Urban Scaling

Allometry in Urban Studies and Spatial Science

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Urban allometry origins, critics and city performance evaluations

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2.1 Scaling laws

Scaling laws empirically test the functional relationship between two quantifiable variables (y, x) which scale with each other. Typically, one of these two variables is the organism, or system, as a whole (x), and the other (y) is a trait of the latter, which could be a physical quantity or a phenomenon. When x and y change at the same rate as each other, the relationship is called *isometric*, or linearly proportional, otherwise, when they do not scale linearly, namely disproportionally, it is called *allometric* [1].

A scaling relationship is a quantitative description of the change of measurable characteristics of a system when the whole system size changes. It is a measure of the *covariation* of a quantifiable trait of a system and the size of the latter. It does not designate their *causal* link, but only the nature of their *association*.

Such allometric relationships are usually following a power law function (eq. 2.1) which can also be written by using the logarithms (eq. 2.2), which allows for an easier interpretation: in a double logarithmic y–x plot if the observations align on a straight line we can describe the y–x link by a power law whose exponent is the slope of such a straight line which can be quantified via regression analysis.

$$y = ax^{\beta} \tag{2.1}$$

$$logy = loga + \beta logx \tag{2.2}$$

The interpretation of a log-log regression coefficient is in terms of percentage (a 1% change in the x, corresponds to a β% change in the y[1]) also called elasticity in economics.

In biology, allometry – called biological scaling – represents the change (e.g. metabolism, surface area, life span, heart beating, etc.) in organisms in relation to proportional changes in their body sizes (i.e., mass).

To my knowledge, the first use of allometric scale was in biology and dates back to 1891 [2]; it got general recognition in the 1920s [3], for later attracting the

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attention of urban scholars who, during the last few decades, started to consistently investigate eventual allometries in urban settlements.

2.2 Urban scaling laws

What are the quantifiable, objective differences among typical small settlements, villages, towns, cities, and megacities within countries? Within a same region, does a typical 2 million inhabitant city, related to a typical 1 million, expect to have double the amount of crime, CO_2 emissions, GDP, built surface, street areas, patents, infections...? If so (isometric relation), or if not (allometric), why? What also happens to density, housing price, and to aspects such as life satisfaction, subjective wellbeing, and physical and mental health?

We will briefly show the state-of-the-art of urban scaling: a relatively recent area of urban science, investigating how measurable characteristics of cities vary (scale) with their sizes. When they scale linearly the relation is isometric (e.g., double population involves double built area); when not (e.g., double population involves more or less than double built area), the relation is allometric involving phenomena such as increasing returns, economies of scale, economies and diseconomies of agglomerations.

Empirical evidence indicates that a type of *universal* behaviour often appears even across countries despite historical unique individual patterns. This universality, the *systematic* scaling of y, makes urban scaling behaviour a main pillar of urban sciences.

Results are statistically robust and often consistent across countries, although attention is needed in keeping a common definition of cities, and methods to measures variables and to estimate the scaling exponent.

2.3 Empirical evidences

Within the best of my knowledge, the earliest allometric laws quantified in the social sciences was in Zipf's work [4] in cities (1949) who found a roughly scaling law between population size and economics variety (service-business, manufactures, retail stores) as well as population density, and it was known [5] since Adam Smith (1776) that occupational specialisation is linked to city sizes.

Since then more empirical findings have been accumulated and we are now reaching a status of knowledge mature enough to establish a new discipline of urban scaling on its own.

I report in Figures 2.1 and 2.2 a summary of empirical knowledge acquired in the last few decades from 46 studies indexed on Scopus.

2.4 Origins?

Reasons explaining why scaling laws empirically appear in cities are still under debate [6–23] and include the following possibilities, sometimes overlapping or mutually linked:

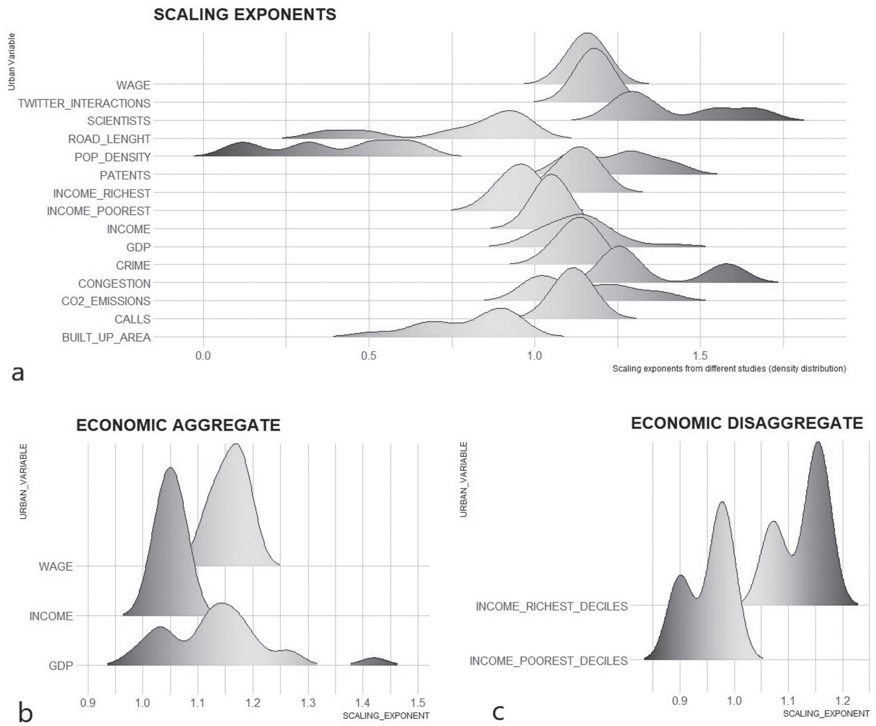

Figure 2.1 Density distribution from empirical results from 46 papers indexed on Scopus, with at least two different studies quantifing the same urban variable scaling exponent. Details about scaling exponents and sources are available at: www.urem.eu/scaling.

1. *Interactions/collaborations*: non-linear scaling of socio-economic variables might be a consequence of the amount of people interactions which typically increase superlinearly with city population size. If the range of (distance-dependent) interactions between citizens and amenities gets bigger, the socio-economic indicators improve, and the infrastructure costs decrease. Studies embracing interaction as a causal link with some urban scaling laws focus on the social network structure of cities and the probability of interactions among people. The more interactions, the more productivity. The probability of finding necessary collaborations is greater in larger populations and could explain some superlinear scaling. When, under constraints (e.g. energy, budget), the links in a network grow with the number of its nodes, scaling laws might emerge [7,15,18,20,24], specifically because *per capita* social connectivity scale-invariantly increases with city size [9].
2. *Densification*: factors such as urbanised area and some services (such as petrol stations) and infrastructures (e.g., streets, gas/electrical/water distribution)

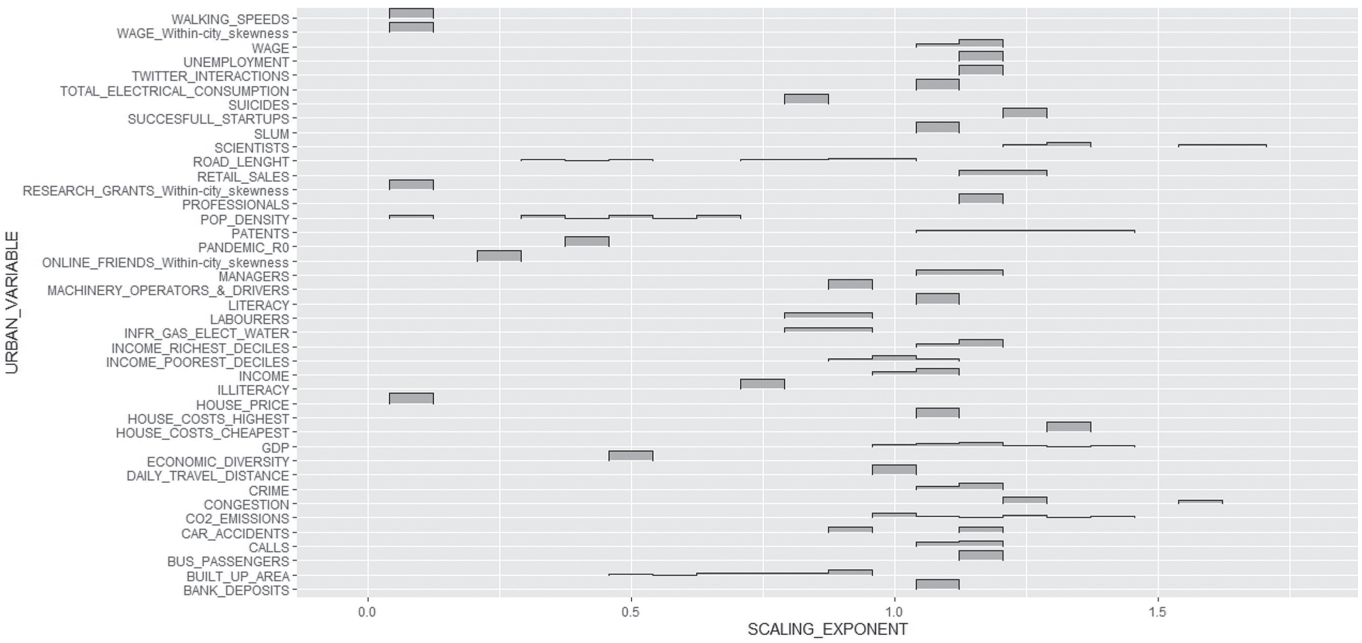

Figure 2.2 All scaling coefficients from 46 papers indexed on Scopus. Details and sources are available at: www.urem.eu/scaling.

might sublinearly scale because density typically increases when a city gets a bigger population. Which in turn is an unplanned free-market consequence of land prices getting higher in the most wanted locations. This spontaneously induces an efficient economy of space [20].

3. *Path-dependency*: cumulative innovations induce city growth, which further induces innovations and so on in a positive-feedback retroactive cycle [21]. When a self-organized system (as a city is) is combined with the Matthew effect (a type of preferential attachment process), power laws could emerge [12].
4. *Geometries and interconnections*: relations between lines, surfaces, and volumes, in a sort of costs–benefits equilibrium in spatial fractal cities (where population – living in 3D buildings – fractality is much larger than road networks) would cause scaling exponents [9,19,25,26]. *If* closer people (nodes in a geographical network) are more likely to be connected, and *if* an urban indicator is the sum of connected node-pairs people activities, and *if* the latter depends on the Euclidean distance between connected, *then* when the urban indicator is an increasing function of the Euclidean distance (e.g. the creative productivity), it scales *superlinearly* (or linearly) with the population size, while when it decreases with the Euclidean distance (e.g., demand for infrastructure) scales *sublinearly* or linearly [22].
5. *Higher complexity*: *if* phenomena depend from the contemporaneous complementarity of various factors; and *if* more complex phenomena require more complementarity (economic complexity, [16,17]); and *if* – as in cultural evolution models, anthropological and urban studies [11,13,27] – the number of factors is proportional to population size; and *if* there is a Gumbel distribution of factors frequency (i.e., rarer factors appears only in big cities); the diversity of factors logarithmically accumulates with population size and generates scaling laws in such phenomena [15].
6. *Social reactor*: Bettencourt [9] proposed the idea of cities as a new type of object in nature that didn't exist before. A complex system we created being between a star and a network. As a star, the bigger the more 'things' attracts and the faster these things 'run'. We created huge social networks set in space-time and we are able to make them evolving and changing without the need to stop them. This allows our extraordinarily inventive and productive nature characterising our species particularly from our urban era. A result of such star-network alike system is the urban scaling law empirically found.
7. *Localisation* versus *urbanisation economies*: some types of industries and occupational types might localise in specific places in order to enjoy agglomeration economies (increasing returns to localisation economies). As these industries are often disproportionately localised in bigger cities, the latter might show increasing returns to scale for localisation economies rather than urbanisation economies per se which might instead follow a constant return to scale [28].
8. *Creative class*: highly talented people (the so-called 'creative class' [29]), could be the driving force for cities' superlinear growth thanks to their creative outputs [30], rather than city population size per se.

Increasing and decreasing returns to scale have been studied by several evolutionary theories such as evolutionary economics, new economic geography, ecological economics, evolutionary transition theory, and evolutionary economic geography.

Rather than a causal variable, the size of the population might be a proxy aggregate variable [31] including all or some of the above potential sub-variables/reasons.

2.5 Planning, city performance, and urban cost–benefit evaluation

Urban scaling allows a new fundamental understanding of cities and, more importantly, in a quantitative way. It gives us tools for a new paradigm in city and regional planning. Knowing such systematic scaling between population size and certain urban factors would lead to more efficient urban management.

Planners and governments can use scaling laws to anticipate consequences of different city sizes, e.g., if a city is expected – or planned – to rapidly grow, or shrink, or be interconnected with other cities reciprocally enjoying spill over (borrow size) effects. Lot of indicators can be inferred and therefore proactively used in planning decisions.

This knowledge adds an important theoretical, empirical, and policies-oriented tool kit which any urban scholar nowadays should, if not dominate, at least be aware of. The impelling need to best accommodate a few billions of new incoming urban dwellers poses us the requirement to well understand what would likely happen to certain factors at different settlement sizes and then, by planning, how to prevent or encourage these factors in an intriguing game between self organization and planning [47]. It also provides scientific inputs to decision-making and policies for the renovation of small settlements (of which the world is full and almost ghost), and offer an intriguing new way to evaluate adjusted performance, costs, and benefits associated with urbanicity levels.

The existence of considerable variations from the regression line were evident since the above-mentioned Zipf work and the successive decades of empirical evidence till nowadays. They suggest the influence of other factors (e.g., history, planning, politics, contingencies, etc.) not taken into account from a simple covariation analysis against population size alone. These variations – which are the residuals from the expected values of equations (2.1) or (2.2) – have been proposed as a fairer evaluation of the performance of cities to measure how well a city is doing *in respect to* a typical city of her size in her region at her time (scale-independent urban indicators, SAMIs [32,33]).

2.6 Points needing attention

1. *Particular* versus *general*: there are two perspectives of viewing cities, an historical one treating each one city as a unique product of historical events which cannot be summarised with a number, being numerically translated and even identical to n other cities just because of having similar population size

[33–35]; and one treating cities as sharing *some* universal features regardless of individualities.

2. *Urban boundaries*: defining cities and therefore their boundaries in different ways is an obvious source of errors which sometimes results in very different scaling coefficients across researches [36]. It is therefore decisive to keep the same definition of city if we wish to *compare* results from different researches.
3. *Aggregative*: most urban scaling research uses only aggregative quantities such us *total* GDP, *total* income, etc. per city, despite the fact that different categories of such quantities (measured, for example, in terms of quantiles, such as the lowest incomes, or the highest) have specific behaviour in relations to city size. In fact, recent researches using disaggregated data, found different scaling coefficients from different categories, often even changing from sub- to superlinear (Figure 2.1b, c) across categories indicating distributional inequalities not possibly emerging from aggregative data [28,37–41].
4. *Categories*: different occupational categories as well as different industry types can show different scaling coefficients [28,41–43]. A city-wide scaling behaviour would be a consequence of specific combinations of these occupational and industry types, partially explaining distances from the expected scaling coefficients (e.g., Cambridge, Oxford, etc.).
5. *Cross-sectional (or transversal, or hierarchical) versus longitudinal (or temporal) scaling*: cross-sectional refers to the classical analysis in urban scaling, namely the functional association between an indicator (e.g. GDP, road areas, crime, CO_2 emissions, etc.) and its change across different city sizes within a region or nation at a given time. It extrapolates elasticities of urban quantifiable factors relative to urban population size at certain times. Longitudinal analysis refers to an indicator change over time for a given city as the latter grows increasing population size. They represent two different things [44].
6. *Phases of economic growth*: some urban properties might have specific scaling coefficients because being in cities during a particular *phase* of their economic growth rather than because they are stuck in universal scaling plots [45]. It will be interesting to investigate if, within regions, an association between population size (or *reciprocal* population size pattern) and economic growth phase is present.
7. *Isolated geographical entities*: studying cities as isolated entities [46] implies underestimation of socio-economic interactions across them and related spillover effects.

Note

1 The % increase of the y can be calculated by the direct substitution of the desired x values in the regression equation, or by: $100[(((100 + p)/100)^b) - 1]$, where p = %increase of the x; or by: $100[(q^b) - 1]$, where q is the multiplicative factor of x (e.g. if x doubles, $q = 2$).

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