

4.1.1 Population

As in the classical overlapping generations model of Samuelson (1958) and Diamond (1965) outlined in Section 2.2.3, time is discrete, starts at date 1, lasts forever, and is denoted by $t = 1, 2, ...$. Each period, a new cohort of individuals enters the economy (newborns) and an old generation dies. The period a generation is born is denoted by $i = 0, 1, ...$. Hence, every generation can be uniquely identified by its age $t - i$. Furthermore, a single cohort comprises M different types of agents indexed by $m = 1, ..., M$ that may be heterogeneous with respect to their income and/or time preferences. For simplicity, all groups of agents have the same population size.

In sum, there is an infinity of agents that can be distinguished by their type and their period of birth (or age, equivalently).

For clarity the following notational convention shall apply. Lowercase letters denote per capita and capital letters aggregate variables. The current period t is denoted by a subscript, whereas the period an agent is born i and its type m are denoted by superscripts. Then, for example, the date t consumption of an agent of type m born in period $i \leq t$ follows by $c_t^{i,m}$. In the case of independence of the realized state at time t the individual's age is used in notation and current and birth periods are dropped. The individual's age $t-i$ is denoted by a subscript. The consumption share of an agent of type m aged $t-i$, for example, is denoted by g_{t-i}^m.

Life expectancy is certain and all individuals live for $N+1$ periods. Furthermore, the number of agent types per generation is fixed. This implies a constant population with a total of $(N+1)M$ agents that coexist in every time step. With regards to the individual's life cycle, every agent lives through two stages in life and receives earnings according to[61]

$$h_t^{i,m} = f_{t-i}^m H_t, \tag{4.1}$$

where f_{t-i}^m is a function capturing the individuals earnings profile over the life cycle and H_t is the amount of aggregate earnings payed to the entire population alive in period t. For the first O periods of their existence, i.e., for $t-i \leq O$, agents are workers and entitled to receive a proportion of aggregate earnings in terms of labor income. As agents do not face a labor-leisure decision during working life and supply labor at a fixed wage rate, their earnings profile is exogenously determined. Moreover, assuming the absence of idiosyncratic income risk, f_{t-i}^m is a deterministic function of age only. In accordance with the empirically documented observations in, for instance, Hubbard et al. (1994), Gourinchas and Parker (2002) or Cocco

[61] Note that, unless otherwise stated, the term "earnings" refers to non-capital (labor and retirement) income in the following.

et al. (2005), this profile can be chosen to replicate the well-known hump-shaped income pattern individuals face through out their working years.

Retirement is exogenous and deterministic. In particular, all individuals retire at age $O+1$. Earnings during retirement are also captured by Equation (4.1), for $O < t-i \leq N$. In this second stage of life the proportion f^m_{t-i} in aggregate earnings H_t, however, is constant and the level of income lower in order to match the empirically observed replacement ratio (see, for example, Cocco et al. (2005) and Gomes et al. (2013)).[62]

Finally, and as outlined in Section 2.1.3, individual heterogeneity is an important property of real-life economies, especially in the context of redistributive taxation. The life-cycle structure of the present setting allows to capture two dimensions of agent heterogeneity: inter-generational and intra-generational heterogeneity. The former arises, since each cohort is situated in a distinct phase of its life-cycle. This, first of all, introduces a natural age-dependent source of heterogeneity, as agents of various age face different remaining lifetimes. Moreover, exogenously specified age-related characteristics may introduce additional inter-generational differences. In the present case, first, the agents' stream of earnings may vary over their life-cycle, as outlined above. This contributes to a heterogeneous distribution of resources across generations. Second, and in line with the empirical evidence documented in Section 2.1.3, individuals' attitude concerning the intertemporal allocation of consumption possibilities may alter with age. This introduces heterogeneity in preferences across generations.

Beyond that, and in line with the previous chapter, intra-generational differences are captured by considering different agent types. Sine the distribution of income across agents within an economy is far from equal, as outlined in Section 2.1.1, heterogeneity within a cohort is, first of all, captured by assuming differences in the income levels across agent types.

[62] Although not modeled explicitly, the fact that individuals receive retirement income may be justified by a pay-as-you-go pension system.

Equation (4.1) expresses this formally by allowing for different levels of the earnings profile f^m_{t-i} for different agent types m. Second, and closely connected, is the fact that individuals' time preferences may also alter across income levels. This implies heterogeneity in preferences across agents. Like in the previous chapter, heterogeneity in patience is formally captured by modeling differences in the agents' subjective time discount factors.

4.1.2 Economic Setup

The life-cycle framework described in the previous section is embedded in a one-sector closed economy. There is one neoclassical production technology that employs capital, labor and technology to produce one type of perishable consumption-investment good. This output can either be consumed or reinvested. Households own the entire capital of the economy. For simplicity, there are no capital adjustment costs and capital immediately depreciates, i.e., it only lasts one period. Hence, the capital stock available at any period equals aggregate investments made in the previous period. This can be interpreted as agents renting their capital to firms. There is no government debt or consumption.

4.1.2.1 Production Technology

There is one representative firm that employs capital, labor and technology to produce output using a constant returns-to-scale Cobb-Douglas production technology:

$$Y_t = (A_t K_t)^\theta L_t^{1-\theta}, \tag{4.2}$$

where Y_t is aggregate production output at date t measured in consumption units.[63] K_t is the stock of capital and L_t is aggregate labor input

[63]The Cobb-Douglas function fulfills all the properties of a neoclassical production function presented in Section 2.2.1.3.

used in production at date t. Technology is capital-augmenting with exogenously given innovations, i.e., A_t is a positive random scaling factor reflecting the technical knowledge of the economy. For tractability, A_t is chosen to be an i.i.d. multinomial random variable with a finite number of possible realizations denoted by A_z, where $z = 1, ..., Z$. Each realization occurs with equal probability $1/Z$. Further, θ and $1 - \theta$ present the output elasticity of capital and labor, respectively. They add up to one in line with the assumption of constant returns-to-scale made above. The range is limited to $0 < \theta < 1$.[64]

Without capital adjustment costs and assuming full capital depreciation, the stock of capital K_t available at period t equals aggregate investments I_{t-1} made in the previous period. Formally, the evolution of capital is, therefore, simply given by

$$K_t = I_{t-1}. \tag{4.3}$$

This identity will subsequently be applied. It is clearly a major simplification compared to the capital process implied by complex capital adjustment technologies with partial depreciation.[65] The specification of (4.3), however, has considerable analytical advantages and is therefore also used in, for instance, Pestieau (1974) and Fischer and Jensen (2014, 2017).

4.1.2.2 Economic Output

Economic output is produced according to the technology described in (4.2) and lasts one period. The economy is closed and there is no govern-

[64] Binomial or multinomial processes are widely used within the discrete-time asset pricing literature (see, for example, Mehra and Prescott (1985), Rietz (1988), He (1991), Dumas and Lyasoff (2012), and Chabakauri (2015)).

[65] Jermann (1998) and Croce (2014), for example, use capital accumulation with convex adjustment costs. This replicates the fact that quick changes in the capital stock are more costly compared to slow changes. As a consequence the price of new capital becomes time-varying and goes up when aggregate investment shall be increased. Considering complex adjustment costs, however, is beyond the scope of the model presented here.

ment debt or consumption. Hence, all units of the consumption-investment good available in period t are either consumed or reinvested into the production process. Put differently, in equilibrium time t production is simply the sum of aggregate consumption C_t and aggregate investment I_t:

$$Y_t = C_t + I_t. \tag{4.4}$$

Further, let $0 < X_t < 1$ be the fraction of aggregate output that is reinvested and $1 - X_t$ be the fraction that is consumed at date t. Then, Equation (4.4) implies that aggregate consumption and investment are given by

$$C_t = (1 - X_t)\, Y_t, \tag{4.5}$$

$$I_t = X_t Y_t, \tag{4.6}$$

respectively. In line with the discussion in Section 2.2.1.3, the fraction X_t is also denoted the aggregate saving rate of private households. It is endogenous to the model and determined in equilibrium.

4.1.2.3 Firms' Optimization

As outlined in Section 2.2.1.3, the assumption of constant returns to scale implies that it is sufficient to consider a single representative firm that combines capital, labor and technology to generate aggregate output according to the neoclassical production technology (4.2). In the present setting, it is convenient to define the firm's optimization problem as the maximization of operating profits net of labor costs only. Hence, in every period the firm's only choice is the amount of labor to hire. The capital investment decision follows from the households' equilibrium behavior. The sequence is: the firm produces output, pays out wages to its employees and distributes the remaining production to the investors (shareholders) in proportion to their ownership of production means. When determining the labor demand in period t the realization of the random factor A_t is known. Hence, in every period the firm's manager simply chooses the amount of labor L_t to be employed in order to maximize profits D_t according to[66]

$$\max_{L_t} D_t = Y_t - \omega L_t, \tag{4.7}$$

where ω is the exogenous and constant wage rate households and firms are assumed to agree upon in equilibrium (see Abel (1983), Nakamura (1999) and Nakamura (2002)).[67] Without loss of generality it is normalized to one, i.e., $\omega \equiv 1$.

Put differently, Equation (4.7) states the classical relationship that profits are given by revenues (output) minus costs (labor). Substituting Equation (4.2) into the maximization problem (4.7) and optimizing with respect to L_t, the following first order condition is derived:

$$\frac{\partial D_t}{\partial L_t} = (1-\theta)\,(A_t K_t)^{\theta}\, L_t^{-\theta} - 1 \overset{!}{=} 0, \tag{4.8}$$

where the normalization $\omega \equiv 1$ has been applied as well. The firms' aggregate demand for labor is derived by multiplying Equation (4.8) with L_t on both sides and using Equation (4.2) again:

$$L_t = (1-\theta)\, Y_t. \tag{4.9}$$

---

[66] Note that, since there is no labor-leisure decision to be made by the households in the present setting, they solely care about the maximization of investment returns through the actions taken by the firm's manager. Beyond that, households possess full transparency about the actions taken by the manager. In this vein, by maximizing profits, the firm's manager acts in the households' best interests, where this conduct can be verified by them without any costs. Despite the fact that there is an apparent agency relationship, there is no agency problem in the present model setting (see Eisenhardt (1989)).

[67] As explained in Section 2.2.1.3, the identity of revenues and aggregate output implies that the price for the consumption good is normalized to one. This is without loss of generality, since the consumption good serves as numeraire in the economy.

As indicated above, agents do not face a labor-leisure decision. Hence, it is assumed that the demand (4.9) is met by the agents' aggregate labor supply in every period. As a result of the normalization of the wage rate, Equation (4.9) also represents the absolute factor income of labor in the economy. That is, aggregate earnings at date t are given by

$$\omega L_t = H_t = (1-\theta)\, Y_t. \tag{4.10}$$

Finally, substituting expression (4.10) in the definition of firm profits (4.7) yields the absolute factor income of capital in the economy

$$D_t = \theta Y_t, \tag{4.11}$$

which is distributed to the shareholders. According to Equations (4.10) and (4.11), factor shares are constant. Beyond that, these equations illustrate the classical result that, in competitive input markets with constant returns-to-scale technology, the factor income shares equal the contribution of each input factor to production output.

Furthermore, Equations (4.10) and (4.11) indicate that earnings and capital income move together. This property finds some support in the empirical literature, since Campbell (1996) finds a high correlation between human capital and market returns, Baxter and Jermann (1997) report a high correlation between labor and capital returns exceeding 92%, and Benzoni et al. (2007) find evidence for a cointegration of aggregate labor income and dividends on the market portfolio. Finally, despite the fact that there is no human capital security explicitly traded on financial markets, the perfect correlation between earnings and capital income in the present setting implies that earnings will be tradable through the real investment possibility (i.e., the stock).

4.1.2.4 Linear Technology and Asset Markets

In order to close the description of the economy, it remains to specify the traded securities and to establish why the production technology is considered to be linear. The latter can be shown by substituting aggregate labor demand (4.9) in the Cobb-Douglas production function (4.2) and solving for output Y_t:

$$Y_t = A_t \Xi_1 K_t, \tag{4.12}$$

where $\Xi_1 \equiv (1-\theta)^{\frac{1-\theta}{\theta}}$. The expression of production in Equation (4.12) depends on the input of capital only, while the exponent on it drops. Hence, the assumptions on the economy made before lead to a production function that is linear in current capital input. Alternatively, using identity (4.3) in (4.12) implies an output technology linear in aggregate investment I_{t-1}, as in Ball and Mankiw (2007) and Fischer and Jensen (2014, 2017).[68]

The economy provides two types of non-redundant assets that are traded in a financial market. First, agents can invest into the production process (4.2) by trading the firms' equity represented by a single stock. In particular, this investment opportunity represents a claim to the firms' capital payout (4.11) in proportion to the share of equity holdings. The gross return on equity investment before tax from period t to $t+1$ follows from above by

$$\begin{aligned} R_{E,t+1} &= \frac{\theta Y_{t+1}}{I_t} = \frac{\theta A_{t+1} \Xi_1 I_t}{I_t} \\ &= \theta \Xi_1 A_{t+1}, \end{aligned} \tag{4.13}$$

where the last expression in the first line follows by substituting Equation (4.12) in combination with identity (4.3). Since it depends on the realization of the random productivity shock A_{t+1} at date $t+1$, which is unknown at the time of investment t, the equity (or stock) investment is risky.

[68] In this respect, the present model framework deviates from the neoclassical paradigm of diminishing marginal productivity presented in Section 2.2.1.3.

Furthermore, due to the linearity property of the production technology aggregate investment is canceled out in Equation (4.13) and the risky return is ultimately only dependent on exogenous quantities.

Second, agents can trade a risk-free one-period bond. The payout of such a security is supposed to be certain, as it is known by the agents one period in advance. The gross return on the risk-free asset before tax between time t and $t+1$ is denoted by $R_{f,t}$. The subscript displays the period at which the return is known to the individuals, not at which it is realized.[69] In contrast to the risky investment, there is no technology that generates the risk-free payout and, hence, the net supply is assumed to be zero. Thus, trading in the risk-free bond requires a market equilibrium that brings about a risk-free rate at which agents are willing to take complementary positions in such an asset. Unlike the risky return (4.13), the risk-free return $R_{f,t}$ cannot be specified directly, but will be determined endogenously in equilibrium. Altogether, investing into the production process (4.2) provides the only way to save for future consumption on a macroeconomic level.

Finally, assets can be traded without transaction costs, do not admit arbitrage and agents do not face any borrowing or short-sales constraints. Since in every time step the number of possible subsequent economic states is larger than the number of traded non-redundant assets, the financial market is incomplete (see, for example, Munk (2013, pp. 91-92)).

4.1.3 Tax System and Redistribution Mechanism

The assumptions regarding the redistributive tax system are based on the specifications in Fischer and Jensen (2015), which have been presented in Section 3.1.2. Contrary to the model described in the previous chapter, however, agents now receive non-capital income, i.e., labor and retirement

[69]This is consistent with the notation used for the return on equity investment in (4.13). In the case of the risky security, however, the point in time at which the return becomes known coincides with the point in time it is realized.

earnings, over their life-cycle. The taxation and redistribution mechanism is, therefore, extended to accommodate these factors. As in the previous chapter, the government still pursues the objective to reduce the disparity in consumption opportunities across agents. In order to achieve its objective, government reduces the disparity in net income, but is confronted with friction costs. The relevant income of individuals now consists of net capital income, i.e., gains from equity and bond investments, as well as non-financial earnings. Aggregate net capital income in the economy is given by $D_t - I_{t-1}$, whereas H_t denotes aggregate earnings. The associated frictions are assumed to differ, in order to account for different capital and labor (or earnings) tax rates.[70]

Due to the overlapping generations framework an additional assumption has to be made concerning the composition of wealth newborn agents are endowed with. For the sake of a simple and stationary model solution, individuals' initial endowment is assumed to consist solely of current (after-tax) labor income, like, for instance, in Cocco et al. (2005) and Fischer et al. (2013). As an immediate consequence, newborns do not receive transfer payments - a fact that must be considered in the design of the government's objective.

Then, denote by $k_t^{i,m}$ the net capital income and by $h_t^{i,m}$ the non-capital income at date t of an agent of type m born in period i before redistribution. Presuming quadratic objective functions again, the government's optimization problem at time t is given by:

$$\min_{\{\{\tilde{k}_t^{i,m},\tilde{h}_t^{i,m}\}_{i=t-N}^{t}\}_{m=1}^{M}} \sum_{m=1}^{M}\sum_{i=t-N}^{t} \left\{ \left(\tilde{k}_t^{i,m} - d_{t-i}\left(D_t - I_{t-1}\right)\right)^2 + \right.$$
$$\left. \kappa_c \left(\tilde{k}_t^{i,m} - k_t^{i,m}\right)^2 + \left(\tilde{h}_t^{i,m} - d_{t-i}H_t\right)^2 + \kappa_l \left(\tilde{h}_t^{i,m} - h_t^{i,m}\right)^2 \right\}, \quad (4.14)$$

[70] For the sake of clarity, in the following the term "labor" is used in the context of the taxation of non-financial income, covering both labor and retirement income.

subject to

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t}\tilde{k}_t^{i,m} = D_t - I_{t-1},\ and \tag{4.15}$$

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t}\tilde{h}_t^{i,m} = H_t, \tag{4.16}$$

where

$$d_{t-i} = \begin{cases} \frac{1}{N\cdot M} & if\ 0 < t-i \leq N, \\ 0 & else, \end{cases} \tag{4.17}$$

and $\tilde{k}_t^{i,m}$ and $\tilde{h}_t^{i,m}$ denote the net capital income and the non-capital income after redistribution, respectively. The parameters $\kappa_c \geq 0$ and $\kappa_l \geq 0$ measure the strength of the frictions in conjunction with capital and non-capital redistribution, respectively. The solution to the government's optimization is given by the following linear feedback rules:[71]

$$\tilde{k}_t^{i,m} = \frac{\kappa_c}{1+\kappa_c} k_t^{i,m} + \frac{1}{1+\kappa_c} d_{t-i}\left(D_t - I_{t-1}\right), \tag{4.18}$$

$$\tilde{h}_t^{i,m} = \frac{\kappa_l}{1+\kappa_l} h_t^{i,m} + \frac{1}{1+\kappa_l} d_{t-i} H_t. \tag{4.19}$$

The redistribution system implied by Equations (4.18) and (4.19) can be realized by imposing a flat tax of $\tau_c = \frac{1}{1+\kappa_c}$ on net capital income together with a flat tax of $\tau_l = \frac{1}{1+\kappa_l}$ on labor and retirement income, while redistributing aggregate tax revenues, according to (4.17), equally among individuals of age $0 < t-i \leq N$. Assuming an equal allocation of tax proceeds ensures that the order of agents according to their pre-tax income level is unchanged by the redistributive tax system. Again, this also implies that agents that earn an income below (above) the average become net recipients (contributors) of transfers. As in Section 3.1.2, the absence of friction

[71]The analytical derivation of the feedback rules is shown in Appendix B.2.2.

costs, i.e., $\kappa_c, \kappa_l = 0$, corresponds to tax rates of 100%. Contrary, in the case of friction costs that tend to infinity, i.e., $\kappa_c, \kappa_l \to \infty$, the tax rates become zero.

Aggregate current tax revenues are immediately redistributed to the agents populating the economy and, hence, equal aggregate current transfer payments. This ensures that each period the tax system is balanced. Since the risk-free security is in zero net supply, disposable tax revenues depend on gains in equity investment and earnings only.[72] Altogether, this implies that government collects an aggregate amount of

$$S_t = \tau_c \left(D_t - I_{t-1} \right) + \tau_l H_t \tag{4.20}$$

in disposable tax revenues at time t. From Equation (4.20) in conjunction with (4.17), the date t transfer payment received by an agent born in period i follows by $s_t^i = d_{t-i} S_t$, which is independent of his earnings history and his type.[73]

Finally, redistribution according to (4.18) and (4.19) apparently affects the distribution of current income levels. As in the previous chapter, this will change the individual consumption and investment behavior. In addition, however, aggregate production and consumption are determined endogenously from the agents' equilibrium behavior in the present setting. In this vein, redistributive taxation will also have a real impact on economic development.

---

[72]When considering “gross” tax revenues, government also collects a positive amount of revenues by taxing bond market gains. The taxation scheme presented here, however, implies immediate tax credits of equal size granted to individuals that hold short positions in the risk-free security. As trading in the bond requires agents that take offsetting positions in that asset, there are no disposable revenues from bond market activities in the aggregate.

[73]Since the individual transfer payment is independent of the agent type, the superscript m is dropped.

4.1.4 The Agents' Optimization Problem

In line with neoclassical theory, all agents are expected utility maximizers with time-additive preferences over a single consumption good, where they form rational expectations about the future uncertain development of the economy based on a full set of information equally available to all of them. Formally, there is an increasing sequence of information sets (a filtration) $\{\mathcal{F}_t : t = 1, 2, ...\}$ underlying the economy and available to all of them in period t. It contains all information regarding the past and current values of the random productivity shock A_t up to time t as well as the consumption and investment histories of all individuals up to period $t-1$. For the agents populating the economy this means that the realization of the stochastic component A_t in the production technology becomes known to them at the beginning of period t. Furthermore, since individuals have access to $\mathcal{F}_t$, they know about the whole history of realizations up to date t. Based on this information set, agents form their (conditional) expectations and make their economic decisions.

4.1.4.1 Heterogeneous Patience and Expected Lifetime Utility

Since agents are heterogeneous with regard to a number of characteristics, their optimization problem differs across them. Heterogeneity between agents, first, stems from the simple fact that each cohort is in a different phase of its life-cycle. Individuals of different age differ in their history and their remaining lifespan, which, in itself, influences consumption and investment behavior. Second, agents are heterogeneous with regards to their income level, as outlined in Section 4.1.1. Third, individuals are assumed to differ in their attitude concerning the intertemporal allocation of consumption possibilities, more precisely, their patience.

In order to formally model heterogeneity in time preferences, agents are assumed to hold different subjective time discount factors denoted by $\delta^m_{t-i} > 0$, as outlined in the previous chapter. The subscript $t-i$ indicates that patience may vary with age, whereas the superscript m implies that it may also differ across agent types. As before, when considering time additive utility, the subjective time discount factor weights the instantaneous utility from consumption at different points in time, i.e., the summands of lifetime utility. In the present context, δ_{t-i}^{m} describes the preferences of individuals regarding consumption between the age of $t-i$ and $t-i+1$. Consequently, for decreasing values of δ_{t-i}^{m} agents' preferences for consumption at age $t-i$ over consumption at age $t-i+1$ increase. That is, individuals become less patient. In contrast, when δ_{t-i}^{m} becomes bigger, patience increases, since agents' preferences for early over late consumption decrease. In the present model the subjective time discount factor is exogenously determined.[74]

Then, assuming that all agents have standard, time separable, instantaneous utility functions $u(c_t^{i,m})$ over consumption $c_t^{i,m}$, the expected present discounted utility, or lifetime utility, $U_t^{i,m}$ at date t of an agent of type m born in period i is given by

$$U_t^{i,m} = \sum_{n=0}^{N-(t-i)} \mathbb{E}_t \left[\left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^{m} \right) u(c_{t+n}^{i,m}) \right], \qquad (4.21)$$

where $\mathbb{E}_t$ is the conditional expectation operator, forming expectations conditional on the agent's time t information, i.e., $\mathcal{F}_t$. Equation (4.21) displays the impact of age on the individual's expected lifetime utility through the changing number of summands. When agents get older, the number of summands decreases until their decisions will only depend on the current

[74]The assumption that the subjective time discount factor is exogenous implies that patience is unaffected by changes in the income or wealth level due to the redistribution mechanism. Considering subjective time discount factors that endogenously depend on the agent's income, consumption or wealth level, therefore, seems to be reasonable and would certainly enrich the model and its implications. However, this would further complicate the model and foil the analytical solution presented below. Moreover, one may also assume that transfer payments are valued differently by the agents than labor and retirement earnings, a fact that might justify the assumption of exogenous patience to some extent. Studies featuring endogenous subjective discount factors are, for instance, Mendoza (1991) and Jahan-Parvar et al. (2013).

instantaneous utility at age N. As long as N is finite, the agents' lifetime will be finite, too. Since only consumption is assumed to generate utility, individuals are not endowed with a bequest motive.[75] Moreover, certainty regarding the lifespan also rules out unanticipated heritage.

4.1.4.2 Evolution of Wealth

Having established the agent's expected lifetime utility one further needs to address how the individual's consumption decision is constrained by the resources available to him over his life cycle. In the present setting, the evolution of agent's wealth (cash on hand) after accounting for taxes consists of four components. First, at date t an agent receives the payout from his equity investment made at period $t-1$. Denote by $\alpha_t^{i,m}$ the share of equity investments in the production process from time t to time $t+1$ of an agent of type m born in period i. Then the period t after-tax income from equity investment of this individual follows by

$$\alpha_{t-1}^{i,m}\left(\left(1-\tau_c\right)D_t+\tau_c I_{t-1}\right). \tag{4.22}$$

Second, agents receive income from their positions in the risk-free security. Let $\beta_t^{i,m}$ be the number of the risk-free bond held by an agent of type m born in period i from time t to $t+1$, then the after-tax income from holdings in the risk-free security at date t is given by

$$\beta_{t-1}^{i,m}\tilde{R}_{f,t-1}, \tag{4.23}$$

where

$$\tilde{R}_{f,t-1} = (1-\tau_c)R_{f,t-1}+\tau_c \tag{4.24}$$

is the gross risk-free return after tax from time $t-1$ to t. Together, Equations (4.22) and (4.23) define the income from financial assets. Third,

[75] This is in line with Hurd (1989) who finds that the marginal utility of bequests is small.

agents receive labor and retirement earnings as defined in Equation (4.1). After accounting for taxes the period t earnings are given by

$$(1-\tau_l)\,h_t^{i,m} = (1-\tau_l)\,f_{t-i}^{m} H_t. \tag{4.25}$$

Fourth, agents receive transfer payments as defined above (see Section 4.1.3) and restated below

$$s_t^i = d_{t-i} S_t. \tag{4.26}$$

Equation (4.25) in conjunction with (4.26) define the non-capital or permanent income of an individual. Finally, bringing all income sources together it follows that the evolution of wealth of an agent of type m born in period $i < t$ is given by

$$\begin{aligned} v_t^{i,m} =& \alpha_{t-1}^{i,m}\left((1-\tau_c)\,D_t + \tau_c I_{t-1}\right) + \beta_{t-1}^{i,m}\tilde{R}_{f,t-1} + \\ & (1-\tau_l)\,h_t^{i,m} + s_t^i. \end{aligned} \tag{4.27}$$

When agents enter the economy they do not own financial assets and their initial wealth consists solely of current (after-tax) labor income. That is, at time t

$$v_t^{t,m} = (1-\tau_l)\,h_t^{t,m} \tag{4.28}$$

is the initial wealth of an agent of type m born in period $i = t$. In line with the previous chapter, Equation (4.27) can be rewritten to display the mode of operation of the redistributive tax system. Substituting (4.25), (4.26), (4.20) in conjunction with (4.17) and rearranging yields

$$\begin{aligned} v_t^{i,m} =& \alpha_{t-1}^{i,m} D_t + \tau_c\left(\frac{1}{N\cdot M} - \alpha_{t-1}^{i,m}\right)(D_t - I_{t-1}) + f_{t-i}^{m} H_t + \\ & \tau_l\left(\frac{1}{N\cdot M} - f_{t-i}^{m}\right) H_t + \beta_{t-1}^{i,m}\tilde{R}_{f,t-1} \end{aligned} \tag{4.29}$$

for the evolution of wealth of an agent of type m born in period $i < t$. The second term of Equation (4.29) represents the net transfers received from aggregate capital output. It shows that the tax rate τ_c is multiplied by $(1/(N \cdot M) - \alpha_{t-1}^{i,m})$, which implies that the effective tax rate on the risky security depends on the agent's share in aggregate equity investments. Richer individuals that possess more equity wealth, i.e., larger $\alpha_{t-1}^{i,m}$, receive less of these net transfers and, consequently, are confronted with higher effective capital tax rates. The same holds true for net transfers received from labor or retirement earnings, the fourth term in expression (4.29). In that case the effective labor tax rate is affected by the agent's share in aggregate earnings. The tax rate τ_l is multiplied by $(1/(N \cdot M) - f_{t-i}^{m})$, which means that individuals with higher earnings, i.e., larger f_{t-i}^{m}, face a higher effective taxation of earnings and, as a result, receive less net transfers from earnings. Put differently, Equation (4.29) shows that effective tax rates are progressive, although the tax system implemented by the government is based on flat tax rates.

4.1.4.3 Maximization Problem

At every point in time t the agents currently alive, i.e., $0 \leq t-i \leq N$, have to make two decisions. The first is the consumption-savings decision, which comprises the decision of how much to consume of the available resources and how much to save. The decision variable is the agent's current consumption $c_t^{i,m}$. The second is the portfolio choice, which represents the decision of how to allocate savings between the different securities available. The associated decision variables are the equity share $\alpha_t^{i,m}$ and the number of the risk-free bond $\beta_t^{i,m}$ to be held from time t to time $t+1$.

Finally, it is assumed that agents have homogeneous instantaneous constant relative risk aversion (CRRA) preferences with common risk aversion parameter $\gamma > 0$. This means, for any agent of type m born in period i, utility from consumption $c_t^{i,m}$ in period t is given by

$$u\left(c_t^{i,m}\right) = \begin{cases} \frac{\left(c_t^{i,m}\right)^{1-\gamma}}{1-\gamma} & if\, \gamma > 0, \gamma \neq 1, \\ \ln\left(c_t^{i,m}\right) & if\, \gamma = 1. \end{cases} \tag{4.30}$$

By putting together Equations (4.21)-(4.30), it follows from the above that the optimization problem at date t of an agent of type m born in period $i \leq t$ is given by

$$\max_{\{\{c_{t+n}^{i,m}\}_{n=0}^{N-(t-i)}, \{\alpha_{t+n}^{i,m}, \beta_{t+n}^{i,m}\}_{n=0}^{N-(t-i)-1}\}} V_t^{i,m} = \sum_{n=0}^{N-(t-i)} \mathbb{E}_t \left[\left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^m \right) \frac{(c_{t+n}^{i,m})^{1-\gamma}}{1-\gamma} \right], \tag{4.31}$$

subject to

$$c_t^{i,m} = v_t^{i,m} - \alpha_t^{i,m} I_t - \beta_t^{i,m}, \tag{4.32}$$

$$\alpha_{N+i}^{i,m} = \beta_{N+i}^{i,m} = 0, \tag{4.33}$$

in combination with constraints (4.27) and (4.28). As explained above, Equation (4.31) is the agent's indirect utility, which is the maximum expected lifetime utility of current and future consumption (Munk (2013)). Equation (4.32) is the agent's budget constraint and (4.33) is the agent's terminal portfolio condition. It describes the fact that agents do not possess any asset holdings at their horizon, i.e., in their last period before they leave the economy. On the one hand, this is a direct consequence of the assumption that individuals do not have a bequest motive. Investing and leaving wealth for successors does not provide them with any utility. Therefore, leaving wealth would not be a utility maximizing behavior. On the other hand, condition (4.33) also constrains agents from leaving debts when they die. Without such a condition, agents would run infinitely into debts as they only care about maximizing their own consumption while being alive. As a result, at age N agents are constrained to consume all their remaining wealth, i.e., $c_{N+i}^{i,m} = v_{N+i}^{i,m}$.

4.1.5 Market Equilibrium

Before turning to the characterization of equilibrium, the concept of human and transfer capital has to be introduced first. As in the previous chapter, in the presence of a stream of permanent income, the expected remaining lifetime consumption (hereinafter referred to as total wealth) of an agent will include the present value of his expected future payments received from this source of income. In the given model, such an income is given by the streams of earnings $h_t^{i,m}$ and transfer payments s_t^i to the individuals. According to that, human capital $p_{h,t}^{i,m}$ is defined as the present value (or price) of expected future earnings (labor and retirement), whereas transfer capital $p_{s,t}^i$ is defined as the present value (or price) of expected future transfers.[76] Both human and transfer capital are determined endogenously and their respective values influence the decisions of individuals. Within the overlapping generation framework they will, moreover, depend on the allocation of resources across agents. They are, therefore, crucial in defining equilibrium.

An equilibrium of the economy described above consists, for every time step $t = 1, 2, ...,$ of the set of individual consumption decisions $\{c_{t+n}^{i,m}\}_{n=0}^{N-(t-i)}$, investment policies $\{\alpha_{t+n}^{i,m}, \beta_{t+n}^{i,m}\}_{n=0}^{N-(t-i)-1}$ and endogenously determined prices $\{p_{h,t+n}^{i,m}, p_{s,t+n}^{i}\}_{n=0}^{N-(t-i)}$, for each individual of generation $i = t, ..., t-N$ and type $m = 1, ..., M$, as well as the endogenously determined risk-free return $\{\tilde{R}_{f,t-1}\}_{t=0}^{\infty}$, such that, for every period t, (i) each agent alive maximizes his expected lifetime utility (4.31), subject to the constraints (4.32)-(4.33) in combination with (4.27)-(4.28), (ii) markets clear and aggregate quantities follow from individual behavior. Market clearing (ii) entails that for any point in time t both the markets for consumption goods and wealth clear, i.e., it formally must hold that

[76] A formal definition of human and transfer capital is derived below in Section 4.2.2.1.

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t} c_t^{i,m} = C_t, \tag{4.34}$$

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t} v_t^{i,m} = Y_t, \tag{4.35}$$

in conjunction with condition (4.4), which also ensures that the sums over individual quantities equal their aggregate counterparts; and asset markets clear

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t} \alpha_t^{i,m} = 1, \tag{4.36}$$

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t} \beta_t^{i,m} = 0, \tag{4.37}$$

i.e., agents hold all outstanding equity shares and the market for the risk-free bond is in zero net supply, respectively. Further, (iii) for the distribution of income streams it must hold that $\sum_{m=1}^{M}\sum_{i=t-N}^{t} f_{t-i}^{m} = 1$ and $\sum_{m=1}^{M}\sum_{i=t-N}^{t} d_{t-i}^{m} = 1$ for all t. This formally closes the model.

Finally, note that an equilibrium solution to the problem described above must be both optimal and feasible. On the one hand, it must ensure maximization of agents' expected lifetime utility according to Equations (4.31)-(4.33). On the other hand, it must ensure clearing on all markets according to conditions (4.34)-(4.37). Firstly, these requirements imply a high complexity of the equilibrium solution that, however, can be directly reduced due to the special model structure. To be precise, it holds true that conditions (4.34)-(4.35) are immediately fulfilled, once conditions (4.36)-(4.37) are fulfilled.[77] Hence, when determining the general equilibrium solution in the following section, it will be sufficient to concentrate on asset market clearing, since clearing on the remaining markets follows immediately from there.

[77] Appendix B.2.1 shows that this relationship holds.

4.2 General Equilibrium Solution

The dynamic general equilibrium model is solved using a "guess and verify" approach. This implies that, first, a conjecture will be made concerning the properties of the model solution. Later, it will then be established that this conjecture is consistent with general equilibrium. Such a procedure is not new to the asset pricing literature: Athanasoulis (2005), for instance, applies a "guess and verify" approach for an infinite horizon, incomplete markets, multiple agent economy where individuals have constant absolute risk aversion (CARA) preferences and finds closed-form solutions. Along these lines, Athanasoulis (2006) obtains analytical solutions for an overlapping generations model with CARA preferences. Gârleanu and Panageas (2015) solve an overlapping generations model where agents have heterogeneous Kreps-Porteus-Epstein-Zin-Weil recursive preferences using a "guess and verify" approach. However, none of these models assume a dynamic general equilibrium production economy.

In the present model a single conjecture is made. In particular, each agent's optimal consumption share in aggregate production g_{t-i}^{m} is assumed to be a deterministic function of age only:

$$g_{t-i}^{m} \equiv \frac{c_t^{i,m}}{Y_t}. \tag{4.38}$$

That is, optimal consumption of each individual is supposed to be an increasing function of aggregate production, which is referred to as the mutuality property (see Munk (2013, p. 257)). Then, in order to show that this conjecture holds, one needs to derive a solution to the optimal consumption policy $c_t^{i,m}$ that confirms the assumption and is consistent with the above stated definition of equilibrium (feasibility). More precisely, in the present model setup it will be necessary to establish the analytical equilibrium solutions to the newborns' marginal propensity to consume out of total wealth (MPCTW) b_0^m, as well as their equilibrium pricing relations for human $p_{h,t}^{t,m}$ and transfer capital $p_{s,t}^{t}$ in order to verify the conjecture.

Following the procedure just described, parts of the solution are found in closed from. To be more precise, these comprise individual consumption growth, the stochastic discount factor as well as the risk-free return. Due to the overlapping generations structure and heterogeneous preferences, the solution to aggregate production, consumption and investment as well as to the individual policy functions will depend on the distribution of consumption among agents. For them, closed-form solutions are not available. However, from the conjecture on age-dependent but deterministic consumption shares, Equation (4.38), in conjunction with the assumption on i.i.d. aggregate risk (see Section 4.1.2.1), follows that agents strive for a linear sharing rule and align their marginal rates of substitution in equilibrium. In this vein, all analytical results can again be found to be dependent on only one endogenous parameter. In the present setting, this parameter is defined by the ratio of consecutive consumption shares of an agent of one specific type and age.

To this effect, denote the ratio of consecutive consumption shares for an agent of type $m = 1$ between age zero and one by

$$\nu \equiv \frac{g_1^1}{g_0^1}. \tag{4.39}$$

In the following, all analytical equilibrium results will be given as a function of this single unknown endogenous parameter. In order to determine ν, the analytical solutions to the individual policies and asset prices in combination with the clearing conditions can be used to derive a nonlinear deterministic equation in this parameter. It constitutes a single equilibrium condition that can be solved for using numerical methods.[78]

[78]Note that the fact that ν is constant only implies that it is time- and state-independent. Through the equilibrium condition it will, however, depend on the distribution of resources across agents. Since this is affected by the redistributive tax system, it will depend on the tax rates. Apparently, the same is true for the agent's consumption share in aggregate production g_{t-i}^m.

The remainder of this section is organized as follows: in Section 4.2.1 analytical solutions for macroeconomic quantities are derived, whereas in Section 4.2.2 analytical results for agents' individual policies are presented. Next, Section 4.2.3 establishes the equilibrium condition and Section 4.2.4 introduces welfare measures used to quantify the impact of the redistributive tax system on individual and macroeconomic level. Finally, following Fischer and Jensen (2014, 2017), in Section 4.2.5 the impact of an active fiscal policy on aggregate production output is discussed.

4.2.1 Aggregate Economic Behavior

This section focuses on the characterization of the equilibrium results with respect to macroeconomic behavior. At the outset, the first order conditions implied by the individual's optimization problem described above will be derived. Based on these, the equilibrium processes for aggregate production, consumption and investment growth are determined. The section closes with presenting the closed-form solutions to the equilibrium stochastic discount factor and the risk-free return.

4.2.1.1 The First Order Conditions

For the given optimization problem, the first order conditions can be derived, as in the model presented in Chapter 3. Again, since the instantaneous utility (4.30) is strictly concave in consumption, second order conditions are satisfied and, therefore, first order conditions provide the optimal choice (see Munk (2013, p. 256)).

After-tax Representation

At time t the first order condition for an agent of type m born in period $t-N < i \leq t$ with respect to consumption at $t+n$ is given by

$$\mu_{t+n}^{i,m} = \left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^{m} \right) \left(\frac{1}{Z} \right)^{n} \left(c_{t+n}^{i,m} \right)^{-\gamma}, \tag{4.40}$$

where $\{\mu_{t+n}^{i,m}\}_{n=0}^{N-(t-i)}$ are the Lagrangian multipliers associated with the agent's constraints and $\{c_{t+n}^{i,m}\}_{n=0}^{N-(t-i)}$ is the optimal equilibrium consumption plan of this individual. Furthermore, the first order conditions at time t for agent type m born in period $t-N < i \leq t$ with respect to his equity and bond investment strategy are

$$1 = \mathbb{E}_t \left[\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}} \tilde{R}_{E,t+1} \right], \tag{4.41}$$

$$1 = \mathbb{E}_t \left[\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}} \right] \tilde{R}_{f,t}, \tag{4.42}$$

respectively, where

$$\tilde{R}_{E,t+1} = (1-\tau_c) R_{E,t+1} + \tau_c \tag{4.43}$$

is the gross risky return after tax from time t to $t+1$.[79] An increase in the tax rate τ_c will lower the after-tax equity return, as long as the pre-tax risky return is larger than one. Its range is then limited to $1 \leq \tilde{R}_{E,t+1} \leq R_{E,t+1}$.

Equations (4.41) and (4.42) are again the basic pricing equations that must hold equally for all individuals currently alive but born before $t-N$. In conjunction with condition (4.40) these equations link prices (or returns) to the optimal consumption plan of an individual agent.

[79] Appendix B.2.3 provides the details of the derivation.

As before, they imply the definition of the stochastic discount factor (SDF) as the agent's marginal rate of substitution over consumption:

$$\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}} = \delta_{t-i}^m \left(\frac{c_{t+1}^{i,m}}{c_t^{i,m}} \right)^{-\gamma} . \tag{4.44}$$

Equation (4.44) is the one-period after-tax SDF induced by agent i, m. It demonstrates the trade-off this individual faces between consumption in period t and period $t+1$, when making his investment decision (Munk (2013)). The willingness to substitute consumption between two periods is influenced by the agent's subjective discount factor, which may vary across agent types m and, furthermore, now depends on the individual's age $t-i$. The SDF will be further determined in Section 4.2.1.3 below.

Pre-tax Representation

In the previous chapter, it was shown that in the presence of taxation on financial investments an alternative representation for the basic pricing equations could be found in terms of a pre-tax SDF. In the present setting featuring overlapping generations and endogenous production, an analogous pre-tax version can be derived.

To this effect, note that Equation (4.41) implies that the pre-tax price of the one-period payout from the equity investment is given by[80]

$$I_t = \mathbb{E}_t \left[\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}} \left((1-\tau_c) D_{t+1} + \tau_c I_t \right) \right] . \tag{4.45}$$

According to Equation (4.45), the pre-tax price of an equity investment is given by the conditional expectation over the discounted future capital payout after tax using the after-tax stochastic discount factor.

[80] See Equation (B.61) in Appendix B.2.3.

Along the lines of Chapter 3, the pre-tax version of this condition can be found by moving around terms:

$$\begin{aligned} I_t =& \mathbb{E}_t \left[\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}} \frac{\tilde{R}_{f,t}}{R_{f,t}} D_{t+1} \right] \\ =& \mathbb{E}_t \left[\frac{Z\lambda_{t+1}^{i,m}}{\lambda_t^{i,m}} D_{t+1} \right], \end{aligned} \tag{4.46}$$

where

$$\frac{Z\lambda_{t+1}^{i,m}}{\lambda_t^{i,m}} = \frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}} \frac{\tilde{R}_{f,t}}{R_{f,t}} \tag{4.47}$$

is a pre-tax stochastic discount factor.[81] As before, it is given by the after-tax version of the stochastic discount factor multiplied by the ratio of after-tax risk-free return to its pre-tax counterpart. According to Equation (4.46), the pre-tax equity price is alternatively defined by the conditional expectation over the discounted future capital payout before tax using the pre-tax stochastic discount factor. Dividing both sides of Equation (4.46) by I_t yields

$$1 = \mathbb{E}_t \left[\frac{Z\lambda_{t+1}^{i,m}}{\lambda_t^{i,m}} R_{E,t+1} \right], \tag{4.48}$$

which is the pre-tax version of Equation (4.41). Similarly, expression (4.47) can be used to derive a pre-tax version of condition (4.42):

$$1 = \mathbb{E}_t \left[\frac{Z\lambda_{t+1}^{i,m}}{\lambda_t^{i,m}} \right] R_{f,t}, \tag{4.49}$$

which demonstrates why (4.47) is considered a pre-tax version of the stochastic discount factor, since it is directly linked to the pre-tax risk-free return.

[81] Details are given in Appendix B.2.4

4.2.1.2 Aggregate Production, Consumption and Investment

In this section, the results presented above are used to determine the processes that specify aggregate economic behavior. For that purpose it will be useful to first show that agents align their marginal rates of substitution in equilibrium under the conjecture (4.38). Afterwards, in conjunction with the first order condition (4.48), an analytical solution to the aggregate saving rate is found. Subsequently, this result is used to derive the equilibrium processes for aggregate production, investment and consumption growth.

Alignment of Marginal Rates of Substitution

By using the stochastic discount factor (4.44) along with the conjecture (4.38) one can rewrite the former expression in order to obtain the following representation:

$$\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}} = \delta_{t-i}^m \left(\frac{g_{(t-i)+1}^m}{g_{t-i}^m}\right)^{-\gamma} \left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}. \tag{4.50}$$

According to this equation, the SDF can be split up into a deterministic individual part and a stochastic common component. At the same time, the pricing relations (4.41) and (4.42) must hold equally for all agents, such that they agree upon one price (or return) for each asset. By taking first order condition (4.42) for any two individuals i, m and j, k currently alive and younger than N, substituting (4.50) and equating them, the following relation is found:

$$\begin{aligned}\delta_{t-i}^m \left(\frac{g_{(t-i)+1}^m}{g_{t-i}^m}\right)^{-\gamma} \mathbb{E}_t\left[\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}\right] =\\ \delta_{t-j}^k \left(\frac{g_{(t-j)+1}^k}{g_{t-j}^k}\right)^{-\gamma} \mathbb{E}_t\left[\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}\right]\end{aligned} \tag{4.51}$$

$$\Leftrightarrow$$

$$\delta^m_{t-i}\left(\frac{g^m_{(t-i)+1}}{g^m_{t-i}}\right)^{-\gamma} = \delta^k_{t-j}\left(\frac{g^k_{(t-j)+1}}{g^k_{t-j}}\right)^{-\gamma}, \tag{4.52}$$

where the last line follows immediately from the first. Equation (4.52) indicates that the deterministic and agent-specific part in (4.50) is equal across all individuals. Then, however, it follows from (4.50), as an immediate consequence of the result just found, that the marginal rates of substitution must be equal for all agents. That is, in equilibrium it must hold that

$$\frac{\mu^{i,m}_{t+1}}{\mu^{i,m}_t} = \frac{\mu^{j,k}_{t+1}}{\mu^{j,k}_t}, \tag{4.53}$$

for any two individuals i, m and j, k currently alive and younger than N. As in the previous chapter, aggregate consumption risk is shared efficiently in equilibrium and distributed such that all individuals possess the same marginal desire to allocate consumption across time and states (Munk (2013)). The fact that agents align their marginal rates of substitution in equilibrium is a consequence of the circumstance that only aggregate risk exists and the mutuality property implied by conjecture (4.38). Accordingly, it will be sufficient to define the stochastic discount factor from the perspective of a single agent of one specific type and age. Let this individual be of type $m = 1$ and age $t - i = 0$, then the preliminary definition of the one-period stochastic discount factor follows from Equations (4.50) and (4.53) by

$$\begin{aligned} \frac{Z\mu_{t+1}}{\mu_t} &\equiv \frac{Z\mu^{i,m}_{t+1}}{\mu^{i,m}_t} = \delta^1_0\left(\frac{g^1_1}{g^1_0}\right)^{-\gamma}\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma} \\ &= \delta^1_0\nu^{-\gamma}\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}, \end{aligned} \tag{4.54}$$

where definition (4.39) was used in order to obtain the second line.

Aggregate Investment Share

Building on the results derived so far, one can now turn to aggregate economic behavior. The aggregate investment share as well as the processes for aggregate production, investment and consumption are found along the lines of Fischer and Jensen (2014, 2017).

Note that from identity (4.3) together with Equation (4.6) capital input at time $t+1$ is given by $K_{t+1} = X_t Y_t$. Substituting this relation into the linear production technology (4.12) and rearranging, accordingly aggregate production growth is given by

$$\frac{Y_{t+1}}{Y_t} = X_t \Xi_1 A_{t+1}. \tag{4.55}$$

In the next step, this result is used along with the expression for the stochastic discount factor (4.54) to restate the first order condition (4.41) according to

$$\begin{aligned} 1 =& \mathbb{E}_t \left[\delta_0^1 \nu^{-\gamma} \left(X_t \Xi_1 A_{t+1} \right)^{-\gamma} \tilde{R}_{E,t+1} \right] \\ =& \mathbb{E}_t \left[\delta_0^1 \nu^{-\gamma} \left(X_t \Xi_1 A_{t+1} \right)^{-\gamma} \left((1-\tau_c)\theta \Xi_1 A_{t+1} + \tau_c \right) \right], \end{aligned} \tag{4.56}$$

where the last line follows from the definition of after-tax equity return and solution (4.13). Equation (4.56) reveals the importance of the linearity of the production technology for finding a tractable model solution - since it implies that the risky return is exogenously specified, only the stochastic discount factor process remains undetermined in expression (4.41).[82] Beyond that, given the information set at time t, i.e., $\mathcal{F}_t$, the aggregate investment share (or saving rate) at date t is known and thus not random. Consequently, X_t can be moved out of the conditional expectations op-

[82] In the case of a nonlinear production technology, the return on capital would also be unspecified. This is again different to the case of an exchange economy presented in Chapter 3, where the macroeconomic production was exogenously given but the risky return was found in equilibrium.

erator $\mathbb{E}_t$. By rearranging Equation (4.56) the analytical solution to the aggregate investment share reads

$$X \equiv X_t = \frac{1}{\nu G_1}, \tag{4.57}$$

where

$$G_1 \equiv \left(\delta_0^1\right)^{-\frac{1}{\gamma}} \mathbb{E}\left[(1-\tau_c)\theta \Xi_1^{1-\gamma} A^{1-\gamma} + \tau_c \Xi_1^{-\gamma} A^{-\gamma}\right]^{-\frac{1}{\gamma}} \tag{4.58}$$

is an exogenous constant that depends on the capital gains tax rate τ_c. Furthermore, the time subscript for the expectations operator and the random variable A in Equation (4.58) are dropped, because of the i.i.d. property of the random scaling factor.[83] Where possible, the time index will generally be suppressed in the following in order to simplify the notation.

Being composed of G_1 and the endogenous parameter ν, it follows that X is also a time- and state-independent constant that is directly dependent on the capital gains tax rate τ_c. Beyond that, the aggregate investment share is indirectly dependent on both tax rates τ_c and τ_l, as redistributive taxation changes the distribution of consumption shares across individuals and, thereby, affects ν. This relation is established through the equilibrium condition, Equation (4.91), which will be derived in Section 4.2.3.[84]

---

[83] Note that the i.i.d. property implies that the realization of A is independent of its past and future realizations and distributed identically in each time step. The unconditional expectation about the future realization is, therefore, identical with its conditional expectation.

[84] The solution to the aggregate investment share in Equation (4.57) has similarities to the result found by Fischer and Jensen (2017, Theorem 1, Item 2). Their model setup, however, is limited to a classical demographic structure, homogeneous preferences and abstracts from labor income. In this vein, the present solution is, first, more general, since it additionally accounts for labor taxation, where the setting featuring exclusively capital income is incorporated in the present model in the case of $\theta \to 1$. Second, due to the heterogeneity introduced by the overlapping generations structure and differences in preferences, the present solution is dependent on the distribution of consumption shares across individuals through the endogenous parameter ν. This is an important property, when studying the impact of redistributive taxation on macroeconomic quantities.

Because of this dependency, however, the relation between the tax rates and aggregate investment behavior cannot be deduced directly from Equation (4.57). In order to derive a deeper understanding of this relation, it will be necessary to use numerical examples. So far, one can just follow that the constant G_1 will be an increasing function in τ_c as long as the following condition holds:[85]

$$\frac{\mathbb{E}\left[A^{-\gamma}\right]}{\mathbb{E}\left[A^{1-\gamma}\right]} < \theta \Xi_1. \tag{4.59}$$

For reasonable parameter settings, like the one chosen in the quantitative analysis below, this relation will be fulfilled. Moreover, it can be stated that an increase in the denominator νG_1 leads to a decrease in X.

Aggregate Growth Rates

Given the results just presented, the specification of the equilibrium processes for macroeconomic growth rates is straightforward. Aggregate consumption and investment growth follow, respectively, from the ratio of Equation (4.5) and the ratio of Equation (4.6) in two consecutive time steps. The finding that X is a time- and state-independent constant, thereby, implies that both ratios are identical and, furthermore, equal to aggregate production growth. Hence, using (4.55) and (4.57), the equilibrium growth rates for production, aggregate investment and aggregate consumption from period t to $t+1$ are given by

$$\frac{Y_{t+1}}{Y_t} = \frac{I_{t+1}}{I_t} = \frac{C_{t+1}}{C_t} = \frac{\Xi_1}{\nu G_1} A_{t+1}, \tag{4.60}$$

respectively. According to Equation (4.60), macroeconomic growth follows an i.i.d. multinomial process in equilibrium. In line with the observations above, it depends on both tax rates. On the one hand, there is a direct impact of the capital gains tax τ_c through the constant G_1. On the other hand,

[85] See Appendix B.2.5 for the derivation.

the growth is indirectly affected by redistributive taxation on labor income and capital gains through the parameter ν. Given the implied dependency on the distribution of consumption, further insights can particularly be derived using numerical examples.

4.2.1.3 The Stochastic Discount Factor and the Risk-free Return

Having established the equilibrium process for aggregate production growth, it will now be possible to specify the stochastic discount factor process and to find a closed-form solution for the risk-free return as well as the equity risk premium.

Equilibrium Stochastic Discount Factor

In order to derive the stochastic discount factor process, just substitute result (4.60) into Equation (4.54) to derive the following i.i.d. multinomial process:

$$\begin{aligned} \frac{Z\mu_{t+1}}{\mu_t} =& \delta_0^1 \nu^{-\gamma} \left(\frac{\Xi_1}{\nu G_1} A_{t+1} \right)^{-\gamma}, \\ =& G_2 A_{t+1}^{-\gamma}, \end{aligned} \tag{4.61}$$

where

$$G_2 \equiv \delta_0^1 \left(\frac{\Xi_1}{G_1} \right)^{-\gamma} = \mathbb{E}\left[(1-\tau_c)\, \theta \Xi_1 A^{1-\gamma} + \tau_c A^{-\gamma} \right]^{-1}, \tag{4.62}$$

is a constant. Since the endogenous parameter ν cancels out itself and only exogenous quantities remain, Equation (4.61) establishes a closed-form solution for the equilibrium stochastic discount factor.[86] Consequently, it is independent of the distribution of resources across agents and, thus, independent of the tax rate on labor income. Surprisingly, since the marginal

[86] Note that, since in the present setting markets are incomplete, the SDF is not unique. To the contrary, there will be many stochastic discount factors that equally satisfy the model's basic pricing equations (see, for example, Munk (2013, Theorem 4.6)).

rate of substitution also turns out to be independent of the subjective time discount factor, the agent's degree of patience seems irrelevant for his willingness to shift consumption through time. However, the agent's degree of patience is already incorporated in the aggregate growth rates, as can be seen in Equation (4.60). In the present production economy these are determined endogenously according to individual behavior. Aggregate growth therefore already reflects its residents' willingness to shift consumption through time.

Finally, the stochastic discount factor is directly affected by the capital gains tax rate τ_c. Similar considerations as for the constant G_1 imply that G_2 will be an increasing function in τ_c as long as condition (4.59) holds. In this case, the agents' marginal rate of substitution increases in τ_c, indicating that future consumption will be valued higher for larger tax rates.

Equilibrium Risk-free Return

As outline above, it can be considered a standard result in asset pricing theory that the gross return on the risk-free security is given by the inverse of the expected stochastic discount factor. Furthermore, it was shown in Chapter 3 that, in the presence of taxation, the relevant return is given by the gross risk-free return after tax. In the context of the present framework, this similarly holds true, as can be deduced from the first order condition (4.42). In particular, when substituting solution (4.61) in condition (4.42), the equilibrium solution to the one-period risk-free rate after tax is given by

$$\begin{aligned}
\tilde{R}_f \equiv \tilde{R}_{f,t} &= \frac{1}{\mathbb{E}_t\left[\frac{Z\mu_{t+1}}{\mu_t}\right]} \\
&= \left(G_2 \mathbb{E}\left[A^{-\gamma}\right]\right)^{-1} \\
&= \left(\frac{\mathbb{E}\left[A^{-\gamma}\right]}{\left(\left(1-\tau_c\right)\theta\Xi_1 \mathbb{E}\left[A^{1-\gamma}\right] + \tau_c \mathbb{E}\left[A^{-\gamma}\right]\right)^{-1}}\right)^{-1} \\
&= \left(1-\tau_c\right)\theta\Xi_1 \frac{\mathbb{E}\left[A^{1-\gamma}\right]}{\mathbb{E}\left[A^{-\gamma}\right]} + \tau_c,
\end{aligned} \tag{4.63}$$

where the time subscript has been dropped again, due to the i.i.d. property of A. Equation (4.63) shows that the equilibrium gross risk-free return after tax is a time- and state-independent constant in the present setting. Moreover, the solution is closed form, since it only depends on exogenous quantities. Beyond that, the risk-free rate after tax is directly dependent on capital taxation. To be precise, it will be decreasing in τ_c, as long as condition (4.59) holds. Nevertheless, it is generally independent of taxation on labor income.

Finally, the closed-form solution to the gross risk-free return before tax follows immediately from Equations (4.24) and (4.63). It reads

$$R_f = \theta \Xi_1 \frac{\mathbb{E}\left[A^{1-\gamma}\right]}{\mathbb{E}\left[A^{-\gamma}\right]}, \tag{4.64}$$

which is a constant independent of both tax rates τ_c and τ_l. Comparing solution (4.64) to condition (4.59) reveals the fact that the latter actually implies a restriction on the gross risk-free return. To be precise, condition (4.59) is equal to restricting the gross risk-free return before tax to be larger than one. Accordingly, the risk-free return after tax will be decreasing in the tax rate τ_c, as long as the pre-tax net risk-free return ($R_f - 1$) is positive. Its range is then limited to $1 \leq \tilde{R}_f \leq R_f$.

Equilibrium Equity Risk Premium and Summary

Lastly, the results established so far can be used to find a closed-form solution for the equity risk premium. That is, the expected excess return a risky investment provides over the risk-free return. Its after-tax version is given by

$$\mathbb{E}\left[\tilde{R}_E\right] - \tilde{R}_f = (1 - \tau_c)\, \theta \Xi_1 \left(\mathbb{E}\left[A\right] - \frac{\mathbb{E}\left[A^{1-\gamma}\right]}{\mathbb{E}\left[A^{-\gamma}\right]} \right), \tag{4.65}$$

which is a time- and state-independent constant in equilibrium. Since the volatility of the risky return decreases with growing taxation, an increase in the tax rate τ_c lowers the after-tax risk premium. Its range is limited to $0 \leq \mathbb{E}\left[\tilde{R}_E\right] - \tilde{R}_f \leq \mathbb{E}\left[A^{-\gamma}\right] - \mathbb{E}\left[A^{1-\gamma}\right] / \mathbb{E}\left[A^{-\gamma}\right]$. Furthermore, an increase in the common risk aversion coefficient γ raises the expected risk premium. This effect reflects the growing compensation demanded by agents with a higher degree of risk aversion for bearing risk.

To sum up, with solutions (4.13), (4.43),(4.63) and (4.64), the capital market is fully determined and only dependent on the model's primitives. Equilibrium asset returns turn out to be generally independent of taxation on labor income. This follows as a consequence of the assumption of a linear production technology and the absence of an endogenous labor-leisure choice. Moreover, pre-tax returns are generally unaffected by taxation in equilibrium. Both after-tax returns, however, depend on the taxation of net capital income. To be more precise, the risky as well as the risk-free return, under condition (4.59), are decreasing functions of τ_c.

4.2.2 Life-cycle Consumption and Investment Behavior

Having established aggregate macroeconomic behavior, the present section derives equilibrium solutions on an individual level. First, the agents' total wealth budget constraints are elaborated. They are used to develop the formal concepts of human and transfer capital and, subsequently, to determine their analytical equilibrium solutions. Afterwards, building on the total wealth budget constraints, the individuals' marginal propensity to consume out of total wealth (MPCTW) is derived and, in a next step, used to determine the agents' optimal consumption policy. Finally, the present section is closed by establishing optimal individual investment policies and refining the representation of net transfer income.

4.2.2.1 Total Wealth Budget Constraint, Human and Transfer Capital

As explained in the previous chapter, in the presence of a stream of permanent income, the expected remaining lifetime consumption of an agent will include the present value of his expected future payments received from this source of income. In the present setting, the concept of total wealth, therefore, incorporates two different elements of non-financial total wealth, human and transfer capital, along with financial wealth. Since the present value of future earnings and transfers affects an individual's consumption decision, the total wealth budget constraint will be the relevant constraint in order to determine his optimal consumption and investment policies.

The concept of total wealth is, for example, used by Epstein and Zin (1991) and Campbell (1993) in order to account for human capital in the analysis of an intertemporal asset pricing model. Considering the present value of permanent income is important, as individuals hold large parts of their total wealth in the form of human capital. In this regard, Lustig et al. (2013), for example, estimate total wealth and find the fraction of human capital therein to be 92%. This is in line with the results of earlier studies, which also identify human capital as the major component of total wealth (see, e.g., Mayers (1972) and Palacios (2015)).

In the given model, the market values of human and transfer capital can be calculated. Moreover, they are decisive for determining the equilibrium. As described above, Equations (4.10) and (4.11) indicate that earnings and capital income move together. This perfect correlation implies that, although there is no human capital security explicitly traded on financial markets, earnings become tradable through the real investment security. Beyond that, transfer income is tradable through a portfolio composed of the risky and risk-free asset. Hence, being similar to capital income, permanent income should be valued the same way. That is, its present value should be derived by discounting future income streams using the equilibrium stochastic discount factor.

Along the lines of the previous chapter, the formal pricing relations for human and transfer capital follow naturally while determining the total wealth budget constraint. The latter is derived by using the SDF and solving forward the dynamic budget constraint (4.32) (Miao (2014)). Accordingly, total wealth of an agent of type m born in period i follows by[87]

$$\begin{aligned} w_t^{i,m} \equiv & \sum_{n=0}^{N-(t-i)} \mathbb{E}_t \left[\frac{Z^n \mu_{t+n}}{\mu_t} c_{t+n}^{i,m} \right] \\ = & a_t^{i,m} + (1-\tau_l)\, h_t^{i,m} + p_{h,t}^{i,m} + s_t^i + p_{s,t}^i, \end{aligned} \tag{4.66}$$

where $a_t^{i,m} \equiv \alpha_{t-1}^{i,m} \left((1-\tau_c)\, D_t + \tau_c I_{t-1} \right) + \beta_{t-1}^{i,m} \tilde{R}_f$ is current financial wealth after accounting for taxes. According to Equation (4.66), total wealth of an agent of type m born in period i equals the present value of his expected life-time consumption. Again, the elements of non-financial total wealth are separated into two components: on the one hand, the above representation distinguishes current earnings and human capital; on the other hand, it considers current transfers and transfer capital separately. Using this representation illustrates the fact that each element of non-financial total wealth can be interpreted as a nontraded asset. In the case of earnings, current income after tax $(1-\tau_l)\, h_t^{i,m}$ can be interpreted as the stochastic dividend and $p_{h,t}^{i,m}$ as the shadow price of the nontraded asset human capital (see Epstein and Zin (1991, Footnote 3)).

By solving for Equation (4.66), the following pricing relations for human and transfer capital result:

$$p_{h,t}^{i,m} \equiv \sum_{n=1}^{N-(t-i)} \mathbb{E}_t \left[\frac{Z^n \mu_{t+n}}{\mu_t} (1-\tau_l)\, h_{t+n}^{i,m} \right], \tag{4.67}$$

$$p_{s,t}^{i} \equiv \sum_{n=1}^{N-(t-i)} \mathbb{E}_t \left[\frac{Z^n \mu_{t+n}}{\mu_t} s_{t+n}^{i} \right], \tag{4.68}$$

[87] Details of the derivation are given in Appendix B.2.6.

respectively. Human capital of an agent of type m born in period i is the sum over his expected future after-tax earnings discounted by the equilibrium stochastic discount factor. Using Equations (4.54) and (4.25) in conjunction with (4.10), the equilibrium solution for human capital is found to be a function linear in aggregate production.[88] Formally, it reads

$$p_{h,t}^{i,m} = \Xi_2 \sum_{n=1}^{N-(t-i)} \rho^n f_{(t-i)+n}^m Y_t, \tag{4.69}$$

where $\Xi_2 \equiv (1-\tau_l)(1-\theta)$ and $\rho \equiv \delta_0^1 \nu^{-1} (\Xi_1/G_1)^{1-\gamma} \mathbb{E}[A^{1-\gamma}]$ are time- and state-independent constants. According to Equation (4.69), the price-to-production ratio of human capital, denoted by $\eta_{h,t-i}^m \equiv p_{h,t}^{i,m}/Y_t$, is a deterministic and age-dependent function. It is directly affected by taxation on labor income τ_l. Moreover, it depends indirectly on both tax rates τ_l and τ_c through the endogenous parameter ν.

Similarly, the transfer capital of an agent of type m born in period i is given by the sum over his expected future transfers discounted by the equilibrium stochastic discount factor. Its equilibrium solution is also a function linear in economic output. It is given by[89]

$$p_{s,t}^{i} = \Xi_3 \sum_{n=1}^{N-(t-i)} \rho^n d_{(t-i)+n} Y_t, \tag{4.70}$$

where $\Xi_3 \equiv \tau_c \theta + \tau_l(1-\theta) - \tau_c \frac{\mathbb{E}[A^{-\gamma}]}{\Xi_1 \mathbb{E}[A^{1-\gamma}]}$ is a time- and state-independent constant. As a result, the price-to-production ratio of transfer capital, i.e., $\eta_{s,t-i} \equiv p_{s,t}^i/Y_t$, is also a deterministic function of age. It depends directly on both kinds of taxation and is, furthermore, affected by τ_l and τ_c indirectly through the endogenous parameter ν.

---

[88] See Appendix B.2.7 for the derivation.

[89] The derivation is shown in Appendix B.2.8.

Although there exists a closed-form solution for the stochastic discount factor, the equilibrium pricing relations (4.69) and (4.70) are not closed form. This is a direct consequence of the fact that aggregate production is dependent on ν. Accordingly, the relation between taxation and human or transfer capital cannot be deduced directly from their analytical solutions. Clearly, one would expect human capital (after tax) to decrease with growing taxation on labor income. In contrast, transfer capital should intuitively rise when tax rates increase. Nevertheless, in the present setting aggregate economic development is affected by the redistributive taxation mechanism as well. The direction and magnitude of this influence are thus decisive in order to determine the actual relation between taxation and the present values of earnings and transfers. This will be studied in the numerical examples below.

4.2.2.2 Consumption Policy

Given the market value of human and transfer capital, one can proceed to solve for the agent's consumption policy. For this purpose, the individual's consumption growth rate in equilibrium will first be derived. Second, the agent's marginal propensity to consume out of total wealth is deduced using the total wealth budget constraint established above. Finally, by making use of these results, the agent's life-cycle consumption strategy can be established.

Individual Consumption Growth

As outlined above, Equation (4.53) shows that agents align their marginal rates of substitution over consumption in equilibrium. Note, however, that this does not imply that they possess the same consumption growth rates.

To demonstrate this, substitute the individuals' marginal rates of substitution, expression (4.44), on both sides of condition (4.53) and rearrange in order to derive consumption growth of agent i,m as a function of consumption growth of agent j,k:

$$\begin{aligned}\delta^m_{t-i}\left(\frac{c^{i,m}_{t+1}}{c^{i,m}_t}\right)^{-\gamma} &= \delta^k_{t-j}\left(\frac{c^{j,k}_{t+1}}{c^{j,k}_t}\right)^{-\gamma}\\ \Leftrightarrow \frac{c^{i,m}_{t+1}}{c^{i,m}_t} &= \left(\frac{\delta^m_{t-i}}{\delta^k_{t-j}}\right)^{\frac{1}{\gamma}}\frac{c^{j,k}_{t+1}}{c^{j,k}_t}. \end{aligned} \tag{4.71}$$

From Equation (4.71) it follows that the consumption growth rates of any two agents are only identical if they hold the same subjective time discount factor, i.e., $\delta^m_{t-i} = \delta^k_{t-j}$. In any other case, individual consumption growth differs across agents. Suppose, for example, that the patience of agent i,m is larger than that of agent j,k, i.e., $\delta^m_{t-i} > \delta^k_{t-j}$. It then follows from Equation (4.71) that the more patient individual faces higher consumption growth from time t to $t+1$ than the individual that is less patient and vice versa.[90]

Next, let the second individual be the one used to define the perspective taken in equilibrium, namely agent type $k = 1$ of age $t-j = 0$, and use expressions (4.38)-(4.39). Then, Equation (4.71) is equivalent to

$$\frac{c^{i,m}_{t+1}}{c^{i,m}_t} = \left(\frac{\delta^m_{t-i}}{\delta^1_0}\right)^{\frac{1}{\gamma}}\nu\frac{Y_{t+1}}{Y_t}. \tag{4.72}$$

[90] Apparently, this requires the range of the coefficient of relative risk aversion to be limited to $0 < \gamma < \infty$. This restriction is reasonable as it implies that agents are neither risk neutral nor infinitely risk averse.

Finally, substituting the solution for aggregate production growth (4.60) it follows that

$$\frac{c_{t+1}^{i,m}}{c_t^{i,m}} = \left(\frac{\delta_{t-i}^m}{\delta_0^1}\right)^{\frac{1}{\gamma}} \frac{\Xi_1}{G_1} A_{t+1} = \left(\frac{\delta_{t-i}^m}{G_2}\right)^{\frac{1}{\gamma}} A_{t+1} \tag{4.73}$$

is the consumption growth of an agent of type m born in period i.[91] It follows an age-dependent i.i.d. multinomial process over the agent's life-cycle. Since it does not depend on the endogenous parameter ν, Equation (4.73) represents a closed-form solution. According to this analytical result, the pattern underlying the development of consumption growth over the individual's life-cycle is directly specified by the age-dependent evolution of his subjective time discount factor.

Marginal Propensity to Consume out of Total Wealth

Building on the recent result, Equation (4.73), one can now turn to determine the agent's marginal propensity to consume out of total wealth. As defined before, the MPCTW is the agent-specific function determining the share of total wealth consumed by an individual at a certain point in time. It will be derived using the total wealth budget constraint.

In this vein, dividing both sides of (4.66) by current individual consumption $c_t^{i,m}$, the agent's total wealth to consumption ratio is given by

$$\begin{aligned} \frac{w_t^{i,m}}{c_t^{i,m}} &= \sum_{n=0}^{N-(t-i)} \mathbb{E}_t \left[\frac{Z^n \mu_{t+n}}{\mu_t} \frac{c_{t+n}^{i,m}}{c_t^{i,m}} \right] \\ &= \sum_{n=0}^{N-(t-i)} \mathbb{E}_t \left[\left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^m \right) \left(\frac{c_{t+n}^{i,m}}{c_t^{i,m}} \right)^{1-\gamma} \right], \end{aligned} \tag{4.74}$$

where expression (4.44) was used in order to derive the second line.

[91] Alternatively, one could have derived the same solution by simply taking the definition of the SDF in Equation (4.44) and equalizing it with the solution for the SDF found in Equation (4.61).

Then, take the reciprocal of the expression just found and substitute individual consumption growth (4.73) from above to derive the closed-form solution for the marginal propensity to consume out of total wealth of an agent of type m and age $t-i$:

$$
\begin{aligned}
b_{t-i}^{m} \equiv & \frac{c_t^{i,m}}{w_t^{i,m}} \\
= & \left\{ \sum_{n=0}^{N-(t-i)} \left[\left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^{m} \right)^{\frac{1}{\gamma}} \left(G_2^{1-\frac{1}{\gamma}} \mathbb{E}\left[A^{1-\gamma}\right] \right)^{n} \right] \right\}^{-1}, \qquad (4.75)
\end{aligned}
$$

where the i.i.d. property of the production shock has been used again.[92] According to Equation (4.75), agent's consumption to total wealth ratio is an age-dependent, but time- and state-independent, function. In line with the findings in Chapter 3, the MPCTW decreases in the subjective time discount factor, which results in a lower share of total wealth currently consumed. As an immediate consequence, the fraction postponed for future consumption grows in δ, representing increasing patience. Considering a coefficient of relative risk aversion of $\gamma = 1$, Equation (4.75) incorporates the special case of logarithmic utility again. The marginal consumption propensity of such an individual will only be dependent on this agent's time preferences (see Merton (1969) and Samuelson (1969)) and independent of any other factors, such as the risk and return properties on the financial market. Beyond that, the marginal propensity to consume out of total wealth will still be a deterministic function, when risk aversion coefficients are larger than one. This results from the i.i.d. property of A. Nevertheless, in this case the characteristics underlying the economy are taken into account by the agents when making their consumption decision, since their marginal propensity to consume out of total wealth will depend on the expectation about the realization of the aggregate shock and the tax rate τ_c.

[92] Appendix B.2.9 details this derivation.

Individual Life-cycle Consumption

Lastly, it remains to establish the agent's equilibrium life-cycle consumption policy. To this effect, note that Equations (4.72) and (4.73), defining individual consumption growth, are linear recurrence equations that can be solved backwards to the agent's date of birth. In this vein, it follows from Equation (4.72), or alternatively from Equation (4.73), that consumption at time t of an agent of type m born in period i can be written as

$$c_t^{i,m} = \left(\frac{\nu}{(\delta_0^1)^{\frac{1}{\gamma}}}\right)^{t-i} \left(\prod_{l=0}^{(t-i)-1} \delta_l^m\right)^{\frac{1}{\gamma}} \frac{Y_t}{Y_i} c_i^{i,m}, \quad or \tag{4.76}$$

$$c_t^{i,m} = \left(\frac{\prod_{l=0}^{(t-i)-1} \delta_l^m}{G_2^{t-i}}\right)^{\frac{1}{\gamma}} \left(\prod_{k=0}^{t-i} A_{i+k}\right) c_i^{i,m}, \tag{4.77}$$

respectively. Equations (4.76) and (4.77) define current consumption as a function of the individual's consumption in the period of his birth. According to this result, the whole consumption sequence of an agent is fully determined as soon as his consumption at age $t-i=0$ is known. At the same time, individual consumption may alternatively be expressed using the MPCTW along with total wealth. In this way, consumption of a newborn individual of type m at date i reads

$$\begin{aligned} c_i^{i,m} =& b_0^m w_i^{i,m} \\ =& b_0^m \left((1-\tau_l)\, h_i^{i,m} + p_{h,i}^{i,m} + p_{s,i}^i\right) \\ =& b_0^m \left((1-\tau_l)\,(1-\theta) f_0^m + \eta_{h,0}^m + \eta_{s,0}\right) Y_i \\ =& g_0^m Y_i, \end{aligned} \tag{4.78}$$

where the second line follows from substituting the agent's total wealth budget constraint (4.66) at age $t-i=0$. Moreover, the third line is due to the definition of individual earnings, Equations (4.1) and (4.10), as well as the analytical results to human and transfer capital, Equations (4.69) and (4.70).

Based on these derivations, the equilibrium life-cycle consumption policy of an agent of type m born in period i can ultimately be derived by combining Equations (4.76) and (4.78):

$$\begin{aligned} c_t^{i,m} =& g_0^m \left(\frac{\nu}{(\delta_0^1)^{\frac{1}{\gamma}}} \right)^{t-i} \left(\prod_{l=0}^{(t-i)-1} \delta_l^m \right)^{\frac{1}{\gamma}} Y_t \\ =& g_{t-i}^m Y_t. \end{aligned} \tag{4.79}$$

Solution (4.79) defines individual utility maximizing consumption behavior in equilibrium. As indicated, it is an affine function of aggregate production; or, in other words, agents establish a linear sharing rule in equilibrium. Since time preferences are considered to be age-dependent in the present setting, the life-cycle pattern of individual consumption will depend on the evolution of agents' subjective time discount factor. Accordingly, the resulting life-cycle profiles may replicate complex shapes, which can be understood best by substituting Equation (4.78) into the alternative representation (4.77):

$$c_t^{i,m} = \left(\frac{\prod_{l=0}^{(t-i)-1} \delta_l^m}{G_2^{t-i}} \right)^{\frac{1}{\gamma}} \left(\prod_{k=0}^{t-i} A_{i+k} \right) g_0^m Y_i. \tag{4.80}$$

According to Equation (4.80), the consumption function is, first, affected by the constant G_2 and the sequence of realizations of the productivity shock A with rising age. Second, and more important, it is the product over the age-specific subjective time discount factors that shapes the consumption pattern. The direction of this effect may change with age, depending on the underlying assumptions regarding the profile of time preferences over the life-cycle. This way, consumption patterns can be replicated that mimic observable facts, like the empirically well documented hump shape indicated above.

Furthermore, Equation (4.80) reveals the fact that individual consumption profiles are affected by the redistribution mechanism in two ways. First, labor taxation enters the solution merely through the consumption share at model age zero g_0^m. Therefore, the tax rate τ_l only affects the consumption level of an agent, but not his life-cycle pattern. Second, and conversely, the taxation of capital gains influences g_0^m as well as the constant G_2. Thus, the tax rate τ_c affects both the level as well as the profile of consumption.

Once again the influence of the taxation system can be separated into a direct and indirect effect. On the one hand, tax rates directly affect individual consumption, as they change directly the newborn's current and expected discounted future incomes, his MPCTW as well as the constant G_2. On the other hand, there exists an indirect effect that arises, due to the fact that different allocations of resources lead to a different endogenous parameter ν (see Section 4.2.3)). This in turn implies different aggregate growth rates (see Equation (4.60)) and, subsequently, a change in human and transfer capital (see Equations (4.69)-(4.70)).

Finally, note that Equation (4.79) implies a consumption share in aggregate production $g_{t-i}^m = c_t^{i,m}/Y_t$ that is actually a deterministic function of age only. It thus confirms the conjecture (4.38) made initially and, accordingly, the results derived based on this assumption. Feasibility of the solution can be ensured by applying the market clearing conditions defined above. This, however, also requires the equilibrium solutions for the individual investment policies, which will be presented in the following section.

4.2.2.3 Investment Policies

As indicated in Section 4.1.4.3, every agent currently alive has to take two decisions in any given time step. The first is the consumption-savings decision, which has just been derived in the preceding section. The second is the decision of how to allocate investments between available securities in order to establish this utility maximizing consumption policy. In the given model, the financial market consists of two securities, risky equity and risk-free one-period bonds. In the following, the optimal individual investment policies for this opportunity set are found.

Individual Bond Investment

Based on the preceding results, individual current consumption is given by solution (4.79) and, alternatively, as function of the marginal propensity to consume (4.75) and total wealth (4.66). On the one hand, using the latter representation, consumption at date t of an agent of type m born in period i can be expressed by

$$\begin{aligned}
c_t^{i,m} =& b_{t-i}^m \Big(\alpha_{t-1}^{i,m}\left(\left(1-\tau_c\right)D_t+\tau_c I_{t-1}\right)+\beta_{t-1}^{i,m}\tilde{R}_f + \\
&\left(1-\tau_l\right)h_t^{i,m}+p_{h,t}^{i,m}+s_t^i+p_{s,t}^i\Big) \\
=& b_{t-i}^m \Big(\alpha_{t-1}^{i,m}\left(\left(1-\tau_c\right)\theta Y_t+\tau_c I_{t-1}\right)+\beta_{t-1}^{i,m}\tilde{R}_f + \\
&\left(1-\tau_l\right)\left(1-\theta\right)f_{t-i}^m Y_t+\eta_{h,t-i}^m Y_t + \\
&d_{t-i}\left(\tau_c\left(\theta Y_t-I_{t-1}\right)+\tau_l\left(1-\theta\right)Y_t\right)+\eta_{s,t-i}Y_t\Big),
\end{aligned} \tag{4.81}$$

where the second line follows by applying the definition of individual earnings (4.1) and transfer payments (4.26), the expressions for aggregate capital output (4.11) and earnings (4.10) as well as the equilibrium results to human (4.69) and transfer (4.70) capital.

On the other hand, it follows from the equilibrium solution (4.79) that the optimal consumption strategy is a function linear in aggregate production. This in return, however, implies that the consumption expression stated on the right-hand side of Equation (4.81) must be linear in current output as well. Therefore, in order to be in line with optimal individual behavior, it follows that all terms that do not involve aggregate output Y_t must vanish from expression (4.81). Formally, this means that the following condition must hold in equilibrium:

$$b_{t-i}^{m}\left(\alpha_{t-1}^{i,m}\tau_c I_{t-1}+\beta_{t-1}^{i,m}\tilde{R}_f-d_{t-i}\tau_c I_{t-1}\right)=0. \tag{4.82}$$

Equation (4.82) is similar to condition (3.54) of Chapter 3. Analogously, it can be rewritten in order to determine the bond market position from time $t-1$ to t of an agent of type m born in period $t-N\leq i\leq t-1$ in terms of his equity investment:

$$\beta_{t-1}^{i,m}=\frac{\tau_c}{\tilde{R}_f}\left(\frac{1}{N\cdot M}-\alpha_{t-1}^{i,m}\right)I_{t-1}, \tag{4.83}$$

where expression (4.17) has been used to substitute d_{t-i}. According to Equation (4.83), the individual investment policy regarding the risk-free security turns out to be a state- and age-dependent function. As observed in the previous chapter, and in line with Fischer and Jensen (2014, 2015, 2017), in the presence of taxation on net capital income, trading on the bond market becomes essential for agents in order to establish their optimal consumption policy. Accordingly, the investment strategy for the risk-free security is induced by the fact that transfers financed by capital taxation are subject to additional macroeconomic risk. Transfers based on investment gains are large (small) when equity returns were high (low) in the recent past, which is the case when economic growth was strong (weak) over the last period. Moreover, as briefly outlined above (see Section 4.1.3), the taxation system is designed in a way that individuals with financial income below (above) the average become net recipients (contributors) of capital transfers. As a result, net recipients have a relatively high exposure to macroeconomic risk, whereas net contributors have a relatively weak (or negative) exposure to that kind of risk. In order to compensate this unequal distribution of risk, agents enter into bond contracts with each other.

Along these lines, Equation (4.83) implies positive bond holdings for individuals that hold an equity share, $\alpha_{t-1}^{i,m}$, that is smaller than that of an average market participant, $1/(N \cdot M)$. The other way around, agents with equity shares larger than on average enter into short positions on the bond market.[93] Hence, there is dynamic trading in the risk-free security, although there is no positive net supply in the bond market. Again, this implies that a market equilibrium is found that produces a risk-free rate at which agents are willing to take complementary positions in such an asset.

Finally, since the agent's optimal bond investment strategy (4.83) directly depends on the agent's optimal equity investment strategy, it follows that clearing on the bond market, condition (4.37), follows directly from clearing on the equity market, condition (4.36). To see this, assume clearing on the equity market, condition (4.36), substitute the agent's optimal bond investment strategy (4.83) into clearing condition (4.37) and take the sum across all individuals. Then, since there is clearing on the stock market, clearing on the bond market follows.[94] Moreover, Section 4.1.5 outlined how clearing on the markets for consumption goods and wealth, conditions (4.34)-(4.35), follows directly from clearing on the asset markets.[95] As a result, for the determination of the general equilibrium solution, it will be sufficient to exclusively concentrate on asset market clearing, since clearing on all remaining markets follows immediately from there.

Individual Equity Investment

The investment strategy of an individual with respect to the risky security is found by equalizing the right-hand sides of Equations (4.79) and (4.81).

---

[93] It will be shown below in detail that transfers financed by capital taxation are subject to additional macroeconomic risk and that net contributors (recipients) of capital transfers enter into short (long) positions on the bond market.

[94] Appendix B.2.10 demonstrates in detail that this relationship holds.

[95] See Appendix B.2.1 for details.

Substituting the solution for the bond market position (4.83) and rearranging for $\alpha_{t-1}^{i,m}$, the equity share held by an agent of type m born in period $t-N \leq i \leq t-1$ from time $t-1$ to t is given by

$$\alpha_{t-1}^{i,m} = \frac{1}{(1-\tau_c)\,\theta}\left(\frac{g_{t-i}^{m}}{b_{t-i}^{m}} - (1-\tau_l)(1-\theta)\, f_{t-i}^{m} - \right.$$
$$\left. \eta_{h,t-i}^{m} - \frac{\tau_c\theta + \tau_l(1-\theta)}{N \cdot M} - \eta_{s,t-i}\right), \qquad (4.84)$$

which is a state- and time-independent function of the individual's age. Equation (4.84) implies that the share of equity holdings increases in the individual's consumption share of aggregate production, g_{t-i}^{m}. Furthermore, recall that the marginal propensity to consume out of total wealth, b_{t-i}^{m}, of an agent is positively linked to his subjective time discount factors. In conjunction with expression (4.84) this implies that patient individuals tend to hold larger fractions of the risky security than impatient ones. Moreover, higher current and expected discounted future permanent incomes negatively affect the equity investment decision. Especially early in life, human and transfer capital are large, since most of the earnings and transfers are still to be received. As a result, positions in the risky security are presumably lowest when agents are young. This may also comprise the fact that individuals enter into short positions on the equity market. Moreover, it should be noted that $\alpha_{t-1}^{i,m}$ depends on both kinds of taxation, directly as well as indirectly through the endogenous parameter ν.

Beyond that, one may also use the bond investment policy in order to make a statement about equilibrium equity investment. Rearranging Equation (4.83) for $\alpha_{t-1}^{i,m}$ yields

$$\alpha_{t-1}^{i,m} = \frac{1}{N \cdot M} - \beta_{t-1}^{i,m}\frac{\tilde{R}_f}{\tau_c I_{t-1}}. \qquad (4.85)$$

From the observations above, then, follows that, since they enter into positive bond holdings, the equity investments of net recipients of capital transfers are reduced by capital taxation. Conversely, net contributors of capital transfers will hold larger shares in the risky security because of their short positions in the risk-free security. This relation might explain low stock market participation rates of poorer individuals, as outlined by Fischer and Jensen (2015).

To sum up, in the previous section it was found that agents, in order to maximize utility, pursue a linear sharing rule for consumption in equilibrium. Subsequently, the present section has shown that, in order to establish this optimal consumption policy in the presence of a redistributive capital gains tax, agents have to dynamically trade on both the bond and the equity markets. Along the lines of the previous chapter and Fischer and Jensen (2015), the linear sharing rule would not be obtainable without dynamic trading in both assets. This can be understood best by looking at Equation (4.81). Without trading in the risk-free asset, term (4.82) would not vanish and, thereby, impede the agents' establishing of their optimal linear consumption policy. When making the investment decision, however, this expression is known to the agents. Hence, they can find the bond market investment strategy given in (4.83) in order to establish the linear sharing rule. In this vein, substituting policy (4.83) into constraint (4.81) it follows that:

$$
\begin{aligned}
c_t^{i,m} =& b_{t-i}^m \left(\alpha_{t-1}^{i,m} \left(1-\tau_c\right)\theta + \left(1-\tau_l\right)\left(1-\theta\right) f_{t-i}^m + \right. \\
& \left. \eta_{h,t-i}^m + \frac{\tau_c\theta + \tau_l\left(1-\theta\right)}{N \cdot M} + \eta_{s,t-i} \right) Y_t,
\end{aligned} \tag{4.86}
$$

which implies that consumption eventually becomes linear in aggregate production.

4.2.2.4 Net Transfers

Based on the equilibrium investment policies, one can now refine the presentation for the net transfers received by any individual.[96] As established so far, at time t the total amount of taxes paid by an agent of type m born in period $t-N \leq i < t$ is is given by

$$\alpha_{t-1}^{i,m}\tau_c\left(D_t - I_{t-1}\right) + \beta_{t-1}^{i,m}\tau_c\left(R_f - 1\right) + f_{t-i}^{m}\tau_l H_t. \tag{4.87}$$

Accordingly, individual tax payments are composed of taxes on equity investment (first term), bond market activities (second term) and labor income (third term). At the same time, following from Equations (4.26) and (4.20) along with Equation (4.17), the same agent receives transfer payments according to

$$\frac{1}{N \cdot M}\left(\tau_c\left(D_t - I_{t-1}\right) + \tau_l H_t\right). \tag{4.88}$$

Note that Equation (4.88) does not include tax revenues from positions in the risk-free security, since there is no net supply in aggregate. Subtracting Equation (4.87) from (4.88), substituting the bond market investment policy (4.83) for $\beta_{t-1}^{i,m}$ and making use of Equation (4.24), the net transfer at date t received by an agent of type m born in period $t-N \leq i < t$ can be written as

$$\tau_c\left(\frac{1}{N \cdot M} - \alpha_{t-1}^{i,m}\right)\left(D_t - \frac{R_f}{\tilde{R}_f} I_{t-1}\right) + \tau_l\left(\frac{1}{N \cdot M} - f_{t-i}^{m}\right) H_t. \tag{4.89}$$

As outlined above, the taxation system is designed in a way that agents with small financial wealth effectively receive a larger fraction of capital transfers than agents that possess high financial wealth. This is reflected in the first term of Equation (4.89). It shows that individuals with an in-

[96]The present section draws on the derivations in Fischer and Jensen (2015, Theorem 1, Item 7).

come from equity investments below (above) that of an average market participant become net recipients (contributors) of capital transfers. In the same way, agents with small labor or retirement income effectively receive a larger fraction of transfers financed by earnings. That is, agents with earnings below (above) an average are net recipients (contributors) of earnings transfers, as displayed by the second term in Equation (4.89).

Building on this representation, it can be shown that capital transfers carry additional macroeconomic risk. To this end, use Equation (4.13) to rewrite the first term of Equation (4.89) to

$$\tau_c \left(1 - \frac{R_f}{\tilde{R}_f} \left(\theta \Xi_1 A_t\right)^{-1}\right) \left(\frac{1}{N \cdot M} - \alpha_{t-1}^{i,m}\right) D_t. \tag{4.90}$$

The level of net capital transfers at date t, first of all, depends linearly on the development of the economy through current capital output D_t.[97] Beyond that, however, expression (4.90) also reveals the fact that the statutory tax rate τ_c is multiplied by $(1 - (R_f/\tilde{R}_f)(\theta \Xi_1 A_t)^{-1})$, which includes the current random productivity shock A_t. This implies that the effective tax rate on capital income itself is dependent on the development of the economy. In boom times, when A_t is large, capital output is large and the effective tax rates on capital income are high as well. Conversely, in times of an economic bust, when A_t is small, capital output is small and the effective tax rates on capital income is low as well. Hence, the transfer mechanism, besides redistributing incomes, also redistributes macroeconomic risk from net contributors to net recipients of capital transfers.

Along with the bond investment policy, it follows that net recipients of capital transfers enter into long positions on the bond market, whereas net contributors take short positions in the risk-free security. Beyond that, as shown by Equation (4.85), it follows that capital transfers reduce (increase) equity investments of net recipients (contributors) of capital transfers.

---

[97] A similar case holds true for earnings transfers.

Finally, newborn agents do not receive transfer payments and enter the economy without physical capital. However, they are already subject to taxation on labor income. Hence, the net tax paid by them is simply given by $\tau_l f_0^m H_t$.

4.2.3 Equilibrium Condition

In the preceding sections the equilibrium solutions for macroeconomic as well as individual behavior were derived. Some of the results were found in terms of closed-form solutions. Others, however, turned out to be dependent on the endogenous parameter ν, which originated from the initially made conjecture (4.38) regarding individual equilibrium consumption. In Section 4.2.2.2 this conjecture and, accordingly, the results that derived from it were confirmed by showing that agent's equilibrium optimal consumption is actually in accordance with the initial conjecture, i.e., a function linear in aggregate production.

Now it finally remains to establish feasibility of the presented solution and to determine the parameter ν. Note that both are mutually dependent. On the one hand, feasibility of the solution requires the market clearing conditions defined above to be fulfilled. On the other hand, the endogenous parameter can be determined by using the agents' equilibrium policies along with the conditions for market clearing. In so doing, market clearing is ensured, and thus is feasibility of the solution. In this context, it has been shown that the initially high complexity of the equilibrium solution of the present model is reduced due to the fact that clearing on the equity market already implies clearing on all other markets. Put differently, for the determination of the general equilibrium solution, it will be sufficient to exclusively concentrate on stock market clearing, since clearing on all remaining markets follows immediately from there.[98]

[98] Appendix B.2.1, along with Appendix B.2.10, show that these relationships hold.

Along these lines, using the agents' equilibrium equity investment strategy (4.84) in conjunction with the clearing condition for the equity market (4.36) and the terminal portfolio condition (4.33), the following equation is found:

$$(1-\tau_c)\,\theta = \sum_{m=1}^{M}\sum_{i=t-N}^{t-1}\left\{\left(\frac{g_{t-i}^{m}}{b_{t-i}^{m}} - (1-\tau_l)(1-\theta)\,f_{t-i}^{m} - \eta_{h,t-i}^{m} - \frac{\tau_c\theta + \tau_l(1-\theta)}{N\cdot M} - \eta_{s,t-i}\right)\right\}. \tag{4.91}$$

Equation (4.91) in combination with the equilibrium results (4.69), (4.70), (4.75), (4.78) and (4.79) form a nonlinear equation in the endogenous parameter ν. Along the lines of Chapter 3, it can be noted that the solution implied by Equation (4.91) is optimal, since it builds on the utility maximizing equilibrium results determined above. On the other hand, the solution must again also be in line with market clearing, i.e., feasible given current economic resources. Clearing on the equity market follows immediately, as the equilibrium condition is based on this restriction. Clearing on all remaining markets follows in return from equity clearing, as outlined above. Therefore, the solution implied by equilibrium condition (4.91) is both optimal and feasible.

In this vein, the equilibrium problem is broken down into a single equation with one unknown. Solving it yields the optimal and feasible equilibrium solution to the endogenous parameter ν. Having established this result, unique solutions to the consumption shares of all agents populating the economy follow. To see this, note that, according to Equation (4.78), the consumption shares of newborns g_0^m are solely determined by exogenously given items except for the parameter ν. That is, a unique solution for them is given as soon as the equilibrium solution to the endogenous parameter ν is determined. Having established the consumption shares of the newborn generation and ν, all other consumption shares can be calculated from Equation (4.79).

Note that the equilibrium condition demonstrates why the previous results, including the endogenous parameter, were said to be dependent on the distribution of resources across individuals. As can be seen from Equation (4.91), the solution to ν basically depends on the current permanent income received as well as on the human and transfer capital held by the agents currently populating the economy. A shift of permanent income from one type of agents to another, or similarly across cohorts, changes the equilibrium condition and, consequently, implies a different solution for ν. As a result, the redistributive taxation of both capital as well as labor income affects macroeconomic and individual behavior, i.e., the equilibrium solution. This is a consequence of the overlapping generations framework and the heterogeneity in preferences considered in the present setting. In asset pricing models with classical demographic structure and homogeneous preferences, this dependency on the distribution of resources would not emerge.[99]

Finally, since the equilibrium condition's dependency on ν is nonlinear, it requires numerical methods in order to solve for the endogenous parameter.[100] To be more precise, for the quantitative analysis presented below a computer-based Newton–Raphson method is applied in order to find a solution to the problem stated above.

---

[99] One might, for instance, consider labor income and labor taxation within the framework presented in Fischer and Jensen (2014). In this context, redistributive taxation on labor income would influence individual behavior the same way as a simple change in initial endowments in the case without permanent income. More importantly, however, aggregate behavior, i.e., economic growth rates, would be left unaffected by any such redistribution.

[100] Note that an analytical solution exists for some simplified special cases. As a result, these cases also imply closed-form solutions for the entire model. Assuming only one type of agents, i.e., $M = 1$, and restricting individuals to live for only two periods, i.e., $N = 1$, a particular simple solution can be directly deduced from the equilibrium condition, Equation (4.91). In this vein, the present model comprises a stochastic version of the seminal overlapping generations production model (with linear technology) introduced by Diamond (1965).

4.2.4 Welfare Measures

The consideration of output quantities primarily gains relevance in view of the fact that it provides individuals with well-being. Besides looking at the effects that redistributive taxation has on aggregate and individual economic quantities, it is of great importance to examine its impact on aggregate and individual welfare. On an individual level, the derivation of a welfare measure is straightforward, since it, apparently, should be based on the agent's expected remaining lifetime utility from consumption in optimum, i.e., his indirect utility. This will be presented in the subsequent Section 4.2.4.1. On a macroeconomic level, however, some further assumptions have to be made with regards to the relevant individual quantities that shall found social (or aggregate) well-being. This will be discussed below in Section 4.2.4.2.

In advance, it can be specified how welfare effects shall generally be measured. As shown above, macroeconomic production and, subsequently, consumption grow at a certain rate (see Equation (4.60)) in the present setting. Note, however, that absolute levels of aggregate quantities cannot be determined without defining the output level of the economy at some point in time. Accordingly, using welfare measures based on absolute values would require further assumptions. This can be avoided by specifying a relative welfare measure, which eventually will not depend on the absolute level of macroeconomic output.

In this vein, following Auerbach and Kotlikoff (1983), Kotlikoff et al. (1999) and Fischer and Jensen (2014), welfare changes due to variation in consumption shall be specified in terms of an equivalent variation, which in the present case is defined by the production equivalent.[101] From the point of view in time t, the selected welfare measure compares expected utility from consumption in a setting without taxation to expected utility from consumption in a setting with redistributive taxation. Technically speak-

[101] The concept of equivalent variation is attributed to Hicks (1939).

ing, it measures the change in current production, Y_t, for the no-taxation setting that is necessary in order to achieve equivalent expected utility levels from consumption in both settings. That way, positive values define welfare gains, whereas negative quantities indicate welfare losses due to redistributive taxation. The formal definition follows below.

4.2.4.1 Individual Welfare Measure

As indicated above, the welfare measure on an individual level can directly be deduced from the agent's indirect utility stated in Equation (4.31). According to that, an individual's well-being is defined by his maximum expected remaining lifetime utility over current and future consumption. In order to measure welfare changes due to redistributive taxation, the previously described approach, building on a production equivalent, is followed.

By using the equilibrium solution for individual consumption (4.79) as well as the MPCTW, Equation (4.74), and applying some basic algebraic manipulations, the expected remaining life-time utility at date t of an agent of type m currently alive, i.e., born in period $t-N \leq i \leq t$, is given by[102]

$$V_t^{i,m} = \frac{\left(g_{t-i}^m\right)^{1-\gamma}}{1-\gamma} \left(b_{t-i}^m\right)^{-1} Y_t^{1-\gamma}. \tag{4.92}$$

This solution is a standard result that arises in the context of CRRA utility and i.i.d. returns (Back (2010)). It depends on the agent's current marginal propensity to consume out of total wealth, his current share in aggregate production as well as on aggregate output of the economy. As a result, the individual's indirect utility is time- and state-dependent and, moreover, affected by the endogenous parameter ν. According to that, individual welfare depends on the distribution of resources across individuals. As outlined above, this is a consequence of the overlapping generations framework and the heterogeneity in preferences considered in the present

[102] The derivation is presented in detail in Appendix B.2.11.

setting. Within the classical asset pricing framework with standard demographic structure and homogeneous preferences, this dependency would not arise.[103]

Since it will be of interest and necessary in the context of aggregate social welfare, the expected life-time utility of newborn cohorts not yet born will be derived as well. This is, the hypothetical expected indirect utility at date t of a newborn individual to be born in period $i > t$ can be written as

$$\mathbb{E}_t\left[V_i^{i,m}\right] = \left(\left(\frac{\Xi_1}{\nu G_1}\right)^{1-\gamma} \mathbb{E}\left[A^{1-\gamma}\right]\right)^{-(t-i)} V_t^{t,m}. \tag{4.93}$$

Based on these results one can now turn to the relative measure of well-being. Recall that welfare was said to be measured as the percentage change in current production needed in the setting without taxation to generate the same level of utility the agent holds in the presence of redistributive taxation. Formally, this means that the following equations must hold, respectively, for agents currently alive and yet unborn

$$V_t^{i,m} = \tilde{V}_t^{i,m}, \quad for\ t - N \leq i \leq t, \tag{4.94}$$

$$\mathbb{E}_t\left[V_i^{i,m}\right] = \mathbb{E}_t\left[\tilde{V}_i^{i,m}\right], \quad for\ i > t, \tag{4.95}$$

where variables with tilde represent values that arise in the presence of taxation, whereas variables without tilde correspond to values in the no-tax setting. Using these conditions along with the equations for indirect utility (4.92) and (4.93), the desired welfare measure follows by

[103] In this regard, Fischer and Jensen (2014, pp. 19-20), for instance, state the following for their framework without overlapping generations and preference heterogeneity: "[...] on the individual household level [...] welfare consequences solely depend on a household's initial endowment relative to the average household's initial endowment, but otherwise not on the distribution of wealth among households [...]."

$$\mathcal{V}_{t-i}^{m} = \begin{cases} \left(\frac{\tilde{g}_{t-i}^{m}}{g_{t-i}^{m}}\right)\left(\frac{b_{t-i}^{m}}{\tilde{b}_{t-i}^{m}}\right)^{\frac{1}{1-\gamma}} - 1 & if\, t-N \leq i \leq t, \\ \left(\frac{\nu G_1}{\tilde{\nu}\tilde{G}_1}\right)^{-(t-i)}\left(\frac{\tilde{g}_0^{m}}{g_0^{m}}\right)\left(\frac{b_0^{m}}{\tilde{b}_0^{m}}\right)^{\frac{1}{1-\gamma}} - 1 & if\, i > t, \end{cases} \tag{4.96}$$

which is a time- and state-independent deterministic function of an individual's age. According to Equation (4.96), individual welfare is affected by the taxation system in two ways, directly and indirectly. First, the fact that individual consumption shares are directly and indirectly dependent on redistributive taxation (see the discussion in Section 4.2.2.2) carries over to individual welfare. Second, there are additional direct effects of capital taxation through the individuals marginal propensity to consume out of total wealth (see Equation (4.75)) and, in case of future generations, the constant G_1 (see Equation (4.58)). Third, there exists an additional indirect effect of both types of taxation on the individual welfare of future generations through the parameter ν that arises, since different allocations of resources lead to different equilibrium solutions (see Section 4.2.3).

4.2.4.2 Social Welfare Measure

As shown above, the production framework established in the present setting implies that there is not only a reallocation of resources between individuals, but also a change in aggregate economic quantities due to redistributive taxation. Consequently, the present setting does not only lead to welfare shifts on individual level, but also produces welfare changes on aggregate level. This, however, brings about the question of a suitable measure for aggregate social welfare.

On the one hand, one could define a representative individual (or a composite consumer) who maximizes a utility function over aggregate consumption and measure welfare from his point of view.[104] By so doing, an

[104]There is a considerable literature on the construction and the existence of a representative

indirect utility over aggregate consumption for the representative agent could be found and the social welfare measure could be derived the same way as in the individual case (see Fischer and Jensen (2014)). In the present setting this is problematic, however, since individuals are heterogeneous regarding a variety of characteristics and the equilibrium solution depends on the allocation of resources between them. This raises the problem of deriving a representative agent (or a composite consumer) in a consistent way, so that his utility maximizing behavior, subject to aggregate endowment, would lead to the same equilibrium asset prices found in the setting featuring overlapping generations and heterogeneous time preferences.[105] Moreover, in the context of overlapping generations, it would still remain questionable whether such measure correctly captures individual well-being in aggregate form.

On the other hand, there is a more straightforward approach that is common in the context of overlapping generations and public finance. It dates back to the seminal "Bergson welfare function" (Bergson (1938)) and is based on the reasonable assumption that aggregate welfare should simply be a function composed of individual welfare. In the present thesis, the social welfare analysis will be based on this concept. In particular, and as it is usually the case in this context, the welfare function is assumed to take an additive form. To be more precise, it shall be given by the weighted sum over the individuals' indirect utilities. Two questions arise in this context:

---

agent (or composite consumer) that dates back to Gorman (1959), Wilson (1968), Rubinstein (1974) and Constantinides (1982).

[105] A specific problem arises in the context of a representative budget constraint. Following the notation used so far, dropping agent individual indices, it would read:

$$C_t = I_{t-1}\Xi_1 A_t + \beta_{t-1} R_{f,t-1} - I_t - \beta_t,$$

which is independent of the tax rates, since a redistributive tax system has no effect in the context of just one agent who pays and receives all revenues. Beyond that, it is also generally independent of permanent income, because of the single agent receiving aggregate production, as consequence of his sole investment decision. These properties of the representative's constraint affect the first order conditions and, hence, lead to different equilibrium pricing relations than in the setting featuring overlapping generations and heterogeneous time preferences.

Who are the relevant individuals to consider in order to quantify aggregate social welfare? And how should their indirect utilities be weighted?

The former question is a matter of short- and long-run effects. Apparently, one could just focus on those agents that are currently alive by solely considering their individual welfare. Such an approach would be exclusively concerned with the short-run implications of a redistributive tax system, since it ignores the effects on future generations entirely. The present model structure featuring overlapping generations, however, implies that there will be an infinite number of future generations populating an infinitely-lived economy. Merely looking at the welfare of the living population totally ignores this fact. In particular, this appears especially myopic when considering the long-run effects a redistributive taxation system has on aggregate and individual quantities in the present setting.[106] In contrast, one may ignore present generations and exclusively take future generations into account, focusing merely on the long-run effects of a redistributive tax system. This procedure is widely applied in the existing literature on optimal taxation and can, for example, be found in Pestieau (1974) and Atkinson and Sandmo (1980). However, it appears at least implausible that a government legitimated through the members of the present population should totally ignore the well-being of the very same. Therefore, based on the recent considerations, it seems reasonable to generally include the expected remaining lifetime utility of both current and future individuals when valuing aggregate social welfare - as, for instance, in Ball and Mankiw (2007).

The latter question is about whether there should be a different weighting on the indirect utilities of different agent types and/or generations. In the

[106] Campbell and Nosbusch (2007) define a social welfare function that considers only generations currently alive. However, they assume an endowment economy without growth in the aggregate output of the consumption good. Alike, Conesa et al. (2009) define the ex ante expected lifetime utility of an average newborn individual as the measure for social welfare. Their model, however, features a stationary equilibrium with constant per capita variables and functions. Contrary to the present study, there are, thus, no long-run effects on future generations' welfare levels.

existing literature, two approaches to this problem exist that are both generally concerned with social discounting of future generations' welfare. On the one hand, based on ethical considerations regarding equality, Ramsey (1927) among others (see also Sen (1961) and Solow (1974)) advocates that no differentiation should be made and consequently zero social discounting should be applied. On the other hand, most studies regarding optimal taxation presume a positive discount rate and, therefore, define social welfare as the discounted sum over future (and current) generations' welfare (see, for example, Pestieau (1974) and Atkinson and Sandmo (1980)). First of all, this helps to overcome some mathematical difficulties that occur with zero discounting (see Koopmans (1967)). Second, and additionally, one may argue that a positive social discount rate just helps to achieve inter-generational equity, as it reflects the value judgments about the inter-generational distribution inherited in the individuals' preferences.

Following the latter approach, social welfare will be defined in the present study as the discounted sum over current and future generations' indirect lifetime utilities. Finding adequate social discount rates, however, is challenging in the present setting, due to the existing heterogeneity in the subjective time discount factors. In order to incorporate agents' preferences about inter-generational distribution, it seems reasonable to assume different social discount rates for the diverse agent types m. These in turn should be based on the groups' patience characteristics. The straightforward approach, proposed here, is to define for each agent type the mean subjective time discount factor across all living cohorts $\bar{\delta}^m$ as its relevant social discount rate.[107] Formally, this can be expressed by the following social welfare function:

$$V_t = \sum_{m=1}^{M} \left\{ \sum_{i=t-N}^{t} V_t^{i,m} + \sum_{i=t+1}^{\infty} \left(\bar{\delta}^m\right)^{-(t-i)} \mathbb{E}_t \left[V_i^{i,m} \right] \right\}. \tag{4.97}$$

[107] For reasonable values of $\bar{\delta}^m$, the numerical results derived below are qualitative robust.

Although there is some flexibility regarding the social discount factor, such a welfare definition can be thought to be constructed according to Rawls's principle of "the veil of ignorance" (Rawls (1999)). Accordingly, given the situation at date t, from a meta perspective each agent would (or at least could) agree in such procedure when taking decisions on "the basis of general considerations", i.e., not being given any special information that specifies and will eventually effect his own particular case.

In order to reduce equation size in the following, the subsequent auxiliary function is used:

$$\varepsilon_{t-i}^{m} = \begin{cases} \frac{\left(g_{t-i}^{m}\right)^{1-\gamma}}{1-\gamma}\left(b_{t-i}^{m}\right)^{-1} & if\ t-N \leq i \leq t, \\ \left(\left(\frac{\Xi_1}{\nu G_1}\right)^{1-\gamma}\mathbb{E}\left[A^{1-\gamma}\right]\right)^{-(t-i)} \frac{\left(g_{0}^{m}\right)^{1-\gamma}}{1-\gamma}\left(b_{0}^{m}\right)^{-1} & if\ i > t. \end{cases} \tag{4.98}$$

Subsequently, given aggregate well-being (4.97), one can now determine the social welfare measure as previously for the individual level. It is defined as the percentage change in production without taxation that would be necessary in order to obtain the same aggregate level of welfare as in the presence of redistributive taxation. This implies that the following equality must hold:

$$V_t = \tilde{V}_t, \tag{4.99}$$

where, once again, variables with tilde denote values that arise in the case of taxation, whereas variables without tilde represent values in the no-tax case. Using this condition and substituting the definition of aggregate welfare (4.97), the social welfare measure finally reads[108]

$$\mathcal{V} = \left(\frac{\sum_{m=1}^{M}\left\{\sum_{i=t-N}^{t}\tilde{\varepsilon}_{t-i}^{m} + \sum_{i=t+1}^{\infty}\left(\bar{\delta}^{m}\right)^{-(t-i)}\tilde{\varepsilon}_{t-i}^{m}\right\}}{\sum_{m=1}^{M}\left\{\sum_{i=t-N}^{t}\varepsilon_{t-i}^{m} + \sum_{i=t+1}^{\infty}\left(\bar{\delta}^{m}\right)^{-(t-i)}\varepsilon_{t-i}^{m}\right\}}\right)^{\frac{1}{1-\gamma}} - 1. \tag{4.100}$$

[108] Details of the derivation are shown in Appendix B.2.12

As proposed, Equation (4.100) defines the percentage change in current production needed in the setting without taxation to generate the same level of aggregate welfare attained in the presence of redistributive taxation.[109] In line with the observations made for the individual welfare measure (4.96), social welfare is time- and state-independent. It is directly affected by redistributive taxation because of the implied changes in individual consumption shares and MPCTWs. Furthermore, as different allocations of resources lead to different endogenous parameters ν, an indirect effect of taxation exists on $\mathcal{V}$. To be precise, different tax rates lead to different growth rates. As earlier, this indirectly affects consumption shares through changes in human and transfer capital. Furthermore, it alters the expected indirect utility of future generations, represented by the term $(\Xi_1/\nu G_1)$ in Equation (4.98).

4.2.5 The Impact of Government Debt

The present section will finally close the analytical treatment of the general equilibrium solution by considering the impact of an active fiscal policy on the model solution. Following Fischer and Jensen (2014, 2017), one could expect the government to try to influence economic growth by investing in the output technology itself and finance its activity by issuing government bonds. However, similar to them, one can show that debt financed government investment programs are neutralized by individual investment behavior in the present setting. Since most of the derivation steps presented above would have to be repeated for this purpose, the following explanations will only concentrate on the decisive steps and results. A more extensive treatment is provided in the Appendix B.2.13.

There is a wide array of literature considering the impact of a debt financed fiscal policy in general equilibrium. Economic literature is traditionally

[109] Note that within the quantitative analysis, this welfare measure will also be adapted, so that it also applies to subgroups of the current and future population.

concerned with this issue and studies examining the influence of government debt in a production framework with overlapping generations date back to Diamond (1965), Pestieau (1974), Atkinson and Sandmo (1980), Auerbach and Kotlikoff (1983), Blanchard (1985) and Gertler (1999). These early studies have focused on deterministic models, where individuals possess perfect foresight and life-cycle features are merely stylized. In contrast, modern macroeconomic and asset pricing models feature aggregate and idiosyncratic risk and may capture realistic individual life-cycles. In this regard, important studies that abstract from an overlapping generations framework are, for example, Aiyagari (1995), Ludvigson (1996) and Aiyagari and McGrattan (1998). Considerable contributions featuring overlapping generations are, for instance, Gomes and Michaelides (2008) and Gomes et al. (2013). Within these approaches, the authors consider debt financed government policies and study the impact on asset prices, risk premiums and agents' consumption and investment behavior in general equilibrium. In contrast to them, the present study can only consider the impact of debt financed fiscal policy as complementary to the preceding analysis. As it will be outlined below, the presented model specification with i.i.d. aggregate risk and tradable permanent income implies that agents will neutralize such government policy and eliminate any general equilibrium impact.

Following the objective to increase macroeconomic growth, the government is assumed to issue government debt to finance investments in the production technology. For the sake of simplicity, the government strategy is given exogenously and designed to establish a constant debt to GDP ratio β^G. In line with Fischer and Jensen (2014, 2017), government bonds are taken to be one-period risk-free bonds that are perfect substitutes for privately issued risky securities. Moreover, it further holds true that the government budget constraint is balanced in every period. That is, government neither builds up wealth, nor debt. The amount of government debt outstanding in every period t is denoted by $\beta^G Y_t$ which implies government investments I_t^G of the same amount and a share of government equity holdings from t to $t+1$ given by $\alpha_t^G \equiv I_t^G / I_t$.[110] Under these assumptions, aggregate private equity and bond holdings are changed to

$$\sum_{m=1}^{M} \sum_{i=t-N}^{t} \bar{\alpha}_t^{i,m} = 1 - \alpha_t^G, \tag{4.101}$$

$$\sum_{m=1}^{M} \sum_{i=t-N}^{t} \bar{\beta}_t^{i,m} = \beta^G Y_t, \tag{4.102}$$

respectively, where bars denote variables (changed) in the presence of the government debt policy. Since the government budget constraint is balanced in every period, any government debt must directly be financed by, and all revenues must directly be paid to the living individuals. Consequently, such government intervention affects the optimization problem, given in Equations (4.31)-(4.33), by altering the individual's budget constraint (4.27). To be precise, it is the amount of transfer payments s_t^i that is changed. In aggregate, disposable transfers are now given by

$$\bar{S}_t = S_t + \alpha_{t-1}^G \left((1-\tau_c) D_t + \tau_c I_{t-1} \right) - \beta^G Y_{t-1} \tilde{R}_f, \tag{4.103}$$

which consist of the original aggregate transfers without government debt (first term), increased by government after-tax equity revenues (second term), but reduced by the repayments of government debts (third term). Using this result, the time t budget constraint of an agent of type m born in period $i < t$ under government debt policy becomes[111]

$$\begin{aligned} \bar{v}_t^{i,m} = & \left(\bar{\alpha}_{t-1}^{i,m} + \frac{1}{N \cdot M} \alpha_{t-1}^G \right) \left((1-\tau_c) D_t + \tau_c I_{t-1} \right) + \\ & \left(\bar{\beta}_{t-1}^{i,m} - \frac{1}{N \cdot M} \beta^G Y_{t-1} \right) \tilde{R}_{f,t-1} + (1-\tau_l) h_t^{i,m} + s_t^i. \end{aligned} \tag{4.104}$$

---

[110] Note that aggregate investment $I_t = I_t^P + I_t^G$ is now split up into a private $I_t^P = \left(X_t - \beta^G\right) Y_t$ and a government $I_t^G = \beta^G Y_t$ part.

[111] The budget constraint of newborn agents (4.28) remains unchanged, since it is independent of transfers.

According to the findings by Fischer and Jensen (2014, 2017), Equation (4.104) illustrates that, induced by the changes in transfers, every agent's exposure to the risk-free security is decreased by the additional term $(N \cdot M)^{-1}\beta^G Y_{t-1}$. In contrast, the exposure to equity of each agent rises by the additional term $(N \cdot M)^{-1}\alpha^G_{t-1}$. These changes are predictable for the agents who can thus react to the government intervention. Technically speaking, it can be shown that under these circumstances, bond and equity investment policies from time $t-1$ to t of any agent of type m born in period $t-N \leq i \leq t-1$ change to

$$\bar{\beta}^{i,m}_{t-1} = \beta^{i,m}_{t-1} + \frac{1}{N \cdot M}\beta^G Y_{t-1}, \tag{4.105}$$

$$\bar{\alpha}^{i,m}_{t-1} = \alpha^{i,m}_{t-1} - \frac{1}{N \cdot M}\alpha^G_{t-1}, \tag{4.106}$$

respectively. That is, individuals adapt their investment decisions by, first, increasing their bond holdings by the amount of $(N \cdot M)^{-1}\beta^G Y_{t-1}$ and, second, lowering their equity investment share by the amount of $(N \cdot M)^{-1}\alpha^G_{t-1}$. Substituting these results into the agents budget constraint (4.104) reveals that individuals' altered investment decisions neutralize the additional effects of the government intervention on an individual level. Eventually, the agent's budget constraint remains unchanged. Moreover, the changed investment policies imply that aggregate private equity (4.101) and bond (4.102) holdings in the setting with government debt turn out to be identical to the original conditions (4.36)-(4.37). Thus, it follows that under the debt financed investment program the optimization problem together with the definition of market equilibrium is identical to the original case without such fiscal policy given by Equations (4.31)-(4.33) and conditions (4.34)-(4.37).

To sum up, in the present setting, when the government finances an investment program by issuing risk-free bonds, agents' exposure to the risk-free security decreases while their exposure to equity increases. The channel through which this takes place is the transfer mechanism, since any government debt is directly financed by and all revenues are directly paid to the living individuals. Agents adapt to these changes by decreasing their equity holdings and increasing their position in the risk-free security. By so doing, they neutralize any effect such government intervention has on individual level and, with it, undo any impact on aggregate level as well. Hence, the optimization problem is unchanged and so is the equilibrium solution. Put differently, by adapting investments, the original budget constraint (4.27) is fulfilled, agents preserve the linear sharing rule (4.79), the equilibrium condition from Equation (4.91) still holds and aggregate investment (4.57) is unchanged. Consequently, the equilibrium solutions remain unchanged and any such government intervention has no effect on economic growth.

4.3 Quantitative Analysis

Building on a numerical analysis, this section illustrates and extends the analytical findings presented in Section 4.2. For this analysis, an empirically plausible parameterization is chosen, which is presented in Section 4.3.1. Based on these assumptions Section 4.3.2 presents some baseline results generated from the model and compares them to the data. Next, Section 4.3.3 studies the impact of varying labor and capital gains tax rates on macroeconomic quantities, individual life-cycle behavior, consumption and wealth disparity as well as individual and social welfare. Finally, an extensive robustness analysis is presented in Section 4.3.4, which studies the effect of different levels of income disparity, diverse specifications for the subjective time discount factor and varying degrees of relative risk aversion.

4.3.1 Baseline Parameterization

Each model period is assumed to correspond to one year, so decisions are made at an annual frequency. In line with the usual assumption in the life-cycle literature, agents enter the model economy at age 20 ($t - i = 0$), when they start their working life.[112] Before that, they are not separately captured, as their economic activity is considered to be part of their parents' decisions (Samuelson (1958)). Individuals stay workers for forty-five years and retire at age 65, that is at model age $t - i = 45$. In retirement, agents are assumed to live for additional 25 years before they die at age 90.[113] This implies that, when entering the economy at age 20, an individual faces a certain horizon of $N + 1 = 70$ periods (see, for example, Gomes et al. (2013)).

In order to study the impact of redistributive taxation on individuals with different proportions in aggregate earnings, the number of different agent types considered for the quantitative analysis is chosen to be $M = 3$. This is reasonable, as it divides the population into three different earnings groups. To be precise, there will be an agent group (agents of type 1 or "high") that is subject to a relatively high income, a group (agents of type 2 or "average") with average earnings and an agent group (agents of type 3 or "low") that receives a relatively small fraction of aggregate earnings.[114]

---

[112] Counting in the model starts at age zero indicating that agents are economically born, i.e., they become economically relevant, at this age. In line with the above assumptions, this implies that a certain model age refers to a real-life age that is 20 years ahead.

[113] In the OECD countries, an average worker retires around the age of 64 (see OECD (2017b)), while the current remaining life expectancy at age 65 is 21 years in high income countries. Beyond that, the projected remaining life expectancy at age 65 for the cohort that entered the labor market at age 20 in 2015 (the cohort born in 1995) is about 24 years, implying a total life expectancy of about 89 years. The parameterization is chosen to match these observations. Nevertheless, it should be noted that the current life expectancy is only about 81 years at birth for high income countries (Source: United Nations, Department of Economic and Social Affairs, Population Division (2017). World Population Prospects: The 2017 Revision). Since, however, in the present model setting no individual dies ahead of time, there is no possibility to reproduce conditional life expectancies for various age groups.

[114] The income groups are of equal size, an assumption that is justified by choosing income

Table 4.1 – This table reports the baseline parameterization for the model featuring production and overlapping generations.

Description	Parameter	Value
Number of agent types	M	3
Agents' economic lifespan	$N + 1$	70
Retirement age	$O + 1$	45
Degree of risk aversion	γ	3
Capital's share of output	θ	30%
Tax rate labor income	τ_l	10%
Tax rate capital gains	τ_c	40%
Productivity shock boom	A^{boom}	8.51
Productivity shock bust	A^{bust}	7.74

Moreover, for the baseline parameterization the agents' common relative risk aversion coefficient is set to $\gamma = 3$. This parameter choice lies within the range of values usually considered in the asset pricing literature. In particular, within their seminal work Mehra and Prescott (1985) reason that γ should be situated between zero and ten. Accordingly, one can find the range of values typically considered in modern life-cycle asset pricing or portfolio choice models (see Dammon (2001), Viceira (2001), Constantinides et al. (2002), Cocco et al. (2005), Campbell and Nosbusch (2007),Gomes et al. (2013), Fischer et al. (2013), Gârleanu and Panageas (2015)). In Section 4.3.4.3, alternative parameter values are considered.

Next, turn to the production side of the model. The capital's share of output (θ) is set to 30%. This is in line with the empirical evidence of the average output share for the U.S. economy (from 1959 to 2016).[115] For

proportions accordingly. The income profile and the particular proportions are given below in Section 4.3.1.1.

[115]The time-series average for the capital's share of output is 29.2% (Source: U.S. Bureau of Economic Analysis, National Income and Product Accounts, "Table 1.10. Gross Domestic Income by Type of Income", annual data, 1959-2016). The underlying calculation is as follows: net operating surplus / (compensation of employees + net operating surplus)

simplicity, the number of different realizations of the productivity shock used in the numerical analysis is restricted to $Z = 2$, as in Fischer and Jensen (2014, 2015, 2017), which implies that markets are complete under the given parameterization. This assumption has no qualitative effects, since the analytical results show that the solution is not affected by the amount of possible realizations but by the unconditional moments implied by them. The specification of the productivity shock is chosen in order to help to replicate the long term historical mean (3.3%) and standard deviation (4.9%) of U.S. gross domestic product (GDP) growth.[116] In this vein, the realizations of the capital augmenting shock are given by $A^{boom} = 8.51$ and $A^{bust} = 7.74$, which are, respectively, referred to as the boom and bust scenarios, like in Fischer and Jensen (2015).

With respect to the tax rates, in the baseline parameter set, the values are chosen in order to match the corresponding shares of tax revenues in GDP observed in the data, as given in Gomes et al. (2013). Since the personal income taxes reported in the underlying data exclude tax credits or transfers, the calculation is based on the "gross" tax revenues collected by the government in the model, which also include revenues collected from long positions in the bond market.[117] Then, to be in line with the empirical numbers, the flat tax rate on labor income τ_l is set to 10% and the flat tax rate on capital gains τ_c is set to 40%.[118]

Together with the specification of the life-cycle earnings profile (Section 4.3.1.1) and the determination of the profile for the subjective time discount factor (Section 4.3.1.2), this set of parameter values is referred to

[116] Source: U.S. Bureau of Economic Analysis, National Income and Product Accounts, "Table 1.1.1. Percent Change From Preceding Period in Real Gross Domestic Product ", annual data, 1930-2016.

[117] Source: U.S. Bureau of Economic Analysis, National Income and Product Accounts, "Table 3.4. Personal Current Tax Receipts".

[118] Gomes et al. (2013) find the same values for capital and labor taxes for their model in order to match the corresponding share of tax revenues in GDP. As pointed out by the authors, the marginal tax rate on labor income in reality is higher than 10%. This discrepancy between model and statutory tax rate is due to the simplifying assumption of a linear tax schedule.

as the baseline parameterization. It is summarized in Table 4.1. In Section 4.3.4 various alternative parameter values are considered.

4.3.1.1 Earnings Profiles

In this section, the life-cycle earnings profiles for the three different agent types ($M = 3$) presumed above are specified. In this regard, the present analysis abstracts from modeling different earnings profiles across income groups, but concentrates on differences in their lifetime income level (i.e., the fraction in aggregate earnings).[119] Applying this simplification, the model properties following from the baseline parameterization will allow to readily separate between inter-generational and intra-generational redistribution effects in the analysis.[120]

The parameterization of the specific labor income profile, f^m_{t-i} for $0 \leq t-i \leq O$, is based on Hubbard et al. (1994), who estimate third-order polynomials in age. The implied deterministic profiles reflect the well-established hump shape of labor income over the life-cycle. The present analysis builds on these result because of the underlying definition of labor income that merely composes household earnings and unemployment insurance.[121] In other studies, the definition of labor income is usually much broader and may additionally include, e.g., public transfer payments (see, for instance, Cocco et al. (2005)). Since the main objective of the

[119] Earnings profiles are usually provided for the three different education groups: college graduates, individuals with high school education but without a college degree, and finally individuals without high school education. The reason for this is the empirical evidence that income-profiles are significantly different for these education groups (see Hubbard et al. (1994) and Cocco et al. (2005)). In the present setting, however, agent groups are assumed to be of equal size within the model, contrary to the distribution of the education groups within the U.S. population.

[120] To be precise, when one of the income groups is subject to average lifetime income, i.e., it possesses a share in aggregate earnings of $1/M$, and life-cycle profiles of time preferences are identical across income groups, the agent type receiving average income will only be subject to inter-generational redistribution. This way, inter-generational redistribution effects can be observed separately, when studying this agent type.

[121] These data have recently been used by Gârleanu and Panageas (2015).

Table 4.2 – This table reports the coefficients for the third-order cohort labor income polynomial in model-age, $F_{t-i} = \bar{\phi} + \phi_1(t-i) + \phi_2(t-i)^2 + \phi_3(t-i)^3$, used in the quantitative analysis. The values depend on the estimates by Hubbard et al. (1994) for high school graduates and are adapted in order to normalize the sum over the earnings shares (incl. retirement income) of all agent types and cohorts to one, i.e., $\sum_{i=t-O}^{t} F_{t-i} + \lambda \sum_{i=t-N}^{t-O+1} F_O = 1$.

$\bar{\phi}$	ϕ_1	ϕ_2	ϕ_3
$1.1360 \cdot 10^{-2}$	$0.8776 \cdot 10^{-3}$	$-0.1151 \cdot 10^{-4}$	$-0.0174 \cdot 10^{-5}$

present study is to analyze the effect of transfers due to redistributive taxation, it seems reasonable to use labor input parameters that are defined generally before transfers, as provided by Hubbard et al. (1994).

The age-profile for retirement income, f_{t-i}^m for $O < t - i \leq N$, is flat. In particular, it is defined as exogenous and constant proportion λ (replacement ratio) of the labor income fraction received in the agents' last working-period. In this vein, aggregate retirement income is simply given as a constant fraction, λF_O with $F_{t-i} = \sum_{m=1}^{M} f_{t-i}^m$, of aggregate earnings H_t. Following Gomes et al. (2013), the replacement ratio λ is equally set to 40% for all agent types, which is consistent with the empirically documented median replacement rate from the U.S. social security system.

Based on the assumptions regarding the underlying labor income and retirement profiles, the specific definition of the deterministic earnings function can now be derived. Since labor income profiles across agent types are heterogeneous only with respect to the implied income levels, but are otherwise identical in shape, it is convenient to, first, define a deterministic earnings function on cohort level, i.e., $F_{t-i} = \sum_{m=1}^{M} f_{t-i}^m$. In a second step, the agent-specific earnings function f_{t-i}^m can then be deduced from this earnings function by scaling it with the group-specific share in aggregate earnings χ^m, where $\sum_{m=1}^{M} \chi^m = 1$. To this effect, it must be ensured that the amount of earnings paid to agents equals the amount of aggregate earnings available in every time step. Formally, this can be

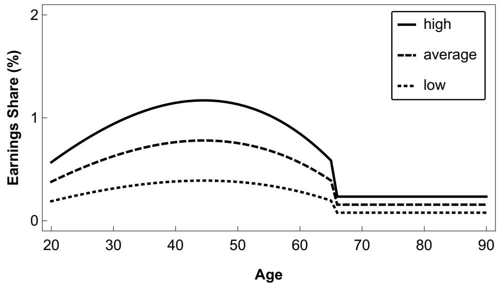

Figure 4.1 – This figure displays the life-cycle earnings profiles f^m_{t-i} for the three income groups (high, average and low income) considered for the baseline parameterization.

defined by the following condition (see Gârleanu and Panageas (2015)): $\sum_{i=t-O}^{t} F_{t-i} + \lambda \sum_{i=t-N}^{t-O+1} F_O = 1$, where the first term corresponds to the proportion of aggregate earnings paid out as labor income and the second term is the fraction of H_t distributed to retirees. In line with the definition of market equilibrium in Section 4.1.5, it just states that the sum over the earnings shares (incl. retirement income) of all agent types and cohorts must sum to one. Using this condition, normalized coefficients for the third-order cohort labor income polynomial reported in Hubbard et al. (1994) are determined. The corresponding values are reported in Table 4.2.

Next, it just remains to specify the group-specific shares in aggregate earnings χ^m. The levels of life-cycle earnings received by the different agent types are chosen to roughly reflect the proportions observed in the U.S. population, divided into three income groups of approximately equal size. In particular, the fraction of aggregate earnings χ^m received by the high income group is set to 50% (51% in the data), the average income group earns 33.33% (26% in the data) and the low income group has 16.67% (22% in the data) of aggregate earnings.[122] In this vein, the individual life-cycle earnings profiles are finally given by the following deterministic function:

[122] Source: "U.S. Census Bureau, Current Population Survey, 2016 Annual Social and Economic Supplement". Divided into three income groups of approximately equal size, the high income group comprises individuals with characteristic "Bachelor's Degree or more", the

$$f^m_{t-i} = \begin{cases} \chi^m F_{t-i} & if\ 0 \leq t-i \leq O, \\ \chi^m \lambda F_O & if\ O < t-i \leq N. \end{cases} \tag{4.107}$$

The resulting life-cycle patterns for the three income groups are illustrated in Figure 4.1. Section 4.3.4.1 considers the impact of different levels of income disparity.

4.3.1.2 Subjective Time Discount Factors

The setting of the baseline parameterization is finally completed by establishing the specification for the individuals' time preferences. As pointed out above (see Section 2.1.3), there is unambiguous evidence that patience alters with age and across income levels. In case of the latter, particularly there also exists a high level of conformity across several studies regarding the relationship, indicating that patience is higher for high income households than for low income households. With respect to the influence of age, however, the existing evidence is ambiguous.

As discussed in Section 2.1.3, although some studies report a positive relationship, most studies find a negative influence of aging on individual patience. That is, patience is found to be typically higher for young adults than for elderly individuals. This relationship is explained by declining mental and physical abilities, decreasing fertility and increasing mortality or generally shortened remaining lifetime with age. Beyond that, however, there is also inconsistency with respect to the relation between discounting of middle-aged individuals and discounting of young adults. On the one hand, building on the mentioned factors, a number of studies predict that discounting is a monotonic function of age, implying patience to be

---

average income group contains the categories "Some College No Degree" and "Associate Degree", and finally the low income group consists of the categories "Less Than 9th Grade", "High School: 9th to 12th Nongrad" and "High School: Graduate (Incl GED)".

Table 4.3 – This table reports the coefficients for the second-order subjective time discount factor polynomial in model-age, $\delta_{t-i} = \bar{\varphi} + \varphi_1(t-i) + \varphi_2(t-i)^2$, and the mean discount factor over the life-cycle implied by this estimate.

$\bar{\varphi}$	φ_1	φ_2	Mean
0.96	$0.4886 \cdot 10^{-2}$	$-0.9429 \cdot 10^{-4}$	0.9759

highest for young adults and constantly declining over the life-cycle (see Trostel and Taylor (2001) and Booij and van Praag (2009)). On the other hand, numerous studies find a hump-shaped pattern of subjective time discount rates over the life-cycle. That is, patience increases until middle age and declines afterwards. In this vein, elderly individuals display the highest degree of impatience, whereas middle-aged individuals are considered most patient (see Harrison et al. (2002), Sozou and Seymour (2003), Read and Read (2004), Chu et al. (2010) and Kageyama (2013)). The reasoning behind this is that, first, young people lack the experience to correctly evaluate uncertainty and, hence, are better off consuming today than postponing benefits. With growing age, individual behavior gets more secure and uncertainty loses its effect on discounting. Therefore, patience goes up early in life. Second, and in line with the other studies mentioned earlier, fertility falls and mortality rises with age at an increasing rate which has an increasingly negative effect on patience over the life-cycle. At a certain age, the second effect starts to dominate the first one, so that in the aggregate the net effect is the indicated hump-shaped age-pattern of discounting. The baseline parameterization follows this reasoning.

Furthermore, since the analysis presented in the previous chapter already focused on heterogeneity in patience across agent types, the following analysis will concentrate on heterogeneity over the life-cycle. Hence, the age-profiles of time preferences will be assumed to be identical across agent types in the baseline parameterization. The impact of different settings for the subjective time discount factors is studied in Section 4.3.4.2.

Then, it remains to formally specify the age-profile of patience. Since the studies presented above all depend on a multitude of different assumptions, the values reported in them are not directly suitable for the present setting (see Read and Read (2004)). The adaptable finding from these contributions is, therefore, limited to the documented relation between age and time preferences.

In order to formalize this relation for the present analysis, a simple second-order polynomial is proposed. In this context, it is noteworthy that the model structure implied by an overlapping generations framework allows for values of the subjective time discount factors that are larger than one (see, for example, Constantinides et al. (2002)). Beyond that, since the individuals' marginal rates of substitution over consumption are actually found to be independent of δ_{t-i} (see Equation 4.61), flexibility in the choice of subjective time discount factors is even larger for the present setting. To be more precise, even for extreme (high) values of δ_{t-i}, individuals might still discount the future, as typically presumed within the standard discounted utility model (see Loewenstein and Prelec (1991) and Frederick et al. (2002)).

Finally, due to the influence of time preferences on aggregate economic growth (see Section 4.2.1.2), the reasonable objective pursued by quantifying δ_{t-i} is to help to match the empirically observed GDP growth rate within the model, while replicating the empirically documented hump-shaped pattern of consumption over an individual's life-cycle. In so doing, the values for the subjective time discount factors will be limited to lie within the generally documented range of values, as documented by, for example, Frederick et al. (2002). Furthermore, it seems reasonable to model a mean subjective time discount factor that is in line with the values typically considered within the asset pricing literature. Based on these considerations, and in order to ultimately estimate the second-order polynomial, the subjective time discount factor is set to 0.96 for newborn agents, reaches its maximum value at model age 20 with 1.02 and drops

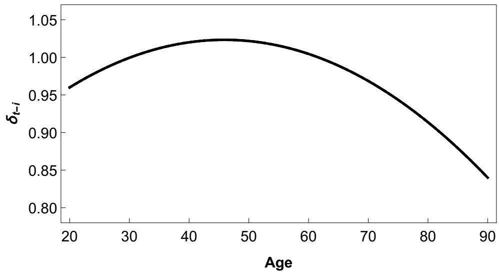

Figure 4.2 – This figure displays the estimated life-cycle profile of the subjective time discount factor δ_{t-i} for the baseline parameterization.

to 0.84 until model age 70.[123] The resulting coefficients for the second-order subjective time discount factor polynomial are reported in Table 4.3. The mean discount factor implied by this estimate is about 0.98, which is well within the range of values usually considered in the asset pricing literature. The implied life-cycle pattern is displayed in Figure 4.2. The baseline parameterization is thus complete.

4.3.2 Baseline Results

In this section, the quantitative model results are presented that follow from the described baseline parameterization. In particular, first, aggregate economic statistics for the model economy are outlined and compared to empirical data. This comprises growth rates, returns and aggregate tax revenues. Second, model implied individual life-cycle profiles are displayed and discussed with respect to the stylized facts documented in other asset pricing and portfolio choice studies. This includes among other things consumption, wealth and income as well as savings and investment life-cycle profiles. Finally, measures for consumption and wealth disparity are presented.

[123] As explained above, this is in line with stylized facts documented within the relevant literature (see Harrison et al. (2002), Sozou and Seymour (2003), Read and Read (2004) and Chu et al. (2010)).

Table 4.4 – This table shows aggregate economic statistics for the model and U.S. data. Panel A reports the share of consumption and investment in aggregate output for the model under baseline parameterization and aggregate U.S. data taken from Gomes et al. (2013). The documented data are adapted to correspond to the setting without government consumption in the model. Panel B reports the mean and standard deviation of GDP for the model under baseline parameterization and the corresponding historical U.S. data (Source: U.S. Bureau of Economic Analysis, National Income and Product Accounts, "Table 1.1.1. Percent Change From Preceding Period in Real Gross Domestic Product ", annual data, 1930-2016).

Panel A: Share of Output (in %)		
	Model	Data
Consumption	70.87	74.56
Investment	29.13	25.44
Panel B: Moments GDP Growth (in %)		
Mean	2.97	3.34
Std. Dev.	4.88	4.91

4.3.2.1 Aggregate Economic Statistics

As outlined above, the parameterization is chosen in order to help to match the first two moments of GDP growth observed empirically. Table 4.4 reports the resulting shares of consumption and investment in aggregate output (Panel A) as well as the mean and standard deviation of GDP growth (Panel B) for the model economy and the corresponding historical U.S. data. The model matches the documented empirical counterparts fairly well. The investment (consumption) share of output is slightly higher (lower) than reported in the data, while the moments of GDP growth predicted by the model are somewhat lower than the observed long term historical U.S. data, but well within the range of observed values.

Next, Table 4.5 reports the asset pricing moments for the baseline model economy and compares them to the empirical U.S. data. The values reveal the apparent problem of the model to match the historically documented figures given the parameterization chosen in the baseline case. The annual

Table 4.5 – This table shows annual asset pricing moments implied by the model economy under baseline parameterization and empirically observed U.S. data taken from Gomes et al. (2013).

Net Returns / Premium	Model	Data
$\tilde{R}_f - 1$	3.20	1.58
$\mathbb{E}\left[\tilde{R}_E\right] - 1$	3.63	8.31
$\sqrt{Var\left[\tilde{R}_E\right] - 1}$	3.02	19.81
$\mathbb{E}\left[\tilde{R}_E\right] - \tilde{R}_f$	0.43	6.74

net risk-free return is double, whereas the annual net risky asset return is only half of its observed empirical counterpart. This results in an annual equity premium that is negligible compared to the reported historical U.S. data, a phenomenon well-known to the literature as the equity premium puzzle, which is ascribed to Mehra and Prescott (1985) (see Section 2.2.2.1). It is caused by the assumptions of standard CRRA preferences and the aggregate stochastic process underlying the economy. From Table 4.5 the particular reason becomes obvious. The standard deviation of equity return is just a small fraction of its empirical counterpart. The reason for this is that in the present model it follows directly from the assumption regarding the standard deviation of GDP growth (see Equation (4.13)). Since the growth rate of output, however, does not vary that strongly, the model can match either one of these figures.

The tax rates for baseline parameterization were chosen in order to match the corresponding shares of tax revenues in GDP observed in the data. To this end, the flat tax rate on labor income τ_l was set to 10% and the flat tax rate on capital gains τ_c was set to 40%. Table 4.6 presents the shares of tax revenues for the model under the baseline parameterization and the corresponding empirical values reported by Gomes et al. (2013). As intended, the model implied values are very close to the documented U.S. data.

Table 4.6 – This table shows the percentage share of tax revenues in GDP by source from the model under baseline parameterization and the corresponding empirical aggregate U.S. data taken from Gomes et al. (2013). The labor value for the model includes tax revenues from aggregate earnings, i.e., labor and retirement income.

Source	Model	Data (1929-2010)
Labor	7.00	6.80
Capital	5.12	5.19

4.3.2.2 Life-cycle Profiles

Figure 4.3 (top panel) plots the mean life-cycle profiles of consumption, wealth and income after tax for the average agent (type 2) under baseline parameterization.[124] In order to derive the profiles, aggregate output (Y) is normalized to one at real-life age 20. Before and including age 65, income corresponds to labor income, whereas from that age onward, it refers to retirement income. The resulting life-cycle profiles are in line with the patterns reported in other general equilibrium (see Gomes and Michaelides (2008)) and portfolio choice (see Cocco et al. (2005), Gomes and Michaelides (2005) and Fischer et al. (2013)) models. As in these studies, the consumption profile shows the empirically well-established hump-shaped pattern (see, for example, Gourinchas and Parker (2002) and Fernández-Villaverde and Krueger (2007)).

The origin of this pattern can be found in Figure 4.3 (mid panel). It shows that individual consumption growth is also hump-shaped over the agent's lifespan. In particular, mean consumption growth is positive until around age 75 and drops sharply from that age onward. This development carries over to the consumption profile illustrated in the top panel. As outlined in Section 4.2.2.2, the consumption growth profile is mainly shaped by the underlying life-cycle profile of the subjective time discount factor

[124] Since the different agent types are identical except for their income levels, the figures for the other two agent types draw a qualitatively similar picture. Therefore, the presentation of the life-cycle profiles in Figure 4.3 focuses on the average agent type.

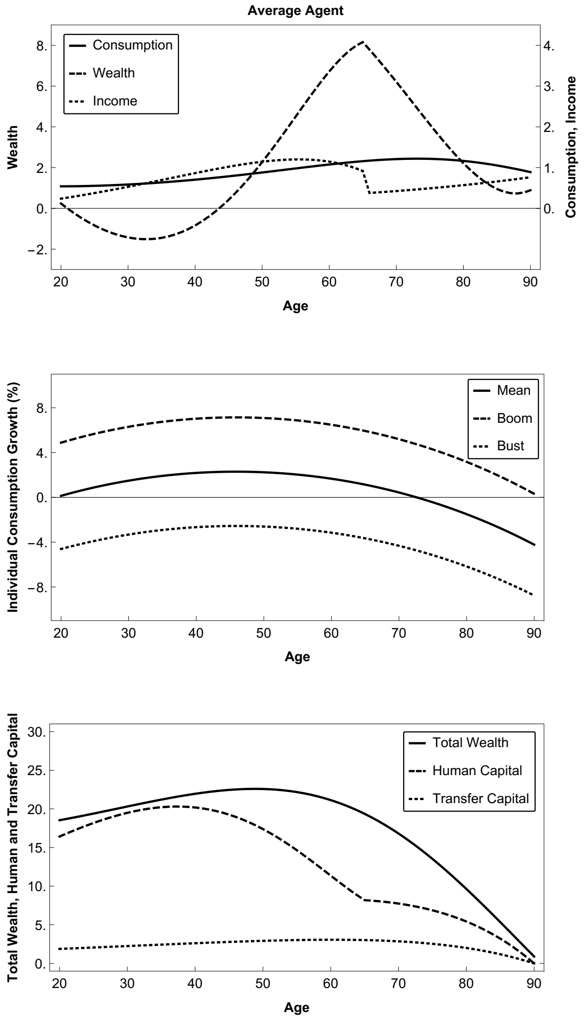

Figure 4.3 – This figure shows mean life-cycle profiles for the average agent (type 2) under baseline parameterization. The top panel displays the model implied mean life-cycle profiles of consumption, wealth and income after tax. Before and including age 65, income corresponds to labor income after tax, whereas from that age onward, it refers to retirement income after tax. The mid panel plots mean individual consumption growth (in %) as well as the corresponding values for the boom and bust scenario over the lifespan. The bottom panel depicts the evolution of mean total wealth, human and transfer capital with age. For the profiles, aggregate output (Y) is normalized to one at age 20.

(see Equation (4.73)). In this vein, the hump-shaped consumption pattern originates mainly due to the underlying profile of time preferences in the present setting.[125]

Since individuals are not borrowing constraint and labor income is tradable through the risky security in the present setting, agents can borrow against their human capital. Therefore, as can be seen from Figure 4.3 (top panel), individuals borrow and hold negative wealth in the first half of their working life, when human capital is still large (see bottom panel). At around age 35, the present value of future earnings starts to decrease and individuals begin to reduce outstanding debts. At around age 45 debts are fully repaid and agents start to save for retirement. From then on, wealth accumulation increases until retirement, from which point on it starts to fall again to finance retirement consumption. The wealth patterns reported by the previously mentioned studies are basically similar, although most of them assume borrowing constrained investors, which implies wealth levels that are positive throughout the whole life-cycle. A notable exception is Cocco et al. (2005) who also find negative wealth levels during working life when considering endogenous borrowing constraints.

Figure 4.4 (left panel) plots the life-cycle profiles of the share of total, equity and bond investment in aggregate investment for the average agent (type 2) under baseline parameterization. In line with the observations for the wealth profile, total investment is negative in the first half of the working life, it becomes positive at around age 45, increases thereafter in order to build up retirement wealth and falls immediately after retirement. Agent's equity investment shows a similar life-cycle pattern, although the peaks are more heavily pronounced. As shown in Section 4.2.2.3, the analytical solution implies that higher current and expected discounted future permanent incomes negatively affect the equity investment decision. Since most

[125] The related literature finds variation in patience over the life-cycle (mostly due to increasing mortality risk) also to be a main driver of the hump shape of the consumption profile. Moreover, it reports liquidity constraints and changes in earnings with age as other relevant factors (see Cocco et al. (2005), Gomes and Michaelides (2008) and Fischer et al. (2013)).

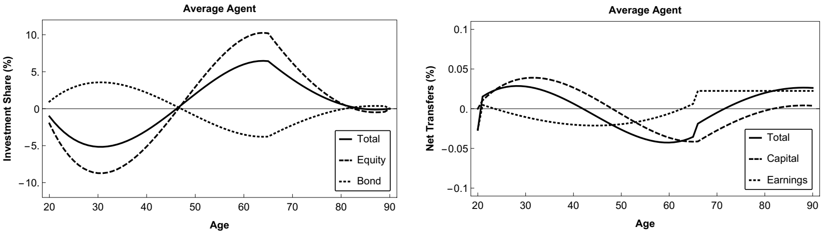

Figure 4.4 – This figure shows investment as well as net transfer life-cycle profiles for the average agent (type 2) under baseline parameterization. The left panel plots the agent's share (in %) of total, equity and bond investment in aggregate investment over his life-cycle. The right panel displays mean net transfers received in percent of aggregate production output over the lifespan, where total transfers are also split up into transfers financed by capital and earnings.

of the earnings and transfers are still to be received, human and transfer capital are large early in life, as displayed by Figure 4.3 (bottom panel). Consequently, holdings in the risky security are lowest when agents are young, including substantial short positions. This observation is in line with Benzoni et al. (2007) who also find that agents should optimally take short positions in the stock market when young. According to the reasoning given here, they also argue that most of the young investor's wealth is still tied up in future labor income. Because of cointegration between labor income and dividends, young agents are overexposed to market risk. In order to compensate this, they take short positions in the stock market.

The holdings in the risk-free bond show a contrary pattern. During the first half of working life positions are positive, they turn negative at around age 45, decrease until retirement and rise immediately thereafter. As found in Section 4.2.2.4, the agent's bond investment policy is driven by the net transfer payments financed by capital taxation. This relationship can be deduced from Figure 4.4 (right panel). In line with the findings from the analytical solution, net recipients of capital transfers enter into long positions on the bond market, whereas net contributors take short positions in the risk-free security. This is the case, because the transfer mechanism, besides redistributing incomes, also redistributes macroeconomic risk from net contributors to net recipients of capital transfers. As a result, net recipients already possess a relative high exposure to macroeconomic risk. In order to compensate this, they enter into long positions on the bond market and decrease their positions in the risky security. The opposite holds true for net contributors.

As explained above, under the assumption of homogeneous life-cycle profiles of earnings and patience, the agent with average income level will only be subject to inter-generational redistribution (see Footnote 120). Therefore, Figure 4.4 (right panel) illustrates the way the redistributive taxation system reallocates incomes across cohorts. Individuals receive positive (inter-generational) net transfers around the first and the last quarter of their lives. The reason for this is that early in life, financial and non-financial income is still low, whereas late in life retirement income is low again and a substantial fraction of retirement savings has already been consumed. These transfers are financed by the middle-aged that are subject to high labor income and have build up substantial financial asset holdings.

In case of the two other income groups, the life-cycle profiles given in Figure 4.4 are quiet similar.[126] Nevertheless, the levels of investment are much larger for high income agents, whereas they are smaller for the low income agents, as one would expect. Beyond that, for the former, net transfer income is negative almost over the whole lifespan, turning them into net contributors of the taxation system. In contrast, low income agents are net recipients since they receive positive net transfers throughout their entire lifetime.

[126]The life-cycle profiles for low (type 3) and high (type 1) income agents are shown in Figures A.1 and A.2 in the Appendix A, respectively.

4.3.2.3 Consumption and Wealth Disparity

This section closes the presentation of the baseline results by looking at the distribution of consumption and wealth across individuals. Since the taxation system is built to reduce consumption disparity and since only consumption provides agents with utility, examining the consumption inequality implied by the model is of special interest. Table 4.7, therefore, reports the percentage consumption shares corresponding to the different quintiles of the consumption distribution as well as the consumption Gini coefficient for the model under baseline parameterization and the relevant empirical U.S. data reported by Krueger et al. (2017). Overall, the model matches the empirically documented consumption distribution pretty well. Although the consumption Gini coefficient implied by the model (0.28) is somewhat lower than the value reported by Krueger et al. (2017) using U.S. data from the 2006 Consumer Expenditure Survey (0.36), it is still within the range of reasonable values reported by other studies. For instance, Krueger and Perri (2006) find a consumption Gini coefficient of around 0.25 between 1980 and 2003, also building on the data from the Consumer Expenditure Survey.[127]

Given the assumption of homogeneous preferences across agent types, the consumption Gini coefficient conditional on age is constant (0.30) in the baseline parameterization. This is contrary to empirical findings that report growing consumption disparity with increasing age (Krueger and Perri (2006)). In the present model this can be achieved, by assuming heterogeneity in the subjective time discount factors across agent types, as for example in Gomes et al. (2013).[128]

Next, turn to the distribution of wealth within the model. In line with the qualitative findings in the relevant literature, wealth inequality in the

[127] See also the discussion in Section 2.1.1.

[128] Section 4.3.4.2 studies the impact of heterogeneous subjective time discount factors across agent types.

Table 4.7 – This table reports the percentage total consumption fractions per consumption quintile and the consumption Gini coefficient for the model under baseline parameterization and U.S. data taken from Krueger et al. (2017) using data from the 2006 Consumer Expenditure Survey.

	Q1	Q2	Q3	Q4	Q5	Gini
Model	8.0	12.9	18.0	25.4	35.6	0.28
Data	6.5	11.4	16.4	23.3	42.4	0.36

model is much larger than consumption inequality (see Section 2.1.1). Nevertheless, the present setting overstates the wealth disparity across agents heavily by implying a Gini coefficient larger than one, compared to a wealth Gini coefficient of around 0.80 observed empirically (see Krueger and Perri (2006) and Favilukis (2013)). The reason for this is that individuals are not borrowing constraint and permanent income is tradable through the two financial securities. Therefore, agents borrow against their present value of future incomes and hold high amounts of negative wealth when young. As a consequence, the percentage fraction of agents with zero or negative wealth holdings within the model (33, 3%) is about twice as high as its empirical counterpart (18.6%).[129] This causes the unreasonably high wealth Gini coefficient under the baseline parameterization. The impact of different parameter settings will be subject of the following sections.

4.3.3 The Impact of Redistributive Taxation

Having established the baseline results one can now turn to study the impact of different model parameterizations. Since the main objective of the present work is to examine the impact of redistributive taxation in general equilibrium, analyzing the implications of varying tax rates is, obviously, of special interest. Building on the general equilibrium overlapping gener-

[129]The percentage of negative wealth holdings is taken from Wolff (2010) based on the data from the 2007 Survey of Consumer Finances.

ations model of the present chapter, the current section gives attention to this central question. In particular, keeping the remaining specifications as presented in Section 4.3.1, the following sections investigate quantitatively the influence of varying labor and capital tax rates on macroeconomic quantities (like the investment fraction and production growth), on individual life-cycle profiles (of consumption, wealth and investment), on consumption and wealth disparity as well as on social and individual welfare.

4.3.3.1 Macroeconomic Effects

As shown in Section 4.2.3, the equilibrium model solution depends basically on the distribution of current and expected present value of future permanent income across the agents currently populating the economy. A shift of permanent income from one type of agents to another, or across cohorts, changes the equilibrium condition and, consequently, leads to a different solution for the endogenous parameter ν. As a result, the redistributive taxation of both capital as well as labor income affect individual behavior. As outlined above, this is a consequence of the overlapping generations framework and the assumption of heterogeneous preferences. Moreover, since aggregate production output is endogenous to the model, individual decisions further determine aggregate economic development, making them also dependent on the tax rates.

The present section investigates the latter relationship by studying the quantitative effects of varying tax rates on macroeconomic quantities. In particular, Figure 4.5 depicts the impact of redistributive labor (τ_l) and capital (τ_c) taxation on the endogenous model parameter (ν, top left panel), the aggregate investment share (X, top right panel), expected economic growth (bottom left panel) and the annual standard deviation of economic growth (bottom right panel).

The top left panel of Figure 4.5 illustrates the relationship between the tax rates and the parameter ν. Recall that the analytical solution concluded

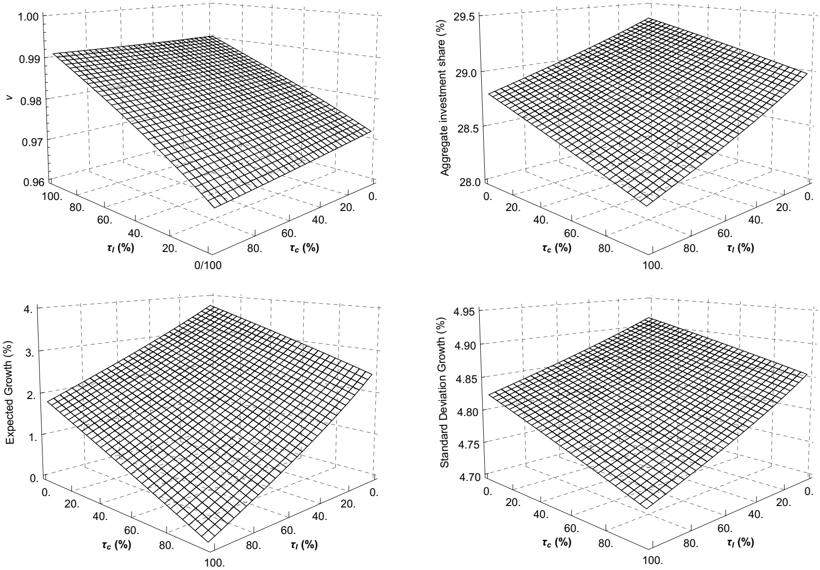

Figure 4.5 – This figure shows the impact of varying labor (τ_l) and capital (τ_c) tax rates on the endogenous model parameter (ν, top left panel), the aggregate investment share (X, top right panel), expected economic growth (bottom left panel) and the annual standard deviation of economic growth (bottom right panel).

that labor taxation has no influence on the stochastic discount factor (see Section 4.2.1.3). Therefore, any equilibrium effect of the tax rate τ_l is only due to the influence of different income distributions across agents. Figure 4.5 (top left panel) depicts that, with growing tax rates on labor income, permanent income is reallocated in a way that the ratio of consumption shares for the reference agent (type $m = 1$) between age zero and one (ν) is increasing. Being subject to a high level of labor income, this agent type is a net contributor to the transfer mechanism with respect to earnings during working years.[130] Consequently, his consumption shares decrease with higher tax rates τ_l when young. In particular, the consumption share

[130]The fact that high income agents are net contributors has been described before in Section 4.3.2.2. Moreover, Figure A.2 in the Appendix A illustrates that agent type 1 is a net contributor to the transfer mechanism with respect to earnings during the entire working life.

when newborn is reduced more strongly than in the subsequent age of life, so that the endogenous parameter ν increases with labor taxation.

The relationship between capital taxation and the relevant ratio of consumption shares is different. Since it reduces capital income, while increasing transfer payments, the tax rate τ_c affects the equilibrium solution through a reallocation effect, too. Capital holdings are zero or negative in the beginning of the individuals' life-cycle, independent of the agent type. Therefore, net transfers from capital are generally positive for young individuals within the present setting and consumption shares increase with the tax rate τ_c. In contrast to labor taxation, however, there is also a second effect of capital taxation. More precisely, under the given parameterization, taxing capital decreases individual consumption growth (see Equation (4.73)) and, hence, increases the individual's marginal rate of substitution between current and future consumption. The price of future consumption raises relative to the price of current consumption and individuals increase present consumption at the cost of future consumption. Overall, both effects lead to an endogenous parameter ν that decreases with increasing τ_c. The effect weakens for increasing labor tax rates τ_l.

The top right panel of Figure 4.5 illustrates the relationship between the tax rates and the aggregate investment share (X). The analytical solution (see Equation (4.57)) has shown that X depends on the labor tax solely indirectly through the reciprocal of the endogenous parameter. Further, and as just described, ν is positively linked to the tax rate τ_l. Therefore, it follows that the aggregate share of output that is reinvested into the production technology decreases with growing labor taxation. Again this effect is only due to the equilibrium influence of different income distributions across agents.

The effect of capital taxation on the aggregate investment share takes place through two channels. First, X also depends on τ_c indirectly through the reciprocal of the endogenous parameter. Since ν decreases (for almost all labor tax rates) with increasing capital taxation, this has a positive effect on the aggregate investment share. Second, as shown in Equation (4.57), X is also affected directly by τ_c through the reciprocal of the constant G_1. For the given parameterization, this constant is positively correlated to the capital tax rate implying a negative effect on the aggregate investment share. The second effect dominates the first one with the result that the aggregate investment share also decreases with growing capital taxation. Overall, this relationship can yet again be reasoned by the influence of τ_c on the stochastic discount factor. As explained above, the marginal rate of substitution between current and future consumption is positively linked to the capital tax rate and, hence, individuals increase present consumption at the cost of future consumption. Since this relation holds for all individuals, it translates onto macroeconomic level. Consequently, current aggregate consumption raises while current aggregate investment drops.

Overall, the present model predicts an investment share on economic level that is negatively related to taxation in general. This observation is in line with the empirical evidence in Blanchard and Perotti (2002) and also carries over to the influence of tax rates on (expected) macroeconomic growth rates, as depicted in the bottom left panel of Figure 4.5. The analytical solution in Equation (4.55) describes a positive linear relationship between the aggregate investment share and aggregate production growth. As a result, since aggregate investment is negatively linked to taxation, GDP growth decreases with both capital and labor taxation.[131] Although the influence on the level of aggregate investment is small, the effect on growth rates is quantitatively substantial. Without any redistributive taxation, annual economic growth is around 3.7%. Implementing a tax rate on labor income of $\tau_l = 40\%$, it decreases to 2.9%, while introducing a capital tax of $\tau_c = 40\%$ leads to a reduced growth rate of 3.2%. Levying both taxes simultaneously, the effects aggravate and economic growth is in the most extreme case ($\tau_l = \tau_c = 100\%$) eventually eliminated completely. Finally,

[131] Equation (4.60) shows that aggregate growth rates are identical in the present model setting. Thus, the described tax influence also holds true for aggregate consumption and investment growth.

the bottom right panel of Figure 4.5 depicts the relation between the standard deviation of economic growth and taxation. With growing tax rates, growth risk is reduced slightly, although it stays at a considerably high level throughout.

The described observations are in line with the results reported in a variety of theoretical and empirical studies. In particular, based on theoretical model approaches, Judd (1985), Chamley (1986), Jones et al. (1997) and Fischer and Jensen (2014) find that taxing capital income reduces aggregate production. The present study extends this result to the taxation of non-capital income. Furthermore, there is broad empirical evidence that confirms the described negative relation between taxation and economic growth in general, as for example Blanchard and Perotti (2002), Romer and Romer (2010), Barro and Redlick (2011), Cloyne (2013) and Mertens and Ravn (2013).

4.3.3.2 Individual Life-cycle Effects

As outlined above, a reallocation of permanent income between agents changes the equilibrium solution. Consequently, redistributive taxation of both capital as well as labor income affects individual behavior. The present section investigates this relationship by studying the quantitative effects of varying tax rates on the individual life-cycle profiles of consumption (Figure 4.6), wealth (Figure 4.7) as well as investment and net transfers (Figure 4.8). The left panels of Figures 4.6 - 4.8 depict the influence of different labor tax rates $\tau_l = \{0, 0.1, 0.4\}$, while the right panels illustrate the impact of different capital tax rates $\tau_c = \{0, 0.2, 0.4\}$.

The analytical solution has shown (see Equation (4.80)) that the individual consumption profile is affected differently by the two taxation types. First, the taxation of labor income merely affects the individual consumption level, but not the shape of the life-cycle pattern. Second, and in contrast, the taxation of capital gains affects both the level as well as the life-cycle

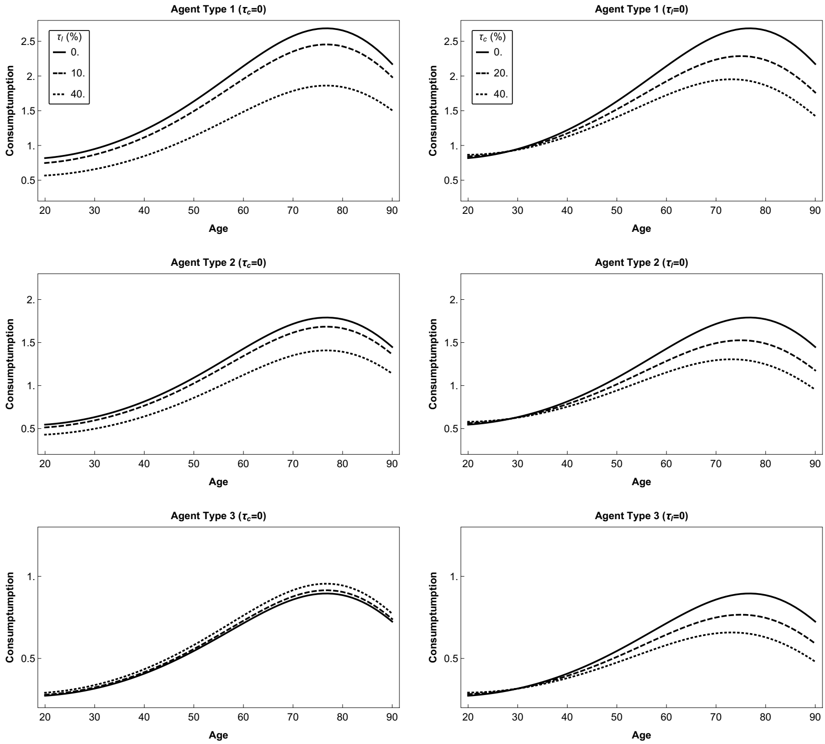

Figure 4.6 – This figure shows the impact of varying labor (τ_l, left panels) and capital (τ_c, right panels) tax rates on the mean life-cycle consumption profiles for the three agent types. The top panel depicts the model implied mean life-cycle profile of consumption for the high income agent (type 1), the mid panel displays the profile for the average agent (type 2) and the bottom panel for the low income agent (type 3). For the profiles, aggregate output (Y) is normalized to one at age 20.

pattern of consumption. These effects are illustrated in Figure 4.6. The left panels show the impact of different labor tax rates on the mean consumption profiles for the high income (type 1, top panel), the average (type 2, mid panel) and the low income (type 3, bottom panel) agent. In line with the analytical observations, the mean consumption levels change for varying τ_l, but the life-cycle patterns remain unchanged. In particular, taxing labor income leads to a reduction in the consumption levels for agents with high income, as one would expect. However, it also reduces the mean consumption level for the agent type receiving average income throughout the entire lifespan. There are two channels that lead to this effect. First, there is the redistribution effect that reallocates labor and retirement income across cohorts and agent types. This changes the amount and the timing of life-cycle earnings and, thus, agents' total wealth and, with it, consumption levels.[132] Second, the previous section has shown that there is a significant effect of taxation on macroeconomic quantities. Although aggregate consumption share in output increases slightly with growing τ_l, the decrease in production growth that accompanies the tax change is substantial. As a consequence, current and future aggregate consumption levels drop. Only for the low income agent the reduction in output is compensated by high net transfers leading to a higher mean life-cycle consumption level.

The right panels of Figure 4.6 show the impact of different capital tax rates on the mean consumption profiles for the high income (type 1, top panel), the average (type 2, mid panel) and the low income (type 3, bottom panel) agent. As described above, both the mean consumption levels as well as the life-cycle patterns change for varying τ_c. The qualitative effects are identical for all agent types. In particular, taxing capital leads to a substantial reduction in consumption from about age 30 until death, i.e., for most of the individuals' lifespan. Nevertheless, there is a slight increase in mean consumption before that, when agents are young. On the one hand, as explained above, increasing τ_c changes the individuals' marginal rate of substitution between current and future consumption. To be more precise, with higher capital tax rates individuals increase present consumption at the cost of future consumption, leading to the observed shift of consumption to early stages in life. On the other hand, production growth also declines with increasing τ_c. As a result, mean consumption levels drop considerably in later phases of life due to the reduction in aggregate output.

[132] As pointed out above, the average agent is only subject to inter-generational redistribution. Therefore, for him, reallocation of labor and retirement income only leads to a change in the timing of life-cycle earnings.

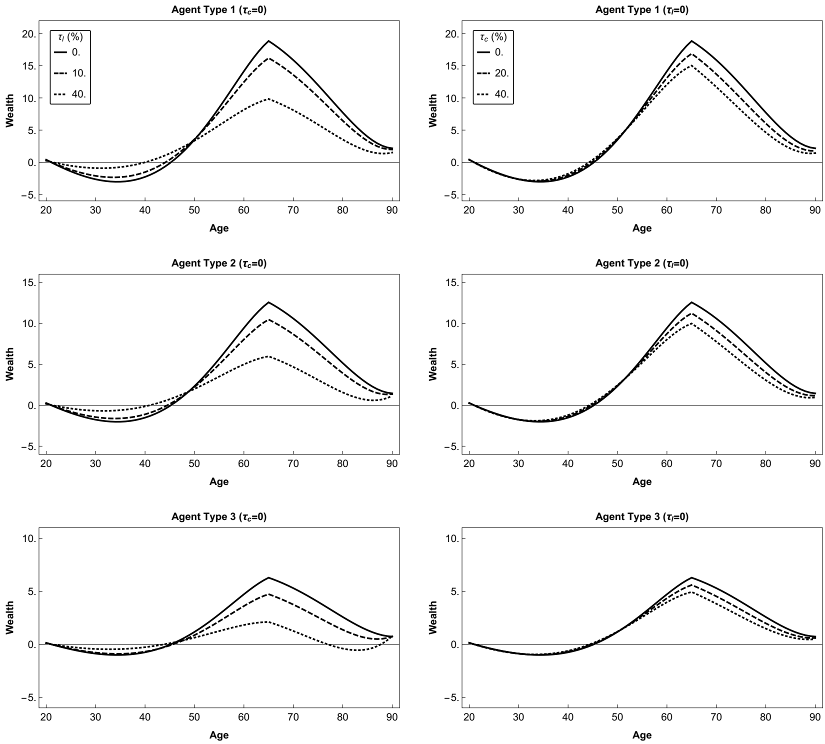

Figure 4.7 – This figure shows the impact of varying labor (τ_l, left panels) and capital (τ_c, right panels) tax rates on the mean life-cycle wealth profiles for the three agent types. The top panel depicts the model implied mean life-cycle profile of wealth for the high income agent (type 1), the mid panel displays the profile for the average agent (type 2) and the bottom panel for the low income agent (type 3). For the profiles, aggregate output (Y) is normalized to one at age 20.

Figure 4.7 plots the mean life-cycle profiles of individual wealth. The observed effects of redistributive taxation are qualitatively the same for high (type 1, top panels), average (type 2, mid panels) and low income agents (type 3, bottom panels). In case of labor taxation (left panels), borrowing is reduced in the early stage of the life-cycle, whereas capital taxation (right panels) first increases borrowing slightly before it decreases it. These effects are caused by the changes in mean consumption levels with growing tax rates, as explained above. Common to both taxation types is the reduction of wealth accumulation in later phases of life. Again, this is caused by the reduction in GDP growth that implies lower production output and, consequently, lower aggregate disposable wealth - especially with increasing age.

Finally, Figure 4.8 depicts the impact of varying tax rates on the life-cycle profiles of the share of equity and bond holdings relative to aggregate investment (top panels) and the received net transfer payments (bottom panels) for the average agent (type 2). When there is only labor taxation (left panels), agents only take positions in the risky asset and bond holdings are zero. Furthermore, in line with the wealth profile, borrowing early in life is reduced at the cost of lowering equity accumulation in the second half of the life-cycle. For the average agent, net transfer income from taxing labor income is negative through most of the working life, but positive in retirement. Varying the labor tax rate only changes the magnitude of net transfers, but leaves the age pattern unaffected.

In the presence of capital taxation (right panels), individuals start to trade the risk-free bond. As found in Section 4.2.2.3, the agent's bond investment policy is driven by the net transfers from capital, as can be gathered directly from Figure 4.8. In line with the analytical solution, net recipients of capital transfers enter into long positions on the bond market, whereas net contributors take short positions in the risk-free security. This is the case, because the transfer mechanism, besides redistributing incomes, also redistributes macroeconomic risk from net contributors to net recipients of capital transfers. Moreover, this also affects equity holdings. In particular, when capital transfers are positive, the additional risk exposure is further compensated by lowering the holdings in the risky security. The other way around, when capital transfers are negative, the implied reduction in exposure to market risk is compensated by increasing equity holdings. As a result, higher capital tax rates τ_c lead to asset positions that are more extreme in both directions.

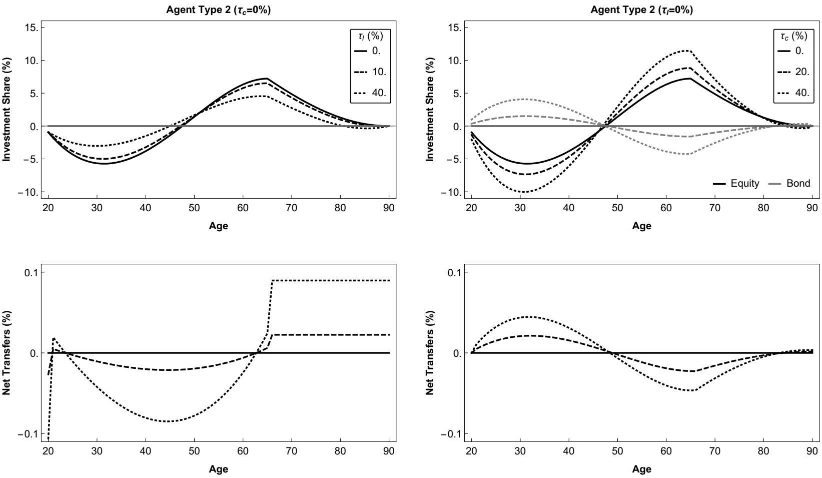

Figure 4.8 – This figure shows the impact of varying labor (τ_l, left panels) and capital (τ_C, right panels) tax rates on the life-cycle profiles of investment (top panels) and net transfers (bottom panels) for the average agent (type 2). For the investment profiles, the agent's share (in %) of equity is represented by the black lines, while the gray lines are his share (in %) of bond investment in aggregate investment.

In case of the two other income groups, the life-cycle profiles given in Figure 4.8 are quiet similar.[133] Nevertheless, as already described above, the levels of investment for high income agents are much larger, whereas they are significantly smaller for the low income agents. Furthermore, for the former net transfer income from labor taxation is negative for the whole working life, whereas low income agents receive positive net transfers from labor and retirement earnings throughout their entire lifetime.

4.3.3.3 Consumption and Wealth Disparity

In line with empirical observations, the baseline results presented in Section 4.3.2.3 have shown that the given model setting features substantial disparity in the distribution of consumption and wealth across agents. This

[133]The life-cycle profiles for high (type 1) and low (type 3) income agents are shown in Figures A.3 and A.4 in the Appendix A, respectively.

documented inequality has been the motivation of the present study and first raised the question of how a reallocation of incomes, in particular by implementing a redistributive taxation system, could influence this distribution of resources (see Section 2.1.1). The results derived so far have shown that the taxation mechanism, implemented to reduce the prevailing inequality, affects both macroeconomic evolution as well as individual life-cycle behavior substantially. The present section investigates whether the tax system achieves the fundamental objective of reducing disparities. Figure 4.9, therefore, illustrates the impact of redistributive labor (τ_l) and capital (τ_c) taxation on consumption (top left), wealth (top right) and equity (bottom) Gini coefficients relative to the case without taxation.

The top left panel shows how the tax rates affect the distribution of consumption across all agents alive at one point in time. The level of the labor taxation possesses substantial influence in reducing consumption disparity. At a tax rate of $\tau_l = 40\%$ ($\tau_c = 0\%$), the consumption Gini drops about 38%, while at a tax rate of $\tau_l = 80\%$ it is reduced by about 71% relative to the case without taxation. In case of capital taxation, however, the influence turns out to be contrary to the intended effect. With growing tax rate τ_c, the consumption Gini increases slightly relative to the no-tax setting. This is surprising, since the taxation system - independent of labor or capital taxation - was designed to reduce income inequalities with the objective to lower consumption disparities within the current population. Beyond that, it is also contrary to the findings in the related literature without overlapping generations (see Fischer and Jensen (2015)). Different in scope, however, the present setting, featuring overlapping generations, presents heterogeneity across agents due to individuals being situated in different phases of their life-cycle, which implies heterogeneity in the agents' consumption and investment behavior. Furthermore, the allocation of resources is decisive in determining the equilibrium. As pointed out above, in the present setting this implies that capital taxation reduces consumption growth and shifts consumption shares from older generations to individuals in earlier stages of life. It turns out that this results in

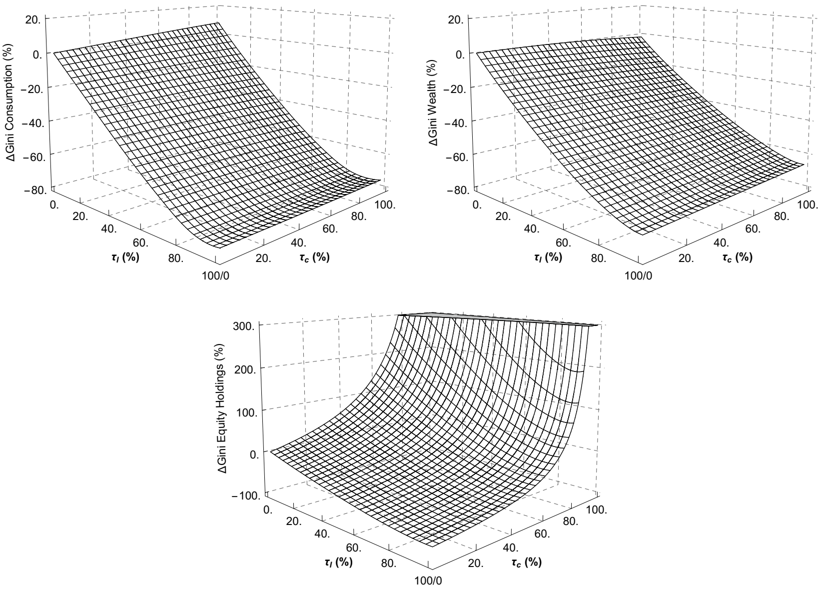

Figure 4.9 – This figure shows the percentage change of the consumption Gini coefficient (top left panel), the wealth Gini coefficient (X, top right panel) as well as the equity Gini coefficient (bottom panel) for varying labor (τ_l) and capital (τ_c) tax rates.

a cross-generational profile of consumption shares that gets steeper with increasing capital tax rates, implying higher inequality. Although there exists a weak positive impact of capital taxation on intra-generational consumption distribution, the overall effect is an allocation of consumption shares across individuals currently populating the economy that is more uneven.

The top right panel of Figure 4.9 depicts the impact of varying tax rates on the distribution of wealth within the current population. As in the case of consumption, labor taxation has a substantial effect on wealth inequality, while the influence of capital taxation is only marginal. Contrary to the observation above, however, is the fact that wealth disparity decreases with both tax rates. This is in line with the expected relationship, since current income (i.e., labor and retirement income after tax, capital gains after tax, and transfers) forms a substantial part of current wealth. Increasing tax rates equalizes current income across agents and consequently leads to a distribution of wealth that is more even. Nevertheless, it is noteworthy that the heterogeneity in consumption and investment behavior across generations implies that, even in the case of an absolutely equal income allocation, an equal distribution of consumption and wealth across all individuals is not derived. Since there is no intra-generational preference but only income heterogeneity in the present parameterization, an equal consumption and wealth distribution within cohorts (across agent types), however, is attainable by choosing an labor tax rate of $\tau_l = 100\%$.

Finally, the bottom panel of Figure 4.9 shows how redistributive taxation affects the distribution of equity holdings by plotting the equity Gini coefficient relative to the case without taxation. In line with the results for consumption and wealth disparity, inequality in equity holdings is decreased by imposing a tax rate on labor income and is further reduced by increasing it. In particular, and according to the observations for the life-cycle profiles of equity investment in Figure 4.8 (top left panel), labor taxation decreases both the short positions in the risky security in the first third of the lifespan as well as the accumulation of equity wealth in the second third of the agent's life. As a result, life-cycle patterns are less pronounced in either direction implying a substantial reduction in inter-generational equity disparity. Beyond that, the redistribution of labor and retirement earnings also implies a higher degree of equality in equity holdings across agent types.

The impact of capital taxation on the distribution of equity holdings lies in sharp contrast to this, as the disparity in equity holdings increases progressively in the capital tax rate. As depicted by the bottom panel of Figure 4.9, assuming a tax rate of $\tau_c = 20\%$ ($\tau_l = 0\%$) implies an increase in the equity Gini coefficient by about 25% relative to the case without taxation, while a tax rate of $\tau_c = 40\%$ leads to an increase by almost 65%. The reason for this is the described redistribution of macroeconomic risk from net contributors to net recipients of capital transfers. As explained above, when net capital transfers are positive, the additional risk exposure is also compensated by lowering the holdings in the risky security. The other way around, when capital transfers are negative, the implied reduction in exposure to macroeconomic risk is compensated by increasing equity holdings. Consequently, higher capital tax rates lead to life-cycle patterns of equity holdings that are more extreme in both directions, as shown in Figure 4.8 (top right panel). As a result, inter-generational equity disparity grows considerably.

Finally, it can be summarized that the taxation on labor income has the intended effect of reducing inequalities in the distribution of consumption among agents. Furthermore, taxing and redistributing labor and retirement income also lowers wealth and equity disparity. Capital taxation, however, only has a small damping effect on wealth inequality, while it even increases consumption inequality slightly. In case of the distribution of equity holdings, the results imply a progressive rise of the disparity in equity holdings with growing capital tax rates; a relation that might explain low stock market participation rates of poorer individuals (Fischer and Jensen (2015)). In general, however, the implementation of redistributive taxation - independent of whether it attains a more equal distribution of resources or not - has its costs. As shown in Section 4.3.3.1, production growth is negatively linked to both tax rates, implying a reduction in future production output with increasing taxation. In other words, redistributive taxation also affects the level of available future aggregate consumption, besides changing the allocation of them. This implies significant welfare effects on macroeconomic as well as individual levels, which are studied in the next section.

4.3.3.4 Welfare Effects

So far, the previous sections have investigated the quantitative effects of redistributive taxation on macroeconomic development, on individual behavior and on the distribution of resources across agents. Particularly, it was concluded that, besides affecting the resource allocation, taxing and reallocating incomes also affects the intertemporal distribution of resources on aggregate level due to changes in GDP growth. Nevertheless, considering the distribution of consumption goods and the disposable quantity on macroeconomic level does not bear meaning until one considers the fact that it ultimately provides individuals and, consequently, society with well-being. To quantify well-being, Section 4.2.4 has introduced the welfare measure in terms of the production equivalent. In particular, it has been defined to measure the change in current production for the no-taxation setting that is necessary in order to achieve equivalent expected utility levels from consumption in both settings. Based on these considerations, the present section investigates the impact of redistributive capital and labor taxation on, first, individual and, second, aggregate welfare.

Individual Welfare

Figure 4.10 shows the impact of varying labor (τ_l, left panels) and capital (τ_c, right panels) tax rates on individual welfare measured by the percentage change in the production equivalent. The top panels depict the implied welfare change for different generations of the high income agent (type 1), the mid panels display the welfare change for different generations of the average agent (type 2) and the bottom panels for different generations of the low income agent (type 3). The horizontal axis indexes the time of birth of an agent relative to the current period. Therefore, generation 0 corresponds to individuals being born in the current period. Furthermore, positive values correspond to the model age $(t-i)$ of individuals currently alive, while negative values indicate agents that will enter the economy in the future. This implies, for instance, that generation 20 corresponds to agents that entered the economy 20 time steps earlier and have a current real-life age of 40 years, whereas generation -10 comprises newborns of a generation that will be economically born in 10 periods.

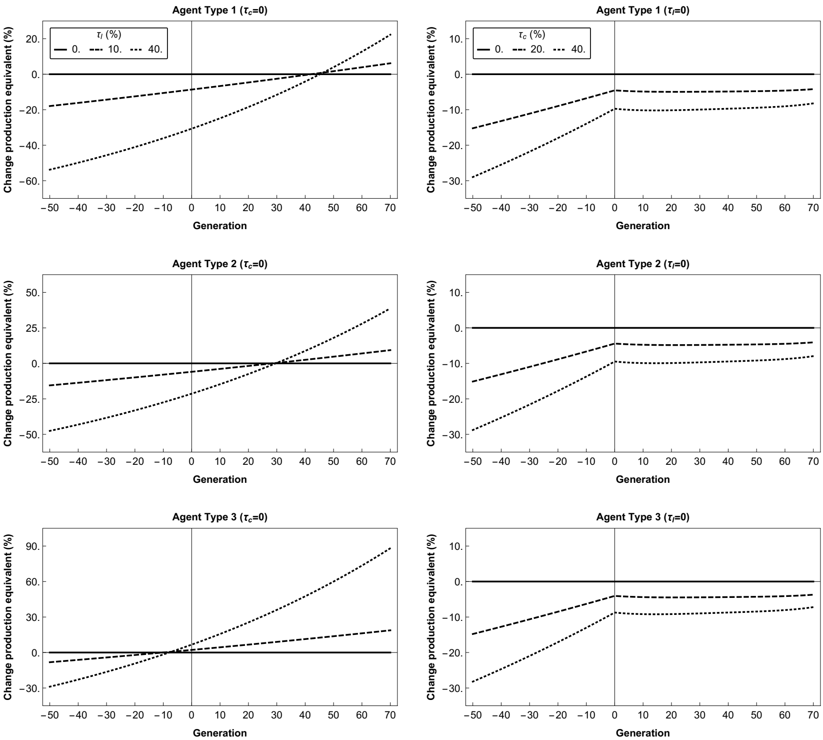

Figure 4.10 – This figure shows the impact of varying labor (τ_l, left panels) and capital (τ_c, right panels) tax rates on individual welfare measured by the percentage change in the production equivalent. The top panel depicts the implied welfare change for different generations of the high income agent (type 1), the mid panel displays the welfare change for diverse generations of the average agent (type 2) and the bottom panel for different generations of the low income agent (type 3). The generation on the horizontal axis indicates the time of birth of an agent relative to the current period. While positive values correspond to the model age ($t - i$) of agents currently living, negative values indicate agents that will enter the economy in the future.

For both labor and capital taxation the general relationship is the same for all agent types. Imposing a tax on income tends to negatively affect especially the welfare of future and younger generations, while it leads to welfare gains, or at least smaller welfare losses, for older generations. As described in Section 4.2.4.1, the reason for this lies in the fact that individual welfare is affected by the taxation system in two ways. On the one hand, the reallocation of incomes has a positive (negative) effect on the consumption of net recipients (contributors) and, consequently, on their welfare. On the other hand, redistributive taxation is negatively linked to production growth and, hence, reduces aggregate future consumption opportunities for all agent types. The latter leads to negative welfare effects especially for young and future generations, since most or all of their income is still to be received while lowered by imposing taxation, as explained above.

In the case of an labor tax, depicted in the left panels of Figure 4.10, redistributive taxation implies positive welfare effects for a number of different generations across income groups. This is due to the reallocation effect described above. As shown in Section 4.3.3.2, the redistribution mechanism favors retirees of all types, turning them into net recipients of earnings transfers. As a consequence, even for the high income agent (type 1) retired generations gain welfare. In case of the average agent (type 2) middle-aged generations are subject to positive welfare changes as well, whereas for the low income agent (type 3) all living generations and some unborn generations entering the economy in the near future have increased welfare. Nevertheless, all other generations are subject to negative welfare effects with losses increasing with declining age (or later time of birth). This is caused by the negative effect of redistributive taxation on aggregate future consumption opportunities.

Moreover, the changes in the welfare level are substantial and differ strongly across agent types. For example, for agent type 1 generation 70, an labor tax rate of $\tau_l = 40\%$ ($\tau_c = 0\%$) produces an increase in the production equivalent of 22.3%, while it creates an increase of 88.1% in the production equivalent for the same generation of agent type 3. In other words, this implies that in the absence of a redistributive taxation system an equivalent level of expected individual utility for agent type 1 (agent type 3) is achieved when the current level of production in this case, Y_t, was 22.3% (88.1%) higher than in the setting with an labor taxation of $\tau_l = 40\%$.

The right panels of Figure 4.10 consider the influence of capital taxation on the individual welfare levels of living and future generations. In contrast to labor taxation, taxing capital gains and redistributing revenues implies negative welfare effects for all agent types and generations. First, the reallocation effect favors those individuals that have small or negative equity holdings. As shown in Section 4.3.3.2, these are especially young workers in the first third of their lives. However, tax revenues from capital gains are smaller compared to revenues collected from labor and retirement earnings, implying a relatively weak impact of the reallocation effect. Second, the negative effect of the drop in future consumption opportunities affects younger agents stronger than middle-aged or elderly individuals which are mostly net contributors to capital transfers. Overall, this results in a quiet similar welfare loss for all living generations independent of the income group. For individuals of future generations, there is no additional positive reallocation effect with later time of birth, as it is the case for living agents. Nevertheless, they are also affected negatively by the reduction in GDP growth. This effect gets stronger with later birth dates. Therefore, the welfare level drops sharply for unborn generations independent of the income type.

Since the reallocation effect is quiet weak in case of redistributive capital taxation, the changes in the welfare level across agent types are only small. For instance, considering agent type 1 generation 70 again, a capital tax rate of $\tau_c = 40\%$ ($\tau_l = 0\%$) generates a decrease in the production equivalent of 8.2%, while it implies a drop of 7.2% in the production equivalent for the same generation of agent type 3. Put differently, an equivalent level of expected individual utility for agent type 1 (agent type 3) is achieved in the absence of a redistributive taxation system when the current level of production in this case, Y_t, was 8.2% (7.2%) lower than the current level of production in the setting with a capital tax rate of $\tau_c = 40\%$.

In general, redistributive labor taxation improves the individual welfare level of agents with relatively low labor or retirement income at the cost of reducing the welfare level of agents with relatively high earnings.

Nevertheless, due to the negative effect on future production output, these welfare gains of present generations are also at the cost of future generations that are subject to substantial welfare losses. Contrary, redistributive capital taxation produces negative welfare effects for all agent types and generations. In line with the findings in Fischer and Jensen (2014), this even implies losses in the production equivalent for net recipients of capital transfers.

Aggregate Welfare

Figure 4.11 shows the impact of varying labor (τ_l) and capital (τ_c) tax rates on social welfare measured by the percentage change in the production equivalent. The top panel depicts the change in social welfare considering all current and future generations. Since the previous results have shown that the impact of redistributive taxation on individual welfare levels of different generations is significantly diverse, the bottom panels split the social welfare measure into the change in the production equivalent considering only current (left panel) or future (right panel) generations.

In particular, Figure 4.11 depicts that the impact of capital taxation on individual welfare directly translates into the effects it has on aggregate welfare. Since it lowers individual welfare levels for all agent types and generations, social welfare (comprising current and future generations) as well as aggregate welfare for all living generations and, moreover, aggregate welfare for all unborn generations decline consistently with growing capital tax rates. Furthermore, in line with the above results, future generations are affected more heavily than current generations, implying stronger welfare losses for the former. For instance, while a capital tax rate of $\tau_c = 40\%$ ($\tau_l = 0\%$) generates a decrease in the production equivalent for living generations (bottom left panel) of 8.9%, it creates a decline of 14.1% in the production equivalent for unborn generations (bottom right panel). Combined, this leads to a drop of 9.3% in social welfare (top panel).

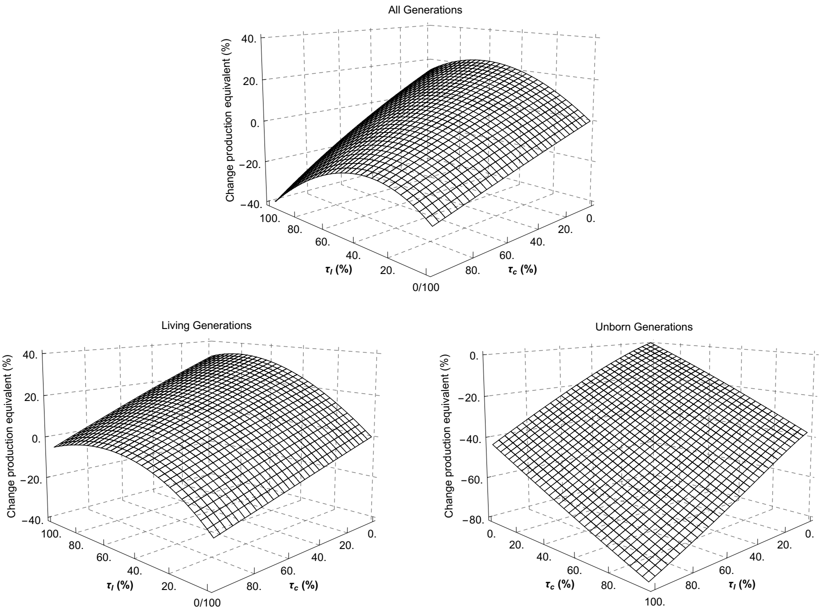

Figure 4.11 – This figure shows the impact of varying labor (τ_l) and capital (τ_c) tax rates on aggregate welfare measured by the percentage change in the production equivalent. The top panel depicts the change in social welfare considering all current and future generations, whereas the bottom panels display aggregate welfare change considering only current (left panel) or future (right panel) generations.

In case of redistributive labor taxation, the aggregate welfare effects are more complex. On the one hand, there are huge welfare gains for a number of agents of different generations, as explained above. On the other hand, taxing labor income and redistributing also has severe negative effects on the welfare level of especially young individuals and future generations. In combination, the top panel of Figure 4.11 shows that when tax rates are small, the positive reallocation effects dominate, implying a rise in social welfare with growing tax rates. The increase, however, turns out to be nonlinear and diminishes with growing labor tax rates. This indicates that the impact of the negative effects caused by declining production growth increases in the tax rate in comparison to the positive reallocation effect. As a matter of fact, the social welfare level eventually reaches its maximum at a tax rate of $\tau_l = 62\%$ ($\tau_c = 0\%$) implying a change in the production equivalent of 23.2%. That is, the same level of expected social welfare than in the setting with an labor tax rate of $\tau_l = 62\%$ is attained in the setting without redistributive taxation, when the current level of production in this case was 23.2% higher. With labor tax rates increasing further, social welfare starts to recede again implying that the negative effects caused by declining production growth ultimately begin to dominate.

Finally, on individual level it is almost exclusively living generations that are subject to positive welfare changes due to redistributive labor taxation. The bottom panels of Figure 4.11 translate this impact into aggregate welfare considering exclusively current (left panel) or future (right panel) generations. It follows straightforwardly from the results on individual level that imposing an labor tax generally increases aggregate welfare of living generations, while it consistently decreases aggregate welfare of unborn generations. This confirms the above observation that welfare gains of current generations are at the expense of future generations.

4.3.3.5 Summary

The previous sections have studied quantitatively the impact of varying labor and capital tax rates on macroeconomic quantities, individual life-cycle profiles, consumption and wealth inequality as well as individual and aggregate welfare. In the present section, the major findings elaborated above are briefly summarized. First, building on the baseline parameterization, Section 4.3.3.1 has investigated the influence of redistributive taxation on the aggregate development of the model economy. In this regard, it is found that the aggregate investment share is negatively related to taxation implying a GDP growth rate that decreases with both capital and labor taxes. Consequently, imposing redistributive taxation reduces future production output, thus lowering aggregate future consumption opportunities.

Second, Section 4.3.3.2 has examined the tax impact on individual quantities analyzing life-cycle profiles. The results imply that solely for the low income agent labor taxation has a positive effect on the mean life-cycle consumption profile due to high transfer incomes. Because of the negative effect on future production output, redistributive taxation affects life-cycle consumption negatively in any other case. Regarding individual investment behavior it is found that imposing an labor tax dampens both borrowing as well as wealth accumulation. Moreover, in the absence of capital taxation, agents only take positions in the risky asset and bond holdings are zero. In order to compensate the reallocation of macroeconomic risk that comes along with redistributive capital taxation, however, individuals start to trade the risk-free bond. Particularly, net recipients of capital transfers enter into long positions on the bond market and reduce their holdings in the risky security, whereas net contributors take short positions in the risk-free security and increase their equity holdings. Since this reduces the equity positions of individuals that already hold little equity wealth, this effect might help to explain the empirically documented low stock market participation rates of relatively poor individuals (Fischer and Jensen (2015)).

Third, the influence of varying labor and capital tax rates on consumption and wealth disparity has been studied in Section 4.3.3.3. In line with the results on individual life-cycle profiles, redistributive labor taxation is found to reduce inequalities in the distribution of consumption, wealth and equity holdings within the current population. In contrast to this, redistributive capital taxation only produces a small reduction in wealth inequality. Beyond that, however, it even increases consumption disparity and, moreover, raises inequality in the distribution of equity holdings sharply.

Fourth, Section 4.3.3.4 has investigated the welfare effects of redistributive taxation. In general, within the current population, labor taxation improves the individual welfare level of agents with relatively low labor or retirement income at the cost of reducing the welfare level of agents with relatively high earnings. On aggregate level, this implies a positive effect on social welfare that peaks at a tax rate of $\tau_l = 62\%$. These welfare gains, however, are at the cost of future generations, due to the negative effects of taxation on future production output. Contrary, redistributive capital taxation produces negative welfare effects for all agent types and generations, thus reducing social welfare consistently. This implies that even net recipients of capital transfers are subject to welfare losses.

4.3.4 Robustness Analysis

The present section investigates the influence of various parameter variations in order to study the robustness of the model results. While generally building on the baseline parameter setting presented in Section 4.3.1, particular parameterizations are changed in the following. To be precise, Section 4.3.4.1 investigates the impact of changes in the assumption regarding the underlying distribution of income levels between agent types. Then, in Section 4.3.4.2, the influence of heterogeneity in the subjective time discount factor across agent types is studied. Finally, Section 4.3.4.3 shows the impact of varying levels of common risk aversion.

4.3.4.1 Impact Earnings Disparity

The following section studies the impact of changes in the earnings disparity across income groups. Since agent types are still assumed to be homogeneous regarding their preferences, varying the shares of earnings received by the different agent types does not change the aggregate consumption-investment decision and, hence, does not lead to a different equilibrium solution. That is, production growth remains unaffected. Apparently, however, varying the allocation of earnings among the three agent types considered affects the distribution of consumption among them and, thus, implies a change in the impact of redistributive income taxation on individual and aggregate welfare. The present section investigates this relationship. In so doing, the earnings fraction of the average agent is kept unchanged (i.e., $\chi^2 = 1/3$), while it is varied for the two other agent types. In particular, the earnings fraction of the high income agent type χ^1 is considered in the interval $[1/3, 2/3)$.[134] The lower bound corresponds to three absolutely identical income groups and ensures that the ordering with respect to earnings remains unchanged. The upper bound corresponds to the highest possible level of income inequality between agent type 1 and 3, as the latter receives almost no earnings income at all.

Figure 4.12 depicts the influence of varying earnings shares on individual welfare measured by the percentage change in the production equivalent under labor ($\tau_l = 10\%$, left panels) and capital taxation ($\tau_c = 40\%$, right panels). The top panels plot the implied welfare changes for different generations of the high income agent (type 1), while the bottom panels plots the welfare changes for different generations of the low income agent (type 3). Since the income share of agent type 2 is kept constant and the equilibrium solution is unaffected, the results for the average agent are unchanged and, therefore, not displayed in Figure 4.12.

The earnings share determines whether and to what extent agents are net recipients or contributors to the transfer system. With increasing income inequality (keeping the original ordering with respect to earnings unchanged), high income agents pay, while low income agents receive higher net transfer payments. As shown in Figure 4.12, this implies that, given a certain tax setting, individual welfare of high income agents decreases stronger, whereas individual welfare of low income agents increases more heavily (or decreases less) with greater levels of earnings disparity. This relation, however, is not linear but increases (declines) for low (high) income agents with growing earnings inequality. As a result, for quiet extreme values of earnings inequality ($\chi^1 = 65\%$), first, even small labor tax rates

[134]This implies that the earnings fraction of the low income agent $\chi^3 = 1 - \chi^1 - \chi^2$ lies in the interval $(0, 1/3]$.

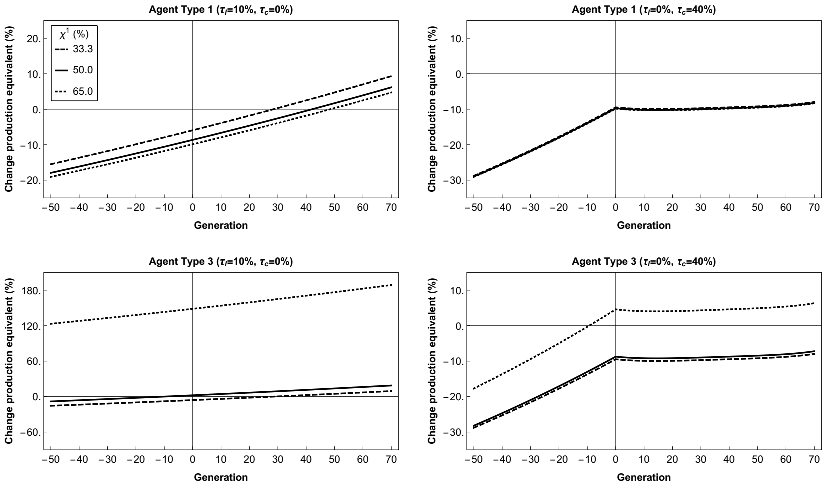

Figure 4.12 – This figure shows the impact of varying earnings shares χ^1 on individual welfare measured by the percentage change in the production equivalent under labor ($\tau_l = 10\%$, left panels) and capital taxation ($\tau_c = 40\%$, right panels). The top panels plot the implied welfare changes for different generations of the high income agent (type 1), while the bottom panels plots the welfare changes for different generations of the low income agent (type 3). The generation on the horizontal axis indicates the time of birth of an agent relative to the current period. While positive values correspond to the model age $(t-i)$ of agents currently living, negative values indicate agents that will enter the economy in the future.

lead to substantial welfare gains for living and yet unborn generations of low income agents. Second, also redistributive capital taxation may have a positive effect on individual welfare for agents with relatively low earnings levels.

These findings also translate into aggregate welfare levels. Figure 4.13, therefore, shows the influence of varying earnings shares (χ^1) in combination with varying labor (τ_l, left panel) and capital (τ_c, right panel) tax rates on social welfare measured by the percentage change in the production equivalent. On the one hand, the left panel of Figure 4.13 depicts a positive link that exists between earnings disparity and the impact of redistributive labor taxation on social welfare. Nevertheless, it also follows that in case of moderate levels of earnings inequality, labor taxation may neg-

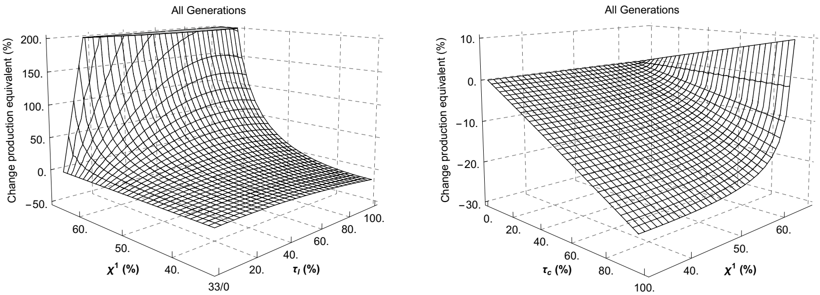

Figure 4.13 – This figure shows the impact of varying earnings shares (χ^1) as well as varying labor (τ_l, left panel) and capital (τ_c, right panel) tax rates on social welfare measured by the percentage change in the production equivalent.

atively affect the production equivalent on aggregate level. On the other hand, and in line with the results on individual level, the right panel of Figure 4.13 illustrates that for quiet extreme values of earnings inequality redistributive capital taxation may have moderate positive effects on the level of social welfare.

Overall, the results show that the original level of earnings inequality is decisive for whether the earnings and capital redistribution system is welfare improving on aggregate level. This holds true, since different levels of earnings disparity affect the size and impact of the reallocation effect, while production growth is unchanged. Beyond that, the findings that welfare gains on individual level for agents with relatively low income are at the cost of agents with relatively high earnings and that any welfare gain due to redistributive taxation is at the cost of some future generations is robust to changes in the earnings disparity.

4.3.4.2 Heterogeneous Subjective Time Discount Factors

So far, the quantitative analysis has considered the fact that patience may vary over the lifespan, but disregarded heterogeneity with respect to preferences across agent types. As outlined above (see Section 2.1.3), however, empirical evidence shows that individuals' attitude concerning the intertemporal allocation of consumption possibilities may also alter across income levels. To be more precise, patience is found to be higher for high income households than for low income ones consistently across several studies (see Samwick (1998), Lawrance (1991) and Booij and van Praag (2009)). Along the lines of this observation, the present section investigates the influence of heterogeneity in the level of the life-cycle profile of the subjective time discount factor across income groups. To affect patience, different values for the constant $\bar{\varphi}^m$ of the second-order subjective time discount factor polynomial presented in Table 4.3 are considered. This implies changes and, thus, heterogeneity in the level of patience, but does not affect its life-cycle pattern. In line with the reported empirical observations, the following analysis will concentrate on settings where either the high income agent (type 1) is relatively more patient or the low income agent (type 3) is relatively less patient than in the baseline parameterization, or both. For high income agents the analysis considers patience constants $\bar{\varphi}^1 = \{0.96, 0.97, 0.98, 0.99, 1.00\}$, whereas it assumes constants $\bar{\varphi}^3 = \{0.92, 0.93, 0.94, 0.95, 0.96\}$ for low income agents.[135] The subjective time discount factor profile of the average agent (type 2) is kept unchanged compared to the baseline parameter setting ($\bar{\varphi}^2 = 0.96$) throughout.

Then, keeping the remaining parameter specifications as presented in Section 4.3.1, in the following the impact of varying levels of patience on selected macroeconomic quantities, individual life-cycle profiles as well as social and individual welfare is studied.

Macroeconomic Effects

Changes in the individual's patience directly affect the agent's consumption-savings decision. To be precise, the results from the analytical solution (Equation 4.75) show that with growing (shrinking) subjective time

[135] This implies mean subjective discount factors over the life-cycle of $\{0.976, 0.986, 0.996, 1.006, 1.016\}$ and $\{0.936, 0.946, 0.956, 0.966, 0.976\}$, respectively.

discount factors the agent's MPCTW decreases (increases), which implies a lower (greater) proportion of total wealth being consumed. In return, the fraction postponed for future consumption, i.e., saving, rises (falls). In the given production economy, this change in individual behavior also affects the investment behavior on aggregate level and is, therefore, decisive in determining macroeconomic production output.

Figure 4.14 depicts the impact of varying levels of patience of the high ($\bar{\varphi}^1$) and low income agent ($\bar{\varphi}^3$) on the aggregate investment share (X, left panel) and expected economic growth (right panel). Confirming the analytical results on individual level, the left panel shows that with increasing (decreasing) subjective time discount factors the share of aggregate output that is reinvested into the production technology grows (falls). The direction of this effect is independent of the agent type; its magnitude, however, differs depending on the agent type that is subject to changes in patience. This is a consequence of the different earnings and, accordingly, total wealth levels that exist across income groups. Although, the same change in the subjective time discount factor affects the MPCTW of all agent types equally, the ultimate effect on consumption and investment quantities differs depending on the individual's total wealth level. As a result, a change in patience of a relatively more wealthy agent (type 1) has a greater impact on the aggregate investment share than the same change in patience of a relatively poor agent (type 3).

As illustrated in the right panel of Figure 4.14, these results directly translate into the impact of varying levels of patience on expected production growth. While the GDP growth rate increases (decreases) generally with larger (smaller) subjective time discount factors, the magnitude of this effect is greater for changes in the patience of the relatively wealthy agent type than compared to the same change in patience of the relatively poor agent type.

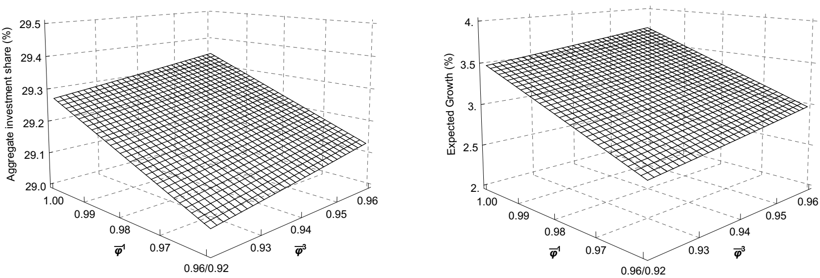

Figure 4.14 – This figure shows the impact of varying levels of patience of the high ($\bar{\varphi}^1$) and low income agent ($\bar{\varphi}^3$) on the aggregate investment share (X, left panel) and expected economic growth (right panel).

Life-Cycle Effects

Figure 4.15 plots the impact of different levels of patience of the high and low income agent on the life-cycle profiles of investment (left panels) and net transfers (right panels) for the three agent types. Throughout the present section, the solid line depicts the profiles for the baseline parameterization with homogeneous life-cycle profiles of the subjective time discount factor ($\bar{\varphi}^1 = \bar{\varphi}^2 = \bar{\varphi}^3 = 0.96$), the dashed line for the case in which the high income agent (type 1) is more patient ($\bar{\varphi}^1 = 1.0$; $\bar{\varphi}^2 = \bar{\varphi}^3 = 0.96$), and the dotted line for the case in which the low income agent (type 3) is less patient ($\bar{\varphi}^1 = \bar{\varphi}^2 = 0.96$; $\bar{\varphi}^3 = 0.92$) than the other two agent types.

On an individual level, a change in patience of a special agent type first of all affects the consumption-savings decision of this very type. In the presence of capital taxation, however, the implied change in savings also affects his tax payments from capital gains and the amount of net transfers received or paid. This in return changes the net transfer income of all other agent types and eventually their investment behavior. As a consequence, the average agent also becomes subject to intra-generational redistribution in the case of heterogeneous levels of subjective time discount factors.

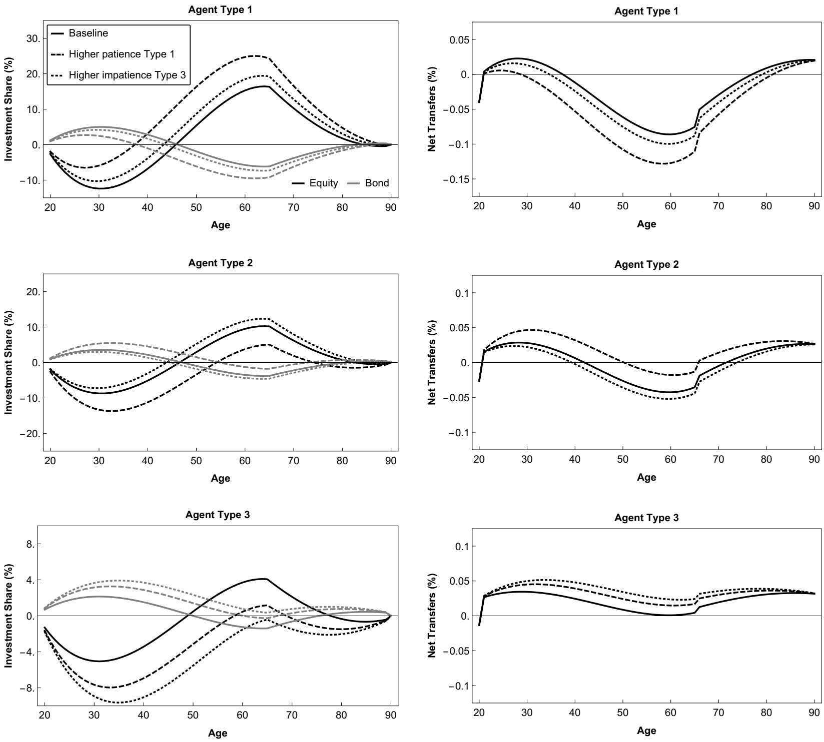

Figure 4.15 – This figure shows the impact of varying levels of patience of the high and low income agent on the life-cycle profiles of investment (left panels) and net transfers (right panels) for the three agent types. For the investment profiles, the agent's share (in %) of equity is represented by the black lines, while the gray lines are his share (in %) of bond investment in aggregate investment. The solid line depicts the profiles for the baseline parameterization with homogeneous life-cycle profiles of the subjective time discount factor ($\bar{\varphi}^1 = \bar{\varphi}^2 = \bar{\varphi}^3 = 0.96$), the dashed line for the case in which the high income agent (type 1) is more patient ($\bar{\varphi}^1 = 1.0$; $\bar{\varphi}^2 = \bar{\varphi}^3 = 0.96$), and the dotted line for the case in which the low income agent (type 3) is less patient ($\bar{\varphi}^1 = \bar{\varphi}^2 = 0.96$; $\bar{\varphi}^3 = 0.92$) than the other two agent types.

In the first case, the high income agent is considered to be more patient and, consequently, increases his savings compared to the baseline parameter setting. As a direct consequence, his amount of taxable capital gains goes up. Since patience and, therefore, the consumption-savings behavior of the other agent types are unchanged, this implies that the amount of net

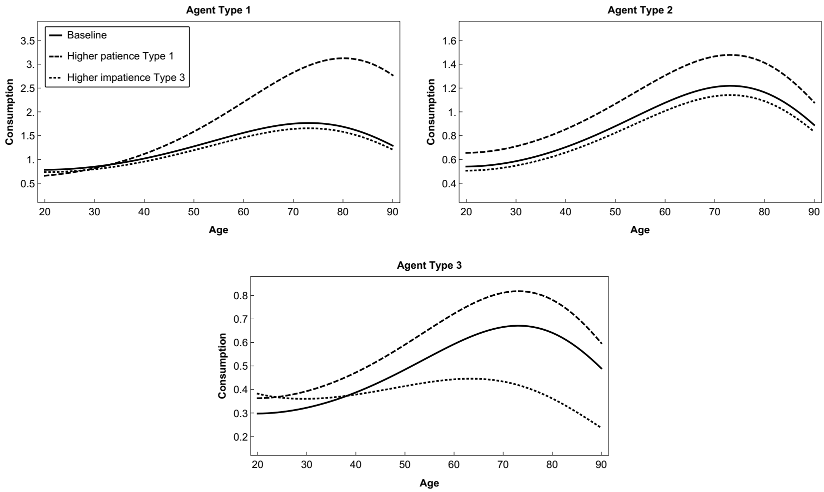

Figure 4.16 – This figure shows the impact of varying levels of patience of the high and low income agent on the mean life-cycle consumption profiles for the three agent types. The solid line depicts the profiles for the baseline parameterization with homogeneous life-cycle profiles of the subjective time discount factor ($\bar{\varphi}^1 = \bar{\varphi}^2 = \bar{\varphi}^3 = 0.96$), the dashed line for the case in which the high income agent (type 1) is more patient ($\bar{\varphi}^1 = 1.0$; $\bar{\varphi}^2 = \bar{\varphi}^3 = 0.96$), and the dotted line for the case in which the low income agent (type 3) is less patient ($\bar{\varphi}^1 = \bar{\varphi}^2 = 0.96$; $\bar{\varphi}^3 = 0.92$) than the other two agent types. For the profiles aggregate output (Y) is normalized to one at age 20.

transfers received by agent type 1 decreases, while the net transfer incomes of the other agent types increase (right panels). In line with the previous results, this change in net capital transfers affects the agents' exposure to macroeconomic risk. As a result, type 1 agents compensate the implied reduction in the risk exposure by decreasing bond and increasing equity holdings, whereas the other agent types compensate the additional risk exposure by higher bond and lower equity holdings (left panels).

For the second case, keeping all other parameters as in the baseline setting, the low income agent is assumed to be less patient and, thus, decreases savings compared to the baseline parameter setting. This leads to a further reduction in the amount of capital wealth held by agent type 3 relative to the mean capital holdings and, hence, implies an increase in his net transfer income at the cost of decreasing the amount of net transfers received by the other agent types (right panels). In order to compensate the change in the implied risk exposure, type 3 agents raise bond and reduce equity holdings, while the other agent types lower their bond and increase their equity holdings (left panels).

Figure 4.16 shows the impact of varying levels of patience of the high and low income agent on the mean life-cycle consumption profiles for the three agent types. As explained above, considering different levels of patience for a special agent type first of all affects the consumption-savings decision and, thus, the life-cycle consumption pattern of the very same type. Nevertheless, the recent results have shown that the other agent types are also affected due to the implied changes in, first, production growth and, second, net transfers. In particular, the relation is as follows: with increasing (decreasing) patience, the high (low) income agent reduces (raises) his consumption ratio and increases (decreases) savings. This adjustment lowers (increments) his mean consumption level early in life, while it increases (decreases) consumption at later ages due to higher (lower) wealth levels. The implied increase (decrease) in the net transfer income received by the other agent types has a positive (negative) effect on their mean consumption levels, whereas the raise (drop) in aggregate investment and, consequently, production growth positively (negatively) affects the mean consumption levels of all agents.

In general, Figure 4.16 illustrates that heterogeneity in the subjective time discount factor leads to individual life-cycle consumption patterns that differ across agent types. As a consequence, intra-cohort consumption disparity is no longer the same for all generations but varies with different age categories. Figure 4.17 depicts the impact of varying levels of patience of the high and low income agent on the consumption Gini coefficient conditional on age. Because of the implied heterogeneity in the life-cycle savings behavior across agent types, in the presence of heterogeneous patience intra-cohort consumption inequality increases with age.

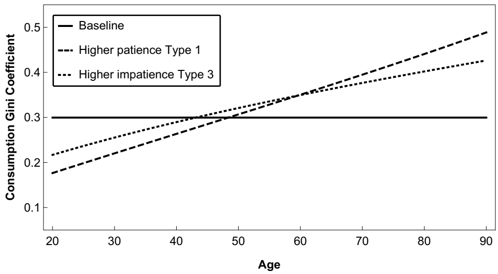

Figure 4.17 – This figure shows the impact of varying levels of patience of the high and low income agent on the consumption Gini coefficient conditional on age. The solid line depicts the profile for the baseline parameterization with homogeneous life-cycle profiles of the subjective time discount factor ($\bar{\varphi}^1 = \bar{\varphi}^2 = \bar{\varphi}^3 = 0.96$), the dashed line for the case in whcih the high income agent (type 1) is more patient ($\bar{\varphi}^1 = 1.0$; $\bar{\varphi}^2 = \bar{\varphi}^3 = 0.96$), and the dotted line for the case in which the low income agent (type 3) is less patient ($\bar{\varphi}^1 = \bar{\varphi}^2 = 0.96$; $\bar{\varphi}^3 = 0.92$) than the other two agent types.

This result is in line with the empirically documented evidence reported by Krueger and Perri (2006) and the theoretical findings in Gomes et al. (2013).

Welfare Effects

Closing the present section, the influence of heterogeneity in patience across agent types on individual and social welfare is studied. As above, welfare is measured by the percentage change in the production equivalent. That is, the change in current production for the no-tax setting that is necessary in order to achieve equivalent expected utility levels from consumption in both the no-tax and tax case. Besides varying the subjective time discount factors, the baseline parameterization further applies. This implies that the tax rates considered for the taxation setting are still given by $\tau_l = 10\%$ and $\tau_c = 40\%$.

Figure 4.18 depicts the meaning of varying levels of patience of the high and low income agent on the change in the production equivalent on individual level due to redistributive taxation. Consistently with the previous analysis, in the first case the high income agent is considered to be more patient (dashed line), while in the second case the low income agent is assumed to be less patient (dotted line) than the other agent types.

For the high income agent (type 1, top left panel), the negative effect of redistributive taxation on individual welfare is intensified in both cases. To be precise, the percentage change in current production needed in the setting without taxation to generate the same level of utility the agent holds in the presence of redistributive taxation further drops. This is a consequence of the increased net transfer payments high income agents have to pay in the considered settings, which imply an aggravated reallocation effect from their point of view.

As shown above, average agents become net recipients of intra-cohort transfer incomes, as the relatively more wealthy agent type becomes relatively more patient. Therefore, in the first case a positive effect on the change in the production equivalent for numerous generations of the average agent (type 2, top right panel), especially for the living cohorts, arises. Since average agents become net contributors to intra-cohort transfers, as the relatively less wealthy agent type becomes relatively less patient, a negative effect on the change in the production equivalent for almost all generations results in the second case.

The effect of the considered patience cases on the individual welfare level of low income agents (type 3, bottom panel) is directly opposed to the previous findings for the high income agent type. To be precise, in both cases a positive effect on the change in the production equivalent for a wide range of generations arises. This is a result of the higher net transfer incomes received by low income agents in both cases, which makes the reallocation system more favorable for them.

In general, the positive (negative) welfare effect on individual level tends to be greater (weaker) for older generations. As shown above in Figure 4.17, in the presence of heterogeneous patience, intra-cohort consumption inequality increases with age. Due to the concavity in utility for consump-

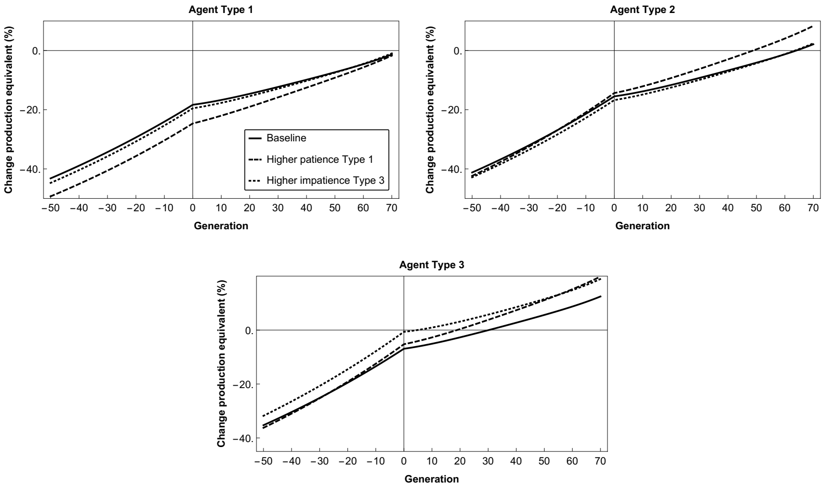

Figure 4.18 – This figure shows the impact of varying levels of patience of the high and low income agent on individual welfare measured by the percentage change in the production equivalent. The top left panel depicts the implied welfare change for different generations of the high income agent (type 1), the top right panel for diverse generations of the average agent (type 2) and the bottom panel for different generations of the low income agent (type 3). The solid line depicts the profiles for the baseline parameterization with homogeneous life-cycle profiles of the subjective time discount factor ($\bar{\varphi}^1 = \bar{\varphi}^2 = \bar{\varphi}^3 = 0.96$), the dashed line for the case in which the high income agent (type 1) is more patient ($\bar{\varphi}^1 = 1.0$; $\bar{\varphi}^2 = \bar{\varphi}^3 = 0.96$), the dotted line for the case in which the low income agent (type 3) is less patient ($\bar{\varphi}^1 = \bar{\varphi}^2 = 0.96$; $\bar{\varphi}^3 = 0.92$) than the other two agent types. The generation on the horizontal axis indicates the time of birth of an agent relative to the current period. While positive values correspond to the model age ($t - i$) of agents currently living, negative values indicate agents that will enter the economy in the future.

tion, this increase in inequality implies that a reallocation of consumption goods has an increasing positive (or decreasing negative) effect on welfare, the lower the initial consumption level of the individual is before transfers. Consequently, with heterogeneous patience the positive (negative) welfare effect on individual level grows (shrinks) with age.

Finally, Figure 4.19 illustrates the impact of varying levels of patience for the high and low income agent on social welfare measured by the percentage change in the production equivalent. The just described results on individual level add to a positive impact of increasing heterogeneity in pa-

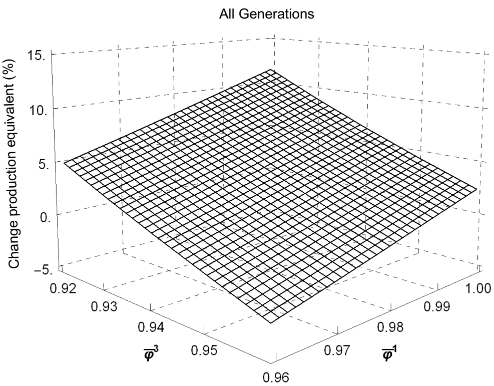

Figure 4.19 – This figure shows the impact of varying levels of patience for the high ($\bar{\varphi}^1$) and low income agent ($\bar{\varphi}^3$) on social welfare measured by the percentage change in the production equivalent.

tience on an aggregate level. That is, with decreasing patience of the low income agent and increasing patience of the high income agent, the social welfare level consistently increases. Again, this is a result of the raise in consumption disparity, especially at higher ages, in combination with the concavity of the utility function. Altogether, therefore, the reallocation effect of the redistributive taxation system becomes more favorable for welfare on aggregate level.

Overall, these results are in line with the findings reported above. Welfare improvements for agent types with relatively low income levels occur at the cost of welfare cuts for agent types with relatively high income levels. In line with the findings in Chapter 3, this effect is actually further amplified when considering patience heterogeneity across income groups according to empirical observations. Moreover, it still holds that any welfare gain due to redistributive taxation happens at the cost of welfare losses of some future generations. Contrary to the case with homogeneous preferences, however, runs the fact that the average agent type becomes subject to intracohort transfers in the presence of heterogeneous patience.

4.3.4.3 Impact Level of Risk Aversion

The robustness analysis finally closes by investigating the influence of different levels of the common risk aversion coefficient γ on the model results. The agents' attitude towards risk is a decisive factor that determines an individual's life-cycle consumption-savings behavior and, in the present model, also influences macroeconomic development. Figure 4.20, therefore, depicts the impact of varying levels of the common relative risk aversion coefficient on the life-cycle profile of expected individual consumption growth (left panel) as well as on expected economic growth and aggregate investment (right panel). Besides considering different levels of risk aversion, the baseline parameterization applies.

According to the analytical solution (Equation (4.73)), individual consumption growth is directly affected by the agent's risk attitude. In particular, while the life-cycle pattern of the subjective time discount factor generally drives the hump-shaped profile of consumption growth, the risk aversion coefficient affects its curvature. The left panel of Figure 4.20 illustrates this influence. The solid line corresponds to the results under baseline parameterization ($\gamma = 3$), the dashed line for individuals with risk aversion $\gamma = 2$ and the dotted line for $\gamma = 5$. The graphs show that the curvature of the life-cycle profile decreases with the degree of risk aversion. Put differently, when γ goes up, individual consumption growth raises early in life at the cost of lowering growth levels throughout the working years.

Again, this change in individual behavior also affects the development of the aggregate economy. In order to finance higher consumption growth early in life, individuals have to further decrease savings when young. This weakens life-cycle wealth accumulation, which itself negatively affects saving quantities. As shown by the dashed line in the right panel of Figure 4.20, on aggregate level this implies that the share of output that is reinvested into the production technology declines with increasing risk aversion. Caused by this reduction in the share of aggregate investment, a drop in macroeconomic growth follows, as indicated by the solid line.

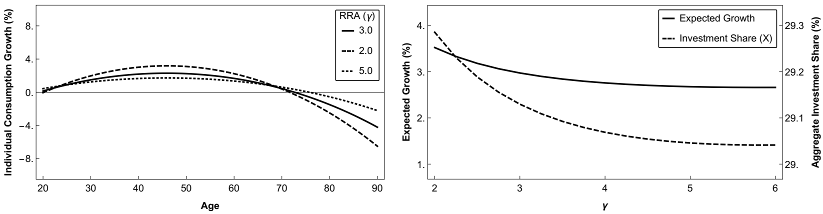

Figure 4.20 – This figure shows the impact of varying levels of the common relative risk aversion coefficient (γ) on expected individual consumption growth over the life-cycle (left panel) as well as on expected economic growth and the aggregate investment share (right panel).

Finally, Figure 4.21 depicts the influence of varying common relative risk aversion coefficients on individual and social welfare measured by the percentage change in the production equivalent. The left panel displays the welfare change for different generations of the average agent (type 2) as a function of the degree of risk aversion.[136] The baseline parameter setting with $\gamma = 3$ is given by the solid line, while the dashed and dotted lines assume $\gamma = 2$ and $\gamma = 5$, respectively. The graphs indicate a positive effect on the change in the production equivalent due to redistributive taxation with increasing risk aversion. This effect is driven by the implied increase in the curvature of the utility function over consumption. Put differently, a reallocation of consumption goods from agents with high to agents with low consumption levels has an increasing positive (or decreasing negative) effect on welfare the higher the curvature of utility.

The right panel of Figure 4.21 shows that this result translates into aggregate levels again. That is, social welfare is positively linked to the agents' risk attitude. In general, one can conclude that with increasing (decreasing) risk aversion, the redistributive taxation system becomes more (less) favorable in terms of welfare. Beyond that, the findings are in line with the previous results and, therefore, qualitatively robust to changes in the degree of risk aversion.

[136] Since the pictures for the other two agent types are qualitatively identical, they are not separately displayed.

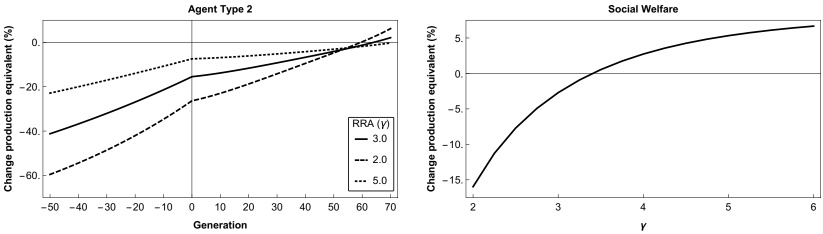

Figure 4.21 – This figure shows the impact of varying levels of the common relative risk aversion coefficient (γ) on individual and social welfare measured by the percentage change in the production equivalent. The left panel displays the welfare change for different generations of the average agent (type 2) for different values of the coefficient of relative risk aversion, where the generation on the horizontal axis indicates the time of birth of an agent relative to the current period. The right panel depicts the change in social welfare due to redistributive taxation as a function of the common degree of risk aversion.

4.4 Conclusion

In this chapter, the simultaneous impact of redistributive capital and labor taxation as well as household heterogeneity is analyzed using a dynamic general equilibrium asset pricing model. The framework features an economy with endogenous linear production technology populated by a finite number of overlapping generations, where every cohort is composed of different types of agents that are heterogeneous with respect to their stream of permanent life-cycle labor income and time preferences. Beyond that, time preferences are explicitly assumed to be age-dependent, i.e., to vary over the individual's life-cycle.

In this context, the derivation of a simple stationary equilibrium solution is facilitated by the restriction on i.i.d. aggregate production risk, with the solution method in this chapter building upon a "guess and verify" approach. Thus, tractable analytical results for the equilibrium processes are derived. Specifically, in the present setting, individual consumption growth, the stochastic discount factor as well as the risk-free return are given by closed-form solutions. The solutions to aggregate production, consumption and investment as well as to the individual policy functions are dependent on the distribution of consumption among agents in equilibrium, which is endogenous to the model. Given age-dependent but deterministic consumption shares, however, implies that agents strive for a linear sharing rule and align their marginal rates of substitution in equilibrium. Therefore, all analytical results are again found dependent on only one endogenous parameter, which can be determined by deriving and solving a deterministic equilibrium condition. Furthermore, the fact that new generations of each agent type are always coming along ensures that, although individuals are heterogeneous with respect to their time preferences, no group of agents dominates the economy in the long run.

The results in this chapter show that the life-cycle profile of individual consumption is directly dependent on the evolution of time preferences with age. In this vein, assuming empirically plausible age-profiles of patience, the empirically well-established hump-shaped pattern of consumption is replicated, while agents are generally allowed to trade freely in the asset markets. Moreover, in line with the findings in Chapter 3, the design of the redistributive tax system implies that capital transfers are pro-cyclical and, hence, carry additional macroeconomic risk. Trying to compensate the unequal distribution of risk across agents, relatively poor (rich) individuals, i.e., net recipients (contributors) of capital transfers, reduce (increase) their stock holdings while they increase (decrease) their bond holdings. This results in dynamic trading in both asset markets.

In the context of the present thesis, the impact of redistributive taxation is of special interest. In this regard, it is shown that, under empirically plausible (baseline) parameterization, aggregate investment is negatively related to taxation implying a GDP growth rate that decreases with both capital and labor taxes. In this way, future production output, i.e., future consumption opportunities, decreases with growing tax rates. As a result, solely for low income agents labor taxation has a positive effect on the mean life-cycle consumption due to high transfer incomes. Since the redistribution effect is much smaller in case of capital taxation, however, the latter is associated with negative effects on consumption across all agent types. Moreover, while labor taxation reduces inequalities in the distribution of consumption, wealth and equity holdings, redistributive capital taxation only produces a small reduction in wealth inequality but actually increases consumption disparity and raises inequality in the distribution of equity holdings sharply. These effects carry over to individual and aggregate welfare. Consequently, labor taxation improves the welfare of agents with relatively low earnings at the cost of reducing the welfare level of agents with relatively high earnings. On aggregate level, this implies a hump-shaped profile of social welfare in the labor tax rates. Contrary, capital taxation produces negative welfare effects across all agent types and generations. That way, even net recipients of capital transfers may be subject to welfare losses. On aggregate level, social welfare is consistently reduced for growing capital tax rates.

Deviating from the baseline parameterization yields further insights. First of all, the level of earnings inequality is found to be decisive for the direction the impact of labor and capital taxation on aggregate welfare takes. This holds true, since different levels of labor disparity affect the size and impact of the reallocation effect, while production growth is unchanged. Nevertheless, it requires implausibly high (low) levels of income inequality for redistributive capital (labor) taxation to gain moderate positive (negative) effects on the level of social welfare. Next, when allowing for different time preferences across agent types, individual life-cycle consumption patterns become heterogeneous across agent types. In line with the existing literature, this implies that intra-generational consumption inequality increases with age. Finally, it is shown that redistributive taxation becomes more (less) favorable in terms of individual and aggregate welfare with increasing (decreasing) coefficients of relative risk aversion.

In any case, however, welfare gains on individual level for agents with relatively low income are at the cost of agents with relatively high earnings. Moreover, any welfare gain due to redistributive taxation is at the cost of future generations.

Finally, although it captures all required dimensions identified in Chapter 2, the realism of the present model can still be enriched in various ways. As in the previous chapter, agents are not constrained with respect to their financial market activities within the present setting. Individuals, thus, borrow against future human and transfer capital, which causes unrealistically high fractions of negative stock holdings and wealth levels especially early in the individuals' lifetime. Therefore, one way of extending the model would be to assume borrowing or short-sales constraints. Related to this is the fact that earnings and transfers are tradable through the asset markets. In the present setting this results from the restriction on i.i.d. aggregate production risk and the absence of unspanned labor income shocks. Allowing for idiosyncratic earnings risk, as in Gomes and Michaelides (2008) or Gomes et al. (2013), would help to overcome these limitations. Finally, the design of the redistributive taxation system is very stylized. Accordingly, exploring the impact of different taxation and redistribution schemes, for example, the effects of progressive statutory tax rates, also offers interesting topics for future research. Nevertheless, complicating the model in either of these directions would foil the tractability of the present model solution.

5

Conclusion and Outlook

Motivated by the ongoing debate on the effects of policy instruments designed to address the inequality in the distribution of income, wealth and consumption, the objective of the present thesis is to quantitatively study the impact of redistributive taxation systems using economic equilibrium models. In this context, Chapter 2 identifies six dimensions that should be captured (ideally simultaneously) by respective modeling approaches. These are: *redistributive taxation, endogenous individual behavior, agent heterogeneity, life-cycle characteristics, endogenous production* and *asset pricing*. The knowledge of the true relationships between all of these aspects and the understanding of the underlying causes is decisive for any policy maker. With respect to agent heterogeneity, especially differences in individual time preferences are found to be of major importance in order to replicate the observable inequality patterns and, therefore, the focus of the present work. In order to enhance comprehensibility, a two step approach based on the successive development of two economic models is followed:

In Chapter 3, a simplified dynamic general equilibrium asset pricing model is presented that abstracts from the aspects of *endogenous production* and partly *life-cycle characteristics*. It builds on an exchange economy with classical demographic structure populated by two agents heterogeneous in their respective patience and initial financial endowments. Beyond that, the approach concentrates on taxation of capital income and ignores labor income taxation. The results indicate that considering heterogeneous time preferences (as observed empirically) worsens consumption and wealth inequality, which is in line with the existing literature (see Krusell and Smith (1998) and Hendricks (2007)), and, thereby, further aggravates the low stock market participation rates for poor individuals. Although the impact of redistributive taxation on wealth inequality is ambiguous, it is capable to reduce consumption disparity and thereby redistributes welfare from high to low income households. Finally, comparing the results to the findings in Fischer and Jensen (2015), who consider different levels of risk aversion instead of time preferences, the general effects of both sources of preference heterogeneity with respect to redistributive taxation are found to be comparable. This justifies the concentration on heterogeneous subjective time preferences in the context of a tax-based reallocation mechanism.

Chapter 4 builds on the model of Chapter 3 but adds the two missing dimensions. In particular, the framework is based on an economy with endogenous linear production technology populated by a finite number of overlapping generations, where every cohort is composed of different types of agents that are heterogeneous with respect to their stream of permanent life-cycle labor income and time preferences. Beyond that, and in line with the empirical evidence, time preferences are explicitly assumed to vary over the individual's life-cycle. Lastly, the approach considers both capital and labor income taxation. The results, first of all, confirm the findings of the previous chapter in that heterogeneity in time preferences (as observed empirically) is associated with higher consumption and wealth inequality, which further lowers stock market participation rates of poorer individuals. Next, building on age-dependent time preferences, the life-cycle profile of individual consumption becomes directly dependent on their evolution. Under empirically plausible age-profiles of patience, the empirically well-established hump-shaped pattern of consumption results. With respect to the impact of redistributive taxation on inequality and welfare, the findings are complex. In particular, labor taxation generally reduces inequalities in the distribution of consumption, wealth and equity holdings, whereas redistributive capital taxation only produces a small reduction in wealth inequality but actually increases disparity in consumption and equity holdings. Moreover, a hump-shaped profile of aggregate welfare in the labor tax rate implies an (welfare) optimal labor tax rate that is positive. In contrast, capital taxation is typically associated with negative welfare effects throughout.

When considered with tax-based policy instruments to address inequality, these findings imply several key issues: First of all, heterogeneity is an essential dimension that should be taken into account when designing redistributive taxation systems. With respect to differences in preferences, heterogeneity in time preferences is of major importance, since it accounts for additional consumption and wealth inequality and, thereby, aggravates the low stock market participation rates of poor individuals. Beyond that, it helps to replicate realistic consumption profiles and is decisive when designing welfare measures. In this vein, heterogeneous patience is an important factor in understanding low participation rates and crucial when studying the equilibrium impact of policy instruments, as in Gomes et al. (2013). Furthermore, the results of Chapter 4 indicate that the idea of redistributing (capital) income streams in order to enhance equality or welfare might be a pitfall. In a variety of cases, redistributive taxation might actually foster inequality in wealth, consumption and participation rates, while simultaneously hampering economic growth. By this means, and in line with the findings in Fischer and Jensen (2014), future consumption opportunities drop and even net recipients of capital transfers might suffer from welfare losses. For raising inequality, however, redistribution becomes increasingly effective and welfare improving at some critical level. This threshold is extremely high in case of capital taxation, but considerably moderate for labor taxation. When effective, however, redistributive taxation is finally always associated with a trade-off between macroeconomic growth and equality (or welfare), an observation that is in line with the empirical evidence in Blanchard and Perotti (2002), Romer and Romer (2010), Barro and Redlick (2011) and Cloyne (2013). This particularly implies that higher equality and/or welfare levels occur especially at the expense of future generations. The knowledge about these relationships, feedback effects and trade-offs is essential for any policy maker when designing redistributive taxation systems.

Apparently, there are several assumptions and simplifications underlying the modeling approaches presented in the course of this thesis. Tackling some of these weakenings can help to validate and refine the results derived above, while producing additionally interesting policy implications. Allowing for borrowing or short-sales constraints as well as idiosyncratic earnings risk, as in Gomes and Michaelides (2008) or Gomes et al. (2013), would be one interesting way of extending the analysis and enriching the results. Moreover, one could endogenize earnings by allowing for an explicit labor-leisure decision of households, or enhance realism by modeling the production and the government sector in a more realistic manner. Finally, real-life taxation and transfer systems are more complex than considered in the present frameworks. Exploring the impact of different and more sophisticated taxation and redistribution schemes, for example, the effects of progressive statutory tax rates, certainly is another interesting way of enhancing the analysis.

To conclude, the present thesis constitutes a step forward towards a better understanding of the complex effects of redistributive taxation systems. Nevertheless, limitations and assumptions still leave space for future research. Hopefully, the ideas and findings developed in the course of this work will contribute to the future development of this meaningful field of research.

Appendix A

Figures

Figure A.1 - Baseline parameterization: Life-cycle profiles of total, equity and bond investment as well as net transfers for the low income agent

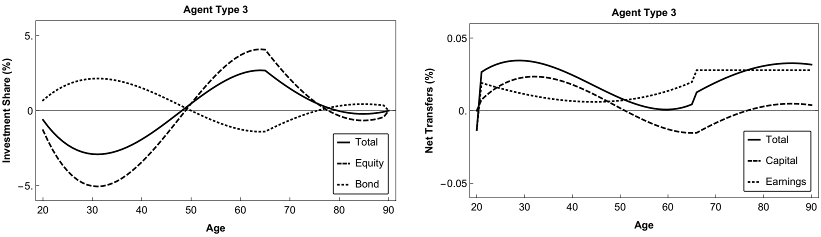

Figure A.1 – This figure shows investment as well as net transfer life-cycle profiles for the low income agent (type 3) under baseline parameterization. The left panel plots the agent's share (in %) of total, equity and bond investment in aggregate investment over his life-cycle. The right panel displays mean net transfers received in percent of aggregate production output over the lifespan, where total transfers are also split up into transfers financed by capital and earnings.

Figure A.2 - Baseline parameterization: Life-cycle profiles of total, equity and bond investment as well as net transfers for the high income agent

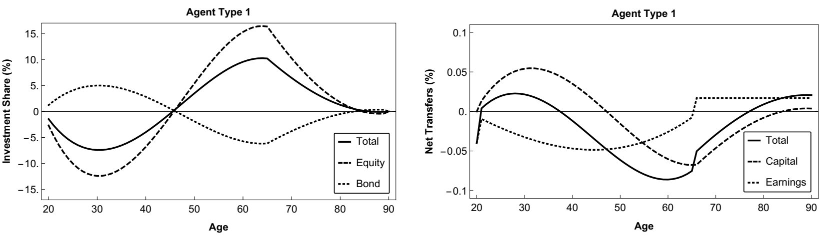

Figure A.2 – This figure shows investment as well as net transfer life-cycle profiles for the high income agent (type 1) under baseline parameterization. The left panel plots the agent's share (in %) of total, equity and bond investment in aggregate investment over his life-cycle. The right panel displays mean net transfers received in percent of aggregate production output over the lifespan, where total transfers are also split up into transfers financed by capital and earnings.

Figure A.3 - Impact of tax rates: Life-cycle profiles of investment and net transfers for high income agent

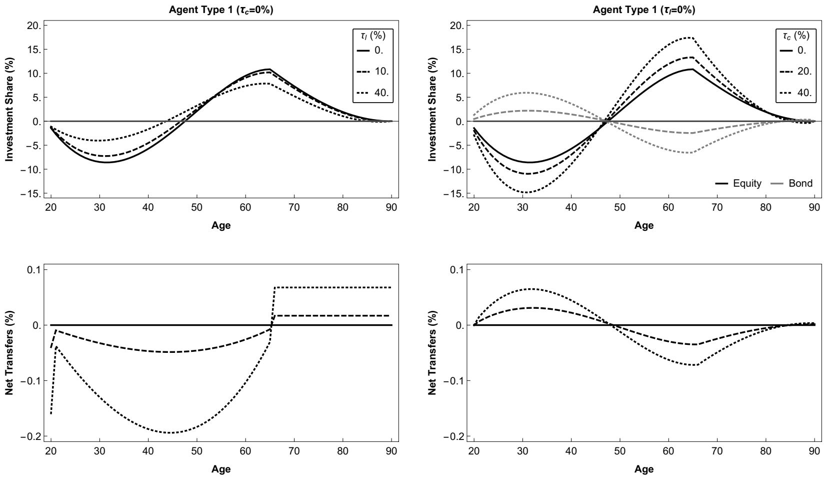

Figure A.3 – This figure shows the impact of varying labor (τ_l, left panels) and capital (τ_c, right panels) tax rates on the life-cycle profiles of investment (top panels) and net transfers (bottom panels) for the high income agent (type 1). For the investment profiles the agent's share (in %) of equity is represented by the black lines, while the gray lines are his share (in %) of bond investment in aggregate investment.

Figure A.4 - Impact of tax rates: Life-cycle profiles of investment and net transfers for low income agent

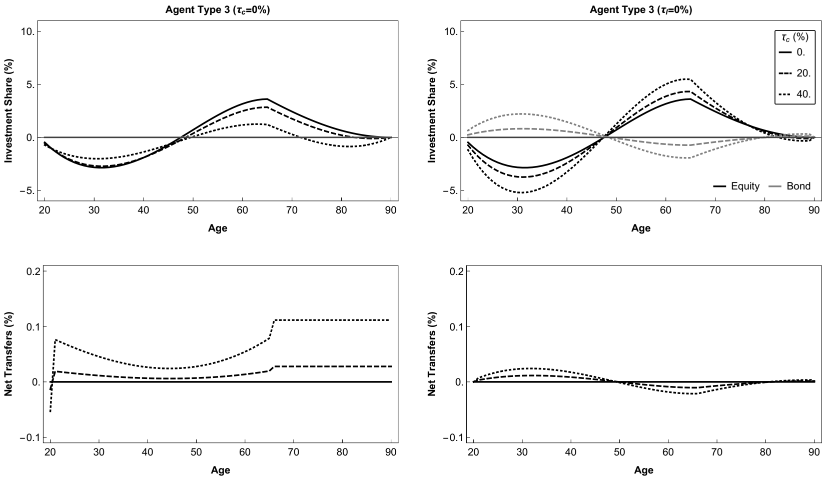

Figure A.4 – This figure shows the impact of varying labor (τ_l, left panels) and capital (τ_c, right panels) tax rates on the life-cycle profiles of investment (top panels) and net transfers (bottom panels) for the low income agent (type 3). For the investment profiles the agent's share (in %) of equity is represented by the black lines, while the gray lines are his share (in %) of bond investment in aggregate investment.

Appendix B

Proofs

B.1 Redistributive Taxation in an Exchange Economy

B.1.1 The Government's Optimization Problem

The government minimizes the quadratic optimization problem (3.1) subject to (3.2). The Lagrangian associated with the constrained problem is given by

$$\begin{aligned}\mathcal{L}^{\mathcal{G}} =&\sum_{m=1}^{M}\left\{\left(\tilde{k}_t^m - \frac{1}{2}\left(P_{k,t} + D_t - P_{k,t-1}\right)\right)^2 + \kappa\left(\tilde{k}_t^m - k_t^m\right)^2\right\} - \\ &\Lambda\left\{\sum_{m=1}^{M}\tilde{k}_t^m - \left(P_{k,t} + D_t - P_{k,t-1}\right)\right\},\end{aligned} \tag{B.1}$$

where Λ is the Lagrangian multiplier. The first order condition with respect to $\tilde{k}_t^m$ is

$$\frac{\partial \mathcal{L}^{\mathcal{G}}}{\partial \tilde{k}_t^m} =2\left(\tilde{k}_t^m - \frac{1}{2}\left(P_{k,t} + D_t - P_{k,t-1}\right)\right) + 2\kappa\left(\tilde{k}_t^m - k_t^m\right) - \Lambda \overset{!}{=} 0. \tag{B.2}$$

Taking the sum over all agent types m at date t the Lagrangian multiplier turn out to be zero, i.e., $\Lambda = 0$. Using this result in conditions (B.2) again and solving for $\tilde{k}_t^m$ yields the linear feedback rule

$$\tilde{k}_t^m =\frac{\kappa}{1+\kappa}k_t^m + \frac{1}{1+\kappa}\frac{1}{2}\left(P_{k,t} + D_t - P_{k,t-1}\right), \tag{B.3}$$

which corresponds to Equation (3.3) in the main text.

B.1.2 Derivation of First Order Conditions

Given the assumptions regarding the economy in the main text (see Section 3.1.1), there exists a finite and discrete state space of possible outcomes Ω underlying the economy, where element $\omega \in \Omega$ denotes one possible state that might be realized. The agent's optimization problem is given by Equation (3.14) subject to constraints (3.15)-(3.16) and (3.10)-(3.11). The associated Lagrangian of agent type m reads

$$\begin{aligned}\mathcal{L}^m =&\sum_{t=0}^{T}\mathbb{E}_0\left[(\delta^m)^t\,\frac{(c_t^m)^{1-\gamma}}{1-\gamma}\right]-\mu_0^m\Big(c_0^m-\alpha_{-1}^m\left(P_{k,0}+D_0\right)+\alpha_0^m P_{k,0}+\\&\beta_0^m\Big)-\sum_{t=1}^{T-1}\Big\langle\mu_t^m,c_t^m-\alpha_{t-1}^m\left(\left(1-\tau\right)\left(P_{k,t}+D_t\right)+\tau P_{k,t-1}\right)-\\&\beta_{t-1}^m\tilde{R}_{f,t-1}-s_t+\alpha_t^m P_{k,t}+\beta_t^m\Big\rangle-\Big\langle\mu_T^m,c_T^m-\beta_{T-1}^m\tilde{R}_{f,T-1}-\\&\alpha_{T-1}^m\left(\left(1-\tau\right)\left(P_{k,T}+D_T\right)+\tau P_{k,T-1}\right)-s_T\Big\rangle,\end{aligned} \tag{B.4}$$

where $\{\mu_t^m\}_{t=0}^T$ are the Lagrangian multipliers associated with the constraints and $\langle\cdot,\cdot\rangle$ is the scalar product over the relevant states, as in Fischer and Jensen (2014, 2015, 2017). Using the representation of the optimization problem stated in Equation (B.4), the first order conditions can be derived by optimizing state-by-state.

Since the number of possible realizations of the random growth rate G_z is limited to $z = 1, 2$ and each realization occurs with equal probability $1/2$, differentiating the Lagrangian for agent type m with respect to consumption at time t and state ω, $c_{t,\omega}^m$, the following first order condition is derived:

$$\begin{aligned}\frac{\partial\mathcal{L}^m}{\partial c_{t,\omega}^m}&=\left(\frac{\delta^m}{2}\right)^t\left(c_{t,\omega}^m\right)^{-\gamma}-\mu_{t,\omega}^m\overset{!}{=}0\\\Leftrightarrow\mu_{t,\omega}^m&=\left(\frac{\delta^m}{2}\right)^t\left(c_{t,\omega}^m\right)^{-\gamma}.\end{aligned} \tag{B.5}$$

This Equation holds equally across states and, hence, it holds true that

$$\mu_t^m = \left(\frac{\delta^m}{2}\right)^t (c_t^m)^{-\gamma}, \tag{B.6}$$

which is Equation (3.20) in the main text.

Next, differentiating the Lagrangian (B.4) for agent type m with respect to his equity share in period t and state ω, $\alpha_{t,\omega}^m$, it follows that:

$$\begin{aligned}\frac{\partial \mathcal{L}^m}{\partial \alpha_{t,\omega}^m} =& -\mu_{t,\omega}^m P_{k,t,\omega} + \sum_{z=1}^{2} \Bigg\{ \mu_{t+1,\omega'(z)}^m \cdot \\ & \left((1-\tau)\left(P_{k,t+1,\omega'(z)} + D_{t+1,\omega'(z)}\right) + \tau P_{k,t,\omega}\right) \Bigg\} \stackrel{!}{=} 0 \\ \Leftrightarrow & \\ P_{k,t,\omega} =& \sum_{z=1}^{2} \frac{\mu_{t+1,\omega'(z)}^m}{\mu_{t,\omega}^m} \left((1-\tau)\left(P_{k,t+1,\omega'(z)} + D_{t+1,\omega'(z)}\right) + \tau P_{k,t,\omega}\right),\end{aligned} \tag{B.7}$$

where $\omega'(z)$ denotes the economic state in the subsequent period $t+1$ that is realized, if the z-th possible growth rate is realized from time t to $t+1$. Given the information set $\mathcal{F}_t$, the realization of state ω at time t is known and the pricing relation (B.7) can be restated in terms of expectations conditional on the information available at time t:

$$P_{k,t} = \mathbb{E}_t \left[\frac{2\mu_{t+1}^m}{\mu_t^m} \left((1-\tau)\left(P_{k,t+1} + D_{t+1}\right) + \tau P_{k,t}\right) \right], \tag{B.8}$$

with $P_{k,T} = 0$. This is Equation (3.21) in the main text.

Finally, differentiating (B.4) for agent type m with respect to his bond investment in period t and state ω, $\beta_{t,\omega}^m$, the following first order condition is found:

$$\frac{\partial \mathcal{L}^m}{\partial \beta^m_{t,\omega}} = -\mu^m_{t,\omega} + \sum_{z=1}^{2} \mu^m_{t+1,\omega'(z)} \tilde{R}_{f,t,\omega} \overset{!}{=} 0$$

$$\Leftrightarrow 1 = \sum_{z=1}^{2} \frac{\mu^m_{t+1,\omega'(z)}}{\mu^m_{t,\omega}} \tilde{R}_{f,t,\omega}. \tag{B.9}$$

Along the lines of Equation (B.8), the pricing relation (B.9) can be restated in terms of expectations conditional on the information set $\mathcal{F}_t$:

$$1 = \mathbb{E}_t \left[\frac{2\mu^m_{t+1}}{\mu^m_t} \right] \tilde{R}_{f,t}. \tag{B.10}$$

Condition (B.10) corresponds to Equation (3.22) in the text.

B.1.3 Derivation of the Pre-tax Stochastic Discount Factor

Since the realization of the uncertain event at date t is known under the information set at time t, i.e., $\mathcal{F}_t$, the stock price $P_{k,t}$ can be moved out of the conditional expectations operator $\mathbb{E}_t$ in the first order condition (B.8) in order to obtain

$$P_{k,t} = \mathbb{E}_t \left[\frac{2\mu^m_{t+1}}{\mu^m_t} (1-\tau) \left(P_{k,t+1} + D_{t+1} \right) \right] + \mathbb{E}_t \left[\frac{2\mu^m_{t+1}}{\mu^m_t} \right] \tau P_{k,t}. \tag{B.11}$$

Rearranging Equation (B.11) for $P_{k,t}$ and substituting condition (B.10) yields

$$\begin{aligned} P_{k,t} &= \frac{(1-\tau)}{1 - \mathbb{E}_t \left[\frac{2\mu^m_{t+1}}{\mu^m_t} \right] \tau} \mathbb{E}_t \left[\frac{2\mu^m_{t+1}}{\mu^m_t} \left(P_{k,t+1} + D_{t+1} \right) \right] \\ &= \frac{(1-\tau)}{1 - \frac{\tau}{\tilde{R}_{f,t}}} \mathbb{E}_t \left[\frac{2\mu^m_{t+1}}{\mu^m_t} \left(P_{k,t+1} + D_{t+1} \right) \right]. \end{aligned} \tag{B.12}$$

Then, using expression (3.8), the term outside the expectations turns out to be equivalent to $\tilde{R}_{f,t}/R_{f,t}$, from which it follows that the pricing relation for the risky stock is alternatively given by

$$P_{k,t} = \mathbb{E}_t \left[\frac{2\mu_{t+1}^m}{\mu_t^m} \frac{\tilde{R}_{f,t}}{R_{f,t}} \left(P_{k,t+1} + D_{t+1} \right) \right]. \tag{B.13}$$

This is equivalent to the first line of Equation (3.24) in the text.

B.1.4 Derivation of the Consumption Share Function

Recall that given one current economic state $\omega \in \Omega$, the assumptions regarding the growth rates G_z imply the realization of only two possible direct successive future states $\{\omega'(z)\}_{z=1}^2$ for every time step $t = 0, 1, \ldots, T$. Then, in order to determine the consumption share function, start by rewriting the pricing equations for the risk-free rate (B.9) and the stock price (B.13) at time t by substituting the definition of the SDF (3.23):

$$1 = \frac{\delta^m}{2} \sum_{z=1}^{2} \left(\frac{c_{t+1,\omega'(z)}^m}{c_{t,\omega}^m} \right)^{-\gamma} \tilde{R}_{f,t,\omega}, \tag{B.14}$$

$$PD_{k,t,\omega} = \frac{\tilde{R}_{f,t,\omega}}{R_{f,t,\omega}} \frac{\delta^m}{2} \sum_{z=1}^{2} \left(\frac{c_{t+1,\omega'(z)}^m}{c_{t,\omega}^m} \right)^{-\gamma} \left(P_{k,t+1,\omega'(z)} + D_{k,t+1,\omega'(z)} \right). \tag{B.15}$$

Equations (B.14)-(B.15) provide the pricing conditions in terms of both agents' consumption. Equating these conditions, respectively, for $m = 1$ and $m = 2$ and substituting $c_t^1 = g_t^1 D_t$ and $c_t^2 = (1 - g_t^1) D_t$ from the definition of consumption shares, the following two equations are obtained:

$$
\begin{aligned}
&\delta^1 \sum_{z=1}^{2} \left(\frac{g^1_{t+1,\omega'(z)}}{g^1_{t,\omega}} \right)^{-\gamma} \left(\frac{D_{k,t+1,\omega'(z)}}{D_{k,t,\omega}} \right)^{-\gamma} \\
=&\delta^2 \sum_{z=1}^{2} \left(\frac{\left(1-g^1_{t+1,\omega'(z)}\right)}{\left(1-g^1_{t,\omega}\right)} \right)^{-\gamma} \left(\frac{D_{k,t+1,\omega'(z)}}{D_{k,t,\omega}} \right)^{-\gamma}, \qquad \text{(B.16)}\\
&\delta^1 \sum_{z=1}^{2} \left(\frac{g^1_{t+1,\omega'(z)}}{g^1_{t,\omega}} \right)^{-\gamma} \left(\frac{D_{k,t+1,\omega'(z)}}{D_{k,t,\omega}} \right)^{-\gamma} \cdot \\
&\left(P_{k,t+1,\omega'(z)} + D_{k,t+1,\omega'(z)}\right) \\
=&\delta^2 \sum_{z=1}^{2} \left(\frac{\left(1-g^1_{t+1,\omega'(z)}\right)}{\left(1-g^1_{t,\omega}\right)} \right)^{-\gamma} \left(\frac{D_{k,t+1,\omega'(z)}}{D_{k,t,\omega}} \right)^{-\gamma} \cdot \\
&\left(P_{k,t+1,\omega'(z)} + D_{k,t+1,\omega'(z)}\right). \qquad \text{(B.17)}
\end{aligned}
$$

Conditional on state ω at date t, Equations (B.16)-(B.17) form a deterministic system of equations in two unknowns, namely the possible consumption shares of agent type $m = 1$ at date $t+1$ in states $\{\omega'(z)\}_{z=1}^{2}$.

Then, for the system of equations to be fulfilled it must either hold that the future payoffs from equity or consumption shares at time $t+1$ conditional on state ω at time t are identical, i.e., $\left(P_{k,t+1,\omega'(1)} + D_{k,t+1,\omega'(1)}\right) = \left(P_{k,t+1,\omega'(2)} + D_{k,t+1,\omega'(2)}\right)$ or $g^1_{t+1,\omega'(1)} = g^1_{t+1,\omega'(2)}$. The former condition implies that the period $t+1$ equity payoff is not uncertain but known at date t. It can be eliminated, since this in return implies that there would be only one non-redundant asset, which contradicts the assumptions made above on financial markets. Therefore,

$$
g^1_{t+1,\omega'(1)} = g^1_{t+1,\omega'(2)}, \quad t = 0, 1, ..., T-1, \qquad \text{(B.18)}
$$

is the only feasible solution. Using this equality in Equation (B.16) and rearranging, it follows that the consumption share at time $t+1$ is just a function of the consumption share in state ω at time t:

$$g^1_{t+1,\omega} \equiv g^1_{t+1,\omega'(z)} = \frac{g^1_{t,\omega}}{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{1}{\gamma}}\right) g^1_{t,\omega} + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{1}{\gamma}}}, \tag{B.19}$$

$$z = 1,2, \quad t = 0,1,...,T-1.$$

Solving backwards until date $t = 0$, the consumption share of agent type $m = 1$ in period t can be expressed in terms of his consumption share at the initial date $t = 0$:

$$g^1_t = \frac{g^1_0}{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right) g^1_0 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}, \quad t = 0,1,...,T, \tag{B.20}$$

which implies that in the present setting consumption shares are generally time-dependent, but state-independent.[137] Equation (3.27) in the main text follows.

Then, in order to proof Equation (3.29) in the text, differentiate Equation (B.20) with respect to t:

$$\frac{\partial g^1_t}{\partial t} = \ln\left(\frac{\delta^1}{\delta^2}\right)\left(\frac{1}{\gamma}\frac{\left(\frac{\delta^2}{\delta^1}\right)^{\frac{t}{\gamma}} g^1_0\left(1-g^1_0\right)}{\left(g^1_0 + \left(1-g^1_0\right)\left(\frac{\delta^2}{\delta^1}\right)^{\frac{t}{\gamma}}\right)^2}\right). \tag{B.21}$$

The assumption of CRRA preferences implies that an optimal solution has the property that consumption will be strictly positive (see, for example, Munk (2013, p. 171)). Therefore, for the agents' consumption shares it must hold that $0 < g^m_t < 1$, for all time steps $t = 0, 1, \ldots, T$ and agent types $m = 1, 2$. In combination with the parameter assumptions

[137] Note that the solution method applied here depends on the assumption of complete markets and the limitation to two individuals. Nevertheless, the general result of time-dependent, but state-independent, consumption shares is independent of these restrictions. A model setup with more agents and incomplete markets can be solved using an equivalent method as the one presented in Chapter 4.

$0 < \delta^m < 1$ and $\gamma > 0$, it follows that the second factor in brackets of Equation (B.21) is always positive. Hence, the sign of the derivative (B.21) can be determined by the sign of its first term, for which it holds that

$$\ln\left(\frac{\delta^1}{\delta^2}\right) = \begin{cases} > 0 & if\ \delta^1 > \delta^2, \\ < 0 & if\ \delta^1 < \delta^2. \end{cases} \tag{B.22}$$

Equation (3.29) in the main text follows immediately.

B.1.5 Properties of the Risk-free Return

In order to show that the one-period risk-free return is a strictly monotonically decreasing function in time, its analytical solution in Equation (3.35) is first restated below:

$$\tilde{R}_{f,t} = \left(\frac{\left(1 - \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right) g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}{\left(1 - \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+1}{\gamma}}\right) g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+1}{\gamma}}} \right)^{\gamma} \left(\delta^1 \mathbb{E}\left[G^{-\gamma}\right]\right)^{-1}. \tag{B.23}$$

Then, differentiating Equation (B.23) with respect to time t and simplifying, the following derivative is obtained:

$$\frac{\partial \tilde{R}_{f,t}}{\partial t} = \frac{\left(1 - g_0^1\right) g_0^1 \left(\frac{\delta^2}{\delta^1}\right)^{\frac{t}{\gamma}}}{\gamma \left(g_0^1 + \left(1 - \left(\frac{\delta^2}{\delta^1}\right)^{\frac{t+1}{\gamma}}\right) g_0^1\right)^2} \left(\left(\frac{\delta^2}{\delta^1}\right)^{\frac{1}{\gamma}} - 1\right) \ln\left(\frac{\delta^1}{\delta^2}\right). \tag{B.24}$$

Since it holds that $0 < g_t^m < 1$, $0 < \delta^m < 1$ and $\gamma > 0$, the first factor in Equation (B.24) is always positive. Hence, the sign of the derivative (B.24) can be determined by the sign of the remaining factors. Assuming that $\gamma < \infty$, it holds for the second factor that

$$\left(\frac{\delta^2}{\delta^1}\right)^{\frac{1}{\gamma}} - 1 = \begin{cases} < 0 & if\ \delta^1 > \delta^2, \\ > 0 & if\ \delta^1 < \delta^2. \end{cases} \tag{B.25}$$

In combination with Equation (B.22) it follows that $\partial \tilde{R}_{f,t}/\partial t < 0$, which implies that the one-period risk-free return after tax is a strictly monotonically decreasing function in t.

B.1.6 Derivation Total Wealth Budget Constraint

The derivation of the total wealth budget constraint follows Miao (2014). First, multiply by $2^n \mu_{t+n}$ on both sides of the dynamic budget constraint (3.15) at date $t+n$ of agent m and take conditional expectations at time t in order to obtain

$$\begin{aligned} \mathbb{E}_t \left[2^n \mu_{t+n} c^m_{t+n}\right] = \\ \mathbb{E}_t \Big[2^n \mu_{t+n} \Big(\alpha^m_{t+n-1} \left((1-\tau)\left(P_{k,t+n} + D_{t+n}\right) + \tau P_{k,t+n-1}\right) + \\ \beta^m_{t+n-1} \tilde{R}_{f,t+n-1} + s_{t+n} - \alpha^m_{t+n} P_{k,t+n} - \beta^m_{t+n}\Big)\Big]. \end{aligned} \tag{B.26}$$

Next, Equation (B.26) is added up over the remaining lifespan of the economy, i.e., $n = t, ..., T-t$, and divided by μ_t on both sides to get

$$\begin{aligned} \sum_{n=0}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} c^m_{t+n}\right] = \\ \sum_{n=0}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} \alpha^m_{t+n-1} \left((1-\tau)\left(P_{k,t+n} + D_{t+n}\right) + \tau P_{k,t+n-1}\right)\right] + \\ \sum_{n=0}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} \beta^m_{t+n-1} \tilde{R}_{f,t+n-1}\right] + \sum_{n=1}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} s_{t+n}\right] + s_t - \\ \sum_{n=0}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} \alpha^m_{t+n} P_{k,t+n}\right] - \sum_{n=0}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} \beta^m_{t+n}\right], \end{aligned} \tag{B.27}$$

where

$$P_{s,t} \equiv \sum_{n=1}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} s_{t+n} \right] \tag{B.28}$$

is Equation (3.46) in the text. Then, note that by using the law of iterated expectations (see, for example, Munk (2013, p. 30)) and substituting the basic pricing condition for the stock (3.21) the first term of Equation (B.27) can be rewritten to:

$$\begin{aligned}
&\sum_{n=0}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} \alpha^m_{t+n-1} \left((1-\tau) \left(P_{k,t+n} + D_{t+n} \right) + \tau P_{k,t+n-1} \right) \right] = \\
&\alpha^m_{t-1} \left((1-\tau) \left(P_{k,t} + D_t \right) + \tau P_{k,t-1} \right) + \sum_{n=1}^{T-t} \mathbb{E}_t \left[\frac{2^{n-1} \mu_{t+n-1}}{\mu_t} \alpha^m_{t+n-1} \cdot \right. \\
&\left. \mathbb{E}_{t+n-1} \left[\frac{2 \mu_{t+n}}{\mu_{t+n-1}} \left((1-\tau) \left(P_{k,t+n} + D_{t+n} \right) + \tau P_{k,t+n-1} \right) \right] \right] = \\
&\alpha^m_{t-1} \left((1-\tau) \left(P_{k,t} + D_t \right) + \tau P_{k,t-1} \right) + \sum_{n=0}^{T-t-1} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} \alpha^m_{t+n} P_{k,t+n} \right].
\end{aligned} \tag{B.29}$$

Applying a similar procedure to the second term of Equation (B.27) and using the basic pricing condition for the bond (3.22) one obtains:

$$\begin{aligned}
&\sum_{n=0}^{T-t} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} \beta^m_{t+n-1} \tilde{R}_{f,t+n-1} \right] = \\
&\beta^m_{t-1} \tilde{R}_{f,t-1} + \sum_{n=1}^{T-t} \mathbb{E}_t \left[\frac{2^{n-1} \mu_{t+n-1}}{\mu_t} \beta^m_{t+n-1} \mathbb{E}_{t+n-1} \left[\frac{2 \mu_{t+n}}{\mu_{t+n-1}} \tilde{R}_{f,t+n-1} \right] \right] = \\
&\beta^m_{t-1} \tilde{R}_{f,t-1} + \sum_{n=0}^{T-t-1} \mathbb{E}_t \left[\frac{2^n \mu_{t+n}}{\mu_t} \beta^m_{t+n} \right].
\end{aligned} \tag{B.30}$$

Finally, substituting the expressions just derived in Equation (B.27) it simplifies to

$$\begin{aligned} w_t^m \equiv & \alpha_{t-1}^m \left((1-\tau) \left(P_{k,t} + D_t \right) + \tau P_{k,t-1} \right) + \beta_{t-1}^m \tilde{R}_{f,t-1} + \\ & s_t + P_{s,t} - \mathbb{E}_t \left[\frac{2^{T-t} \mu_T}{\mu_t} \left(\alpha_T^m P_{k,T} + \beta_T^m \right) \right] \\ = & \alpha_{t-1}^m \left((1-\tau) \left(P_{k,t} + D_t \right) + \tau P_{k,t-1} \right) + \beta_{t-1}^m \tilde{R}_{f,t-1} + s_t + P_{s,t}, \end{aligned} \tag{B.31}$$

where the last term in the second line can be dropped due to the agent's terminal portfolio condition (3.16). Equation (B.31) corresponds to Equation (3.44) in the main text.

B.1.7 Derivation Price-Dividend Ratio Transfer Capital

The derivation of the solution to the price-dividend ratio for the nontraded transfer capital asset follows analogously to the derivation of the price-dividend ratio for the stock in Section 3.2.2.2. Again, start by dividing both sides of the pricing Equation (3.46) by D_t in order to define the price-dividend ratio, $PD_{s,t} \equiv P_{s,t}/D_t$, by :

$$PD_{s,t} = \mathbb{E}_t \left[\frac{2\mu_{t+1}}{\mu_t} \left(PD_{s,t+1} + \frac{s_{t+1}}{D_{t+1}} \right) G_{t+1} \right]. \tag{B.32}$$

Then, using the equilibrium SDF process (3.33) as well as current individual transfers (3.9) and simplifying, one obtains

$$\begin{aligned} PD_{s,t} = & \delta^m \left(\frac{g_{t+1}^m}{g_t^m} \right)^{-\gamma} \left(\mathbb{E}_t \left[PD_{s,t+1} G_{t+1}^{1-\gamma} \right] + \right. \\ & \left. \frac{\tau}{2} \left(PD_{k,t+1} + 1 \right) \mathbb{E}_t \left[G_{t+1}^{1-\gamma} \right] - \frac{\tau}{2} PD_{k,t} \mathbb{E}_t \left[G_{t+1}^{-\gamma} \right] \right), \end{aligned} \tag{B.33}$$

where the result of state-independent stock price-dividend ratios has been used (see Section 3.2.2.2). Having established Equation (B.33), the transfer capital price-dividend ratio can be derived by backward induction starting from the horizon. Since there are no future payoffs at the terminal date T, it holds that $P_{s,T} = PD_{s,T} = 0$ and the price-dividend ratio at date $T-1$ is given by

$$PD_{s,T-1} = \delta^m \left(\frac{g_T^m}{g_{T-1}^m}\right)^{-\gamma} \frac{\tau}{2}\left(\mathbb{E}\left[G^{1-\gamma}\right] - PD_{k,T-1}\mathbb{E}\left[G^{-\gamma}\right]\right), \quad \text{(B.34)}$$

where the assumption of i.i.d. consumption growth has been used again. Equation (B.34) shows that the transfer capital price-dividend ratio at date $T-1$ is state-independent. Applying this result to Equation (B.33) and iterating backwards through time, $PD_{s,t}$ is found to be generally state-independent for all time steps t and given by the following recurrence equation:

$$\begin{aligned} PD_{s,t} =& \delta^m \left(\frac{g_{t+1}^m}{g_t^m}\right)^{-\gamma} \Big(PD_{s,t+1}\mathbb{E}\left[G^{1-\gamma}\right] + \\ & \frac{\tau}{2}\left(PD_{k,t+1}+1\right)\mathbb{E}\left[G^{1-\gamma}\right] - \frac{\tau}{2}PD_{k,t}\mathbb{E}\left[G^{-\gamma}\right]\Big) \\ =& \tilde{R}_{f,t}^{-1}\Bigg(PD_{s,t+1}\frac{\mathbb{E}\left[G^{1-\gamma}\right]}{\mathbb{E}\left[G^{-\gamma}\right]} + \\ & \frac{\tau}{2}\left(PD_{k,t+1}+1\right)\frac{\mathbb{E}\left[G^{1-\gamma}\right]}{\mathbb{E}\left[G^{-\gamma}\right]} - \frac{\tau}{2}PD_{k,t}\Bigg), \end{aligned} \quad \text{(B.35)}$$

where the last line is due to the definition of the risk-free rate given in Equation (3.34). Furthermore, from the recurrence Equation of the stock price-dividend ratio (3.41) it follows that

$$PD_{k,t+1} + 1 = PD_{k,t}R_{f,t}\frac{\mathbb{E}\left[G^{-\gamma}\right]}{\mathbb{E}\left[G^{1-\gamma}\right]}. \quad \text{(B.36)}$$

Substituting this result into Equation (B.35), the price-dividend ratio for the nontraded transfer capital at date t reads

$$PD_{s,t} = \tilde{R}_{f,t}^{-1}\left(PD_{s,t+1}\frac{\mathbb{E}\left[G^{1-\gamma}\right]}{\mathbb{E}\left[G^{-\gamma}\right]} + \frac{\tau}{2}\left(R_{f,t}-1\right)PD_{k,t}\right), \qquad \text{(B.37)}$$

which is Equation (3.47) in the main text. As in the case of the stock, an explicit representation can be derived by starting at the horizon (using the fact that $PD_{s,T} = 0$) and solving the recursive definition (B.37) by backward induction. Solution (3.48) in the main text follows.

B.1.8 Derivation Marginal Propensity to Consume out of Total Wealth

To demonstrate the derivation of the MPCTW, first note that the n-period expression for the SDF follows from its one-period counterpart (3.23) by:

$$\begin{aligned}\frac{2^n\mu_{t+n}}{\mu_t} &= \frac{2\mu_{t+1}}{\mu_t}\frac{2\mu_{t+2}}{\mu_{t+1}}\cdots\frac{2\mu_{t+n}}{\mu_{t+n-1}} \\ &= \delta^m\left(\frac{c_{t+1}^m}{c_t^m}\right)^{-\gamma}\delta^m\left(\frac{c_{t+2}^m}{c_{t+1}^m}\right)^{-\gamma}\cdots\delta^m\left(\frac{c_{t+n}^m}{c_{t+n-1}^m}\right)^{-\gamma} \\ &= \left(\delta^m\right)^n\left(\frac{c_{t+n}^m}{c_t^m}\right)^{-\gamma}. \qquad \text{(B.38)}\end{aligned}$$

Then, divide both sides of the total wealth budget constraint (3.43) by current consumption c_t^m and substitute the expression just derived, Equation (B.38), in order to obtain

$$\begin{aligned}\frac{w_t^m}{c_t^m} &= \sum_{n=0}^{T-t}\mathbb{E}_t\left[\frac{2^n\mu_{t+n}}{\mu_t}\frac{c_{t+n}^m}{c_t^m}\right] \\ &= \sum_{n=0}^{T-t}\mathbb{E}_t\left[\left(\delta^m\right)^n\left(\frac{c_{t+n}^m}{c_t^m}\right)^{1-\gamma}\right], \qquad \text{(B.39)}\end{aligned}$$

which is Equation (3.51) in the text. Next, note that the n-period expression for individual consumption growth follows from its one-period counterpart (3.50) by:

$$\frac{c_{t+n}^m}{c_t^m} = \frac{c_{t+1}^m}{c_t^m}\frac{c_{t+2}^m}{c_{t+1}^m}\cdots\frac{c_{t+n}^m}{c_{t+n-1}^m}$$
$$= \left(\frac{\delta^m}{\delta^1}\right)^{\frac{n}{\gamma}} \frac{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right)g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}\right)g_0^1 + \left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}}\prod_{l=1}^{n} G_{t+l}. \tag{B.40}$$

Using this expression along with Equation (B.39) it follows that:

$$\frac{w_t^m}{c_t^m} =$$
$$\sum_{n=0}^{T-t}\mathbb{E}_t\left[(\delta^m)^n\left(\left(\frac{\delta^m}{\delta^1}\right)^{\frac{n}{\gamma}}\frac{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right)g_0^1+\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}\right)g_0^1+\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}}\prod_{l=1}^{n}G_{t+l}\right)^{1-\gamma}\right] =$$
$$\sum_{n=0}^{T-t}\mathbb{E}_t\left[\left(\frac{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right)g_0^1+\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}\right)g_0^1+\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}}\right)^{1-\gamma}\left((\delta^m)^{\frac{1}{\gamma}}\left(\delta^1\right)^{\frac{\gamma-1}{\gamma}}\right)^n\left(\prod_{l=1}^{n}G_{t+l}^{1-\gamma}\right)\right] =$$
$$\sum_{n=0}^{T-t}\left(\frac{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right)g_0^1+\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}\right)g_0^1+\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}}\right)^{1-\gamma}\left((\delta^m)^{\frac{1}{\gamma}}\left(\delta^1\right)^{\frac{\gamma-1}{\gamma}}\right)^n\mathbb{E}_t\left[\left(\prod_{l=1}^{n}G_{t+l}^{1-\gamma}\right)\right] =$$
$$\sum_{n=0}^{T-t}\left(\frac{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}\right)g_0^1+\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t}{\gamma}}}{\left(1-\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}\right)g_0^1+\left(\frac{\delta_2}{\delta_1}\right)^{\frac{t+n}{\gamma}}}\right)^{1-\gamma}\left((\delta^m)^{\frac{1}{\gamma}}\left(\delta^1\right)^{\frac{\gamma-1}{\gamma}}\mathbb{E}\left[\left(G^{1-\gamma}\right)\right]\right)^n, \tag{B.41}$$

where the second equality is due to power law, the third equality follows since only future production growth $\{G_{t+l}\}_{l=1}^n$ is unknown at time t, and the last equation is a consequence of the i.i.d. property of G. Finally, taking the reciprocal of expression (B.41), Equation (3.52) in the main text follows.

B.1.9 Optimal Expected Lifetime Utility

In order to derive the solution for the indirect utility at date t of an agent of type m, start by rearranging Equation (3.14) and use the definition of the MPCTW as the reciprocal of Equation (3.51):

$$\begin{aligned} V_0^m =& \sum_{t=0}^{T} (\delta^m)^t \, \mathbb{E}_0 \left[\frac{(c_t^m)^{1-\gamma}}{1-\gamma} \right] \\ =& \sum_{t=0}^{T} \mathbb{E}_0 \left[(\delta^m)^t \left(\frac{c_t^m}{c_0^m} \right)^{1-\gamma} \right] \frac{(c_0^m)^{1-\gamma}}{1-\gamma} \\ =& \, (b_0^m)^{-1} \frac{(c_0^m)^{1-\gamma}}{1-\gamma}. \end{aligned} \tag{B.42}$$

Finally, recall that the definition of individual consumption shares implies $c_t^m = g_t^m D_t$ and apply this to Equation (B.42) in order to derive:

$$V_0^m = (b_0^m)^{-1} \frac{(g_0^m)^{1-\gamma}}{1-\gamma} D_0^{1-\gamma}, \tag{B.43}$$

which is agent m's maximum expected lifetime utility given by Equation (3.61) in the main text.

B.1.10 Clearing on the Stock Market

Given clearing on the market for consumption goods, clearing on the stock market follows. To see this, substitute budget constraint (3.15) along with optimal bond investment (3.55) into the clearing condition for consumption (3.17). Dividing both sides by D_t and simplifying the following condition is obtained:

$$\begin{aligned}
g_t^1 + g_t^2 = 1 = \\
\left(\alpha_{t-1}^1 (1-\tau) + \frac{\tau}{2}\right)(PD_{k,t}+1) - \alpha_t^1 PD_{k,t} + \frac{\tau}{\tilde{R}_{f,t}}\left(\frac{1}{2} - \alpha_t^1\right) PD_{k,t} + \\
\left(\alpha_{t-1}^2 (1-\tau) + \frac{\tau}{2}\right)(PD_{k,t}+1) - \alpha_t^2 PD_{k,t} + \frac{\tau}{\tilde{R}_{f,t}}\left(\frac{1}{2} - \alpha_t^2\right) PD_{k,t} = \\
\left(\left(\alpha_{t-1}^1 + \alpha_{t-1}^2\right)(1-\tau) + \tau\right)(PD_{k,t}+1) - \left(\alpha_t^1 + \alpha_t^2\right) PD_{k,t} + \\
\frac{\tau}{\tilde{R}_{f,t}}\left(1 - \left(\alpha_t^1 + \alpha_t^2\right)\right) PD_{k,t}.
\end{aligned} \tag{B.44}$$

Since $PD_{k,t} = PD_{s,t} = 0$, clearing on the stock market at the horizon $T-1$ follows:

$$\begin{aligned}
1 =& \left(\left(\alpha_{T-1}^1 + \alpha_{T-1}^2\right)(1-\tau) + \tau\right) \\
\Leftrightarrow 1 =& \alpha_{T-1}^1 + \alpha_{T-1}^2.
\end{aligned} \tag{B.45}$$

Then, applying this result to Equation (B.44) at $T-1$, clearing at date $T-2$ follows:

$$\begin{aligned}
1 =& \left(\left(\alpha_{T-2}^1 + \alpha_{T-2}^2\right)(1-\tau) + \tau\right)(PD_{k,T-1}+1) - PD_{k,T-1} \\
\Leftrightarrow 1 =& \alpha_{T-2}^1 + \alpha_{T-2}^2.
\end{aligned} \tag{B.46}$$

Continuing that way and working backwards through time, clearing on the stock market follows subsequently for any time step t.

B.2 Redistributive Taxation in a Production Economy with Overlapping Generations

B.2.1 Clearing on the Market for Wealth and Consumption Goods

Given the present model structure, the initially high complexity of the equilibrium solution can be reduced due to the fact that clearing on the asset markets, conditions (4.36)-(4.37), readily imply clearing on the markets for consumption goods and wealth, conditions (4.34)-(4.35).

To see this, assume clearing on the asset markets, substitute the evolution of individual wealth, Equation (4.27), into clearing condition (4.35) and apply the terminal portfolio condition (4.33) to get:

$$\begin{aligned}\sum_{m=1}^{M}\sum_{i=t-N}^{t} v_t^{i,m} =& \sum_{m=1}^{M}\sum_{i=t-N}^{t-1} \alpha_{t-1}^{i,m}\left(\left(1-\tau_c\right)D_t+\tau_c I_{t-1}\right) \\ &+\sum_{m=1}^{M}\sum_{i=t-N}^{t-1} \beta_{t-1}^{i,m}\tilde{R}_{f,t-1}+\left(1-\tau_l\right)\sum_{m=1}^{M}\sum_{i=t-N}^{t} f_{t-i}^{m} H_t \\ &+\sum_{m=1}^{M}\sum_{i=t-N}^{t} d_{t-i} S_t. \end{aligned} \tag{B.47}$$

Making use of the equilibrium condition for the distribution of income streams, $\sum_{m=1}^{M}\sum_{i=t-N}^{t} f_{t-i}^{m}=1$ and $\sum_{m=1}^{M}\sum_{i=t-N}^{t} d_{t-i}^{m}=1$, as well as clearing on the bond and equity market, conditions (4.36)-(4.37), Equation (B.47) simplifies to

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t} v_t^{i,m} = \left(\left(1-\tau_c\right)D_t+\tau_c I_{t-1}\right)+\left(1-\tau_l\right)H_t+S_t. \tag{B.48}$$

Next, using the definition of aggregate earnings (4.10), capital income (4.11) and tax revenues (4.20), it follows that:

$$\begin{aligned}\sum_{m=1}^{M}\sum_{i=t-N}^{t} v_t^{i,m} =& \left(1-\tau_c\right)\theta Y_t + \tau_c I_{t-1} + \left(1-\tau_l\right)\left(1-\theta\right)Y_t + \\ & \tau_c\left(\theta Y_t - I_{t-1}\right) + \tau_l\left(1-\theta\right)Y_t \\ =& Y_t, \end{aligned} \tag{B.49}$$

which demonstrates that clearing with respect to wealth, condition (4.35), is a direct consequence of asset market clearing.

Then, in order to show that clearing on the market for consumption goods results from clearing on all other markets, substitute agent's budget constraint (4.32) into equation (4.34) and apply the terminal portfolio condition (4.33) to obtain:

$$\begin{aligned}\sum_{m=1}^{M}\sum_{i=t-N}^{t} c_t^{i,m} =& \sum_{m=1}^{M}\sum_{i=t-N}^{t} v_t^{i,m} - \sum_{m=1}^{M}\sum_{i=t-N}^{t-1} \alpha_t^{i,m} I_t - \sum_{m=1}^{M}\sum_{i=t-N}^{t-1} \beta_t^{i,m} \\ =& Y_t - I_t, \end{aligned} \tag{B.50}$$

where the second line results from clearing on the asset markets as well as clearing with respect to wealth. Finally, by making use of the definitions of aggregate consumption (4.5) and investment (4.6), it follows that:

$$\begin{aligned}\sum_{m=1}^{M}\sum_{i=t-N}^{t} c_t^{i,m} =& Y_t - I_t \\ =& \left(1-X_t\right)Y_t \\ =& C_t, \end{aligned} \tag{B.51}$$

which proves that clearing on the market for consumption goods, condition (4.34), is a direct result of clearing on the other markets.

To sum up, first, clearing with respect to wealth originates from equity and bond market clearing. Second, clearing on the market for consumption goods results from clearing on the asset markets along with clearing with respect to wealth.

B.2.2 The Government's Optimization Problem

The government minimizes the quadratic optimization problem (4.14) subject to (4.15) and (4.16). The Lagrangian associated with the constrained problem is given by

$$\begin{aligned}\mathcal{L}^{\mathcal{G}} =& \sum_{m=1}^{M}\sum_{i=t-N}^{t}\left\{\left(\tilde{k}_t^{i,m} - d_{t-i}\left(D_t - I_{t-1}\right)\right)^2 + \kappa_c\left(\tilde{k}_t^{i,m} - k_t^{i,m}\right)^2 + \right.\\ &\left.\left(\tilde{h}_t^{i,m} - d_{t-i}H_t\right)^2 + \kappa_l\left(\tilde{h}_t^{i,m} - h_t^{i,m}\right)^2\right\} - \\ &\Lambda_c\left\{\sum_{m=1}^{M}\sum_{i=t-N}^{t}\tilde{k}_t^{i,m} - \left(D_t - I_{t-1}\right)\right\} - \Lambda_l\left\{\sum_{m=1}^{M}\sum_{i=t-N}^{t}\tilde{h}_t^{i,m} - H_t\right\},\end{aligned} \tag{B.52}$$

where Λ_c and Λ_l are the Lagrangian multipliers. The first order conditions with respect to $\tilde{k}_t^{i,m}$ and $\tilde{h}_t^{i,m}$ are

$$\frac{\partial\mathcal{L}^{\mathcal{G}}}{\partial\tilde{k}_t^{i,m}} = 2\left(\tilde{k}_t^{i,m} - d_{t-i}\left(D_t - I_{t-1}\right)\right) + 2\kappa_c\left(\tilde{k}_t^{i,m} - k_t^{i,m}\right) - \Lambda_c \overset{!}{=} 0, \tag{B.53}$$

$$\frac{\partial\mathcal{L}^{\mathcal{G}}}{\partial\tilde{h}_t^{i,m}} = 2\left(\tilde{h}_t^{i,m} - d_{t-i}H_t\right) + 2\kappa_l\left(\tilde{h}_t^{i,m} - h_t^{i,m}\right) - \Lambda_l \overset{!}{=} 0, \tag{B.54}$$

respectively. Taking the sum over all agent types m and all generations $i \leq t$ alive at date t the Lagrangian multipliers turn out to be zero, i.e., $\Lambda_c, \Lambda_l = 0$. Using this result in conditions (B.53) and (B.54) again and solving for $\tilde{k}_t^{i,m}$ and $\tilde{h}_t^{i,m}$ yields the linear feedback rules

$$\tilde{k}_t^{i,m} = \frac{\kappa_c}{1+\kappa_c} k_t^{i,m} + \frac{1}{1+\kappa_c} d_{t-i} \left(D_t - I_{t-1} \right), \tag{B.55}$$

$$\tilde{h}_t^{i,m} = \frac{\kappa_l}{1+\kappa_l} h_t^{i,m} + \frac{1}{1+\kappa_l} d_{t-i} H_t, \tag{B.56}$$

which are Equations (4.18) and (4.19) in the main text.

B.2.3 Derivation of First Order Conditions

Along the lines of proof B.1.2, there exists a finite and discrete state space of possible outcomes Ω underlying the economy, where element $\omega \in \Omega$ denotes one possible state that might be realized. The agent's optimization problem is given by Equation (4.31) subject to the constraints (4.32)-(4.33) and (4.27)-(4.28) and the associated Lagrangian at time t of an agent of type m born in period $t - N \leq i \leq t$ can be written as

$$\begin{aligned}
\mathcal{L}_t^{i,m} = & \frac{(c_t^{i,m})^{1-\gamma}}{1-\gamma} + \sum_{n=1}^{N-(t-i)} \mathbb{E}_t \left[\left(\prod_{l=0}^{n-1} \delta^m_{(t-i)+l} \right) \frac{(c_{t+n}^{i,m})^{1-\gamma}}{1-\gamma} \right] - \\
& \mu_t^{i,m} \Big(c_t^{i,m} - \alpha_{t-1}^{i,m} \left((1-\tau_c) D_t + \tau_c I_{t-1} \right) - \beta_{t-1}^{i,m} \tilde{R}_{f,t-1} - \\
& (1-\tau_l) h_t^{i,m} - s_t^i + \alpha_t^{i,m} I_t + \beta_t^{i,m} \Big) - \sum_{n=1}^{N-(t-i)} \Big\langle \mu_{t+n}^{i,m}, c_{t+n}^{i,m} - \\
& \alpha_{t+n-1}^{i,m} \left((1-\tau_c) D_{t+n} + \tau_c I_{t+n-1} \right) - \beta_{t+n-1}^{i,m} \tilde{R}_{f,t+n-1} \Big\rangle - \\
& \sum_{n=1}^{N-(t-i)} \Big\langle \mu_{t+n}^{i,m}, (1-\tau_l) h_{t+n}^{i,m} + s_{t+n}^i - \alpha_{t+n}^{i,m} I_{t+n} - \beta_{t+n}^{i,m} \Big\rangle,
\end{aligned} \tag{B.57}$$

where $\alpha_{t-1}^{t,m} = \beta_{t-1}^{t,m} = s_t^t = 0$ and $\alpha_{N+i}^{i,m} = \beta_{N+i}^{i,m} = 0$. $\{\mu_{t+n}^{i,m}\}_{n=0}^{N-(t-i)}$ are the Lagrangian multipliers associated with the constraints and $\langle \cdot, \cdot \rangle$ is the scalar product over the relevant states as in Fischer and Jensen (2014, 2015, 2017). Having established (B.57), the first order conditions can be derived by optimizing state-by-state.

Recall that there is a finite number of realizations $z = 1, ..., Z$ of the random productivity shock A_z and that each realization occurs with equal probability $1/Z$. Then, differentiating the Lagrangian (B.57) at date t for an agent of type m born in period $t - N < i \leq t$ with respect to consumption at $t + n$ and state ω, $c^{i,m}_{t+n,\omega}$, the following first order condition is derived:

$$\begin{aligned} \frac{\partial \mathcal{L}^{i,m}_t}{\partial c^{i,m}_{t+n,\omega}} &= \left(\prod_{l=0}^{n-1} \delta^m_{(t-i)+l}\right) \left(\frac{1}{Z}\right)^n \left(c^{i,m}_{t+n,\omega}\right)^{-\gamma} - \mu^{i,m}_{t+n,\omega} \stackrel{!}{=} 0 \\ \Leftrightarrow \mu^{i,m}_{t+n,\omega} &= \left(\prod_{l=0}^{n-1} \delta^m_{(t-i)+l}\right) \left(\frac{1}{Z}\right)^n \left(c^{i,m}_{t+n,\omega}\right)^{-\gamma}. \end{aligned} \tag{B.58}$$

Since this equation holds equally across states, the relation can also be written as

$$\mu^{i,m}_{t+n} = \left(\prod_{l=0}^{n-1} \delta^m_{(t-i)+l}\right) \left(\frac{1}{Z}\right)^n \left(c^{i,m}_{t+n}\right)^{-\gamma}, \tag{B.59}$$

which is Equation (4.40) in the main text. Next, differentiating the Lagrangian (B.57) at date t for an agent of type m born in period $t-N < i \leq t$ with respect to his equity share in period t and state ω, $\alpha^{i,m}_{t,\omega}$, it follows that:

$$\begin{aligned} \frac{\partial \mathcal{L}^{i,m}_t}{\partial \alpha^{i,m}_{t,\omega}} &= -\,\mu^{i,m}_{t,\omega} I_{t,\omega} + \sum_{z=1}^{Z} \mu^{i,m}_{t+1,\omega'(z)} \left(\left(1-\tau_c\right) D_{t+1,\omega'(z)} + \tau_c I_{t,\omega}\right) \stackrel{!}{=} 0 \\ \Leftrightarrow I_{t,\omega} &= \sum_{z=1}^{Z} \frac{\mu^{i,m}_{t+1,\omega'(z)}}{\mu^{i,m}_{t,\omega}} \left(\left(1-\tau_c\right) D_{t+1,\omega'(z)} + \tau_c I_{t,\omega}\right), \end{aligned} \tag{B.60}$$

where $\omega'(z)$ denotes the economic state in the subsequent period $t+1$ that is realized, if the z-th possible growth rate is realized from time t to $t+1$.

Given the information set $\mathcal{F}_t$ the realization of state ω at time t is known and the pricing relation (B.60) can be restated in terms of expectations conditional on the information available at time t:

$$I_t = \mathbb{E}_t \left[\frac{Z \mu_{t+1}^{i,m}}{\mu_t^{i,m}} \left((1-\tau_c) D_{t+1} + \tau_c I_t \right) \right]. \tag{B.61}$$

Dividing both sides of relation (B.61) by I_t and using Equation (4.13) one derives at

$$1 = \mathbb{E}_t \left[\frac{Z \mu_{t+1}^{i,m}}{\mu_t^{i,m}} \left((1-\tau_c) R_{E,t+1} + \tau_c \right) \right]. \tag{B.62}$$

This is condition (4.41) in the main text. Finally, differentiating (B.57) for an agent of type m born in period $t - N < i \leq t$ with respect to his bond share in period t and state ω, $\beta_{t,\omega}^{i,m}$, the following first order condition is found:

$$\begin{aligned} \frac{\partial \mathcal{L}_t^{i,m}}{\partial \beta_{t,\omega}^{i,m}} &= -\mu_{t,\omega}^{i,m} + \sum_{z=1}^{Z} \mu_{t+1,\omega'(z)}^{i,m} \tilde{R}_{f,t,\omega} \stackrel{!}{=} 0 \\ \Leftrightarrow 1 &= \sum_{z=1}^{Z} \frac{\mu_{t+1,\omega'(z)}^{i,m}}{\mu_{t,\omega}^{i,m}} \tilde{R}_{f,t,\omega}. \end{aligned} \tag{B.63}$$

Like for condition (B.61), Equation (B.63) is restated in terms of expectations conditional on the information set $\mathcal{F}_t$, which yields pricing relation (4.42) in the text:

$$1 = \mathbb{E}_t \left[\frac{Z \mu_{t+1}^{i,m}}{\mu_t^{i,m}} \right] \tilde{R}_{f,t}. \tag{B.64}$$

B.2.4 Derivation of the Pre-tax Stochastic Discount Factor

Given the information set at time t, i.e., $\mathcal{F}_t$, the aggregate investment at date t is known and thus not random. Consequently, it follows that I_t can be moved out of the expectations operator $\mathbb{E}_t$ in the first order condition (B.61) in order to obtain

$$I_t = \mathbb{E}_t\left[\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}}\left(1-\tau_c\right)D_{t+1}\right] + \mathbb{E}_t\left[\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}}\right]\tau_c I_t. \tag{B.65}$$

Rearranging Equation (B.65) for I_t yields

$$\begin{aligned} I_t &= \frac{(1-\tau_c)}{1-\mathbb{E}_t\left[\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}}\right]\tau_c}\mathbb{E}_t\left[\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}}D_{t+1}\right] \\ &= \frac{(1-\tau_c)}{1-\frac{\tau_c}{\tilde{R}_{f,t}}}\mathbb{E}_t\left[\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}}D_{t+1}\right], \end{aligned} \tag{B.66}$$

where the second line follows from condition (B.64). Using expression (4.24) it can be shown that the term outside the expectations is equivalent to $\tilde{R}_{f,t}/R_{f,t}$, from which it follows that

$$I_t = \mathbb{E}_t\left[\frac{Z\mu_{t+1}^{i,m}}{\mu_t^{i,m}}\frac{\tilde{R}_{f,t}}{R_{f,t}}D_{t+1}\right]. \tag{B.67}$$

This is equivalent to the first line of Equation (4.46) in the text.

B.2.5 Derivation of Condition (4.59)

Equation (4.58) will be increasing in the capital gains tax as long as its first derivative with respect to τ_c is larger than zero, i.e.,

$$\frac{\partial G_1}{\delta \tau_c} = \left(\frac{\left(\delta_0^1\right)^{-\frac{1}{\gamma}} \Xi_1}{\gamma} \right) \frac{\left(\theta \Xi_1 \mathbb{E}\left[A^{1-\gamma}\right] - \mathbb{E}\left[A^{-\gamma}\right]\right)}{\mathbb{E}\left[(1-\tau_c)\theta \Xi_1 A^{1-\gamma} + \tau_c A^{-\gamma}\right]^{\frac{1+\gamma}{\gamma}}} > 0. \quad \text{(B.68)}$$

This is true if either only one factor or all three factors in (B.68) are larger than zero. Recall that the following assumptions regarding the model parameters were assumed: $\delta_{t-i}^m, \gamma > 0$ and $0 < \theta, \tau_c, \tau_l < 1$. From this it follows that the first factor in (B.68) is always positive, i.e.,

$$\left(\frac{\left(\delta_0^1\right)^{-\frac{1}{\gamma}} \Xi_1}{\gamma} \right) > 0. \quad \text{(B.69)}$$

Next, the third term is also positive if it holds that

$$\begin{aligned} (1-\tau_c)\theta \Xi_1 \mathbb{E}\left[A^{1-\gamma}\right] + \tau_c \mathbb{E}\left[A^{-\gamma}\right] &> 0 \\ \Leftrightarrow \frac{\mathbb{E}\left[A^{-\gamma}\right]}{\mathbb{E}\left[A^{1-\gamma}\right]} &> -\frac{(1-\tau_c)\theta \Xi_1}{\tau_c}. \end{aligned} \quad \text{(B.70)}$$

On the one hand, the left hand side is always positive, since the realization of A is assumed to be positive for every state z. On the other hand, the restrictions on the parameters imply that the right-hand side is never positive. Hence, it follows that the third factor in (B.68) is always positive. Then, finally, Equation (4.58) will be increasing in the capital gains tax as long as the second factor is positive, i.e.,

$$\begin{aligned} \theta \Xi_1 \mathbb{E}\left[A^{1-\gamma}\right] - \mathbb{E}\left[A^{-\gamma}\right] &> 0 \\ \Leftrightarrow \frac{\mathbb{E}\left[A^{-\gamma}\right]}{\mathbb{E}\left[A^{1-\gamma}\right]} &< \theta \Xi_1. \end{aligned} \quad \text{(B.71)}$$

This is condition (4.59) given in the main text.

B.2.6 Derivation Total Wealth Budget Constraint

The derivation of the total wealth budget constraint follows Miao (2014). First, multiply by $Z^n \mu_{t+n}$ on both sides of the dynamic budget constraint (4.32) at date $t+n$ of an agent of type m born in period i and take conditional expectations at time t in order to obtain

$$\mathbb{E}_t\left[Z^n \mu_{t+n} c_{t+n}^{i,m}\right] = \mathbb{E}_t\Big[Z^n \mu_{t+n}\Big(\alpha_{t+n-1}^{i,m}\left(\left(1-\tau_c\right) D_{t+n} + \tau_c I_{t+n-1}\right) + \beta_{t+n-1}^{i,m} \tilde{R}_f + \left(1-\tau_l\right) h_{t+n}^{i,m} + s_{t+n}^{i} - \alpha_{t+n}^{i,m} I_{t+n} - \beta_{t+n}^{i,m}\Big)\Big]. \tag{B.72}$$

Next, Equation (B.72) is added up over the periods of the agent's remaining lifespan, i.e., $n = 0, ..., N-(t-i)$, and divided by μ_t on both sides to get

$$\begin{aligned}\sum_{n=0}^{N-(t-i)} \mathbb{E}_t\left[\frac{Z^n \mu_{t+n}}{\mu_t} c_{t+n}^{i,m}\right] = {}& \sum_{n=0}^{N-(t-i)} \mathbb{E}_t\left[\frac{Z^n \mu_{t+n}}{\mu_t} \alpha_{t+n-1}^{i,m}\left(\left(1-\tau_c\right) D_{t+n} + \tau_c I_{t+n-1}\right)\right] + \\ & \sum_{n=0}^{N-(t-i)} \mathbb{E}_t\left[\frac{Z^n \mu_{t+n}}{\mu_t} \beta_{t+n-1}^{i,m} \tilde{R}_f\right] + \sum_{n=1}^{N-(t-i)} \mathbb{E}_t\left[\frac{Z^n \mu_{t+n}}{\mu_t}\left(1-\tau_l\right) h_{t+n}^{i,m}\right] + \\ & \left(1-\tau_l\right) h_t^{i,m} + \sum_{n=1}^{N-(t-i)} \mathbb{E}_t\left[\frac{Z^n \mu_{t+n}}{\mu_t} s_{t+n}^{i}\right] + s_t^i - \\ & \sum_{n=0}^{N-(t-i)} \mathbb{E}_t\left[\frac{Z^n \mu_{t+n}}{\mu_t} \alpha_{t+n}^{i,m} I_{t+n}\right] - \sum_{n=0}^{N-(t-i)} \mathbb{E}_t\left[\frac{Z^n \mu_{t+n}}{\mu_t} \beta_{t+n}^{i,m}\right],\end{aligned} \tag{B.73}$$

where

$$p_{h,t}^{i,m} \equiv \sum_{n=1}^{N-(t-i)} \mathbb{E}_t \left[\frac{Z^n \mu_{t+n}}{\mu_t} (1-\tau_l) h_{t+n}^{i,m} \right], \tag{B.74}$$

$$p_{s,t}^{i} \equiv \sum_{n=1}^{N-(t-i)} \mathbb{E}_t \left[\frac{Z^n \mu_{t+n}}{\mu_t} s_{t+n}^{i} \right], \tag{B.75}$$

are Equations (4.67) and (4.68) in the text, respectively. In line with Appendix B.1.6, by making use of the Law of Iterated Expectations (see, for example, Munk (2013, p. 30)) and applying the basic pricing conditions (4.45) and (4.42), Equation (B.73) simplifies to

$$\begin{aligned} w_t^{i,m} \equiv & \alpha_{t-1}^{i,m} \left((1-\tau_c) D_t + \tau_c I_{t-1} \right) + \beta_{t-1}^{i,m} \tilde{R}_f + (1-\tau_l) h_t^{i,m} + p_{h,t}^{i,m} + \\ & s_t^i + p_{s,t}^i - \mathbb{E}_t \left[\frac{Z^{N-(t-i)} \mu_{N+i}}{\mu_t} \left(\alpha_{N+i}^{i,m} I_{N+i} + \beta_{N+i}^{i,m} \right) \right] \\ = & \alpha_{t-1}^{i,m} \left((1-\tau_c) D_t + \tau_c I_{t-1} \right) + \beta_{t-1}^{i,m} \tilde{R}_f + (1-\tau_l) h_t^{i,m} + \\ & + p_{h,t}^{i,m} + s_t^i + p_{s,t}^i, \end{aligned} \tag{B.76}$$

where the last term in the second line can be dropped due to the agent's terminal portfolio condition (4.33). This is equivalent to Equation (4.66) in the main text.

B.2.7 Derivation Human Capital

As indicated by condition (4.67), human capital at date t of an agent of type m born in period i is the sum over his expected future after-tax earnings discounted by the equilibrium SDF. Substituting the stochastic discount factor, expression (4.54), the market value of human capital is given by

$$
\begin{aligned}
p_{h,t}^{i,m} &= \sum_{n=1}^{N-(t-i)} \mathbb{E}_t \left[\frac{Z^n \mu_{t+n}}{\mu_t} (1-\tau_l) h_{t+n}^{i,m} \right] \\
&= (1-\tau_l) \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t \left[\left(\frac{Y_{t+n}}{Y_t} \right)^{-\gamma} h_{t+n}^{i,m} \right]. \qquad \text{(B.77)}
\end{aligned}
$$

Using the expression for individual earnings (4.1) along with the definition of aggregate earnings (4.10) yields

$$
\begin{aligned}
p_{h,t}^{i,m} &= (1-\tau_l)(1-\theta) \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t \left[\left(\frac{Y_{t+n}}{Y_t} \right)^{-\gamma} Y_{t+n} \right] f_{(t-i)+n}^m \\
&= (1-\tau_l)(1-\theta) \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t \left[\left(\frac{Y_{t+n}}{Y_t} \right)^{1-\gamma} \right] f_{(t-i)+n}^m Y_t. \qquad \text{(B.78)}
\end{aligned}
$$

Recall that Equation (4.60) finds production growth to be given by $Y_{t+1}/Y_t = \Xi_1 (\nu G_1)^{-1} A_{t+1}$. Then, Equation (B.78) can be rewritten to

$$
\begin{aligned}
p_{h,t}^{i,m} &= (1-\tau_l)(1-\theta) \sum_{n=1}^{N-(t-i)} \left\{ \left(\delta_0^1 \nu^{-1} \left(\frac{\Xi_1}{G_1} \right)^{1-\gamma} \right)^n \cdot \right. \\
&\qquad \left. \mathbb{E}_t \left[\left(\prod_{l=1}^{n} A_{t+l} \right)^{1-\gamma} \right] f_{(t-i)+n}^m \right\} Y_t \\
&= (1-\tau_l)(1-\theta) \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-1} \left(\frac{\Xi_1}{G_1} \right)^{1-\gamma} \mathbb{E}\left[A^{1-\gamma}\right] \right)^n f_{(t-i)+n}^m Y_t, \qquad \text{(B.79)}
\end{aligned}
$$

where the last line follows from the assumption of i.i.d. production shocks. This is the result presented in Equation (4.69).

B.2.8 Derivation Transfer Capital

Condition (4.68) implies that the transfer capital at date t of an agent of type m born in period i is the sum over his expected future transfer payments discounted by the equilibrium stochastic discount factor. By using the SDF expression (4.54) and Equations (4.26) and (4.20) the present value of future transfers follows by

$$\begin{aligned}
p_{s,t}^{i} &= \sum_{n=1}^{N-(t-i)} \mathbb{E}_t\left[\frac{Z^n \mu_{t+n}}{\mu_t} s_{t+n}^{i}\right] \\
&= \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t\Bigg[\left(\frac{Y_{t+n}}{Y_t}\right)^{-\gamma} \left(\tau_c \left(\theta Y_{t+n} - I_{t+n-1}\right) + \right. \\
&\quad \left. \tau_l \left(1-\theta\right) Y_{t+n}\right)\Bigg] d_{(t-i)+n} \\
&= \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t\left[\left(\frac{Y_{t+n}}{Y_t}\right)^{-\gamma} \left(\tau_c \theta + \tau_l \left(1-\theta\right)\right) Y_{t+n}\right] d_{(t-i)+n} - \\
&\quad \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t\left[\left(\frac{Y_{t+n}}{Y_t}\right)^{-\gamma} \tau_c I_{t+n-1}\right] d_{(t-i)+n}.
\end{aligned} \tag{B.80}$$

Then, note that from Equations (4.6) and (4.57) aggregate investment follows by $I_t = XY_t = (\nu G_1)^{-1} Y_t$. Using this and applying some algebraic manipulations, Equation (B.80) is equivalent to

$$\begin{aligned}
p_{s,t}^{i} &= \left(\tau_c \theta + \tau_l \left(1-\theta\right)\right) \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t\left[\left(\frac{Y_{t+n}}{Y_t}\right)^{1-\gamma}\right] Y_t d_{(t-i)+n} - \\
&\quad \tau_c (\nu G_1)^{-1} \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t\Bigg[\left(\frac{Y_{t+n-1}}{Y_t}\right)^{1-\gamma} \cdot \\
&\quad \left(\frac{Y_{t+n}}{Y_{t+n-1}}\right)^{-\gamma}\Bigg] Y_t d_{(t-i)+n}.
\end{aligned} \tag{B.81}$$

Next, substitute production growth from Equation (4.60) to obtain

$$p_{s,t}^{i} = \left(\tau_c\theta + \tau_l\left(1-\theta\right)\right) \sum_{n=1}^{N-(t-i)} \left\{ \left(\delta_0^1 \nu^{-1} \left(\frac{\Xi_1}{G_1}\right)^{1-\gamma}\right)^n \cdot \right.$$
$$\left. \mathbb{E}_t \left[\left(\prod_{l=1}^{n} A_{t+l}\right)^{1-\gamma} \right] Y_t d_{(t-i)+n} \right\} -$$
$$\tau_c \Xi_1^{-1} \sum_{n=1}^{N-(t-i)} \left\{ \left(\delta_0^1 \nu^{-1} \left(\frac{\Xi_1}{G_1}\right)^{1-\gamma}\right)^n \cdot \right.$$
$$\left. \mathbb{E}_t \left[\left(\prod_{l=1}^{n-1} A_{t+l}\right)^{1-\gamma} \left(A_{t+n}\right)^{-\gamma} \right] Y_t d_{(t-i)+n} \right\}. \qquad \text{(B.82)}$$

Finally, using the i.i.d. property of production shocks this is equivalent to

$$p_{s,t}^{i} = \left(\tau_c\theta + \tau_l\left(1-\theta\right)\right) \sum_{n=1}^{N-(t-i)} \left\{ \left(\delta_0^1 \nu^{-1} \left(\frac{\Xi_1}{G_1}\right)^{1-\gamma} \mathbb{E}\left[A^{1-\gamma}\right]\right)^n \cdot \right.$$
$$\left. Y_t d_{(t-i)+n} \right\} -$$
$$\tau_c \frac{\mathbb{E}\left[A^{-\gamma}\right]}{\Xi_1 \mathbb{E}\left[A^{1-\gamma}\right]} \sum_{n=1}^{N-(t-i)} \left\{ \left(\delta_0^1 \nu^{-1} \left(\frac{\Xi_1}{G_1}\right)^{1-\gamma} \mathbb{E}\left[A^{1-\gamma}\right]\right)^n \right.$$
$$\left. Y_t d_{(t-i)+n} \right\}, \qquad \text{(B.83)}$$

from which Equation (4.70) in the main text follows.

B.2.9 Derivation Marginal Propensity to Consume out of Total Wealth

The derivation of the MPCTW follows from Equations (4.74) and (4.73). To demonstrate this, first note that the term $(c_{t+n}^{i,m}/c_t^{i,m})$ can alternatively be written as

$$\frac{c_{t+n}^{i,m}}{c_t^{i,m}} = \frac{c_{t+1}^{i,m}}{c_t^{i,m}} \frac{c_{t+2}^{i,m}}{c_{t+1}^{i,m}} \cdots \frac{c_{t+n}^{i,m}}{c_{t+n-1}^{i,m}}. \tag{B.84}$$

Next, substituting Equation (4.73), individual consumption growth from time t to $t+n$ reads:

$$\begin{aligned}\frac{c_{t+n}^{i,m}}{c_t^{i,m}} &= \left(\frac{\delta_{t-i}^m}{G_2}\right)^{\frac{1}{\gamma}} A_{t+1} \left(\frac{\delta_{t+1-i}^m}{G_2}\right)^{\frac{1}{\gamma}} A_{t+2} \cdots \left(\frac{\delta_{t+n-1-i}^m}{G_2}\right)^{\frac{1}{\gamma}} A_{t+n} \\ &= \left(\frac{1}{G_2}\right)^{\frac{n}{\gamma}} \left(\prod_{l=1}^{n} A_{t+l}\right) \left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^m\right)^{\frac{1}{\gamma}}.\end{aligned} \tag{B.85}$$

Using this expression along with the agent's total wealth to consumption ratio, Equation (4.74), it follows that:

$$\begin{aligned}\frac{w_t^{i,m}}{c_t^{i,m}} &= \sum_{n=0}^{N-(t-i)} \mathbb{E}_t \left[\left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^m\right) \cdot \right. \\ &\qquad \left. \left(\left(\frac{1}{G_2}\right)^{\frac{n}{\gamma}} \left(\prod_{l=1}^{n} A_{t+l}\right) \left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^m\right)^{\frac{1}{\gamma}} \right)^{1-\gamma} \right] \\ &= \sum_{n=0}^{N-(t-i)} \mathbb{E}_t \left[\left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^m\right)^{\frac{1}{\gamma}} \left(\frac{1}{G_2}\right)^{n\frac{1-\gamma}{\gamma}} \left(\prod_{l=1}^{n} A_{t+l}^{1-\gamma}\right) \right] \\ &= \sum_{n=0}^{N-(t-i)} \left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^m\right)^{\frac{1}{\gamma}} \left(\frac{1}{G_2}\right)^{n\frac{1-\gamma}{\gamma}} \mathbb{E}_t \left[\left(\prod_{l=1}^{n} A_{t+l}^{1-\gamma}\right) \right] \\ &= \sum_{n=0}^{N-(t-i)} \left(\prod_{l=0}^{n-1} \delta_{(t-i)+l}^m\right)^{\frac{1}{\gamma}} \left(G_2^{1-\frac{1}{\gamma}} \mathbb{E}\left[A^{1-\gamma}\right] \right)^n,\end{aligned} \tag{B.86}$$

where the second equality is due to power law, the third equality follows since only future productivity shocks $\{A_{t+l}\}_{l=1}^{n}$ are unknown at time t, and the last equation is a consequence of the i.i.d. property of these shocks. Finally, taking the reciprocal of expression (B.86), Equation (4.75) in the main text follows immediately.

B.2.10 Clearing on the Bond Market

The initially high complexity of the equilibrium solution is further reduced due to the fact that clearing on the bond market, condition (4.37), follows directly from clearing on the equity market, condition (4.36).

To see this, assume clearing on the equity market, condition (4.36). Then, use the clearing condition for the bond market (4.37) along with the terminal portfolio condition (4.33) to obtain:

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t-1}\beta_t^{i,m} = 0. \tag{B.87}$$

Substituting optimal bond investment (4.83) on the left hand side and applying some algebraic manipulations yields:

$$\begin{aligned}\sum_{m=1}^{M}\sum_{i=t-N}^{t-1}\beta_t^{i,m} &= \sum_{m=1}^{M}\sum_{i=t-N}^{t-1}\left(\frac{\tau_c}{\tilde{R}_f}\left(\frac{1}{N\cdot M}-\alpha_t^{i,m}\right)I_t\right)\\ &= \frac{\tau_c}{\tilde{R}_f}I_t\left(\frac{1}{N\cdot M}\sum_{m=1}^{M}\sum_{i=1}^{N}1-\sum_{m=1}^{M}\sum_{i=t-N}^{t-1}\alpha_t^{i,m}\right).\end{aligned} \tag{B.88}$$

Next, by making use of clearing on the stock market (4.34), basic arithmetics and the terminal portfolio condition (4.33) it follows that:

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t-1}\beta_t^{i,m} = \frac{\tau_c}{\tilde{R}_f} I_t\left(1-1\right)$$
$$=0. \tag{B.89}$$

In other words, clearing on the bond market is a direct consequence of stock market clearing.

B.2.11 Derivation of Individual Welfare Measure

In order to derive the solution for the indirect utility at date t of an agent of type m born in period $t-N \leq i \leq t$, rearrange Equation (4.31) and apply the MPCTW Equation (4.74) and individual consumption (4.79):

$$\begin{aligned} V_t^{i,m} &= \sum_{n=0}^{N-(t-i)} \mathbb{E}_t\left[\left(\prod_{l=0}^{n-1}\delta_{(t-i)+l}^m\right)\frac{\left(c_t^{i,m}\right)^{1-\gamma}}{1-\gamma}\left(\frac{c_{t+n}^{i,m}}{c_t^{i,m}}\right)^{1-\gamma}\right] \\ &= \frac{\left(c_t^{i,m}\right)^{1-\gamma}}{1-\gamma}\sum_{n=0}^{N-(t-i)}\mathbb{E}_t\left[\left(\prod_{l=0}^{n-1}\delta_{(t-i)+l}^m\right)\left(\frac{c_{t+n}^{i,m}}{c_t^{i,m}}\right)^{1-\gamma}\right] \\ &= \frac{\left(g_{t-i}^m\right)^{1-\gamma}}{1-\gamma}\left(b_{t-i}^m\right)^{-1} Y_t^{1-\gamma}. \end{aligned} \tag{B.90}$$

The hypothetical expected indirect utility at date t of a newborn individual to be born in period $i > t$ follows from Equation (B.90) by

$$\begin{aligned}
\mathbb{E}_t\left[V_i^{i,m}\right] &= \frac{\left(g_0^m\right)^{1-\gamma}}{1-\gamma}\left(b_0^m\right)^{-1}\mathbb{E}_t\left[Y_i^{1-\gamma}\right] \\
&= \frac{\left(g_0^m\right)^{1-\gamma}}{1-\gamma}\left(b_0^m\right)^{-1}\mathbb{E}_t\left[\left(\frac{Y_i}{Y_t}\right)^{1-\gamma}\right]Y_t^{1-\gamma} \\
&= \left(\left(\frac{\Xi_1}{\nu G_1}\right)^{1-\gamma}\mathbb{E}\left[A^{1-\gamma}\right]\right)^{-(t-i)}\frac{\left(g_0^m\right)^{1-\gamma}}{1-\gamma}\left(b_0^m\right)^{-1}Y_t^{1-\gamma},
\end{aligned} \tag{B.91}$$

where the last line is derived by substituting production growth in the second line and applying the independence property of A again. Equations (B.90) and (B.91) are the solutions (4.92) and (4.93) stated in the text, respectively.

Let variables with tilde represent values that arise in the presence of taxation, whereas variables without tilde shall correspond to values in the no-tax setting. Further, denote by $(\mathcal{V}_{t-i}^m + 1)$ the (gross) equivalent valuation factor of production output necessary to attain an individual's welfare level in the case without taxation that equals the individual's welfare level in the setting with taxation. Then, by using condition (4.94) and substituting indirect utility (B.90) at date t it follows that

$$\begin{aligned}
\frac{\left(g_{t-i}^m\right)^{1-\gamma}}{1-\gamma}\left(b_{t-i}^m\right)^{-1}\left(Y_t\left(\mathcal{V}_{t-i}^m+1\right)\right)^{1-\gamma} &= \frac{\left(\tilde{g}_{t-i}^m\right)^{1-\gamma}}{1-\gamma}\left(\tilde{b}_{t-i}^m\right)^{-1}Y_t^{1-\gamma}, \\
\left(\mathcal{V}_{t-i}^m+1\right)^{1-\gamma} &= \frac{\frac{\left(\tilde{g}_{t-i}^m\right)^{1-\gamma}}{1-\gamma}b_{t-i}^m}{\frac{\left(g_{t-i}^m\right)^{1-\gamma}}{1-\gamma}\tilde{b}_{t-i}^m}, \\
\mathcal{V}_{t-i}^m &= \left(\frac{\tilde{g}_{t-i}^m}{g_{t-i}^m}\right)\left(\frac{b_{t-i}^m}{\tilde{b}_{t-i}^m}\right)^{\frac{1}{1-\gamma}} - 1.
\end{aligned} \tag{B.92}$$

Equation (B.92) describes the percentage change in production without taxation that would be necessary for an agent of type m born in period $t-N \leq i \leq t$ in order to obtain the same level of utility as in the presence of redistributive taxation. Using condition (4.95) and substituting indirect utility (B.91) at date t, similar steps lead to the measure for unborn individuals. That is,

$$\mathcal{V}_{t-i}^{m} = \left(\left(\frac{\nu G_1}{\tilde{\nu}\tilde{G}_1}\right)^{-(t-i)} \left(\frac{\tilde{g}_0^m}{g_0^m}\right)\left(\frac{b_0^m}{\tilde{b}_0^m}\right)\right)^{\frac{1}{1-\gamma}} - 1, \tag{B.93}$$

for an agent of type m to be born in period $i > t$. Equations (B.92)-(B.93) are solution (4.96) in the main text.

B.2.12 Derivation of Social Welfare Measure

In order to derive the social welfare measure, start by substituting agents' indirect utilities (4.92) and (4.93) in the aggregate welfare function (4.97) :

$$\begin{aligned}
V_t =& \sum_{m=1}^{M} \left\{ \sum_{i=t-N}^{t} V_t^{i,m} + \sum_{i=t+1}^{\infty} \left(\bar{\delta}^m\right)^{-(t-i)} \mathbb{E}_t \left[V_i^{i,m} \right] \right\} \\
=& \sum_{m=1}^{M} \left\{ \sum_{i=t-N}^{t} \left(\frac{\left(g_{t-i}^m\right)^{1-\gamma}}{1-\gamma} \left(b_{t-i}^m\right)^{-1} Y_t^{1-\gamma} \right) + \right. \\
& \sum_{i=t+1}^{\infty} \left(\bar{\delta}^m\right)^{-(t-i)} \left(\left(\left(\frac{\Xi_1}{\nu G_1} \right)^{1-\gamma} \mathbb{E}\left[A^{1-\gamma}\right] \right)^{-(t-i)} \cdot \right. \\
& \left. \left. \frac{\left(g_0^m\right)^{1-\gamma}}{1-\gamma} \left(b_0^m\right)^{-1} Y_t^{1-\gamma} \right) \right\} \\
=& \sum_{m=1}^{M} \left\{ \sum_{i=t-N}^{t} \varepsilon_{t-i}^m + \sum_{i=t+1}^{\infty} \left(\bar{\delta}^m\right)^{-(t-i)} \varepsilon_{t-i}^m \right\} Y_t^{1-\gamma},
\end{aligned} \tag{B.94}$$

where

$$\varepsilon_{t-i}^m = \begin{cases} \frac{\left(g_{t-i}^m\right)^{1-\gamma}}{1-\gamma} \left(b_{t-i}^m\right)^{-1} & if\ t-N \leq i \leq t, \\ \left(\left(\frac{\Xi_1}{\nu G_1} \right)^{1-\gamma} \mathbb{E}\left[A^{1-\gamma}\right] \right)^{-(t-i)} \frac{\left(g_0^m\right)^{1-\gamma}}{1-\gamma} \left(b_0^m\right)^{-1} & if\ i > t. \end{cases} \tag{B.95}$$

Denote by $(\mathcal{V} + 1)$ the (gross) equivalent valuation factor of production output necessary to attain an aggregate welfare level in the case without taxation that equals the aggregate welfare level in the setting with taxation. Again, variables with tilde represent values that arise in the presence of taxation, whereas variables without tilde display values in the no-tax case. Then, by using condition (4.99) and substituting social welfare (B.94) at date t it follows that

$$\frac{\sum_{m=1}^{M}\left\{\sum_{i=t-N}^{t}\varepsilon_{t-i}^{m}+\sum_{i=t+1}^{\infty}\left(\bar{\delta}^{m}\right)^{-(t-i)}\varepsilon_{t-i}^{m}\right\}}{\left(Y_t\left(\mathcal{V}+1\right)\right)^{\gamma-1}}$$

$$=\frac{\sum_{m=1}^{M}\left\{\sum_{i=t-N}^{t}\bar{\varepsilon}_{t-i}^{m}+\sum_{i=t+1}^{\infty}\left(\bar{\delta}^{m}\right)^{-(t-i)}\bar{\varepsilon}_{t-i}^{m}\right\}}{Y_t^{\gamma-1}}$$

$$\Leftrightarrow \mathcal{V}+1=\left(\frac{\sum_{m=1}^{M}\left\{\sum_{i=t-N}^{t}\bar{\varepsilon}_{t-i}^{m}+\sum_{i=t+1}^{\infty}\left(\bar{\delta}^{m}\right)^{-(t-i)}\bar{\varepsilon}_{t-i}^{m}\right\}}{\sum_{m=1}^{M}\left\{\sum_{i=t-N}^{t}\varepsilon_{t-i}^{m}+\sum_{i=t+1}^{\infty}\left(\bar{\delta}^{m}\right)^{-(t-i)}\varepsilon_{t-i}^{m}\right\}}\right)^{\frac{1}{1-\gamma}} \tag{B.96}$$

is the social welfare measure, Equation (4.100), that indicates the percentage change in production without taxation that would be necessary in order to obtain the same aggregate level of welfare as in the presence of redistributive taxation.

B.2.13 Impact of Government Debt

As described in the main text, in every period t government issues an amount of government debt $\beta^G Y_t$ to finance government investment in the production, where β^G is the exogenously given and constant debt to GDP ratio. Aggregate real investment $I_t = I_t^P + I_t^G$ is therefore split up in a private $I_t^P = (X_t - \beta^G)Y_t$ and a government $I_t^G = \beta^G Y_t$ part. The share of government holdings in the risky investment from time $t-1$ to t is denoted by $\alpha_{t-1}^G \equiv I_{t-1}^G / I_{t-1}$. In the presence of the government debt policy (changed) variables are labeled with a bar. Government bonds are perfect substitutes to the privately issued one-period risk-free bonds. Further, government neither builds up wealth nor debt, so the government budget constraint is balanced in every period. It can be written as

$$
\begin{aligned}
\beta^G Y_{t-1} R_{f,t-1} + I_t^G + \bar{S}_t = \\
\beta^G Y_t + \alpha_{t-1}^G D_t + \beta^G Y_{t-1} \left(R_{f,t-1} - \tilde{R}_{f,t-1} \right) + \\
\tau_c \left(1 - \alpha_{t-1}^G\right) \left(D_t - I_{t-1}\right) + \tau_l H_t,
\end{aligned} \tag{B.97}
$$

where the right-hand side represents government revenues, whereas the left-hand side states government expenditures. The former is composed of public debt (first term), revenues from government real investment (second term), revenues from taxation on bond (third term) and equity (fourth term) market activities as well as on labor income (fifth term). Government expenditures include the repayment of public debts (first term), government real investment (second term) and aggregate transfer payments (third term). Since any government debt must be directly financed by and all revenues must be directly paid to the living individuals, it follows by simplifying constraint (B.97) that aggregate disposable transfers are given by

$$
\bar{S}_t = S_t + \alpha_{t-1}^G \left(\left(1 - \tau_c\right) D_t + \tau_c I_{t-1} \right) - \beta^G Y_{t-1} \tilde{R}_{f,t-1}, \tag{B.98}
$$

which is Equation (4.103) in the main text, where S_t corresponds to the definition of aggregate disposable transfers as given in Equation (4.20). Compared to the setting without government debt, disposable transfers in the presence of the given debt policy $\bar{S}_t$ are changed by the capital market activities taken by the government. To be precise, transfers are increased by the revenues collected from government equity investment activities (second term), but reduced by the repayments of government debts (third term). In line with the derivations made without government debt policy, the further steps are as follows:

1. The production side remains unaffected. That is, the definition of factor outputs given in Equations (4.10)-(4.11) as well as the equity return (4.13) remain unchanged.

2. The government intervention affects the optimization problem, given in Equations (4.31)-(4.33), by altering the individual's budget constraint (4.27), since individual transfers s_t^i change due to $\bar{S}_t/(N \cdot M)$. The time t budget constraint of an agent of type m born in period $i < t$ under government debt policy becomes

$$\bar{v}_t^{i,m} = \left(\bar{\alpha}_{t-1}^{i,m} + \frac{1}{N \cdot M}\alpha_{t-1}^G\right)\left(\left(1-\tau_c\right)D_t + \tau_c I_{t-1}\right) + \left(\bar{\beta}_{t-1}^{i,m} - \frac{1}{N \cdot M}\beta^G Y_{t-1}\right)\tilde{R}_{f,t-1} + \left(1-\tau_l\right)h_t^{i,m} + s_t^i. \tag{B.99}$$

This is constraint (4.104) in the text. Since they are independent of transfers, the budget constraints of newborn agents (4.28) remain unchanged.

3. Regarding the definition of the market equilibrium, solely the asset market clearing conditions differ as agents now do not hold all outstanding equity shares and government bonds imply a positive net supply for the risk-free security. That is, aggregate private equity and bond holdings are given by

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t}\bar{\alpha}_t^{i,m} = 1 - \alpha_t^G, \tag{B.100}$$

$$\sum_{m=1}^{M}\sum_{i=t-N}^{t}\bar{\beta}_t^{i,m} = \beta^G Y_t, \tag{B.101}$$

respectively, which are Equations (4.101)-(4.102) in the text.

4. Building on the "guess and verify" approach presented in Section 4.2, the same solution method can be applied as before. Then, going through the subsequent derivation steps shown in Sections 4.2.1.1 and 4.2.1.2, identical first order conditions (4.40)-(4.42), the same definition of the stochastic discount factor (4.54) and the same equation for production growth (4.55), as in the case without government intervention are found. It follows that the solution for the aggregate investment is still constant and given by Equation (4.57), i.e., $X = (\nu G_1)^{-1}$. This result in turn can be used to show that the solutions for aggregate production growth (4.60), the stochastic discount factor (4.61) and the after-tax risk-free return (4.63) remain unchanged as well. Finally, it follows that the government equity share is constant, since $I^G_{t-1}/I_{t-1} = \beta^G/X \equiv \alpha^G$.

5. Recall that the total wealth budget constraint is composed of the current wealth budget constraint, Equation (B.99), and the present value of expected future after-tax earnings as well as the present value of expected future transfers. It can be shown that the solutions to human and transfer capital stay unchanged compared to the solutions found in the setting without government debt, Equations (4.69) and (4.70), respectively. The former proof is straightforward, the latter is presented in the following.

   By using the unchanged expressions for the SDF (4.54), individual transfers (4.26) and the aggregate investment share (4.57) together with the new equation for aggregate transfers (B.98) the pricing relation for transfer capital can be written as

$$
\begin{aligned}
\bar{p}^i_{s,t} &= \sum_{n=1}^{N-(t-i)} \mathbb{E}_t\left[\frac{Z^n \mu_{t+n}}{\mu_t}\bar{s}^i_{t+n}\right] \\
&= \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t\left[\left(\frac{Y_{t+n}}{Y_t}\right)^{-\gamma} S_{t+n}\right] d_{(t-i)+n} + \\
&\quad \alpha^G(1-\tau_c) \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t\left[\left(\frac{Y_{t+n}}{Y_t}\right)^{-\gamma} D_{t+n}\right] d_{(t-i)+n} - \\
&\quad \alpha^G\left(\tilde{R}_f - \tau_c\right) \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t\left[\left(\frac{Y_{t+n}}{Y_t}\right)^{-\gamma} I_{t+n-1}\right] d_{(t-i)+n},
\end{aligned}
\tag{B.102}
$$

where the last term in the last equality is due to relation $\beta^G Y_t = (\beta^G/X) I_t = \alpha^G I_t$. The first term is equivalent to the pricing equation without government debt (4.68), so it remains to be shown that the last two terms cancel each other out. Start by the second term in Equation (B.102) and substitute the unchanged result for capital output (4.11) and production growth (4.60) in order to obtain

$$
\begin{aligned}
\alpha^G (1-\tau_c)\,\theta \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-\gamma}\right)^n \mathbb{E}_t\left[\left(\frac{Y_{t+n}}{Y_t}\right)^{-\gamma} Y_{t+n}\right] d_{(t-i)+n} &= \\
\alpha^G (1-\tau_c)\,\theta \sum_{n=1}^{N-(t-i)} \left(\delta_0^1 \nu^{-1} \left(\frac{\Xi_1}{G_1}\right)^{1-\gamma} \mathbb{E}\left[A^{1-\gamma}\right]\right)^n d_{(t-i)+n} Y_t &= \\
\alpha^G (1-\tau_c)\,\theta \sum_{n=1}^{N-(t-i)} \rho^n d_{(t-i)+n} Y_t&,
\end{aligned}
\tag{B.103}
$$

where the the i.i.d. property of production shocks has also been used again. Then, turn to the third term in Equation (B.102). By substituting the expression $I_t = (1/\nu G_1) Y_t$ for aggregate investment, applying some algebraic manipulations and the independence property of A_t one obtains

$$
\begin{aligned}
&-\alpha^G \left(\frac{1}{\nu G_1}\right)\left(\tilde{R}_f-\tau_c\right)\sum_{n=1}^{N-(t-i)}\Bigg\{\left(\delta_0^1\nu^{-\gamma}\right)^n\cdot\\
&\mathbb{E}_t\left[\left(\frac{Y_{t+n-1}}{Y_t}\right)^{1-\gamma}\left(\frac{Y_{t+n}}{Y_{t+n-1}}\right)^{-\gamma}\right]d_{(t-i)+n}\Bigg\}Y_t=\\
&-\frac{\alpha^G\left(\frac{1}{\upsilon G_1}\right)^{1-\gamma}\mathbb{E}\left[A^{-\gamma}\right]\left(\tilde{R}_f-\tau_c\right)}{\left(\frac{\Xi_1}{\nu G_1}\right)^{1-\gamma}\Xi_1^{\gamma}\mathbb{E}\left[A^{1-\gamma}\right]}\sum_{n=1}^{N-(t-i)}\Bigg\{\Bigg(\delta_0^1\nu^{-1}\cdot\\
&\left(\frac{\Xi_1}{G_1}\right)^{1-\gamma}\mathbb{E}\left[A^{1-\gamma}\right]\Bigg)^n d_{(t-i)+n}\Bigg\}Y_t=\\
&-\alpha^G\left(\tilde{R}_f-\tau_c\right)\frac{\mathbb{E}\left[A^{-\gamma}\right]}{\Xi_1\mathbb{E}\left[A^{1-\gamma}\right]}\sum_{n=1}^{N-(t-i)}\rho^n d_{(t-i)+n}Y_t. \qquad \text{(B.104)}
\end{aligned}
$$

Next, by using the equilibrium solution for the risk-free return before tax (4.64) and subsequently the solution for the risk-free return after tax (4.63) this expression can be simplified to

$$
\begin{aligned}
&-\alpha^G\frac{\left(\tilde{R}_f-\tau_c\right)}{R_f}\theta\sum_{n=1}^{N-(t-i)}\rho^n d_{(t-i)+n}Y_t=\\
&-\alpha^G\left(1-\tau_c\right)\theta\sum_{n=1}^{N-(t-i)}\rho^n d_{(t-i)+n}Y_t, \qquad \text{(B.105)}
\end{aligned}
$$

which, except for its sign, is identical to expression (B.103). Consequently, both terms cancel each other and the solution to transfer capital is solely given by the first term in Equation (B.102). Hence, it is the same as in the setting without government intervention, i.e., $\bar{p}_{s,t}^i = p_{s,t}^i$.

Finally, it follows that the total wealth budget constraint is solely changed regarding agent's current transfer payment $\bar{s}_t^i$.

6. Since the solution for the individual consumption policy, Equation (4.79), is independent of current transfers, it remains unchanged.

7. Turning to the individual investment decisions, these are changed due to changes in the current transfer $\bar{s}_t^i$. Taking the same derivation steps as presented in Section 4.2.2.3, the bond $\bar{\beta}_{t-1}^{i,m}$ and equity $\bar{\alpha}_{t-1}^{i,m}$ investment policies with debt financed fiscal policy turn out to be

$$\bar{\beta}_{t-1}^{i,m} = \beta_{t-1}^{i,m} + \frac{1}{N \cdot M} \beta^G Y_{t-1}, \tag{B.106}$$

$$\bar{\alpha}_{t-1}^{i,m} = \alpha_{t-1}^{i,m} - \frac{1}{N \cdot M} \alpha_{t-1}^G, \tag{B.107}$$

   which are Equations (4.105) and (4.106) in the main text, respectively.

8. Substituting the new decision rules into the asset market clearing conditions under the government debt policy, Equations (B.100) and (B.101), these turn out to be identical to the original conditions, Equations (4.36) and (4.37). Hence, the equilibrium condition, Equation (4.91), is identical to the setting without government intervention.

Altogether, it follows that under the debt financed investment program the optimization problem together with the definition of market equilibrium is identical to the original case without such fiscal policy. This equally implies that the equilibrium condition, Equation (4.91), remains unchanged. Consequently, the equilibrium solution remains unchanged and any debt financed government intervention has no effect.

Bibliography

Abel, A. B., 1983. Optimal investment under uncertainty. American Economic Review 73, 228–233.

Aiyagari, S. R., 1995. Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting. Journal of Political Economy 103, 1158–1175.

Aiyagari, S. R., McGrattan, E. R., 1998. The optimum quantity of debt. Journal of Monetary Economics 42, 447–469.

Alan, S., Browning, M., 2010. Estimating intertemporal allocation parameters using synthetic residual estimation. Review of Economic Studies 77, 1231–1261.

Allais, M., 1947. Economie et Intérêt. Imprimerie Nationale, Paris.

Alvaredo, F., Atkinson, A. B., Piketty, T., Saez, E., 2013. The top 1 percent in international and historical perspective. Journal of Economic Perspectives 27, 3–20.

Alvaredo, F., Chancel, L., Piketty, T., Saez, E., Zucman, G., 2017. World inequality report 2018. World Inequality Lab, Paris.

Arrow, K. J., 1951. An extension of the basic theorems of classical welfare economics. In: *Proceedings of the second Berkeley symposium on mathemat-*

ical statistics and probability, The Regents of the University of California, Berkeley, 507–532.

Arrow, K. J., Debreu, G., 1954. Existence of an equilibrium for a competitive economy. Econometrica 22, 265–290.

Aspromourgos, T., 1986. On the origins of the term 'neoclassical'. Cambridge Journal of Economics 10, 265–270.

Athanasoulis, S. G., 2005. Asset pricing from primitives: Closed form solutions to asset prices, consumption, and portfolio demands. Journal of Economic Dynamics and Control 29, 423–447.

Athanasoulis, S. G., 2006. Asset prices and consumption in a model of perpetual youth. Journal of Business 79, 2835–2868.

Athanasoulis, S. G., Shiller, R. J., 2001. World income components: Measuring and exploiting risk-sharing opportunities. American Economic Review 91, 1031–1054.

Atkinson, A. B., Piketty, T., Saez, E., 2011. Top incomes in the long run of history. Journal of Economic Literature 49, 3–71.

Atkinson, A. B., Sandmo, A., 1980. Welfare implications of the taxation of savings. Economic Journal 90, 529–549.

Attanasio, O. P., Banks, J., Tanner, S., 2002. Asset holding and consumption volatility. Journal of Political Economy 110, 771–792.

Attanasio, O. P., Browning, M., 1995. Consumption over the life cycle and over the business cycle. American Economic Review 85, 1118–1137.

Attanasio, O. P., Pistaferri, L., 2016. Consumption inequality. Journal of Economic Perspectives 30, 3–28.

Auerbach, A. J., 1979. The optimal taxation of heterogeneous capital. Quarterly Journal of Economics 93, 589–612.

Auerbach, A. J., Kotlikoff, L. J., 1983. National savings, economic welfare, and the structure of taxation. In: *Behavioral simulation methods in tax policy analysis*, University of Chicago Press, Chicago, 459–493.

Back, K., 2010. Asset pricing and portfolio choice theory. Financial Management Association Aurvey and Synthesis Series, Oxford University Press, Oxford and New York.

Ball, L., Mankiw, N. G., 2007. Intergenerational risk sharing in the spirit of Arrow, Debreu, and Rawls, with applications to social security design. Journal of Political Economy 115, 523–547.

Bansal, R., Yaron, A., 2004. Risks for the long run: A potential resolution of asset pricing puzzles. Journal of Finance 59, 1481–1509.

Barro, R. J., Redlick, C. J., 2011. Macroeconomic effects from government purchases and taxes. Quarterly Journal of Economics 126, 51–102.

Barro, R. J., Sala-i Martin, X., 2004. Economic growth. MIT Press, Cambridge, Mass, second ed.

Baxter, M., Jermann, U. J., 1997. The international diversification puzzle is worse than you think. American Economic Review 87, 170–180.

Beaumont, P. M., Culham, A. J., Kercheval, A. N., 2013. Asset market dynamics in equilibrium models with heterogeneous agents: Analytical results. Advances in Economics and Business 1, 49–56.

Becker, R. A., 1980. On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households. Quarterly Journal of Economics 95, 375–382.

Benhabib, J., Bisin, A., Zhu, S., 2011. The distribution of wealth and fiscal policy in economies with finitely lived agents. Econometrica 79, 123–157.

Benzoni, L., Collin-Dufresne, P., Goldstein, R. S., 2007. Portfolio choice over the life-cycle when the stock and labor markets are cointegrated. Journal of Finance 62, 2123–2167.

Bergson, A., 1938. A reformulation of certain aspects of welfare economics. Quarterly Journal of Economics 52, 310–334.

Bernoulli, D., 1738. Specimen theoriae novae de mensura sortis. Papers of the Imperial Academy of Sciences in Petersburg V, 175–192.

Bhamra, H. S., Uppal, R., 2014. Asset prices with heterogeneity in preferences and beliefs. Review of Financial Studies 27, 519–580.

Blanchard, O., 1985. Debt, deficits, and finite horizons. Journal of Political Economy 93, 223–247.

Blanchard, O., Perotti, R., 2002. An empirical characterization of the dynamic effects of changes in government spending and taxes on output. Quarterly Journal of Economics 117, 1329–1368.

Bloch, D., Pisu, M., Joumard, I., 2013. Tackling income inequality. OECD Journal: Economic Studies 2012, 37–70.

Blundell, R., Browning, M., Meghir, C., 1994. Consumer demand and the life-cycle allocation of household expenditures. Review of Economic Studies 61, 57–80.

Booij, A. S., van Praag, B. M., 2009. A simultaneous approach to the estimation of risk aversion and the subjective time discount rate. Journal of Economic Behavior & Organization 70, 374–388.

Bosch-Domènech, A., Silvestre, J., 1999. Does risk aversion or attraction depend on income? An experiment. Economics Letters 65, 265–273.

Branger, N., Konermann, P., Schlag, C., 2013. Optimists, pessimists, and the equity premium: The role of preferences and market (in)completeness. Unpublished Working Paper.

Brav, A., Constantinides, G. M., Geczy, C. C., 2002. Asset pricing with heterogeneous consumers and limited participation: Empirical evidence. Journal of Political Economy 110, 793–824.

Breeden, D. T., 1979. An intertemporal asset pricing model with stochastic consumption and investment opportunities. Journal of Financial Economics 7, 265–296.

Calvet, L. E., Campbell, J. Y., Sodini, P., 2007. Down or out: Assessing the welfare costs of household investment mistakes. Journal of Political Economy 115, 707–747.

Calvet, L. E., Campbell, J. Y., Sodini, P., 2009a. Fight or flight? Portfolio rebalancing by individual investors. Quarterly Journal of Economics 124, 301–348.

Calvet, L. E., Campbell, J. Y., Sodini, P., 2009b. Measuring the financial sophistication of households. American Economic Review 99, 393–398.

Campbell, J. Y., 1993. Intertemporal asset pricing without consumption data. American Economic Review 83, 487–512.

Campbell, J. Y., 1996. Understanding risk and return. Journal of Political Economy 104, 298–345.

Campbell, J. Y., 1999. Asset prices, consumption, and the business cycle. In: *Handbook of Macroeconomics*, Elsevier, vol. 1, 1231–1303.

Campbell, J. Y., 2000. Asset pricing at the millennium. Journal of Finance 55, 1515–1567.

Campbell, J. Y., 2003. Consumption-based asset pricing. In: Constantinides, G. M., Harris, M., Stulz, R. M. (eds.), *Handbook of the economics of finance*, Elsevier, Amsterdam, vol. 1B, 803–887.

Campbell, J. Y., 2006. Household finance. Journal of Finance 61, 1553–1604.

Campbell, J. Y., Cochrane, J. H., 1999. By force of habit: A consumption–based explanation of aggregate stock market behavior. Journal of Political Economy 107, 205–251.

Campbell, J. Y., Mankiw, N. G., 1989. Consumption, income, and interest rates: Reinterpreting the time series evidence. NBER Macroeconomics Annual 4, 185–216.

Campbell, J. Y., Nosbusch, Y., 2007. Intergenerational risksharing and equilibrium asset prices. Journal of Monetary Economics 54, 2251–2268.

Cass, D., 1965. Optimum growth in an aggregative model of capital accumulation. Review of Economic Studies 32, 233–240.

Chabakauri, G., 2013. Dynamic equilibrium with two stocks, heterogeneous investors, and portfolio constraints. Review of Financial Studies 26, 3104–3141.

Chabakauri, G., 2015. Dynamic equilibrium with rare disasters and heterogeneous Epstein-Zin investors. Unpublished Working Paper.

Chamley, C., 1986. Optimal taxation of capital income in general equilibrium with infinite lives. Econometrica 54, 607–622.

Chen, H., Joslin, S., Tran, N.-K., 2012. Rare disasters and risk sharing with heterogeneous beliefs. Review of Financial Studies 25, 2189–2224.

Christiansen, C., Joensen, J. S., Rangvid, J., 2008. Are economists more likely to hold stocks? Review of Finance 12, 465–496.

Chu, C. Y. C., Chien, H.-K., Lee, R. D., 2010. The evolutionary theory of time preferences and intergenerational transfers. Journal of Economic Behavior & Organization 76, 451–464.

Clarke, S., 1991. The marginalist revolution in economics. In: Clarke, S. (ed.), *Marx, Marginalism and Modern Sociology*, Palgrave Macmillan UK, London, 182–206.

Cloyne, J., 2013. Discretionary tax changes and the macroeconomy: New narrative evidence from the United Kingdom. American Economic Review 103, 1507–1528.

Cobb, C. W., Douglas, P. H., 1928. A theory of production. American Economic Review 18, 139–165.

Cocco, J. F., Gomes, F. J., Maenhout, P. J., 2005. Consumption and portfolio choice over the life cycle. Review of Financial Studies 18, 491–533.

Cochrane, J. H., 1991. Production-based asset pricing and the link between stock returns and economic fluctuations. Journal of Finance 46, 209–237.

Cochrane, J. H., 1996. A cross-sectional test of an investment-based asset pricing model. Journal of Political Economy 104, 572–621.

Cochrane, J. H., 2005. Asset pricing. Princeton University Press, Princeton, revised ed.

Colander, D., Holt, R., Rosser, B., 2004. The changing face of mainstream economics. Review of Political Economy 16, 485–499.

Coller, M., Williams, M. B., 1999. Eliciting individual discount rates. Experimental Economics 2, 107–127.

Conesa, J. C., Kitao, S., Krueger, D., 2009. Taxing capital? Not a bad idea after all! American Economic Review 99, 25–48.

Constantinides, G. M., 1982. Intertemporal asset pricing with heterogeneous consumers and without demand aggregation. Journal of Business 55, 253–267.

Constantinides, G. M., 1992. A theory of the nominal term structure of interest rates. Review of Financial Studies 5, 531–552.

Constantinides, G. M., 2005. Theory of valuation: Overview and recent developments. In: Bhattacharya, S., Constantinides, G. M. (eds.), *Theory of Valuation*, World Scientific, 1–23, second ed.

Constantinides, G. M., Donaldson, J. B., Mehra, R., 2002. Junior can't borrow: A new perspective on the equity premium puzzle. Quarterly Journal of Economics 117, 269–296.

Constantinides, G. M., Duffie, D., 1996. Asset pricing with heterogeneous consumers. Journal of Political Economy 104, 219–240.

Copeland, T. E., Weston, J. F., Shastri, K., 2014. Financial theory and corporate policy. Pearson, Harlow, Essex, fourth ed.

Cox, J. C., Ingersoll, Jr., J. E., Ross, S. A., 1985a. An intertemporal general equilibrium model of asset prices. Econometrica 53, 363.

Cox, J. C., Ingersoll, Jr., J. E., Ross, S. A., 1985b. A theory of the term structure of interest rates. Econometrica 53, 385–408.

Cox, J. C., Ross, S. A., 1976. The valuation of options for alternative stochastic processes. Journal of Financial Economics 3, 145–166.

Croce, M. M., 2014. Long-run productivity risk: A new hope for production-based asset pricing? Journal of Monetary Economics 66, 13–31.

Cvitanić, J., Malamud, S., 2010. Relative extinction of heterogeneous agents. B.E. Journal of Theoretical Economics 10, 1–21.

Dai, Z., Maydew, E., Shackelford, D. A., Zhang, H. H., 2008. Capital gains taxes and asset prices: Capitalization or lock-in? Journal of Finance 63, 709–742.

Dammon, R. M., 2001. Optimal consumption and investment with capital gains taxes. Review of Financial Studies 14, 583–616.

Danthine, J.-P., Donaldson, J. B., 2015. Intermediate financial theory. Third ed.

Debreu, G., 1951. The coefficient of resource utilization. Econometrica 19, 273–292.

Diamond, P. A., 1965. National debt in a neoclassical growth model. American Economic Review 55, 1126–1150.

Diamond, P. A., 1973. Taxation and public production in a growth setting. In: *Models of economic growth: proceedings of a conference held by the International Economic Association at Jerusalem*, Wiley, New York, 215–240.

Dieckmann, S., Gallmeyer, M., 2005. The equilibrium allocation of diffusive and jump risks with heterogeneous agents. Journal of Economic Dynamics and Control 29, 1547–1576.

Donkers, B., van Soest, A., 1999. Subjective measures of household preferences and financial decisions. Journal of Economic Psychology 20, 613–642.

Dumas, B., 1989. Two-person dynamic equilibrium in the capital market. Review of Financial Studies 2, 157–188.

Dumas, B., Lyasoff, A., 2012. Incomplete-market equilibria solved recursively on an event tree. Journal of Finance 67, 1897–1941.

Eisenhardt, K. M., 1989. Agency theory: An assessment and review. The Academy of Management Review 14, 57–74.

Epstein, L. G., Zin, S. E., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. Econometrica 57, 937–969.

Epstein, L. G., Zin, S. E., 1991. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis. Journal of Political Economy 99, 263–286.

Farmer, R. E., Nourry, C., Venditti, A., 2011. Debt, deficits and finite horizons: The stochastic case. Economics Letters 111, 47–49.

Favilukis, J., 2013. Inequality, stock market participation, and the equity premium. Journal of Financial Economics 107, 740–759.

Fernández-Villaverde, J., Krueger, D., 2007. Consumption over the life cycle: Facts from Consumer Expenditure Survey data. Review of Economics and Statistics 89, 552–565.

Fischer, M., Jensen, B. A., 2014. Losing welfare by receiving transfers. Unpublished Working Paper.

Fischer, M., Jensen, B. A., 2015. Taxation, transfer income and stock market participation. Review of Finance 19, 823–863.

Fischer, M., Jensen, B. A., 2017. The debt tax shield, economic growth and inequality. Unpublished Working Paper.

Fischer, M., Kraft, H., Munk, C., 2013. Asset allocation over the life cycle: How much do taxes matter? Journal of Economic Dynamics and Control 37, 2217–2240.

Fisher, I., 1930. The theory of interest: As determined by impatience to spend income and opportunity to invest it. Macmillan, New York.

Fisher, J., Johnson, D., Smeeding, T., Thompson, J., 2018. Inequality in 3-D: Income, consumption, and wealth. Finance and Economics Discussion Series 2018-001, 1–49.

Frederick, S., Loewenstein, G., O'Donoghue, T., 2002. Time discounting and time preference: A critical review. Journal of Economic Literature 40, 351–401.

Friedman, M., 1957. A theory of the consumption function, vol. 63 of *General Series*. Princeton University Press, Princeton.

Fuchs-Schündeln, N., Krueger, D., Sommer, M., 2010. Inequality trends for Germany in the last two decades: A tale of two countries. Review of Economic Dynamics 13, 103–132.

Gârleanu, N., Panageas, S., 2015. Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing. Journal of Political Economy 123, 670–685.

Gertler, M., 1999. Government debt and social security in a life-cycle economy. Carnegie-Rochester Conference Series on Public Policy 50, 111–117.

Giannetti, M., Koskinen, Y., 2010. Investor protection, equity returns, and financial globalization. Journal of Financial and Quantitative Analysis 45, 135–168.

Gollier, C., Zeckhauser, R., 2005. Aggregation of heterogeneous time preferences. Journal of Political Economy 113, 878–896.

Gomes, F., Michaelides, A., 2005. Optimal life-cycle asset allocation: Understanding the empirical evidence. Journal of Finance 60, 869–904.

Gomes, F., Michaelides, A., 2008. Asset pricing with limited risk sharing and heterogeneous agents. Review of Financial Studies 21, 415–448.

Gomes, F., Michaelides, A., Polkovnichenko, V., 2013. Fiscal policy and asset prices with incomplete markets. Review of Financial Studies 26, 531–566.

Gorman, W. M., 1959. Separable utility and aggregation. Econometrica 27, 469–481.

Gourinchas, P.-O., Parker, J. A., 2002. Consumption over the life cycle. Econometrica 70, 47–89.

Grossman, S. J., Shiller, R. J., 1981. The determinants of the variability of stock market prices. American Economic Review 71, 222–227.

Guiso, L., Paiella, M., 2008. Risk aversion, wealth, and background risk. Journal of the European Economic Association 6, 1109–1150.

Guvenen, F., 2006. Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective. Journal of Monetary Economics 53, 1451–1472.

Guvenen, F., 2011. Macroeconomics with heterogeneity: A practical guide. Economic Quarterly 97, 255–327.

Halek, M., Eisenhauer, J. G., 2001. Demography of risk aversion. Journal of Risk and Insurance 68, 1–24.

Hansen, L. P., Jagannathan, R., 1991. Implications of security market data for models of dynamic economies. Journal of Political Economy 99, 225–262.

Hansen, L. P., Richard, S. F., 1987. The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models. Econometrica 55, 587–613.

Harrison, G. W., Lau, M. I., Williams, M. B., 2002. Estimating individual discount rates in Denmark: A field experiment. American Economic Review 92, 1606–1617.

Harrison, J., Kreps, D. M., 1979. Martingales and arbitrage in multiperiod securities markets. Journal of Economic Theory 20, 381–408.

Hartog, J., Ferrer-i Carbonell, A., Jonker, N., 2002. Linking measured risk aversion to individual characteristics. Kyklos 55, 3–26.

Havranek, T., Horvath, R., Irsova, Z., Rusnak, M., 2015. Cross-country heterogeneity in intertemporal substitution. Journal of International Economics 96, 100–118.

He, H., 1991. Optimal consumption-portfolio policies: A convergence from discrete to continuous time models. Journal of Economic Theory 55, 340–363.

Heathcote, J., Perri, F., Violante, G. L., 2010. Unequal we stand: An empirical analysis of economic inequality in the United States, 1967-2006. Review of Economic Dynamics 13, 15–51.

Heaton, J., Lucas, D., 1999. Stock prices and fundamentals. NBER Macroeconomics Annual 14, 213–242.

Hendricks, L., 2007. How important is discount rate heterogeneity for wealth inequality? Journal of Economic Dynamics and Control 31, 3042–3068.

Henry, J. F., 2012. The making of neoclassical economics. Routledge Revivals, Routledge, London.

Hicks, J., 1939. Value and capital: An inquiry into some fundamental principles of economic theory. Clarendon Press, Oxford.

Hong, H., Stein, J. C., 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. Journal of Finance 54, 2143–2184.

Hubbard, R. G., Skinner, J. S., Zeldes, S. P., 1994. The importance of precautionary motives in explaining individual and aggregate saving. Carnegie-Rochester Conference Series on Public Policy 40, 59–125.

Hurd, M. D., 1989. Mortality risk and bequests. Econometrica 57, 779–813.

IMF, 2017. Fiscal Monitor: Tackling inequality. World Economic and Financial Surveys, Washington, DC.

Inada, K.-I., 1963. On a two-sector model of economic growth: Comments and a generalization. Review of Economic Studies 30, 119–127.

Jahan-Parvar, M. R., Liu, X., Rothman, P., 2013. Equity returns and business cycles in small open economies. Journal of Money, Credit and Banking 45, 1117–1146.

Jäkel, M. F., 2006. Pensionomics: On the role of PAYGO in pension portfolios, vol. 572 of *Lecture Notes in Economics and Mathematical Systems*. Springer, Berlin and New York.

Jermann, U. J., 1998. Asset pricing in production economies. Journal of Monetary Economics 41, 257–275.

Jevons, W. S., 1871. The theory of political economy. Macmillan, London, third ed.

Jones, L. E., Manuelli, R. E., Rossi, P. E., 1997. On the optimal taxation of capital income. Journal of Economic Theory 73, 93–117.

Judd, K. L., 1985. Redistributive taxation in a simple perfect foresight model. Journal of Public Economics 28, 59–83.

Kageyama, J., 2013. Why are children impatient? Evolutionary selection of preferences. Theoretical Economics Letters 3, 18–25.

Kapteyn, A., Teppa, F., 2003. Hypothetical intertemporal consumption choices. Economic Journal 113, C140–C152.

Keeley, B., 2015. Income Inequality: The gap between ich and poor. OECD Insights, OECD Publishing, Paris.

Koopmans, T. C., 1965. On the concept of optimal economic growth. In: *The Econometric Approach to Development Planning*, North Holland, Amsterdam, 225–300.

Koopmans, T. C., 1967. Objectives, constraints, and outcomes in optimal growth models. Econometrica 35, 1–15.

Kotlikoff, L. J., Smetters, K., Walliser, J., 1999. Privatizing social security in the United States: Comparing the options. Review of Economic Dynamics 2, 532–574.

Kreps, D. M., Porteus, E. L., 1978. Temporal resolution of uncertainty and dynamic choice theory. Econometrica 46, 185.

Krueger, D., Mitman, K., Perri, F., 2017. On the distribution of the welfare losses of large recessions. In: *Advances in Economics and Econometrics*, Cambridge University Press, Cambridge, vol. 2 of *Econometric Society Monographs* 143–184.

Krueger, D., Perri, F., 2006. Does income inequality lead to consumption inequality? Evidence and theory. Review of Economic Studies 73, 163–193.

Krusell, P., Smith, J. A. A., 1998. Income and wealth heterogeneity in the macroeconomy. Journal of Political Economy 106, 867–896.

Kydland, F. E., Prescott, E. C., 1982. Time to build and aggregate fluctuations. Econometrica 50, 1345–1370.

La Croix, David de, Michel, P., 2002. A theory of economic growth: Dynamics and policy in overlapping generations. Cambridge University Press, Cambridge and New York.

Lawrance, E. C., 1987. Transfers to the poor and long run savings. Economic Inquiry 25, 459–478.

Lawrance, E. C., 1991. Poverty and the rate of time preference: Evidence from panel data. Journal of Political Economy 99, 54–77.

Lengwiler, Y., 2005. Heterogeneous patience and the term structure of real interest rates. American Economic Review 95, 890–896.

Lengwiler, Y., 2006. Microfoundations of financial economics: An introduction to general equilibrium asset pricing. Princeton Paperbacks, Princeton University Press, Princeton, second ed.

Lengwiler, Y., Malamud, S., Trubowitz, E., 2005. Asset pricing in heterogeneous economies. Unpublished Working Paper.

Lettau, M., Ludvigson, S. C., 2005. Expected returns and expected dividend growth. Journal of Financial Economics 76, 583–626.

Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Review of Economics and Statistics 47, 13–37.

Ljungqvist, L., Sargent, T. J., 2004. Recursive macroeconomic theory. MIT Press, Cambridge, Mass., second ed.

Loewenstein, G., Prelec, D., 1991. Negative time preference. American Economic Review 81, 347–352.

Long, J. B., Plosser, C. I., 1983. Real business cycles. Journal of Political Economy 91, 39–69.

Longstaff, F. A., Wang, J., 2012. Asset pricing and the credit market. Review of Financial Studies 25, 3169–3215.

Lucas, Jr., R. E., 1978. Asset prices in an exchange economy. Econometrica 46, 1429–1445.

Ludvigson, S., 1996. The macroeconomic effects of government debt in a stochastic growth model. Journal of Monetary Economics 38, 25–45.

Lustig, H., van Nieuwerburgh, S., Verdelhan, A., 2013. The wealth-consumption ratio. Review of Asset Pricing Studies 3, 38–94.

Malinvaud, E., 1987. The overlapping generations model in 1947. Journal of Economic Literature 25, 103–105.

Mankiw, N., Zeldes, S. P., 1991. The consumption of stockholders and nonstockholders. Journal of Financial Economics 29, 97–112.

Marshall, A., 1890. Principles of economics. Macmillan, London.

Mayers, D., 1972. Nonmarketable assets and capital market equilibrium under uncertainty. In: Jensen, M. C. (ed.), *Studies in the theory of capital markets*, New York, 223–248.

Mehra, R., 2012. Consumption-based asset pricing models. Annual Review of Financial Economics 4, 385–409.

Mehra, R., Prescott, E. C., 1985. The equity premium: A puzzle. Journal of Monetary Economics 15, 145–161.

Mendoza, E. G., 1991. Real business cycles in a small open economy. American Economic Review 81, 797–818.

Menger, C., 1871. Grundsätze der Volkswirthschaftslehre. Braumüller, Wien.

Mertens, K., Ravn, M. O., 2013. The dynamic effects of personal and corporate income tax changes in the United States. American Economic Review 103, 1212–1247.

Merton, R. C., 1969. Lifetime portfolio selection under uncertainty: The continuous-time case. Review of Economics and Statistics 51, 247–257.

Miao, J., 2014. Economic dynamics in discrete time. MIT Press, Cambridge, Mass.

Modigliani, F., 1966. The life cycle hypothesis of saving, the demand for wealth and the supply of capital. Social Research 33, 160–217.

Mossin, J., 1966. Equilibrium in a capital asset market. Econometrica 34, 768–783.

Munk, C., 2013. Financial asset pricing theory. Oxford University Press, Oxford, first ed.

Nakamura, T., 1999. Risk-aversion and the uncertainty–investment relationship: A note. Journal of Economic Behavior & Organization 38, 357–363.

Nakamura, T., 2002. Finite durability of capital and the investment-uncertainty relationship. Psychological Aspects of Economic Behavior 48, 51–56.

OECD, 2015. In it together: Why less inequality benefits all. OECD Publishing, Paris.

OECD, 2016. Inequality update - November 2016. OECD Publishing, Paris.

OECD, 2017a. Bridging the gap: Inclusive growth (2017 Update Report). OECD Publishing, Paris.

OECD, 2017b. Pensions at a glance 2017: OECD and G20 indicators. OECD Publishing, Paris.

Outreville, J. F., 2015. The relationship between relative risk aversion and the level of education: A survey and implications for the demand for life insurance. Journal of Economic Surveys 29, 97–111.

Palacios, M., 2015. Human capital as an asset class implications from a general equilibrium model. Review of Financial Studies 28, 978–1023.

Pålsson, A.-M., 1996. Does the degree of relative risk aversion vary with household characteristics? Journal of Economic Psychology 17, 771–787.

Pastor, L., Veronesi, P., 2016. Income inequality and asset prices under redistributive taxation. Journal of Monetary Economics 81, 1–20.

Pestieau, P. M., 1974. Optimal taxation and discount rate for public investment in a growth setting. Journal of Public Economics 3, 217–235.

Piatti, I., 2014. Heterogeneous beliefs about rare event risk in the Lucas orchard. Unpublished Working Paper.

Piketty, T., Goldhammer, A., 2014. Capital in the twenty-first century. Belknap Press of Harvard University Press, Cambridge, Mass.

Ramsey, F. P., 1927. A contribution to the theory of taxation. Economic Journal 37, 47–61.

Ramsey, F. P., 1928. A mathematical theory of saving. Economic Journal 38, 543–559.

Rawls, J., 1999. A theory of justice. Belknap Press of Harvard University Press, Cambridge, Mass., revised ed.

Read, D., Read, N., 2004. Time discounting over the lifespan. Organizational Behavior and Human Decision Processes 94, 22–32.

Rietz, T. A., 1988. The equity risk premium a solution. Journal of Monetary Economics 22, 117–131.

Rogers, A. R., 1994. Evolution of time preference by natural selection. American Economic Review 84, 460–481.

Romer, C. D., Romer, D. H., 2010. The macroeconomic effects of tax changes: Estimates based on a new measure of fiscal shocks. American Economic Review 100, 763–801.

Romer, C. D., Romer, D. H., 2016. Transfer payments and the macroeconomy: The effects of social security benefit increases, 1952-1991. American Economic Journal: Macroeconomics 8, 1–42.

Ross, S. A., 1976. The arbitrage theory of capital asset pricing. Journal of Economic Theory 13, 341–360.

Ross, S. A., 1978. A simple approach to the valuation of risky streams. Journal of Business 51, 453–475.

Rubinstein, M., 1974. An aggregation theorem for securities markets. Journal of Financial Economics 1, 225–244.

Rubinstein, M., 1976. The valuation of uncertain income streams and the pricing of options. Bell Journal of Economics 7, 407.

Samuelson, P. A., 1958. An exact consumption-loan model of interest with or without the social contrivance of money. Journal of Political Economy 66, 467–482.

Samuelson, P. A., 1969. Lifetime portfolio selection by dynamic stochastic programming. Review of Economics and Statistics 51, 239–246.

Samwick, A. A., 1998. Discount rate heterogeneity and social security reform. Journal of Development Economics 57, 117–146.

Sardadvar, S., 2011. Neoclassical growth theory and standard models. In: Sardadvar, S. (ed.), *Economic Growth in the Regions of Europe*, Springer, Berlin and Heidelberg, vol. 1 of *Contributions to Economics*, 9–22.

Sen, A. K., 1961. On optimising the rate of saving. Economic Journal 71, 479.

Sharpe, W. F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance 19, 425–442.

Sialm, C., 2006. Stochastic taxation and asset pricing in dynamic general equilibrium. Journal of Economic Dynamics and Control 30, 511–540.

Sialm, C., 2009. Tax changes and asset pricing. American Economic Review 99, 1356–1383.

Solow, R. M., 1956. A contribution to the theory of economic growth. Quarterly Journal of Economics 70, 65–94.

Solow, R. M., 1974. The economics of resources or the resources of economics. American Economic Review 64, 1–14.

Solow, R. M., 1999. Neoclassical growth theory. In: *Handbook of Macroeconomics*, Elsevier, vol. 1, 637–667.

Sozou, P. D., Seymour, R. M., 2003. Augmented discounting: Interaction between ageing and time-preference behaviour. Proceedings of the Royal Society of London B: Biological Sciences 270, 1047–1053.

Sundaresan, M., 1984. Consumption and equilibrium interest rates in stochastic production economies. Journal of Finance 39, 77–92.

Swan, T. W., 1956. Economic growth and capital accumulation. Economic Record 32, 334–361.

Thimme, J., 2017. Intertemporal substitution in consumption: A literature review. Journal of Economic Surveys 31, 226–257.

Trostel, P. A., Taylor, G. A., 2001. A theory of time preference. Economic Inquiry 39, 379–395.

Vasicek, O. A., 2005. The economics of interest rates. Journal of Financial Economics 76, 293–307.

Veblen, T., 1900. The preconceptions of economic science. Quarterly Journal of Economics 14, 240–269.

Viceira, L. M., 2001. Optimal portfolio choice for long-horizon investors with nontradable labor income. Journal of Finance 56, 433–470.

Vissing-Jørgensen, A., 2002. Limited asset market participation and the elasticity of intertemporal substitution. Journal of Political Economy 110, 825–853.

Vissing-Jørgensen, A., Attanasio, O. P., 2003. Stock-market participation, intertemporal substitution, and risk-aversion. American Economic Review 93, 383–391.

von Neumann, J., Morgenstern, O., 1944. Theory of games and economic behavior. Theory of Games and Economic Behavior, Princeton University Press, Princeton, N.J.

Walras, L., 1874. Elements of pure economics: Or the theory of social wealth. Allen and Unwin, London, second ed.

Wang, J., 1993. A model of intertemporal asset prices under asymmetric information. Review of Economic Studies 60, 249–282.

Wang, J., 1994. A model of competitive stock trading volume. Journal of Political Economy 102, 127–168.

Wang, J., 1996. The term structure of interest rates in a pure exchange economy with heterogeneous investors. Journal of Financial Economics 41, 75–110.

Weil, P., 1989a. The equity premium puzzle and the risk-free rate puzzle. Journal of Monetary Economics 24, 401–421.

Weil, P., 1989b. Overlapping families of infinitely-lived agents. Journal of Public Economics 38, 183–198.

Weil, P., 2008. Overlapping generations: The first jubilee. Journal of Economic Perspectives 22, 115–134.

Wilson, R., 1968. The theory of syndicates. Econometrica 36, 119–132.

Wolff, E. N., 2010. Recent trends in household wealth in the United States: Rising debt and the middle-class squeeze - An update to 2007. Unpublished Working Paper.

Wolff, E. N., 2017. Household wealth trends in the United States, 1962 to 2016: Has middle class wealth recovered? Unpublished Working Paper.

Zeldes, S. P., 1989. Optimal consumption with stochastic income: Deviations from certainty equivalence. Quarterly Journal of Economics 104, 275–298.

University of Bamberg Press

The consideration of the distribution of income, wealth and consumption within countries around the globe reveals one basic and consistent picture: inequality in the allocation of resources is a prevailing global phenomenon and subject to an ongoing negative trend since, at least, the last three decades. In general, governments are equipped with a range of instruments and tools to influence distribution, with redistributive tax systems being considered the most direct, powerful and popular instrument in this context. Besides its influence on resource allocation, however, tax and transfer policies also possess substantial effects on many economic areas, for example, economic growth, financial markets and individual as well as aggregate welfare.

Although widely discussed, the effects of taxation and redistribution as well as the underlying causes are not yet fully understood. By the means of economic equilibrium models the present work, therefore, addresses the influence of redistributive taxation within a closed economy populated by heterogeneous agents. The results show that rising tax rates on labor income or capital gains are generally associated with decreasing economic growth rates and, hence, diminishing future consumption possibilities. Moreover, in a variety of cases, redistributive taxation might foster inequality in wealth, consumption and participation rates. Even when effective, redistributive taxation is generally associated with a trade-off between macroeconomic growth and equality (or welfare).

ISBN 978-3-86309-703-5

9 783863 097035

www.uni-bamberg.de/ubp/