Chunxia Qi · Lianghuo Fan · Jian Liu · Qimeng Liu · Lianchun Dong *Editors*

Recent Advances in Mathematics Textbook Research and Development

Proceedings of the 4th International Conference on Mathematics Textbook Research and Development

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Preface

Textbooks are vital curriculum resources for teaching and learning mathematics in classrooms around the world. They play an integral role in the design of learning opportunities for students by teachers, support teachers' learning and define mathematics as a school subject. The development, the reform, the content and the use of textbooks have always been an important theme in mathematics education research. In the past years, an increasing international interest in textbook research and scientifically grounded development of textbooks has been noticeable. Developing an understanding of textbooks and their development and of how they are incorporated into teachers' professional work, how they promote and influence curricular reforms, how they support students' learning have become important endeavors in the field. The challenges of digitalization even emphasise and increase these issues.

The Fourth International Conference on Mathematics Textbook Research and Development (ICMT 4) continues the successful series that started in 2014 with the first ICMT held in Southampton (UK) and was proceeded in 2017 by ICMT2 in Rio de Janeiro (Brazil) and in 2019 by ICMT 3 in Paderborn (Germany).

This volume documents recent issues and the latest development of research on mathematics textbooks presented at the Fourth International Conference on Mathematics Textbook Research and Development (ICMT4), held at Beijing Normal University (China) from 14 to 17 November 2022.

The conference is about all issues related, but not limited to the history, development, content, use of mathematics textbooks and other curriculum resources in all forms (print, digital, online, hybrid, etc.) The themes of the conference are:

- **Development of textbooks** (concepts, task design, learning-teaching-trajectories, design-based research, digital textbooks)
- **The relationship between curriculum reform and textbook development**
- **Content and language of textbooks**
- **Use of textbooks** (students, teachers, parents, etc.) **and student achievement**
- **Historical-perspectives on textbooks**
- **Comparative studies of textbooks**
- **Textbook policies** (governmental educational policies, distribution and market strategies)
- **Meta Research on Mathematics Textbooks** (issues, methods, directions, etc.)
- **Interdisciplinarity in textbooks**
- **Challenges and roles for textbooks under pandemic context**

5 plenary lectures, 4 symposia, 1 workshops, 68 oral communications and 7 posters with participants from all the five continents reflect the richness and large scope of research on mathematics textbooks related to these themes.

ICMT4 was organised by an International Programme Committee:

- Chunxia Qi (Beijing Normal University, China)—Chair

- Lianghuo Fan (East China Normal University, China; University of Southampton, UK)—Co-Chair
- Jian Liu (Beijing Normal University, China)– Chair of LOC
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- Weizhong Zhang (Zhejiang Normal University)

The contributions reflect the work that was done in preparation for the conference based on a peer-review process among the contributors of the conference. We would like to thank all reviewers for their valuable work that helped to improve the quality of all papers in this volume.

Chunxia Qi
Lianghuo Fan
Jian Liu

Plenaries

A Review and the Development of Chinese Elementary School Mathematics Textbooks

Ningzhong, Shi
Northeast Normal University, China

Through the review of the 100-year history of Chinese mathematics curriculum and teaching materials, it can be seen that Chinese mathematics education began in 1867 (the sixth year of Tongzhi's reign) with the establishment of the School of Arithmetic in the Tongwen Hall in Beijing; from the promulgation of "The Constitution of the Elementary School" in 1904 to the "Compulsory Education Mathematics Curriculum Standards (2011 Edition)", a total of 20 curriculum standards were issued (including charters and syllabus); formal elementary school mathematics textbooks, which began with The Mathematics of Penmanship, edited by Dickowen and Zou Liwen and published by the American and Chinese Printing House in 1875, have been published in more than 200 sets since then.

Since the new century, the Chinese mathematics curriculum has undergone a process from the Double Basics (basic knowledge and skills) to the Four Basics (basic knowledge, skills, ideas and activities experiences) and from the Four Basics to Core Literacy, and it emphasises on that students should not only acquire knowledge and skills, but also increase their ability to observe the real world with mathematical insight, think about the real world with mathematical thinking and express the real world with mathematical language.

Traditional mathematics teaching materials guide students to know integers, fractions and decimals with the help of the authentic contexts, and guide students to master the calculation methods of integers, fractions and decimals through practical meaning. This kind of mathematics teaching material is not conducive to students' understanding of the essence of mathematics and the formation and development of their mathematical literacy. Therefore, the concept of "counting units" is introduced in the "Compulsory Education Mathematics Curriculum Standards (2022 Edition)" as an instrumentality to effectively solve the above problems in traditional mathematics teaching, hoping that teaching materials in the future will achieve the consistency of the cognition and operations of numbers, which is a process of mathematization.

Students' Learning of Mathematics From Text(books) and Other Resources

Sebastian Rezat
University of Paderborn, Germany

There has been a growing interest in students' use of textbooks and other learning resources in the past decade. This is grounded in the increasing availability of digital resources and a changing perspective on students' agency for their learning, which includes selecting and using resources. To encourage student agency in selecting appropriate resources and using them effectively for their learning, there is a need to understand how students learn mathematics and how their use of resources influences this. Therefore, it is essential to know their preferences and gain insight into how they use textbooks and other resources.

A significant challenge for investigating students' use of textbooks and other resources is developing methodologies beyond students' self-reports about their choice and use of resources. To provide deep insight into the role of resources in the learning processes and the resource-mediated learning processes themselves, there is a need for methods to reconstruct students' learning processes when interacting with their mathematics textbooks and other resources.

In the talk, I focus on three issues: First, I discuss the methodological issue of gathering data on students' use of mathematics textbooks and other resources that provide deep insight into the process of using them and how this influences their learning of mathematics. Second, I give an overview of what we know about how students use their mathematics textbooks and other resources for learning mathematics. Third, I discuss what circumstances may encourage student agency for using their mathematics textbooks and other resources for learning mathematics.

Categories for Historical Analyses of Mathematical Textbooks

Gert Schubring
Bielefeld University, Germany

Following the analyses by Thomas Kuhn in his studies on Revolutions in Science (1962), research on textbooks has developed into an important means for accessing the so-called normal science. Yet, analyses of historical textbooks, in particular mathematical ones, use not to care much for methodologies for such research and follow rather ad-hoc set-in approaches, focussing on a particular textbook, without any refinement or attempts to situate the specific case within its larger production context.

The lecture will focus on methodologies for contextualised analyses of mathematical textbooks. I will discuss the conception of "context", which has been elaborated recently by Alain Bernard and Christine Proust for textual analyses in history of science (2014), thus making operational a hitherto rather vague or pluri-valent notion, and then discuss its applicability to mathematical textbooks.

Historical textbook analysis thus proves to constitute an interdisciplinary research area. Connecting textual issues by interfaces with developments in the academic discipline, with categories of social history and systems theory, with refined approaches of hermeneutical analysis. Categories specific for textbook analyses are constituted by questions like: who is the author of a textbook? Studies reveal that the person named as author on the cover page might represent a collective of authors, having contributed more or less indirectly—thus, Sylvestre-François Lacroix, the French bestseller-author of the first half of the nineteenth century, spoke of "common property" of text, when he copied from other textbooks.

A key category, which determines not only the use of textbooks, but which also configures the elaboration of textbooks—but which is rather rarely considered for textbook analysis—is constituted by the relation between orality and the written. It is thus that the teacher enters textbook analysis as a decisive structural pattern. The lecture will thus expose the notion of the "textbook triangle", inter-relating the textbook, the teacher and the student and reveal its structural and historical variability for analysing mathematics textbooks for various cultures.

Decoding the Tacit Messages of Instructional Resources in Mathematics Teaching and Learning

Janine Remillard
University of Pennsylvania, USA

Around the world, teachers and schools are increasingly using a vast array of mathematics textbooks, curriculum materials and supplementary resources to support mathematics instruction. These instructional resources vary in format (physical or digital), intended interactional mode (direct use by teachers or students) and how they are used in different contexts and educational systems. They are all arguably designed to support students' mathematics learning, but they also signal important and often tacit messages about relationships between teachers, students and mathematics. This address discusses three different analyses of mathematics instructional resources for the elementary level aimed at uncovering and critically examining some of these tacit messages. An analysis of written communication directed towards the teachers explores how teachers are positioned and teaching is conceptualised by the textbook authors. A multimodal analysis of lesson guides from three cultural contexts examines messages about assumed relationships between teachers, students, the text and mathematics. Finally, an analysis of the student-facing features of online mathematics platforms examines how students are positioned as learners of mathematics and considers the impact of their use alongside other instructional resources. Taken together, these three studies raise questions about how the designs and uses of instructional resources have critical implications for agency and power relations within the human endeavour of mathematics teaching and learning.

Looking Back on Mathematics Textbook Research in Korea: Challenges and Future Directions

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Given the significance of textbooks in mathematics teaching and learning, research on textbooks has recently attracted noticeable attention. The purpose of this paper is to explain the overall research trend of mathematics textbooks in Korea for the international context and to discuss the challenges and future directions of textbook research.

This paper presents an analysis of 418 peer-reviewed papers published in the seven Korean professional journals in mathematics education. The categories of mathematics textbook research are divided into textbook analysis, textbook use and textbook development. The intersections between the categories are specified to reveal what may have been hidden under the most prevalent textbook analysis research. Each category is subdivided into main topics and sub-topics. Furthermore, various cross-analyses are conducted among the topics, school levels, publication years, mathematics content strands and research methods including research participants. In addition, research on foreign textbooks or comparative studies between different textbooks is further analysed.

Building on the review on mathematics textbook research, this paper analyses what has been catalysts for the rapidly increasing textbook research and discusses the challenges and future directions of textbook research. These include methodological considerations, diversification of mathematics textbook research topics, international comparative studies on mathematics textbooks, alternative forms of textbooks, systematic process of textbook development, textbook selection and evaluation and devices to facilitate effective use of textbooks by teachers and students. As such, this paper is expected to activate international comparative or collaborative studies on mathematics textbooks to better notice what gaps exist across different education systems and to search for alternative approaches.

Contents

Symposia—Symposium 1: Textbook Development and Mathematics Curriculum Reform: Evidence-Based Approach in NCM 1-6 Textbook Development in China

The Retrospection of Development of NCM in the Primary grades: A Case Study from Evidence-Based Perspective

Qimeng Liu, Yaoyao Dong, Zhiyong Xie, and Jian Liu(✉)

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Abstract. The calling for transforming textbook development into evidence-based field have been increasingly concerned, while the method and path of how to develop evidence-based textbook still remain. New Century Mathematics (NCM) as one of the textbooks with largest market share and influence, its' development history has witnessed the trajectory of mathematics curriculum reform in China in the last 40 years. This study aimed to retrospect the process of NCM textbook development in primary grades from an evidence-based perspective. Historical research design was employed to explore in what way and how does NCM get to evidence-based textbook development. Qualitative research methodology of the semi-structure interview, focus group interviews, and documentary research have been applied to collect data. The result indicated that in the process of getting to evidence-based textbook development, there are three key features should be mentioned, including Situation + question string establishes the textbook style; Form a complete chain of evidence from fragmented to systematic; Achieving learning-centeredness with the Five Unifications. There were six steps which contained several rounds of data collection. They are preliminary practical investigation, fundamental research, systematic revision, learning and teaching trail, review and revision, and typography and submission for validation. It manifests how does NCM carry out evidence-based textbook. By drawing on the retrospection of development of NCM textbook, the study provides a critical analysis of contemporary methods and issues for mathematics textbook development in China, and puts forwards evidence-based paradigm of textbook development which may provide an inspiration for the whole mathematics textbook research field.

1 Introduction

It is a general consensus that textbook development serves as an important channel for curriculum development (Fan 2013). While the development of mathematics textbook in primary stage occurred within a flawed system. The lack of evidence about effectiveness of textbooks was embodied in the process of textbook development that rely on the experience and knowledge of the authors to develop the content of textbooks and do not field-test the material with students (Reys and Reys 2006).

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Since the late last century and early this century, the calling for transforming textbook development into evidence-based field have been increasingly concerned. (e.g. NCTM 2010). The transformation of evidence-based design in textbook development did not only stay at the level of theoretical discussion and appeal. Many of research groups and publishers set up new projects to explore the new version of evidence-based textbook (e.g. Lappan et al. 2002; Isaacs et al. 2001).

However, though the evidence-based 'label' has been marked on many mathematics textbooks, there were still some prominent flaws that not been overcame. It can be summarized as follow. (1) Too policy-oriented, lack of reflection and promotion from textbook developer (NCTM 2010). (2) The isolation between textbook development and research (Reys and Reys 2006). (3) The evidence was too scattered to help develop and revise textbook systematically (ASCD 2004). (4) The evidence serves for winning competition of textbook selection, not for improvement of itself (Koedel and Polikoff 2017).

The 'lack of evidence' phenomenon also exists in the process of mathematics textbook development in China along with the curriculum reform in the new century (Li 2008), which echoes the problem that encountered by international research field. Drawing on the retrospection of development of NCM textbook would reflect how Chinese mathematics textbook developers deal with the problem of evidence-based design in the development of textbook.

The current study intends to demonstrate that how does NCM develop evidence-based mathematics textbook in primary stage. With this purpose in mind, this paper addresses the following research questions:

1. How does NCM develop mathematics textbook in primary grades based on evidence?
2. What are the key features in the development of NCM textbook?

2 Theoretical Babsis of Current Study

According to The Scientific principle 4 raised by National Research Council (2002), the scientific research required the development of a logical chain of reasoning from evidence to theory and back again that is coherent and shareable. Mathematics textbooks, or its more broadly stem, curriculum resources, are regarded as scientific instruments for the implementation of change in mathematics classrooms (e.g. Cai and Howson 2013). For the research of the retrospection of development of NCM in the Primary Grades as a case study, Participatory Action Research (PAR) and Triangulation would be involved to frame the theoretical basis of the research. PAR seeks to understand and improve the world by changing it. At its heart is collective, self-reflective inquiry that researchers and participants undertake, so they can understand and improve upon the practices in which they participate and the situations in which they find themselves. The reflective process is directly linked to action, influenced by understanding of history, culture, and local context and embedded in social relationships (Baum et al. 2006). While Triangulation is a method used to increase the credibility and validity of research findings (Cohen et al. 2000). It refers to the use of multiple methods or data sources in qualitative research to develop a comprehensive understanding of phenomena. The theoretical basis of PAR and Triangulation provides a well explicit chain of analyzing the evidence basis of textbook development for the current study. To be more specific, the textbook would be analyzed from those four aspects in terms of mathematical content, students' learning, teachers' teaching and the goal achievement.

3 Method

3.1 Participants

The study target on the development of NCM textbook in primary grades. As there were 6 grades in Chinese elementary stage, three chief editors and six editors 1–6 sub-volumes of NCM textbooks would participate in this study.

3.2 Data Collection

The study adopts both narrative inquiry and documentary research as the way of collecting, evaluating and synthesizing evidence. The narrative inquiry provides an up-down perspective on subjective statements, while documentary research affords bottom-up evidence support. Informal conversations and semi-structured group interviews were carried out to collect data for narrative inquiry. The conversations and interviews were conducted by one-to-many WeChat conference call for about 30 min in each section. The interview data was collected by real-time audio recording. 11 Meeting Minutes and Records were reserved. 23 Articles and 5 working papers from nine editors about textbook compilation, investigation report and meeting minutes were gathered for documentary research.

3.3 Data Analysis

We adapted Schoenfeld's (1985) verbal protocol analysis to analyze the process of textbook development. This method is an event-based sampling. Compared to time-based sampling, the processes are not divided into fixed time segments (e.g., 30s), which are then coded. Instead, new codes are set when the interviewees' behavior or topic changes. At first, the recorded interviews are segmented into "macroscopic chunks of consistent behavior or topics" (Schoenfeld 1985, p. 292) that are called episodes in which "an editor or group of editors is engaged in one large topic or closely related body of topics in the service of the same goal". In a second step, the episodes are then characterized in terms of content.

There are three major levels of coding—open coding, axial coding and selective coding (Corbin and Strauss 2008). Open coding help breaking data apart and delineating concepts to stand for blocks of raw data. Axial coding enables researchers to identify any connections that may exist between codes. Selective coding was manipulated to illustrate the mode of NCM to develop textbook.

The numerous codes at the lowest level (open coding) of the spiral are analyzed using constant comparison, in which we employed event-based sampling according to the particular year. Some or all of them were then clustered into a smaller number of groups of focused codes or categories. In this study, we integrated those codes into different categories in terms of the changing of the version of textbook (axial coding). Besides, the higher level of codes were refined to building on and enhancing their overall conceptual and explanatory power (selective coding), which resulted in four three characteristics (Bryant 2017).

3.4 Validity and Trustworthiness

In this study, interviews, article, reports and minutes provided multiple data sources. Member checking helped to provide credibility (Kownacki et al. 2020). By giving participants the opportunity to review the interpretation and conclusions drawn by the researchers for comparison with the meaning they intended. A transcript of each participant's individual interview or minutes of group interview were provided for their review and feedback to endorse findings. In keeping with participatory modes of research, participants were informed and involved in the research process (Brown et al. 2002). To analyze the interrater agreement, independent coders with various backgrounds including mathematics education and mathematics textbook editing were trained. The consistency of both coders was above 90%.

4 Results

4.1 How Does NCM Develop Mathematics Textbook in Primary Grades Based on Evidence

Referring to the result of axial coding, for research question 1, as the iteration of NCM textbook, we gradually form the framework of development and revision process of NCM textbook that integrated fragments into a whole.

For the 5th edition of NCM textbook's development, there were six steps which contained several rounds of data collection. They are priliminary practical investigation, foundamental research, systematic revision, learning and teaching trail, review and revision, and typography and submition for validation. It could help provide sufficient evidence to develop and revise textbook. There are two steps which should be specifically noted as they are central to the evidence-based research (Fig. 1).

1. Fundamental reseach

First is the fundamental reseach, it inludes comparative research, thematic research, and students analysis. In the comparative researh, the writting team would compare different editions of textbook from countries all over the world, such as Primary math and power math from UK, New Discovering Mathematics and Targeting mathematics from sigapore, and so on.

In thematic reseach, we will consider many factors that should be taken into account to improve and innovate NCM textbook. For example, in this series of symposium, we have a study on the design Framework for Project-Based learning in NCM textbook; Mathematics teachers' perspectives on NCM textbooks' design, and so on.

And in student analysis, there are many studies that were employed to understand the myths of students when using textbooks. For example, we have study on students' opportunities to learn reasoning and arguments in algebra in NCM textbooks.

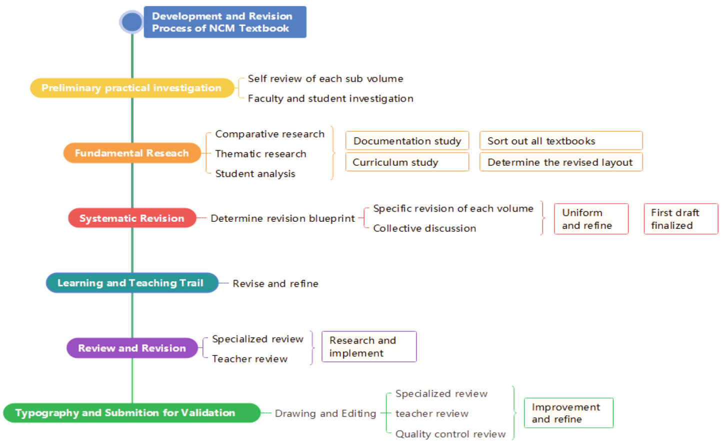

Fig. 1. Development and Revision Process of NCM Textbook

2. Learning and teaching trail

Another one that should be illustrated is learning and teaching trail. This is a sophisticated system, just like a series of pilot studies that put textbook into practice gradually, to make sure that reflections from everyone involved in this system could be manifested, captured, rehearsed.

Learning and teaching trail contains clarifying research questions, sampling, training and discussion, instruments design for investigation, individual interview, group teaching observation, and whole-class teaching. All of findings would be assembled in to a research report to help editors revise and refine textbook (Fig. 2).

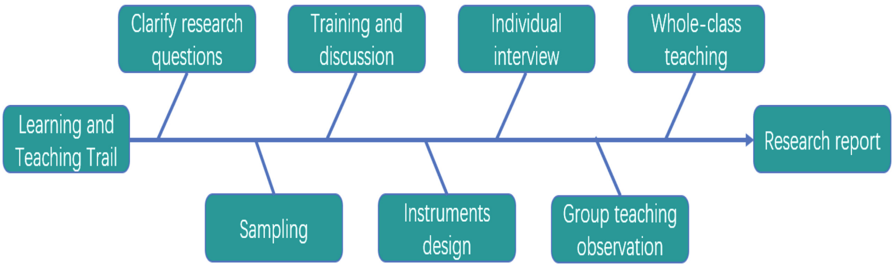

Fig. 2. Learning and teaching trail of NCM Textbook

4.2 What Are the Key Features in the Development of NCM Textbook

The results of qualitative analysis indicated that in the process of getting to evidence-based textbook development, there are three typical characteristics the NCM hold, which were Form a complete chain of evidence from fragmented to systematic; Situation +

question string establishes the textbook style; Achieving learning-centeredness with the Five Unifications.

1. Form a complete chain of evidence from fragmented to systematic;

As the version change of NCM textbook, the evidence gained from the process of textbook development were more and more refined and elaborated, which gradually form a chain of evidence that integrated fragmented into a whole. For the initiative of the 1st version of NCM textbook, like CMP and UCSMP, unit design, field trials and data feedback from teachers and students over several years were the key elements of evidence that lead the process of textbook development and revision (Lappan et al. 2002; Isaacs et al. 2001). However, with the continuous deepening of the understanding of the connotation of evidence, design-research embodied in the whole process of textbook development become the theoretical basis for obtaining evidence to revise textbook. Priliminary practical investigation, foundamental research, systematic revision, learning and teaching trail, review and revision, and typography and submition for validation are all surrounded around design-research. Evidence collected from each section is all important episode that been unified into one framework that taken into consideration throughout the entire process of textbook revision.

2. Situation + question string establishes the textbook style;

There is a common consensus that the impact of curricula on learning principally through textbook presentation, including problem contexts, types of problems, and sequences of presenting the concepts (Ding and Li 2010). 'Situation + question string' is an overall scenario design which considering problem contexts, types of problems and sequences of presenting together to help student learning in deep in mathematics. From this stand point of view, 'Situation + question string' is more than a way of learning and teaching, it is also a systematical reflection model of students' learning process, which is access to evidence of students' thinking. It represents a basic narrative style that containing 'problem contexts-mathematics modeling-explanation and application'. Following this model, the key learning goal was triggered by the course content carried in a specific situation. Students' thought, strategies and confusions were showed when they entered the situation and followed the guidance of question string (Liu et al. 2014). It provides evidence from varies sources to help editors modified textbook.

3. Achieving learning-centeredness with the Five Unifications

In the development and using of NCM textbook, the process of occurring on mathematics knowledge, the process of students' mathematics learning, the process of teachers' guide to students, the process of achieving were unified under the process of unfolding context + question string, which utimately form the model of 'Five Unifications'. 'Five Unifications' was firstly proposed by the editor-in-chief of NCM, which was enlightened by curriculum theory claimed by IEA (Liu et al. 2014). Teachers, students, curriclum and learning practice get involed in the same senario to achieving such a goal of aligning the teaching process with the learning process. The selection and organization of mathematics textbook contents is a mean of dialogue with National Curriculum that implement planed curriculum. The concerning about students' learning in textbook, on the other hand, represent the carry out of intended curriculum that provide a substantial of activities and learning opportunities in published curriculum materials. Students not only gain an understanding of important mathematical concepts and mathematical ideas, but more importantly, tell them how to learn mathematics, how to explore and pose mathematical problems, and how to analyze and solve problems in the process of doing mathematics. Furthermore, the teachers' teaching supported by NCM textbook fully reflected in the field trail in classroom, in which teachers could apply the well- designed activities by textbook designers under the clear guideline. And the examination and inspection of goal achievement in the process of students' communication, sharing, discussion, and questioning mapped out how students gradually learn to think systematically, multidimensionally, and mathematically, and meet the requirement of National Curriculum from cognitive and non-cognitive aspects. At the same time, such highly inclusive model will provide convenience for textbook editors to accurately understand and grasp the pros and cons of textbook as well as getting first-hand data directly from students' thinking process.

5 Conclusion and Discussion

Compared with the existing evidence-based research of mathematics textbook, we can see the unique features that accompanying with the development of NCM textbook. Firstly, NCM provides a systematic chain of evidence to carry out evidence-based research on textbook development. Secondly, the situation + question string restores the authentic situation for students within their mathematics learning in real world that help obtaining comprehensive information on students' mathematical thinking, attitudes and values. Thirdly, the 'Four Unifications' theoretical system constructed with the development of textbook in decades well integrates curriculum theory (planed, intended, enacted, attained) into textbook research. Overall, NCM textbook development process is a process of dialogue with the National Curriculum. While basically following the direction of National Curriculum, the textbook writing group will implant the up-to-date, evidence based research as well as their own understanding and claim into the textbook, which hold a reflective perspective to provide evidence for improve National Curriculum in turn.

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A Design Framework for Project-Based Learning Textbook in Primary Mathematics

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Abstract. Project-based learning (PBL) is an effective approach to develop students' twenty-first century core literacy. The development of PBL textbooks in mathematics can contribute to the implementation of PBL and the cultivation of students' mathematical literacy, thereby promoting the reform of classroom instruction. Relying on the New Century Mathematics (NCM) that is one of the mainstream textbooks in China, this study takes the lead in integrating mathematics PBL into the national curriculum. It combines literature analysis and expert argumentation, focusing on five key processes of mathematics PBL (include Create the real-life context; propose and split a driving task; design and implement subtasks; display and exchange the product; reflection and revision) and its design principles, thus forming a design framework, which can provide inspiration for the development of textbook materials and teaching of mathematics PBL.

1 Introduction

Project-Based learning (PBL), as an important means of cultivating core literacy, has received extensive attention in the field of education in most countries. Mathematical Project-Based learning (Math-PBL) helps to develop students' high-level thinking ability (He and Qi 2018), and is an effective way to develop students' mathematical literacy (Xu 2021). At present, PBL has entered the K-12 education system in many countries (Han et al. 2016), but it has not yet been substantively applied in China with extensive and in-depth classroom practices. In this regard, the development and implementation of Math-PBL textbooks will help the implementation of PBL in conventional mathematics classrooms, so that can force the reformation of means of learning and teaching, and truly promote the reformation of literacy-oriented classroom teaching.

Based on this context, the current study relies on the one of the mainstream primary school mathematics textbooks in China, editorial board of the Chinese New Century Primary School Mathematics Textbook (NCPM), to integrate Math-PBL into the national curriculum for compulsory education. In this regard, the research question of this study is "What is the design framework for Project-Based textbook in Primary Mathematics", which includes key processes and design principles corresponding to different processes.

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2 Theoretical Background

PBL can be a learning method (Buck Institute for Education 2015), a teaching method (Xia 2017), or a curriculum development form (Krajcik et al. 2007; Qi and Zuo 2021). The current study regards PBL as a form of curriculum development. On this basis, combined with the existing studies on PBL characteristics, design elements, curriculum elements and basic processes, we summarized the key processes of PBL textbook design: (1) Create the real-life context (Mayer 2016); (2) Propose and split a driving task (Buck Institute for Education 2015; Krajcik et al. 2007; Mayer 2016; Qi and Zuo 2021); (3) Design and implement subtasks (Xu 2021); (4) Display and exchange the product (Buck Institute for Education 2015; Xu 2021); (5) Reflection and revision (Buck Institute for Education 2015; Xia 2020).

Because the characteristics of Math-PBL are consistent with the essential characteristics of PBL, the key processes of Math-PBL are also the same as that of PBL in general. In addition to considering the consistent "overt plot" of project tasks, it is also necessary to fully consider the design of the "covert plot" of step-by-step mathematics learning in the design of Math-PBL (He and Qi 2017). And the key to "covert plot" is to point to the core concepts of mathematics.

The above five key processes and two plots will be the theoretical basis for this study to construct the design prototype for project-based learning textbook in primary mathematics.

3 The Construction of Design Framework for Project-Based Learning Textbook in Primary Mathematics

Based on the literature analysis and the expert review, this study focuses on the five key processes and two plots of Math-PBL, and constructs a design framework of Math-PBL for primary school students.

3.1 Create the Real-Life Context

The design principle of this phase is that the context is real and derived from life, that is, it should be closely connected with the students' life experience (Teng et al. 2018). Such a context allows students to face challenges from the real world, especially to pay more attention to mathematical problems in life, so that we can improve their awareness of mathematical application and creative problem solving. At the same time, the created context needs to consider the interest (Gao 2020), and can stimulate students' curiosity and enthusiasm for learning, and arouse students' learning interest. In addition, the context created needs to have a certain complexity, leaving room for eliciting the driving task. In addition, for the Math-PBL, we should attach importance to the triggering of students' mathematics learning cues when creating a context.

3.2 Propose and Split a Driving Task

Proposing a driving task is to formulate the core task or problem in the project based on the context. Combined with existing research (Buck Institute of Education 2008;

Kuang 2017), effective driving tasks should stimulate students' interest in learning (Xia 2019), and the solutions and answers should be open. The designed driving tasks should be challenging to a certain extent, which can drive students to carry out continuous exploration and lay the groundwork for the necessity of subtask splitting. It is necessary to guide students to think about how to complete the driving task. In this process, they should continuously split and form sub-tasks to reach a consensus. It is also possible for students to try to solve the problem first, and experience the process of polishing from a rough solution to a refined solution. At least 2 subtasks should be split, but it should not be too many for primary school students, and it is appropriate to form 2 ~ 3 subtasks in the end.

Designing the driving task envisions what the student output will be. As the "output carrier" of students' Math-PBL, products should not only reflect the learning objectives of the project, but also contain the learning requirements of the mathematical core concept (Boss and Larmer 2021; Buck Institute of Education 2008; Xia 2021). For example, in a Math-PBL course around statistical content, the products could be a statistical survey proposal, a recipe menu with nutritional data, etc. Combined with the characteristics of primary school students, the products should be designed to be completed by most students through individual or group efforts.

3.3 Design and Implement Subtasks

The completion of subtasks needs to go through the process of designing and implementing the plan. Among them, in the process of "Design Plan", the question strings should be used to guide the students to plan ahead and implement the subtasks. In the "Implementation Plan" part, use the question strings to guide students to implement subtasks in an orderly manner according to the design plan. As the Math-PBL, it is necessary to clarify the "mathematical learning focus" in each subtask, integrate mathematical core concepts into subtasks, and establish the relationship between project activities and the hidden knowledge of the subject concepts behind it (Thomas, 2000). In this way, children can comprehensively use mathematics to solve problems around their understanding of a certain core concept of mathematics.

3.4 Display and Exchange the Product

After completing the subtasks, it is necessary to clarify the display unit of the product, such as individual or group. At the same time, determine the requirements for the presentation of the product, such as oral reports, poster presentations, etc. In this section, the assessment rubric is presented to allow students to talk about the products (Hu and Xiao 2021; Capraro et al. 2016; Mayer 2016), including assessing your own or the work of others. At the same time, the design of the rubric needs to include the assessment of PBL clues and mathematics learning clues, especially at a high level, there are higher requirements for the depth of students' mathematics learning.

3.5 Reflection and Revision

This phase is the important part of PBL and is often at last. On the one hand, combined with existing research (Buck Institute of Education 2015; Xia 2020; Lin and Yuan 2021), it is necessary to guide students to reflect on the product of Math-PBL in combination with the assessment rubric, and improve their products in the cycle of critical feedback and revision. On the other hand, students should also be guided to reflect on their own performance in the entire project learning process (Xia 2020), and their mathematics learning process (Table 1).

Table 1. A design framework for project-based learning textbook in primary mathematics

Processes	Principles
Create the real-life context	The context is real and closely related to the students' life experience Can stimulate students' curiosity and enthusiasm for learning, and arouse students' learning interest Have a certain degree of complexity, leaving room for eliciting the driving task Trigger students' mathematical learning cues
Propose and split a driving task	The driving task is interesting and open The driving task is challenging and can drive students' continuous exploration Provide students with the opportunity to think and try how to complete the driving task, present the process of students splitting it into 2 ~ 3 subtasks, and gradually guide the reaching of consensus The requirements for the completion of products are clearly defined, including the application of the mathematical core concepts
Design and implement subtasks	Design the "Design Plan" part, and use the question strings to guide students to plan ahead and pre-set the implementation of the subtasks Design the "Implementation Plan" part, and use the question strings to guide students to implement subtasks in an orderly manner according to the design plan Clarify the "mathematics learning focus" in each subtask, and integrate the mathematical core concept into subtasks
Display and exchange the product	Clarify the display unit of the product, such as individuals or groups Clarify the requirements for the presentation of the product, such as oral reports, poster presentations, etc
Reflection and revision	Requires the use of assessment rubric to reflect on the lack of the product and try to revise Design a self-checklist for students to reflect on the PBL and mathematics learning process

4 Conclusions

The presented paper introduces how PBL can be integrated into the mathematics subject and national curriculum. The five key processes and corresponding principles for the "PBL Textbook in Primary Mathematics" are preliminarily determined. This framework can provide reference for textbook developers or mathematics curriculum researchers to develop primary Math-PBL materials, and can also provide inspiration for the secondary development of textbook for mathematics teachers.

On this basis, we will further collect empirical data through the paradigm of design research to iterate this design framework.

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Evidence-Based Approach in Textbook Revision: Understand Students' Adaptive Strategy Use in Addition of Two-Digit Numbers

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Abstract. Addition strategies that build on decomposing and composing numbers in flexible ways contribute to students' overall number sense. In many instances, these adaptive strategies are easier and faster than standard algorithms. Therefore, it is best to have students learn a variety of methods that they can select from as needed. Empirical studies suggest that students often choose efficient strategies provided they know any appropriate strategies for a given problem.

Addition strategies that build on decomposing and composing numbers in flexible ways contribute to students' overall number sense. In many instances, these adaptive strategies are easier and faster than standard algorithms. Therefore, it is best to have students learn a variety of methods that they can select from as needed. Empirical studies suggest that students often choose efficient strategies provided they know any appropriate strategies for a given problem (Franziska and Lipowsky 2009; Humphreys and Parker 2015; Parrish 2014). However, the flexibility of using these adaptive strategies varies depending on many elements: the textbooks, instructional methods, students' familiarity with all these strategies etc. Adaptive strategies are more meaningful and would be faster and more accurate while many students and teachers resort to standard algorithms that they don't fully understand (Biddlecomb and Carr 2011; Fuson and Beckmann 2012/2013). Furthermore, Heinze (2018) found that students are more likely to develop adaptive use of these strategies from the implicit approach than the explicit approach when the strategies are not too complex to be generated by students. For the past few decades, Chinese mathematics education has received global attention for its leading performance in various academic assessments (Liu 1992; OECD 2004, 2013). While they excelled in mathematics computation under the extremely utilitarian exam-oriented education system, students also exhibited a lack of interest in exploring various strategies due to the expectation of fluency and accuracy (Leung 2001; Fan et al. 2004). Chinese students outperformed U.S. students on tasks involving computation and simple problem solving, but not on the tasks involving complex problem solving (Cai and Silver 1995). In recent years, Chinese researchers and educators have aimed to respond to this concern. New Century Mathematics (NCM) as one of the textbooks with the largest market share and influence, has been paying close attention on diversifying strategies in their textbook

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revision. They always refer to empirical studies and classroom practices in their textbook revision process. This study is one piece of their "trail teaching and trail learning" approach in textbook revision.

In this study, with 83 Chinese first graders learning by textbooks with different instructional approaches, we investigate accuracy and adaptivity of students' strategy use when adding two-digit numbers. The strategies that we are interested include: analogy strategy, stepwise strategy, split strategy, compensation strategy, flexible strategy, and standard algorithm. Table 1 summarizes the strategies for two digits addition and examples for each strategy.

Table 1. Adaptive strategies in two digits addition

Analogy	Stepwise strategy	Split strategy	Compensation strategy	Flexible strategy	Standard algorithm
$\underline{5+2=7}$ $50+20=70$	$\underline{36+23}$ $36+20=56$ $56+3=59$	$\underline{36+23}$ $30+20=50$ $6+3=9$ $50+9=59$	$\underline{27+98}$ $27+100=127$ $127–2=125$	$\underline{27+98}$ $25+100=125$	$\underline{27+98}$ 27 $\underline{+98}$ 125

1 Method

1.1 Subjects

83 Children from an elementary school in Beijing, China participated in the study. There were 42 children in the experimental group (mean age of 7 years and 4 months), and 41 in the control group (mean age of 7 years and 3 month). These two classes have two different teachers. Teacher A for the experimental group has 6 years of teaching experience and holds a master degree in education, while teacher B for the control group has 12 years of teaching experience and holds a bachelor degree in Education.

1.2 Materials and Procedure

Although our study addressed the influence of different instructional approaches on children's adaptive strategy use, we did not collect data of instructional quality or other variables concerning the teaching and learning processes taken place during the lessons. Instead, we carefully selected the textbook units and considered possible revision in these units. We decided to use unit 5 and unit 6 in NCM textbook 1B. In the current textbook, the strategies laid out in the sequence of counting sticks, placing counters, using number lines, and introducing standard algorithm. The possible revision ideas would be offering more examples for all adaptive strategies that mentioned above.

For the data collection, we used a test with 10 addition items. The items were categorized as: adding tens, two digits adding one digit without carrying, two digits adding one digit with carrying, two digits adding two digits without carrying, two digits adding two digits with carrying. All items passed a check by experienced primary teachers to ensure that the item presentation was comprehensible for first graders. One researcher from the research team offered a training workshop to a group of teachers before data collection. The pretest had taken place in the second half of Grade one before the standard algorithm for addition was introduced. The pretest was administrated in a classroom by trained teachers in the school. In each classroom, 6 teachers presented while each teacher supervised 6–7 students at the same time. Teachers walked around the room to provide positive feedback and encouragement without providing answers. Students were given enough time to complete the work. For anyone who completes the task earlier, they are free to quietly leave the classroom without disturbing others. The test time took about one school lesson (45 min) including instruction and collection of the test booklets.

For the intervention, the control group teacher followed the current textbook for both units. The experimental group received specific strategies instruction on top of the current textbook. Stepwise strategy and split strategy were introduced to this group in unit 5, while compensation strategy and flexible strategy were introduced in unit 6. Due to the Covid outbreak, only one session of unit 6 was completed before the post test. In other words, students only had one lesson of learning about compensation strategy and flexible strategy.

For the post test, different testing items in the same categories were administrated to students. For the post test, students were encouraged to use as many strategies as possible. Similar to the pretest, in each classroom, 5 teachers presented while each teacher supervised 6–7 students at the same time. Teachers walked around the room to provide positive feedback and encouragement without providing answers. Students were given enough time to complete the work. For anyone who completes the task earlier, they are free to quietly leave the classroom without disturbing others. The test time took about 60 min including instruction and collection of the test booklets.

2 Results and Discussion

The descriptive statistics on each category conducted at the pretest are presented in Table 2. The frequencies and proportion of the strategies used for each category are listed. A series of Chi-square test of independence were performed to examine the relation between the strategies that students use and the group for each category. No significant differences were found among the groups for each category in the pretest.

The pretest analysis showed that the sampling was unbiased. To examine whether the two group performed differently after the intervention, another series of Chi-square test of independence were performed to examine the relation between the strategies that students use and the group for each category. No significant differences were found in adding tens, two digits adding two digits with carrying (Table 3).

A chi-square test of independence was performed to examine the relation between the strategies that students use and the group. Significant X2 $(2, N = 79) = 12.87$, $p < .05$. Experimental group were more likely to use different strategies even though majority of students from both group use standard algorithm (Table 4).

A chi-square test of independence was performed to examine the relation between the strategies that students use and the group. Significant X2 $(2, N = 81) = 14.83$,

Table 2. Frequencies and proportion of strategies used on each category in pretest

Item category	Strategies used	Control Group (N = 42)		Experimental Group (N = 41)	
		Absolute frequency	%	Absolute frequency	%
Adding tens	Operation error	2	4.8	2	4.9
	Self-correction	4	9.5	3	7.3
	Stepwise	18	42.8	17	41.4
	Split	0	0	0	0
	Compensation	0	0	0	0
	Flexible	0	0	0	0
	Standard algorithm	20	47.6	20	48.8
Two digits adding one digit without carrying	Operation error	7	16.7	5	12.1
	Self-correction	0	0	4	9.7
	Stepwise	0	0	0	0
	Split	11	26.2	12	29.2
	Compensation	0	0	0	0
	Flexible	2	4.8	2	4.9
	Standard algorithm	19	45.2	24	58.5
Two digits adding one digit with carrying	Operation error	3	7.1	0	0
	Self-correction	0	0	3	0
	Stepwise	10	23.8	7	17.1
	Split	14	33.3	12	29.3
	Compensation	0	0	0	0
	Flexible	0	0	0	0
	Standard algorithm	20	47.6	22	53.6
Two digits adding two digits without carrying	Operation error	0	0	0	0
	Self-correction	2	4.8	1	2.4
	Stepwise	3	7.1	6	14.6
	Split	20	47.6	21	51.2

(*continued*)

$p < .05$. Experimental group were more likely to use different strategies even though majority of students from both group use standard algorithm.

Table 2. (*continued*)

Item category	Strategies used	Control Group (N = 42)		Experimental Group (N = 41)	
		Absolute frequency	%	Absolute frequency	%
	Compensation	0	0	0	0
	Flexible	0	0	0	0
	Standard algorithm	21	50.0	20	48.8
Two digits adding two digits with carrying	Operation error	13	30.9	11	26.8
	Self-correction	3	6.9	5	12.2
	Stepwise	2	4.6	4	9.7
	Split	20	47.6	22	53.6
	Compensation	1	2.3	0	0
	Flexible	0	0	0	0
	Standard algorithm	19	45.2	16	39.2

Table 3. Frequencies and proportion of strategies used for two digits adding two digits without carrying

Strategies used	Control Group (N = 38)		Experimental Group (N = 39)	
	Absolute frequency	%	Absolute frequency	%
Operation error	0	0	0	0
Self-correction	1	2.6	0	0
Stepwise	0	0	0	0
Split	16	42.1	27	69.2
Compensation	0	0	2	5.1
Flexible	0	0	3	7.7
Standard algorithm	31	81.5	35	90.0

3 Discussion

Majority of both groups use standard algorithm. Overall, over 60% of the items are processed by standard algorithm both at the pretest and the posttest. Both groups of students feel comfortable using the standard algorithm and the accuracy of using standard algorithm are very high which is consistent with previous studies. Our sample shows a tendency of using adaptive strategies if students are encouraged to do so. Significant differences were found only for two categories: two digits adding one digit, and two digits adding two digits without carrying. Although research on textbook use shows that textbooks have a significant impact on teachers' delivery and the lesson structure

Table 4. Frequencies and proportion of strategies used for two digits adding one digit

Strategies used	Control Group (N = 39)		Experimental Group (N = 39)	
	Absolute frequency	%	Absolute frequency	%
Operation error	2	5.1	0	0
Self-correction	6	15.4	2	5.1
Stepwise	0	0	6	15.4
Split	10	25.6	16	41.0
Compensation	0	0	1	2.6
Flexible	3	7.7	3	7.7
Standard algorithm	30	76.9	33	84.6

and contents (Freeman and Porter 1989; Johansson 2003), we interpret our results with caution in mind that many aspects can affect students' use of adaptive strategies. Students are more likely to use more strategies when they are more comfortable with problems. Another possible explanation is that two digit additions are not complex enough to detect students' use of adaptive strategies. Future studies should examine these strategies on three digits addition and subtraction for higher grades.

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Opportunities to Learn Mathematical Modelling in NCM Primary Textbook: A Case Study of NCM Textbook for Grade 6

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Abstract. Mathematical modelling has become an important part of mathematics curriculum standards in many countries and needs to be focused and developed in textbooks. In order to obtain specific details about mathematical modelling in the mathematics textbooks and compare mathematical modelling across Oriental and Western mathematics textbooks, this study provides a unified analytical framework to present mathematical modelling within textbooks. Meanwhile, this paper examined two volumes of New Century Mathematics textbooks based on this new framework and connected the results with prior research to discuss the features of Chinese textbooks design regarding to mathematical modelling. Moreover, this framework could also be applicable to examine mathematical modelling in the textbooks in the future studies.

1 Introduction

Mathematical modelling, as a mathematical thought and mathematical concept, may be used to solve various problems in the real world every day, which is conducive to the cultivation of students' creativity, communication and other key competencies (COMAP and SIAM, 2016). In a comparative study of Chinese and American students' problem-solving competencies, Cai (2002) found that Chinese students are better than American students in solving general problems, but weaker than American students in solving open-ended problems and using diversity strategies. Therefore, it is important to explore how to cultivate students' awareness and competency of mathematical modelling in mathematics teaching in China.

Textbooks play a very important role in mathematics teaching, reflecting the intent of mathematics curriculum standards and giving specific examples (Schmidt et al., 2001). In many countries, mathematical modelling, to largely extent, becomes an essential part of the core literacy of mathematics that students must gain and underlies the content of mathematics curriculum (Cai & Xu, 2016). With the development of Chinese mathematics curriculum standards, the emphasis on mathematical modelling is gradually increasing, and the description of the mathematical modelling process is gradually

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improving (Huang et al., 2019). In a sense, the first step to improving primary school students' mathematical modelling competency is to clarify how mathematical modelling presents in primary school mathematics textbooks.

2 Literature Review

Mathematical modelling is considered to be the process of applying mathematics to solve real-world problems, with a two-way transformation between the mathematical world and the real world (Greefrath and Vorhölter, 2016). Some researchers have carried out various studies on mathematical modelling in mathematics textbooks, but have not formed a complete analytical framework. For instance, Zwaneveld et al. (2017) established an analytical framework including both modelling activities and modelling purposes to analyse mathematical modelling in Dutch high school textbooks. They found that conceptualising, mathematising, solving and interpreting were the main modelling activities in mathematics textbooks; verifying, validating, iterating, communicating and reflecting hardly occur or are fully absent. Stillman et al. (2013) analysed high school mathematics textbooks for Year 11 and 12 and found that the textbooks could provide a foundation for teaching mathematical modelling, enhance understanding of the real world, and develop a critical disposition towards the surrounding world through mathematical modelling. In this paper, we focus on the opportunities that sixth-grade mathematics textbooks provide for students to learn mathematical modelling and try to answer the following research questions: (1) What modelling processes are presented in the mathematical modelling tasks of Chinese mathematics textbooks? (2) What modelling processes are introduced in the mathematical modelling tasks of mathematics textbooks? (3) How mathematical modelling tasks are distributed across different context areas in Chinese mathematics textbooks?

3 Analytical Framework

The analytical framework of this paper mainly contains three main dimensions: characteristics of the mathematical modelling tasks, the mathematical modelling processes, and the context areas.

Mathematical modelling tasks are different from applied problems and have some characteristics in terms of context and solutions. Mathematical modelling tasks in the early and middle grades are considered to have two characteristics: realistic significance and value; open-ended and varied answers (COMAP and SIAM, 2016). English (2003) proposed that the tasks for developing students' mathematical modelling competency should provide opportunities for model exploration and application.

Mathematical modelling is considered to be a process of applying mathematics to the real world, so the modelling cycle has become the focus of mathematical modelling education research. Many researchers have proposed models of the mathematical modelling cycle, such as four-step modelling cycle (Blum & Kirsch, 1989), five-step modelling cycle (Maaß, 2006) and seven-step modelling cycle (Blum & Leiß, 2007). The five-step modelling cycle contains Simplifying, Mathematising, Working mathematically, Interpreting and Validating (Kaiser, 2007).

The use of meaningful contexts is a critical feature of modelling tasks and appropriate contexts help students understand the close relationship between mathematics and the real world (English, 2021). OECD (2018) used four context areas to define real-world situations: personal, occupational, societal and scientific.

4 Pilot Study and the Framework in This Study

We used the frameworks based on the previous works to conduct a pilot coding scheme for the tasks in sixth-grade mathematics textbooks and found that a few coding in the framework is not suitable for sixth-grade textbooks. For example, there are almost no occupational context tasks in the sixth-grade textbook, which may be related to the lack of knowledge in the occupational area of primary school students.

In summary, we adjusted the analytical framework according to the pilot study. Due to the limitation of cognitive level and contextual knowledge of sixth graders, this framework simplifies the mathematical modelling process and removes the occupational context areas. Table 1 shows the analytical framework for coding tasks in the mathematics textbooks.

Table 1: Analytical Framework for Coding Mathematical Modelling Tasks

Dimension	Sub-dimension	Coding
Characteristics of Mathematical Modelling Tasks	Realistic Situation	RS
	Open and Varied	OV
	Applied Models	AM
Mathematical Modelling Process	Simplifying	SI
	Mathematising	MA
	Working mathematically	WM
	Interpreting and Validating	IV
Context Areas	Personal	PE
	Societal	SO
	Scientific	SC

5 Methods

5.1 Textbook Selection

This paper chose two sixth-grade textbooks of the New Century Mathematics Primary Textbooks published in 2011 by Beijing Normal University Press for two main reasons: (1) This series textbooks reflect the advanced rationale of new national mathematics curriculum reform. (2) Many key members of this series also belong to the team that was responsible for the national mathematics curriculum reform. (3) Sixth-grade textbooks contain all the mathematical knowledge and skills needed for primary school students, which are used to solve real-world problems.

5.2 Unit of Analysis

The unit of analysis used in this paper is a "task" in narratives and exercise sets. This series textbooks designed based on "context + question strings", so that each "task" was an entire narrative unit which contained a "context" and several "question strings" or a question, including context problems, steps to solve problems and diverse thinking.

5.3 Procedure

To code the two selected mathematics textbooks, we asked a master student and a doctoral student, both of whom majored in mathematics education to conduct the pilot study and official analysis. To ensure the correct understanding of mathematics textbook design, we refer to teacher's edition of textbooks in the process of analysis.

We chose 2 units from each of the two textbooks for the pilot study. Firstly, two students discussed the frameworks to ensure that their understanding was coherent. Then, they divided tasks and code the tasks independently. What's more, they discussed and adjusted the results of coding to reach a very high consistency between their codes. When finishing the pilot study, they continued to divide tasks and code the tasks independently in remaining units of the two textbooks.

To check the reliability, we compared the codes of raters to calculate the interrater agreement of our coding scheme. The first reliability value indicated whether each task offered a mathematical modelling opportunity to students. The interrater agreement was 99.9%. The second reliability value indicated the determinations of what mathematical modelling processes do the tasks involve and what context do it belong to. The interrater agreement was 95.2%.

6 Results

6.1 Mathematical Modelling Tasks in Textbooks

There were about 737 tasks in the sixth-grade mathematics textbooks, of which 229 tasks (31.1%) were narratives and 508 tasks (68.9%) were exercise sets. Table 2 reports the frequencies and percentages of characteristics of mathematical modelling tasks in the tasks.

We defined a mathematical modelling task that contains all three characteristics. There were 34 tasks (4.6%) in total, of which 13 mathematical modelling tasks (38.2%) in narratives and 21 mathematical modelling tasks (61.8%) in exercise sets.

6.2 Process of Mathematical Modelling Tasks in Textbooks

Table 3 shows the frequencies and percentages of the number of mathematical modelling processes per task in textbooks.

Table 4 summarizes the frequencies and percentages of occurrence of mathematical modelling processes in the tasks.

Table 2: Frequencies and Percentages of the Characteristics

Characteristics	Narratives	Exercise Sets	Total
Realistic Situation	86 (37.6%)	206 (40.6%)	292 (39.6%)
Open and Varied	66 (28.8%)	66 (13.0%)	132 (17.9%)
Applied Models	118 (51.5%)	387 (76.2%)	505 (68.5%)

Table 3: Frequencies and Percentages of the Number of Processes per Task

Number of Processes	Narratives	Exercise Sets	Total
0	3 (23.1%)	9 (42.3%)	12 (35.3%)
1	2 (15.4%)	4 (19.0%)	6 (17.6%)
2	3 (23.1%)	3 (14.3%)	6 (17.6%)
3	3 (23.1%)	4 (19.0%)	7 (20.6%)
4	2 (15.4%)	1 (4.8%)	3 (8.8%)

Table 4: Frequencies and Percentages of Processes in Tasks

Processes	Narratives	Exercise Sets	Total
Simplifying	3 (23.1%)	5 (23.8%)	8 (23.5%)
Mathematising	10 (76.9%)	10 (47.6%)	20 (58.8%)
Working mathematically	6 (46.2%)	5 (23.8%)	11 (32.4%)
Interpreting and Validating	6 (46.2%)	6 (28.6%)	12 (35.3%)

6.3 Context Areas of Mathematical Modelling Tasks in Textbooks

Table 5 shows the percentages of context areas of mathematical modelling tasks.

Table 5: Frequencies and Percentages of Context Areas

Context Areas	Narratives	Exercise Sets	Total
Personal	2 (15.4%)	5 (23.8%)	7 (20.6%)
Societal	9 (69.2%)	14 (66.7%)	23 (67.6%)
Scientific	2 (15.4%)	2 (9.5%)	4 (11.8%)

7 Discussion

Firstly, more than one-third of the tasks in NCM textbooks for grade 6 are context-based, but the proportion is still low compared with foreign mathematics textbooks (e.g., Zwaneveld et al., 2017). Appropriate realistic situation tasks can be used to stimulate students' learning motivation (English, 2021), so it is necessary to arrange appropriate realistic situation for the tasks in Chinese mathematics textbooks.

Secondly, NCM textbooks for grade 6 have some mathematical modelling tasks, but the number of mathematical modelling processes per task is not large and the distribution is not balanced. There are only a few mathematical modelling tasks that cover all mathematical modelling processes, especially the lack of simplifying, which may lead to difficulties in solving tasks for primary school students (Zwaneveld et al., 2017).

What's more, most of the mathematical modelling tasks in NCM textbooks for grade 6 are based on societal context area, but there is a lack of tasks based on scientific context area. English (2021) found that tasks based on scientific context area could cultivate students' interdisciplinary mathematical modelling competency, so more scientific tasks should be designed into textbooks.

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Opportunities to Learn Reasoning and Proving in the Algebra of NCM Mathematics Textbooks

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Abstract. Mathematics textbooks play an essential role in providing students with learning opportunities for reasoning and proving (RP). Researchers showed that tasks used by teachers were directly related to those provided by mathematics textbooks. Yet, little is known about the learning opportunities offered by elementary textbooks for students to learn RP in China. This study analyzed six volumes of New Century Mathematics (NCM) textbooks, focusing on RP's opportunities in written tasks based on a revised analytical framework. The results showed that the textbooks we analyzed in this study provide students with more opportunities to engage in the RP of the algebraic domain, especially in narrative sets. In addition, in terms of sub-components and representation, the opportunities provided by textbooks were different in narrative sets and exercise sets.

1 Introduction

Reasoning-and-proving is fundamental for deep learning in mathematics and plays an essential role in fostering students' conceptual understanding (Zhou et al., 2021). In some countries' curriculum documents, reasoning-and-proving was emphasized as critical component of school mathematics education. For example, the newest version of *the full-time compulsory education mathematics curriculum standards (2022 version)* in China sets for Grade 1–9 students is about reasoning-and-proving: "[Students should] make observations, conjectures, experiments, [and] justifications in the practice of mathematics." In conjunction with policy documents calling for an expanded role of reasoning-and-proving, international researchers have also identified reasoning-and-proving as an area of focus (Hanna & de Villiers, 2012). Some researchers paid attention to teaching on reasoning-and-proving (Harel & Sowder, 2007; Krummheuer, 2007; Stylianides, 2016), and other researchers focused on textbook analysis (Bieda et al., 2013; Fujita & Jones, 2014; Stylianides, 2009).

Textbooks not only connect the intentions of curriculum standards with students' achievement (Schmidt et al., 2001) but also act as the instantiation of the implementation of curriculum standards (Valverde et al., 2002). Thus, textbooks have an essential role in everyday mathematics lessons. In China, in the mathematics subject, mathematics textbooks are teachers' primary references for their daily teaching activities, as well as

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the main resources from which students learn mathematics (Zhang & Qi, 2019). Previous studies have shown that many students in different countries face severe difficulties with reasoning and proving (Fujita & Jones, 2014; Harel & Sowder, 2007). Some international researchers thought that the main reason why elementary students did not perform well in reasoning-and-proving tasks was that they were not given enough opportunities to engage in reasoning-and-proving activities at the elementary level (Bieda et al., 2013; Stylianides, 2016). Therefore, it's necessary to conduct some studies to understand how much Chinese mathematics textbooks provide opportunities for students to engage in reasoning-and-proving activities. In addition, a few studies have analyzed either the whole textbooks (Stylianides, 2009; Zhang & Qi, 2019) or the geometry domain (Fujita & Jones, 2014; Otten et al., 2014; Wong, 2018). However, few studies have explored how many opportunities the algebraic domain provides for students to engage in reasoning-and-proving activities.

Based on the above arguments, this study will focus on the algebraic domain of elementary mathematics textbooks to find out how many opportunities they provide for students to engage in reasoning-and-proving activities.

2 Analytic Framework and Method

The analytic framework of this study originated from the integration of both Stylianides' (2007) and Stylianides' research (2009). Stylianides (2007) defined the component of an argument, wherein the modes of argument representation were proposed and regarded as one key aspect of an element in an argument. Stylianides (2009) constructed one analytic framework of reasoning-and-proving in analyzing textbooks, including the subcomponents and purposes of reasoning-and-proving. According to the previous studies, the analytic framework of this study was proposed, including the subcomponents of reasoning-and-proving tasks, the purposes of subcomponents, and the representation modes of reasoning-and-proving tasks. The specific research questions are: (1) How are RP tasks distributed in mathematics textbooks? (2) How are RP sub-components, purposes of sub-components, and representation modes of RP tasks distributed in mathematics textbooks' narrative and exercise sets? (Table 1).

Table 1 The analytic framework of this study

	Reasoning		Proving	
Subcomponents	Identifying patterns	Making conjectures	Justifying arguments	Evaluating justifications
Purposes	Obtained by coding from the bottom up			
Representation	Obtained by coding from the bottom up			

In this study, we chose six volumes of mathematics textbooks (Grade 4–6) published in 2014 by Beijing Normal University Press, considering that this set of mathematics textbooks better reflect the advanced education ideas of the new national mathematics curriculum reform (Zhang & Qi, 2019). Further, we mainly focus on the algebraic domain in these textbooks. To maintain a uniform standard, we used a similar unit of analysis as the existing relevant studies. Specifically, a task with a separate context is considered a unit of analysis, regardless of how many sub-questions it has (Wijaya et al., 2015; Stylianides, 2009). To ensure the accuracy of coding, we adopted the independent two-person (undergraduate students majoring in primary mathematics education) coding and provided teachers' guidebooks for reference. When the coding was inconsistent, we discussed with an expert to ensure every coding and finally determined the coding results (The independent two-person coding is 95%).

3 Results and Discussion

We presented the results of the RP opportunities in six volumes of mathematics textbooks concerning RP tasks, RP tasks in the narrative, and exercise sets.

3.1 Reasoning-and-Proving Tasks

In total, there are 1005 tasks in six volumes of mathematics textbooks, of which 125 are RP tasks (12.44%). Further, classified by narrative and exercise sets, we identified 233 tasks in narrative sets, of which 56 were RP tasks (24.03%). In exercise sets, there were 772 tasks, of which 69 were RP tasks (8.9%). In comparison, there are more RP tasks in narrative sets. This result differed from some international studies (Bieda et al., 2013; Otten et al., 2014; Stylianides, 2009; Wong, 2017). Stylianides (2009) found about 40% of tasks in U.S. middle mathematics textbooks involved a sub-component of PR. The key reason for this difference may be due to differences in mathematics contents, as more emphasis is placed on proof in the mathematics textbooks of middle school (Stylianides, 2009).

3.2 RP Sub-Components and Purposes of Sub-Components in Different Sets

According to Tables 2 and 3, we found that the opportunities textbooks provided, in terms of sub-components, differed in narrative sets and exercise sets. There is a higher percentage of identifying patterns and making conjectures in a narrative set. However, there is an equal percentage in identifying patterns, making conjectures, and making arguments in exercise sets. These results showed that the textbook series paid more attention to the process of developing conjectures by inductive reasoning in a narrative set (Zhang & Qi, 2019). However, in exercise sets, the textbook series also focused more on making arguments by proving reasonable explanations (Stylianides, 2005). Making arguments is an important foundation for developing students' logical reasoning and deductive reasoning (Stylianides, 2008). Thus, it would be better if textbooks in narrative sets provided more opportunities to make arguments. In addition, there are few evaluating arguments, both in narrative sets and in exercise sets. Compared with other international studies (Bieda et al., 2013; Zhang & Qi, 2019), we have the same conclusions. One possible reason is that any such opportunities to evaluate arguments were presented in teaching activities that were not sampled (Bieda et al., 2013). However, evaluating arguments, an important aspect of RP has been regarded as an essential foundation for developing students' metacognitive skills (Stylianides & Stylianides, 2009). Thus, we should also take consideration into designing tasks involving evaluating arguments. Further, we found that in the purposes of sub-components, the purposes of identifying patterns were mainly to develop conjectures. The main purposes of making conjectures, arguments, and evaluations were to solve questions and understand concepts in narrative sets and exercise sets. All these purposes were very important for students to learn better mathematics (Fujita & Jones, 2014).

Table 2 Percentages of RP sub-components and purposes of sub-components in the narrative set and the exercise set

	Narrative sets		**Exercise sets**
Identify pattern	40 (42.11%)	**Identify patterns**	35 (35%)
Conjecture precursor	32	Conjecture precursor	28
Evaluation precursor	1	Solving questions	4
Solving questions	3	Understanding concepts	3
Understanding concepts	4	**Making conjectures**	30 (30%)
Making conjectures	35 (36.84%)	Argument precursor	4
Conjecture precursor	2	Solving questions	24
Argument precursor	1	Understanding concepts	2
Evaluation precursor	3	**Making arguments**	34 (34%)
Solving questions	10	Argument precursor	2
Understanding concepts	19	Evaluation precursor	1
Making arguments	16 (16.84%)	Solving questions	16
Understanding concepts	10	Understanding concepts	15
Solving questions	6	**Evaluating arguments**	1 (1%)
Evaluating arguments	4 (4.21%)	Understand concepts	1
Understand concepts	3	Solving questions	0
Solving questions	1		

Notes. There is a total of 95 RP questions in the narrative sets of grade 4–6 textbooks; there is a total of 100 RP questions in the exercise sets of grade 4–6 textbooks

3.3 RP Representation Modes by Grades in Different Sets

Based on Table 3, we found that the opportunities textbooks provided, in terms of representation modes, existed differences in narrative sets and exercise sets. There is a higher percentage in the arithmetic and number representation in the narrative set. However, there is an equal percentage in the arithmetic and number, tables and graphics, and textual and integrated representations in the exercise set. In previous studies, few studies are focusing on representation modes of RP tasks in the analysis of textbooks, but appropriate representation modes did play important roles in building understanding, communicating, and demonstrating reasoning (Fennell & Tom, 2001). Thus, combined with specific mathematics contents, textbook design should promote the appropriate use of representation.

Table 3 Percentages of RP representation by grades in a narrative sets

	Narrative sets	Exercise sets
1. Arithmetic & Number	30(53.57%)	17(24.64%)
2. Tables & Graphics	17(30.36%)	25(36.23%)
3. Textual representations	2(3.57%)	16(23.19%)
4. Integrated representations	7(12.50%)	11(15.94%)

Notes. There are 56 RP tasks in narrative sets of grade 4–6 textbooks; there is a total of 69 RP tasks in exercise sets of grade 4–6 textbooks

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Study on Learning Opportunities of Geometric Transformation in NCM Textbooks

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Abstract. Spatial geometry is closely related to our lives. Geometric transformation raises geometry to a dynamic level, which helps students construct and manipulate the psychological representation of multidimensional objects. Student academic performance is closely related to opportunities to learn (OTL). Textbooks are the most important learning resource for students' differences in OTL. Based on the perspective of OTL, this study has constructed an evaluation framework, studied the characteristics of the NCM textbook, a mainstream textbook of primary school mathematics in China, in terms of geometric transformation compilation, and discussed the compilation of textbooks.

1 Introduction

Spatial geometry is closely related to our lives. Geometry is a necessary tool for us to understand and explain our geometric world. At the same time, it provides us with the necessary knowledge and ways to design and creatively solve problems (Boulter & Kirby, 1994). The geometric transformation raises geometry to a dynamic level. Spatial reasoning and spatial visualization through geometric transformation are helpful in constructing and manipulating the psychological representation of two-dimensional objects (NCTM, 2000). For primary school students, geometric transformation is an important learning content, composed of translation, rotation, axisymmetry, similarity, and projection (Ministry of Education of the People's Republic of China, 2001; NCTM, 2000). Research shows that to succeed in higher-level mathematics learning, students should begin to contact in primary school and have the ability to geometric transformation before the end of Grade 8 (Flanders, 1987; Zorin, 2011).

When students cannot achieve specific educational goals, a seemingly reasonable question is whether they have received fair education to achieve the abilities expressed in these goals. Therefore, the concept of opportunities to learn (OTL) came into being; that is, students have enough time to learn (Liu, 2009).

Hiebert and Grouws (2007) emphasized that the most important factor in student achievement is OTL, and textbooks are an indispensable teaching medium that affects students' OTL (Begle, 1973; Grouws et al., 2004). Content analysis is a key link in textbook analysis, and the OTL level is usually used as an indicator to measure the potential effectiveness of the reviewed materials. Therefore, an important factor affecting

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students' outcomes is OTL based on textbook content (Tornroos, 2005). Since textbooks are the primary source of curriculum materials (Growws et al., 2004), Schmidt (2002) pointed out that the OTL differences of students occur in primary schools and textbook differences affect students' performance to a large extent. The key to improving student performance is to let them participate in more cognitive activities fairly (Boston & Smith, 2009; Stein & Smith, 1998). Therefore, students must have the opportunity to learn valuable mathematical concepts and complete tasks with high cognitive requirements. Examine the treatment of these concepts to ensure that students receive appropriate learning opportunities. In this study, OTL is defined as how to deal with a concept in the curriculum, including the importance of mathematical concepts in textbooks, the nature of demonstrations, the types of tasks presented for students' learning, and the level of cognitive needs required by students to complete the provided tasks.

However, few studies have explored the frequency in the content of geometric transformation in primary school mathematics textbooks. This study takes a set of mainstream textbooks in China, namely the Chinese new century primary school mathematics textbook (NCM), as examples to study their opportunities to learn geometric transformation. Hence this study focuses on the OTL of geometric transformation content in the NCM textbooks and puts forward the following research questions:

1. What is the arrangement of the geometric transformation content in the NCM textbooks?
2. To what extent does the geometry transformation in the NCM meet the OTL of the students?

2 Analytical Framework

Based on previous literature (Liu, 2009; Wijaya, 2015), we propose an OTL framework of geometric transformation in textbooks that includes three aspects: the representativeness of the situation, the type of information, and the cognitive level, as shown in Table 1.

Representativeness of the situation means familiarity of children with learning situations. Only when children learn in a familiar environment can they have a fair chance to learn. For example, the situation considers the differences between urban and rural areas, gender, and region, indicating that this indicator is "Strong".

The type of information means that the tasks of textbooks include the degree to which the geometric transformation content information required by the solution is included. For example, tasks contain less information on geometric transformation than needed, so students must derive additional data, indicating that this indicator is 'missing'.

The cognitive level means the cognitive demand level of geometry transformation, van Hill proposed a 5-level framework, which was later revised to a 3-level framework. For example, "Learners recognize, name, compare, and draw geometric transformation figures according to the appearance of geometric transformation", indicating that this indicator is "visual level".

Table 1. The OTL analysis framework for geometric transformation in the textbook.

Category	Subcategory	Explanation
Situation representativeness	Strong	-The situation considers the differences between urban and rural areas, gender, and region
	Ordinary	-The situation considers the differences between urban and rural areas, gender, region, and other two groups
	Weaker	-The situation does not consider or only considers the differences between urban and rural areas, gender, region, and other groups
Type of information	Matching	-The tasks contain exactly the geometric transformation information needed to find the solution
	Missing	-The tasks contain less geometric transformation information than needed, so students need to derive additional data
	Superfluous	-The tasks contain more geometric transformation information than needed so students need to select information
Cognitive level	Visual level	Learners recognize, name, compare and draw geometric transformation figures according to the appearance of geometric transformation
	Descriptive level	Understand the geometric transformation object through the properties of geometric transformation, and solve the problem according to the properties of geometric transformation
	Theoretical level	Learners can use the "If then" idea to develop geometric transformation and its logical reasoning ability in problem-solving

3 Methods

3.1 Textbook Selection

To answer the research questions, we analyze the contents related to geometric transformation in the fourth edition of the NCM textbooks (first published 2012) for two main reasons: (1) This textbook is the mainstream textbook in China and is widely used. (2) The compilation of this set of textbooks is based on the national education policy documents and reflects the advanced rationale of the new national mathematics curriculum reform of China.

3.2 Procedure

To encode the mathematics textbooks, we invited a master's student and an associate professor majoring in mathematics education to conduct an analysis. First, the two researchers discussed the framework to ensure that their understanding was consistent. Then they divide the tasks and code them independently. More importantly, they discussed and adjusted the coding results to achieve very high consistency between the codes. Upon completion of the pilot study, they continued to divide and independently code tasks in the remaining units of the textbooks. When two coders complete 15% of the tasks, the reliability of coding is calculated, and Cohen of the geometric transformation textbook analysis is 0.72. These results show that the coding is reliable (Landis & Koch, 1977).

4 Results

4.1 Proportion of Geometric Transformation Content in Textbooks

As shown in Table 2, the geometric transformation content of the NCM textbook has been compiled 11 times, including 4 times axial symmetry, 3 times translation, and 4 times rotation, accounting for 3.96% of the total compilation content.

Table 2. Proportion of geometric transformation content in the NCM textbooks.

Series textbooks	Axial symmetry	Translation	Rotation	Percentage
NCM4	4	3	4	3.96%

4.2 Dimension of Geometric Transformation in NCM Textbooks

As shown in Table 3, the frequency of the three dimensions of geometric transformation of OPL in the NCM textbooks ranges from 0 to 4. Among them, the frequency of the dimensions of translation and rotation is lower. Most of the content meets the requirements, and the content of the cognitive level dimension needs to be improved.

Table 3. The frequency of dimensions of geometric transformation in the NCM textbooks.

Dimension	Situation representativeness			Type of information			Cognitive level		
	Strong	Ordinary	Weaker	Matching	Missing	Superfluous	Visual	Descriptive	Theoretical
Axial symmetry	2	2	0	3	1	0	2	2	0
Translation	0	3	0	3	1	0	1	3	0
Rotation	0	4	0	4	0	0	1	3	0

5 Discussion

Based on the above results, there are four enlightenments. First, from the analysis of the NCM textbooks, it is found that China attaches great importance to the content of geometric transformation; Second, the NCM textbooks provide a higher OPL in geometric transformation; Third, there is still room for improvement in the cognitive level of geometric transformation in the NCM textbooks; Finally, combined with case analysis, it is found that the compilation style of the NCM textbooks reflects the oriental culture.

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Mathematics Teachers' Perspectives on Textbooks' Design in Multiplication Tables

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Abstract. The introduction of the multiplication tables (or nine-times table) is of significance to children's learning of basic multiplication fact. To fulfil the potential of multiplication tables in mathematics teaching and learning, different mathematics textbooks adopt different designs, presenting distinct features regarding the order of introducing multiplication tables, classroom activities and tasks for building connections, etc. This study collected interview data from 18 primary school teachers with different teaching experience, and aimed to reveal teachers' viewpoints regarding the differences in textbooks' design in multiplication tables and how they make impacts on the teaching and learning of multiplication at classroom level.

1 Background

International assessment programme and researches have documented Chinese students' good performance on recalling multiplication fact and computing arithmetic operation tasks in general (Cai 2002; LeFevre and Liu 1997; OECD 2014). Although it is well accepted that teaching of multiplication tables should focus less on automatic recall individual facts but more on building relations in between by reasoning, it is challenging for teachers to put this idea into practice (Cao et al. 2015; Zhang et al. 2019). Since textbooks serve as teachers' key resource to organize daily teaching (Cai et al. 2014), it is worthwhile examining what opportunities and/or limits textbook may provide.

2 Literature Review

2.1 Learning Multiplication Tables in China: Textbooks and Background

In China, different versions of mathematics textbooks for primary school students have been developed according to national curriculum standards, which is referred to as "one curriculum standard, many series of textbooks." (Li and Li 2018). Different series of textbooks shared mathematics contents in common, but each textbook series has unique design in the presentation of mathematics contents. Out of all mathematics textbooks in China, PEP and NCM are two most extensively used textbook series.

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In addition, Shanghai curriculum follows the general framework of national reform, but Shanghai, as a pioneer, is often given the privilege of experimenting with curriculum and textbooks with innovative designs (Cheng 2011). Therefore, Shanghai Math textbooks usually have features different from all the other textbooks aligned with national curriculum standards. Although Shanghai Math is mainly used in Shanghai, its design has many reform-based features about mathematics teaching and learning.

For all three mathematics textbooks series (i.e., PEP, NCM, and Shanghai Math), multiplication operations and multiplication tables are included in Grade 2. On top of the mathematics agenda of an eight-year old Chinese student is to learn the meaning of multiplication and single-digit whole number multiplication fact.

2.2 Textbook Designs: Learning Trajectory of Multiplication Tables

When focusing on the arrangement of the chapters of learning multiplication fact in different textbook series clear distinction can be identified regarding the order of introducing multiplication tables, classroom activities and tasks for building connections, etc. Taking seven textbook series from four countries as an example (Table 1), when looking into the order of introducing multiplication tables and range of digits included, significant difference can be identified.

Table 1 Order of learning multiplication fact in different textbook series

Country	Textbook series	Order of learning multiplication fact
China	PEP	5,2,3,4, 6,7,8,9
	NCM	5,2,3,4, 6,7,8,9
	Shanghai Math	10,5,2,4,8,7,3,6,9
USA	California	2,4,5,10,0,1,3,6,7,8,9
Singapore	My Pals are here	2,5,10,3,4,6,7,8,9
Netherlands	De Wereld in Getallen	2,10,5,3,4,6,7,8,9

Overall, two kinds of learning trajectories can be distinguished in terms of learning multiplication fact. Or it was better considered as the two pole of a continuum. One is mainly following the order of positive integer, and the underlying rationale is the power of the natural order of numbers and students' familiarity (Zhang et al. 2019). The other is starting with foundational facts, such as 2, 5, 10, 0 and 1, and solving unknown facts on the basis of the known by reasoning (Van de Walle et al. 2019). As shown in Table 1, three different textbook series in China seem to locate their decisions somewhere in this continuum. However, little was known about how primary school mathematics teachers view the different designs of multiplication tables and how they adapt textbooks' design into classroom practices.

2.3 Research Questions

This study aims to answer the following research questions: (1) What adaptations have teachers made when using NCM textbook to teach multiplication tables? (2) How do teachers make comparisons among three textbooks' design on multiplication tables?

3 Method

3.1 Participants

By convenience sampling, 18 primary school mathematics teachers in Daqing City, Heilongjiang Province, were included in the current study. Participants were selected with the consideration of their teaching experiences and the locations of their schools. All participants are female and have at least one year's expedience in teaching Grade 2, and they all used New Century Mathematics (NCM) primary textbook series for daily teaching. For anonymity, the teachers were called T1 to T18.

3.2 Data Collection

Semi-structured online interview was conducted with the participating teachers.Each teacher was interviewed for about 90 min. The interview protocol mainly includes two aspects: (1) personal teaching practices of multiplication tables and adaptations when using NCM textbook, (2) comparison of three textbooks series.

3.3 Data Analysis

The interview data was firstly transcribed into Chinese text and thematic analysis was conducted. Two researchers coded the data of two teachers respectively, and discussed the categories and subcategories. When the decisions differed, they were discussed at length until 100% agreement was reached.

4 Results

4.1 Adaptations of NCM Textbook

By analyzing teachers' responses, we identified four aspects with regard to the design of teaching multiplications tables: (1) contexts for introduction of multiplications, (2) processes for developing multiplication tables, (3) tasks for understanding and investigating multiplication tables and (4) tasks for exercises.

4.2 Contexts for Introduction of Multiplications

One adaptation is to change the contexts designed in the NCM textbook. For example, the context for multiplication of fives in NCM textbook is to help two squirrels count the pine cones. Teacher Tong did not use this context but asked students to observe the number of fingers in one hand. "When I teach the formula of multiplication of five, I only use the context of fingers. Every child has five fingers, which is easier for children to understand." [Tong, 00:08:59] As is stated by her, the reason of making this adaptation is make it easier for students to make sense of the mathematics contents.

Another example is the context design for multiplication of fours. NCM textbook introduced a story that Little Bear would like to invited friends to have meals at home and wanted to prepare some red fruits and put every four red fruits on a stick. Teacher Zhang used bunches of small sticks to represent the red fruits. "We eat candied haws in winter, don't we? When children saw the string of red fruits, they immediately thought of the scenes of candied haws. They were very excited and wanted to string the red fruits. However, I was worried that the bamboo sticks would hurt children's hands, so I decided to use small sticks to represent red fruits." [Zhang L, 00:27:44] Here Teacher Zhang considered the feasibility of realizing the actions of putting red fruits on a stick.

4.3 Processes for Developing Multiplication Tables

NCM textbook designed detailed processes of developing multiplication tables from contexts. Teachers made some changes when following the designed processes. "When developing the multiplication of fives, it is supposed to be a processes of investigations. But most students could say the multiplication arithmetic correctly without doing investigations. So students simply write them down without necessarily develop the understanding. What we can do is to let them go for next step, which is to represent the multiplications of fives in dot paper. Students got stuck and then I can ask them go back to rethink the processes of developing multiplication tables and get conceptual understanding about multiplication tables." [Rong, 00:14:21]. Here Teacher Rong chose to skip the processes of developing multiplication tables at first but then guided students to revisit the processes.

4.4 Tasks for Understanding and Investigating Multiplication Tables

Teachers added tasks to encourage multiple representations. "There are two ways of interpreting the result of 6 times 7 in the textbook (See Fig. 1). But I think more ways are needed. I can ask students to give more ways and ask them whether there are some other ways of doing this. I think the textbook should give more spaces for this part". [Rong, 00:37:32].

4.5 Tasks for Exercises

Teachers redesigned the exercise tasks to make it more open-ended. "Students are asked to fill the two blanks here (See Fig. 2). But I would like to drop 6 and leaves four blanks here for students to fill. Or I simply ask students to give a number sentence that is equal to the results of 6 times 9" [Rong, 00:30:43].

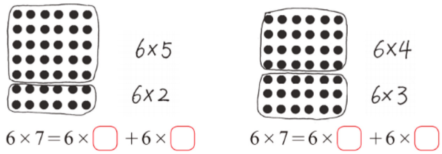

Fig. 1. Tasks of 6 times 7 in NCM textbook

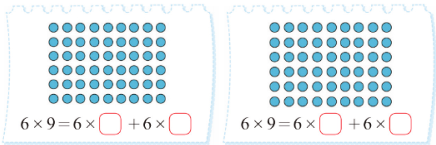

Fig. 2. Tasks of 6 times 9 in NCM textbook *Note* Here students are asked to provide two ways of calculating 6×9 and visualize the calculation methods using the dots.

4.6 Comparisons Among Three Textbook Series

When making comparisons among three textbook series, teachers mentioned three aspects: (1) contexts for introduction of multiplications, (2) processes for developing multiplication tables and (3) tasks for exercises.

4.7 Contexts for Introduction of Multiplications

Most teachers highlighted the importance of contexts for introducing multiplication table. "Both PEP and NCM designed contexts for multiplication tables, but Shanghai Math directly used the number line and students need to develop multiplication table from the number line, which is too abstract for students" [Zhang L, 03:40:35].

4.8 Processes for Developing Multiplication Tables

Most teachers noticed the differences in the order of presenting multiplication tables but their seemed reluctant to make comprehensive comparisons between NCM and Shanghai Math. "After the multiplication of twos and fours, the Shanghai Math jumped to the multiplication of eights. I think this is a bit incoherent, not as orderly as that in NCM which follows the order from small numbers to large numbers. Children could get the way orderly thinking in this process, but Shanghai Math breaks the order, making it harder for students to understand" [Zhang L, 01:33:07]. "NCM follows starts from small numbers and then increased the difficulty little by little. I feel that the Shanghai Math gives me a jumping feeling" [Zhang C, 01:00:49]. When interpreting the orders of multiplication table, both teachers focused mainly on orderly thinking without noticing the connections between multiplications of twos, fours and eights.

Regarding multiplication table for one number, most teachers felt Shanghai Math's design is redundant. "I don't see the point of giving the third column in Shanghai Math (See Fig. 3). It is not necessary to swap the first number and second number in the multiplication. Students all know that 2 times 6 is equal to 6 times 2. And we know the

Commutative Law of Multiplication. So this is unnecessary" [Zhang L, 01:35:16]. When making comparisons, Teacher Zhang mainly focused mainly on students' knowing rather than their understanding.

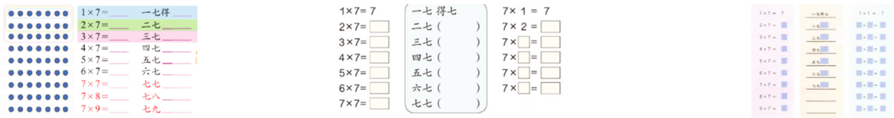

Fig. 3. Multiplication of sevens in NCM (left), PEP (middle)and Shanghai Math (right)

4.9 Tasks in Exercises

Most teachers noticed the volume of exercise tasks. "I found that there are more exercise tasks in PEP than the other two textbook series" [Sun, 00:56:50].

Some teachers mentioned NCM could borrow some ideas from PEP in context selections of exercise tasks. "PEP used Tangram in exercise tasks of seven's multiplication table. Our students learned Tangram previously. Here Tangram could be included in the learning of multiplication tables" [Rong, 01:16:40].

5 Conclusions and Discussion

Teachers' adaptation of textbook reflects teacher beliefs in mathematics teaching and learning, and teachers' comparisons among different textbook series could also show teachers' ability to learn from textbooks (Nicol and Crespo 2006; Shinno and Mizoguchi 2021). This study finds that teachers' focus in using mathematics textbooks is different from that in analyzing textbook series. When teachers used one textbook series without critical reflections, their ability in learning from other textbook series could be diminished.

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A Study on the Primary School Mathematics Teachers' Attention Regarding the Adoption of NCM Textbooks

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Abstract. Textbook is one of the most important resources for teachers to implement teaching, teacher's attention to the textbooks profoundly effects their teaching practice. Based on the Concern-based Adoption Model, according to the theme of teachers' attention of NCM textbook, a six-stage model is established, including the attention stages of information acquisition, personal competence, management and control, student results, collaborative study and reflection evaluation. Revised stage of attention questionnaire was used to investigate the current situation of primary school mathematics teachers' NCM textbooks attention, and explore whether and how different teachers care of the textbooks. Then through the investigation of typical teachers, we found ways to improve the teachers' attention of NCM textbooks, so as to lay the foundation for teachers to use textbooks better and implement the concept of curriculum reform thoroughly.

1 Introduction

Mathematics textbooks are often known as the core of mathematics teaching resources (Pepin and Haggerty, 2003). Existed research showed that the teaching of teachers, especially the implementation of the curriculum by mathematics teachers, is largely dependent on the content presented in the textbook (Wijaya, van den Heuvel-Panhuizen, & Doorman, 2015). The stages of teachers' attention of textbooks largely determines whether teachers can implement the ideas throughout the textbooks into the curriculum. Li Qing (2018) conducted a survey about whether the teacher relies on the textbook and the way the teacher uses the textbook in China, she found that it depends largely on the discipline the teacher taught, mathematics teachers are more dependent on textbooks.

The concept "teacher's concern" originated in the late 1960's, when Fuller put forward the hierarchical structure of the teachers' concern, including three levels: self, task and impact. Later, Hall et al. (1977) on the basis of the hierarchical structure extends the Fuller model, put forward Concern-based Adoption Model (CBAM), one of which is the Stages of Concern (SoC) theory. The self-level contains information and individual concern; task-level corresponds to management issue. The SoC theory summarizes

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the orderly process of changes in teachers' concern. Since the theoretical framework was proposed, empirical studies have been used as evidence to support the sequence of teachers' concern stages (Hall & Hord, 1987). SoC theory of CBAM provides the corresponding assessment questionnaire focuses on concern stage (SoCQ) initialed at the university of Texas in the 1970s with the development research center RDCTE, then developed and tested (Hall et al., 1977) in southwest education development laboratory (SEDL). The practice based on the theory were spreaded worldwide, including empirical studies in the United States, Australia, Belgium, Britain, Canada and China Hong Kong, their research focus on stage of concern theory implemented in the background of curriculum reform, which further developed the theory itself (B. and Palsha, 1992; Cheung, Hattie and Ng, 2001;Cheung, 2002; Bullard, Rutledge, & Kohler-Evans, 2017; Fischer, McCoy, Foster, Eisen and Lawrenz, 2019).

This study focus on the primary school mathematics teachers' stage of attention of the textbooks. On the one hand, we can analyze the current situation and influencing factors of teachers' attention to the textbooks with the help of theory and existing tools; On the other hand, this study can also expand the applicable situation of Concern-based Adoption Model (CBAM) and the Stages of Concern (SoC) theory.

2 Research Design

2.1 Participant

This study has gone through four stages, the participants were composed of four groups of teachers. Firstly, experts were invited to judge the rationality of the framework and tools (N = 22); next, pre-service and in-service teachers were selected to complete the pre-test questionnaire (N = 123); then, teachers who use the New-Century Primary School Mathematics Textbook published by Beijing Normal University were selected to participate in the fomal test (N = 1980); finally, teachers with more than 6 years of textbook use experience were selected for in-depth interviews to refine the key influencing factors (n = 34).

2.2 Questionnaire

The questionnaire in this study consists of two parts. The first part involves basic demographic variables, including 14 questions such as teacher's gender, teaching seniority, teaching grade and participation in training etc. In the second part, the SoCQ was translated and used to measure teachers' concern of textbooks (el-saleh and Kamel, 2011). Meanwhile, the information about teachers' attitudes towards textbooks and their influencing factors was obtained.

2.3 Research Question

- What is the current situation of mathematics teachers' attention of textbooks?
- What factors affect teachers' attention of the textbooks?

3 Research Result

According to descriptive statistical results, a total of 2237 primary school mathematics teachers answered the questionnaire. After manual verification, among the 1980 teachers who effectively participated in the survey, there were 361 male teachers and 1619 female teachers, which was in line with the current situation that female teachers accounted for the majority of primary school teachers in China. The teaching experience of the teachers participating in this survey is evenly distributed in six intervals of 0–2 years, 3–5 years, 6–8 years, 9–14 years,15–20 years and over 20 years.

By calculating the mean of the items in each dimension and synthesizing the scores of the six dimensions, the highest points of teachers' attention of textbooks are the management & control and student results dimension respectively, while teachers' reflection evaluation of textbooks is relatively weak (Fig. 1).

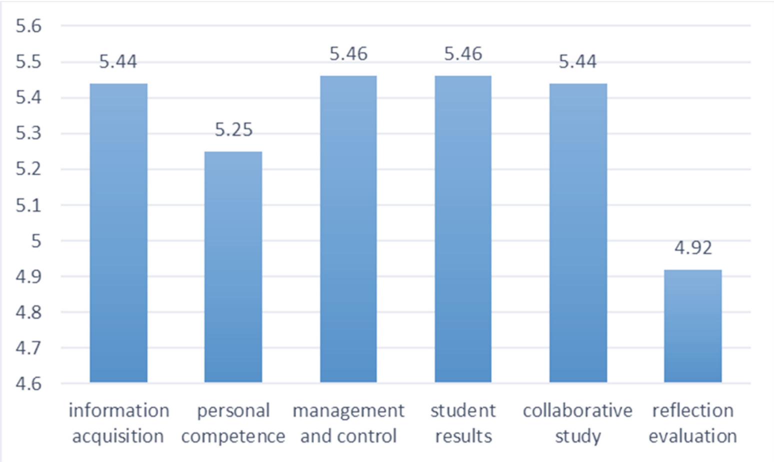

Fig. 1: Teacher's stages of attention of the NCM mathematics textbooks

Based on the existing research, we then take the teaching experience as the factor influencing the performance of teachers' textbook attention stage, and the results of one-way ANOVA showed that teachers with different seniorities have significant differences in six stages of textbook attention. When teachers have 6–8 years and 15–20 years teaching experiences, their performance tends to go up slightly. That may because that teachers who have been teaching for 6–8 years generally have just completed a complete teaching cycle. At this time, teachers will reassess the content and value of textbooks, their personal use of textbooks, and the impact of using this edition of textbooks on students' development. Teachers who have been teaching for more than 15 years pay more attention to reflection and evaluation, which reflects that teachers pay more attention to the optimization of textbook content and teaching methods with the accumulation of experience, so as to improve the use effect (Fig. 2).

After that, we took the school location as a factor influencing teachers textbooks attention to make the single factor variance analysis. According to the results, teachers

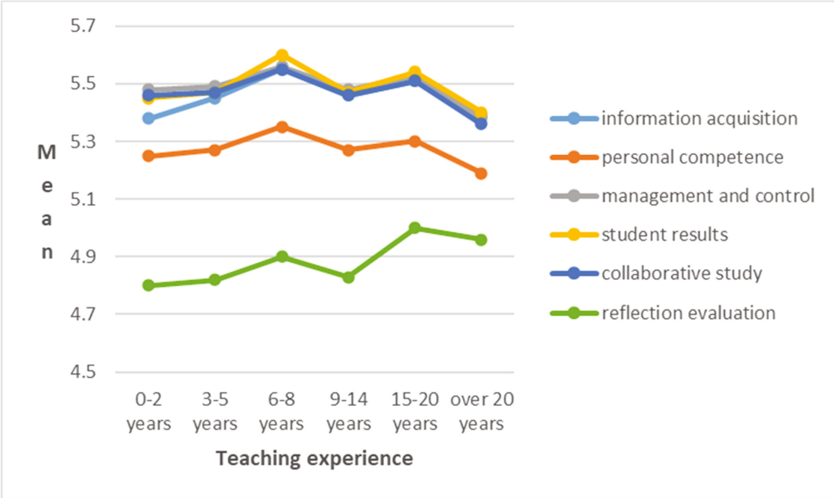

Fig. 2: Different teaching experience teacher's stages of attention of the NCM mathematics textbooks

within different location of schools showed significant differences in five dimensions, except for the personal competence dimension (Fig. 3).

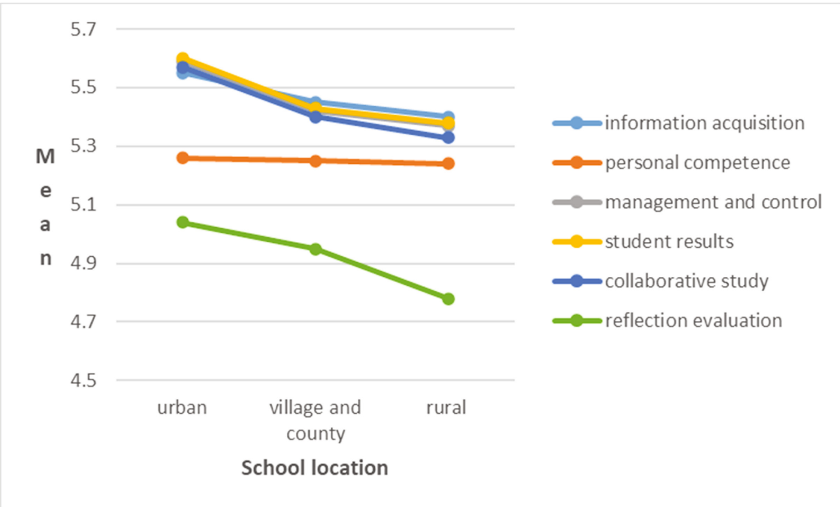

Fig. 3: Different school location teacher's stages of attention of the NCM mathematics textbooks

Teachers in urban schools will more actively understand the basic information and value of textbooks, pay more attention to the use of textbooks in the classroom and the construction of textbook research community. Teachers in urban schools will also pay more attention to how to adjust the current situation of textbook use, so as to obtain better results.

Finally, we focus on 34 typical teachers, through self-report on the attention stages and influencing factors of textbooks usage at all stages, understanding the changes of teachers' attention of textbooks with the increase of the teaching experience, and summarise what factors affect the changes of teachers' attention of textbooks, responding to and explaining the previous research results, and providing suggestions for teachers to improve the usage of textbooks.

Combined with the qualitative data of the typical groups in the this part, we found that: In the first two years of using textbooks, teachers focus on the acquisition of textbook information, they want to know how to use textbooks in the process of lesson preparation. After using textbooks for 3–5 years, teachers still pay high attention to the information and value of textbooks and their own management and regulation of textbooks. However, compared with the previous stage, they pay more attention to the effect of students' harvest, and also involve the reflection and adjustment of textbooks. When using textbooks for 6–10 years, teachers pay special attention to the impact of textbook use on students in addition to continuing to pay attention to and interpret the compilation and writing sketches of textbooks. Teachers pay attention to the effect of using textbooks on cultivating students' high-order thinking, emotional attitude and learning habits, the promotion of textbooks on the learning process, and whether students' academic performance has been improved. Teachers who have used textbooks for more than 11 years gradually pay more attention to textbooks than information acquisition, textbook use management and learning effect evaluation in a general sense. At this stage, teachers pay most attention to the generation of learning paths and the impact of textbooks on students. Teachers hope to study the process of students' knowledge formation with the help of the contents of textbooks and the learning situation of students.

4 Conclusion and Suggestion

Through this study, we found that:

(1) Primary school mathematics teachers maintain a high degree of attention of the textbooks.
(2) Primary school mathematics teachers who have different teaching experience have significant differences in the performance of each attention stage. Teachers of different school locations have significant differences in other attention stages except for their attention to personal competence.
(3) The factors affecting primary school mathematics teachers' attention to textbooks include teachers' understanding of textbooks, teachers' personal professional quality, students, training and environmental factors.

Therefore, we put forward three suggestions to promote teachers' attention of the textbooks. The first is to interpret the intention and explore the value of textbooks. The second is to pay attention to students' learning path and adjust textbooks according to their learning situation. The third is to unblock experience sharing channels and establish research and cooperation platforms.

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Study on the Influence of Textbooks on Teachers' Teaching: An NCM Story

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Abstract. In this paper, we focus on NCM (New Century Mathematics) textbooks' influence on teachers' teaching. Based on the conceptual framework of Shulman and Fan, a survey with a questionnaire and interview was conducted. The data were collected from a stratified sample from Haidian District, Beijing. The results revealed the following: the textbooks facilitated mathematics teachers' teaching, especially in the process of Transformation; the design of activities and columns of the NCM textbooks facilitated mathematics teachers' teaching more obviously than other features; and compared with teacher characteristics, school characteristics played a more significant role on teachers' response about how textbooks and the features facilitating their teaching.

1 Introduction

Mathematics textbooks, as auxiliary materials for mathematics teaching and learning, have been around since ancient times. Compared to the long history of mathematics textbooks, the study of the influence of mathematics textbooks, or more generally, the study of mathematics textbooks, is much shorter. As Cronbach (1955) pointed out more than 60 years ago, although textbooks are most popular in the classroom, textbook-centric research is "scattered, uncertain, and often trivial". In the 1980s, researchers became increasingly aware that school textbooks remained a largely unexplored area and that more research was needed in this area. Textbooks have a huge impact on teachers and teaching which play a role in teachers' teaching by delivering teaching information and providing a curriculum environment that encourages or inhibits teachers to use different teaching strategies (Fan and Kaeley 2000). NCM (New Century Mathematics) textbooks play an important role in mathematics education reform, so it is of great significance to explore the influence of this teaching material on teachers for educational practice. Therefore, this study proposes the following research questions:

1. Do NCM textbooks facilitate teachers' teaching?
2. What are the features of NCM textbooks and which of them facilitate teachers' teaching more?

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2 Literature Review

Fan identified the field of textbook analysis in a textbook review study, accounting for 63 percent of all identified studies (including textbook comparisons, 29 percent) and textbook use (25 percent), while much less was found in other fields (12 percent). The survey shows that the important role of textbooks in teaching and learning has been widely recognized by researchers. Researchers generally believe that textbooks, as the main carrier of curriculum, play a leading role in modern education of different subjects.

On the use of textbooks researchers have explored the role of textbooks in teaching. Some reported that differences in the use of textbooks may or may not exist across different groups of teachers. Birgit et al. (2016) found that inexperienced or non-professional teachers relied more on textbooks than experienced teachers because they lacked time to prepare for lessons. On the other hand, Johansson's study of three Swedish teachers (2006) shows that, despite the differences among these teachers, textbooks largely play a similar guiding role in their teaching.

As key teaching materials, textbooks are intended to be used in different stages of teaching. According to Shulman (1987), teachers' pedagogical reasoning and actions for teaching involve a cycle through activities of Comprehension, Transformation, Instruction, Evaluation and Reflection. With regard to above stages, there are some related researches, but they are relatively independent.

These studies, however, focus more on the way textbooks are used than on the impact of textbook use. The effects of using textbooks seem to have been largely ignored. But the latter involves value judgment, especially in the question of whether textbooks promote or hinder teachers' teaching. However, this value judgment has rarely been examined in previous literature. Therefore, it is of great significance for teaching reform and practice exploration to study the actual influence of NCM textbooks on teachers.

3 Methodology

The conceptual framework was based on Fan's et al. (2021), drawing partly on Shulman's (1987) model of pedagogical reasoning and action. Both qualitative and quantitative methods were used.

3.1 Conceptual Framework and Research Instruments

The conceptual framework of this research includes two parts: the first is how textbooks facilitate teachers' teaching, more details can be seen in Fan et al. (2021). Fan referred to Shulman's (1987) model of pedagogical reasoning and action and discussed the teachers' teaching from five aspects, including Comprehension, Transformation, Instruction, Evaluation and Reflection. In Fan's research, he designed a questionnaire with 9 items in the dimension Comprehension, 8 items in Transformation, 6 items in Instruction, 2 items in Evaluation, and 1 item in Reflection. We quoted his questionnaire but designed one more item, that is dividing his first item about"Understanding of internal and external connections of mathematics" into two parts:

1. Understanding of internal connections of mathematics.

2. Understanding of external connections with the real world of mathematics.

Fan's questionnaire had a Cronbach's α of 0.98, indicating high reliability, while the questionnaire of this research was 0.99, and Cronbach's α of each dimension was bigger than 0.9, which also indicates high reliability (Sharma, 2016). Besides, the CFI was 0.89 and the TLI was 0.88, indicating the high validity of the questionnaire (Tucker and Lewis 1973). That's Questionnaire A.

The second part of this conceptual framework was about the features of NCM textbooks that facilitate teachers' teaching. To understand the specific features of NCM textbooks, the researchers conducted in-depth interviews with the chief editors of NCM textbooks. Through the interpretation of the official documents of the textbooks, the researcher firstly proposed the main arrangement and style features of 8 NCM editions of the textbooks. Further, the chief editor will be asked to sort and select these features, among which the top five will go into further research tool development. The main features of the teaching materials communicated with the chief editor in the Interview include the following 8 aspects:

[1] The presenting way of the "context+question string";
[2] Design of activities and columns provide chance of ample observation, experimentation, speculation, reasoning, and reflection;
[3] The organizational content with mathematical ideas and methods as the main line;
[4] Focusing on stimulating students' interest in learning;
[5] Providing personalized learning opportunities for students by setting open questions;
[6] Rich realistic situations;
[7] Diversity of problem-solving strategies;
[8] Full opportunities for cooperation, communication, and sharing.

The top five items are all top five items of the three editors, just with only a small adjustment to the order. Questionnaire B with textbooks features is developed around these five, which is similar to Questionnaire "how textbooks facilitate teachers' teaching". The questionnaire also adopts the 4-point Likert scale(4 for 'Strongly Agree' and 1 for 'Strongly Disagree'). First of all, the features of NCM textbooks are investigated overall. Further, we will investigate the effects of each feature one by one (Table 1).

This questionnaire B's Cronbach's was 0.99 and Cronbach's α of each dimension was bigger than 0.9. Besides, the CFI was 0.84 and the TLI was 0.82, while the CFI and TLI of each 5 features were bigger than 0.9, indicating the questionnaire can be accepted for further research.

3.2 Sample

This research selected the key experimental area of NCM textbooks: Haidian District, Beijing. And stratified sampling in Haidian District is conducted. Combined with the judgment of the teachers and researchers on the schools and the distribution of the test results at the end of the last semester in the district, about 5 high-performing schools and 7 medium- performing schools were selected to participate in this study, altogether 12 schools. We obtained a total of 225 samples, excluding 29 invalid samples, and 196 actual effective samples.

Table 1. The questionnaire B about the features of the NCM textbooks in teachers' teaching of mathematics

Item	Description
B1–6	The presenting way of " context+question string" in textbooks is helpful to my teaching A. Helpful for my understanding of the teaching purpose, mathematical content, and characteristics of students B. Helpful for my preliminary design teaching C. Helpful for me to flexibly adjust and practice teaching D. Helpful for me to evaluate students' learning E. Helpful for me to reflect on teaching
B7–12	Helpful to me because the activities and columns designed in textbooks provide chance of ample observation, experimentation, speculation, reasoning, and reflection;
B13–18	Helpful to me because the mathematical ideas and methods designed in textbooks are mainline to organize the content
B19–24	Helpful to me because textbooks focus on stimulating students' interest in learning
B25–30	Helpful to me because open questions designed in the textbooks provide learning opportunities for different students

3.3 Data Collection and Analysis

Questionnaires were collected online. To improve the reliability of the questionnaire, the method of anonymity was adopted, namely, the school name was handled anonymously, and each school was sent in the way of one link. The effective response rate of the questionnaire was 100%, and the effective rate of the sample was 87%.

4 Results and Discussion

The results and discussion will be carried out in two parts. The first two subsections are about "how textbooks facilitate their teaching". The last subsection is about "what features of NCM textbooks facilitate their teaching."

4.1 How Textbooks Facilitated Teachers' Teaching

The average score of teachers on questionnaire A was 3.51, which indicated NCM textbooks are of great help in facilitating teachers' teaching in general. As for the sub-dimensions, the mean score of transformation was the highest (M = 3.56), followed by comprehension (M = 3.52) and instruction (M = 3.51), and then evaluation and reflection (M = 3.43). That is to say, the textbooks were most helpful to teachers in the stage of preparation for teaching (i.e., comprehension and transformation) and by contrast, and least helpful in evaluation and reflection, whose findings are same as Fan's et al. (2021).

As to the dimension Transformation, 6 of 8 questions of transformation all scored high, even though the low score of question "11. Textbooks help me set teaching objectives for each lesson." (M11 = 3.52) and "17. Textbooks help me choose appropriate teaching methods for classroom teaching." (M17 = 3.54) are all above 3.5, indicating that textbooks play a significant role in helping teachers design and transform their teaching, especially in promoting teachers' mastery of classroom teaching structure, selection and use of classroom representations, and organization of classroom teaching.

The conclusion of this section includes 3 items of evaluation and reflection dimensions. This dimension has a relatively low score, which shows that the textbooks played a less positive role in facilitating teachers' 'formative assessment', 'summative assessment' and 'teaching reflection'.

4.2 Different Teachers' Responses on How Textbooks Facilitated Their Teaching

The study detected if there existed differences between different teacher groups. The results showed that no significant differences were found between teachers with different years of teaching experience, with different years of continuous use of the textbook, with different educational background and different job titles. It seems that NCM textbooks played largely a similar facilitation role in Haidian teachers' teaching no matter how many years they had taught, how many years they had continuously used NCM textbooks, and what their educational background or job title was. It also further explains the extensive and comprehensive promoting effect of NCM textbooks on teachers' teaching (Table 2).

Table 2. Different groups of teachers' responses on facilitation of textbooks

Dimension	Teacher characteristics							
	Years of teaching experience		Years of continuous use of the textbook		Educational background		Job titles	
	χ^2	*df*	χ^2	*df*	χ^2	*df*	χ^2	*df*
Comprehension	0.170	2	0.090	2	3.285	2	1.471	3
Transformation	0.502	2	0.980	2	2.769	2	1.070	3
Instruction	0.243	2	0.017	2	5.295	2	1.647	3
Evaluation and Reflection	0.442	2	2.103	2	4.751	2	1.826	3
Textbooks facilitating teaching	0.163	2	0.4	2	4.421	2	1.337	3

4.3 Different Schools' Responses on How Textbooks Facilitated Their Teaching

As shown in Table 3, in terms of school characteristics based on student performance levels (high-performing vs. medium-performing), the average value of teachers from high-performing schools was significantly higher than that of teachers from medium-performing schools ($\chi^2 = 5.246$, $p < 0.05$), indicating that traditional high-performing schools can make more use of NCM textbooks to play the effect of them. In addition, school-level differences are shown in various dimensions: Comprehension ($\chi^2 = 7.558$, $p < 0.01$), Transformation ($\chi^2 = 4.242$, $p < 0.05$), Instruction ($\chi^2 = 5.437$, $p < 0.05$), Evaluation and Reflection ($\chi^2 = 4.082$, $p < 0.05$).

Table 3. Different schools' responses on facilitation of textbooks

Dimension	School characteristics	
	High-performing/Medium-performing	
	χ^2	*df*
Comprehension	7.558**	1
Transformation	4.242*	1
Instruction	5.437*	1
Evaluation and Reflection	4.082*	1
Textbooks facilitating teaching	5.246*	1

*$p < 0.05$
** $p < 0.01$

4.4 Teachers' Responses on the Facilitation Role of Different Features of NCM Textbooks in Their Teaching

Teachers agreed that some features of textbooks were helpful to their teaching, especially the design of activities and columns. In addition, the arrangement type of context+question string and open questions were also helpful to teachers' teaching. Look specifically at the impact of each features for teachers' Comprehension, Transformation, Instruction, Evaluation and Reflection, we can find from the results that the features of NCM textbooks helped teachers' teaching from several aspects and covered the cycle of teaching. Moreover, there was no difference between schools and teacher groups in responses on the features of NCM textbooks facilitating their teaching.

To reveal whether there were significant differences in the roles of these five features of textbooks in facilitating teachers' teaching, we used Friedman test, and the results showed that there were significant differences in the roles of these five features ($\chi^2 = 11.023$, $p < 0.05$). To further examine if there were significant differences between any two of the five features of NCM textbooks, Wilcoxon signed-rank test were conducted, and the results showed that the feature of Design of activities and columns played a significantly more facilitating role in teachers' teaching than the feature of Mathematical ideas and methods designed ($Z = 2.362, p < 0.05$) and the feature of Stimulating students' interest in learning ($Z = 2.646, p < 0.01$), the feature of Open questions designed played a significantly more facilitating role in teachers' teaching than the feature of Stimulating students' interest in learning ($Z = 2.175, p < 0.05$) (Table 4).

Table 4. Teachers' response on the facilitation role of different features of NCM textbooks in their teaching($N = 196$)

Features	M	SD
Context+question string	3.62	0.503
Design of activities and columns	3.63	0.488
Mathematical ideas and methods designed	3.60	0.516
Stimulating students' interest in learning	3.59	0.516
Open questions designed	3.62	0.499

5 Summary and Conclusions

From the research, NCM textbooks has a big facilitating effect to teachers' teaching, especially in teachers' Comprehension, Transformation, Instruction. And the concrete arrangement style, features, especially the design of activities and columns, context+question string, open questions of textbooks have more significant effect in facilitating teachers' teaching. Further strengthening the research of textbook, especially how to promote teachers' evaluation and reflection on students' learning and their own teaching, can and should become the direction of further research.

Despite the increasing number of high-performing schools in China, what factors remain to be improved in promoting effect of NCM textbooks on teachers' teaching in medium-performing schools is worth further study.

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The Factors Facilitate Teachers' Use of Textbooks: An NCM Story

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Abstract. Studying the influence mechanism of textbooks on teachers is of great significance to the compilation and improvement of textbooks. Based on the questionnaire survey on the impact of NCM (New Century Mathematics) mathematics textbooks on teachers this study selected 16 teachers for semi-structured interviews to explore how textbooks influence teachers and the factors facilitate teachers' use of textbooks. Through the analysis of the text, it is found that the textbooks all contribute to teaching but in different ways. Besides the teachers' internal motivation, teaching experience and teaching and research activities affect teachers' use of textbooks.

1 Introduction

Textbooks are among the most widely used and trusted written resources by students and teachers for school based learning in all parts of the world (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996). Teachers often rely heavily on textbooks for many decisions such as what to teach, how to teach it, what kinds of tasks and exercises to assign to their students (Robitaille & Travers, 1992). Research on teachers using textbooks bears much significance and it is well acknowledged that "However significant the impact of a textbook on curriculum, it cannot be forgotten that at the end of the line, the practitioner decides how it is used" (Pratt, Anderson & Tomkins, 1982). So exploring how this played out is important. In the new curriculum reform, the compilation of textbooks has changed a lot. The textbook of NCM has great influence in the whole country. Therefore, it is of great significance to study the influence of this version of teaching material on teachers to understand the practical effects of use by teachers and students. So we propose the following research questions: 1. How NCM textbooks influence teachers? 2. What underpins these effect?

2 Literature Review

Mathematics textbooks have existed since ancient times, but until the 1980s few studies of their actual content had been published. More recently, however, Fan, Zhu and Miao (2013) have analyzed 111 journal articles and other scientific publications concerned with mathematics textbooks which have been published in recent decades. Their sample

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is quite comprehensive, which suggests that this area had not been much explored. The influence of textbooks on mathematics teaching has been described both as indirect, when the textbook facilitates teaching for teachers (Pepin & Haggerty, 2001), and direct, when teachers choose to adopt textbooks exclusively without additional materials. Teachers can also choose additional content, or teach using a different structure from the textbook (Sosniak & Stodolsky, 1993), or take more factors into account, for example assessment data, when preparing mathematics teaching (Sullivan, Clarke, Clarke, Farrell, & Gerrard, 2013).They provide the teacher and the learners a framework to ensure a sense of structure and progress (Richards, 2005; Ur, 1996). Thus, teachers' long-term use of textbooks and the effect of textbook use on teachers' professional development should be attended to. Although an extensive amount of research has been done on the actual materials use, little of it provides empirical evidence of the effects of materials on their users, far less of that on teachers. It is true to say that the influence of textbooks on classroom teaching practices remains very poorly understood to date. Many researchers have pointed out that too little attention has been given to how teachers actually make use of textbooks; exactly what the presence of textbooks signals about their use has not been adequately studied or analyzed (Stodolsky, 1989: 159).

3 Method

3.1 Frame

In this study I used Remillard's frame which examine the influence that curriculum materials have on student learning. In this framework, they conceptualized these various meanings of curriculum as unfolding in a series of temporal phases from the printed page (the written curriculum), to the teachers' plans for instruction (the intended curriculum), to the actual implementation of curricular-based tasks in the classroom (the enacted curriculum) (See Fig. 1). In our research, we focused on what kind of influence textbooks have on teachers and what specific factors affect these changes in this process.

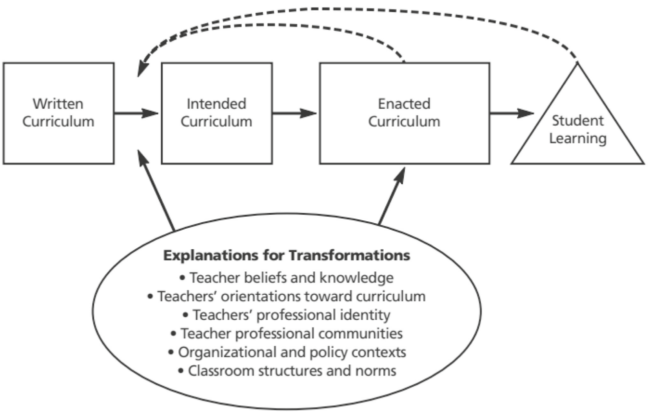

Fig. 1. Temporal phases of curriculum

3.2 Sample

In order to improve the representativeness of the sample, we adopted a strict multi-stage stratified random sampling method, first selecting regions, then selecting schools, and finally selecting teachers. First, we selected the haidian district, which is more representative in terms of the years of use of textbooks and the level distribution of schools. Schools were then divided into two categories based on interviews with teaching and research staff and school test scores: excellent schools and mediocre schools. The classification results were verified by local education experts. Then, we randomly selected 2 excellent schools and 2 ordinary schools from the two categories in each region, and randomly selected 2 senior teachers and elementary or intermediate teachers from each school, and conducted interviews with 16 teachers. Table 1 provides an overview of the participating teachers.

Table 1 Profile of participating teachers

n	School level	Teachers characteristic			n	School level	Teachers characteristic		
		Gender	Teaching years	The title			Gender	Teaching years	The title
A1	High-per	Female	3	Second-grade	C1	Ordinary	Female	3	Second-grade
A2	High-per	Female	5	Frist-grade	C2	Ordinary	Female	2	Second-grade
A3	High-per	Male	14	Senior	C3	Ordinary	Female	14	Frist-grade
A4	High-per	Female	17	Senior	C4	Ordinary	Female	15	Senior
B1	High-per	Female	2	Second-grade	D1	Ordinary	Female	4	Second-grade
B2	High-per	Female	7	Frist-grade	D2	Ordinary	Male	12	Senior
B3	High-per	Female	11	Senior	D3	Ordinary	Female	2	Second-grade
B4	High-per	Male	15	Senior	D4	Ordinary	Female	12	Senior

3.3 Research Instruments

Based on the conceptual framework, a structured interview designed to obtain qualitative and in-depth information. The structured interview was designed to gather indepth information by asking teachers questions focusing on how textbooks facilitated their teaching of mathematics and reason causes this effect. OEQ 1 Please use metaphors to discribe the role of textbooks for you. OEQ 2 Can you describe how you used textbooks in your last class? OEQ 3What causes this effect?

4 Result

4.1 The New Ideas in the Textbook Changed the Teacher's Vision

NCM provides teachers with a new vision. All the teachers mentioned that they learned by reading the textbooks. The textbooks enable teachers and learners to develop in harmony with the introduction of new ideas. A3 said," through the design of exploration activities and question string in NCM, I felt that it was very important for students to learn how to solve problems, which was more in line with the learning characteristics of students compared with the traditional lecturing learning."Having changed the teacher's mindset, the textbook provides the teacher with a complete picture of what change looks like and an actionable plan. Lay the foundation for subsequent behavior change.

4.2 Make an Impact Through Student Feedback

In addition to the influence of reading materials, teachers often mentioned the influence of students. On the one hand, the influence of students will help teachers to understand the textbooks on a deeper level, on the other hand, it will also promote teachers to use the textbooks creatively. B3 said, "there were math games in the textbook. I took the children to play that game after class, and I felt their enthusiasm was very high. And in this process, I have a deep impression of the content of this lesson. Therefore, I also feel the necessity of such activities as textbook design, and I may also need to design more math games in my classroom in the future."

4.3 The Factors Facilitate Teachers' Use of Textbooks

Through the analysis of the interviews, it is found that the promoting influence of NCM textbooks on teachers is produced under the influence of these factors.

4.4 The Influence of Internal Motivation

The active thinking of teachers themselves will drive teachers to understand the textbook and students, and obtain new educational teaching ideas and put them into practice. D2 gave an example, "In the past, I felt that this pattern designed by students was not good. So in recent teaching at this time, I realized that the teaching material of the second question is to see how a small unit of a good on a single integrated design. So, I thought, in fact, textbooks want to give us such an idea, that is, the beauty of mathematics."

4.5 The Influence of Personal Teaching Experience

The accumulation of teaching experience will influence the effect of textbooks. Among them, novice teachers and senior teachers in the novice stage all feel that the design of the textbook is redundant or too simple due to their insufficient understanding of the textbook or students. C1 said: "Sometimes when I read this textbook by myself, I will feel that the setting of this question is not necessary at the beginning. Or I think maybe it's too easy, I don't think this section is necessary. But two years later I found that I didn't understand the design intention of the textbook."

4.6 The Influence of Professional Communities

Almost all teachers mentioned the influence of research activities on their use of materials, including expert lectures, district-level guidance, interaction between different schools and regular school research activities. Different types of teaching and research activities have brought different influences to teachers. Among them, expert lectures play the most important role in guiding theories. District-level training undertakes more practical guidance of putting theories and concepts into practice. On the other hand, school-level training and in-school teaching and research are specific to each class, which is equivalent to a more detailed level than the district level, with more seminars on teaching methods.

5 Discussion

5.1 All Had an Impact but in Different Forms

In the questionnaire, teachers also chose textbooks to promote their influence, but through interviews, it was found that these influences were different. Novice teachers' description of the use of teaching materials focuses on the process of using teaching materials, which is a one-way effect of teaching materials on teachers. Senior teachers put emphasis on the interaction between learning and teaching materials, which is an interactive use of teaching materials. In the interview, six novice teachers mentioned in the process of describing the use of textbooks， "first look at the string of questions in the textbooks, and then think about why they are designed in this way. Look at the task hints and exercises, which are very helpful in the course of lesson preparation. I can quickly understand what the teaching points of this lesson are."They are all step-by-step instructions for using the textbook. And there are four senior teachers in the description of the use of the situation mentioned recall learning, to establish the relationship between learning and teaching materials. Therefore, teachers' own ideas and accumulation of teaching experience will lead to their different ways of using textbooks.

5.2 The Cooperation of Teaching Research and Teaching Materials

In the description of the teaching materials, teachers reveal that the teaching materials provide a leading direction for their own teaching, but also leave their own space to play. In order to give full play to the value of this space, the design and evaluation of teaching and research activities should be strengthened. Although the four types of teaching and research activities provide teachers with three different levels of guidance, it is inevitable that there will be omissions, and the theme of these activities is not unified planning and design, so there will be a tendency of fragmentation of teaching and research. Therefore, only by systematically designing teaching and research activities at the regional or school level and matching them with the concepts, ideas and methods of the teaching materials can the teaching materials play the greatest role.

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Symposia—Symposium 2: Designs and Practices of Problems-Based Middle School Mathematics Textbooks

Pursuing Mathematics Textbooks' Education—Orientation Through Building Up Systematic Problems-Based Designs: An Overall Introduction to the Ideas of Developing Mathematics Textbooks for Grade 7-9 Published by Beijing Normal University Press

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Abstract. Textbook development should be based on careful researches and will experience many iterated progressive revising cycles. After this time's revising cycle, Mathematics Textbooks for Grade 7-9 Published by Beijing Normal University Press (shorted as "BNUM (7-9)" hereafter) are "education-oriented" and have systematic "problems-based" designs for the seek of possessing sufficient educational values. This paper firstly explains the conception of "education-oriented textbooks" and its main features. Then, it illustrates the idea about "problems-based designs" and why do those designs work. Lastly, it clarifies design principles, structures and layouts, and characteristics of "problems-based" in BNUM (7-9).

1 Introduction

School textbooks have received increasing attention in the international research community of mathematics education over the last decades (Fan 2013). It is generally recognized that developing and using textbooks can serve as an important channel for promoting changes in teaching and learning mathematics (Ball and Cohen 1996). In order to avoid developing textbook only at an experience-based level, making it a scientific endeavour becomes more and more important. Therefore, textbook development should be based on careful researches.

In mainland China, the new curriculum reform that started at the beginning of 21 century has been carried out for over twenty years. Many changes related to "the instructional triangle" have taken place, especially after the implement of Mathematics Curriculum Standards of Nine-year Compulsory Education (2011 version). "Four Basics" and "Four Abilities" have been highlighted. Teaching focuses more on ability training and lifelong development than knowledge learning. Learning is usually exploratory or cooperative rather than mouth-to-ear-like. The interaction environment in the classroom is often student-centered instead of teacher-centered. Those changes have revealed necessities

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of studying the textbook development and some specific directions in textbook development period. Besides, the publishing policy of "one standard, many versions" has also promoted the advancements of studying textbook development and the improvements of published textbooks' quality.

Keeping pace with tendencies mentioned above, BNUM(7-9) go through their 20 years along with comprehensive studying and progressive revising many times. During these years, as writers and editors, we have carried out several iterated revising cycles of "researching-authoring-reviewing-experimenting-reviewing-revising" in order to make BNUM(7-9) reach contemporary educational demands and learning requirements better. In this time's revising cycle, based on the understanding of current mathematics education, after careful discussions and studies several times, we have elaborated "education-orientation" as the basic conception of BNUM(7-9) and thus built up systematic "problems-based" designs for the seek of possessing sufficient educational values with all volumes of BNUM(7-9).

2 Education-Oriented Mathematics Textbooks

2.1 The Conception of "Education-Oriented Textbooks"

Making textbook development a scientific research process is based on the premise of having a scientific view of "textbook". Thus, before we began this time's revising, we firstly determined what is the advanced conception of "textbook".

Generally, the view of the nature of "textbook" is depending on our basic perspective to it. If we regard what presented in textbooks as the purpose of the education, we will think that the contents in textbooks are supposed to be conformed to disciplinary logics and arranged in line with knowledge structures. Then, there will exist "knowledge-oriented textbooks". On the contrary, if we regard what presented in textbooks as the means of the education, we will think that the contents in textbooks are supposed to be conformed to psychological logics and arranged in line with sequences of experience generating process. Then, there will exist "experience-oriented textbooks".

In recent years, with the renewal of educational ideas, many researchers realize that there is a necessity of integrating those two views. We also hold this argument since we regard "learning to learn" both as purpose and means of educating. Starting from this standpoint, we have formed our conception of "education-oriented textbooks".

Based on our conception, we think that the nature of textbooks is learning materials. Their designs should pay equal attention to learning process and results and reconcile disciplinary logics and psychological logics at the same time. Beyond showing knowledge and their structures, their presentations should also show the processes of how knowledge develop and form by means of inspiring students to explore and learn through their own approaches.

2.2 Main Features of Education-Oriented Mathematics Textbooks

The nature of education-oriented mathematics textbooks are materials which support students' self-driven mathematics learning. They facilitate the occurrence and maintenance of exploring, communicating, and cooperating in the class. They offer diverse and sufficient opportunities for students to interact with mathematics deeply through mathematical thinking and behaviours.

Therefore, education-oriented mathematics textbooks are supposed to have at least five basic features:

- Present mathematics and its experiential learning process simultaneously.
- Advance formations of knowledge and their structures by using heuristic presenting approaches.
- Emphasize cultivating mathematical thinking and general competencies.
- Focus on extensive uses of mathematics through inspiring students to explore the real world.
- Support students in finding their own agencies and taking advantage of their creativities.

3 Pursuing Mathematics Textbooks' Education-Orientation Through Problems-Based Designs

3.1 What Is "Problems-Based Designs"?

Before developing textbooks, there is a question need to be answered first: "What are the chosen contents and their presenting modes in them?" According to the conception and main features of education-oriented mathematics textbooks, the developed textbook series must take specific considerations not only about content selection and mathematical treatments, but also many aspects of students' learning and cognitive development.

In terms of contents, this kind of textbooks should contain not only mathematics knowledge and their structures required by curriculum standard, but also tools of facilitating revealing their process of development. Besides, contexts which encourage multiple ways of thinking and materials which improve learning motivation are also important elements in them.

In terms of presenting modes, this kind of textbooks should offer enough opportunities for students to explore, communicate, conjecture, and prove. By using them, students can learn meaningfully through accomplishing challenging tasks. As a result, they will acquire required knowledge, integrate different knowledge, and develop their mathematical thinking gradually.

Based on the above ideas, we regard "problems-based" as the reasonable idea throughout all volumes of BNUM(7-9). "Problems-based designs" means that the textbooks are always centered around problem posing, analysing, and solving and often along with the reflections on those three aspects. In problem posing, they lead students generate learning needs and agencies. In problem analysing, they guide students explore the development process and learning approaches of knowledge. In problem solving, they help students perceive significances and appliances of knowledge. In reflections, they support students gain profound understanding and enhance mathematical thinking. (See in Fig. 1.)

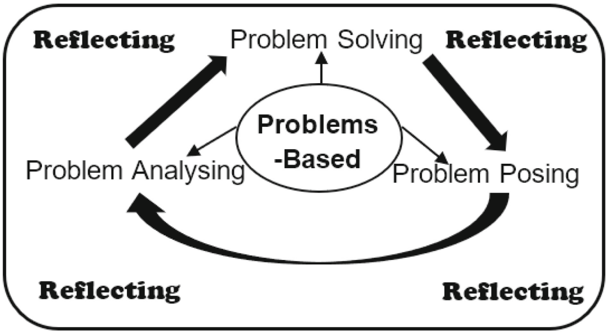

Fig. 1: Four Aspects of "Problems-Based" Idea

3.2 Why Problems-Based Designs Facilitate the Achievement of Mathematics Textbooks' Education-Orientation?

"Problems are the heart of mathematics." (Halmos 1980), the start point of applying mathematics in the real word is usually problem-like. Furthermore, for most people, the mathematics they encounter daily is also problem-like. Thus, what students need most from mathematics learning is adequate mathematics powers that help them solve problems whenever they face them.

Problems-based mathematics textbooks can support students to attain this kind of mathematics powers because of its six internal features:

- Advocate self-driven perceiving, building, and understanding to problems.
- Concern with processing forms of knowledge and students' well-around development.
- Organize contents in line with development process of practical experience.
- Facilitate active engagement of students' thinking in various ways.
- Raise students' problem consciousness and innovative thinking continuously.
- Reflect the development and learning significances of mathematics as cultural process.

It's not hard to see that the above features conform to the features of education-oriented mathematics textbooks. Therefore, designs centered around "problem-based" are conductive to realize "education-oriented".

4 Systematic Problems-Based Designs in bnum(7-9)

4.1 Design Principles of "Problems-Based" in BNUM(7-9)

Before this time's revising, we firstly formulated design principles of "problems-based". They played as basic directions of authoring and reviewing and thus enlightened writers' individual revising and discussing in community.

There are three core related questions we considered most when we determined those principles: (1) As a series of education-oriented mathematics textbooks, what are the stylistic rules of them? (2) How can we make "problem-based" more explicit in textbooks? (3) How can we make "problem-based" represent "education-oriented" better?

After progressive discussions, we reached our five principles:

- To arrange knowledge from different domains in flexible hybrid mode and arrange core contents, learning approaches, and inquiry strategies in spiral-up mode.
- To build up logical and hierarchical relationships of knowledge and relationships of types of methods and degree of difficulty of exploring.
- To explicate four aspects of "problems-based" idea by applying fixed patterns of presenting problems, analysis, and solutions.
- To problematize knowledge and their exploring processes and make problems constructive in order to emphasize "what to learn" and "how to learn" equally.
- To connect knowledge learning and their appliances through diversified problems with different kinds of contexts.

4.2 Structures and Layouts of "Problems-Based" in BNUM(7-9)

Structures of textbooks refer to organization modes in macro-grained level of all volumes in the series, including elements and their relationship of the contents. Layouts of textbooks refer to presentation patterns in medium-grained level, that is, the programs and segments in every chapter and lesson. In most cases, although chapters or lessons are different, they share same layouts.

In terms of structures, BNUM(7-9) have 6 volumes and each volume is used for one semester. Every volume contains algebra, geometry, statistics, and cross-domain contents. All volumes adopt hybrid mode for knowledge from different domains and adopt spiral-up mode for related topics in the same domain. Besides those explicit structures, there are some special implicit structures centered on "problems-based". Most of them are attached to the contents arranged by the explicit structure while some of them are presented as thematic lessons around solving one problem. These implicit structures are pointed to four aspects of "problems-based" idea respectively, methodizing practical experiences, and exploring problems creatively, etc.

In terms of layouts, BNUM(7-9) emphasize that the arrangements of programs and segments in chapters and lessons need to comply with possible cognitive sequence of students. Every chapter has 2 or 3 overall questions that can guide the learning of the whole chapter as the forward. Every lesson is composed of different inquiry-oriented contextual or mathematical questions and activities. After presenting all lessons of the chapter, there are a string of summative or reflective problems named "Summary and Reflection". For the problem-solving thematic lessons, BNUM(7-9) emphasize learning of basic steps, heuristic strategies, reminder questions, and meta-cognition methods of problem solving. They are all carried out through problem solving stages in Pólya's model and with summary of key points and main strategies.

4.3 Characteristics of "Problems-Based" in BNUM(7-9)

Based on the conception of "education-oriented" and aimed at realizing "problems-based", after this time's revising, BNUM(7-9) reveal some characteristics. This paper will only list them briefly as following while other papers in our symposium will illustrate them in detail.

- Set up proper exploratory problems to guide and connect students' processes and results of discovering, inquiring, and understanding of mathematics knowledge.

- Chose and arrange structures and layouts of the contents flexibly in the light of learning necessities and cognitive rules.
- Label explicitly of the teaching and learning about problem posing, analysing, solving and the reflections on those three aspects.
- Improve students' cognitive completeness of mathematics through the whole learning processes, including perceiving, understanding, applying, and structuring of the knowledge.
- Emphasize problems' heuristic role to students, and support students to obtain learning opportunities and develop mathematical thinking.
- Emphasize problems' demonstrative role to teachers, and facilitate teachers to promote communication in class and advance teaching approaches.

Textbook development is a process full of gradual improvements. Although we elaborated the orientation and ideas above and carried out related systematic designs in this time's revising, with further researches and extensive use of BNUM(7-9), there is still room for their better improvements.

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Problem Posing Tasks in the BNU Middle School Mathematics Textbooks in China

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Abstract. Problem posing tasks presented in textbook series are of great importance for teachers' daily teaching and students' learning. In this paper it is investigated the problem posing tasks in the textbook series published by Beijing Normal University Press (BNU). A coding framework was developed which includes four main categories. The amount of problem posing tasks for each grade, content stands and problems types were reported. And three concrete examples of problems posing tasks regarding function were illustrated. In conclusion, various learning opportunities were provided in BNU textbook series in order to promote students' ability of problem posing.

1 Background

Over the past several decades, problem posing has been received increasing attention at the level of intended curriculum in many countries (Cai et al, 2016). In China, evidence was found by analysing the mathematics curriculum standards over 60 years that the importance of problem posing has gradually increased (Chen, Xie & Cai, 2021). According to Mathematics curriculum standards of nine-year compulsory education (MoE, 2022), one of the curriculum objects refers to enhancing students' ability of using mathematics knowledge and methodology to identify, pose, analyse and solve problems.

Textbooks, as the implemented curriculum, serve as a bridge between national mathematics curriculum standards (syllabus) and daily teaching and learning at classroom level. For Chinese teachers in particular, textbooks are their main resource for day to day instruction (Li, Chen, & Kulm, 2009) and they pay much attention to study and understand textbooks carefully and thoroughly (Cai & Wang, 2010). Although textbooks are generally designed in alignment with national mathematics curriculum standards in which problem posing is heavily emphasized, gaps has been identified in terms of small proportion of problem posing tasks integrated in textbooks (Guo, Yan & Zhou, 2022). Therefore, it is necessary look closely into individual textbook series in order to illustrate what opportunities textbooks have been provided for teachers and students to put posing problem into practice. In the current study, we focus on the mathematics textbooks of Grade 7–9 published by Beijing Normal University Press (Ma, 2013), or BNU textbook series for short. This textbook series are widely used in China and characterised by their design of problem strings. The research questions are (1) *what types of problem posing tasks are included in the BNU textbook series*, and (2) *what types of learning opportunities offered by such activities?*

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2 Literature Review

A problem-posing task is an activity which requires students to generate new problems based either on a given situation or on a given mathematical expression or diagram (Cai & Jiang, 2017). Problem posing can occur before, during, or after the solution of a problem, including both the generation of new problems and the reformulation or variation of given problems (Silver, 1994). An increasing number of researches have paid attention to the presentation of problem-posing tasks in mathematics textbooks (Cai & Jiang, 2017; Hu et al., 2015; Jia & Yao, 2021; Xu & Cai, 2019). These studies are in fact related when considering their (co)authors' cooperation, and they shared the same coding framework or part of it. For example, Cai, Jiang, Hwang, Nie, and Hu (2016) examined how mathematical problem posing is integrated in Chinese and US elementary mathematics textbooks regarding grade level, content area, task types and representations of given information. By focusing on whether students need to provide information as givens and whether there is sample questions provided, four problem-posing types were distinguished: (1) posing a problem that matches the given arithmetic operation(s), (2) posing variations of a question with similar mathematical relationship or structure, (3) posing additional questions based on the given information and a sample question, (4) posing questions based on given information, and (5) unconstrained problem posing tasks. In later studies (Cai & Jiang, 2017; Jia & Yao, 2021), the first four types were used. The studies mentioned above, including their coding framework, are served as the basis for our study. However, as the focus of the current study is on BNU textbook series used by middle school students, the coding framework was further adapted based on the existing problem-posing task in the textbooks by.

3 Method

By constant comparison analysis, the coding framework was developed to identify problem-posing tasks. According to whether there is existing sample question(s) and specific context information, four main categories were distinguished (Table 1). Firstly, when there are specific context information and sample problem(s) provided, students are asked to provide related but varied problems; secondly, when there is only context information, students are asked to come up with problems; thirdly, students are expected to give the matched context according to the given arithmetic operation, equation(s) and function(s); fourthly, there is only some general information given in the textbook, students need to come up with both specific context and targeted problems. Compared to the existing coding framework, this crosstab is more practical when deciding the types of problem-posing tasks. Furthermore, we focused on specific mathematical topics and further examine what opportunities the textbook series offered in order to improve students' ability of posing problems over time.

Table 1: The main category of coding framework

		If sample question(s) provided	
		Yes	No
If specific context information provided	Yes	Generating varied problem	Generating specific problem
	No	Generating matched context	Generating both specific context and problem

4 Results

4.1 Amount of Problem-Posing Tasks

As show in Table 2, there are 453 (12.68%) problem posing tasks in total in the BNU textbook series, and nearly half of them were distributed in Grade 7, the beginning phase of students' middle school education.

Table 2: The amount of problem-posing tasks in Grade 7, 8 and 9

	Grade 7	Grade 8	Grade 9	Total
All tasks	1240	1323	1010	3573
Problem posing tasks	213 (17.18%)	132 (9.98%)	108 (10.69%)	453 (12.68%)

When zooming in where these problem-posing tasks were arranged exactly, it was found that most of them were identified in the the section of "Think", "Practice" and "Discussion", which is followed by the section of introduction of the mathematics topics and exercises done in class. Only 6 (1.32%) were in the example problem illustrated in detail. According to Mathematics curriculum standards of nine-year compulsory education, four strands were distinguished from the content perspective. About half problem posing tasks are related to the content strand of shape and geometry, one third regarding number and algebra, 55 (12.14%) activities are about statistics and probability and only 18(3.97%) are comprehensive practice.

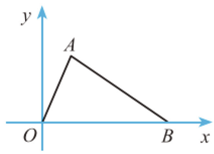
4.2 Types of Problem-Posing Tasks

In the BNU textbook series, there are 59 (13.02%) tasks asking for *Generating varied problem*, 132 (29.14%) tasks asking for *Generating matched context*, 178 (39.29%) tasks asking for *Generating specific problem*, and 84 (18.54%) tasks asking for *Generating both specific context and problem.*

For *Generating varied problem*, we take a problem posing task in the chapter of liner function for example (Fig. 1). It is followed by an example of *Generating both.*

specific context and problem in the chapter of inverse proportional function (Fig. 2). In the end, a problem posing task of *Generating varied problem* regarding quadratic function is illustrated (Fig. 3).

10.（1）右图可以用来反映这样一个实际情境：一艘船从甲地航行到乙地，到达乙地后旋即返回．这里横坐标表示航行的时间，纵坐标表示船只与甲地的距离．你认为，船只从甲地到乙地航行的速度与返航的速度是否相同？说说你的理由．



（第 10 题）

（2）请再给该图赋予一个实际背景，提出一个具体的问题．指出实际背景中横坐标、纵坐标所表示的意思，写出 A，B 两点的坐标，并解决你所提出的实际问题．

Fig. 1. Example of Generating varied problem (requirements for analogous problems) regarding liner function. *Note.* For English translation: (1) The graph can be used to represent such a real context: A ship travels from city A to city B, and it will return immediately upon arrival. The horizontal axis of the coordinate system represents time and the vertical axis represents the distance between the ship and City A. Were the speed of the ship before and after reaching city the same? Provide your explanation. (2) Try to find another real world context according to the given graph, and pose a concrete problem. Make clear the meanings of the horizontal and vertical axis of the coordinate system and the coordinates of point A and B. Solve the problems you proposed in the end.

1. 你能举出现实生活中有关反比例函数的几个实例吗？

Fig. 2. Example of Generating both specific context and problem (without specific requirements) regarding inverse proportional function. *Note.* For English translation: Can you provide several real life examples regarding inverse proportional function?

数学理解

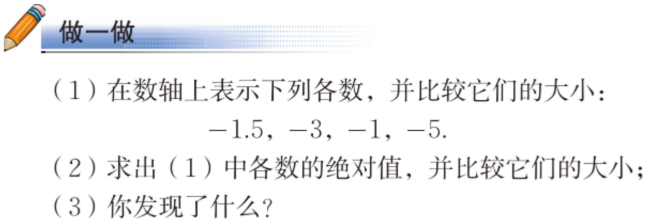
3．如图，题目中的灰色部分是被墨水污染了无法辨认的文字．请你根据已有信息添加一个适当的条件，把原题补充完整并求解．

已知二次函数 $y = ax^2 + bx + c$ 的图象经过点 A（0, a），B（1, −2），．求该二次函数的表达式．

Fig. 3. Example of Generating varied problem (requirements for changing given conditions) regarding quadratic function. *Note.* For English translation: The ink stain makes the texts unrecognizable. Add an appropriate condition to make the problem complete, and then solve it. Given that the quadratic function $y = ax^2 + bx + c$ passes through the points A (0, a) and B (1, -2), find the expression of the quadratic function.

4.3 Learning Opportunities Offered by Problem-Posing Tasks

Regarding learning opportunities offered by the problem-posing tasks in the textbooks, it was found that the textbook developers tried to support students' problem posing behaviour through three different levels. Firstly, students are invited to make conjecture according to given information. For example, in the textbook used in the first semester in Grade 7, students are asked to compare several numbers, that is, -1.5, -3, -1 and -5 by using number line; then they need to compare these numbers' absolute value. With experience of solving such concrete problems, students are invited to make a conjecture by asking "What do you find?" (Fig. 4).


做一做

（1）在数轴上表示下列各数，并比较它们的大小：

$-1.5,\ -3,\ -1,\ -5.$

（2）求出（1）中各数的绝对值，并比较它们的大小；

（3）你发现了什么？

Fig. 4. Examples of learning opportunity regarding make an conjecture.

For the second level, students are encouraged to pose concrete problems and solve them as well. For example, it was given in the textbook that Students in Grade 7 went to excursion. Team A left earlier with the speed of 4 km/h, while Team B left one hour later with the speed of 6 km/h. There is a student travel between these two teams with the speed of 12 km/h for information transfer. Within such open-ended context, students were asked to pose problems and solve them accordingly.

After students have accumulated some experience of posing problems, for the third level, students are guided to reflect the process of solving a certain problem, and then come up with problems related to be further explored. Such opportunities are often given in the content stands of comprehensive practice, with the aim of helping students to developing strategies of posing problems like using generalization and analogy.

5 Conclusions and Discussion

Although the amount of problem posing tasks in BNU textbook series takes only a small proportion, which is similar to what has been found in other studies; various types of problem posing tasks were identified in the textbooks for each grade and three different level of learning opportunities were provide in order to promote students' sense and abilities of problem posing step by step. It seems that the BNU textbook series attached greater importance to enhance students' ability to pose problems and choose context and hints carefully in order to invite students at different achievement levels to pose problems and gain strategies of posing problems at the same time.

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Understanding the Feature of Geometrical Graphics in Thinking and Answering Reasoning Questions

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1 Introduction

In recent years, the importance of geometry in cross-domain applications has become prominent. This is not only because geometry is used in a wide variety of fields, such as from robotics to computer graphics for film, from crystallography to architecture, from neuroscience to the nature of our universe, but because new geometric concepts are emerging in different fields (Malkevitch 2009; Whiteley 1999).

These phenomena pose a problem for curriculum designers: what geometry courses should be selected and what should be omitted from school mathematics courses (Jones 2000). Reflecting on geometry textbooks, we will find that the quality of textbooks still needs to be improved. Some textbooks focus primarily on the names and properties of shapes, followed closely by specific formats for giving geometric proofs (Brantlinger 2011).

This tendency was also very serious in geometry courses and textbooks before the reform of the Chinese curriculum in the early 21st century (Liu Xiaomei 2002).

The "Mathematics Curriculum Standards for Compulsory Education (Experimental Draft)" promulgated/promegeit/ in 2001 proposed to develop students' reasoning ability including plausible reasoning and deductive reasoning, and put forward the goal of developing students' "spatial concept". The compilation of teaching materials also changed the "geometry = proof" situation. In the "Mathematics Curriculum Standards for Compulsory Education (2011 Edition)" (referred to as "Standards [2011]"), the content in "Graphics and Geometry" in the third semester is divided into "nature of graphs", "changes of graphs", and "graphs and geometry". Based on learning these contents to realize learning objectives of "explore and master the basic properties and judgments of intersecting lines, parallel lines, triangles, quadrilaterals and circles, master basic proof methods and basic drawing skills; explore and understand translation, rotation, and axisymmetry of planar figures; recognize projections and views; explore and understand planar Cartesian coordinate systems and their applications", so as to develop students' reasoning ability, spatial concept and geometric intuition. How does the problem-based textbook design concept lead students to learn the course content through questions in geometry field? In particular, how to break the validity of "geometry = proof" with a focus on reasoning ability? What percentage of questions lead students' reasoning?

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2 Research Framework

The textbook is task-driven by problem scenarios, and inspires students to "actively participate in positive thinking, and form practical innovation ability and problem-solving ability in activities such as comparison, analysis, synthesis, classification, sorting, and abstraction" (Wang Panfeng and Song Yaqin 2018).

As materials for teachers to teach and students to learn, textbooks are designed with various types of questions.

Based on cognitive process dimension of Bloom's taxonomy of educational objectives, Shen Jiliang et al., Yu Guowen et al., Ma Yuhui et al. divided the types of questions into recall, understanding, application, evaluation, analysis, comprehensive,etc.from the perspective of teachers' classroom questions; according to the function of questions, Xu Fenying and others divided mathematical thinking problems into four categories: connection, comprehension, expansion and induction;from the perspective of learning opportunities provided by questions, Wang Rong divided the questions in the textbook into two types of opportunities: recognizing, understanding, abstraction and generalization, inquiry and discovery, reflection and induction.

There are also many studies focusing on the cultivation of teachers' problem-based teaching and students' ability to problem raising, or the questions that have certain functions, while studies systematically classifying all the questions in textbooks are limited.

Based on the above analysis, all the problems in the textbook are classified into types. An analysis framework is formed, and corresponding researches are carried out on this basis. These types of problems are particularly necessary to better understand and use of the content of the textbook.

Type	Question connotation
Memory Question (MQ)	Enable students to retrieve or extract relevant information from long-term memory through questions, including recognition and recall of knowledge
Comprehension Question (CQ)	Enable students to understand the meaning of the concepts learned and related materials, in order to promote students' self-construction of knowledge through questions
Judgmental Question (JQ)	Students are required to make reasonable judgments on issues based on existing facts or conclusions through questions, including intuitive judgments and judgments that have been reasoned (1-step reasoning or multi-step reasoning)
Abstraction Question (AbQ)	Students are required to generalize to slightly general conclusions through reasoning in specific facts or situations
Applicability Question (ApQ)	Students are required to establish connections between old and new knowledge through questions, and enable them to use knowledge to solve new questions, including questions with practical background and application of theorem proving questions

(*continued*)

(*continued*)

Type	Question connotation
Extensive Question (ExQ)	Students are required to think on a larger scale through questions or extend questions on the basis of existing problems
Evaluative Question (EvQ)	Enable students to make informed evaluations of existing reasoning processes through questions

3 Framework-Based Analysis

3.1 Statistics on Geometric Part Questions

In the Beijing Normal University version of the textbook, the course content involving the understanding of the nature of two-dimensional graphics mainly includes: intersecting lines and parallel lines, congruence of triangles in the seventh grade, Pythagorean theorem, proof of parallel lines, proof of a triangle, parallelograms in the eighth grade, special parallelogram, graphic similarity in the ninth grade, eight units (chapters) in total. The types and quantities of questions in each chapter are as follows:

	Types of question		MQ	CQ	JQ		AbQ	ApQ		ExQ	EvQ
					Intuitive	Reasoned		Practical contexts	Theorem proving applications		
Chapter grade	Grade7	Intersecting line and Parallel lines	1	13	13	9	3	5	0	0	1
		Triangle	2	7	18	21	12	1	1	0	1
	Grade8	Pythagorean Theorem		2	3	5	1	6	0	1	0
		Proof of parallel lines	1	1	4	12	3	0	4	2	2
		Proof of a triangle	4	9	6	12	4	0	3	1	2
		Parallelogram	1	3	9	4	4	3	3	4	2
	Grade9	Special parallelogram	2	4	9	15	5	3	0	3	2
		Graphic similarity	1	3	7	22	7	6	0	3	1
		Total	12	42	69	100	39	24	11	14	11
		Proportion (%)	3.73	13.04	21.43	31.06	12.11	7.45	3.42	4.35	3.42

3.2 Analysis Based on Statistics

Data analysis are as follows:

Among the seven types of questions above, the latter five are all related to reasoning, in addition to MQ and CQ,, accounting for 52.49%, 12.11%, 10.87%, 4.35% and 3.42% respectively, Among them, the three parts of JQ, AbQ and application theorems in ApQ are more closely related to reasoning,They are all questions that require making judgments or drawing conclusions based on certain reasons..

In terms of quantity, the number of JQ is the largest, followed by AbQ and ApQ. It can be seen that the number of three types of questions closely related to reasoning is also the largest, which runs through the learning process of junior high school students.

(1) Deductive and Inductive Reasoning Complement Each Other

Judgmental questions are classified into two types. One is based on observed phenomena. Which can be called intuitive reasoning,such as “On the night of the Lantern Festival, the beams of the house are lit up with colored lights. Which wire is longer, the one with yellow colored lights or the one with red colored lights?” (From textbook of the seventh grade). One is a process that draws new conclusions based on existing conclusions, such as “If two lines are intercepted by a third line, if the congruent angles are equal, are the interior angles equal? Do congruent interior angles complement each other?” (From textbook of the seventh grade). This type of reasoning runs throughout, and the purpose is to accompany the learning of knowledge, that is, the understanding of the nature of graphics, and cultivate students’ awareness and ability to reason, because this is the basic element of students’ deductive reasoning.

The reasoning of abstract problems from a special situation to a wider range of conclusions is mainly inductive reasoning, which is a prominent manifestation of the “reasoning ability” proposed by the Mathematics Curriculum Reform, including deductive reasoning and plausible reasoning in teaching materials. For example, “Draw a number of right-angled triangles on paper, measure their three sides, and see what is the relationship between the squares of the lengths of the three sides?”(From textbook of the eighth grade).

(2) Focus on the cultivation of reasoning awareness

The geometry section of the textbook focuses on cultivating students’ sense of reasoning. With the help of judgmental questions (through reasoning), students try to present their thinking process in a mathematical language in a realistic way. While giving evidence, they can fully mobilize their mathematical thinking, so as to promote students’ flexible response to what they have learned.

(3) Emphasis on students’ reasoning in problem solving

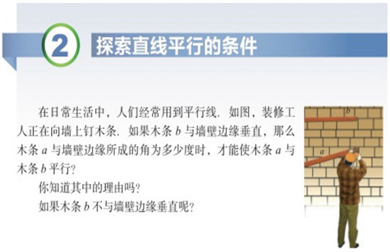
Under the new curriculum reform, mathematics teaching attaches great importance to the cultivation of students’ reasoning ablity. In the process of problem solving, plausible reasoning is used to explore ideas and discover conclusions, and deductive reasoning is used to verify conclusions. Many questions in textbooks like applied questions mainly include questions with practical background and questions proved by applying theorems.

On the one hand, focusing on stimulating students' interest in learning through situations, guiding them to use life experience to fully think and reason about the questions, and developing plausible reasoning ability. For example, "How can you check if the door frame you just installed in your house (or classroom) is rectangular?" (From textbook of the ninth grade). On the other hand, students are guided to use existing knowledge to reason and prove questions and cultivate their deductive reasoning ability. For example, "According to the basic facts and theorems given earlier, can you explain the proof idea of this conclusion in your own words?" (From textbook of the eighth grade).

3.3 Examples of Question Design for Reasoning Development

Logical reasoning based on plausible reasoning

The parallel lines are often used in everyday life. As shown in the figure, a decorator is nailing a strip of wood to a wall. If batten b is perpendicular to the edge of the wall, what is the angle between batten and the edge of the wall in degrees that will make batten a parallel to batten b?


2 探索直线平行的条件

在日常生活中，人们经常用到平行线．如图，装修工人正在向墙上钉木条．如果木条 b 与墙壁边缘垂直，那么木条 a 与墙壁边缘所成的角为多少度时，才能使木条 a 与木条 b 平行？

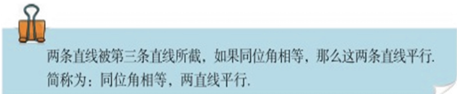
你知道其中的理由吗？

如果木条 b 不与墙壁边缘垂直呢？

Do you know the reason?

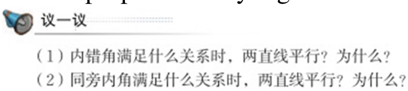
What if batten b is not perpendicular to the edge of the wall?

Two lines are intercepted by a third line. If the congruent angles are equal, then the two lines are parallel. Simplified: If congruent angles are equal, the two lines are parallel.


两条直线被第三条直线所截，如果同位角相等，那么这两条直线平行．

简称为：同位角相等，两直线平行．

One of the judging conditions for obtaining parallel lines by plausible reasoning (this proposition is also one of the basic facts of the subsequent deductive proof, i.e., an axiom). On this basis, the textbook asks the following questions to guide students to obtain new propositions by logical reasoning on the basis of existing propositions.


议一议

（1）内错角满足什么关系时，两直线平行？为什么？

（2）同旁内角满足什么关系时，两直线平行？为什么？

Discussion:

(1) What is the relationship between the interior angles when the two lines are parallel? Why?
(2) What are the relationships between same-side interior angles when the two lines are parallel? Why?

Such questions are posed and arranged in the second semester of 7th grade to better guide students to logically draw new propositions on the basis of existing conclusions.

Thinking:

Two lines are intercepted by a 3rd line. If the corresponding angles are equal, are the alternate interior angles equal? Are the same-side interior angles complement?

想一想

两条直线被第三条直线所截，如果同位角相等，那么内错角相等吗？同旁内角互补吗？

4 Conclusion

4.1 Diverse Forms of Reasoning

As the table shows, reasoning questions are widely distributed in textbooks and have various forms, mainly including judgmental questions that involve one-step and multi-step reasoning, abstract questions that involve reasoning, and applied questions that reason in the actual context. Depending on the purpose of the question, different forms are given to reasoning, so as to promote students' understanding of the problem and the improvement of reasoning ability.

4.2 Grade Difference in Reasoning Question

After entering junior high school, students' reasoning ability needs to be gradually cultivated. Intuitive reasoning with the help of real objects or graphics is the beginning of the cultivation of reasoning consciousness and ability. However, with the increase of grades, the proportion of reasoning and judgment based on existing facts gradually increases. So in the textbook, there are gradually fewer JQ by means of intuition than by means of reasoning.

The acquisition of geometric concepts is mostly abstracted from concrete objects in the real world. Therefore, AQ are fully reflected in Grade 7. Generally speaking, the proportion of AQ in the three grades is basically the same; although the proportion of ApQ, ExQ and EvQ all have the highest proportions in Grade 8, if the number of geometric units in Grade 8 is considered, the number of questions in these three aspects is basically balanced.

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From Big Ideas to Big Questions: Practices and Reflections on Teaching Units from a Competency Perspective

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1 Big Ideas and Teaching

With the introduction of "Core Competency for Student Development in China", Chinese education has officially entered the era of "competency." Although the curriculum standards have described the core competencies in different subjects to different degrees, there is still a large gap between them and specific subject knowledge, which is not enough to provide classroom teachers with the directions to implement the core competencies in their teaching practice. To make core competencies come into practice, it is necessary to find bridges that effectively link competency and learning content.

This bridge is the "big idea."

2 The Big Idea

In the field of curriculum and instruction, the idea of the broad concept of discipline big ideas can be traced back to the pedagogy advocated by J.S. Bruner.[1] D.P. Ausubel was already using this term when he mentions the "prior organizers" of the curriculum in 1963. In his 1981 book Evaluation to Improve Learning, Benjamin S. Bloom, the famous American educational psychologist known for his Taxonomy of Education, suggested that a crucial part of education is for scholars and teachers to take abstract concepts of a subject area and help students apply them to various contexts.

In recent years, some scholars have explored the big idea in more depth, and the representative views are as follows.

M. Whiteley (2012) emphasizes that big ideas are the building blocks of understanding and can be thought of as meaningful patterns for establishing connections among fragmented knowledge.[2] Edward Clark (1997) argues that big ideas provide the cognitive framework or structure for building learners' understanding.[3] Olson (2008) states

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that big ideas is the "take home message" which is the central concept that remains long after concrete experiences and facts have been forgotten.[4] H.L. Erickson adds that big ideas are abstract generalizations; they are deep, transferable ideas that arise on the basis of facts; they describe relationships between concepts. Big ideas have the characteristics of generality, abstraction, timelessness, and universality.[5] G. Wiggins & J. McTighe give a more systematic account of big ideas. They argue that big ideas are concepts, themes, or issues that give meaning and connection to individual facts and skills, and can be thought of as meaningful patterns that allow us to connect the knowledge acquired. It goes beyond individual knowledge and skills and can be applied to new contexts within or beyond the discipline. They liken the big idea to the bolts of a car, without which the wheels and other parts will be scattered and useless.[6]

There is also a strong emphasis on big ideas in mathematics education research.

In 1993, mathematics education experts Catherine Fosnot and Deborah Schifter co-authored their book Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenge of Reform. In it, they define the big idea from the perspective of the mathematics discipline: the big idea is the discipline-centered organizing views of mathematics, the principles that define the order of mathematics.

Since then, Catherine Fosnot has elaborated on the big idea in greater depth: Big ideas are something closely linked to the core structure of the mathematical discipline and are also indicators of changes in learners' thinking - showing changes in thinking perspectives, logic, and in establishing mathematical relationships. The development of mathematical big ideas has brought progress across cultures over the centuries - characterized by a paradigm shift in thinking. This is because structural changes in thinking are often a feature of the learning processes. Thus, big ideas are "big" because they are themselves the most critical ideas in mathematics, but also because they facilitate great leaps in the development of the structure of children's thinking.[7]

The American Association for the Advancement of Science (AAAS) draws on the definition and formulation of the big idea in Principles and Standards in the Mathematics Curriculum (2000), published by the National Association of Teachers of Mathematics, to suggest that the big idea is "the core of scientific learning that links many pieces of scientific knowledge into a coherent whole."[8]

In 2005, Randall I. Charles, a tenured professor at San Jose State University in California, published a 16-page paper in NCSM in which he comprehensively discusses the definition of the big idea, its meaning, and the areas in which it can be applied. He states that a big idea is an idea or statement that lies at the heart of the discipline of mathematics and that links many understandings of mathematics into a coherent whole.

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[4] Olson, J.K. Concept-Focused Teaching: Using Big Ideass to Guide Instruction in Science [J].Science anar Children, 2008: 45.

[5] Sheng H. Big Idea and Curriculum Construction based on Big Idea [J]. Contemporary Education Sciences,2015,18:27—31.

[6] Wang X. The connotation, Significance and Acquisition Approach of the "Big Concept" of discipline [J]. Teaching and management, 2018,8:86—88.

[7] https://davidwees.com/content/teaching-to-big-ideas/

[8] National Council of Teachers of Mathematics. Principles and Standards for School Mathematics [M]. Reston, VA: NCTM. 2000: 17.

Charles proposes that a big idea should have three characteristics: first, it should be a point of view or statement; second, it must be central to learning mathematics; and finally, it must be connective, able to link many detailed understandings into a whole.

In discussing the importance of big ideas, Charles cites Hiebert's 1992 observation that "new knowledge is truly understood when we make connections between the new and the known. And the number and strength of the connections between old and new knowledge determines the depth of our understanding." Charles argues that big ideas are extremely important precisely because of their strong connective character and their ability to help us better make connections so that we can understand the discipline as a whole.

At the same time, Charles outlined 21 big ideas for teaching and learning mathematics in primary and secondary schools, and explained in detail the specifics of these ideas. This list of 21 ideas were cited by the Department of Mathematics at the University of Cambridge in 2015, to which 2 big ideas were added, succeeding to 23 big ideas that more comprehensively summarize the mathematics curriculum in primary and secondary schools (Table 1).

In summary, big ideas are not detailed knowledge in a discipline, but a overarching disciplinary view that reflects the essence of the discipline. Big ideas are relatively stable consensus.

For example, the statement "the introduction of symbols makes mathematics more universal" can be called a big idea in mathematics, which is both a concrete expression of the core competency of "abstraction" and a simplification of Algebraic Equations. It is also a simplifying of the content of Algebra.

It can be seen that big ideas are actually the further interpretation and decomposition of relatively abstract core competencies based on concrete knowledge. Thus, learners can aim to understand and internalize these big ideas, and gradually achieve the core competency by picking up the steps. At the same time, the big ideas are like "anchors" wedged into each part of the learning, steadily fixing the learning of subject knowledge in the direction of competency. It can be said that the big idea plays the role of both knowledge organizer and competency transmitter, and is the "messenger" linking knowledge and competency (Fig. 1).

3 The Importance of Units in Teaching

In 1931, the American educator H.C. Morrison proposed his approach to unit planning in his book The Practice of Secondary School Teaching.

In traditional classroom teaching, the focus is often on knowledge gained from individual lessons. However, the formation of disciplinary core competency is obviously different from the acquisition of knowledge, which is not marked by memorization, but by the influence on students' way of thinking and behavior. This requires students to take a more macro view of the subject matter they are learning, and teachers need to lead students to stand above the knowledge and change their perspective so that they can feel the essence of the subject matter behind the knowledge.

If the classroom is artificially fragmented by one and another 'lesson' instead of a longer teaching unit, students will be stuck with specific knowledge, and it will be difficult to form a larger and more comprehensive view due to the lack of any macroscopic connectedness.

Taking on a holistic approach to teaching, a unit is a well-designed and arranged teaching activity based on certain objectives and topics, as opposed to the simple transfer of knowledge and arrangement of skills training in most cases of traditional classroom teaching. Of course, the unit can be either a unit in the textbook or a reorganization of content based on certain topics or objectives.

As understood, big ideas are relatively stable, reflecting the essence of the subject, and having a unifying view of the subject. So, teaching based on these big ideas cannot follow traditional methodology focused on individual lessons, but must adopt these bid ideas as organizing factors in unit planning as a whole. This is done with the goal that all the scattered knowledge will eventually be brought together creating a holistic understanding of the content from the students. Structuring teaching into these units also makes knowledge more transferable and can establish certain connections between the present learning and practical life, which is helpful for students to better connect what they learn with their life and future, enhancing their motivation to learn.

This shows that using units in teaching is not only a technical endeavor, but one that reflects the ideas of core competencies. To implementation of core competancies is not possible without the use of big ideas.

4 From Big Ideas to Big Questions

4.1 Translating Big Ideas into Big Questions

The big ideas are so critical, yet one of the problems we face is that they are highly specialized and condensed, which are understandable only for teachers or experts and can hardly be presented directly to students. Because the understanding of big ideas requires comprehensive understanding the overall unit. Without continuous guidance and stimulation, students can easily get caught up in the trivial skills and knowledge, resulting in a neglect of the big ideas.

Therefore, to enable students to understand big ideas, designing "big questions" around the big ideas are required.

For example, we can transform the big idea "introducing symbols makes mathematics more universal" into the question: "Why are letters used in mathematics?" Students can be guided by such questions to consciously reflect on what they have learned, form their own opinions, and eventually gain insight and understanding of the big ideas.

Thus, the big questions are in a sense, "translations" of the big ideas at the student level, and the big ideas and the big questions are two sides of the same coin. Big ideas are the one side for teachers, while big questions are the other side for students.

4.2 The Basic Characteristics of the Big Question

This all now begs the question, what kind of question can be classified as a "big question."

First, as a "translation" of big ideas at the student level, it should be highly consistent with the big idea. In other words, the ideal big question should eventually lead to the big idea (although in most cases it is difficult for students to reach it), i.e., the big question is actually a kind of questioning of the big idea itself.

Second, the big question should be a question without a strict answer and one which students' understanding of will deepen as they continue to learn. In the process of learning, the big question needs to be asked repeatedly, naturally and at the right time, so that students are continually stimulated to think and answer from different perspectives.

Again, the big question should bring out some other important questions and cause students to reflect continuously on their prior experiences; it is central and overarching. When students try to answer this question, they tend to think of other questions related to it, and then continuously mobilize their own prior experiences to complete the reconstruction of their thinking.

Finally, unlike the highly specialized nature of big ideas, big questions should be relatively easy to understand and students should be able to comprehend their meaning without much guidance from the teacher (Fig. 2).

We can look at the following two sets of examples.

- Big Idea: Number field expansion is the inevitable product of arithmetic pursuit of completeness and closure.
- Big Question: Why do we keep expanding the range of numbers?
- Big Idea: Studying variance in graphical transformations is an important way to understand graphs.

Big question: How can we understand change and variance in the changing. Motion of a graph?

5 Case Study

How can we design the unit teaching activities through driving big questions led by big ideas? The following is an example of a unit on factorization. We will discuss the designing process of the unit.

5.1 Come up With a Big Idea

Decomposition is an important way of thinking that people use in the process of understanding and describing the world. When recognizing a large whole or completing a complex thing, it is usually decomposed into several parts to make it simple and approachable. Function differentiation and spatial decomposition in mathematics also uses this way of thinking, a specific embodiment of the idea of reduction.

This way of thinking based on factorization also plays an important role in the process of students' mathematical learning. Factoring is not only the inheritance and development of elementary school factorization, but also an important foundation for the subsequent learning of operations of fractions, solving quadratic equations, etc. It also carries the function of cultivating students' core competencies in areas like mathematical abstraction and mathematical operations. In teaching principles of factorization, the previous methodologies focused more on the training of methods and skills, but neglected the consideration of the purpose of these princples. In fact, the same mathematical object expressed in different forms provides us with different information and perspectives, and different information and perspectives are useful for solving specific problems. *The product* is an important mathematical structure, both in simple mathematical objects such as numbers and algebraic equations, and in more complex mathematical objects such as vectors and matrices. Students will only understand the value of factorization when they realize that the product is different from other mathematical forms.

It is based on the above thinking that we set the big idea of the unit as the following: "**Different forms of the same number or equation can provide different information and perspectives.**" Such a big idea may make students appreciate what information the different forms of a polynomial before and after factorization can provide to us respectively, and then realize the meaning of factorization in comparison. The teaching of big idea focuses more on the factorization operation than on the different forms of formulas before and after factorization. Enable students to understand the significance of factorization in comparison.

5.2 Transform Big Idea into a Big Question

In order for students to reach an understanding of the big ideas, we designed corresponding big questions.

"What are the advantages and disadvantages of expressing the same equation in both 'sum' and 'product' forms?"

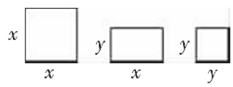
This question replaces "information and perspective" in the big idea with "advantages and disadvantages of **'sum' and 'product'**", allowing students to have a clearer entry point for their thinking. This allows students to effectively integrate their prior learning experiences with what they have just learned about multiplying whole numbers. In arguing for these strengths and weaknesses, it is natural to start with the different forms that will provide different information and perspectives, which helps the understanding of big idea.

In addition, "strengths and weaknesses" are in some sense individual subjective perceptions, and students need to argue their points from different perspectives combined with different examples when answering the question, which makes the question highly open-ended. At the same time, in order to gain perspectives on argumentation and accumulate examples of arguments, students need to pay high attention to what they learn in each lesson of the unit in order to keep gaining new experiences, which plays an important role in promoting learning and understanding throughout the unit. Finally, the question is simple and straightforward, and students will be able to rely on prior experience to make certain responses even at the beginning of the unit.

5.3 Unit Task Design

To help students better think about and answer the big questions, we also designed "big tasks" and several "subtasks" for the unit. When designing the unit tasks, you can use common materials in textbooks to create mathematical problems that include certain real-world situations to stimulate students' interest in completing the tasks and to gain thinking about the big questions in the process.

For example, the problem of interpreting quadratic polynomials using the area of a graph has appeared in different versions of textbooks. Using this material, we introduce the context of "codes" and design a learning task based on the following rules.

> In cryptography, letters are often corresponding to numbers, and the numbers are encrypted to convey important information: for example, the 26 letters correspond to the numbers 1~26 in sequence, **if it is agreed to use a number of pieces of paper A, B and C to put together a square to convey the information.**
>
> (1) Please design a scheme that shows how to convey the letter *l*.
>
> (2) In the scheme you designed, how many sheets of A, B, and C are needed to convey the letter *w*?
>
> (3) In the scheme you designed, if you use 4 sheets of A, 20 sheets of B and 25 sheets of C, which letter's message can be conveyed?
>
> 
>

In the above example, ingeniously integrated into the background of the password, this interesting setting fully mobilized the students' enthusiasm for learning, and stimulated the students' interest in observing the real world from the perspective of mathematics. Students can design their own password rules according to their own learning experience and their understanding of the problem. It not only makes full use of students' previous learning experience, but also gives students full freedom of thinking, so that students can think about the real world with mathematical thinking. Finally, students need to express their own cryptographic scheme, which is to express the real world in mathematical language.

For example, some students have made such designs. If the information needs to be decrypted, the above figure needs to be spelled out as a square. The area expression consisting of 4 sheets of A, 20 sheets of B and 25 sheets of C can be transformed to obtain the Equation $4x^2 + 20xy + 25y^2 = (2x + 5y)^2 x^2 + 20xy + 25y^2 = (2x + 5y)^2$ If the coefficients of x are "tens" and the coefficients of y are "single digits," then the cipher text "25" corresponds to the letter "*y*"; conversely, if the message "*w*" is to be transmitted, then the square has a side length $2x + 3y$. If the message "w" needs to be transmitted, we need to construct a square with side length of "w" and expand the expression of its area to obtain the equation $(2x + 3y)^2 = 4x^2 + 12xy + 9y^2$. The number of sheets of paper A, B and C required can be obtained from the coefficients of each of the expressions. In this way, the students will be able to think about the different forms of the "sum" and "product" of an equation in the process of encryption and decryption, which are mutually inverse.

Another example. We know that expressing the equation in the form of a "product" is an important idea for solving higher order equations, so we can design a task where the agreed information is hidden in the positive integer solution of the equation. If the encryption rule is x-2, then we get x=26 and the message is the letter "z"; if the encryption rule is x2-76, then we get a solution of the equation as x=10 and the message is the letter "j." If students are given more difficult encryption rules like x3 – x=24, they would first need to decompose it, using the decomposition of the "product" in the form x3–x=x(x–1)(x+1) and then combining it with 24=2 × 3 × 4 to get a solution of x=3, which conveys the information of the letter "c."

> In cryptography, letters are often corresponding to numbers, and the numbers are encrypted to convey important information: for example, 26 letters in sequence correspond to the numbers 1~26, **if both parties agree to use the positive integer solution of a certain equation to pass the information**, at this time you have the key in hand $a^3 - a$.
>
> (1) If the letter *l is* passed, what is the plaintext?
>
> (2) If the plaintext is 6840, what is the message being delivered?

This task has also designed an interesting application background, combining cipher and factorization. In the process of understanding, thinking and completing this task, students will experience a complete process of observing the real world with mathematical vision, thinking about the real world with mathematical thinking and expressing the real world with mathematical language, Students' core qualities are gradually cultivated.

Students will continue to think about and answer the big questions by completing several sub-tasks to accumulate different perspectives on the "sum" and "product" of an equation. We can further propose a big task for the unit: design a cryptographic scheme based on your understanding of the characteristics of the "sum" and "product" of an equation, and explain the principles of the design.

> **Big tasks**
>
> A cipher is a sequence of symbols that conceals the true content of a message according to a specific rule and turns the message to be conveyed into a message code that cannot be read by anyone other than the communicating parties. In this chapter, you will be designing a cryptographic scheme based on your understanding of the "sum" and "product" characteristics of an equation and explaining the principles of the design.

To complete this big task, students need to use factorization as a tool and use the characteristics of “sum” and “product” to design a reasonable way to help them think deeply and answer the big question in the process of completing the task. Through such large tasks, students incorporate their understanding of the different forms of “sum” and “product” in this chapter into their own designs and use them appropriately. By designing these tasks with the big questions in mind, students will not forget to keep thinking about and answering the big questions as they go through the unit, and thus gain an understanding of the big ideas behind the questions. The process of completing tasks with the help of the big questions will also further improve the core discipline quality of students.

It is important to note that students may not be able to answer the big questions to the same extent at different stages, and not every student has a deep sense of the big questions, which is a true reflection of differing abilities in mathematics. In any case, the “big question-led units” take students away from mere knowledge and skills training and lead them toward the formation of disciplinary competency.

In the process of practice, we have also gradually explored the basic elements, characteristics of each element and the relationship between them. This culminated in an operational model for planning units (Fig. 3).

Under the continuous effect of such big questions, those big ideas that reflect the essence of the subject will gradually converge in students’ minds and become internalized. During this process, the core competencies will take root and grow in the students’ minds. In this way, our children will gain the strength to face the future in today’s schools!

Table 1. 23 big ideas of mathematics curriculum in primary and secondary schools

Serial number	Fields	Big ideas
1	Numbers	The set of real numbers is infinite, and each real number can be associated with a unique point on the number line
2	The Base Ten Numeration System	The base ten numeration system is a scheme for recording numbers using digits 0–9, groups of ten, and place value
3	Equivalence	Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value
4	Comparison	Numbers, expressions, and measures can be compared by their relative values
5	Operation Meanings & Relationships	The same number sentence can be associated with different concrete or real-world situations, and different number sentences can be associated with the same concrete or real-world situation. The same concrete or real-world situations, and different number sentences can be associated with the same concrete or real-world situation
6	Properties	For a given set of numbers there are relationships that are always true, and these are the rules that govern arithmetic and algebra
7	Basic Facts & Algorithms	Basic facts and algorithms for operations with rational numbers use notions of equivalence to transform calculations into simpler ones
8	Estimation	Numerical calculations can be approximated by replacing numbers with other numbers that are close and easy to compute with mentally. Measurements can be approximated using known referents as the unit in the measurement process

(*continued*)

Table 1. (*continued*)

Serial number	Fields	Big ideas
9	Patterns	Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways
10	Variable	Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations
11	Proportionality	If two quantities vary proportionally, that relationship can be represented as a linear function
12	Relations & Functions	Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set. A special rule (function) assigns each member of one set to a unique member of the other set
13	Equations & Inequalities	Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found
14	Shapes & Solids	Two- and three-dimensional objects with or without curved surfaces can be described,classified, and analyzed by their attributes
15	Orientation & Location	Objects in space can be oriented in an infinite number of ways, and an object's location in space can be described quantitatively
16	Transformations	Objects in space can be transformed in an infinite number of ways, and those transformations can be described and analyzed mathematically

(*continued*)

Table 1. (*continued*)

Serial number	Fields	Big ideas
17	Measurement	Some attributes of objects are measurable and can be quantified using unit amounts
18	Data Collection	Some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that needs to be collected and how best to collect it
19	Data Representation	The type of data determines the best choice of visual representation
20	Data Distribution	There are special numerical measures that describe the center and spread of numerical data sets
21	Chance	The chance of an event occurring can be described numerically by a number between 0 and 1 inclusive and used to make predictions about other events
22	Proof	Mathematical statements can be proved or disproved using previously established statements, self-evident truths or assumed statements. This may be This may be through the use of physical objects, diagrams, manipulatives or algebra
23	Classification	Abstract and concrete mathematical items can be grouped according to their characteristics

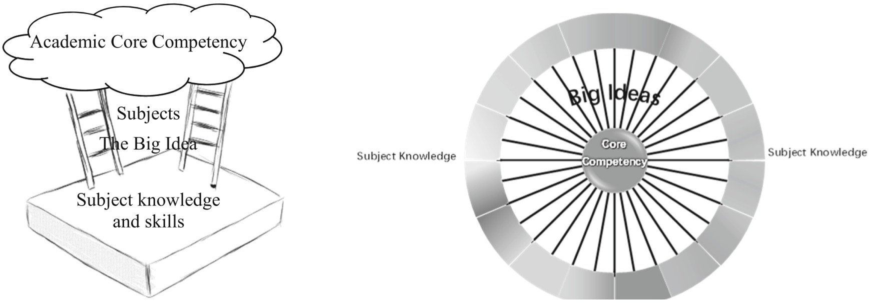


Fig. 1: Two "metaphors" for big ideas

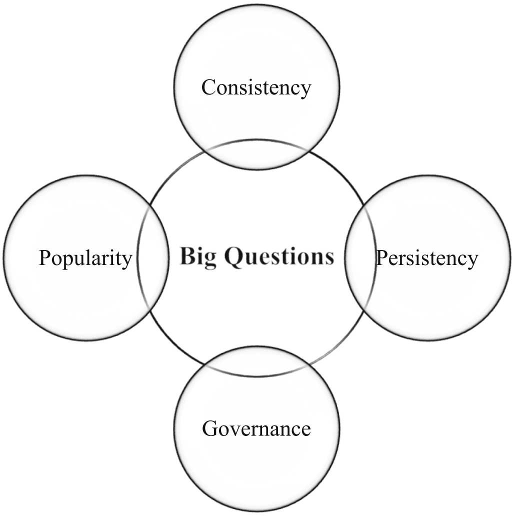


Fig. 2. The basic characteristics of the big question

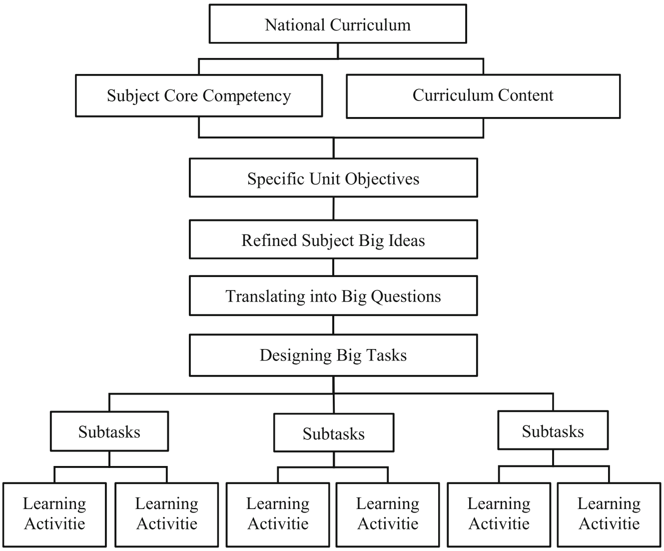


Fig. 3. Unit design flow chart

![Check for updates]

A Study on the Teaching Design of "Problem Solving" for Junior Middle School Mathematics Teachers—Take the Example of "Applying Linear Equation in One Variable—Discount Sales"

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Abstract. The teaching materials about problem solving are selected from the junior middle school mathematics textbook of Beijing Normal University Edition. The teachers present the teaching design by reference to the teaching materials. Then, with the guidance of experts and peer discussion, they make continuous improvement to carry out the teaching design that can reflect the compilation idea of the textbook. In the reflection, they have a deeper understanding of the textbook and the compilation intention of the textbook, so as to promote the professional growth of teachers and the effective implementation of the textbook.

1 Aims of the Study

Based on the content of "Applying linear equation in one variable ---discount sales" in "linear equation in one variable" of the junior middle school mathematics textbook, The teaching design of the research object is discussed and improved continuously, so that teachers can reflect the idea of textbooks better and the general method of problem solving in the teaching design, so as to promote the professional growth of teachers and the effective implementation of teaching materials.

2 Research Process

2.1 The Research Subject

A teacher with 7-year experience was selected as the research subject to write the teaching design and reflections independently in this case. A four-member expert team consists of textbook writers, teaching researchers and superfine teacher, provincial subject leaders, etc. was formed, and six other teachers from different regions participated in the workshop together.

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2.2 Research Idea

2.3 Research Questions

The main problems discussed and solved in this case are: 1. What is the teaching objective written by teacher W? (including the understanding of the teaching materials and the conceptual cognition of the problem-solving-based teaching design); 2. How to design the process of problem solving? 3. How is the quality (validity) of the teaching design based on the analysis of textbooks and interview?

2.4 Research Methods

This case presents the comparison and reflection of three instructional designs written by teacher W. The whole research process went through two workshops and two revisions, the first discussion mainly addressed the above question 1, the second discussion mainly addressed the above question 2, 3.

Researchers have constructeded various frameworks to describe general problem-solving procedures (e.g., Krutetskii 1976; Pólya 1971). Among these frameworks, Pólya's four-stage framework is used most widely in mathematics instruction (e.g., Schoenfeld 1983, 1985). Thus, we mainly draw on Pólya's framework.

3 Research Results

3.1 The Establishment of Teaching Objectives

The first draft of the teaching objectives developed by the teacher according to their own understanding is:(1) To understand the quantitative relationship between cost, selling price, profit, and profit margin, and be able to repeat them.(2) To be able to accurately identify the equation in the specific discount problem and solve the equation, according to the solution of the equation to explain and analyze the specific phenomenon of discount sales (3) Through investigation, experience and analysis, students can fully feel the mathematics around them, and try to analyze the discount phenomenon in life with the eyes of mathematics, Implementing rational consumption concept (4) To explore the equation relationship from the problem situation, experience the process of using linear equation in one variable to solve practical problems, and develop the ability of abstraction, generalization, problem analysis and problem solving.

Expressions From the "understand…. " to the expression "understand quantitative relationships and be able to repeat them" reflect that Teacher W focused on teaching and memorizing to achieve knowledge objectives, while the expressions of the ideological and emotional objectives were rather hollow and not operational. After the joint discussion, Teacher W had a deeper understanding of the teaching materials and further comprehended the writing intention of the teaching materials. The overall positioning of the teaching design has risen from the teaching of specific knowledge methods to the

Table 1. Analytical framework

Analysis Dimension		Analysis method
Formulation of segmental teaching objectives		Present teacher's teaching objectives, analyzing the dimensions of objectives, understanding the idea of teaching design and so on
Problem solving and process design	Problem posing	First analyze if there is a corresponding teaching session for problem solving (There may not be explicit link titles), and then analyze how to do
	Understand the meaning of question	
	Problem solving	
	Review and Reflection	
Teacher Reflection	Teaching concept	The teacher is the organizer of learning, the guide and the collaborator. Integrated design reflects the general method of solving problems. Changes in teaching methods
	Student View	The teacher recognized that students are the main body of learning, whether the design of the problem strings could provoke students to think about the general approach to problem solving

induction of general methods of problem solving The teaching objectives of this segment were finally clarified as follows: (1) Combining with problem situations, guide students to discover and understand the meaning of each quantity in the situation, exploring the relationship among all quantities, and guide them to visually express the relationship among all quantities using geometric figures such as diagrams; (2) to be able to represent the same quantity with different expressions, establish equivalence, and learn to analyze and solve problems from different perspectives; (3) experiencing the process of using linear equation in one variable to solve real-world problems, summarize and understand the general steps and methods of using linear equation in one variable to solve real-world problems.

	Teacher W
problem posing	No issues were raised in the first draft
	The second draft has questions posed. The teacher introduces the topic of discounted sales through the price of water glasses and leads students to express the relevant terms before asking the required questions
Understand the meaning of the problem	The third draft creates the situation of Xiao Ming's mother opening a clothing store to solve the problem of discounted sales
	All of them are designed to help understand the meaning of the questions through question strings The design of the first draft is scripted, and the problem design is superficial and lacks directivity, which cannot trigger in-depth thinking
	The second draft adds question 1 to draft 1 for guidance and follow-up question 2 to guide student thinking about the visual representation of the relationship between the quantities
	The third draft further clarifies the relationship between the questions on the basis of the second draft, and revises the "Pursuit 1", adding the requirement of expressing the relationship between quantities in terms of equations before finding the equivalence and listing equations, guiding students to read the question stem to obtain information and understand the meaning of the question
Problem solving	The first draft the process exactly as the textbook. While the problem is solved, it does not reflect the general idea of problem solving

(*continued*)

(*continued*)

	Teacher W
	The problem-solving process in the second draft is the same as in The first draft, but 2 follow-up questions are added afterwards, asking students to try to draw diagrams to represent the relationships between the quantities and guiding them to ask and solve other problems in the context of the scenario
	The 3rd draft modified the questioning to focus on the solution of a situational problem, deleting the two new follow-up questions which added in the 2nd draft, and added the questions of setting up an unknown number flexibly to guide
Review and Reflection	The first draft: "What did you learn? What are some of your takeaways? Then guide students to work together to summarize the formulas related to discounted sales"
	Draft 2, 3: "(1) What quantities did you learn today? How do you go about expressing the relationship between quantities?; (2) What do you think is the essence of solving application problems by equations? (3) What steps are involved in the process of solving application problems with equations?

3.2 Problem Solving Process Design

Based on the general thinking process of problem solving, the process design of the teacher will be analyzed from four aspects: problem posing, understanding the problem, problem solving, and retrospective reflection. The following Table 1 presents the comparison of the design of the three drafts of teacher W in each aspect.

3.3 Analysis of Problem Posing

The textbook is accompanied by pictures of discounted clothing sales, posing questions, and then guiding students through the process of analyzing and understanding the problem, solving it, and then reviewing and reflecting on it in order to build an overall structure for problem solving.

In the first draft, teacher W designed a longer preparation activity, explaining the quantities that would be encountered in the sales problem, such as price, cost, selling price, discount, profit, etc., one by one and expressing the relationship between them in an equation before explaining it as an example problem. After workshops, teacher W gradually adjusted the design, and there was a significant improvement in the idea of teaching design, and the problem-solving based design was reflected in the third draft.

Teacher W reflection: The deficiency of my first draft is to use the form of fill-in-the-blank questions to summarize the one-way sales formula, so that students apply the formula to solve the problem stiffly, without guiding them to understand the two-way relationship between quantities. Students are still confused about the problems with many solutions and complex situations. After the first discussion, in the second draft design, the familiar living resources around the students, such as the price of the teacher's water cup, were introduced to discuss the amount of sales problems, which was relatively natural for students to accept, but there was not enough space for students to think. The transfer of knowledge is the main. In the third draft, it was changed to the situation of Xiao Ming's mother's clothing store to directly raise questions, so as to access to the questions in the textbook easily and quickly, which reflects the general idea of problem solving.

3.4 Analysis for Understanding the Meaning of the Question

After two seminars, the teacher changed her original understanding of the textbook. Teacher W made significant progress in understanding the textbook and its writing intentions, and also made greater progress in the concept of teaching design, but there was still a situation of over-traction, leaving insufficient space for students and lacking overall awareness.

Teacher W's reflection: The design of the first draft lacks directivity and Operability, mainly through teaching and training mechanically such as do exercises and explaining; in the analysis of the equivalence relationship only found an equivalence relationship, not multi-angle thinking whether there are other equivalence relationships? In the form of expression, there were only written equations instead of block diagrams. After discussion, I have a deeper understanding of the content and writing intention of the textbook. The second and third drafts were designed with additional questions before question 1 in the first draft: "“40%”, “20% off” and “15 yuan” in the scene repectively mean which two quantities are related?" These questions guided students to analyze the relationship between quantities. Compared with the two drafts after the first draft, the questions are more in-depth and focus more on guiding students to sort out and express the relationship between quantities, laying a good foundation for subsequent problem solving.

3.5 Analysis of Problem Solving

This case is expected to reflect the process of problem solving through Problem 3 and three follow-up questions to enhance the visualization and the variety of ways to establish the equivalence relationship. For example, "Question 3: With the help of the graph of the relationship between the quantities, if you set the cost of each garment to be x, can you express the other quantities using algebraic equations containing x? Following up with Question 1: What kind of equivalence can you find by reviewing the known conditions? Following up with Q.2: Is there another way to express the price and the selling price with the help of the graph of the relationship between the quantities? Following up with Q.3: What other equivalence can you find? What kind of equation can you find? (Based on the profit of 15, the equation is $(1 + 40\%)$ x. 80% - x = 15; based on the equal selling price, the equation is $(1 + 40\%)$ x. $80\% = x + 15$; based on the equal markup, the equation is $(1 + 40\%)\ x = (x + 15)/80\%$.

Teacher W's reflection: In the first draft, the design was completed according to the textbook, and only provided answers to the questions, without guidance from the general method. In the second draft, students were further guided to sort out the relationship between each quantity and represent it in a diagram, which opened up their minds and inspired them to look for equivalent relationships from the perspective of different unknown quantities and list different equations, and then let them ask other questions and solve them based on the same situation. The disadvantage is that the students are not guided to general thinking and induction after the end of each question. Draft 3 was further revised. In the second draft, the requirement for students to ask other questions and solve them was deleted, but too much emphasis is placed on intuitive representation and the diversity of equivalence relations which leads to a large span of thinking in problem design, emphasis on skill training, and there are still deficiencies in the guidance of thinking in problem solving.

3.6 Review and Reflection

As a result of the workshop, the researcher expects to ask the following three questions in the retrospective reflection session. Question 4: What is the essence of solving application problems by equations? Question 5: What is the general procedure for solving actual problems using linear equation in one variable? Question 6: Along the path of thinking from Problem 1 - Problem 2 - Problem 3, can you summarize the general approach to problem solving? Through the three problems step by step, guide the students to reflect on finding equal relation, writing equations to solve word problems and the general method of problem solving.

Teacher W's Reflection: After the seminar, changes were made, not only to review the knowledge points and sort out the inner connection of knowledge, but also to pay more attention to the essence of equations and the generalization of the method of solving application problems by equations.

3.7 Research Conclusions and Discussion

Conclusions: Through the guidance of experts and the cooperation of partners, after two discussions and two improvements, teacher W's third draft of teaching design has basically reflected the teaching design idea based on problem solving, and has achieved the aims of the study. From the design analysis of problem solving process and teacher reflection, it can be seen that teacher W has made significant progress in understanding the textbook and editing idea, and the concepts of teaching and students have also been improved.

Discussion: 1. To strengthen teachers' understanding of textbook and promote their effective implementation. 2.The teaching design based on problem solving should be designed along the thinking path of "asking questions -- understanding questions -- solving problems -- reviewing and reflecting". 3. Pay attention to the visualization of mathematical representation, implement the cultivation of core literacy.

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The Influence of Teachers' Teaching Idea and Behaviour on Students' Problem Posing: An Explorative Study

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Abstract. This paper explored how teachers applying problem posing as an teaching activity at classroom level based on "Conjecture, Proof and Extension", which is the synthesis and practice of textbook series published by Beijing Normal University Press (BNU). The students' ability to pose problems is evaluated from five dimensions. It was analysed both horizontally among different teachers and vertically between two different lessons given by the same teacher. It was found that teachers' teaching behaviour has a significant impact on students' problem posing, and there is a great difference between the same teacher's two lessons. After the second round of collective discussion, teachers have a deeper understanding of how to apply problem posing at classroom level.

1 Research Background

Problem posing is a crucial part in scientific research. The Torrance Tests of Creative Thinking(TTCT) apply fluency, flexibility and originality to evaluate the ability(Torrance 1968). By the 1980s, problem posing was seen as a relatively independent teaching activity. Skinner asked elementary school students to pose plenty of math problems and share them to form materials for subsequent problem-solving activities (Skinner 1991). Silver thought "problem posing" refers to creating a new problem in a mathematical situation or elaborating the problem in the process of solving it (Silver 1994). Silver and Cai focused on evaluation dimensions on the fluency, novelty and complexity (Silver and Cai 2005). Li and Cai evaluated problem posing from four dimensions: flexibility, novelty, depth and relevance (Li and Cai 2020). Therefore, through the fluency, flexibility, novelty, depth, clarity of five dimensions, we choose different teachers to teach the same lesson, and analyze (1) *what different teaching treatment teachers will have in the classroom teaching?* And (2) *What are the effects of different teachers' teaching on students' problem posing?*

2 Methods

2.1 Participants

We selected 15 key math teachers from 5 urban districts of Qingdao to teach. These teachers are the experienced teachers with good classroom control. And they have had some attempts to apply the "problem posing" in daily teaching.

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2.2 Lesson Selection

We chose the synthesis and practice of "Conjecture, Proof and Extension" of textbook series published by Beijing Normal University Press (BNU). This lesson is mainly to explore the relationship between the perimeter and the area of the graph, trying to let students experience the general idea of research "Conjecture, Proof and Extension" of the results. From the perspective of teaching content, this lesson is suitable for teaching through problem posing, which is convenient for us to study how teachers' behaviour affects students' problem posing (Fig. 1).

2.3 Research Implementation

Before teaching, we firstly trained teachers on problem posing, which helped teachers master relevant knowledge, implementation methods and paying attention to problem posing, so as to facilitate teachers to implement teaching. On the other hand, the content of the course was discussed in a uniform manner, so that the teachers' understanding reached almost the same level. Each teacher completed the teaching design independently and taught different classes separately.

After the first lesson, further relevant training will be conducted to help teachers analyze the problems encountered in the first lesson. Through the cooperation and mutual assistance among teachers, they can improve their understanding of problem posing, and change their behaviour, and make classroom teaching more conducive to the direction of students' problem posing. On this basis, 10 of them choose other classes for the second time. By collecting the data of problems posed from different classes, the influence of the change of teachers' ideas and behaviours on students' problems posed is comparatively analyzed.

In order to better study the teaching activity of "problem posing" in mathematics, the changes from teachers and students brought about by classroom performance are analyzed and studied from two aspects.

Compared the differences among different teachers on the same lesson horizontally. Through the data collection and analysis of problems posed by each class, the differences of problems posed by different classes are compared and the causes are analyzed.

Compared the same teacher before and after two lessons vertically. Teacher's ideas and behaviour changes on problem posing are analyzed (Fig. 2).

2.4 Data Collection and Analysis

Videos are collected for horizontal and vertical comparison during the two lessons. It helped us to analyze the behaviour changes of the same teacher's two lessons, and the influence of the change of teacher's teaching behaviour on problems posing.

After the first class, we conducted a questionnaire survey on 10 teachers who needed to give the second class. The survey problems mainly include, in the first two links of this class, "how do teachers guide students to pose problems", "how to deal with the problems posed by students", "how long do teachers pose problems for students", and "how teachers to think about teaching mode after teaching", etc.

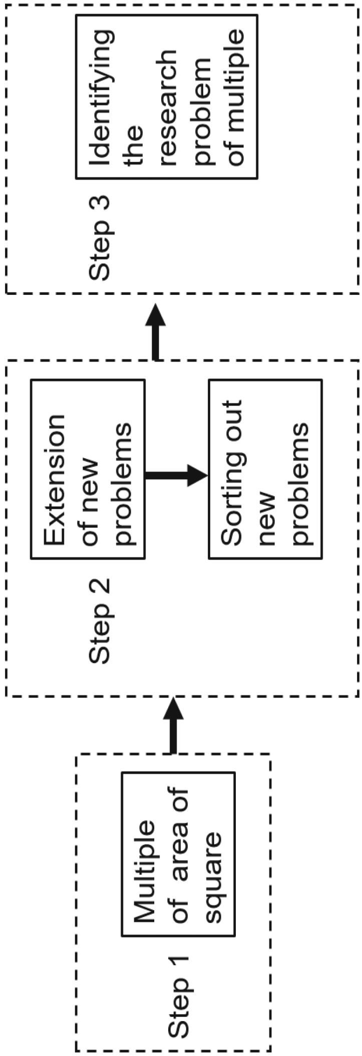

Fig. 1. Three steps to help students experience conjecture, proof and extension.

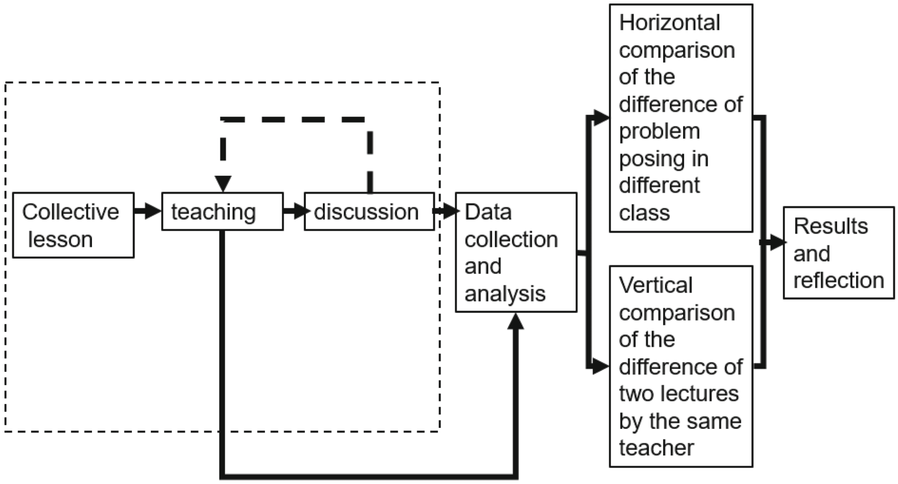

Fig. 2. The sections of research implementation.

Collect all the problems posed by students in each link of class on the spot, some of which are orally expressed and recorded by teachers on the blackboard; and some students submit problems on the spot via slips of paper. According to our requirement of problem classification, the instructor sorted out all the problems posed by the students in the class and filled in the following form.

We selected five dimensions of fluency, flexibility, novelty, profundity and clarity to establish the following index system for evaluating students' ability to propose mathematical problems. The specific description is shown in Table 1.

Table 1: Five dimensions for analysis

Dimensions	Meaning	Viewpoint
Fluency	Fluency in asking problems	Number of valid problems
Flexibility	Flexibility in asking problems	Types of valid problems
Novelty	The novelty of the problem	The number of novel problems
Profundity	The depth of the problem	The amount of information contained in the problem and the degree of connection
Clarity	The clarity of the problem	How clearly the problem is expressed

Fluency means that we use "the number of students who pose effective problems and the percentage of the total number of students", "the number of effective problems and the average number of effective problems posed by each person" as the standard for the overall evaluation of students' ability to pose problems. "The number of students who ask 2 or more problems and the percentage in the total number of students" is used as the evaluation index of the individual students' ability to ask problems.

Table 2: Numbers changing in clear problems

Class	1	2	3	4	5	6	7	8	9	10
First lesson	49	23	3	76	84	29	23	30	15	48
Second lesson	92	63	25	99	65	34	76	32	53	58

Flexibility means we measured "the kinds of graphs and multiplications involved in effective problems" from each class.

Novelty means we define the following problems as novelty problems: "the problem involves other shapes (non-square, rectangular)" "The same problem involves two shapes".

Profundity means we define the following problems as novel problems: "number of problems that summarize a type of graph", "number of problems that are multiples of letters".

Clarity means we divided the effective problems posed by students into clear problems and unclear problems according to whether they were expressed clearly and logically, counted the number of clear problems and unclear problems in each class, and calculated their proportion in all effective problems.

It should be noted that the situation of students in different classes of different schools are different. Therefore, when studying these five dimensions, we made corresponding adjustments according to the actual situation of students. Teachers also paid more attention to the classroom situation of their classes when asking questions and giving guidance. After data collection, we communicated with each other through discussion and reflection. From the perspective of quantification, the results of the study are analyzed and sorted out to provide reference research ideas and methods for problem posing.

3 Results

3.1 Horizontal Comparison

The percentage of students who were able to ask problems varied widely, from 92% in the highest class to 24% in the lowest. The proportion of students who can ask 2 or more problems varies greatly, with 71% of students in some classes asking 2 or more problems, and only 22% in others.

Among the valid problems posed by students of different classes, the number of types of graphs involved is higher than the number of types of multiplication involved. There was a great difference in the number of graph types, but there was no obvious difference in the number of multiple types among classes.

Among the valid problems posed by students of different classes, the number of problems involving other figures varies greatly. Among the 91 valid problems posed by students of the highest class, 52 problems involve non-square and non-rectangle. There was little difference between classes in the number of problems involving more than two shapes in the same problem.

Compared with the two lectures of the same teacher, it was noticed that the class of 7 teachers' second lecture improved the graph generalization in problem posing. The other teachers did not change their teaching too much.

Among the problems posed by students in different classes, there are obvious differences between clear problems and unclear problems. Some classes had 100% clear problems, while others had only 18%.

3.2 Vertical Comparison

Compared with the same teacher who taught two different parallel classes before and after, the proportion of those who posed two or more problems increased greatly, and the number of effective problems increased in most classes.

Compared with the same teacher's two lectures before and after, the number of problems involving other figures increased significantly, and all the experimental class problems in the second lecture contained more figures.

Compared with the same teacher before and after the two lectures, the number of clear problems has increased obviously compared with the first lecture (Table 2).

Comparing two parallel classes in the same school, the number of effective problems, the number of clear problems, the number of two or more problems, and the number of generalized graphic problems in the class with a teacher is higher than that in the class without a teacher.

4 Conclusions and Discussion

Through the above studies, we get the following conclusions.

Firstly, the teacher's teaching idea will affect the teacher's class teaching behavior, and thereby has an impact on students' problems posing activities. Through the uniform training of teachers before and after teaching, teachers' understanding of problems posing can be improved, so that teachers are more willing to accept and apply the teaching of problems posing in practice.

Secondly, teachers' orientation of curriculum objectives will obviously affect students' problems. If the teacher aims at mastering specific knowledge, the limited time in class will reduce the time for students to pose problems, which is bound to have an impact on students' problems. In our experimental research, teachers who taught for the first time did not give students enough time in the link of problem posing, which affected the fluency and flexibility of students' problem posing.

Thirdly, teachers' classroom guidance language will influence students' problem posing to a certain extent. In our experiment, some teachers used the following guidance in the problem posed in link 2, "Which graph do you think meet the requirements?" "What problems can you ask?" Because the guidance language of teachers is more clear, the demonstration is more clear, which affects the flexibility of students to pose problems. Therefore, in practical teaching, we suggest the formation of clear language to help students to pose problems.

Fourthly, the depth of students' understanding will affect the profundity and novelty of their problems. Therefore, teachers should pave the way for students to truly understand the nature of the problem, so as to pose more in-depth and innovative problems.

Finally, we hope to get the specific paradigm of teaching practice and guide teaching through experimental research.

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Symposia—Symposium 3: Students' Agency in Selecting and Using Curriculum Resources

Symposium—Towards Innovative Practices in Mathematics Education: Teachers' and Students' Choice and Use of Digital Resources

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Abstract. In this symposium we investigate students' agency of selecting and using (digital) resources for developing their own learning paths. For that, we first review the literature related to students' selection and use of resources in mathematics education in different pedagogical settings (presentation 1). Second, we develop insights from the different studies that participate in this symposium (presentation 2–6), at school as well as at university level. Results show that attempts have been made to provide students opportunities to develop agency of their mathematics learning, in particular with the development and provision of numerous digital tools and learning resources at university level and related to innovative pedagogical approaches. At the same time, it is not obvious how these tools and resources help students to develop deeper conceptual understandings. Certainly, students often 'demand' more student-centered and autonomous education approaches (e.g., at university level), also in mathematics education. Further, it seems that authentic problem-based education approaches are more motivating for students. These 'innovative' approaches necessitate particular types of structure and support for students. Moreover, they require different ways of providing resources that students can and want to interact with, and that help students to navigate through the curriculum to develop their own learning paths. At the same time, teachers also need support on how to orchestrate student learning with the available resources in such environments, so to be able to attend to students' individual needs. The symposium comprised altogether six presentations:

Birgit Pepin & Sebastian Rezat: Students' agency of selecting and using (digital) resources for developing their own learning paths: An overview

Annalisa Cusi & Agnese I. Telloni: Learning through digital curriculum resource design: students' reflections on their role as designers

Vilma Mesa, Lelia Burley-Sanford, Xinyi Hao, & Carlos Quiroz: Interactive features in university textbooks and their use by teachers and students

Sebastian Rezat: Fostering university students' reading and understanding of mathematical text in a flipped classroom approach with a digital marking tool

Birgit Pepin & Ulises Salinas: Challenge/problem-based mathematics learning at university level: The case of the modeling week

Farzad Radmehr: Problem-posing: An inclusive activity for improving teaching and learning of mathematics at university level

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1 Student Agency of Selecting and Using (Digital) Resources for Developing Their Own Learning Paths: An Overview

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We present our results from a review of the mathematics education literature with respect to students developing agency when selecting and using digital curriculum resources for their learning of mathematics. This is done to position the presentations of this symposium in the field of knowledge of student work with digital (curriculum) resources at primary, secondary and tertiary level.

Using the 'lens of resources' (Pepin & Kock, 2019; Trouche et al., 2019), Pepin and Kock ask the following research questions: (1) what kinds of resources do students select and use; and (2) which factors influence their selection and use of resources. This was done in order to develop insights into learning environments that may foster student agency (in the selection and use of digital resources).

In terms of methodology, we conducted a literature review. This review mainly relied on the procedures of a thematic synthesis (Xiao & Watson, 2019) with the overarching aim to build on the current body of literature, to summarize what is known about students' selection and use of resources and related influential factors.

In terms of results, and answering the first research question, we contend that at primary and secondary level the textbook used to be the main learning resource for students, with teachers mediating students' textbook use and thus not providing agency to students to choose their own learning resources or even select content within the resource (e.g., Pepin & Haggarty, 2001). Rezat (2013) challenged this view with regard to textbooks, and in recent years this situation has drastically changed due to the increased availability of digital resources. In particular through the internet, students have access to a plethora of different digital resources (e.g., Wikipedia, YouTube-videos, learning platforms, chats, discussion forums, social media) that are intended to support them in their learning of mathematics or can be used as resources to get assistance when learning mathematics. In this situation it can be argued that primary and secondary students have the opportunity to choose among the different resources, and instrumentalize and orchestrate them for their learning of mathematics.

As Rezat (2013) has already shown for students' use of traditional mathematics textbooks, even a single resource is not necessarily used as a whole in a predictable way, but lower and upper secondary students make choices of opportunities to learn within it. These choices depend on different factors such as the design of the resource, on students' goals and beliefs, on their previous knowledge about using this or a similar resource, and on the pedagogical approaches that provide the context for students' learning activities.

At tertiary/university level, there is an increasing number of studies focusing on university students' selection of (digital) resources to study mathematics from the plethora of resources available (curriculum resources, social resources, cognitive resources, and general resources). In earlier studies in the UK and Norway (e.g., Pampaka et al. 2016), students selected mainly traditional resources, and this was an issue for the transition from school to university mathematics. In more recent studies, for example, Anastasakis and Lerman (2021) explored the range of resources mathematics and engineering students used when studying mathematics and the ways undergraduates combine these tools. Results from this survey (N = 628) showed that students in their sample used mostly their notes, the university's virtual learning environment (VLE), other students and textbooks. Mathematics students were found using more online encyclopedias, the university's VLE, instant messaging, and other students, whereas engineering students reported using more textbooks. In the Netherlands, a survey (N = 403) on the range of resources selected and used by students in first year Calculus and Linear Algebra courses was administered to engineering students (Kock & Pepin, 2018). Results showed three "communities" of students regarding the relative importance of the resources: (1) students who considered lecturer explanations of content and of problem solving as the most important resources; (2) students who considered the textbook as the most important resource; (3) students who considered other resources, such as materials created by the teacher (e.g., lecture notes), worked solutions, past examinations and the university's DLE as the most important. During the COVID-19 pandemic, Kempen and Liebendörfer (2021) collected survey data (N = 89) on a fully digitalized Linear Algebra course for mathematics, physics, and computer science students, as well as for students in a program for a teaching degree. Results showed that students rated (digitalized) traditional parts of mathematics teaching (lecture, full-class tutorial, small-group tutorial and lecture notes) as highly useful. DRs (videos, webpages and chat) were also generally rated as useful, as well as external videos and videos from the teaching team. Communication with other students was rated as the most useful resource. In another innovative context, Pepin and Kock (2021) studied the selection of mathematics resources in a challenge/problem-based course where students worked in multidisciplinary groups on authentic challenges set by external stakeholders (e.g., business/industry). Here students mostly used resources outside the realm of curriculum resources offered to them in traditional courses. These included "pieces of knowledge" obtained from various sources, scientific papers, websites, mathematics software, peers and experts in the field. Social resources took on a special role, and the most prominent was the role of the academic coaches: they gave discipline-specific feedback and helped students to re-focus on the project aims, when the students' ideas went into different directions.

Regarding the second research question, in our review, we identified four factors that are likely to influence students' choice and use of resources: (1) the availability and nature of resources (which would include, e.g., the structure and visual appearance of resources; the nature of the tasks implemented in digital resources); (2) the nature and structure of the course (including, e.g., nature of problems as part of the course) and its associated pedagogical approaches (e.g., teacher mediation of the use of resources); (3) institutional framing; and (4) student beliefs (e.g., about the nature of mathematics and.

In pedagogical terms, considering the four factors influencing students' choice and use of DRs, we contend that learning environments beneficial for students developing and exercising agency can be characterized by, or include, the above-outlined characteristics: (1) student-centeredness; (2) active engagement of students; (3) authenticity of tasks/problems; (4) forms of working that foster dialogue/communication amongst students; and (5) the nature of the DR itself that fosters student agency. We contend that these environments can be called 'agentic': they are environments where students are provided with opportunities to exercise and develop agency. Moreover, we argue that 'agency' in the kinds of digital learning environments we reviewed was 'distributed', in particular, the 'resource agency' was often underestimated.

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Learning Through Digital Curriculum Resource Design: Students' Reflections on Their Role as Designers

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Abstract. The focus of this contribution is on the role that students could play as designers of digital curriculum resources (DCR) (Pepin et al., 2017). Our study focuses on data collected within an educational programme that involved 7 upper secondary school students (grade 12).

The focus of this contribution is on the role that students could play as designers of digital curriculum resources (DCR) (Pepin et al., 2017). Our study focuses on data collected within an educational programme that involved 7 upper secondary school students (grade 12). The programme was set up by the Department of Industrial Engineering and Mathematics of the University of Ancona and was aimed at guiding students in deepening their knowledge of specific mathematical topics using GeoGebra as a tool to design DCR (Cusi & Telloni, 2022; Cusi, Gagliani Caputo & Telloni, 2022).

To frame the processes in which students are involved when they work on DCR-design, we refer to Chevallard's (1991) notion of praxeology, which is structured in two main levels (García et al., 2006): (1) the praxis, which includes different kinds of tasks to be faced and techniques used to face them; (2) the logos, which includes the "discourses" that describe, explain and justify the techniques.

We interpret the DCR-design as a particular task that the students involved within the educational programme.

Our study has been set to characterize the students-designers' praxeologies related to this task, by investigating both the techniques adopted by students-designers in facing their task and the discourses through which they justify the use of these techniques by focusing on both their pragmatic and epistemic value (Artigue, 2002). A further aim was to investigate how students-designers interpret their experience of being both students and designers.

These aspects were analysed by means of a written questionnaire and semi-structured interviews, developed at the end of the educational programme to collect students-designers' reflections on both the process of DCR-design and on the learning process that could be boosted through it.

In our analysis we focused on students-designers' ways of: (1) describing DCR-design main phases and making the faced difficulties explicit; (2) sharing the didactical aims related to the design of their DCR and the ways in which the DCR was designed

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taking into account the future user's perspective; (3) reflecting on their learning as being both students and designers; (4) discussing the role played by their experience of DCR-design in affecting their vision of the Mathematical content at stake.

Our analysis enabled us to identify four main techniques adopted by students-designers in creating a specific DCR (a revision guide for students) and the corresponding justifying discourses:

a) enabling students to choose the objects to display on their screens (graphs, symbolic formulas or other representations) and the order in which they are displayed to give them the possibility to manage the information on which to focus;
b) enabling students to see multiple representations at the same time and interact with them, to support the coordination of different representation registers and the consequent development of a solid knowledge;
c) enabling students to observe the effects of the variation and covariation of specific parameters using the sliders, to enable students to identify classes of problems/phenomena;
d) providing not completely explicit feedback on the reasons' underlying specific students' mistakes to scaffold their metacognitive reflections on the mistakes they made.

When students-designers reflect on their experience, they interpret DCR-design as a particular problem-solving activity, stressing on the role played by DCR-design in enabling the designer to reflect on aspects that usually are taken for granted and to autonomously discover key-aspects of knowledge construction while facing the difficulties connected to the design process. Moreover, they declare that playing the role of designers of DCR positively affected their vision of both the Mathematical content at stake and of Mathematics in general.

The results of our analysis highlighted the effectiveness, from an educational perspective, of involving students in activities within which they play the double role of designers and learners, since this experience gives them the opportunity to reflect on both their DCR-design to justify it by clearly referring to the learning that each of the adopted techniques could support and the ways in which acting as designers could foster the designers' learning itself.

Currently, we are deepening our investigation of the emergence and development of specific praxeologies related to DCR-design by combining the analysis of the answers to the written questionnaire and of the a-posteriori interviews with the analysis of the audio and video-recordings of the students-designers' work while they are carrying out their design process (Cusi, Gagliani Caputo & Telloni, 2022).

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Interactive Features in University Textbooks and Their Use by Teachers and Students

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Abstract. The expansion of the field of interactive textbooks, thanks to the evolution and accessibility of production tools, has significantly facilitated basic research on these resources. There is building evidence in post-secondary education, that user interaction with learning resources (apps, videos, assessment systems) facilitate learning (Stains et al., 2018). Moreover, the field has been able to advance both theories and methods to investigate the use of these resources using real time data (Trouche, 2020a, 2020b; Trouche et al., 2020). In this context, we ask: to what extent does a teachers' use of a particular textbook feature influence their students' use of that feature?

The feature, reading questions, has been added to textbooks written in PreTeXt to entice students into reading the material in the textbooks prior to coming to class. The feature collects students' responses directly in the textbooks and delivers them to teachers in real time; perusing the responses allow teachers to make decisions to alter plans for their lessons. Using data from four instructors who each taught twice a university course using one of two interactive textbooks, either calculus (Boelkins, 2021) or abstract algebra (Judson, 2021), we analyzed the relationship between their utilization schemes of reading questions and those of their students.

We used the instrumental approach (Rabardel, 2002) and our prior findings that identified four instructor utilization schemes (Mesa et al., 2021) and three student utilization schemes, (Quiroz et al., 2022). The four instructor schemes related to (T1) completing questions individually to assess own knowledge before planning (self-evaluating); (T2) requiring students to complete the reading questions before class to plan instruction; (T3) using the reading questions during class; and (T4) requiring students answer the questions to assess correctness and give a grade. The three student schemes related to (S1) familiarizing with content before class; (S2) studying by practicing and doing homework; and (S3) self-evaluating own understanding of content.

Our data come from answers provided by teachers and their students about the use of this feature via surveys and periodic logs answered over the course of one semester of teaching. We had 62 different units of text from the four instructors, and 215 units of text from about 100 students of these instructors.

In our analyses we identified two of the three student schemes, and two of the four instructor schemes. Students familiarized with content before class (S1) (e.g., "To see what this section will be covering and to prepare myself for the unit") stated by Student

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08 of Teacher 25, teaching calculus in the seventh semester of data collection (S08, T-725, AC) and self-evaluated their own understanding of the content (S3) (e.g., “I think about these questions with deep thought and occasionally try to answer them if I find I have enough information to.” S06, T-726, AC)”. Teachers required students to complete the questions for lesson planning (T2):

> [the] arrows [show] the workflow that happens immediately before class, in the period between when I have a copy of my notes ready to go and when class actually happens. These changes are mostly based on last-minute changes, such as when I get to look over students’ prep assignments, or if we have technological issues to deal with. (T-725, AC)

Or assigned the questions during instruction (T3) (e.g., “We ended up doing the preview question together in class. I think that was good, some students were lost trying to label the different pieces,” T-523, AA). More significantly, however, we found that teachers’ instrumentations related to students’ instrumentations of the reading questions. The two abstract algebra instructors used both schemes and their students also used both schemes; one calculus instructor who instrumented the questions for planning only, had their students use both schemes; and for the second calculus instructor, who only instrumented the questions for classroom instruction, their students only instrumented them for self-evaluating. These findings suggest that instructors’ use of textbook features, even when not done explicitly, signal that those features are important, and that for these groups of students, understanding the material was more important than completing work for assessment purposes. We found it interesting that no instructor used the questions for assessment purposes and that their students did not instrument the feature for practicing or doing homework. In part this might be due to teachers’ approach to teaching the content; all teachers emphasized groups work at the board over lecturing and understanding over rote learning. We believe that the content is playing an important role, as we expand our analysis to include sections using a linear algebra textbook, which demands more knowledge of terminology, that we anticipate having instrumentations related to assessment.

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Fostering University Students' Reading and Understanding of Mathematical Text in a Flipped Classroom Approach with a Digital Marking Tool

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Even in the digital age, learning mathematics at an academic level still requires much interaction with mathematical texts. Understanding and developing disciplinary literacy skills at all levels is an increasing matter of interest (e.g., Rezat et al. 2022; Rezat and Rezat 2017; Shanahan and Shanahan 2008; Wiesner et al. 2020). We regard disciplinary literacy as a prerequisite for using symbolic resources effectively and, thus, an important component of students' agency in selecting and using resources. Therefore, fostering students' disciplinary literacy skills can be regarded as one way of fostering students' agency in selecting and using learning resources.

In our project, we focus on reading as one component of disciplinary literacy. Research has investigated how readers engage in reading mathematical text and unveiled different reading styles (e.g., Berger 2019; Österholm 2006; Weinberg et al. 2022). Berger (2019) has described behavior that successful readers show when reading mathematical text as a reading style termed "close reading" (Berger 2019). According to Berger (2019) close reading comprises the following strategies: skimming to see what is known and what is new; careful reading of the entire text, including definitions, theorems, proofs, worked examples, and highlighting what is new; making connections by relating what is read to relevant theorems, worked examples, or to prior knowledge; working examples and exercises by doing worked examples on one's own and comparing with the solution provided in the textbook or solving exercises after reading the text.

The main aim of our study was to foster students' close reading of mathematical text. We have developed and implemented an approach that builds on the main idea of the flipped classroom but interprets it in a different way using the digital tool "AnnoPy". AnnoPy is a marking and commenting tool that has been developed for bridging individual and collective aspects of reading of texts and allows for showing the aggregated overlapping results of all individual markings of the text based on predefined categories. Students were asked to read the course materials before the lecture in AnnoPy and mark the text using three categories of markings: (a) familiar content, (b) important content, and (c) content difficult to understand. The lecture was prepared by focusing particularly on the aggregated markings of difficult-to-understand content to provide clarification and further explanation.

After implementing the course for one semester, we surveyed $n = 282$ students' opinions of the course concept to find out if students' thought that the course design and,

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in particular, the use of AnnoPy contributed to developing their mathematical reading literacy. As we noticed that students marked more text in AnnoPy at the beginning of the semester than near the end, we also asked students about their marking behavior. Furthermore, we also wanted to find out about students' reading strategies and therefore included some questions about their reading behavior. The questionnaire contained 25 five-level Lickert items, three interval-scaled items, and five open items altogether. Exemplary descriptive results of seven Lickert items related to the course design and the marking and reading behavior are presented in Figs. 1 and 2.

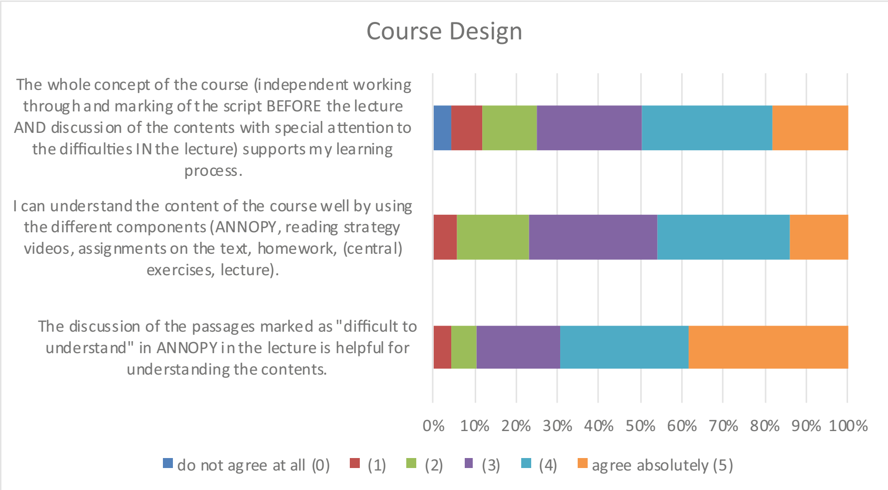

Fig. 1. Diagram with the results of three questions from the survey about students' opinions about the course design.

The results show that about 50% of the participants agreed with the two statements that the course concept supported their learning and that the resources supported their understanding of the contents. 70% of the students agreed with the statement that the discussion of difficult-to-understand passages was helpful in understanding the content. In terms of marking behavior, the descriptive results support our hypothesis that students put more effort into marking at the beginning than at the end of the semester and that this is not because they thought that the script was better to understand near the end. Furthermore, 40% and 65% of the students showed reading behavior that is related to close reading.

These findings show that the use of AnnoPy in the flipped classroom approach in the course was evaluated as helpful for students. If and how reading behavior is influenced by the use of AnnoPy and the flipped classroom approach and if students' disciplinary reading skills improved needs to be further investigated.

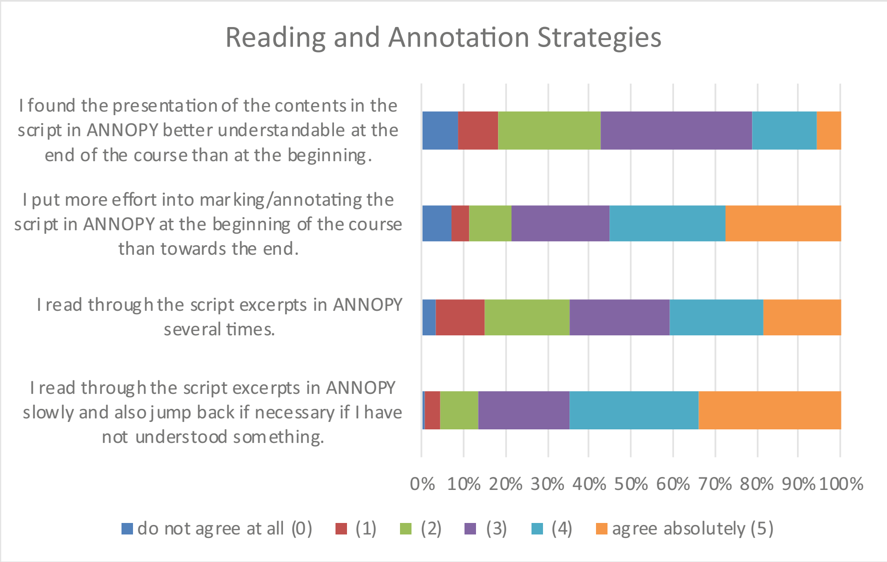

Fig. 2: Diagram with the results of four questions from the survey about students' marking and reading behavior.

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Challenge-Based Mathematics Learning at University Level: The Case of the Modelling Week

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There are new needs and concerns in society and industry that require suitable preparation of engineering students for the challenges of this century (e.g., global warming and sustainability). Consequently, new educational approaches have been developed that are more relevant in contributing to the solution of challenges through collaboration between industry and universities (van Uum & Pepin, 2022). One of these approaches is Challenge-Based Education (CBE) (Membrillo-Hernández et al., 2019). In this context, it is important to analyze what occurs in these innovative learning environments at the three levels of the curriculum: intended, enacted, and attained. Our research focuses on observing and analyzing what happens at the boundary between enacted and attained level, considering the point of view of the students themselves. We ask two research questions (RQ): (1) *How did students perceive the role of available resources to help them solve their challenge during a one-week challenge-based mathematics course?* (2) *How did students experience their learning in terms of mathematics and professional skills?*

In terms of theoretical frames, we used (1) the CBE approach, and (2) the lens of resources. (1) In CBE, teams of students are given a challenge linked to an issue in society. The challenges are typically provided by stakeholders external to the university (e.g., from industry) and have an open character. The stakeholders collaborate with university lecturers and tutors to support the students (Gallagher & Savage, 2020). We focus on mathematics students' learning experiences in CBE (Malmqvist et al., 2015), which we interpret as the conjunction of two processes: A process of being affected and getting new knowledge or skills, and a process of developing and applying knowledge or skills; in both processes students integrate and use resources and collaborate in different ways. (2) Taking the 'lens of resources', we draw on the Instrumental Approach (Trouche, 2004) to analyze both how different resources influence students' practice and understanding (instrumentation process), and how students in turn modify and adapt the resources according to their own needs (instrumentalization process).

The study took place in a course for first year master's students at a Dutch technical university, the Modelling Week (MW). This week was part of a compulsory course in the Applied Mathematics Master's program and allowed students to work for a week on problems provided by stakeholders from outside the university (Problem Owners-PO). The data collection instruments were the following: (1) interviews, (2) exit cards (filled in three times during the MW), which consisted of five questions to be answered by students regarding their perceptions about their work and learning, and hurdles/difficulties

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experienced; (3) selected observations of the student teams; (4) drawings that students were asked to make after the course. These were schematic representations of how students used and integrated different resources throughout the MW.

The analysis focused on the work and perceptions of a group of 7 students regarding their challenge: to improve an algorithm to plan taxi trips for elderly and disabled people by including stochastic start/arrival times. In terms of results and regarding our first RQ, we observed the different ways in which each student represented their experiences in the MW (e.g., Fig. 1). In the drawings we can observe the different resources that the students had been using throughout the work with their problem; the interactions with the tutor and the PO were identified as important resources for the students.

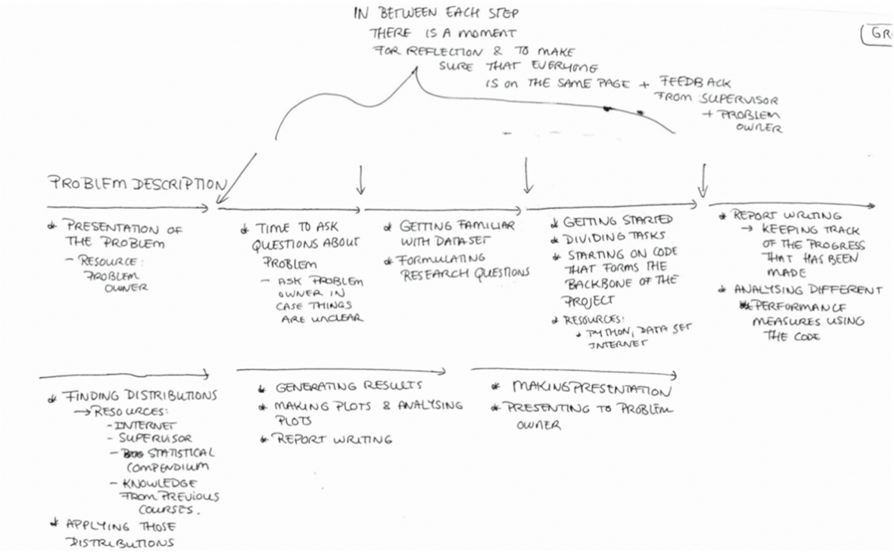

Fig. 1. Schematic representation of resource system from a student

Regarding our second RQ, the students' responses on the exit cards, in addition to the interviews, allowed us to identify perceptions of their learning in mathematics and professional skills. Students pointed out that they learned "how to convert a question from reality problem into a math question" and "making a plan of work together. Also including all team members, also the ones that are less active". It is noteworthy that a student stated that she did not learn "anything new" in terms of mathematics. However, our observations showed that the students had to operationalize mathematics knowledge in the context of their challenge and that this was not a trivial task for them. Vergnaud (2009) made clear that knowledge is not a static concept but rather a conceptual field which includes the operationalization of knowledge in different situations. The student may not have recognized this as mathematics learning and that raises the need to consider mathematical knowledge and learning in CBE in a broader sense.

Hence, results showed (1) the implication and importance of PO and tutor in student learning experiences, and (2) the development and application of both knowledge and skills to face such challenges. As we could observe, these two processes are closely related to the use of resources, where two processes are involved: instrumentation and instrumentalization.

Finally, in the context of the transition to a student-centered (CBE) approach, our results pointed to a new conceptualization of the tutor/PO role in terms of interaction with students, and of the kinds of knowledge needed for students to develop into 'valuable' engineers.

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Problem-Posing: An Inclusive Activity for Improving Teaching and Learning of Mathematics at the University Level

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Problem posing (PP) could be defined in mathematics as "the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems" (Stoyanova and Ellerton 1996, p. 1). The PP activities have been used in research and practice at school mathematics and teacher education programs in the past few decades. However, less attention has been paid to this type of activity at the university level to improve mathematics teaching and learning. Furthermore, this type of task is barely used in university textbooks, such as those for calculus (e.g., Hass et al. 2018; Hughes-Hallett et al. 2017). This is despite several benefits highlighted in the literature for engaging students with PP activities.

Engaging students in PP activities is one of the ways to activate higher order thinking as students to pose a meaningful mathematical problem need to interpret, infer, compare, and analyze the given information (Radmehr and Drake 2018; Radmehr and Vos 2020). Furthermore, many students found PP activities enjoyable and accessible at different stages of education, from the primary to the tertiary level. Previous research also suggests that the PP activities benefit students' mathematical learning. It could help students develop their creativity, conceptual understanding, and positive attitudes towards mathematics (e.g., See Nedaei et al. 2022 for a review of literature on the possible benefits of PP activities). Furthermore, several studies reported that students' problem-solving and PP competency are related (e.g., Silver and Cai 1996), and recently, Xie & Masingila (2017) pointed out that PP and problem solving mutually reinforce one other. More recently, PP activities were found beneficial to elicit students' mathematical understanding and misunderstanding (Nedaei et al., 2022).

In this paper, reflecting on my past research with colleagues in New Zealand (Radmehr and Drake 2017, 2019) and Iran (Nedaei et al. 2019, 2022; Radmehr et al. 2023) on how upper secondary and tertiary students engage with PP activities, I will discuss how PP activities could be used in the teaching and learning of mathematics at the university level. In particular, I will discuss how to design inclusive PP activities for students with different prior knowledge and experience. I will also share how university students perceived their engagement with this type of activity. The suggested design is inspired by the empirical taxonomy of problem-posing processes that Christou et al. (2005) have developed and has been used to explore students' mathematical understanding of integral calculus (Nedaei et al., 2022; Radmehr and Drake 2017, 2019). Furthermore, I will discuss how PP activities could be integrated with inquiry-based learning in the context

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of teaching and learning graph theory (Radmehr et al. 2023). I also discuss how PP activities could elicit students' mathematical understanding and how students' posed problems could be assessed, for example, by focusing on dimensions such as solvability, language and context (See for example, Cankoy and Özder 2017; Nedaei et al. 2022).

I hope this presentation could encourage tertiary mathematics educators and mathematics lecturers to use PP activities more often in research and practice alongside problem-solving activities.

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Workshops

Research on the Quality Monitoring and Evaluation Index of Mathematics Textbooks in Basic Education

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Abstract. The construction of evaluation criteria is an important grip for the quality assurance system of textbooks. Major countries around the world have strategically reviewed and strengthened the construction and development of textbooks and made efforts to improve the quality of teaching materials. In particular, continuously increased the development and improvement of textbooks management methods, quality standards, and evaluation criteria, and have developed characteristics in terms of the basis for the development of textbooks evaluation criteria, evaluation content dimensions and specific indicators, evaluation methods and tools, etc. Analyzing the characteristics of the evaluation standards of textbooks in the world's major countries and learning from the experience of the management of textbooks in developed countries will help improve the quality standards of basic education textbooks in China and strengthen and improve the development, management, and construction of teaching materials in China.

Textbooks are an important vehicle for reflecting the will of the state and passing on national culture. The construction of high-quality textbooks to meet the development of the new era is an important undertaking for the Party and the State. This study focuses on the quality monitoring and evaluation of basic education textbooks, based on survey research and comparative research, and integrates modeling methods and case studies to explore the index system for monitoring and evaluating the quality of basic education textbooks. To address the multi-level and multi-subject characteristics of teaching materials quality monitoring and evaluation, this study combines the recent theories of total quality management, evaluation frontier technologies, and curriculum theory to construct a textbook quality monitoring and evaluation index system, reflecting the performance of textbook quality through the coupling relationship between the elements of textbook quality, examining the implementation of the correct value orientation of textbooks, and monitoring the effectiveness of textbooks in cultivating roots and forging souls, enlightening and increasing wisdom. The project is expected to form a quality monitoring and evaluation index system for basic education textbooks based on the theory of total quality management (TQM), reflecting the state's authority and supported by empirical data, which is expected to be an effective supplement and improvement to the existing system of textbook evaluation, provide strong evaluation theoretical support for the construction of textbook, and echo the strategic needs of national education development.

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1 Basis of Topic Selection

1.1 Present Studies

The issue of quality monitoring and evaluation of textbooks is an important issue raised by Gao. L. & Zhang. T. et al. (2002) for basic education, is one of the key clues to understanding the issue of building textbooks in China. The academic community has taken up this issue, focusing on three main perspectives:

One is the textbook evaluation perspective, i.e., the dimension of textbook analysis and assessment, which examines how the high level of quality of textbooks is influenced by which elements, with curriculum textbook research as the mainstream.

The other is the textbook quality perspective, i.e., the study of textbook quality and its guarantee from the connotation of textbook quality, with editorial and pedagogical studies as the mainstream. The core of textbook quality is to meet the needs of students and teachers and to facilitate teaching and learning, including both intrinsic and extrinsic quality (Kong & Shi, 2007). Intrinsic quality is the basis of textbook quality evaluation (Mei, 2014). The quality evaluation of publications currently conducted in the country mainly focuses on the quality of editors and proofreaders, and it is more important to strengthen the evaluation of the intrinsic quality of textbooks (Zhang et al., 2017). ISO quality management is the way out for quality assurance of textbook publishing (Zhang & Kong, 2005). Grasping the construction of primary and secondary school teaching materials in the new era…the standards and requirements of good textbooks must be clarified (Zheng, 2017).

The third is the perspective of textbook management, starting from improving textbook management effectiveness, researching the effectiveness of textbook management, and quality monitoring, with management and quality research as the mainstream. Textbook construction must actively respond to the challenges of the times and meet national requirements (Shen, 2019), and there should be a clear understanding of the problems in the preparation and construction of textbooks (Han, 2019). The construction of teaching materials is a matter of national interest, and the transmission of mainstream and authoritative knowledge and values by teaching materials is important for achieving moral education (Luo, 2019; Hao, 2020). To achieve a large country of teaching materials to a strong country of teaching materials (Li et al., 2020). To this end, textbook quality construction should follow the rules and strengthen research to promote the quality of textbooks (Tian, 2019); deepen research on e-textbooks and textbook evaluation (Li et al., 2019). Educational administrations and parents are also textbook users; Theoretical research on the quality management of textbooks should be strengthened, and scientific, objective, and authoritative evaluation criteria for textbooks are the core issue in conducting total quality management (TQM) of textbooks, and the development of quality standards for textbooks must be strengthened (Zhang, Wang, & Tian, 2020). Zhao, J. (2005) and Kang, N. & Kong, F. (2018) constructed quality standards for chemistry and music textbooks.

1.2 Academic Value and Application Value

In China, during the 13th Five-Year Plan period, great achievements have been made in the construction of basic education textbooks, with the establishment of a high-profile

National Textbook Committee (NTC) and the newly formed Textbook Bureau of the MOE (Xu, 2020). However, the development of the times, knowledge patterns and competition logic, and the trend of diversification and modernization of textbooks have made the original empirical textbook evaluation method and administrative textbook quality monitoring mechanism highlight many drawbacks, and there is an urgent need to improve the existing mechanisms, develop textbook quality updating mechanism, and establish a textbook incentive mechanism. At the same time, the underlying theory of textbook quality is weak, the synergy between textbook management organizations is pale, and there is a disconnect between policy and practice. The reason for this is that the evaluation standards and methods of textbooks stop at the results formed in the middle of the curriculum reform. Scientific, objective, and authoritative criteria for evaluating the quality of textbooks are the basic basis for the validation of teaching materials (Tian et al., 2020). The "Ninth Five-Year Plan" project developed the textbook evaluation tool for the evaluation system of compulsory education textbooks in China (Gao, 2002). The quality monitoring measures developed by the "Tenth Five-Year Plan" project (Kong, 2007) on the research and practice of the development mechanism and quality monitoring of new curriculum materials for basic education. Research on the practical value of the ideological function of textbooks on the fundamental task of moral education (Zhao, 2017) proposed by the "13th Five-Year Plan" project on the measures of textbooks on the implementation of the fundamental task of moral education. As well as the study on the innovative development of excellent Chinese traditional culture in the construction of textbooks (Tian, 2018), which is a major topic of the "13th Five-Year Plan", "constructing a feedback mechanism for the quality of textbooks to achieve an effective interface between the national will represented by policymakers and the public will" can help solve the above problems. This research will go a step further, teachers and students are textbook users, and education administrative departments, and parents are also textbook users, the quality of textbooks not only needs to meet the needs of teachers and students but also to meet the needs of the country represented by the education administrative departments, and the connotation and evaluation criteria of textbook quality need to be developed. Only clear and consistent, scientific, and objective criteria for evaluating the quality of textbooks can better avoid the different tracks of understanding the quality standards of textbooks and the information asymmetry between the public and the government and can respond to the challenges of the times and meet the national requirements.

The above discussion focuses on theoretical explanation and management level, while textbooks are public goods. The study of textbook quality monitoring and evaluation cannot be limited to textbook construction itself. It is valuable to explore the evaluation of textbooks with the methods of management and evaluation, but it should be placed at the height of state authority, and the quality monitoring and evaluation of textbooks should be considered one of the main clues of the many processes, and the construction of textbook should be viewed in an integrated way.

In summary, this study will take the development of a quality monitoring and evaluation index system for basic education textbooks as a clue, place the evaluation of textbooks in the context of state authority, and systematically explain how the elements of textbook quality affect the quality of textbook, and further investigate what value orientation these elements have, to examine the requirements of textbook in terms of the fundamental task of establishing moral education. The findings of this study can provide theoretical references and tools for monitoring line and evaluating the quality of basic education textbooks as well as other materials evaluation.

2 Research Content

2.1 Research Subjects

The object of this study is the quality monitoring and evaluation index system of basic education textbooks in the context of the new era, which includes four aspects. First, the connotation and legal and academic basis of quality monitoring and evaluation of the textbook. The second is a comparative study of basic education textbooks' American, German, Canadian, and Russian quality monitoring and evaluation standards. Third, to build measurable and feasible quality monitoring and evaluation index system for the textbook that reflects the will of the state. Fourth is the application strategy of the basic educational textbook's quality monitoring and evaluation index system.

2.2 Ideas and Aims

This study is based on a full understanding and thorough grasp of the basic theories of quality monitoring and evaluation in textbooks and incorporates the latest theories of total quality management (TQM) and new techniques of monitoring and evaluation from related disciplines.

1. Clarify the theoretical basis of quality monitoring and evaluation of teaching materials. It will study the theoretical basis of textbook quality monitoring and evaluation from multiple disciplines, perspectives, and dimensions, explore the connotation and nature, types and values, status and functions of textbook quality, monitoring, and evaluation, and explain the necessity and importance of improving the quality monitoring and evaluation standards of basic education textbook in the context of the new era. The study of quality monitoring and evaluation of basic education textbooks is based on multiple perspectives, including total quality management (TQM), curriculum theory, and evaluation theory, respectively, to provide firm theoretical support for the construction of a textbook quality monitoring and evaluation index system that matches the state's authority.
2. Review the current situation of quality monitoring and evaluation of basic education textbooks and sort out the Chinese experience. Review the status quo of quality monitoring and evaluation of basic education textbooks, analyze the existing system navigation mechanism, development quality monitoring mechanism, review and evaluation mechanism, textbook use mechanism, feedback adjustment mechanism, and refine Chinese experience in textbook quality monitoring and evaluation.
3. Compare the quality monitoring and evaluation standards of basic education textbooks in four countries. To study the quality monitoring and evaluation standards of basic education textbooks in the U.S., Germany, Canada, and Russia, and to summarize the experiences that can be used as learning and reference for the quality monitoring and evaluation of basic education materials in China.

4. Construct a quality monitoring and evaluation index system and application strategy for a basic education textbook. Construct a system of indicators for monitoring and evaluating the quality of basic education textbooks based on state authority and oriented to the modernization of education, as well as operational norms and theoretical and juridical bases. Taking mathematics and language textbook and digital textbook as examples, we conduct typical case studies on the specific application of textbook quality monitoring and evaluation indicators and form the basic theory of total quality management (TQM) of teaching materials.

3 Preliminary Conclusion

3.1 Basic Features and Trends in the Quality Evaluation Criteria of Mathematics Textbooks in Major Countries

Through literature research and field studies, we conducted an in-depth study of mathematics textbooks development, use and management policies and implementation, basic theories and methods of teaching materials, validation systems, and evaluation criteria frameworks in major countries (regions) such as the United States, Germany, Canada, Russia, the United Kingdom, Japan, Singapore, Belgium, Denmark, Korea, and (China) Hong Kong, focusing on the latest evaluation criteria for mathematics textbooks in major developed countries. In general, the quality of mathematics textbooks in major countries is evaluated in terms of the following basic features and general trends.

1. Theoretical and methodological support for the evaluation of textbooks has become a consensus in all countries, focusing on new developments in line with students, national standards, and the construction of textbook discourse and the function of discourse

In the early days, the evaluation criteria for teaching materials were process-analytic and descriptiveness, typified by the 'Sussex (UK) program' (Michael Eraut, 1975). After the 1980s, more textbook studies re-examined the status and role of textbooks in education and teaching from a new perspective. Or based on cognitive, psychological, and pedagogical developments, such as the 'Bielefeld Raster' (Eckhardt Fuchs, 2014) and the 'Vienna Kriterienkatalog' (Eckhardt Fuchs, 2014) in Germany, the 'Measurement Instrument for Learning Material Quality' (MILQ) (Arno J.C. Reints, 2009) in the Netherlands, emphasis on examining the quality of teaching materials from the student's or teacher's perspective, based on the laws of student psychological development and teacher education; or based on learning theory and factual logic, for example, the German 'Reutlingen Raster' (Eckhardt Fuchs, 2014)argues that prior educational and theoretical preferences in the evaluation of the quality of teaching materials can exist with different understandings and therefore produce different results in some cases due to the choice of terminology; or according to a Wishlist approach or a combination of qualitative and quantitative approaches, for example, the 'Sussex programme' in the UK and the 'Levanto Tool' (Switzerland) (Eckhardt Fuchs, 2014) argue that textbook quality evaluation is based on the preferences and desires of different user groups, and the possibility of giving some weight to different indicators and learning functions so that uncertainties are diminished or masked. In other words, the shift from this period begins with the establishment of textbook evaluation standards from new perspectives and methods, and new requirements for the preparation, review, and evaluation of textbooks. However, these studies start from the theory and do not recognize the need to start from the selection and evaluation, and the methods are more complicated and have more indicators. Thus, since the 1990s, and especially in the last two decades or so, the analysis and evaluation of textbooks have shifted more towards the analysis of coherence based on the actual needs of the country and the content of curriculum standards or documented policies. For example, using the U.S. 'Project 2061 Materials Evaluation Tool' (Zhang, 2009), 'Quality Instructional Materials Tool' (Zhai, 2019), 'Common Core State Standards for Mathematics' (2013, CCSSO & Achieve, etc.), 'Instructional Materials Evaluation Tool' (CCSSO, CGCS, SAP & Achieve, 2016), High-Quality Instructional Materials (HQIM) (Mississippi First & Mississippi Department of Education, 2019), Textbook Validation Standards in Russia (Liu, 2007), Hong Kong's 'Basic principles of quality textbooks' (Hong Kong Education Department, 2016), and evaluation tools such as 'Textbook Improvement' in Japan (MEXT, 2017). Secondly, Taiwan's 'Textbook Validation Standards' (Taiwan Institute of Education, 2014), Hong Kong, 'Guidelines for Teaching Resources' (The Publishers Association, 2015), and Korea's "Validated Textbook Standard" a representative, focusing on developing students' psychology and needs, emphasizing students' life experiences as the main focus, and providing quality materials to promote students' development. In addition, Germany's 'Textbook Quality: A Guide to Textbook Standards' (Ivan Ivić, 2017) and Pakistan's 'Textbooks Review Criteria' (Khalid Mahmood, 2009), use the theory of verbal genre and the theory of quality management as a new type of textbook evaluation theory, we analyzed the complexity of textbooks in depth based on the fact that they are public goods and developed a textbook evaluation tool that fits the reality of the country (the reality of the region) (see Table 1).

Different research objectives and research content in the field of textbook research use different theoretical frameworks (Wang, 2017). In other words, the theoretical analysis and construction of textbooks are the basis and prerequisite for the evaluation of textbooks and are related to the cognitive and methodological value of the effects of the evaluation. A consensus has emerged in many countries to focus on theoretical support for the evaluation of educational materials. At least three theoretical requirements for constructing a scientific, concise, and applicable evaluation framework are found in combing through the above evaluation criteria for teaching materials: One is the need to comply with the laws of physical and mental development of students and the principles of learning and teaching; the second is to use national curriculum documents and policies, educational theories or the foundation of laws and regulations as the basis for constructing evaluation criteria; the third is to build its evaluation criteria by drawing on and absorbing interdisciplinary systems and discourse systems, functions, etc., such as quality science, management science, and linguistics.

2. The development and updating of evaluation standards for textbooks are mainly based on their textbook review systems, which vary from country to country and involve diverse evaluation subjects, but what is common is that the evaluation standards are based on laws and regulations and the authorization system is relatively complete.

Table 1. Teaching material evaluation theories and methods in major countries

Region	Country	Textbook evaluation tools	Year	Author / Place	Theory / Methodology	Theoretical content/basis
Europe	Germany	Textbook Quality: A Guide to Textbook Standards	2013	Leibniz Institute for Educational Media | Georg Eckert Institute	Vygotsky's theory of cultural psychological tools	Support or amplify individual cognitive abilities and development
					Bachkin's Words Genre Theory	The textbook is a complex narrative structure
		AAER	2017	Carl-Christian Fey		Interdisciplinary analysis of New Tools
		Bielefelder Raster	1986	Universität Bielefeld	Psychology of form and perception, communication aesthetics and physiology	
		Reutlinger Raster	1986	Martin Rauch & Lothar Tomaschewski	Learning theory and factual logic	The evaluator's previous education and theoretical preferences can be understood very differently

(*continued*)

Table 1. (*continued*)

Region	Country	Textbook evaluation tools	Year	Author / Place	Theory / Methodology	Theoretical content/basis
		Wiener Kriterienkatalog	1998	Richard Bamberger	Psychology	From a faculty development perspective
	Russia	Textbook Validation Standards	2004	Federal Textbook Committee		Consistent with national policies and the basic direction of education modernization
	Switzerland	Levanto Tool	2012	Interkantonalen Lehrmittelzentrale der Schweiz	Wishlist Method	Asking user groups about their preferences and desires results in forming the basis
	United Kingdom	Sussex Programs	1975	Sussex,Michael,Len,George	Descriptive	Five key elements
		Guidelines for Teaching Resources	2015	The Publishers Association		Make students participatory
	Netherlands	MILQ	2009	Arno J.C.Reints&Hendrianne J.Wilkens	Psychology, Pedagogy	The possibility of giving some weight to different items and learning functions
North America	United States	Project 2061 Materials Evaluation Tool	1997	AAAS	Science literacy is the center	Strengthening basic scientific literacy
		Quality Instructional Materials Tool	2017	EdReports.org	Principles of Curriculum Standards Alignment Strategy	Textbook evaluation standards are aligned with the Common Core State Standards
		Common Core State Standards for Mathematics	2013	CCSSO, Achieve,etc		Constructivist view and progressive educational thinking
		Instructional Materials Evaluation Tool	2016	CCSSO, CGCS, SAP & Achieve		Useful tools for understanding this consistency
		High-Quality Instructional Materials	2019	Mississippi First & Mississippi Department of Education		Support teachers in selecting and implementing high-quality teaching materials

(*continued*)

Table 1. (*continued*)

Region	Country	Textbook evaluation tools	Year	Author / Place	Theory / Methodology	Theoretical content/basis
	Canada	Guidelines for Approval of Textbooks	2008	Ontario Ministry of Education		"Learning for all students" is the main focus
Asia	Japan	Compulsory education books for school materials identification standards	2017	NEXT	Basic Law of Education, School Education Act, and Study Guide	Respect for independent persons and the public spirit in the harmony of knowledge, virtue, and body
	Korea	Validated Textbook Standard	2013	Korea Ministry of Education		Objectivity and correctness of the facts, concepts, theories, etc. used
	Hong Kong	Basic principles of quality textbooks	2016	Hong Kong Education Bureau		Quality textbooks with a student-centered curriculum
	Taiwan	Textbook Validation Standards	2014	Taiwan Institute of Education	Pedagogy, Psychology	Connect with students' life experiences
	Pakistan	Textbooks Review Criteria	2009	Curriculum Wing (CW) of the Ministry of Education	Quality Management	Teaching materials should meet the needs of the primary users

In Germany, each federal state develops its state syllabus according to the actual situation of the state, and the publishers convene professionals in the relevant disciplines or teaching fields to prepare textbooks according to the syllabus. The content of the teaching materials is designed and organized by the writers in each subject area according to the competencies and subject requirements specified in the syllabus and curriculum plans. Second, the development of educational materials must be consistent with the legal basis and determine the consistency of educational objectives (Sun, 2020). In the UK, the publication and selection of primary and secondary school textbooks do not need to be validated by the government. Writers simply need to write the textbooks by the national curriculum standards and they can be distributed publicly (Yang, 2018). To improve the quality of textbook development, the British Publishers Association and the British Educational Suppliers Association jointly published guidelines for textbook development in 2015.

In Russia, the Federal Textbook Review Agency consists of three parts: the departments of the Ministry of Education and Science of the Russian Federation, the Textbook Validation Center, and the Russian Textbook Validation Committee. Textbooks of compulsory subjects for the whole Federation are issued by the Federal Education, while textbooks with strong national, regional and local expression are issued by local or republican education departments (Shen, 2001).

In Alberta, Canada, the Alberta Department of Learning (the provincial executive) approves textbooks for core subjects (e.g., math, English, etc.) under the "authority" of the Minister of Learning, and there is a list of alternative textbooks and resources for schools or teachers to choose from, which means that Alberta manages and monitoring the source of the materials, thus ensuring the quality of the materials. The Ontario Ministry of Education has developed the 'Ontario Ministry of Education Guidelines for the Approval of Educational Materials (2008)', which provides details on the "legal basis" for educational materials (Hu, 2017).

The evaluation standards for textbooks issued in the United States can be divided into those issued by industry organizations, those issued by local education departments, and those issued by educational consulting organizations, depending on the organization that issues them. The evaluation standards for textbooks issued by industry organizations are generic standards for printing, binding, publishing, and distribution of textbooks, drawing on the technical specifications of the publishing industry. For example, the U.S. Advisory Committee on Textbook Specifications has three bodies under it: the National Textbook Review Association, the Book Manufacturers Association, and the American Publishers Association. They determine the durability of primary and secondary school materials and other relevant performance factors by conducting relevant research and testing and making recommendations on the manufacturing process and standards for materials to ensure appropriate quality and performance standards. The Common Criteria Framework for the Vetting of Instructional Materials, published by the National Association of State Textbook Administrators, presents informative criteria for evaluating instructional materials in five areas: content, equity and attainability, evaluation, organization and presentation, and instructional design and support (NASTA, 2012). The Educational Advisory Service (EAS) publishes the Textbook Evaluation Standards, which are textbook evaluation standards developed by non-profit educational organizations and institutions that bring together individuals with extensive educational and publishing experience to provide a third-party independent evaluation of the quality of textbooks published in the United States in various subjects. The American Association for the Advancement of Science (AAAS) has launched a series of evaluation criteria and tools in support of Project 2061 (AAAS, 2010), including materials evaluation criteria that address both the content analysis and instructional analysis dimensions. Nearly 30 states in the U.S. have implemented textbook review systems, with the textbook review standards and management practices of California, Florida, and Texas having a large impact on the United States (Yang, 2013). The state of Massachusetts is an open area with no centralized textbook procurement and the selection and purchase of textbooks and other instructional materials is a local district activity, however, centers such as the National Center on Accessible Educational Materials (AEM) exist to identify and produce accessible materials that meet certain standards (AEM, 2014). The Mississippi Department of Education (MDE) has adopted High Quality Instructional Materials (HQIM) to ensure that teachers have access to high quality materials. In 2019, the Department of Education (MDE) worked with Mississippi teachers and EdReports to develop and publish Mississippi's standards for reviewing high-quality instructional materials (MDE, 2019). Based on the reform of curriculum standards and the change of the traditional textbook review system, Tennessee's textbook review system has made innovations in clarifying the main responsibilities of the three levels of textbook review institutions, setting textbook review standards based on standards, giving the public the right to monitor and comment on the textbook review, and improving the feedback mechanism for textbook review, to provide institutional guarantee for the supply of quality textbooks (Bamberger, 1998).

Textbooks in Hong Kong are approved by the Education Bureau of the HKSAR Government based on formal quality standards and consistency with the Hong Kong curriculum. The Hong Kong Government's primary role in the supply of textbooks is to review the materials submitted by publishers and to include those materials that meet the requirements of the relevant curriculum guides and prescribed standards in the "List of Applicable Titles" and "List of Titles for Use of Electronic Teaching Materials" for schools to choose from (Hong Kong Education Bureau, 2016). In Singapore, the Ministry of Education (MOE) is given "approval authority" for textbooks, hiring publishers to develop materials based on the syllabus, and the quality of teaching materials is subject to a rigorous materials review process. The review process is repeated several times before the materials are approved and placed on a list of materials approved by the Ministry of Education for schools to select (Charlotte Eyre, 2016). Specifically, Singapore schools are not legally empowered to use educational materials, but if they do, they must use approved materials. Secondly, in Singapore, as in Hong Kong, the development of electronic resources has tended to improve the approval system.

Despite geographical differences and different policies, it has become a consensus across countries to continuously strengthen the revision of textbook policies and the implementation of policies to guarantee the quality of textbooks (see Table 2).

Many countries have well-developed systems and standards for the review and validation of educational materials. The textbook review system involves the public education system at the national level, by intervening to ensure the development of management practices, quality standards, and evaluation criteria for its textbooks. In other words, the criteria for evaluating textbooks contain the expectations of state power organs, academic institutions, social and cultural groups, etc. about the value orientation of textbooks. The textbook system is not only the institutional text in the national textbook review and selection process, but also the institutional practices behind the textbook system, such as the educational system, publishers, economic market, and third-party institutions, to fully understand how the textbook system forms a holistic picture in each country's context.

3. Textbook evaluation tools focus on normative, scientific, educational, and concise, and evaluation dimensions and criteria are refined and classified according to different perspectives, purposes, roles, and natures, and are more normative and scientific

Europe, represented by Germany, remains relatively mature in the study of textbook evaluation tools. Extensive evaluation methods had already emerged in Germany in the 1980s, such as the Bielefelder Raster, which includes five dimensions (metatheory, materials design, subject matter science, subject pedagogy, and educational science), each containing 10 to 30 categories, which are subdivided into individual questions within most dimensions, so that the Bielefeld Raster includes approximately 450 individual questions. These questions are theoretically justified and the evaluator is guided step by step to increasingly specific units of analysis. In this way, the subjective and normative parts can be minimized and a more transparent evaluation result can be ensured. Richard Bamberger points out that the Bielefeld Raster is too extensive and complex and thus hardly used by schools, publishers, authors, and teachers (Bamberger, 1998). The Reutlinger Raster consists of nine categories (bibliographic information, purpose, content, pedagogy, object, design, text, task, image, picture, or text), each formalized by three to 13 declarative sentences. Karin Brock points out that "the declarative form of the categories changes the character of the analysis compared to Bielefeld's Raster. Through the declarative form and strict quantification, uncertainties are concealed and qualitative features disappear" (Karin Brock, 2001). Based on this two Raster, the Vienna Textbook Institute has developed a practical and short evaluation guide for the teacher system, consisting of a general list of 40 criteria, each of which can be answered on a scale of 1 to 5. In contrast to the Raster of Bielefeld and Reutlingen, this Raster is explicitly developed from the perspective of teaching and learning for teachers. The brevity and conciseness of the standard seem to lend themselves to practical use. In recent years, the University of Belgrade and others published "Textbook Quality: A Guide to Textbook Standards", which consists of seven dimensions (quality of textbooks, quality standards of textbooks as student books, quality standards of thematic units, quality standards of content of textbooks, quality standards of pedagogical aspects of textbooks, quality standards of language of textbooks, quality standards of electronic components of textbooks and quality standards of hanging textbooks) with a total of 43 indicators, covering the selection, organization, and presentation of knowledge in subjects, acquisition, and consolidation of skills, development, and penetration of values, building a more comprehensive and systematic quality standard of textbooks. In addition, the Augsburger Raster, based on a more recent (mainly post-PISA) review of educational quality publications, proposes eight dimensions to describe different aspects of educational materials and other educational media, each dimension consisting of 7 to 12 items. The Swiss Interkantonale Lehrmittelzentrale developed the ‘Levanto tool’ with 52 indicators in three dimensions (pedagogical and pedagogical domains, subject matter content domains, and formal design domains) for cantonal and national textbook publishers. The tool is designed to support the development of new teaching materials and the evaluation of existing materials by establishing certain criteria. The program was developed using a wish list approach, where user groups were asked about their preferences and desires, and the results formed the basis.

In North America, represented by the United States, instructional materials evaluation tools can be divided into those issued by industry organizations, those issued by local ministries of education, and those issued by educational consulting organizations, depending on their issuing agencies. ‘The Quality Instructional Materials Tool’, published by an Educational Consulting Agency that evaluates instructional materials as a third party, contains 64 indicators in three dimensions (text quality, design of learning activities, and support for teacher and student development), highlighting the supportive evaluation of instructional materials for teacher and student teaching and learning, and emphasizing “evidence-based” evaluation of instructional materials through a rigorous and standardized process. Taking the Instructional Materials Evaluation Tool (IMET) led by the U.S. federal government and jointly developed by CCSSO, the Council of College-City Schools (CGCS), the Alliance for Student Achievement (SAP), and Achieve, a third-party educational organization, as an example, the IMET consists of three dimensions (non-negotiable alignment criteria, alignment criteria, and quality criteria) to build the Instructional Materials Evaluation Tool for grades K-12. Based on the in-depth integration of theoretical connotations and goals of curriculum materials alignment, the most effective way of change is identified to provide materials for states, districts, and schools to successfully secure reliable evaluation results in practice.

The The‘Basic Principles of Quality Textbooks’ in Asia, represented by the Hong Kong region, consists of 7 specific guidelines with 44 indicators, which mainly describe the main principles of selecting, writing, and evaluating quality textbooks, with Hong Kong guidelines or subject-specific textbook development guidelines for different learning areas or subject curriculum details. It is worth noting that the Korean and Russian textbook standards are more focused on the correctness of the content to provide a solid foundation for students to build a scientific and cultural system of knowledge. At the same time, Russia intensified Soviet-style academic censorship of textbook content to guard against Western use of Russian textbooks to distort Russian history; and published patriotic textbooks to strengthen patriotic education.

The construction of the textbook evaluation criteria is the core and main element of the evaluation. Different countries and regions develop their unique and different dimensional evaluation criteria for textbooks based on their economic, cultural, scientific and technological, remediation, and educational development. Therefore, the development of a definite, clear, operable, and concise textbook evaluation dimension and index system is a key element in improving the reliability and validity of evaluation and guaranteeing the quality of textbook construction. In other words, how to construct specific textbook evaluation dimensions and evaluation index systems in response to different evaluation purposes is the key point of current research on textbooks around the world. To sum up the index combing analysis, there are a hundred different evaluation dimensions and indicators for the evaluation of textbooks in different countries due to different geographical, economic, and political backgrounds, but the evaluation dimensions include at least the following five aspects.

One of them is the evaluation of the editorial design (including language, fonts, illustrations, graphic elements, etc.) of the printed text of the educational material for correctness, standardization, science, and conciseness.

Second, the evaluation of the organization and layout of the materials, including the rationality and appropriateness of the content and structure of the materials.

Third, the evaluation of the teaching and learning attributes of the textbooks, including the impact of the correctness, volume, difficulty, presentation, exercises, and homework settings of their content on the physical and mental development of students.

Fourth, an evaluation of the physical characteristics of the materials, such as printing quality, typographic design, color coordination, etc.

The fifth is to cater to the emergence of modern digital media and to evaluate the management, design, and functionality of electronic textbooks.

Evaluation indicators include at least four features: First, the evaluation of textbooks focuses on the relevance of the materials themselves; Second, evidence-based indicators and procedures for evaluating educational materials; Third, the evaluation index of textbooks is scientific and reasonable, operational and comprehensive in coverage; Fourth, the teaching guidelines of the material evaluation are strong.

4. Most countries and regions (provinces/states) use "content", "instructional design", "publication design", and "bias issues" as the main dimensions of the evaluation criteria framework for textbooks.

The evaluation criteria frameworks for textbooks developed in different countries and regions in the context of their own cultural, historical, economic, and educational development are distinctive from their form to their content, and the evaluation frameworks in different provinces (states) of the same country vary significantly, even in the formulation of words on the same dimension. For example, the terms used for the instructional dimension include instructional characteristics (signs), teaching and learning design, and instructional attributes. In-depth analysis revealed that almost all of the evaluation indicator frameworks addressed the dimensions or elements of "content," "teaching," "publication," and "avoidance of bias and discrimination". The criteria for evaluating Russian textbooks include three dimensions (Liu, 2007): the basic idea of the textbook, the content of the textbook, and the instructional materials of the textbook, among which the "content" dimension concerns the scientific, contemporary, complete, systematic, linguistic, aesthetic, and adaptation to the student's level and the connection with supplementary textbooks. The content on bias and discrimination generally involves specific indicators of ethnicity, race, national culture and multiculturalism, religious beliefs, gender or age, socio-economic status, family structure, disadvantaged groups such as the disabled, language use, etc., to avoid discrimination, exclusive or wrong values, inappropriate language, photos and illustrations in teaching materials, and to eliminate the appearance of violence, mysterious content and all kinds of advertisements, etc. (Hu, 2015). In some countries' frameworks for evaluation criteria for educational materials, even if bias and discrimination are not included as evaluation dimensions, the specific evaluation indicators involved are integrated into other dimensions (e.g., the content of materials, language use, values, etc.).

Pakistan proposes criteria for evaluating teaching materials from a qualitative perspective, arguing that teaching materials should meet the needs of their primary users, i.e., teachers and students, and proposes to evaluate the quality of teaching materials in terms of performance, characteristics, reliability, consistency, durability, usefulness, aesthetics, and perceived value, with the quality of content and quality of expression at the core (Mahmood, 2009).

Researchers in Turkey, on the other hand, have constructed textbook evaluation systems from the perspectives of both teachers and students, respectively. The evaluation system from the teacher's perspective is divided into five dimensions: physical appearance, content selection, presentation, language, teaching aids, and guidance effect on students, with no secondary dimensions and direct follow-up on 18 specific criteria. The student perspective evaluation system has no overarching dimensions and is presented directly on six specific criteria, including that the illustrations were useful, the content was difficult to understand, it was difficult to interconnect the topics, the learning questions were useful, the topics were organized in a way that helped me think, and the material enhanced my interest in the course (Ali Yildirim, 2006).

5. Specific evaluation indicators emphasize the presentation of up-to-date knowledge, interdisciplinary integration, and assessment of learning progress, focus on the development of key competencies, and advocate a variety of teaching strategies and learning styles, thus highlighting the inquiry, constructive, activity, and dialogue nature of the materials

With the introduction of 21st-century skills, key competencies, core literacy, cross-disciplinary interdisciplinary education, and reforms in teaching and learning methods, more emphasis has been placed on the specific indicators of the evaluation criteria for teaching materials on providing as many appropriate opportunities, pathways, and strategies as possible to create appropriate learning environments to support the cultivation and development of students' abilities such as critical thinking, effective communication, innovation, problem-solving, and scientific argumentation, and to achieve interdisciplinary or cross-disciplinary integration. The design of tasks and problems in the materials should help students experience and achieve cooperative learning, discovery learning, inquiry learning, and interactive teaching and learning. The content and design of the materials should promote the organic integration of teaching and learning strategies with assessment strategies and provide adequate and appropriate assessment methods and tools.

For example, the sub-dimension "Accuracy and current state of knowledge in textbooks" of the German textbook content quality standard require the inclusion of accurate and up-to-date knowledge of the subject to which it relates, following the current state of development of the subject and providing the most up-to-date knowledge in the field. The other two sub-dimensions "horizontal relevance: linking topics that are similar to related content"; "Vertical relevance: cross-grade, cross-sectional, and cross-level linkages of related content within the same discipline" emphasizes the integration of the same content across disciplines and years. It can be seen that, under the goal of cultivating students' literacy, the emphasis on the content and form of textbooks should be more inquisitive, constructive, active, integrated, thematic (theme), and dialogical (interactive), which provides an important basis and creates a wider space for the formation and strengthening of the characteristics of textbooks.

3.2 Improve and Innovate the Quality Evaluation Standard of Basic Education Textbooks in China

The evaluation criteria for textbooks are not only an important part of the teaching materials management methods and regulations but also a ruler to monitor the quality of teaching materials, as well as a basis, requirement, and guidance for the preparation and publication of teaching materials. To address the weaknesses of the evaluation criteria for textbooks in China and to critically draw on international experience, this study has developed the " Evaluation criteria for the quality of basic education mathematics textbooks in China (Preliminary Draft)".

With the design concept of focusing on the content to form textbook quality evaluation criteria as comprehensive, scientific, and concise, and with the goal of fairness, impartiality, and quality assurance, the content framework of mathematics textbook quality evaluation is constructed, mainly including two dimensions of intrinsic quality and extrinsic quality. Among them, the intrinsic quality includes five sub-dimensions: the value orientation of the textbook, the consistency of the textbook with the curriculum standards, the selection of the content of the textbook, the presentation of the content of the textbook, and the guidance of the textbook for teaching and learning; Extrinsic quality includes two sub-dimensions: editing and design of teaching materials, and feasibility; Several specific indicators and levels of conformity are included under each sub-dimension (see Table 3). In particular, we add the dimension of "value orientation of textbooks" to fill the gap; we emphasize the consistency between textbooks and curriculum standards, strengthen the role of curriculum standards in guiding the development of textbooks, and create space for the formation of textbook characteristics. At the same time, we strive to transfer the results of reform in the field of curriculum and teaching to the actual classroom teaching through the development and evaluation of textbooks and give full play to the role of the bridge and link of textbooks. It is expected that the design of this evaluation criterion will provide a reference for the soundness and improvement of the review criteria and merit assessment of mathematics textbooks in primary and secondary schools in China.

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Table 2. Textbook review systems in major countries (regions)

Region	State	Textbook Review System
Europe	Germany	Each federal state develops its state syllabus based on the actual situation of the state, and each state is reviewed by the Ministry of Education before teachers can choose to use it
	Russia	The Federal Ministry of Education issues compulsory subjects, and materials with strong local, national or regional characteristics are issued by local or republican education departments
	UK, Finland, Sweden, Netherlands	The publisher is the main developer, writer, and organizer of teaching materials. The power to select teaching materials rests with local school principals and teachers
	France	The textbook recognition committee of each county will determine that they are qualified before they can be used by schools and teachers of their choice
North America	United States	Develop curriculum standards at the national level, while state education boards and third-party agencies develop and publish catalogs of qualified materials based on national or state curriculum standards on a selective basis, depending on the state's situation
	Canada	Development and publication of a catalog of qualified teaching materials by the Education Commission of each state Ministry of Education for schools and teachers to choose from
Asia	Singapore, Korea, Hong Kong	Prepared and distributed by the Ministry of National Education (regional education authorities)
	Japan	Development and distribution of educational materials by private educational materials publishers

Table 3. Evaluation criteria for the quality of basic education mathematics textbooks (preliminary draft)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
Intrinsic Quality	**1. Value orientation of textbooks**	1.1 According to the nature and tasks of mathematics subjects and the age characteristics of students, infiltrate the core socialist value system, strengthen education on patriotism, revolutionary traditions, excellent Chinese culture, and Chinese national community consciousness, enhance the cultivation of students' basic moral qualities, reflect national and ethnic foundation values, and reflect the achievements of human knowledge accumulation and innovation					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		1.2 Mathematics textbooks should contribute to the promotion of national identity and national cultural identity; mathematics textbooks should highlight Chinese elements, reflect excellent Chinese culture, reflect the cultural value of mathematics and the beauty of mathematics					
		1.3 Facing all students and promoting their all-round development					
		1.4 Reflect the spirit of the times, encourage students to explore and innovate, and promote their personality development					

(*continued*)

Table 3. *(continued)*

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		1.5 Demonstrate adherence to the equal rights and duties of all citizens					
		1.6 The content, examples, tasks, and models explained in the materials should be derived from a range of socio-cultural contexts, and the different socio-cultural contexts of the users of the textbooks should be taken into account when selecting such content					

(continued)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		1.7 The content of textbooks should take into account students' specific characteristics, such as emotions, preferences, and motivations; they should not be limited to a neutral presentation of academic knowledge and, where possible, content should be tailored to students					
		1.8 The content of the textbook and its interpretation should be socially relevant, showing society the specific value of its content and how that content is relevant to daily life					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		1.9 All value content disseminated by the textbook should be consistent with the character and goals of the mathematics discipline at a given educational level, and the meaningful message of that content should be consistent and capable of creating a value system					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		1.10 Textbooks shall not discriminate based on special status such as race, national origin, ethnicity, language, culture, religion, social status, gender, age, physical or mental disability, illness, homeless status, etc., and textbooks shall not in any explicit or implicit way disseminate racism, nationalism, chauvinism, sexism, racial or religious hatred, or any form of religious or other interest of individuals or groups, textbooks must not contain names or labels that may be offensive to members of any society or ethnic group, and **textbooks should respect diversity**					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
	2. Textbook Consistency with curriculum alignment	2.1 The overall design reflects the basic concepts of the curriculum standards					
		2.2 Arrangement and presentation reflect the course objectives defined by the curriculum standards					
		2.3 The breadth of coverage is in line with the content scope specified in the curriculum standards					
		2.4 The level of difficulty meets the requirements specified in the curriculum standards					
	3. Selection of textbook content	3.1 The design of the content has a holistic nature					
		3.2 Rigorous and precise content, reasonable structure, rich in modernity					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		3.3 The content is close to the reality of life and deals with the problems encountered in daily life					
		3.4 The choice of content material is broad and can be combined with students' life experience					
		3.5 The content is contemporary and reflects the connection with social progress, technological development, and information technology					
		3.6 The depth and breadth of content match the different cognitive and linguistic levels of students					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		3.7 The content has sufficient complexity and is based on the student's existing knowledge and skill base					
	4. Presentation of textbook content	4.1 Content presentation cautiously treats network language					
		4.2 Content is presented using language, symbols, and terminology that students can understand					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		4.3 The presentation follows the cognitive development of students, and the content arrangement and contextual design are conducive to stimulating students' interest, enhancing their engagement in learning, promoting active study and deep understanding, and fostering the spirit of innovation					
		4.4 Illustrations are closely related to the content of the text, which is conducive to students' understanding and learning					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		4.5 The expression of the content is clear, accurate, vivid, and readable; there is no unnecessary repetition and cross					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		4.6 The thematic unit structure of the textbook is clear, relevant, consistent, and holistic: each thematic unit and its components must have a clear function and purpose, clearly label its most important parts and what is expected of the students, connect content-wise or logically to the preceding and following parts of the textbook, and establish an overall connection between knowledge in a given area; Thematic unit content must be presented with a clear and coherent structure, using appropriate language and graphic devices; coherence involves interconnected themes and issues, a logical flow and the development of thematic content, language, and illustrations that are consistent with the thematic structure and flow; The structure of thematic units (text, illustrations, summaries, questions, and tasks, etc.) must be visually marked in a clear manner and throughout the book; avoid overuse of certain components, or visual monotony and poor structure					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		4.7 Activity design is based on the curriculum standard expectation of knowledge and ability requirements, learning activities (including experiments, discussions, etc.) are diverse, designed with clear purposes and reasonable requirements, and are operable					
		4.8 Reasonable column design and a lively form					
		4.9 Exercises contribute to the development of student's abilities such as comprehension, application, and expansion, and are appropriate in number and difficulty					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		4.10 Pay attention to the connection between textbooks and exercise books, and between this subject and related subjects					
		4.11 Smooth articulation within and between chapters					
		4.12 No sexist content					
		4.13 Respect and care for people with disabilities					
		4.14 No scientific and common sense errors, no descriptions involving violence, pseudoscience, mystification, offensiveness					
		4.15 No commercial promotion of any kind					
		4.16 Appropriate language selection and use					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		4.17 Effective use of various appropriate media and platforms					
		4.18 Color and layout design applications are vivid, practical, and focused					
		4.19 illustrations, charts, photographs, etc. are clear and accurate, in line with the requirements of the layout and related regulations					
		4.20 font, font size, line spacing, white space, and other designs in line with the actual students of the corresponding school section					
	5. The guidance of textbooks for learning and teaching	5.1 Textbooks are attractive and stimulate students' desire, interest, and curiosity to learn					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		5.2 Provide "scaffolding" to help students to learn, guide their learning and thinking, and deepen their understanding					
		5.3 Provide students with hands-on learning opportunities and design a variety of practical activities and assignments, such as individual, group, oral, written, pictorial, physical, in-class, and out-of-class, to encourage and support students to conduct investigative learning activities and develop their ideas and opinions					

(*continued*)

Table 3. *(continued)*

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		5.4 Guide and support teachers to choose appropriate instructional strategies according to course objectives and teaching objects, and inspire teachers to design and organize learning activities with active student participation					
		5.5 Provide curriculum resource clues to guide teachers and students through their teaching tasks					

(continued)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		5.6 Guide and support teachers to choose appropriate ways to instruct students to assess and summarize the process and results of learning according to different learning objectives and contents					
External Quality	**6. Editing and design of textbooks**	6.1 Textbook size, shape, paper quality, and print quality to meet the convenience of students to carry, practical, durable, and environmentally friendly					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		6.2 The quality of printing, cover, binding, and packaging of textbooks is excellent and can satisfy the frequent and lasting use and preservation of most students					
		6.3 The layout design is clean and attractive, with appropriate text and graphics, high-quality illustrations, appropriate quantity, and clear images					
		6.4 Ability to treat cartoon characters and virtual pictures with caution					
		6.5 Consistent identification of various symbols, punctuation, numbers, and units of measurement using standardized					

(*continued*)

Table 3. (*continued*)

Category		Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
		6.6 Text error rate of not more than 0.25 parts per million					
		6.7 The components of teaching materials resources (supporting teaching materials, teaching aids, learning aids, CD-ROMs, digital resources, etc.) are well designed and convenient to match, and these materials are clearly organized, logical, easy to use, and reasonably priced					
		6.8 The links and websites provided by the textbook comply with the relevant regulations and usage policies					

(*continued*)

Table 3. (*continued*)

Category	Specific indicators	Failure 0 points	Basic Qualified 1 point	Qualified but not distinctive 2 points	Qualified and locally distinctive 3 points	Qualified and distinctive 4 points
7. Feasibility	7.1 The extent to which textbooks are aligned with the development of the times and educational needs					
	7.2 The degree of adaptation of textbooks to the economic and social development of the areas where they are used					
	7.3 Degree of conformity of textbooks to actual use					
	7.4 Degree of adaptation of textbooks to teachers' and students' levels					

Oral Communications

Investigating the Features of Nepali 8th Grade Mathematics Textbook

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Abstract. A textbook is considered to be one of the primary resources for teaching and learning mathematics around the world. The role of textbooks is even more important in developing nations such as Nepal, where the textbook is the main curriculum resource. Thus, it is important to have rich mathematical tasks in a textbook to enhance the teaching and learning of mathematics. The rich mathematical task includes various features such as the usage of appropriate representations, a combination of contextual vs. non-contextual tasks, a balanced number of open vs. closed-ended problems, and utilization of high vs. low cognitive demand tasks. Using a multi-dimensional framework, we examined 1359 mathematics problems from an 8th-grade textbook published by the government of Nepal. The study revealed that the textbook did not include problems based on the various features of rich tasks. 87% of the problems are of low cognitive demand and non-contextual, and almost all problems are closed-ended.

1 Introduction

Textbooks are one of the crucial elements of curriculum reforms and are considered to have a significant role in implementing curricula around the globe (Valverde et al. 2002). In fact, "good textbooks are more important for high attainment in mathematics than factors such as setting or expensive IT equipment" (Howson, p. 649). In the last couple of decades, studies have emphasized the importance of problem-solving and mathematical reasoning in textbooks, which further highlights the importance of textbooks in teaching and learning mathematics (Fan 2013). The emergence of technology in the last few decades enhanced traditional textbooks in multiple ways. However, in Nepal, due to limited access to modern technology among other several reasons, traditional textbooks are an integral part of teaching and learning mathematics. Reports show mathematics textbooks are used as a primary (sole for most cases) resource by many teachers in Nepal.

The National curriculum framework (NCF 2007) of Nepal outlines the overarching goal of school education. The NCF aims to produce critical thinkers and innovators to fulfill the need for a highly qualified workforce. The National Assessment of Student Achievement (NASA) in Nepal follows the assessment guidelines similar to the Programme for International Student Assessment (PISA) to assess students' success in grades 5, 8, and 10 in three different areas: mathematics, reading, and sciences. Students' performance in mathematics is on a downward trend, as shown by the recent NASA

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results. Most eighth-graders are incompetent in solving cognitively high-demand tasks and contextual problems (ERO, 2013). Furthermore, the majority of statistics problems in school textbooks in Nepal demand mathematical calculation, whereas other phases of problem-solving such as data collection, question formation and result interpretation are rarely emphasized (Jones and Basyal 2019).

Studies (Manandhar et al. 2022; Khanal et al. 2021) show that students are encouraged and competent in routine, low-demand, and non-contextual problem-solving. Moreover, it is documented that mathematics textbook authors in Nepal need more didactical knowledge and skill that is required to write appropriate textbooks (Basyal and Mainali 2023). Therefore, it is natural to ask questions such as: does the mathematics textbook equally include various types of problems? Do the textbooks contain tasks that require higher vs. lower cognitive skills? Do textbooks consist of problems that enhance both the conceptual understanding and procedural fluency of students? Are the given tasks connected to the real-life context? In this regard, this study sought to investigate the types of mathematical tasks included in an 8th-grade mathematics textbook. In particular, we examined the various features which can be used to pose mathematical tasks while presenting the problems in the textbook. In particular, we investigated various features of the problems, such as contextual vs. non-contextual, open-ended vs. closed-ended, and high cognitive demand vs. low cognitive demand. We also investigated the uses of representations in mathematical tasks.

2 Brief Literature Review

The textbook is one of the most important resources for teaching and learning mathematics because it provides an opportunity to enhance both teaching and learning. In fact, the rich mathematical tasks help engage students and enhance mathematical reasoning (Gracin 2018). Features of rich tasks consist of: essential, authentic, rich, engaging, active, feasible, equitable, and open tasks (Stenmark 1991). Incorporating these various features is equally important for rich mathematical tasks. Many of these rich task features are directly connected with mathematical problems related to such as cognitive demands, context, response type, and representation (Gracin 2018).

Many earlier studies reported that textbooks need to include tasks with the right balance of the cognitive load, open and closed-ended problems, contextual and non-contextual problems, and problems with multiple representations. For example, Stein and Lane (1996) reported that a high cognitive demand problem positively impacts students' conceptual understanding of mathematics. Open-ended problems stimulate students' mathematical creativity by stimulating divergent and flexible thinking (NCTM 2000). Contextual problems enhance accessibility by making the problem transparent, suggesting solution strategies, and can be solved on different levels (Van Den Heuvel-Panhuizen 2005). Various representational forms in mathematics help build an understanding of mathematical concepts (Zazkis and Liljedahl 2004). Thus, it is apparent that the appropriate usage of these various features while posing mathematical tasks in the textbook are essential components for effective teaching and learning mathematics.

2.1 Theoretical Framework

The mathematics problems are categorized based on the cognitive complexity students face while working to produce a solution. The four levels of cognitive demand of a task are: (1) memorization, (2) procedures without connections, (3) procedures with connections, and (4) doing mathematics. The former two are categorized as low cognitive demand-task, and the latter two are considered high cognitive demand-task (Stein & Smith, 1996). Furthermore, mathematics problems can also be categorized as contextual if a problem is context-embedded and non-contextual if a problem does not include a context. Context-embedded problems refer to what extent real-world experiences are incorporated into tasks, while non-contextual problems are more generic math tasks that associate more computational processes, not connected to real-life experiences. Normally, open-ended math tasks refer to problems with several or many correct answers, whereas closed-ended mathematical problems have only one answer. Similarly, representation is a crucial element for teaching and learning mathematics and is normally divided into four categories: symbolic, visual, verbal, and combined (NCTM 2000). The symbolic representation indicates the use of symbols and formulas in the problem. The verbal representation includes written and spoken languages. The visual representation includes various types of visual forms such as pictures, diagrams, and other figural representations. Mathematics problems normally require more than one mode of representation labeled as a combined form.

3 Methods

Eighth grade is a crucial doorstep for high school mathematics in the Nepali education system. Also, the NASA results show a downward mathematics performance trend by Nepali 8th graders. The entitled "My Mathematics" grade 8 textbook is published and distributed by the government of Nepal, and the book is used in almost all public schools in Nepal. Thus, we chose to examine the problems in the 8th-grade mathematics textbooks that students are expected to work with.

We developed tentative guidelines based on the related literature and theoretical framework. Then, each researcher independently coded a sample of 100 problems into four categories: usage of representation, high and low cognitive demand tasks, contextual vs. non-contextual problems, open vs. closed-ended problems. The initial analysis of the 100 problems indicated 87% agreement in terms of coding by both researchers. Both researchers discussed discrepancies encountered for the rest of the 13% problems and reached a general consensus. For the analysis of the remaining problems, we agreed to follow the same guidelines that emerged based on the first 100 problems.

4 Results

The analysis of the data revealed that low cognitive-demand problems make up the majority of the problems. In the high cognitive demand category, only 13% of problems are procedures with connection, and 87% of problems ask students to either recall the memorized facts or to find a solution to a problem without making a connection to the underlying concepts. No high-level 'doing mathematics' problems are identified in the textbook.

Table 1: Distribution of features (in %) of the tasks in grade 8 textbook

Cognitive Demand	Memorization (6.18)	Procedures without Connections (80.79)	Procedures with Connections (12.88)	Doing Mathematics (0)
Representation	Symbolic (35.47)	Visual (12.66)	Verbal (22.15)	Combined (29.73)
Response Type	Open ended (0.59)	Close Ended (99.41)		
Context	Contextual (13.24)	Non-contextual (86.76)		

Problems are represented in many different formats: symbolic, verbal, visual, and combined form. Most problems are given in the symbolic form, and most symbolic forms problems are concentrated in algebra chapters. Fewer problems are in visual form, and most visual form problems are given in geometry chapters.

Most problems (87%) are non-contextual and contextual problems are concentrated in chapters such as profit and loss, unitary method (ratio and proportion), and simple interest. The summary of the findings is presented in Table 1. Almost all problems are closed-ended. Only eight problems out of 1359 are open-ended.

5 Discussion, Implication, and Limitation

The rich mathematical task requires a combination of various features in mathematics textbooks. The study, however, revealed that there is a lack of balance in different features necessary for the rich mathematical tasks in 8th grade mathematics textbook. Most tasks in grade 8 textbooks are of low cognitive demand confirming the earlier results (Basyal et al. 2023). Moreover, most of the problems are non-contextual and closed-ended. The fact is that open-ended problems stimulate students' mathematical creativity by stimulating divergent and flexible thinking (NCTM 2000). The incorporation of visual representation in mathematical tasks is relatively lower than other types of representations. Thus, there is a disbalance in the utilization of different modes of representation. However, it is important to have balance in utilizing various types of representations in teaching and learning mathematics (Mainali 2021). The 8th-grade textbook likely needs to include various modes of representation as well as more open-ended problem to be a more effective textbook.

Students focus more on drill and practice strategy to solve mathematics problems in Nepal (Manandhar et al. 2022; Khanal et al. 2021). One of the reasons for focusing more on drill and practice approaches could be the lack of incorporation of various important features of mathematical tasks in the textbook. Furthermore, lack of high cognitive demand and contextual problems likely cause the students' underperformance in NASA assessments. Thus, the study suggested that the 8th-grade mathematics textbook requires various features to make the textbook a more effective resource both for teachers and students. For example, algebraic tasks can be made richer by incorporating visual representations; geometry tasks could be made with higher cognitive demand by connecting them to students' lives. Similarly, the use of more open-ended problems likely promotes higher-order thinking in students.

We believe studies with various grade-level textbooks are required to get better insights into the different features incorporated in mathematics textbooks in Nepal. Although the study is based on only one textbook, we hope the findings of this study provide some implications for further studies in this domain of mathematics textbooks.

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A Comparative U.S.-China Study of Quality Criteria for Junior High School Mathematics Instructional Materials

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Abstract. In the context of the continuous deepening of curriculum reform and the urgent need to improve the current situation of textbook evaluation, China has paid more and more attention to the quality evaluation of instructional materials. This study takes the U.S. Quality Instructional Materials Tool and the Chinese Compulsory Education Textbook Review Tool as comparative objects, and systematically compares the quality criteria of junior high school mathematics instructional materials from the perspectives of evaluation bases, contents, procedures and methods, with a view to shedding light and making reference to the construction of quality criteria of junior high school mathematics instructional materials in China.

1 Research Background

Textbook construction is a national matter, and the textbook reform in the new era should take the quality of textbooks as the core task of textbook construction. on January 7, 2020, the National Textbook Committee issued the National Plan for Building Textbooks in Universities, Schools and Colleges (2019–2022), which for the first time provides a systematic design for the construction of textbooks in various school sections and subject areas in China's education sector. The improvement of teaching materials quality needs to be based on scientific and reasonable standards, and the construction and research of quality standards for teaching materials is an important condition to guarantee the effectiveness of evaluation. In the early 20th century, theoretical research on textbook evaluation began to appear in foreign countries, and many countries have paid great attention to the establishment of textbook quality standards, and the United States, Japan, Germany, and the United Kingdom have all formed relatively complete textbook quality standards. Both the United States and China focus on the consistency of teaching materials and curriculum standards in the development of teaching materials,

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which are standardization-oriented and have commonality in the development of mathematics teaching materials. Therefore, it is important to compare the quality standards of mathematics teaching materials in China and the United States, learn from the advanced experience, and improve and innovate the evaluation standards of middle school mathematics teaching materials, which will be important to improve the quality of teaching materials and promote the construction of teaching materials in China.

2 Theoretical Framework

Yang (2019) compared the textbook validation systems in China and Japan in terms of the institutions, procedures, mechanisms of textbook validation and the basis of textbook evaluation criteria.hu (2019) dissected the characteristics of textbook quality standards in developed countries in terms of the basis for the development of textbook quality standards, evaluation content dimensions and specific indicators, and evaluation methods and tools. Shi (2021) compared the textbook validation system in China and the United States in four aspects: evaluation subjects, criteria, procedures, and management mechanisms. Therefore, this study refers to the framework of Hu (2019) and others, and takes the Compulsory Education Textbook Review Tool provided by the National Bureau of Textbooks in China and the Quality Textbook Tool of the Education Reporting Organization in the United States as the objects of the study. A comparative analysis method was used to compare the quality standards of middle school mathematics textbooks in China and the United States at four levels: evaluation basis, subject, procedure, and content, with the expectation of providing lessons for the construction of mathematics textbooks in basic education in China.

3 Results

3.1 Evaluation Basis

In terms of materials evaluation, both China and the United States emphasize curriculum standards as the main basis for materials evaluation. The Compulsory Education Textbook Review Tool requires that the design of the materials reflect the basic concepts of the curriculum standards, that the arrangement and presentation reflect the goals set by the standards, that the breadth of coverage meet the range of content set by the standards, and that the level of difficulty meet the requirements set by the standards. The Rules for Reviewing and Validating National Curriculum Teaching Materials for Primary and Secondary Schools (2018) and the Measures for Managing Teaching Materials for Primary and Secondary Schools (2019) also set out specific requirements for instructional materials. The current specific requirements for quality criteria for compulsory education instructional materials in China are also contained in these two documents. Therefore, in addition to meeting the requirements of the curriculum standards, instructional materials also emphasize compliance with the discourses and requirements of the relevant educational policy documents regarding instructional materials. The Quality Instructional Materials Tool emphasize the alignment of materials with the standards and focus on three major “mathematical shifts”. In addition, the Quality Instructional Materials Tool has a companion "Evidence Guide" that complements the criteria by elaborating details for each indicator including the purpose of the indicator, information on how to collect evidence, guiding questions and discussion prompts, and scoring criteria. It not only facilitates the evaluators to understand the design intention of the criteria, but also forms an "evidence-based" instructional material evaluation method, which helps to strengthen the objectivity and scientific validity of the evaluation results.

3.2 Evaluation Procedures

There are significant differences between China and the United States in the process of textbook evaluation. In China, the national education administrative department is mainly responsible, and there is no corresponding regulation on the specific process, but in some education regulations, the evaluation institutions, criteria, personnel and other aspects are stipulated. For example, the "National Curriculum Textbook Approval for Primary and Secondary Schools"[5] divides the review process of evaluation into four steps, including the process of submission - initial review - revision - re-review. Each state in the U.S. evaluates educational materials according to its own criteria and develops an evaluation process that is appropriate for the state. The Quality Instructional Materials Tool conducts continuous quantitative evaluation of instructional materials through three levels. Instructional materials are considered high quality only if it meets the expectations of Gateway 1 and Gateway 2 before it is evaluated in Gateway 3 and scores 18 and above in Gateway 3. Unlike others, The Quality Instructional Materials Tool not only counts the final score of materials, but also includes the score of each gateway in the evaluation as an important basis for determining whether instructional materials are qualified or not. This approach is a response to the fact that a single total score does not accurately reflect the level of the material and "ignores the details" in the evaluation of materials. This kind of evaluation method, which focuses on both the results and the "process", is worth learning from.

3.3 Evaluation Content

Both the Compulsory Education Textbook Review Tool and the Quality Textbook Tool use a tower structure. The Compulsory Education Textbook Review Tool constructs an evaluation standard of 38 indicators from five aspects: ideological orientation, fit with the curriculum, selection and presentation of content, guidance for teaching and learning, and editing and design, assigns points to evaluate instructional materials. The Quality Instructional Materials Tool has a total of 40 indicator materials quality criteria consisting of three areas: focus and coherence, rigor and mathematical practice, and pedagogical support and usability. The first two dimensions focus on the alignment of the materials with the curriculum standards. The third dimension concerned with how user-friendly the materials are and the level of support for teaching and learning. The evaluation dimensions and indicators show that both China and the United States not only focus on the content dimension, pedagogical dimension, and material design dimension of the materials, but also on the highly subjective judgmental content of the materials, such as national politics and value orientation. For example, they both propose to respect and care for people with disabilities and to provide support for students with special needs.

The differences in evaluation criteria between China and the United States are also relatively obvious. The criteria are still too general when implemented into concrete work, and the evaluation results are easily influenced by subjective factors in China. The Quality Instructional Materials Tool requires quantitative scoring of each indicator, and the emphasis on evidence better ensures that the results are fair and objective and more actionable. Meanwhile, the U.S. are carefully specified for each subject, for example, the EdReports.org has separate evaluation criteria for English and mathematics. Our current quality criteria take into account disciplinarity in the content dimension, but it is difficult to truly implement the specific requirements of subjectivity by using a set of criteria to uniformly review and evaluate instructional materials in various disciplines and stages.

3.4 Evaluation Methods

The analysis revealed that both the Compulsory Education Textbook Review Comment Form and the Quality Instructional Materials Tool emphasize the use of a combination of qualitative and quantitative evaluation methods. Reviewers of the Quality Instructional Materials Tool quantitatively scored participating materials based on evidence-based guidelines and also required qualitative descriptions of the strengths and weaknesses of the materials, providing evidence for the scores. When evaluating instructional materials, reviewers must score relevant indicators based on the evidence collected and pass three gateways in order to be considered high-quality materials. Qualitative descriptions are not subjective impressions of reviewers, but objective judgments based on evidence. The Compulsory Education Textbook Review Tool emphasizes the review of instructional materials after rating them. However, since the instructional materials are evaluated using common criteria for each subject, the descriptions of the indicators are not specific. Most of them are based on researchers' experiences, impressions, and feelings, and the evaluation results are one-sided. The results of different researchers sometimes contradict each other, which reduces the usefulness of the evaluation results in guiding the development, revision, selection, and use of instructional materials.

The comparison shows that there are commonalities and differences between the criteria of China and the United States. Both emphasize the alignment of instructional materials with curriculum standards, the evaluation of content dimensions, pedagogical dimensions, and technical specification dimensions of instructional materials, and the combination of qualitative and quantitative evaluation methods. The United States focuses on the relevance of instructional materials and proposes that instructional materials should meet the learning needs of students at different levels of cognitive development; It emphasizes evidence-based evaluation, and the indicators mostly present specific and actionable characteristics. On the contrary, China focuses more on the content selection of instructional materials and less on the guiding concern of teaching and learning. The description of indicators is relatively rough and vague, and the characteristics of mathematics are not strong. However, China emphasizes the education of ideas and values, such as the education of the excellent Chinese culture, while the United States emphasizes more on the understanding of knowledge and providing support for students' learning, laying a solid foundation for students to enter university studies or to improve their vocational skills.

4 Implications for Quality Criteria of Junior High School Mathematics Instructional Materials In China

The Quality Instructional Materials Tool is the current representative standard in the United States, and learning from its advanced experience has certain implications for the high-quality development of junior high school mathematics teaching materials in China.

First, constructing scientific quality criteria for junior high school mathematics instructional materials. Many scholars in China have written and discussed about the construction of instructional materials evaluation tools, but there is a lack of systematic, practical and highly recognized tools; relevant textbook writing units and publishing institutions have some evaluation criteria to assist their work, but they are mostly internal materials and cannot be promoted. Therefore, our first task, at this stage, is to develop localized, mathematical textbook quality criteria for junior high school mathematics with the characteristics of mathematics subjects.

Second, a combination of quantitative and qualitative analysis is used to evaluate the method. Looking at the current state of research and practice in China, we can find that there are still some problems with the evaluation methods of instructional materials. Whether studying foreign ones or the evaluation methods adopted by researchers based on the actual development of our instructional materials, there is the drawback of staying on the surface; empirical evaluation is dominant and the evaluation is highly subjective.

Third, the evaluation indicators constructed are specific and operable. In recent years, China has gradually paid attention to the specificity and operable of the quality criteria of instructional materials, but the criteria are mainly constructed on the basis of borrowing from general evaluation models. First, there are more macro indicators, and micro indicators still need to be improved. Second, there are fewer disciplinary indicators, and localized indicators still need to be strengthened. Finally, the indicators highlight the general, and the targeted indicators need to be enriched.

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Comparative Study of the Content of Quadrilateral in the Three Editions of Junior High School Mathematics Textbook

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Abstract. "Quadrilaterals" is an important branch of mathematics in junior middle school. This study used qualitative and quantitative research methods to analyze and compare the "Quadrilaterals" content in Chinese and American Mathematics Textbooks from the macro layout structure design and micro content difficulty. It is concluded that the arrangement order of "Quadrilaterals" chapters in the three editions of textbooks is basically the same, the GMH version has a wide range of content, while the BNUP version has a low depth of content. The discuss of this study are as follows: properly add the expansion chapters of teaching materials, appropriately enrich the teaching introduction links, optimize the course structure and tap the beauty of mathematics.

1 Background

As an important tool for teachers' teaching and students' learning, textbooks have a long history of international research. Textbooks are not only examples of implementing curriculum standards (Valverde et al. 2002), but also an important medium for students to acquire knowledge (Zhang and Qi 2019). Therefore, as the basis of textbook development and implementation, the importance of textbook content comparison is self-evident. From the mathematics curriculum standards in China and US (MOE 2022; CCSSM 2010) and various editions of mathematics textbooks, we can know that "quadrilaterals" is an important content of middle school geometry. Therefore, this study aims to compare the differences of quadrilaterals content in Chinese and American textbooks, and put forward suggestions on the construction of quadrilateral knowledge system and the development of textbooks.

2 Research Design

The research object of this study is the "quadrilaterals" content of junior middle school mathematics textbooks widely used in two countries. The two Chinese textbooks are published by the People's Education Press and Beijing Normal University Press, and the American version is published by Glencoe McGraw Hill press (hereafter "PEP", "BNUP" and "GMH"). The specific research framework is as follows (Fig. 1).

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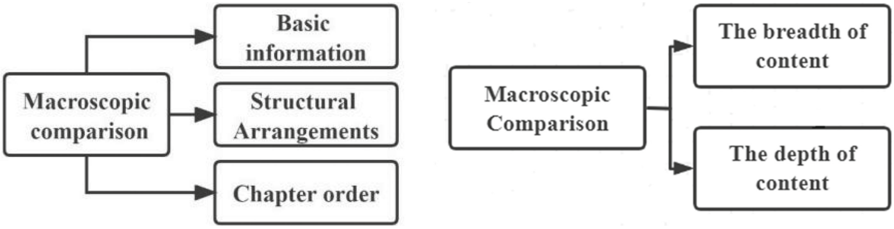

Fig. 1. Research framework

At the micro level, the difficulty of the textbook content is influenced by the breadth and depth of the content (Kong and Shi 2006). The examples reflect the basic requirements of the teaching material in the degree of knowledge mastery, and affect the difficulty of the teaching material. Therefore, this study selects three dimensions of the breadth of content, the depth of content and the difficulty of examples to quantitatively analyze the content difficulty of the textbook.

The number of knowledge points is selected to depict the breadth of content, which include the definitions, properties and theorems involved in the text of the textbooks. The depth of content is described by the way of the presentation of mathematical knowledge points in the textbooks. In this study, the content of quadraterals are divided into seven themes: polygons, parallelograms, rectangles, rhombis, squares, trapezoids and kites, and triangles.

Then, P_j, Q_j, M_j represents the number of knowledge points that are presented by the way of intuitive description, induction and deduction in theme j. $P = \sum_{j=1}^{7} P_j, Q = \sum_{j=1}^{7} Q_j, M = \sum_{j=1}^{7} M_j$ represents the total number of the knowledge points that are presented by different ways respectively. The content depth of every edition of the textbook is recorded as follows: $H = \frac{1\times P+2\times Q+3\times M}{P+Q+M}$ and the relative content depth of the theme j in the same edition of the textbook is recorded as $H_j = \frac{1\times P_j+2\times Q_j+3\times M_j}{P_j+Q_j+M_j}(j = 1, 2, 3, 4, 5, 6, 7)$.

3 Macroscopic Comparison

3.1 Basic Information

Comparing the background information of quadrilateral content in the three editions of textbooks, the results are shown in Table 1.

Table 1 shows that the pages of the PEP textbook is significantly less than the other two editions, because there are fewer quadrilateral chapters. The color of the GMH textbook comes from the diverse pictures and characteristic sections, and there are many white spaces on the left side of the textbook. The color of the BNUP textbook is mostly used to decorate titles, sections' border, theorems and photos of the title page. PEP textbooks are mostly used for decorative thinking, exploration and exercises.

Table 1. Comparison of "Quadrilaterals" in three editions of textbooks

Edition	PEP	BNUP	GMH
Chapter(s)	Grade8 volume II 18 Parallelograms	Grade8 volume II Parallelograms Grade9 volume I Special parallelograms	Chapter 6 Quadrilaterals
Press	People's Education Press	Beijing Normal University Press	Glencoe McGraw—Hill Press
Pages	40–69	Grade8 volume II 134–161 Grade9 volume I 1–29	390–457
Color	Paperback, partly color, mainly black and white	Paperback, partly color, mainly black and white	Hardcover, color
Number of pictures	34	41	70
Blank space	Less	Less	Moderate

3.2 The Chapter- and Lesson-Level Structure

The comparsion of the chapter and lesson-level structure of three editions of textbooks, is shown in Fig. 2.

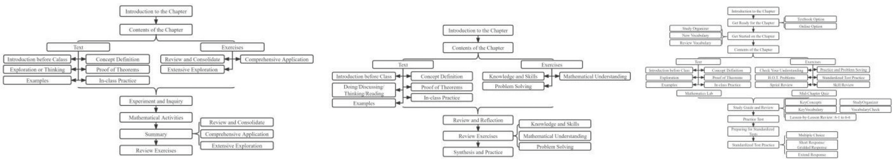

Fig. 2. The chapter- and lesson-level of "Parallelograms" in PEP, BNUP and GMH

Comparing Fig. 2, the structural arrangements of three textbooks is roughly the same: total-divide-total. However, there are some differences between Chinese and American textbooks in the beginning and end of the chapter, exercises and the content. GMH has more test questions in chapter introduction, after-class and end-of-chapter review, and more content in pre-chapter preview and end-of-chapter review.

3.3 Chapter Order

Comparing the chapter order of "Quadrilaterals" in three textbooks, as shown below (Table 2).

Table 2. Comparison of chapter order of the three editions of textbooks

Edition	PEP	BNUP	GMH
Chapter(s)	Grade8 volume II 18 Parallelograms	Grade8 volume II Parallelograms Grade9 volume I Special parallelograms	Chapter 6 Quadrilaterals
Lesson Outline	18.1 Parallelograms 18.1.1 Properties of parallelograms 18.1.2 Conditions for Parallelograms 18.2 Special parallelograms 18.2.1 Rectangles 18.2.2 Rhombus 18.2.3 Squares Mathematical activities Summary Review exercises 18	6.1 Properties of parallelograms 6.2 Conditions for Parallelograms 6.3 The neutrality lines of triangles 6.4 Angles of Polygons Review and reflection Review exercises 1.1 Properties and Conditions of Rhombus 1.2 Properties and Conditions of Rectangles 1.3 Properties and Conditions of Squares Review and reflection Review exercises	6.1 Angles of Polygons Extend 6.1 Spreadsheet Lab Angles of Polygons 6.2 Parallelograms Explore 6.3 Graphing Technology Lab: Parallelograms 6.3 Tests for Parallelograms Mid-Chapter Quiz 6.4 Rectangles 6.5 Rhombi and Squares 6.6 Trapezoids and Kites Study Guide and Review Practice Test Preparing for Standardized Tests Standardized Test Practice

The contents of "Quadrilaterals" in the three editions are basically the same. The three textbooks are arranged in the same order. However, there are some differences in some chapters in the three editions. GMH has Trapezoids and Kites chapter. Independent chapters such as "extend" and "explore" are added before and after some chapters. BNUP arranges a separate chapter for The neutrality lines of triangles. PEP puts the chapters and sections of Angles of Polygons into Triangles for learning.

4 Microscopic Comparison

4.1 The Breadth of Content

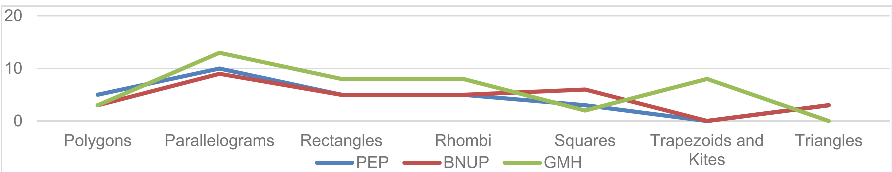

Fig. 3. Comparison of knowledge points of seven themes in PEP, BNUP and GMH

The GMH do not contain the relevant knowledge of the triangle median line and the triangle oblique midpoint, but there are more parts about the trapezoid and kite. The content breadth is also significantly higher than the other two editions of the textbooks (Fig. 3).

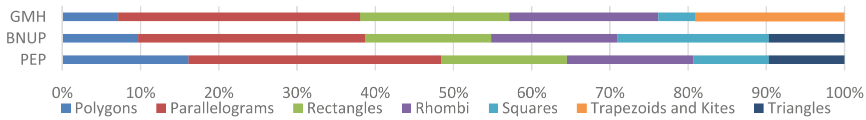

Fig. 4. Proportion of knowledge points of each theme in PEP, BNUP and GMH

Analying of every textbook, the relative breadth of different themes can be found, in the common contains five themes, every textbook is given priority to with parallelogram, the relative breadth of polygon in PEP is higher than others (Fig. 4).

4.2 The Depth of Content

Table 3. Comparison of the content depth level of the three editions of textbooks

Edition	P	Q	M	Sum	The depth of content
PEP	16	4	11	31	2.19
BNUP	9	5	17	31	1.84
GMH	16	2	24	42	2.26

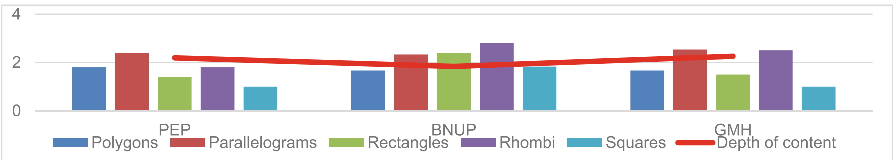

Fig. 5. Comparison of the depth of each theme content of PEP, BNUP and GMH

Based on the five themes included in every textbook, we found that except BNUP, the ordinary "parallelogram" has the highest relative depth. Most topics have lower relative depth than the overall content depth (Table 3, Fig. 5).

5 Conclusion and Discussion

5.1 Conclusion

1. On the macro comparison of textbooks, the content of quadrilaterals in PEP is less than in BNUP and GMH. The color of GMH is richer, and GMH has more blank space. The structural arrangements of three textbooks is roughly the same: total-divide-total.

GMH has more test questions in chapter introduction, after-class and end-of-chapter review, and more content in pre-chapter preview and end-of-chapter review. The three editions of the textbooks are arranged in the same order: parallelogram, special parallelograms. GMH has added Trapezoids and Kites and some expanded chapters.
2. From the micro comparison of textbooks, GMH contains the most extensive knowledge points in terms of the content breadth of the three editions of teaching materials, and the content of "parallelograms" in each edition of textbook is the most extensive; In terms of content depth, BNUP is the lowest, and the content of "parallelograms" in each edition is more difficult than the overall content.

5.2 Discussion

1. **Properly add the expansion chapters of the textbook and pay attention to the exploration process before the knowledge points.**

The American GMH will add "extend", "explore" and other independent chapters before and after some chapters, so that students can expand and apply the knowledge points when learning important chapter knowledge points. It is a process of students' independent exploration and divergent thinking. Setting up expansion chapters and using auxiliary tools such as mathematical software are conducive to students' deeper and comprehensive understanding of knowledge points, and cultivate students' thinking ability to look at problems from different angles and methods.

2. **Appropriately enrich the teaching introduction links and pay attention to the cultivation of the core literacy of intuitive imagination and reasoning ability.**

For special parallelograms, the definition method of genus plus species difference is adopted in three textbooks, but the introduction and presentation of concepts are slightly different. For example: the introduction of rectangles in PEP leads to the definition of rectangles by maximizing the area of parallelogram with a certain side length, so as to feel the connection between parallelogram and rectangle, and further cultivate students' intuitive imagination literacy. As for the definition of rectangle and diamond, GMH has more comprehensive conditions. However, PEP and BNUP are defined by the minimum conditions that can form the rectangle and the diamond shape. The emphasis is to let students deduce all the properties of the rectangle and the diamond shape independently, so as to improve students' reasoning ability.

3. **Optimize the course content structure and focus on the value of geometry learning.**

It is found that the content of "parallelograms" is the core of three textbooks, but GMH tends to present knowledge in a "wide and shallow" way, while PEP and PNUP tend to present knowledge in a "narrow and deep" way, which may be due to the inconsistent development direction caused by the value orientation and demand for parallelograms education in different countries (Ma et al., 2018). Two textbooks from China can increase "exploratory" and "realistic" problems, so that students can experience the practicality and practicability of geometry learning, use creativity to solve problems, and focus on the important value of geometry learning to cultivate students' literacy.

4. **Fully Explore the Beauty of Mathematics and Stimulate Students' Interest in Learning.**

Quadrilateral is an important part of cultivating students' core literacy of intuitive imagination. Special quadrilateral can help students establish the ability of analysis and deconstruction of graphics and excavate the symmetrical beauty of geometric graphics. In China's two textbooks, some quadrilateral illustrations can be added to the exercises, and specific situations can be used to solve mathematical problems. The compilation of teaching materials can appropriately increase the reading content, let students understand the application of special quadrilaterals, broaden students' knowledge, make the teaching materials readable and stimulate students' interest in learning.

6 Research Prospect

The quadrilaterals contain a lot of reasoning and proof (RP), and the research on reasoning and proof (RP) (Stylianides 2009) is more carried out in Western countries (Fan et al. 2018), while only a few studies are concentrated in Eastern countries. In the further study, we will analyze the reasoning and proof (RP) of quadrilaterals of three textbooks, and provide suggestions for textbook arrangement and teachers' teaching.

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A Study on Function Concept among Freshmen in Senior High School in Inner Mongolia and Suggestions to Textbook Compilation

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Abstract. The concept of function is the first important and difficult knowledge that high school students encounter when learning mathematics. The purpose of this study was to examine the understanding of the concept of function among freshmen at a senior high school in Inner Mongolia, China. This study adopted a combination of a questionnaire and a test concerning the concept of function in senior high school. The results show that freshmen have the following problems in the understanding of function concepts: understanding of the correspondence definition of function concepts is not thorough enough, the understanding of the concept and interrelationship of the three elements of functions is not deep enough, and there are major problems in understanding whether two functions are the same function, the graphics, and domain of abstract functions. Finally, provide suggestions to mathematics textbook writers to improve the content of function concepts.

1 Introduction

The Ordinary Senior High School Mathematics Curriculum Standard (2017 Edition) introduces the content of functions as: "The study of this unit can help students establish a complete mathematical concept, not only understand functions as a mathematical language tool to describe the dependencies between variables, but also understand functions as the correspondence between sets of real numbers; can use algebraic operations, and function images reveal the main properties of functions; in real problems, functions can be used to build models and solve problems. The contents include the concept of functions, properties of functions, and the formation and development of functions."

Faced with the abstract and difficult concept of function, what problems exist in the cognitive process of learning this concept? What are the reasons for these problems? How to rationalize the content of mathematics textbooks based on the cognitive difficulties of students' function concepts? From CNKI, many researchers investigate the level of understanding and status of functions for high school students (Hao 2006; Wang 2007; Song 2010; Wang, 2014; Chen 2015; Lu 2016; Ma 2019; Li 2011; Yuan 2017; Zhen 2012; Qin 2014; Zhou and Shen 2011; Lin and Shi 2018). The open literature focuses mainly on three aspects: the definition, the representation method, and the application.

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Although there are many studies on the status of understanding the concept of function for high school students, there are few studies concerning the cases in Inner Mongolia. To study the situation of understanding the concept of function among high school students in Inner Mongolia, we survey the "Function Concept Test Paper for Fresh men in Senior High School", analyse the survey results, and finally give suggestions for textbook compiling.

2 Research Methods

2.1 Research Object

In this study, students from an ordinary senior high school in Inner Mongolia were selected as research objects, 5 classes were randomly selected, 180 test papers were distributed and 156 valid test papers were obtained, with a recovery rate of 86.7%. The test time is 28–30 September 2021.

2.2 Questionnaire with Test

There are 18 questions in this questionnaire with test, Questions 1 and 2 are about the learning attitude of the function, and Question 18 requires students to write their views on the learning function content. Questions 5, 6, 8, 9, 10, 11, 12, and 15 are examining the three elements of a function. Questions 13, and 14 examine the representation of a function. Questions 7, and 16 examine the three elements of a function. Questions 16, and 17 examine the three elements of a compound function.

2.3 Research Procedures

Research tools contain a pretest questionnaire with test, a formal questionnaire with test, and an interview outline. The pretest questionnaire with test is based on the teaching experience of the authors, a reference to existing questionnaires involved in the open literature related to understanding the concept of the function of high school students, and is also discussed with some high school math teachers. Based on the results of the pretest and the overall situation of the questionnaire, and the interview, some questions are revised, and finally, the formal questionnaire is established.

3 Data Analysis and Discussion

3.1 Understanding the Concept of Function

From Table 1, on Question 1, we can find that 49.4% of the students choose "Not Sure"; 12.8% of the students do not like to learn functions, and only 33.7% like to learn functions, which also shows that more than 62.2% of the students encountered difficulties or doubts in the learning function process. In Question 2, 82.1% of the students think learning functions are important for future mathematics learning, which shows that the majority of freshmen recognize that functions are important. On Question 3, only 34% of the students were able to give the function definition clearly, and two-thirds could not remember the function definition clearly or could not give the definition.

Question 4 is “Please explain and imagine the notation function”. The more you imagine and detail the explanation, the better. The purpose of Question 4 is to require students to be able to explain or repeat the concept of function or write related mathematical concepts of functions. Statistics show that only 74 students answered this question, accounting for 47.4%. Among the students who answered, we found that 12 students could write the function definition correctly, and 5 students write the wrong definition of function (incomplete, missed keywords, or misunderstood). One student wrote the definition of the variable version of the function concept, which was introduced in junior high school, indicating that he could not write the set version of the function concept. Thirty-eight students wrote linear functions, quadratic functions, proportional functions, and inverse proportional functions. Seven students showed monotonicity, due to they have just learned monotonicity. It can be seen that few can fully write or recite the definition of the set version concept of functions. Most students can only write the functions they have already learned in high school. The above data show that most students have not yet formed a complete concept network of function, have not fully grasped the set version and the variable version of the function concept, and have not yet established the interconnection between mathematical knowledge and logical thinking framework.

Table 1: Statistics of the answers to Questions 1–3

Item No	Question	Yes	Not sure	No	None	Total
1	Do you like to learn about functions?	51	77	20	8	156
2	Is the learning of functions important for future mathematics learning?	128	15	5	8	156
3	Can you give the definition of the function explicitly?	53	84	9	10	156

3.2 Understanding the Relationship Between the Three Elements of a Function

Question 5: Determine whether the following statements are correct. If correct, explain why; if incorrect, give a counter-example.

(1) If the domain and value of both functions are the same, the two functions are the same; (2) If the domain and mapping of both functions are the same, the two functions are the same; (3) If the range and mapping of both functions are the same, the two functions are the same.

The three elements of a function refer to the domain, range, and mapping. The range is uniquely determined by the domain and the mapping. Thirty-one students correctly judged the 3 subquestions of Question 5; 6 students and 3 students could give counterexamples or wrong counterexamples, respectively. This shows that most of the students who can judge correctly are not able to give correct counter-examples and understand the three elements standing on conceptual memorization and have not reached the level of understanding and application. This also shows that the students’ understanding of the concept of functions is not good, especially for judging whether two functions are the same or not, and cannot correctly give counterexamples. For counter-examples, some students even give function values, not functions.

From the answers to Questions 6, 8, 9, 10, 11, 12, and 15, it can be found that the students have the following problems:

(1) The set of points and the interval cannot be well distinguished, such as $\{a, b\}$ and (a, b), resulting in the wrong calculation of the domain or range; many students are confused $\{a, b\}$ with $[a, b]$.

(2) $f(x)$ and $f(a)$ cannot be well distinguished. Many students think that the two independent variables are different or different functions, and some students think that the two are "range" or "sets", and wrongly write "$f(a) \in f(x)$".

(3) In the answer to Question 9, "Given a function $y = f(x)$, how many intersections between $y = f(x)$ with the straight line $x = a$? And explain the reason." Most students answered that there is one intersection point, since the 1-to-1 mapping, and did ignore whether a belongs to the domain of the function; there are also two intersection points, but no reason is given. It can be seen that the student's understanding of important concepts such as the image, the mappings, and the domain is in an isolated state and lacks integrating understanding. Questions 10, 11, and 12 are as follows:

Question 10. Is there a function that maps each positive number to 1, each negative number to -1, and 0 to 0?

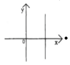
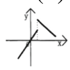
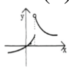
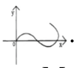
Question 11. Is there a function that maps each prime number to 1, each non-prime number to -1, and 0 to 0?

Question 12. Is there a function that maps each rational number to 1 and each irrational number to 0?

The statistics show that only 7 students answered all three questions, accounting for 4.5%, and only one student thought the answer was a "piecewise function", but no example was given. Most of the students leave Question 8 blank or answer incorrectly, and no counterexamples were given. It can be seen that the student's understanding of the function concept and the three elements has not reached the application level, nor can they thoroughly understand the connotation of mappings and the set version of the concept of function.

3.3 Understanding of Representation of a Function

Question 14: Do the following mappings represent a function? Explain why.

(1) $x^2 + y^2 = 1$. (2) $x^2 + 2y = 1$. (3) $x = 1$. (4) $y = 1$. (5) $xy = 1$. (6) $y = \sqrt{x-2} + \sqrt{2-x}$. (7) $y = \sqrt{x-2} + \sqrt{1-x}$. (8) . (9) . (10) . (11) .

Most of the students cannot well analyse the domain, range, and mapping from the given formulas or figures. The ability to read pictures and compute needs to be improved; for example, the students cannot derive the mapping formulas from Equation (1); ignore the range and the domain in formula (7) cannot empty. Some students cannot use 1-to-1 or n-to-1 mappings to correctly answer the sub-questions (9) (10) (11). Some students misjudge (3) (4), and although students are familiar with linear functions, they know little about constant functions. The answers given by the students cannot either support their answers or are nonsense for the questions. Therefore, many students have many misunderstandings about the concept of functions, three elements, and representation methods. They do not understand the set version of definitions of function, and they only stay in the variable version concept of function which was introduced in junior high school. Students also have problems with reading images. Some students mistakenly consider (9) and (11) as functions. In particular, some students did not have a full understanding of the mapping in the concept of a function and did not pay attention to the “2-to-1” mapping of the quadratic function.

3.4 Application Level of the Concept of Compound Function

Questions 16: the domain of the function $f(x-2)$ is $(-3,4]$, give the domain for $f(3x-2)$.

Questions 17: If $f(f(x)) = x$, please write expressions of $f(x)$, the more the better.

Questions 16 and 17 examine the concept and three elements of composite functions. Two students answered, but the results were wrong. For Question 17, 13 students wrote a correct example: $y = x$ and some students wrote wrong examples, such as $y = |x|$,$y = x^2$, $y = x^4$, $y = |1-x| + 1$. Most students do not understand the concept of composite functions and cannot calculate the domain. Students’ computation to the domain of simple functions cannot be transferred to composite functions.

4 Results and Suggestions

The above results clearly show the weakness of students who are learning the concept of functions in Inner Mongolia, despite the sample size and the regional limitations of the sample. Many freshmen cannot accurately understand the concept of function. Their understanding of the three elements is superficial and is at the level of conceptual knowledge, not at the level of understanding and application. The students apply the graphs and the three elements in isolation, lacking connections. One affection for the above problems is related to mathematics teachers and teaching, which is out of our discussion. However, students’ learning difficulties are impacted by textbooks, and we will present some useful suggestions to mathematics textbook writers for consideration to improve the contents of the concept of function.

Starting from function models, such as linear function and quadratic function, textbooks should show the domain, range, and mapping through examples and gradually inductive the concepts. Give deep explanations of the three elements, particularly demonstrate function $f : A \to B$ which its domain and range are point set or interval, and explain why the domain cannot be empty and the range can be small than set B. Show different mappings with graphics, such as 1-to-1, 2-to-1, and n-to-1, to help students understand the correspondence definition of function concepts.

Demonstrate the similarities and differences of the three representation methods of functions. Introduce some special functions, such as absolute value function, constant function, Dirichlet function, Riemann function, etc. Explain the compound function.

Moreover, introduce the history of function concepts and enhance the scientific spirit and mathematical interest. Design activities for mathematical modelling by applying basic functions to discover or solve mathematical problems.

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An Investigation of the Use of Technology in Two U.S. Middle School E-Textbooks

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Abstract. This study examines a unit involving ratios appearing in the Desmos and Illustrative Mathematics (IM) e-textbooks at grade 6 for the presence of technology tasks and how technology is used in those units. Specifically, we build on a framework used by Sherman and Cayton (2015) to examine the role of mathematical action technology (MAT) as amplifier or reorganizer to solve individual problems or examine relationships. We found that MAT was typically used as a moderate level amplifier in both textbook units. MAT in the Desmos unit was not designed to be used to solve more than one specific problem while MAT in the IM unit did not include multiple representations. Neither e-textbook unit asked students to use MAT to develop and test conjectures.

1 Background

Research on U.S. middle school mathematics textbooks has focused on content or the cognitive demand of mathematics tasks (Jones and Tarr 2007). Other researchers have provided examples of technology use at the middle school level (e.g., Hollenbeck and Fey 2009), but we know of no middle school textbook or e-textbook analyses involving the use of mathematical action technologies (MATs) (Dick and Hollebrands 2011) as amplifier or reorganizer like those completed recently at the high school level and summarized below. Hence, we address this research gap with this study.

Sherman et al. (2020) examined technology tasks in 20 U.S. high school mathematics textbooks with an amplifier/reorganizer framework. They considered technology as amplifier tasks as those involving the use of MAT to complete calculations and/or construct mathematical representations that students could complete on their own. They categorized tasks involving technology as reorganizer if the MAT could be used to complete tasks that would be too difficult or impossible for students to complete on their own. Out of a total of 10,100 tasks 1,318 (13%) involved technology. Out of these technology tasks, 829 were substantial, where technology was an essential part of the task. Of these 829 tasks, roughly one-third or 280 were coded as technology as reorganizer with the remainder categorized as technology as amplifier. In sum, technology as reorganizer tasks constituted 2.8% of total tasks in this collection of U.S. high school mathematics textbooks.

Although commercial textbooks still dominate the textbook market in the U.S., open educational resources (OER) in mathematics are now being used by 14% of districts in a recent survey (Seaman and Seaman 2020). This led us to examine an OER,

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Illustrative Mathematics (*IM*) (https://illustrativemathematics.org/math-curriculum/6-8-math/), that includes MAT as java-based applets in an online format. Furthermore, due to the popularity of the *Desmos* graphing calculator in the U.S., we chose to examine the *Desmos* e-textbook (https://www.desmos.com/curriculum). Many of the lessons in this e-textbook are available for free preview leading us to consider this curriculum as an OER. We focused on ratios in our analysis due to its importance as a foundational middle school mathematical idea that connects to both elementary school topics and high school content (Lamon 2007). Ratios are introduced in both resources at grade 6. This study was designed to investigate the following research questions.

1. What is the prevalence of MAT in the *IM* and *Desmos* e-textbooks in one unit involving ratios in grade 6?
2. What types of MAT tasks appear in the *IM* and *Desmos* e-textbooks in one unit involving ratios in grade 6?

2 Methods

2.1 Framework

We converted the framework developed by Sherman and Cayton (2015) into a series of questions with three potential answers: not present; amplification; and reorganization. We then moved through several cycles consisting of coding sample *IM* and *Desmos* lessons and revising the framework to include components of the two e-textbooks that we encountered but were unable to code. Sherman and Cayton originally designed the framework to examine tasks that involved the analysis of relationships among different mathematical representations or different mathematics content; however, we encountered the use of MAT to solve individual problems in both e-textbooks. Hence, we broke the framework into two areas as seen in Table 1 – MAT to solve *problem(s)* and MAT to examine *relationship(s)*. We considered the Java-based applet appearing in the e-textbooks to be the MAT and questions or imperatives involving the MAT to be prompts.

As seen in Table 1, we labelled each row of the Sherman and Cayton (2015) framework using four different categories that we derived from the framework: utilization; observation; exploration; and conjecturing. Observation, exploration, and conjecturing were each further broken down into three subcategories: not present; amplification; and reorganization. We coded a framework question as amplification if it involved static examples and reorganization if it involved dynamic dragging. Utilization consisted of whether the activity could be completed without MAT (amplification) or if it was difficult or impossible to complete without technology (reorganization). Furthermore, we categorized each unit of analysis overall as low/moderate/high amplification or reorganization depending on the number of questions present as seen in Table 2. We also listed the questions that were not present for each category. *Desmos* acknowledges that their curriculum was influenced by *IM*, and we do see some similarities in the title of the two focus units, yet the nature of the sections and slides were different enough to warrant comparisons.

Table 1. Area, category, number of questions, and sample questions for framework

Area	Category	Questions	Sample Question
Problem(s)	Utilization	1	Could students achieve the same goal without the MAT?
	Observation	3	Does the MAT contain multiple representations one or more of which can be statically/dynamically manipulated?
	Exploration	3	Does the MAT support the exploration of other problems statically or dynamically?
	Conjecturing	2	Do the prompts guide students to use the MAT to test conjectures?
Relationship(s)	Utilization	1	Could students achieve the same goal without the MAT?
	Observation	4	Do one or more of the components of the MAT allow for dragging?
	Exploration	5	Are students required to make or explain observations/generalizations in terms of the MAT?
	Conjecturing	2	Does the MAT provide feedback or allow students to test/refine conjectures?

Table 2. Area, level, and number of questions present

Area	Amplification or Reorganization Level	Number of Questions Present
Problem(s)	Low	1–3
	Moderate	4–6
	High	7–9
Relationship(s)	Low	1–4
	Moderate	5–8
	High	9–12

2.2 Unit of Analysis

In our initial examinations of the *IM* e-textbook, we noticed that when MAT were incorporated into a lesson it was done at the level of a section. That is, the section contained a Java-based applet and a collection of questions or imperatives involving that applet. This led to a section in the student textbook to be the unit of analysis for *IM*. There was a total of 17 lessons and 55 sections in the *IM* unit involving ratios.

Each *Desmos* lesson consists of 10–16 screens/slides with each screen/slide has the capability to gather student input and engage students in a variety of different digital tools such as a graphing calculator, dynamic geometry, tables, etc. The organization and design of *Desmos* lent itself to using a screen/slide as the unit of analysis. We examined the eight lessons (1, 4, 5, 7, 9, 10, 12, and 13) that were available to preview for free consisting of 102 screens/slides. Although Desmos claims to be based upon the *IM* curriculum, yet as our analysis shows there are differences between the two e-textbooks.

2.3 Coding

Both *IM* and *Desmos* contain teacher resource materials (content designed for teachers to support them in implementing the curriculum) as well as student textbook materials (content designed for students). We focused exclusively on the student textbook materials and MAT at the level of a slide in *Desmos* or a section in *IM*. We did not analyse examples of conveyance technology in the two e-textbooks. *IM* also has physical textbooks that can be purchased from the publisher Kendall Hunt, but we focused only on the student e-textbook available online. *Desmos* is available for purchase, but several lessons are available as free previews. *Desmos* does have paper resources (e.g., assessment resources), but these are only available to those who purchase the curriculum; therefore, we did not analyse these. We limited our analyses of *Desmos* and *IM* to these free resources within Grade 6 within one unit encompassing ratios. In *Desmos*, we examined eight lessons involving a total of 102 screens/slides. In *IM* we examined 17 lessons involving a total of 55 sections.

We coded screens/slides in Desmos independently and discussed our results until we came to agreement on our interpretations or revised framework components. The first author coded the lessons from *IM* while both authors coded screens/slides from *Desmos*. We calculated a *MAT density score* for the target unit of the *Desmos* curriculum by counting the total number of screens/slides involving MAT divided by the total number of screens/slides in the unit and multiplying by 100. We calculated the *MAT density* score for the target unit in IM by counting the total number of sections involving MAT divided by the total number of sections and multiplying by 100. We also examined whether MAT was required or optional in solving problems as there were differences between the e-textbooks in this area.

3 Results

Desmos contained 33 slides/screens involving MAT out of a total of 102 slides/screens for a MAT density score of 32.4%. IM contained seven sections that involved MAT out of a total of 55 sections for a MAT density score of 12.7%. Thus, there was a higher incidence of MAT in Desmos than IM for the unit introducing students to ratios. The summary findings for the unit involving ratios for both curricula appear in Table 3. The majority of the MAT uses across both e-textbooks was to solve one or more problems. There were no instances of a high level of amplification in either e-textbook unit. There were no instances of MAT to examine relationships in IM and only two instances in Desmos. When MAT was used to examine relationships within the Desmos unit it was used as a reorganizer. When students were asked to use MAT to solve problems in IM it was only used as a moderate level amplifier while in Desmos it was used as a moderate level amplifier in 22 screens or 67% of the time. Low level amplification use was seen primarily in two lessons of Desmos.

Table 3. Findings across both e-textbooks for the ratio unit

E-Textbook	Problem(s)	Relationship(s)	Amplification		Reorganizer
			Low	Moderate	
Desmos	31	2	9	22	2
IM	7	0	0	7	0

There was some variability in the *Desmos* MAT screens/slides that were coded as low amplification in the unit, but a common missing component was the exploration category of the framework. That is, there were no alternate pathways present in the MAT for solving problems, the MAT did not promote the solution of other problems, and there were no prompts to explore other problems. The main difference between low and moderate amplification in *Desmos* involved the presence of alternate pathways. Thus, MAT in *Desmos* was designed to answer only a specific problem and students were expected to have a correct procedure to implement instead of testing conjectures and learning from the feedback they received using the MAT.

IM MAT sections that were coded as moderate amplification could have been used to explore other problems, but prompts did not encourage this. Additionally, unlike *Desmos*, *IM* MAT was missing two components of the Observation category: presence of multiple representations and connections among these representations. Similar to *Desmos*, *IM* did not prompt students to test conjectures. In *IM* the MAT use was optional while in *Desmos*, it was required by students to solve problems.

4 Discussion

Like the work of Sherman et al. (2020) we found that tasks involving MAT as reorganizer were not as prevalent as tasks involving MAT as amplifier. Moreover, we found no use of technology as reorganizer in *IM*. Technology as reorganizer tended to occur at the beginning of *Desmos* lessons and given the open-ended nature of these prompts it is not clear that students would necessarily be discovering the relationships that we identified from these MAT uses. We found that all the MAT uses in the *Desmos* unit were difficult or impossible to complete without technology, which, according to the Sherman et al. (2020) framework would have made them instances or technology as reorganizer. With the inclusion of the other categories in our framework, however, we found these instances to be either low or moderate levels of amplification. Our findings suggest the importance of more nuance in categorizing technology tasks in textbooks.

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Investigating Teachers' Interactions with Textbooks: Findings from Secondary Mathematics Teachers in Shanghai, China

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Abstract. We conducted a study on how Shanghai secondary mathematics teachers interact with textbooks. A conceptual framework was established for the study. Data were collected from 133 teachers in 13 randomly selected schools through a questionnaire survey and follow-up interviews with 24 of them. Results show that Shanghai mathematics teachers interacted with textbooks to a varying extent, and they adapted most frequently on mathematical thinking and methods, and least frequently on affects, attitudes and values. Meanwhile, mathematics teachers in high-performing schools and private schools adapted textbooks more frequently, while there was overall no statistically significant difference between teachers with different demographic features. Furthermore, teachers interacted more frequently with student books and teacher manuals than with exercise books. Implications and suggestions are discussed.

1 Introduction

It is widely recognized that mathematics textbooks, as an essential potentially implemented curriculum resource, play an important role in student achievement due to their mediating role between intended curriculum and implemented curriculum (e.g., Love and Pimm 1996). Meanwhile, teachers' interactions with textbooks would influence the gap between potentially implemented curriculum and implemented curriculum, which leads to researchers' interest in teachers' use (e.g., Thompson and Senk 2014) and interactions with textbooks.

However, almost all studies in this line were case studies or of small scale and conducted in Western educational settings. In this study, focusing on secondary schools in Shanghai, China, we will address the research questions as follows:

(1) To what extent do Shanghai mathematics teachers interact with textbooks?
(2) Do different groups of teachers interact with textbooks differently?
(3) Do teachers interact with different types of textbooks[1] differently?

[1] In this study, textbooks refer to student (text)books, teacher manuals (guides), and student exercise books.

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2 Methodology

2.1 Conceptual Framework

To investigate teachers' interactions with textbooks, we established a conceptual framework in terms of the whole process of teaching, through synthesizing previous studies of textbook use (e.g., Amador and Earnest 2018), models regarding teaching processes (e.g., Shulman 1987) and the Shanghai curriculum (Shanghai Municipal Education Commission 2004). Our framework consists of four aspects: objectives, contents, pedagogical approaches and assessment (see Table 1).

Table 1. A conceptual framework for analysing the teacher-textbook interaction

Aspects	Description
1. Objectives	Adjusting the teaching goals
2. Contents	Adding/excluding/modifying examples, adding mathematical thinking and methods, adding connections within mathematics and with other disciplines, adding/excluding/modifying contexts, and contents regarding affects, attitudes and values
3. Pedagogical approaches	Adjusting the sequence, ways of expressions, and teaching methods
4. Assessments	Making adaptations for formative assessments and summative assessments

2.2 Research Instruments

Largely following the aforementioned conceptual framework, we designed a questionnaire to obtain the quantitative data. We also designed an outline for follow-up structured interviews to obtain qualitative data for further information.

The questionnaire consists of three parts. Part 1 collects the demographic information of participating teachers. Part 2 comprises 17 items, using a 4-point Likert scale with 4 representing 'Frequently' and 1 representing 'Never' regarding the first 11 items and 4 representing 'Strongly agree' and 1 representing 'Strongly disagree' concerning the last 6 items. Part 3 is an open-ended question, asking teachers to offer one example to explain how they made adaptations in teaching. The interview consists of five questions to ask interviewees about different aspects of teacher-textbook interactions with instructional examples.

To ensure the validity and reliability, a pilot study with four secondary mathematics teachers was conducted to refine the research instruments. The Cronbach's α of our questionnaire was 0.94, suggesting a high internal consistency.

2.3 Data Collection

Using a multi-stage stratified random sampling method, we first selected districts, then schools, and finally teachers. Eventually, five high-performing schools and eight ordinary schools distributed in nine different districts agreed to take part in the study. We sent the questionnaire to all 137 mathematics teachers working there, and received back from 133 of them, with a response rate of 97.1%. Among them, on a voluntary basis, 24 teachers joined the one-to-one interviews later. The audio records of the interviews were later transcribed into Chinese written texts and translated into English. To keep anonymity, the notations 'T1' to 'T24' were used to refer to the 24 interviewees.

3 Results and Discussion

3.1 Overall Features of How Teachers Interacted with Textbooks

Table 2 presents an overall result on how Shanghai teachers interacted with the textbooks in teaching.

Table 2. Teachers' interaction with textbooks ($N = 133$)

	Student book		Teacher manual		Exercise book		Overall	
	M	SD	M	SD	M	SD	M	SD
1. Objective								
Adjustment on teaching goals	3.32[a]	.705	3.31[b]	.713	3.21[b]	.731	3.28[a]	.691
2. Contents								
Addition/exclusion/modification of examples	3.33[a]	.652	3.38[b]	.601	3.20[a]	.698	3.30[a]	.596
Adding contents about mathematical thinking and methods	3.38[a]	.613	3.37[b]	.612	3.31[b]	.657	3.35[a]	.590
Adding contents about connections within mathematics and with other disciplines	3.21[a]	.690	3.23[b]	.674	3.17[b]	.703	3.20[a]	.654
Addition /exclusion /modification of contexts	3.12[a]	.711	3.14[b]	.699	3.04[a]	.741	3.09[a]	.680
Addition /exclusion /modification of contents on affects, attitudes and values	2.75[a]	.788	2.72[b]	.797	2.70[b]	.791	2.73[a]	.777
3. Pedagogical approaches								
Adjustment on sequenced	3.15[a]	.660	3.20[b]	.661	3.12[a]	.671	3.15[a]	.629
Adjustment on ways of expression	3.15[a]	.687	3.16[b]	.711	3.05[a]	.725	3.12[a]	.648
Adjustment on teaching methods	3.12[a]	.689	3.06[b]	.710	3.04[b]	.717	3.07[a]	.675
4. Assessment								
Adaptations for formative assessment	3.26c	.706	3.29	.705	3.20	.726	3.25[c]	.675
Adaptations for summative assessment	3.26[c]	.684	3.26	.703	3.24	.698	3.25[c]	.681

Note: [a] $N = 130$, [b] $N = 131$, [c] $N = 132$, [d] we mean reordering the content presented in the textbooks

The results revealed that teachers made adaptations most frequently with contents about *mathematical thinking and methods*, *examples* of the textbooks, and *teaching goals*. For example, T6 pointed out in the interview that she emphasized the mathematical method of "classified discussion" (the method of listing several possible cases and discussing what outcomes would they lead to) by adding discourses about the specific mathematical methods besides the student books. Meanwhile, teachers interacted with the textbooks least frequently on the contents related to *affects, attitudes and values*, and they made considerably fewer adjustments on *teaching methods* and *context*.

When it comes to teachers' reactions and attitudes towards interaction with textbooks, results show that generally Shanghai teachers did not interact with textbooks frequently, and they would exchange ideas about the adjustments with their colleagues.

3.2 Different Groups of Teachers' Responses on Interaction with Textbooks

We also compared the responses on teacher-textbook interactions from different teachers in terms of school characteristics and demographic features using chi-square tests. It was found that there were statistically significant differences between teachers from high-performing schools and ordinary schools in terms of adaptations of student books on the *examples* ($\chi^2 = 14.873, p < .001$), *ways of expression* ($\chi^2 = 15.723, p < .001$), and *contents about connections within mathematics and with other disciplines* ($\chi^2 = 7.157, p < .05$). Regarding the interaction with exercise books, there was only a statistically significant difference on the interaction with *examples* ($\chi^2 = 8.811, p < .05$). Generally, teachers from high-performing schools adjusted on these aspects more frequently than those from ordinary schools, so as to better suit the need and level of their students.

We also found that teachers from private schools gave significantly higher ratings than those from public schools in adding *contents about connections within mathematics and with other disciplines* for teacher manuals ($\chi^2 = 7.489, p < .05$), and *adding, excluding or adjusting the contexts* in student books ($\chi^2 = 8.055, p < .01$).

Furthermore, no significant difference was found between teachers with different teaching experiences except adding *contents about connections within and outside mathematics* (student book: $\chi^2 = 24.302, p < .001$; teacher manual: $\chi^2 = 23.350, p < .001$), where teachers who had taught for 10 to 20 years rated higher than other teachers. And there were no significant differences between different groups of teachers in terms of urbanicity, genders and educational backgrounds.

3.3 Teachers' Responses on Interaction with Different Types of Textbooks

Friedman tests showed that for teachers' interactions with the three types of textbooks, there were significant differences in many aspects ($p < .05$), especially on *teaching goals* ($\chi^2 = 15.966, p < .001$) and *examples* ($\chi^2 = 19.780, p < .001$) of student books.

Moreover, there was no significant difference between teachers' interactions with student books and their interactions with teacher manuals in all the items except the adaptations on *teaching methods*, where teachers made more adaptations on student books than on teacher manuals ($Z = -2.309, p < .05$).

4 Summary and Conclusions

Employing a questionnaire survey and follow-up interviews, this study investigated the features of how Shanghai secondary teachers interact with the assigned textbooks. The conclusions are as follows:

Firstly, Shanghai teachers generally made adaptations on textbooks to a varying extent in their teaching. Although teachers' fidelity to textbooks was generally considered as a feature of Chinese teachers (Li et al. 2009), this study showed that Shanghai teachers use textbooks flexibly instead of mechanically following textbooks, as they interacted with various aspects of the textbooks; meanwhile, teachers in Shanghai would interact with textbooks to some degree for innovative teaching.

Secondly, teachers would interact most frequently with *mathematical thinking and methods* and least frequently with texts regarding *affects, attitudes and values*. Moreover, *mathematical thinking and methods*, which was stressed in the curriculum reform (Shanghai Municipal Education Commission 2004), was adapted quite frequently, indicating that it deserves more attention in textbooks. In this regard, it would be helpful to explain how mathematical thinking and methods works in margin notes. As for the result that teachers interact least frequently with texts about *affects, attitudes and values*, one possible reason is that it is hard to demonstrate these explicitly in textbooks and teachers may not realize that they are making use of and adapting contents with regard to *affects, attitudes, and values*. Hence, we suggest textbook developers offer specific and easy-to-follow instructions about nurturing students' positive affect, attitudes and values in teacher manuals, so as to enhance teachers' understanding and inspire them to make intentional interactions.

Thirdly, mathematics teachers in high-performing schools and private schools made more adaptations of textbooks, while overall no significant difference was found between teachers of different genders nor with different educational backgrounds. Such results were also largely consistent with previous findings that teachers' adaptations of curriculum materials were influenced by school setting (Burkhauser and Lesaux 2015) and students' learning needs (Choppin 2011). In our opinion, this result may be due to the effort of the teaching research groups in Shanghai schools, which coordinate teachers' teaching practice in relation to teacher-textbook interactions (Chen 2020).

Finally, in contrast to the rather low frequency of teachers' interactions with exercise books, they interacted more frequently with student books and teacher manuals. According to our previous study (Fan et al. 2021), teacher manuals provided more facilitation to mathematics teaching, hence a possible explanation is that teachers interacted with student books and teacher manuals more frequently because they were used more frequently in the first place. On the other hand, as exercise books consist of problems related to the essential knowledge that requires all students to understand, it is reasonable that to build up a "solid base" of students' mathematical knowledge, contents in exercise books are adapted less frequently.

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On the Design and Development of Digital Mathematics Textbooks: Findings from a Systematic Review

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Abstract. This paper presents a systematic survey of research on digital mathematics textbooks (DMTs), with a particular focus on the design and development of DMTs. A total of 65 studies on DMTs were identified and collected from different data sources including journal articles, dissertations and theses, conference proceedings and research reports over the last three decades, and among them, about one third focused on the design and development of DMTs. The results show that the forms of DMTs have gradually moved from CD-based to web-based, tablet-based, and combined-platform-based, which often incorporate multi-media, multi-representation, feedback and evaluations, interactives, and hyperlinks features. Most of them were on one lesson/chapter, with a limited and narrow scope of mathematics content. There is a need for more collaborations between teachers, researchers, textbook designers, and IT developers in the design and development of DMTs. The paper concludes with suggestions for directions for future research.

1 Introduction

Research on mathematics textbooks has grown rapidly over the last two decades or so (Fan et al., 2013). Along with the rapid development of modern digital technology, research on digital mathematics textbooks, or DMTs, has received growing attention (e.g., Rezat et al., 2021). Some particularly noticeable examples include that, in 2017, *ZDM–Mathematics Education* published a special issue, Volume 49, Issue 5, on digital curricula in mathematics education (https://link.springer.com/journal/11858/volumes-and-issues/49-5), and since 2004, all ICMEs (except ICME-12) provided a DG or TSG focusing on research on mathematics textbooks which calls for contributions related to DMTs. Given the development, it is timely for a systematic review to understand the current status and trends of research on DMTs.

DMTs have been defined and categorized in different ways (e.g., Pepin et al., 2015). In order to conduct a comprehensive review, we use the term *digital textbooks* to refer to textbooks with digitalization which has one or more of the following digitalization-related features for presenting, teaching and learning of the intended mathematics curriculum: electronic, interactive, dynamic, online or hybrid. As a part of a systematic review of DMT research aiming to investigate its general trend and current status, this

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paper particularly focuses on research on the design and development of DMTs with the purpose to examine the features and mathematical contents presented in the design and development of digital mathematics textbooks.

2 Methods

As part of the large research effort, we first searched, collected and screened 142 relevant studies on DMTs published in a timespan from 1990 to 2020. They were obtained from different data sources, including journal articles, dissertations and theses and other relevant publications (e.g., conference proceedings, reports). Among them, we eventually identified 65 articles that meet the criteria for inclusion in our survey. Then, using a thematic analysis approach, we established a framework to categorise all the studies on DMTs into four areas: analysis and comparison, use, design and development and others. After this, two researchers coded each article into one of the four areas. The percentage of agreement between two coders was 100%. The whole procedure was depicted in Fig. 1.

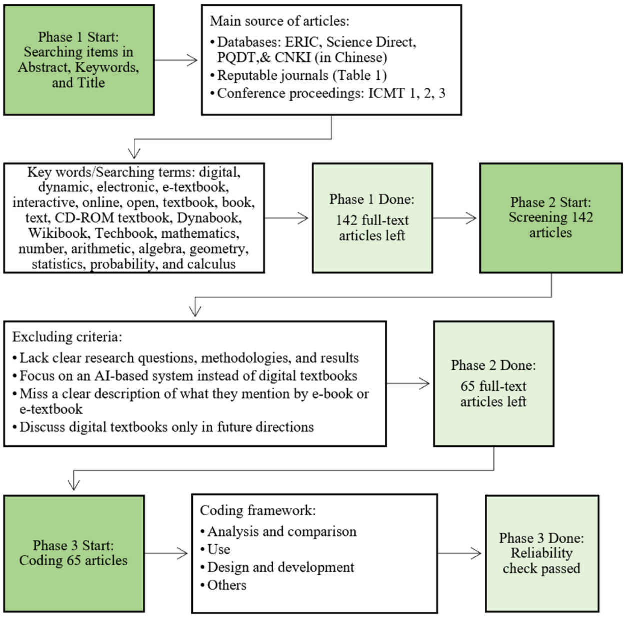

Fig. 1. Research framework and procedure

Table 1 displays the sources of literature included in the survey. Overall, 59% of the articles were from journals, and the majority (89%) of the relevant literature was published after 2010. It indicates that substantial progress in DMT research has been made over the past ten years, especially in the late 2010s.

In terms of research areas, the majority of the articles focused on the use of DMTs. Moreover, about one third of the articles (or 21 articles to be exact) were on the design and development of DMTs, which is the focus of this article. Below we shall report the results in this specific area in terms of the digital presentation features, mathematical contents presented, and the collaborative nature of development.

Table 1: Sources of literature included in the survey

	1990–2000	2001–05	2006–10	2011–15	2016–20	Subtotal
Journal articles	1	3	0	11	23	38 (59%)
Dissertations	0	0	0	6	4	10 (15%)
Others	1	0	2	6	8	17 (26%)
Subtotal	2 (3%)	3 (5%)	2 (3%)	23 (35%)	35 (54%)	65

3 Results

3.1 Digital Presentations

Single-mode presentation. Along with the development of technology in the past few decades, the presentational format of DMTs gradually shifted from CD-based to web-based, tablet-based, or knowledge management system-based.

Some earlier studies introduced the development of CD-based interactive DMTs (e.g., Mississippi Consortium for International Development, 2009). In fact, the first journal paper on DMTs was published in 1995, describing the development of CD-based interactive textbooks in the Renaissance project, where Harding et al. (1995) developed an interactive hypertext version of an existing Calculus textbook with a set of multi-media courseware modules.

After 2000, web-based DMTs accounted for a significant portion (e.g., Yerushalmy, 2005). For example, in the *Dynabook* project, the web-based digital text with rich interactives was designed and developed. It went through iterative development phases with multi-faceted feedback cycles (Phillips & John, 2013). Moreover, there existed DMTs based on the knowledge management system, in which DMTs can be managed and produced in an intelligent way (e.g., Qiao, 2018). For example, Chen and Wang (2012) proposed a dynamic geometry textbook system, *Electronic Geometry Textbook*, which was capable of automatically (1) checking the consistency of the presentation structure, the completeness, and redundancy of the contents, (2) managing and producing contents in multiple versions, and forms in a one-size-fits-all style and (3) providing access to external geometry software.

Recently, a growing number of researchers have paid attention to develop DMTs on tablet platforms instead of the web (e.g., Lu et al., 2015). For example, Hoch et al. (2018) developed an iPad-based interactive textbook, the "*Alice: fractions*", which incorporated digital manipulatives and adaptive demands.

Multi-mode presentation. In the design and development process of digital textbooks, researchers usually mentioned that they incorporated multiple representations (e.g., text, picture, diagram, speech, symbol) and media (e.g., animations, simulations, videos, slides). For example, *VisualMath: The Function Web Book* demonstrated an increasing degree of openness to interaction within interactive diagrams and three different visual and text designs (Yerushalmy, 2005). In addition, Zhao (2014) developed an interactive e-Textbook with two modules using multiple representations and media: (1) introduction: graphs, videos and games, and (2) explanation: slides, videos and animations. Focusing on six elements: text, picture, slide, video and animation, subjective question and interaction group, Mou and Wu (2015) developed an electronic textbook writing software, which manifested the prevalence of multiple representations and media in the development of DMTs.

Other presentation features. Many DMTs also incorporated immediate, adaptive, automatic, or real-time feedback and evaluations, such as real-time evaluations and feedback (Zhao, 2014), step-by-step pop-up hints and immediate feedback (Lu et al., 2015) and automatic feedback (Hoch et al., 2018). Moreover, meaningful hyperlinks or hypertext were employed in the development of DMTs for a long time, such as in the *Renaissance* project (Harding et al., 1995), Ellipse in Geometer's Sketchpad (Wang, 2013) and Circular Cone in FounderFX (Lu et al., 2015). Researchers have also paid attention to interactive tools (Phillips & John, 2013), interactive exercises (Hoch et al., 2018), homework with an online sketchpad (Zhao, 2014), as well as the note-taking feature, drag and measure feature and click feature (e.g., Lu et al., 2015; Wang, 2013).

3.2 Mathematical Contents

Instead of developing a set of coherent DMTs, researchers often focused on developing digital textbooks for one lesson or one chapter. For example, Wang (2013) designed a digital textbook for one lesson, ellipse, based on Geometer's Sketchpad. Gueudet et al. (2016) concentrated on developing the functions chapter of a Grade 10 web-based interactive e-textbook. Hoch et al. (2018) developed an interactive textbook only for fractions. There is a lack of research on a series of digital textbooks that cover a wide range of mathematics content for a whole grade or education level.

3.3 Collaboration in Design and Development

Many researchers have focused on using wiki technology in which students build their own DMTs to support their mathematical learning. For example, Katz and Thoren (2014) described three types of wiki-textbook that students constructed: lectured-based for Calculus II, exploration-based for Vector Calculus, and inquiry-based for Modern Geometry. Olsher and Even (2019) mentioned that new Grade 7 wiki-textbooks prepared by school teachers integrated technological tools, exhibited re-structuring content presentation, had materials for students with low-achievement, and embedded organizing tools to make DMTs user-friendlier.

Several projects evidenced that collaborations between heterogeneous author teams were essential in the development of DMTs. For example, in the creation of *Numbers cBook* (part of the MC-squared project), Bokhove et al. (2014) suggested the UK Communities of Interest, which consisted of several heterogeneous stakeholders (e.g., teachers, designers, researchers, and teacher-educators) with different backgrounds, gave rise to new creative feature. Gueudet et al. (2016) documented various collaborations among teachers, IT developers, and members of the *Sésamath* board, which were essential and beneficial at different stages in the *Sésamath* project.

4 Concluding Remarks

Our investigation indicates that substantial progress in DMT research has been made over the past ten years, especially in the late 2010s. In terms of the design and development of DMTs, the study revealed the main findings as follows.

First, the forms of DMTs have gradually moved from CD-based to web-based, tablet-based, and combined-platform-based, with multi-representation, feedback and evaluations, interactives, and hyperlinks features. We think this trend clearly reflects the rapid advancement of modern digital technology over the last decades and will continue, particularly by integrating artificial intelligence (AI) technology, in future.

Second, research on the design and development of DMTs has been so far largely set in a small-scale research setting, and in a sense, incoherent and fragmented, with a narrowed scope of mathematics contents. Further research is highly needed to expand from one learning unit to a whole integrated and coherent range of mathematics contents in DMTs.

Finally, it is important to have collaborations between researchers and practitioners with different expertise and backgrounds, which is, to a great extent, essential and beneficial in developing DMTs.

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Examining Geometry in the Textbook and Teacher's Guide at Middle School in the Light of Van Hiele's Model and Didactical Phenomenology

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Abstract. This paper reports about a study undertaken on the mathematics textbook and teacher's guide used at 6th grade in Ethiopia, with a particular focus on geometry. The study draws on a framework composed of van Hiele's model of geometrical thinking and Freudenthal's didactical phenomenology. The interest is to find out which levels of geometrical thinking are emphasized and whether phenomena familiar to Ethiopian children are used as context. Document analysis method is used. The results show that geometry topics mostly focus on analysis and informal deduction levels. The approach is usually contrary to van Hiele's suggestion of order. Ethiopian grade six students are demanded to demonstrate formal deduction level. The study's implications are set out.

1 Introduction

This paper reports about a study undertaken on the mathematics textbook used at 6th grade in Ethiopia, with a particular focus on geometry. Earlier studies reveal that textbooks are important resources, which hugely influence the mathematics classroom and teachers rely on the textbook for their teaching of mathematics (e.g., Lepik, Grevholm, & Viholainen, 2017). In Ethiopian school system, there is only one mathematics textbook per grade level. The textbook and teacher's guide examined in this study are prepared by Ministry of Education of the Federal Democratic Republic of Ethiopia (MEFDRE, 2011a & 2011b). Children at this stage often experience geometry to be difficult (e.g., Gravemeijer, Figueiredo, van Galen, Keijzer, Munk, 2016). Gravemeijer et al. (2016) examined children's learning of geometry at upper primary, where they expose the students' transition from informal context-related solution to the formal mathematics. They stress that in students learning of angle it is important to start from experiencing an activity, which is dynamic, where they tell changing of directions. Later the students will be introduced to the static concept of angle, which is a result of directional change (ibid.). According to Gravemeijer et al. (2016), this experience can have motivational effect. The research questions are set out in the following section.

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2 The Theoretical Framework

The theoretical framework borrows idea from van Heile's model of geometrical thinking (Crowley, 1987) and from didactical phenomenology theory of Freudenthal (Treffers, 1987). Doorman et al. (2020) are critical of the traditional approach of teaching of geometry, which is characterized by a focus on structure. Crowley (1987) states that van Hiele's model consists of five levels—visualization, analysis, informal deduction, formal deduction and rigor. The levels are related to the students' particular grade level. For example, Crowley (1987) suggests that the important levels for students at middle school are the first three. In Japan students are introduced to deductive proofs in geometry at 8th grade (Miyazaki et al., 2017). According to Miyazaki and colleagues several students find deductive proofs to be difficult. In van Hiele's model, we pay attention to the properties of the model. For example, the stages are sequential, it is not possible for a person to skip one stage and move to the next (Crowley, 1987). In his theory of didactical phenomenology, Freudenthal, points out another very important idea in the learning of geometry, that is, the starting point for the concept formation, the phenomenon (e.g., Treffers, 1987). Freudenthal is also critical of the teaching of geometry by isolating it from reality (e.g., Doorman et al., 2020). Freudenthal stresses that line and other entities which we defined and use in geometry arise from practical activities. He emphasizes that children should experience practical activities and the mathematical entities in this order so that appropriate learning can take place (ibid.). The current study attempts to answer the research questions: which levels of geometrical thinking are emphasized in the grade six textbook? Are phenomena familiar to Ethiopian children used as starting point for the teaching of geometry?

3 Methods

Qualitative methods were used to examine the data sources (the 6th grade textbook and teacher's guide). Particularly, document analysis method was used where following Bowen (2009), the three stages—skimming, thoroughly examining and interpreting data—were undertaken. The sections and subsections of the geometry topics were thoroughly examined to expose the structure of the chapter. Then, the relevant data were extracted, and were translated from Amharic language into English. The data are further analysed using the first four levels in van Hiele's model (visualization, analysis, informal deduction, formal deduction) and utilization of practical activity or phenomenon. These five pillars were also used in undertaking coding and form the categories (see Table 1). The next section provides exemplar data. For the purpose of analysis, the data are identified by numbers in square brackets as in [5], [6]. Together with the data, the coding is included by using curly brackets, as in {analysing}. In the results section, these data segments are referred to using these numbers.

4 Data Presentation

The last chapter of 6th grade textbook is about geometry and measurement (MEFDRE, 2011a). The chapter has four sections, the first three sections are about geometry—Angeles, Constructing triangles, and Congruence of triangles. The teacher's guide is

Table 1. Main categories, coding and Exemplar data.

Main categories	Codes	Exemplar data
Visualization	Visualizing	See [5]
Analysis	Analysing	See [6]
Informal deduction	Establishing interrelationships of properties, classifying	See [4] [5] [7]
Formal deduction	Constructing proofs, understanding deduction	See [5] [7] [10]
Phenomenon	Stating, using phenomenon	See [8], [11]

structured similarly. The current section presents data from these topics of the textbook and teacher's guide. The data includes text which are paraphrased from these sources.

[4] The section about angles begins with stating that students have learnt about angles in grade 5, and that they now "*learn about angles more broadly*" (p. 98). Then, it provides an activity composed of true or false questions, measuring angles with a protractor, adding angles, classifying angles. The subsection of the section about angles has similar features. For example, the second subsection, which sets out about "parallel lines and angles", starts with a definition. It provides a definition of a vertically opposite angles. After providing definitions, it gives examples. [5] Then, it provides an axiom which states "*Axiom 6.1: When two parallel lines are crossed by a transversal line, the corresponding angles are congruent*" (p. 106). Then, the textbook provides examples and activities which apply the axiom {understanding deduction}. [6] One of the activities has illustration showing two parallel lines crossed by a transversal. The angles are named by numbers 1–8. For example, "*angles 1 and 5 are corresponding angles*". The text part of the activity is as follows, "*based on the figure {visualizing} if angle 1 is 60^0, what is measure of angle 5? {analysing} Why? Find measures of the remaining angles {searching for interrelationship}*" (p. 106). [7] The subsection presents a theorem, which states, "*When two parallel lines are crossed by a transversal line, the alternate interior angles are congruent*" (p. 107). The theorem's proof provides reasons for each statement and applies the axiom stated in [5] above. Then, follows another theorem with its proof. The subsection ends with a set of exercises. The teacher's guide mainly focuses on giving guidance about how the teacher can implement what the textbook states *{establishing interrelationships}.*

[8]. Similar to the other sections and subsections, the first subsection of the topic about angles, ends with a set of exercises. Among 7 exercises one is connected with real-life. It states, "*Weavers use basic geometric shapes in design of clothes … angles formed by intersecting lines {stating phenomenon} …*" (p. 204). Then, it asks measure of an angle if its vertically opposite angle is given. However, the question does not make any connection with the statement about the design. [9] The second section of the chapter is about constructing triangles. After a brief text about the previous lesson, it provides a task for groupwork where students are asked to engage in *"constructing triangles"* (p. 112) [10]. The third section is about *"congruence rules for triangles: SAS, SSS and ASA"* (p. 118). It starts with brief information about the topic, followed by this task:

let us take two triangular figures *{visualizing}*. Note that [it is given that corresponding sides are congruent]. Now, measure each of the angles. … name which rule of congruence holds *{applying rules of relationship}*. What does this show you*{understanding deduction} (p. 118)*?

This task is given as an activity. It provides an illustration of the two triangles. The textbook provides the three rules of congruence of triangles. After each rule there is one example followed by a task, which is named as activity. [11] In the teacher's guide under the subtopic *"congruence rules for triangles: SAS, SSS and ASA"*, it states:

"By cutting out shapes and placing one of the triangles over the other {using phenomenon}, enable students to identify the congruent ones {visualizing}. … By giving triangles of the following type, ask students to name congruent angles and sides" {establishing interrelationship} (p. 200).

5 Results

The data analysis reveals that the textbook usually follows the traditional approach, and the textbook rarely uses practical situations as starting point for the teaching of geometry (cf. Treffers, 1987). Geometry tasks should be based on direct experience with the phenomenon such that students will have the opportunity to explore mathematics in the phenomenon (Freudenthal in Treffers, 1987). For instance, the topic about angles uses hardly any practical situations as starting point [4] and [5]. It has a static approach to the concept contrary to the dynamic approach (cf. Gravemeijer et al., 2016). The transition from dynamic experience of angle, where students can tell angle as changing of directions, to the static one, where they see it as a result of directional change is important (ibid.). Students might miss motivation when they miss such experiences (ibid.). The topics, in general, are characterized by a focus on structure (cf. Doorman et al., 2020). Consistent with Freudenthal, Doorman and colleagues are critical of such approach that limits intuition, which bases on direct experience with phenomenon. In some geometry topics, practical situations are used [8], [10]. The teacher's guide gives guidance about how to use them [11], but not as a starting point for teaching a concept.

The textbook and the teacher's guide provide suggestions of using a practical activity of learning about congruence of triangles, where the students are supposed to work at the visualization level [10] and [11]. Then, suggests going to a more analytic and informal deduction level [11]. However, the textbook and teacher's guide rarely follow such pattern. For example, while students have the chance to experience visualization level, it is often intended to occur after the analysis level [4] [5] and [6]. The textbook and teacher's guide usually focus on analysis and informal deduction levels. In some topics, the students are demanded to be at formal deduction stage [5] and [7]. This is contrary to studies about geometry learning, which suggest that students will be at this stage after middle school (e.g., Crowley, 1987; Miyazaki et al., 2017). The teacher's guide provides general information about formative assessment, but no guidance about formative assessment of students' understanding of geometry.

6 Concluding Remark

The topics usually focus on analysis and informal deduction levels. However, there are cases where 6th grade students are expected to be at deductive level. Besides, the approach is usually contrary to van Hiele's suggestion of order. The use of phenomena familiar to Ethiopian children, as starting point for the teaching of geometry, are very rare. The study has implications; the textbook and teacher's guide need to set out the model employed to organize the topics. The teacher's guide also needs to provide guidance about assessment, which is specific to geometry, because of its peculiar nature regarding the levels of geometrical thinking. Though important results were obtained, it is difficult to know from this study alone about what happens in the classroom. Future studies will explore mathematics classrooms to expose the implementation of the curriculum and examine how the Ethiopian students experience the lessons of deductive proofs. It would also be interesting to undertake a comparative study with other cultures.

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Model Construction of GeoGebra Integrating High School Mathematics Teaching Under the Embodied Cognitive Horizon—Taking the "Elliptical Standard Equation" for Example

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Abstract. Since education has entered a new era, information technology and classroom teaching have entered the stage of "deep integration". Through the "elliptical standard equation" (the first class) teaching process, based on the three characteristics of the embodied teaching environment, construct the embodied cognitive horizon GeoGebra fusion high school mathematics teaching mode from the theoretical basis, teaching objectives, operation procedures, teaching evaluation, and analysis the classroom teaching four links "situational perception, embodied exploration, operation experience, reflection monitoring" with teaching advice, to provide a reference for basic education teaching and material development.

1 Preface

Since the era of educational informatization 2.0, information technology means has had an increasingly profound influence on classroom teaching, and it has also brought new development to the teaching reform. According to the *Mathematics Curriculum Standards for Ordinary High Schools* (2020 Revision) (hereinafter referred to as the Curriculum Standards), attention should be paid to the deep integration of information technology and mathematics curriculum, and the traditional teaching process (MOE, 2020) should be optimized.

In the classroom teaching system, teaching mode is the basis of classroom teaching activities, information technology integrating teaching activities is to promote teachers' accurately teaching, guide students' deep thinking, to effectively develop students' core literacy. In the varied process of graphics or data, find the constant law of mathematics to stimulate students' meaningful thinking. However, most of the current middle school teaching modes do not consider the integration of information technology and mathematics curriculum. Under the condition of normalized epidemic prevention and control, "online + offline" hybrid teaching can solve the problem of education teaching, but eventually, all will return to the main line of classroom teaching, so the research with embodied cognitive theory and cognitive law of middle school students, explore how to analysis element of teaching mode, build the effective mechanism of teaching mode in the Geogebra dynamic integration of the teaching environment, to provide a reference for basic education teaching and material development.

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2 An Overview of the Embodied Cognitive Theory

In the 16th century, the French philosopher René Descartes systematically expounded the spiritual dualism in *Meditationes deprima philosophia*, believing that the mind and body are independent and mutually exclusive, setting the tone of classical cognitive science. Since the 17th century, the discussion of physical and mental dual positions has been divided: French intuitive phenomenologist Merleau-Ponty proposed that the body is the body of mental cognition, body, heart, and world are a complete unity (Merleau-Ponty, 2001). In the 18th century, linguists and Johnson's cognitive science paradigm between "embodied" and "away" (Lakoff et al., 1999). Since the embodied cognitive theory is widely spread, but cannot form a unified view. Academia generally thinks embodied cognitive theory refers to emphasizing physical activity and its experience has an important influence on human cognitive activities.

The theory of embodied cognition breaks the view of physical and mental dualism in traditional cognitive science, believing that the body and experience interact in the mental cognition process and the environment (Ye, 2015). When the theory was introduced to China, Professor Ye Haosheng of Guangzhou University, whose thought is widely recognized by Chinese scholars, proposed that "the body is the provider of cognitive content, physical attributes determine the way and steps of cognitive process, cognition, body and external environment constitute an organic whole" (Ye, 2010).

3 The Exploration of the Teaching Mode Under the Embodied Perspective

In this study, the effective mechanism of mathematics teaching mode in the teaching environment of information technology integration is constructed under the embodied cognitive perspective (Fig. 1).

4 Theoretical Basis

The mathematics teaching environment supported by physical cognitive theory needs to meet three basic characteristics, namely physical, situational and generative (Cao. 2021), the integration of information technology into the teaching environment brings new support points to the three properties (Abrahamson et al., 2016).

Physical refers to the information technology can play an important role in the body and its experience to a certain extent when integrated into the classroom teaching environment. It can transform mathematical knowledge and the real environment, to promote meaningful learning for students and unload the cognitive load of students (Dienes, 1971).

Situational is to consider the practical needs of students in teaching activities and can arrange and implement teaching content to use technology to solve practical problems. Students use information technology to independently discover or create mathematical knowledge, promote learning and cognition in the environment (Núñez et al., 1999).

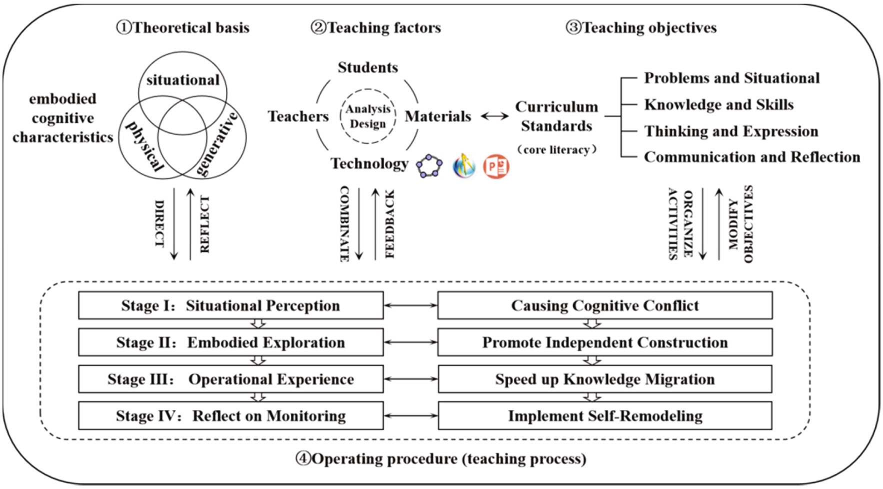

Fig. 1. Model construction of information technology integrated mathematics teaching under the physical cognitive horizon

Generative emphasizes the effective interaction mechanism between teaching elements such as body, heart, environment, and technology, and carries out the interaction between teachers, students, teaching, learning, and technology. After information technology is integrated into the mathematics teaching environment, the generation of mathematical knowledge actively participates in the teaching situation created by the classroom.

4.1 Teaching Content

This part is selected from General High School Textbook: Mathematics (B·2019 Edition) Selective Compulsory Volume 1, Chapter 2, 2.5.1 Ellipse. In traditional teaching, the teachers often focus on the derivation process of the standard equation of the cone curve, but weaken the output of the concept, and ignore the combination of "number" and "shape". Students' mastery of the elliptical inquiry process will directly affect the subsequent learning of the cone curve. Therefore, this part will review the "standard equation" (the first class) and give teaching suggestions.

4.2 Teaching Objectives

"Curriculum Standards" emphasizes the cultivation of students' core mathematical literacy. According to its level and requirements, four-dimensional teaching objectives are set from four aspects, including "problem and situation, knowledge and skills, thinking and expression, communication and reflection".

The teaching objectives of this class are: Problems and Situational Objectives: to experience the historical development of ellipses and the situational perception of parallel light irradiation, combined with the Dandelion two-sphere model, the first definition of ellipses for autonomous induction; Knowledge and Skills Objective: To master the first definition of an ellipse, clarify the relevant concepts (such as focus, etc.) and the derivation process, experience the basic method of establishing the curve equation; Thinking and Expression Objectives: cultivate the ability to solve practical problems by combining numbers and equivalence normalization, cultivate students' ability to discover, understand and use laws; Communication and Reflection Objectives: perceiving the close connection between mathematical knowledge and real life, Through the cooperative learning between teachers and students, cultivate students' ability to summarize, teamwork and the awareness of active communication.

4.3 Operating Procedure (Teaching Process)

According to the characteristics of embodied cognition, the classroom teaching of information technology integration is divided into four continuous links. In the first-class design of "Elliptic Standard Equation" (Fig. 2 below), students contact the existing knowledge and experience. In the dynamic software demonstration environment, the intuitive and vivid feeling process of formation and change of the concept, and experience of the idea of "combination of number and shape".

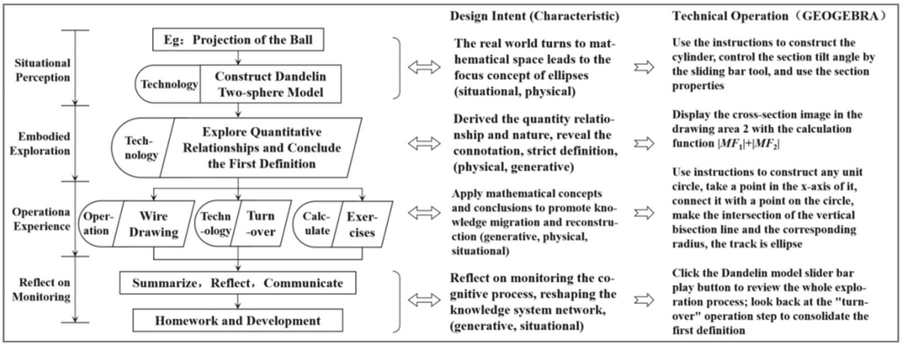

Fig. 2. "Standard teaching roadmap for elliptics" (the first-class)

Stage1: perceiving the situation, causing cognitive conflict

The class of "standard equations for ellipses" (class 1) introduces shadow generation cases of natural light irradiation on the sphere (such as volleyball, etc.) in real-life situations. Volleyball forms an oval shadow on the ground, the situation is transformed into three-dimensional spatial graphics through GeoGebra dynamic software, and then the Dandelion two-ball model (light reversible principle) is used to construct the optical properties (Fig. 3 below) for subsequent learning and study.

Stage2: embodied exploration to promote independent construction

In the inquiry learning of the Dandelion two-sphere model, the teacher starts the section rotation of the "β" button to demonstrate the impact of plane angle on the cylindrical section and initially transitions from the geometry on the section to the concept definition of the ellipse (Fig. 4).

Teachers can guide students to click on the cross-section, cutting points, and related lines, and stimulate the learning desire to think and write, to gradually judge the quantity

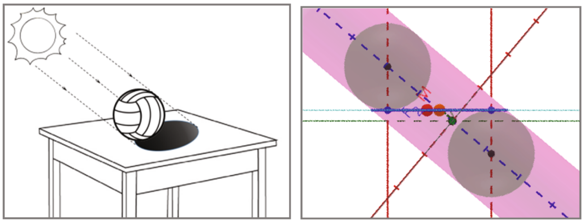

Fig. 3. Application of the two-sphere model in context creation

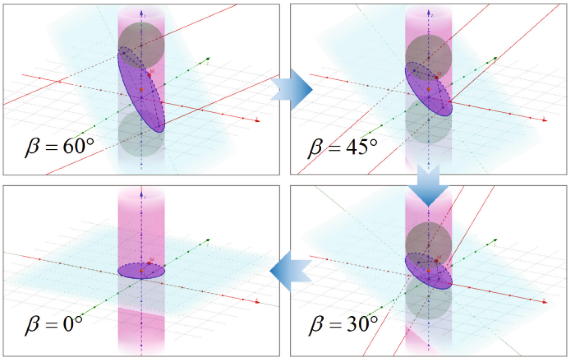

Fig. 4. Dynamic demonstration activity when the cross-section angle changes

and location relationship between the line segments. Teachers can verify the conclusions obtained by the students through the numerical operation function in the GeoGebra (Fig. 5), and guide students to summarize the first definition of the ellipse.

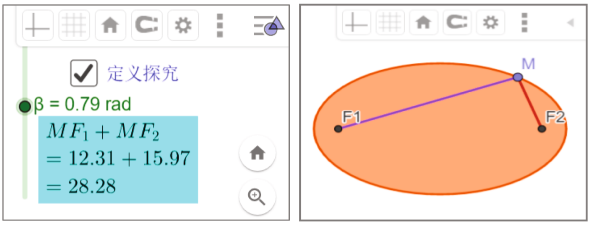

Fig. 5. GeoGebra data calculation for the first ellipse

Stage3: Operational experience, to speed up knowledge migration

According to the first definition of the elliptical inquiry process, the textbook's introduction of the elliptical "drawing method" and GeoGebra's "turnover" operation demonstration (Fig. 6) as the innovative application of this lesson, to complete the deeper knowledge migration and construction. As with traditional teaching, exercises and variant exercises should also be set up to explore the trajectory definition of the curve.

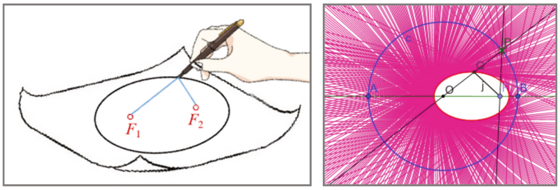

Fig. 6. The conceptual application of the "standard equations for ellipses"

Stage4: reflect on monitoring, implement self-remodeling

"elliptical standard equation" (the first class) build optimized Dandelin double ball model, using GeoGebra greatly reduce the basis of the model teaching difficulty, natural transition 3-dimensional space to a two-dimensional plane, to deepen the students' first definition of elliptic perceptual cognition and rational construction.

5 Conclusion

The GeoGebra integration of high school mathematics teaching emphasizes physical experience and information technology to mobilize students' participation in learning in the teaching environment, providing an operable and improved teaching paradigm for the basic education reform in the new era. However, now the integration of modern information technology and curriculum is still, to a certain extent, affected by teaching content, teaching conditions, teaching operation, and activity organization. Teachers should consider how to create the appropriate situation to develop students' subjective initiative and independent construction.

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A Comparative Analysis of the Integration of Mathematics and Art in Chinese and Singaporean Secondary School Mathenatics Textbooks

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Abstract. This comparative study aims to understand the integrative phenomenon between mathematics and art in Chinese and Singaporean secondary school mathematics textbooks. The art-mathematics relationship in the textbooks was analyzed from six dimensions: (1) art to provide context; (2) art in calculation and measurement; (3) art to master concepts; (4) art to use technological resources; (5) mathematical analysis of art; and (6) creating art with mathematics. It was found that the degree of integration of art and mathematics in the textbooks was not high, which mainly focused on mastering concepts, providing context and artistic creation, while the degree of integration in using technological resources and artistic analysis that involving mathematical creativity and critical thinking needed to be enhanced.

1 Introduction

Mathematics and art are two subjects with distinctive different features. Mathematics is the gymnastics of thinking, which is synonymous with abstraction, rigor and rationality; while art is the leap of inspiration, which is imaginative, vivid and sensual. Moreover, mathematics is the pinnacle of rationality and a model of science; while art is the center of the sensual world and a masterpiece of aesthetics (Huang 1994). However, there is a natural internal relationship between mathematics and art. Incorporating fine arts content into mathematics will not only help students to understand mathematical meaning and achieve better mathematics achievements, but also facilitate the development of aesthetical skills (Brezovnik 2015; Björklund and Ahlskog-Björkman 2017). Futhermore, mathematics textbooks play a critical role in students' mathematics learning, and their quality can directly affect students' learning outcomes and mathematics performance (Peng and Song 2015; Wang and Yang 2016). Therefore, considering the crucial roles of math textbooks, it is essential to analyze the integrative phenomenon of mathematics and art in math textbooks so as to promote students' mathematics learning.

In recent years, interdisciplinary mathematics education has steadily acquired greater importance (Williams et al. 2016). With the development of interdisciplinary mathematics education, the integration of art and mathematics has attracted widespread attention (Brezovnik 2015; Portaankorva-Koivisto and Havinga 2019; Nutov 2021). However,

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there is little research on the integration of mathematics and art in textbooks, and almost no comparative study of integrative phenomenon of mathematics and art in different mathematics textbooks. For a long time, Singapore students are the best performers on many international mathematics test projects, such as TIMSS and PISA (Mullis et al. 2016). Chinese students also achieved great grades in the PISA test in 2018. It is said that students' excellent results could be due at least partly to certain characteristics of the textbooks used there (Oates 2014). Therefore, this paper aims to compare the integration of mathematics and art in Chinese and Singaporean secondary school mathematics textbooks, so as to provide some implications for the development of mathematics textbooks, and puts forward the following research questions: (1) What art forms are embedded in Chinese and Singaporean mathematics textbooks? (2) What roles do these art forms play in the two textbooks? (3) What is the degree of integration between mathematics and art in the two textbooks?

2 Methods

2.1 Selection of Textbooks

The purpose of this paper is to conduct a comparative study of the integration of mathematics and art in secondary school mathematics textbooks in China and Singapore. In the case of Singapore, we analyse "Math Insights Secondary" (Grades 7–9), a textbook developed by Hoon, Hoon & Sum in 2007, which is documented as being used in 80% of secondary schools in Singapore (SGBox.com, 2014). In the case of China, we choose "Mathematics of Compulsory Education Curriculum Standard Experimental Textbooks" (Grades 7–9), which are published by the People's Education Press in 2013.

2.2 Analytical Framework

This paper adopts the analytical framework of Jose Diego-Mantecón et al. (2019) and divides the art cases in textbooks into six dimensions according to their roles: (1) art to provide context; (2) art in calculation and measurement; (3) art to master concepts; (4) art to use technological resources; (5) mathematical analysis of art; and (6) creating art with mathematics.

In the first dimension, art to provide context, the art form is used only to provide a situation for the introduction of new mathematical knowledge or exercise, which helps students have a preliminary understanding of knowledge. In the second dimension, art in calculation and measurement, art element is used to calculate or measure the mathematical property of an object. In the third dimension, art to master concepts, which not only requires students to understand a mathematical concept, more importantly, to be able to identify and apply this mathematical idea to solve mathematical problems. In the fourth dimension, art to use technological resources, it requires students to use mapping software or the Internet to analyze or verify mathematical concepts or properties. In the fifth dimension, mathematical analysis of art, the artistic object is used to analyze the mathematical knowledge or mathematical phenomena embedded in some works of art (usually paintings and architecture). In the sixth dimension, creating art with mathematics, mathematics is used to create works with artistic characteristics, and mathematical concepts or mathematical properties are used and tested in the process of art creation.

2.3 Inter-Rater Reliability

This paper adopts the method of content analysis. Two researchers code on the basis of the analytic framework developed by Jose and finally reach an agreement (both the forms and roles of art). Through the Kappa test of SPSS 21.0, the consistency coefficient is more than .85, which indicates a high reliability.

3 Results and Discussion

After referring to the Spanish scholar Jose's classification of art forms, we divided the art cases in the Chinese and Singaporean mathematics textbooks into seven categories: architecture, sculpture, painting, pattern design, traditional culture, nature, and other mathematical arts. Finally, a total of 179 art cases were identified in the two editions of the textbooks, including 48 cases in the Singaporean textbooks and 131 cases in the Chinese textbooks. Tables 1 and 2 show the integration of mathematics and art in the textbooks of the two countries.

3.1 The Art-mathematics Relationships in Chinese Textbooks

Table 1. Percentages of each art-mathematics dimension per each art form in Chinese textbooks

	Architecture (%)	Sculpture (%)	Painting (%)	Pattern Design (%)	Traditional culture (%)	Nature (%)	Others (%)	Total (%)
Context	36.36	33.33	–	7.35	7.69	28.57	–	11.27
Calculation	18.18	33.33	25.00	1.47	–	–	–	9.86
Mastery concepts	36.36	33.33	50.00	48.53	92.31	71.43	64.29	45.07
Art and technology	–	–	–	4.41	–	–	–	2.82
Analysis of works of art	9.09	–	25.00	2.94	–	–	–	4.23
Art creation	–	–	–	35.29	–	–	35.71	26.76
Total	22	3	4	68	13	7	14	131

Among these 131 art forms, the number of pattern design was the largest, with a total of 68, which was more common in the transformation of geometry. 48.53% of pattern design was used to master concepts, for instance, students were asked to recognize the shape of the circle through the pictures of the Olympic rings. Pattern design for art creation appeared in 35.29% of the activities, students were usually required to use the combination of translation, symmetry and rotation to design some beautiful patterns. However, sculpture, painting and natural art were rarely found, with no more than 10 of each, and they were mainly used to master concepts.

A total of 22 allusions of architecture had been found in the textbooks, with the same percentage of 36.36% in providing contexts and mastering concepts. For example, students can acquire a preliminary concept of symmetry through the picture of the Forbidden City, and understand the stability of triangles by showing the pictures of steel bridges. 15 allusions were found in the other mathematical arts such as fractals, tessellations, and golden spiral triangles, which were mainly for mastering concepts (64.29%) and art creation (35.71%). For instance, Koch snowflake pattern took into account the two artistic functions, which could not only enable students to master the concept of fractal, but also require students to use their imagination to design some fractal patterns. Besides, there were 13 cases of traditional cultures in Chinese textbooks, such as tangram, paper-cutting, and Chinese knots, which were mainly used to master concepts (92.31%).

3.2 The Art-mathematics Relationships in Singaporean Textbooks

Table 2. Percentages of each art-mathematics dimension per each art form in Singaporean textbooks

	Architecture (%)	Sculpture (%)	Painting (%)	Pattern Design (%)	Traditional Culture (%)	Nature (%)	Others (%)	Total (%)
Context	50	100	30	–	16.67	33.33	9.09	33.33
Calculation	–	–	–	25	33.33	–	36.36	15.38
Mastery concepts	33.33	–	10	25	33.33	66.66	45.45	28.21
Art and technology	–	–	–	–	–	–	–	–
Analysis of works of art	8.33	–	–	–	–	–	–	2.56
Art creation	8.33	–	60	50	16.67	–	9.09	20.51
Total	12	2	10	4	6	3	11	48

In these 48 art forms, architecture, painting, and other arts all accounted for more than 20%. While sculpture, pattern design, and nature art rarely appeared in Singapore textbooks, with no more than five of each. There were 12 cases in architecture, which mainly included some famous buildings, such as the Parthenon, the Leaning Tower of Pisa, and the U.S. Pentagon. Of these buildings, 50% appeared as providing context, for example, the Leaning Tower of Pisa was shown to introduce the concept of ratio. Art for mastering concepts were presented in 33.33% of the architecture, such as the picture of the Egyptian pyramids to allow students to identify the pyramid. 11 other kinds of mathematical art had appeared in the Singaporean textbooks, they were some unique mathematical phenomena such as fractal, tessellation, Penrose triangle, Fibonacci sequence, and golden spiral. In 45.45% of the cases, art appeared for mastering concepts. For example, the Koch snowflake was presented to understand the fractals, and some tessellations used the concepts of translation, rotation and symmetry. 36.36% of mathematical art was for calculation and measurement, such as calculating the side lengths through the Pythagorean theorem to draw the spiral triangle pattern. A total of 10 paintings had appeared in the Singapore textbooks, and they were mainly used for art creation (60%), such as the series of illusory figures created by Escher and Sword. Another 30% of the paintings appeared as contexts, such as the square in Joseph's painting, which contributed to the concept of quadrilateral. In traditional culture, there were 6 allusions, including Singapore's first national tapestry and Chinese tangram, which appeared mainly as art forms for calculation, measurement and mastery of concepts (33.33%). These two dimensions often appeared together, for instance, students were asked to identify the figures in tangram and calculate their areas.

4 The Integration Degree of Mathematics and Art

In the previous content, we have made a longitudinal comparison of the roles played by different art forms, and this section will make a horizontal comparison on the degree of integration of mathematics and art in Chinese and Singaporean textbooks in terms of the six dimensions.

The Chinese textbooks used the highest percentage of mastering concepts (43.84%), with almost every art form containing mastery of concepts as its primary role. Although the percentage of artistic creation accounted for 26.03%, it only appeared in pattern design and other mathematical art, while the other five art forms were not involved. In contrast, art provided the highest percentage of providing context (33.33%) in Singapore textbooks, followed by mastery concepts (28.21%), and then art creation (20.51%). It was worth noting that almost all art forms serve to provide context for the introduction of new knowledge as well as mastery of concepts. Compared with art creation in the Chinese textbook, the art creation in Singapore involves five art forms, which were more evenly distributed.

In general, the degree of integration of mathematics and art in the textbooks of the two countries varied widely, and the roles of art forms were unevenly distributed. The Chinese textbooks had a better integration of mathematics and art in the mastery of concepts, while the Singaporean textbooks played a greater role on providing context. However, the overall degree of integration of art and mathematics in the two textbooks was not high, and the integrative phenomenon of mathematics and art was mainly identified in providing context and mastering concepts. Although textbooks in both countries focused on art creation, they lacked the art for using technology and artistic analysis, two dimensions of art-mathematics that required students to develop their mathematical analysis and creativity.

5 Conclusions and Implications

This study analyzed the relationship between art and mathematics in Chinese and Singaporean secondary school mathematics textbooks. As results shown, the overall degree of integration of mathematics and art in the two textbooks was not high. Firstly, in terms of content distribution, both textbooks showed the same characteristics, that was, the integration of mathematics and art was mainly presented in graphics and geometry, while there were few art forms in number and algebra, and no art forms in statistics and probability. In terms of art forms, the two textbooks were mostly related to art in daily life. Most of art forms in Chinese textbooks were pattern design and architecture, while Singaporean textbooks were rich in architecture, painting and other arts. In terms of the roles of art, the art allusions in the two textbooks mainly focused on mastering concepts, providing context and art creation. However, when it came to activities involving students' critical thinking and creativity in mathematics, that were, art for using technology and artistic analysis, the degree of integration of art and mathematics was quite low.

Based on the above analysis, the following are some implications for the integration of art and mathematics in future textbooks: First, both countries should increase the proportion of art components in the number and algebra, statistics and probability. Second, the two countries should try to integrate various art forms into the textbooks so that the art forms are more evenly distributed. Third, the textbooks in both countries should focus on cultivating students' critical thinking and creativity in mathematics, making full use of technological resources to vividly reproduce the art of mathematics and allowing students to feel the beauty of mathematics.

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Analysis of Higher-Order Thinking Skills Content in Chinese High School Mathematics Textbook

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Abstract. This study is a case study with quantitative analysis methods. It aims to explore the proportion of contents for HOTS within Chinese mathematical textbooks.The study defined HOTS in mathematics and provided an assessment framework based on the revised Bloom's taxonomy. The assessment framework was then used to categorize all exercise questions in the Chinese senior high school mathematics textbook. The instrument content validity was obtained through the focus group discussion forum, and the Delphi technique. The result shows that HOTS accounts for 48.18%, dominated by questions classified as analyzing (C1) totaling around 40.08%, there are still a few questions classified as evaluating (C2) of around 6.48% or creating (C3) around 1.62%, which reflects China's emphasis on cultivating students' HOTS.

1 Introduction

Today's education puts forward higher requirements for students' thinking, especially higher-order thinking skills (HOTS). It is one of the necessary qualities for students to adapt to social development. Knowledge can not be transferred from teacher to student because students are supposed to have the ability to actively seek, process, construct and use knowledge. Textbook as a source of learning is important to note when planning and implementing learning so that it is a significant material for teachers and students to cultivate higher-order thinking skills. Pratama (2018) states that the more higher-order thinking content covered in textbooks, the more likely it is to teach students higher-order thinking skills. However, there are few studies on the analysis of HOTS content in textbooks, especially in China. Therefore, an issue of interest is that there are a lot of HOTS contents in Chinese high school mathematics textbook. In this regard, the study aims to explore the proportion of contents for HOTS within Chinese mathematical textbooks.

2 Literature Review

2.1 Higher-Order Thinking Skills (HOTS)

According to different cognitive levels, human thinking skills can be divided into low-order thinking skills (LOTS), and higher-order thinking skills (HOTS). HOTS is a way of thinking at a higher level than memorizing and can be defined in different ways

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and viewpoints, such as Marzano's taxonomy, SOLO taxonomy, and Bloom's taxonomy. According to Bloom's taxonomy, the cognitive level can be divided into six levels: knowledge, comprehension, application, analysis, synthesis, and evaluation. The last three levels are known as higher-order thinking skills. But they do not specify the generative mechanism, his students Anderson revised Bloom's taxonomy later. Judging from the revision of Bloom's taxonomy, HOTS includes the level of analyzing, evaluating, and creating, while LOTS includes the level of remembering, understanding and applying. Commonly speaking, the most popular definition of HOTS is derived from Bloom's taxonomy.

Considering HOTS in mathematics, the National Council of Teachers of Mathematics (NCTM) (1989) defined mathematical HOTS as the ability required to solve unconventional problems. Tanujaya (2016) points out that HOTS in mathematics includes at least critical thinking and creative thinking. Samritin (2016) regards problem-solving and mathematical reasoning as two types of mathematical HOTS. Bayasut (2019) believes that critical thinking in mathematics is an embodiment of HOTS in mathematics. Mulyatna (2021) defines mathematical HOTS as a kind of problem-solving and logical reasoning ability, which includes logical reasoning ability, analysis, evaluation, and creation.

Mathematical higher-order thinking skills are hard to define but can be easily identified by their characteristics. It is also can be trained in the learning process. Therefore, based on the characteristics of the revised Bloom's taxonomy and mathematical disciplines, this study treats HOTS in mathematics as reasoning ability (see Table 1), that is, the ability of individuals to perform mathematical processes or complex tasks or math problems involving connection, problem-solving, and mathematical reasoning.

2.2 Textbook

Teaching materials are the main materials for teachers' teaching and students' learning. In China, textbooks are important teaching resources to realize the goal of the mathematics curriculum and develop students' core literacy in mathematics. Chinese high school mathematics textbooks are divided into compulsory textbooks and selective compulsory textbooks, including four main lines: function, geometry and algebra, probability and statistics, mathematical modeling activities, and mathematical exploration activities. That is to say, mathematics textbooks in China aim at developing students' core literacy of mathematics. The content of the textbooks is presented in a variety of ways, with integrity and local wisdom, reflecting the law of "teaching and learning".

In mathematics learning itself, HOTS is one of the most major skills to be developed, since mathematics will equip students with the ability to think logically, analytically, systematically, critically, creatively, and the ability to work together. The design of teachers' teaching activities is directly influenced by the teaching materials used by teachers. Therefore, it is urgent to analyze the higher-order thinking skills content in mathematics textbooks and find out how HOTS opportunities are given by mathematics textbooks.

3 Methodology

This was a case study, and quantitative analysis methods were employed. Combining the revised Bloom's taxonomy and the characteristics of mathematics itself, we adopt the following assessment framework (see Table 1) that is divided into three levels and six dimensions from Recall to Reason, containing 19 cognitive processes. Meanwhile, all study samples were coded according to the framework.

Table 1. Assessment framework of mathematical higher-order thinking skills

Criteria	Cognitive Level		Cognitive process	Verbs Operational
LOTS	Level I Recall	remembering(A)	recognising, recalling	remembering,listing,repeating,imitating,knowing,mentioning,identifying
	Level II Understand	understanding(B1)	interpreting,exemplifying,classifying,summarising,inferring,comparing,explaining	explaining,clarifying,accepting,reporting,describing,distinguishing,repeating
		applying(B2)	executing,implementing	using,demonstrating,illustrating,operating,clarifying,checking
HOTS	Level III Reason	analyzing(C1)	differentiating,organising,attributing	comparing,checking,critiquing,assessing,analysing,categorising,differentiating
		evaluating(C2)	checking,critiquing	evaluating,assessing,refuting,deciding,concluding,supporting,checking
		creating(C3)	creating,planning,producing	constructing,designing,creating,developing,writing,arranging,formulating

The study divides Recall and Understand into lower-order thinking skills and Reasoning into higher-order thinking skills, which are reflected in three ways, namely, analyzing, evaluating, and creating.

4 Method

The samples, considering their comprehensiveness and representativeness, are selected from the chapter exercises and chapter review questions in Chapters 3 and 4 of the latest version of the Chinese high school mathematics textbook (Compulsory 1). The review questions are more comprehensive compared to the exercise questions, such as Exercise 3.1 and Review Question 3 (Exercise 3) in Chapter 3. Since function counts a great deal in mathematics, these two chapters are the classic chapters of the function part. Through analysis and sorting, 247 valid samples were finally collected and coded. Each sample was categorized from A to C3, according to the assessment framework. To ensure the validity of the coding, three graduate students from the Faculty of Education of Beijing Normal University coded separately, discussed, and revised to determine the final coding. At the same time, we use the Delphi method, relevant experts were invited to comment to ensure the accuracy of the coding. An example of coding is given below:

Exercise: A person drives a car at a speed of 60km/h from Point A to point B 150km away, then stays at point B for one hour, and returns to point A at a speed of 50km/h. Q: Please express the distance x(km) between the car and point A as a function of time t(h)(starting from point A). Then express the speed v(unit: km/h) as a function of time t, and then draw the images of these two functions respectively.

Comment: The code of this question is C1, because this question has a real problem situation, the students are not familiar with this situation, so students need to understand the question by themselves, and then extract valid information from the questions to get the expressions of the two functions.

5 Result

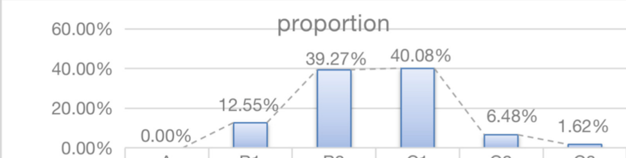
As the results of the analysis of problem coding for each exercise (see Table 2), the cognitive level of different parts was different in quantity and also has obvious characteristics. For instance, C1 exists in each section whereas other levels are not, even, though there is no A. We can also see that among all levels, C1 has the largest number, followed by B2. At the same time, the number of HOTS questions in exercise 3 or 4 is higher than the LOTS questions in its partial exercise (such as 3 vs. 3.4and 4 vs. 4.5).

Table 2. The number of cognitive levels in each exercise

Exercise / Congnitive-Level	3.1	3.2	3.3	3.4	3	4.1	4.2	4.3	4.4	4.5	4	Sum
A	0	0	0	0	0	0	0	0	0	0	0	0
B1	5	1	0	0	0	13	2	6	1	3	0	31
B2	19	6	2	0	5	4	16	20	9	4	12	97
C1	7	13	3	3	16	6	6	6	13	8	18	99
C2	4	1	0	2	1	0	1	0	3	4	0	16
C3	1	1	0	1	0	0	0	0	0	0	1	4
Sum	36	22	5	6	22	23	25	32	26	19	31	247

The following table shows the ratio of HOTS and LOTS in all exercises (see Table 3). We can find that the proportion of HOTS content is 48.18% (40.08% + 6.48% + 1.62%), while LOTS takes 51.82%. Furthermore, the most significant problem is C1: the analysis stage of the reasonable level. Among the problems reflecting HOTS, there are relatively few hierarchical problems in evaluation and creation. Finally, the study found that there was no recall level in the textbook, which indicates that the questions also reached the lowest level of understanding.

Table 3. The proportion of HOTS and LOTS



	A	B1	B2	C1	C2	C3
数量	0	31	97	99	16	4

6 Conclusion and Discussion

6.1 Higher-Order Thinking Skills Occupy a High Proportion in a Chinese Mathematics Textbooks

Through the coding analysis of textbooks, HOTS account for 48.18%, dominated by questions classified as analyzing (C1) totally around 40.08%, there are still a few questions classified as evaluating (C2) of around 6.48% or creating (C3) around 1.62%, which reflects China's emphasis on cultivating students' HOTS.

The reason for the high proportion of HOTS in Chinese textbooks is that China takes much emphasis on developing students' core literacy, such as logical reasoning, mathematical modeling, and mathematical abstraction, which are all related to students' HOTS. Problems in real situations can usually have multiple solutions, so some problems also belong to C2. At the same time, some questions are open and product-oriented, which can test students' creativity and belong to C3. It can be seen that the requirements of core literacy of mathematics in the curriculum standard are similar to the cultivation of HOTS, which is the reason why exercises of HOTS occupy a relatively high proportion in Chinese mathematics textbooks.

6.2 The Proportion of Higher-Order Thinking Skills in Different Knowledge Points is Different

We find that although the materials selected for this study both have HOTS questions. HOTS questions in chapter 3 accounted for 59%, while in chapter 4, the questions on HOTS accounted for 42.45%, less than half of the total exercises. This shows that the proportion of HOTS in different knowledge points is different.

The reasons may have two aspects, one aspect is that in the concept of function and property chapter, functions involved are relatively simple, and students can achieve C, so the proportion of HOTS questions is higher, especially in section 3.4, which has lots of functions applied in practical situation's questions. Another aspect is that there is more LOTS question (especially B2 level) in chapter 4, over 47.7%. As a result, there are more exercises in Chapter 4, so we have a big denominator in terms of HOTS proportions, so the proportion of HOTS questions is lower. Because the exponential function and logarithmic function in Chapter 4 for students is relatively difficult, questions at the B2 level mainly aim to guide students to grasp the knowledge.

6.3 Different Levels of HOTS Have Significant Differences in Each Exercise

According to Table 2, C1 exists in each section, whereas other levels are not. In almost every part of the exercise, C2 is less than C1 and more than C3. In Chapter 3, three of the first four exercises contain C3, while exercise 3 does not, but in Chapter 4, C3 only appears in exercise 4. And the number of them is relatively small.

One reason is that though Chinese mathematics textbooks emphasize higher-order thinking skills, they contain a lot of general questions about HOTS, which leads to C1 taking the most, while C2 and C3 level belongs to the top level of HOTS and are naturally less. Otherwise, C2 and C3 level requires students to be creative, and critical and to have the strong problem-solving ability, which is relatively difficult for most Chinese students. As a result, the proportion of grade C3 is relatively low in textbooks.

In conclusion, HOTS is needed by students, it is necessary to analyze the HOTS content in mathematics textbooks to find out how HOTS opportunities are given by mathematics textbooks, in that teachers integrate all resources to cultivate students' HOTS. In the future, more empirical studies are needed to explore the cultivation of HOTS combined with teaching materials.

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A Comparison of the Contents of Statistics and Probability in Mathematics Curriculum Standards Between China and England

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Abstract. This paper compared the topic of probability and statistics in two curriculum standards between China and England. Using content analysis and comparison methods, this study focuses on the comparison of overall characteristics, content distribution, content extension, information and communication technology. The findings show that the contents of the two curriculum standards are both based on statistics, supplemented by probability. The content distribution and content extension of Chinese curriculum standards are both lower than those of England curriculum standards. For information and communication technology, both the two curriculum standards advocate the effective use of information and communication technology and recommended that teachers should make judgments about the use of technology.

1 Introduction and Background

The curriculum has been a major lever for promoting change and improvement in school math programs (Hirsch and Reys 2009). Researcher regard the curriculum can be considered to have three forms: intended, implemented (enacted), and attained (assessed) curriculum (Cai 2014). Intended curriculum refers to the formal written documents that set system-level expectations for learning (Wang and McDougall 2019), provides teachers and curriculum developers with guidance and guides the development of textbooks and evaluation reports to monitor school programs. Usually, scholars take the official curriculum and textbooks represent the intended curriculum (Cogan and Wolfe 2001). Official curriculum or standards is a broad statement of learning expectations expected by an educational authority, It can provide organized views and information for the systematic teaching and learning process (Rezat et al. 2021). Researchers have conducted some studies on curriculum standards comparison and have also achieved some results (Cao and Yan 2015; Schmidt and Houang 2012). The main focus is on when and how specific topics were introduced and developed (Lloyd et al. 2017). Although analyzing the important features of different curricula is critical to understanding their effects on students' learning (Nie et al. 2009). The field of curriculum research still lacks a cross-national analysis of the intended curriculum, especially on the topic of statistics and probability. And there are key issues that need to be addressed for further study (Lloyd et al. 2017).

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Probability and statistics are important topics in the mathematics curriculum and textbooks. There is no doubt that this part has a broad application in various areas of science and technology as well as in the real life, especially in such an information-based and data-based society (Woolfson 2008). “Statistical literacy” is important to be an effective citizen (Gal 2005). Many countries have increased their emphasis on the study of probability and statistics (Chi and Practice 2022). The comparison can provide us with the opportunity to recognize some distinguishing features and give us some inspirations for future curriculum development.

In this paper, we focus on comparing the overall characteristic, content distribution, content extension, information and communication technology (ICT) in the mathematics curriculum standards between China and England on the topic of Statistics and Probability. The results will delineate two different curriculum models, thereby leading to some reflection on curriculum design in this topic. Three research questions are intended to address:

1. What are the overall characteristics of the two curriculum standards in the design of this topic?
2. How about the content distribution and content extension between the two curriculum standards under this topic?
3. How ICT is treated in the two mathematics curriculum standards under this topic?

2 Research Method and Procedure

The method of text analysis and comparison is adopted in the research method. Text analysis methods were adopted to collect and analyse the data, and the comparison method is adopted to explore similarities and differences between the two mathematics curriculum standards. The study conduct text analysis and comparative research from both horizontal and vertical perspectives. Horizontal indicators are divided into content extension and content extension, and vertical indicators are mainly based on grades and stages.

2.1 Mathematics Curriculum Standards Selection

Based on the years of compulsory education in China, the first three stages in England and the Chinese mathematics curriculum are selected to be the compared object for comparison (Grades 1–9). The curriculum standards are both official guideline documents issued by the government. Basic information on the mathematics curriculum standards of the two countries is presented in Table 1.

2.2 Data Collection and Analysis

We define the smallest collection unit named issue, which refers to every meaningful word in the topic of statistics and probability, such as “calculation”, “graph” and “mean”. In this study, issues were recorded according to two categories. One is the content issue which shows the knowledge list of the study, one is the extent issue shows the requirement

Table 1. Mathematics Curriculum Standard Information

Country	Mathematics Curriculum Standards	Publisher	Code
China	Mathematics Curriculum Standards for Compulsory Education (2011-year Edition)	Ministry of Education of the People's Republic of China	CH-PRC
England	Mathematics programmes of study: key stages 1,2 and 3 (National Curriculum in England)	Department for Education	EN- NCE

Table 2. The classification of statistics and probability of knowledge content

Category	Description
Data Collection and Classification	Measure, record, collect data, count, classify, summarize and organize data
Date Representation and Description	Use certain statistical tools to show data in graphs or tables; or to describe the data of information represented by graphs and tables
Data Analysis and Computation	Processing data means performing necessary calculations on the data, extracting information from the data and making simple judgments, including calculating the mean, median, mode, range, variance, etc.; understanding central and discrete trends
Solve Statistical Problems	Use statistical or probabilistic methods to solve practical problems
Possibility	Know random phenomena, understand the randomness of events, and make guesses and inferences about random events
Calculate the Probability	The relationship between random events is calculated, and the probability of different random events is calculated
Statistical Inference	Estimate unknown population-related information based on sample information and data

expected to achieve (Peng 2019). We determined to divide the content of the topic into 7 themes (see Table 2). Their meanings are described as presented in Table 2.

To grasp the extent of content distribution, each issue is matched to 1, 2, 3, or 4 according to the requirement in the mathematics curriculum standards (Cao and Wang 2016). The new content extent coding is shown in Table 3.

Content breadth refers to the scope and scope of the course content distribution. We use the total number of content issues to represent the breadth of content. Content depth refers to the overall level of learning expectations of curriculum standards for content issues. We represent content depth by the total average of content issues coding

Table 3. Coding information for content extent

Coding Value	Requirement Description
Level 1	Intuit, imitate, observe, feel, try, count
Level 2	Perceive, recognize, understand, know, explain
Level 3	Explore, construct, internalize, master, calculate, interpret
Level 4	Apply, verify, problem solving

calve. The comparison of information technology is the main focus on the concept words related to information technology in the two curriculum standards.

3 Findings and Discussions

3.1 Overall Characteristics

On the overall layout, the two curriculum both have set overall goals and then followed the general goal. Regarding content distribution, they are both based on statistics, supplemented by probability. The Chinese curriculum has divided the content by stage and all the content for compulsory while England curriculum is divided by school year and provides non-statutory content. Probability is rarely involved in the primary school stage, and fewer probability issues are involved in the third stage. The above characteristics show that the two curriculum standards in this topic both attach great importance to the basic work of data, which is consistent with their overall goals.

3.2 Content Distribution

There are 64 content issues in total in England curriculum, while there are 49 content issues in total in the Chinese curriculum. At each stage, the England curriculum standards also cover more issues than the Chinese curriculum standards. Indicating that the England curriculum covers a wider range of content, which means the English curriculum has a greater breadth of content at each stage than the Chinese curriculum.

To clearly understand how the content is distributed in the two curriculum standards, content issues are set according to the theme, and the results are shown in Table 5. Both curriculum standards have the most issues in the theme of Data Representation and Description. More specifically, pictograms, block diagrams, timetable, line graph, and venn diagram is unique content issues in England curriculum standards. Chinese curriculum only holds more content issues in the theme of data Analysis and Computation. This gap feature is even more pronounced in the topic of Solving Statistical Problems. Especially, there are no content issues in the theme of Statistical Inference in England curriculum, while there are four content issues in the Chinese curriculum.

Table 5. Content distribution by theme

Theme	CH-PRC	EN-NCE
Date Collection and Classify	9	13
Date Representation and Description	20	23
Data Analysis and Computation	10	9
Solve Statistical Problems	1	9
Possibility	4	5
Calculate the Probability	1	5
Statistical Inference	4	0

Table 6. Content depth by stage

MCS	Stage 1	Stage 2	Stage 3	Overall average depth
CH-PRC	1.429	2.059	2.280	2.082
EN-NCE	1.308	2.227	2.878	2.516

3.3 Content Extension

The content extension of the two curriculum standards is measured by content depth according to the previous assignment criteria, results by stages obtained in Table 6.

The overall average depth of the Chinese curriculum and England curriculum are 2.082 and 2.516, respectively. It shows that the overall depth of England curriculum is higher than that of the Chinese curriculum. Due to the different presentation styles of the academic year and requirements in the first two stages, if we only consider stage 3, this feature emerges more obvious.

The content depth was calculated by theme (see Table 7). The content depth of the Chinese curriculum is higher than that of England curriculum in the two themes of Data Collection and Classification and Data Analysis and Computation, while in other topics, the content depth is lower.

In another way, when the seven themes are divided into two types of statistics and probability, the first four themes represent Statistics, and the last three themes represent Probability, this description also seems to be more obvious and reliable. For the Probability topic only involved in stage 3 in England curriculum standards, it is necessary to reflect on the depth of probability in Chinese curriculum standards.

3.4 Information and Communication Technology (ICT)

The development of information technology has had a great impact on the value, goals, content and teaching methods of mathematics education (Pepin 2017). When we focus on the form and extension of the use of ICT results show that both curriculum standards

Table 7. Content depth by theme

MCS	Data collection and classification	Date representation and description	Data analysis and computation	Solve statistical problems	Possibility	Calculate the probability	Statistical inference
CH-PRC	1.414	2.050	2.200	2.000	1.250	2.000	1.250
EN-NCE	1.231	2.130	2.111	2.555	2.800	2.600	–

place explicit constraints on ICT, which can be understood from two perspectives, guiding, teaching and learning. For overall guidance, the England curriculum specified that calculators should not be used as a substitute for good written and mental arithmetic. While Chinese the curriculum described ICT as a reasonable tool and advocate a rational use of information technology. For teaching and learning, all the two curricula mentioned that teachers should use their judgment about when ICT tools should be used both in primary and secondary schools. The detailed content shows that the Chinese curriculum has more flexibility in the use of ICT in the theme of statistics and probability in the intended curriculum.

4 Summary and Conclusions

This study examines the overall distribution characteristics, content distribution, content extension and ICT of Chinese and English curriculum standards on the topic of statistics and probability. From the perspective of the overall characteristic, the two curriculum standards adhere to different concepts, but they all choose the concept design of stacking. The Chinese curriculum standard pays more attention to integrity and sees greater curriculum flexibility while the England curriculum was more flexible in content. In terms of content distribution and content extension, results show that the content depth and content breadth in England curriculum are both higher than the Chinese curriculum standards. In terms of ICT, both curricula hold that teachers should make judgments about the use of technology while the Chinese curriculum holds a more supportive attitude than in England.

Although the curriculum standards can help us to understand the curriculum design philosophy, there is a gap between the intended curriculum and the attained curriculum. In addition, curriculum standards are constantly being revised and updated, and the actual teaching and teaching material development are also affected by many factors. So what is in the practical course, may be related to educators, teachers, courses and many other factors. This study just conducts a basic comparison, which leads to some reflection. How to combine the existing comparison results to better conduct the teaching of this topic is still a topic worthy of further study.

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A Comparative Study of the U.S. Evaluation Criteria for Quality Instructional Materials Tool—Grades 1–5 as an Example

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Abstract. As a guiding document for the development, publication, distribution and selection of teaching materials, the evaluation standards for teaching materials are of great significance in ensuring the quality and guiding the healthy development of teaching materials. EdReports.org, a private, independent, non-profit organization that develops and publishes the Quality Instructional Materials Tool: K-8 Mathematics (QIMT) based on the original vision of "equity, excellence, and support for teachers", which provides standards for evaluating mathematic materials for grades K-12. In this paper, we conduct an in-depth study of the background and original intention, framework and content, process and approach of the QIMT (1~5), and finally conclude three experiences and insights that can be used for reference in our country.

Since the 21st century, the basic education of our country is moving towards the educational reform and development mode driven by "curriculum standard". Mathematics materials have formed a "multiple-standards" and diversified development, with multiple versions of teaching materials coexisting and forming a benign competition mechanism, which has injected vitality into the development of teaching materials. How to select teaching materials suitable for different groups of students has become an important issue in elementary mathematics education. America has formed a sound system of developing and reviewing teaching materials from the national level to the state and school levels, and the development of teaching materials also focuses on consistency with curriculum standards, which is standardization-oriented and has commonality with the development of elementary mathematics teaching materials in China. This study selects the U.S. elementary mathematics materials assessment standards as the object of study. By examining the question "What are the main contents of the U.S. elementary mathematics materials assessment standards? What is the position of the assessment criteria? What can it teach us?" to provide references for the construction of a quality assessment framework for elementary school mathematics teaching materials in China.

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1 Background and Original Intention

1.1 Education Policies Drive the Implementation of Curriculum Standards

American evaluation standards for mathematics teaching materials have been constructed and developed over a long period of time, and three types of evaluation standards, national, local state, and private, have been formed according to different research and development organizations. QIMT belongs to the third category. The evaluation covers 12 million students in approximately 1,084 districts, with partner states and districts including Mississippi, Rhode Island, Palm Beach County School District and others. From 2015 till now, EdReports.org has selected a series of high-quality math materials, including Fishtank Plus Math, Jump Math and so on, which have received industry acclaim.

On June 2, 2010, in response to the global wave of deepening basic education reform and driven by the No Child Left Behind Act (NCLB), the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) released the Common Core State Standards for Mathematics (CCSSM), which set out the basic knowledge and skills that students need to understand and master at all grade levels. However, as Professor Richard Elmore notes that "to get to the heart of teaching and learning, it is essential that teachers and students have high-quality content." Thus the Sunnylands Mathematics Strategy Group, a group of mathematics educators, the lead authors of the CCSSM and others, funded by several philanthropic foundations, established EdReports.org, which published the QIMT after conducting research on commonly used scoring criteria and their use.

1.2 The Current Evaluation of Mathematics Materials is not Optimistic

Marquis Alvaradous recalls the years as a math teacher saying, "I persevered without high-quality materials, but I shouldn't have had to" The negative impact of low-quality materials on students' confidence in mathematics cannot be underestimated. The data shows that the number of mathematics materials that are aligned with the CCSSM is only a handful, which goes against the original idea of the policy bill. From the political science perspective, teaching materials are not only a "transmission system" of "facts", but also the result of the struggle and mutual compromise among political, economic and cultural activities. So the knowledge written into materials is subject to the complex set of factors mentioned above, the evaluation of materials quality cannot guarantee complete objectivity and truthfulness. Since its inception, EdReports.org has maintained its original intent and vision of providing teachers and students with high-quality materials to improve student learning outcomes that are relatively less subject to the complex factors mentioned above.

2 Framework and Content

The evaluation framework of the QIMT consists of three levels of focus and coherence, rigor and mathematical practice, instructional supports and usability indicators, each of which is broken down into different dimensions and specific indicators (Table 1).

Table 1. Focus and Coherence

Criterion	Maximum Points
Materials do not assess topics before the grade level in which the topic should be introduced	2
Students and teachers using the materials as designed devote the large majority 4 of class time in each grade K-8 to the major work of the grade	4
Each grade's instructional materials are coherent and consistent with the Standards	8

The first two criterions, both of which are "non-negotiable" criteria. Only if the total score for both dimensions is 6 can the assessment be continued; if not, the assessment is terminated. On the one hand, the assessment should be targeted to the "key shifts" in the CCSSM in order to prepare students for college or career; on the other hand, the assessment should focus on both making connections between new knowledge and students' prior experiences and requiring that knowledge within the same grade level, so that students can continue to reinforce old knowledge by learning new knowledge. The reorganization of experiences also contributes to the shaping and expansion of students' cognitive frameworks (Table 2).

Table 2. Rigor and the Mathematical Practices

Criterion	Maximum Points
Materials can reflect the balances in the Standards and help students meet the Standards' rigorous expectations, by giving appropriate attention to: developing students' conceptual understanding; procedural skill and fluency; engaging applications	8
Materials meaningfully connect the Standards for Mathematical Content and the Standards for Mathematical Practice	10

Criterion 1 focuses on assessing whether the material engages students in all aspects of rigor, and emphasizes the balance of them. Criterion 2 focuses on the relationship between content and practice in the materials. Materials should clearly identify and address the full range of the eight standards of mathematical practice, which in most cases are used to enrich the content standards. In addition, materials should develop students' logical reasoning through the construction of their own arguments, the analysis of others' arguments, the use of mathematical terminology, to develop students' perspective-taking skills (Table 3).

There are some not-scored extended criteria in Gateway 3. The first two criterions assess whether the materials provide supportive support for student learning and teacher professional development. Based on the basic view of TQM theory, if teaching materials are considered as a commodity, teachers' and students' satisfaction with the materials is reflected in whether they support the development of users and their social utility.

Table 3. Instructional Supports and Usability Indicators

Criterion	Maximum Points
Materials are well designed and take into account effective lesson structure and pacing	8
Materials support teacher learning and understanding of the Standards	8
Materials offer teachers resources and tools to collect ongoing data about student progress on the Standards	10
Materials support teachers in differentiating instruction for diverse learners within and across grades	12
Materials support effective use of technology to enhance student learning. Digital materials are accessible and available in multiple platforms	Not Scored

Criterion 3 emphasizes the assessment function of the materials, which allows students to learn and develop better through the process assessment in the materials, and provides a solid foundation for students to develop good self-control and reflection skills. Criterion 4 aims to illustrate that the structure and content of materials can be flexible and balanced as the needs of different people, languages, abilities and so on. Criterion 5 focuses more on digital materials and the use of instructional technology, with the ultimate goal of better serving teachers and students (Fig. 1).

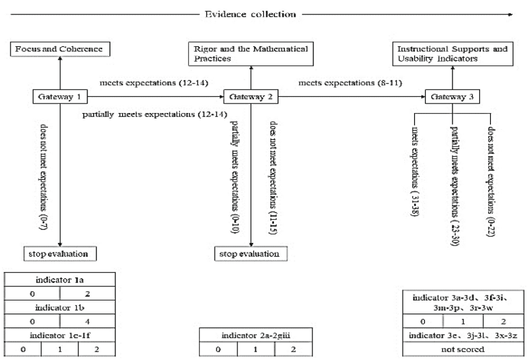

Fig. 1. Evaluation Process

3 Process and Approach

3.1 Process: Three Gateways

The entire assessment is conducted in a "pass-through" format, the following figure shows that.

3.2 Approach: Evidence-Based

It's not enough to select high-quality materials, the teachers should understand why these materials are high-quality and how to use them effectively. EdReports.org provides the Evidence Guides and other three materials to work collaboratively with QIMT, to provide efficient guidance for reviewers. There are four main steps: i. Reviewers must comprehend the intent behind the indicator; ii. Provide examples and counterexamples of evidence from multiple perspectives; iii. Reviewers regularly discuss to present their theoretical basis and evidence for a given indicator; iv. The rationale for scoring and what must be included in the evidence to support the scoring of each level of the indicator are described. In addition to this, publishers participate in the evaluation process by providing background information and research findings about the materials in the form of documents.

4 Insights and Reflections

Teaching materials play a critical role in improving the quality of student learning and closing gaps. The assessment background, content, process, and approach of QMIT provide richer experiences and references for China to build evaluation standards for basic education materials in the new era.

4.1 Pay Attention to the Feelings of the Subject, Support the Development of Teachers and Students

The QIMT reverses the previous view that the quality of materials is measured only by student achievement. Materials are not just a vehicle for transmitting knowledge, but also for teachers and students seeking to develop themselves, stimulate subjectivity, and develop independent learning habits. The teacher is not the role of lecturing from the materials, but is also a learner and beneficiary who developing professional and core competencies in the learning of the materials.

4.2 Avoid Subjective Experience, Seek Evaluation Evidence

In the evaluation process, reviewers are prone to assess materials based on their own subjective experiences, which can affect the final evaluation results. For this reason, QIMT uses an evidence-gathering approach, which not only gives the reviewers a reasoned and evidence-based approach, but also inspires the publishers to improve the quality of their materials. It's worth noting that the collection of evidence is not limited the content in the materials, but should also seek evidence from the specific use process in the classroom, students' academic performance, parents' feelings, community feedback, and other channels, which can more comprehensively confirm the reasonableness and accuracy of the scores.

4.3 Close to the Core Standards, Combined with Content and Practice

The only two "non-negotiable" standards were established at the beginning of Gateway1, which illustrates that the requirement for consistency with the CCSSM is rigid and uncompromising, the teaching materials are still indispensable for the teaching of basic knowledge and skills. In addition, the CCSSM emphasizes the need to link mathematical practices to mathematical content. The students who lack an understanding of a particular piece of knowledge will not be able to apply it well to the practice of mathematics. The intersection of practice and content connected emphasizes core and generative concepts in the mathematics curriculum that can radically improve the quality of curriculum, instruction, assessment, professional development, and student achievement in mathematics. The organic integration of the practice standards with content standards as a focus of assessment is an important contributor to deepen the development of mathematics materials, even the basic education mathematics.

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A Study on the Value Orientation of Chinese Excellent Traditional Culture Contents in Mathematics Textbooks: Based on the Analysis of the Primary School Mathematics Textbooks Published by PEP

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Abstract. The integration of Chinese excellent traditional culture into mathematics textbooks is an important measure to realize the fundamental task of moral education. Through the analysis of the excellent traditional Chinese culture content and its value orientation in the fourth grade mathematics textbooks of The People's Education Press, it is found that the selection of cultural content is diversified, especially focus of state level of culture. The value structure of cultural education is complete, highlighting patriotism and concerns about science and culture; The teaching value of culture emphasizes on promoting problem raising, problem solving and deeper understanding through examples and cultural situations. There are various forms of cultural integration and its depth needs to be strengthened. The integration of Chinese excellent traditional culture into future mathematics textbooks can be improved from the following aspects: rationally examining the educational value and teaching value of Chinese excellent traditional culture, and constructing the target system; Based on the essence of the value of Chinese excellent traditional culture, clear principles of the integration; Optimize the content composition of cultural choice, highlight the value inheritance power of mathematics textbooks; Adjust the form structure of integration.

1 Introduction

The Chinese excellent traditional Culture is the genetic and spiritual lifeblood of the Chinese nation, and a powerful spiritual force for realizing the great rejuvenation of the Chinese nation and building a "community with a shared future for mankind" (Xue 2017). In 2021, the publication of "The Guide of Chinese Excellent Traditional Culture into Primary and secondary School Curriculum Textbooks" (2021) calls for the creative transformation of Chinese excellent traditional culture in curriculum textbooks. In this context, we pay more and more attention to this question: how do modern curriculum and textbooks constructed under the influence of western curriculum and teaching paradigm highlight the background of Excellent Traditional Chinese culture and how to inherit this culture? Therefore, it is of practical significance to analyse the content and value

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orientation of Chinese excellent traditional culture in textbooks. However, the current related studies tend to focus on the humanities and mainly focus on the obvious aspects of Chinese excellent traditional culture. Therefore, this study clarifies the value orientation of Chinese excellent traditional culture contents in primary school mathematics textbooks and makes specific analysis, so as to grasp the positioning of Chinese excellent traditional culture contents in primary school mathematics textbooks and provide ideas for the optimization of subsequent primary school mathematics textbooks.

2 The Definition of Value Orientation of Chinese Excellent Traditional Culture Content in Textbooks

Culture in a broad sense refers to the sum of material and spiritual production capacity acquired by human beings in the process of social practice and material and spiritual wealth created. It becomes a certain cultural entity in the long-term development of the world. Chinese excellent traditional culture (content) refers to the culture formed in the long-term development of the Chinese nation, has a positive historical role, and embodies the characteristics of the Chinese nation's ideas, activities and behavior habits (Li and Wang 2018). It can be seen that "tradition" lies in the accumulation of history. On the one hand, "excellence" is reflected in the fact that culture itself is scientific and true, can be renewed and developed, and shows the deep purposeful value. On the other hand, culture can promote the development of a country and a nation, showing surface instrumental value. The purpose value focuses on the relationship between the subject and the object, the cultivation of human beings, including the relationship between the subject and itself, the subject and other things, the subject and other things, etc., while the instrumental value focuses on the promoting effect of this culture on mathematics teaching. This is exactly the value tendency contained in the excellent traditional Chinese culture itself.

3 Research Methods and Analytical Framework

This study selects two volumes of *Compulsory Education Mathematics Textbook* (Grade 4) published by People's Education Press 2012 as the research object. Based on the above understanding of value orientation, combined with the textbook analysis framework of relevant scholars (Hou and Zhang 2018; Fan et al. 2018; Zhang and Fang 2004) and relevant policy documents (2021), this study constructed the value orientation analysis framework of Chinese excellent traditional culture in primary school mathematics textbooks. The framework includes cultural content, value orientation and organizational presentation, as well as secondary and tertiary dimensions. Two coders conduct pre-analysis and discussion, adjust the original framework after several arguments, determine the analysis dimension and coding, and improve the operational definition expression. Finally, formal coding and the reliability and validity of coder are investigated.

Table 1: Analysis framework of Value orientation of CETC in primary school mathematics textbooks

Level 1 dimension	Level 2 dimension	Level 3 dimension
Cultural content	Cultural aspects of real life Cultural aspects of material states Values	Customs; regulations All kinds of literature and art works; scientific and technological inventions Mode of thinking; traditional ideas
Value orientation	Purposeful value Instrumental value	Personal morality; ethical morality; national feelings; scientific spirit; natural care; cultural inheritance Provide information; stimulate interest; solve problems; deepen understanding; broaden horizons
Organizational presentation	Organization Format of the representation Integration mode	The text; sample; exercises; extension column Words and mathematical symbols; pure image; text and pictures; words, pictures and forms Contribution model; transformation model; action model

4 Results and Discussion

4.1 Diversified Selection of Cultural Content, Especially Focusing on Culture at the Physical Level

The content of Chinese excellent traditional culture in primary school mathematics textbooks involves the culture of real life, the culture of physical state and the culture of values. The selection of content is characterized by diversity, especially focusing on culture at the level of physical state. Among them, physical culture accounts for 70%, involving traditional architectural culture such as Tian ‘anmen’. And Zhaozhou Bridge, etc. There are also tangram, calculation, counting and other traditional mathematical culture; It also includes traditional scientific and technological products such as Chang ‘e I and literary and artistic products such as Journey to the West. Real-life culture accounts for 10 percent, including Chinese facial makeup in operas and traditional food. The culture of values accounted for 20 percent, involving traditional spirit of respect for elders and selfless dedication, such as making tea for the elders and donating books to disaster areas.

4.2 The Value Structure of Cultural Education Is Complete, Highlighting National Feelings and Concerns for Science and Culture

On the whole, the cultural education value of integrating Chinese excellent traditional culture into mathematics textbooks takes into account both the purposeful value and the instrumental value. In terms of the purpose value, it covers six parts of the relationship between human and the outside world. In terms of the instrumental value, it also involves all aspects of the whole process of mathematics teaching, with a complete value structure. In terms of purposeful value, the most attention was paid to national feelings (44.19%), followed by scientific spirit and cultural inheritance (20.93%), and less attention was paid to social ethics (9.30%), personal morality (2.33%) and natural care (2.33%). First of all, it shows that the most important purpose of incorporating Excellent Traditional Chinese culture into textbooks is to guide national consciousness. For example, converting the area of the Forbidden City into hectares and square meters permeates the consciousness of loving the cultural heritage of the motherland. Secondly, the balance between scientific spirit and cultural inheritance shows that the integration of excellent traditional Chinese culture is no longer limited to the natural science of mathematics, but also shows concern for humanities and arts.

4.3 The Teaching Value of Culture Emphasizes on Promoting Problem Raising, Problem Solving and Deeper Understanding Through Examples and Cultural Situations

In promoting the instrumental value of mathematics teaching, "deepening understanding" accounted for the largest proportion, accounting for 37.50%; Next came "problem solving", accounting for 27.50%, and "providing information", accounting for 17.50%. "Broaden horizons" and "arouse interest" account for 12.50% and 5.00% respectively. This teaching value is often related to its placement in textbooks. In terms of organization and arrangement, Chinese excellent traditional culture is mostly presented in exercises (45.00%) and examples (22.50%), followed by text (17.50%) and extended columns (15.00%). More specific analysis shows that sample questions and text are often associated with the value of "providing information" and "solving problems", while exercise questions are often associated with the value of "deepening understanding".

4.4 There Are Various Forms of Cultural Integration and Its Depth Needs to Be Strengthened

In terms of presentation form, the presentation of Chinese excellent traditional culture contents in the Primary school mathematics textbooks involves various forms of text, pictures, charts and mutual combination. Among them, the combination of text and picture is the most (75.00%), some involve the combination of text and mathematical symbols (5.00%), pure picture (10.00%), and the combination of text, picture and table (10.00%). This shows that the excellent traditional Chinese culture often needs to convey the value consciousness through pictures. However, the pictures presented often only show the surface of the culture, which needs to show the deep state and emotion.

In terms of integration methods, the contribution mode is the main mode for the integration of Chinese excellent traditional culture into primary school mathematics textbooks (55.00%), indicating that Chinese excellent traditional culture is mainly presented by factual cultural content such as knowledge, concept, theme and viewpoint. However, it did not clearly guide the students to think, that is, they failed to move towards the transformation mode (32.50%). On the basis of transformation mode, there were fewer action modes that guided students to take culture as the theme, understand the essence of mathematics and culture in action, and put it into practice (12.50%). This reflects that the current integration focuses on the level of "learning mathematics with the help of culture", which has not been able to better cut into the essence of mathematics and culture, and the depth of integration needs to be strengthened.

5 The Outlook of Integrating Chinese Excellent Traditional Culture into Primary School Mathematics Textbooks

5.1 Examine the Purpose Value and Instrumental Value Rationally and Construct the Target System

Although some attempts have been made to integrate Chinese excellent traditional culture into primary school mathematics textbooks, it is still in a vulgar, superficial and formalized stage (Guo 2020). The above analysis also found that the value is biased. The reason is related to the unclear value orientation of integration. Therefore, it is the key point of improvement to rationally examine the value orientation of integrating Chinese excellent traditional culture into primary school mathematics textbooks and construct its target system. At the same time, we should also clearly recognize the limitations of integrating Chinese excellent traditional culture into mathematics textbooks.

5.2 Based on the Essence of the Value of Chinese Excellent Traditional Culture to Clarify the Principle of Integration

It can be seen from the analysis that the content of excellent traditional Chinese culture in current textbooks is outdated and not strongly related to mathematics itself. Therefore, in the process of integration, we need to be goal-oriented, reinterpret the connotation of Chinese excellent traditional culture with the characteristics of The Times, and reveal the essence of mathematics subject with Chinese discourse. For example, the strategy selection process of Tian Ji horse racing is presented dynamically in the form of multimedia to narrow the distance between this traditional cultural content and students and help students understand.

5.3 Optimize the Content Composition of Cultural Choice to Highlight the Value Inheritance Power of Mathematics Textbooks

It is found that the selection of cultural content is obviously biased, which is not conducive to the realization of the overall value of Chinese excellent traditional culture. Therefore, the content composition of cultural choice needs to be optimized. On the one hand, enhance the proportion of cultural aspects of real life and values in textbooks. For example, considering the characteristics of China as a multi-ethnic country, we can use rich multi-ethnic costumes to understand the symmetry and transformation of graphics. On the other hand, there is a need to enhance the relationship between culture at the level of real life, culture at the level of physical state and culture at the level of value.

5.4 Adjust the Form Structure of Integration and Reserve Space for the Realization of Cultural Value

The integration of Chinese excellent traditional culture in mathematics textbooks is gradually reduced in the three-level mode of "contribution -- transformation -- action", which reflects that the cultural orientation of mathematics textbooks is often too obscure and does not clearly guide students to think. In fact, this leaves room for the cultivation of recessive traditional thoughts and values, which needs to be explored by teachers. From contribution mode to transformation mode, teachers need to make full use of textbooks and guide students to further think about the cultural significance of mathematical knowledge on the basis of learning. For example, after the problem is solved, add: "What do you think about this?". From transformation mode to action mode, we can use some specific cultural practices.

The rational analysis of the value orientation of Chinese excellent traditional culture in textbooks provides a way of thinking for the optimization of mathematics textbooks. However, it remains to be further explored and studied how students acquire the value of cultural content in mathematics textbooks.

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A Comparative Study of the Contents of “Plural” in Chinese and Japanese High School Mathematics Courses

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Abstract. Select China taught A high school math teaching material with book research mathematics textbook edition of the "plural" content as the research object, from chapter theme page, knowledge structure, column. The results show that there is a close connection between old and new knowledge in the theme page of the textbook chapter, the column design structure is rich, the basic knowledge is emphasized and the content distribution is focused, and the difficulty of example exercises is relatively low. The theme page of the chapter focuses on the introduction of mathematical history, the column design is simple, the content of knowledge expansion is extensive and comprehensive, and the comprehensive difficulty of examples and exercises is high. On this basis, we get some enlightenment on the compilation of complex content in Chinese textbooks.

1 Introduction

Teaching materials are an important carrier of courses, as well as a concrete embodiment of course objectives and teaching content. Plural number system as a secondary school for the last time expand, its instrumental determines it can not be ignored in the middle school mathematics problem solving function, complex connected algebra, triangle and how a few knowledge of the contents, and various ways of representation and a variety of operation, greatly broaden the student’s point of view, in solving many math problems, there is a new way to combine structure, number and form, transformation and dialectical unity. In recent years, Chinese scholars have conducted comparative studies on the plural contents of textbooks in different countries, but they lack integrity and openness. For example, Wu Jun and Hu Pengyan have studied the plural contents in China and Singapore (Wu, 2016).Wang Fenping made a comparison between Chinese and English plural contents (Wang, 2011). Xie Jun compared the plural contents of China, Japan and Singapore (Xie, 2012).

Belong to east Asia between the two countries, and are influenced by Confucianism, culture and education background have a certain similarity, mathematical education communication has a long history, in recent years the world organization for economic cooperation and development (OECD) to carry out the program for international student assessment of PISA, the Japanese students are among the best in math test scores times, therefore, this study selects the complex contents of high school mathematics textbooks in China and Japan as the research object, and analyzes and compares them from

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four aspects: chapter theme page, column design, knowledge structure and difficulty of example exercises, in order to provide theoretical support and practical guidance for the compilation and revision of textbooks and teachers' teaching in China.

Chinese textbooks are selected for this study. The required Volume 2 of Mathematics A Edition of ordinary high school textbooks published by People's Education Press in 2019, in which the plural number is in chapter 7. Japanese textbooks are selected from high school Mathematics ii (Masaki Kawanaka ほ Kasakawa 13 math ii.) and Mathematics iii (Revised Mathematics iii) revised textbooks published by Heisei Publishing House in 29th, in which the plural numbers are mainly located in the second section of Mathematics ii and the first chapter of Mathematics iii.

2 Research Results and Analysis

2.1 Chapter Theme Page

The topic page of the Chinese People's Education Edition is composed of the chapter title, the figure before the chapter and the introduction. The design of the figure before the chapter is a Map of China's space industry, indicating that the expansion of the number system is developing continuously, just like the space industry, which is another leap in human understanding of logarithms. Chapter Introduction introduces the historical background of complex numbers through unitary quadratic equations with real coefficients, how to square negative real numbers and the difficulties encountered by mathematicians in solving the root of cubic equations with unitary real coefficients. The theme page of the Japanese Mathematical Research Edition of Mathematics ii and Mathematics iii complex chapters is composed of chapter titles, pictures of characters, chapter contents and chapter introduction. Mathematics II chapter before the figure character is a mathematician Galois, chapter preface introduces the 16th century Carl dano and ferrari has put forward three and four, the solution to the equation then gaussian developed in the early 19th century the fundamental theorem of algebra, then Abel found that on average five times, there is no radical solutions of eventually by Galois in-depth study of the nature of this kind of problem; The figure before the chapter of Mathematics iii is mathematician Gauss. The preface mainly tells about Gauss's transformation of complex numbers into points on the coordinate plane, proving that complex numbers can be understood intuitively on the plane. The content of the introduction is simple and clear. The comparison shows that the human Education edition pays more attention to the relevance and extensibility of knowledge, while the mathematical Research edition pays more attention to the permeability of mathematical history and mathematical culture, so that students can better understand the historical development of complex numbers.

2.2 The Arrangement of Knowledge Structure

The human education edition has three sections and independent chapters, with long length. The human education Edition attaches importance to the basic knowledge of the plural. The contents of complex numbers in mathematical Research edition are distributed in mathematics ii and mathematics iii textbooks. The complex numbers in

Mathematics ii is not an independent chapter, but only the first section of the second chapter. In mathematics iii, “complex number plane” consists of four sections and an independent chapter, which is based on the extension and application of complex number after mathematics ii. The first two sections of the concept of complex numbers and the four operations of complex numbers and number research edition of mathematics ii complex content distribution is basically the same, but mathematics ii more than the square root of negative numbers, in comparison, this part of the human edition is more detailed and complete. In mathematics ii, the complex number is arranged in the chapter of “complex number and equation”, which only occupies a small part of the content. It is to provide basic knowledge for the solution of quadratic equation, remainder theorem, factor theorem and higher order equation, so as to extend the solution of equation to the complex number range. The section of “trigonometric representation of complex numbers” in human Education edition is arranged in Mathematics iii by the number research edition. The plane of complex numbers in mathematics iii is to expand and perfect the content of complex numbers in mathematics ii, so that it constitutes a more complete knowledge system, paying attention to the gradient of complex numbers. On the whole, the human Education edition focuses on basic knowledge and content distribution, while the compilation of knowledge of the digital research edition is relatively in-depth and rich, with a wide range of knowledge and a spiraling form of content.

2.3 The Framework of Column Design

The two editions of textbooks set up prechapter diagram and chapter introduction, each section of content knowledge points are set up sample questions, at the end of the chapter set review questions. Knowledge points need to pay attention to the use of human education version of the prompt column, the number of research version of the supplement column. Human Education edition mainly through “thinking”, “exploration” and other ways to introduce new content, “main knowledge” using blue bold font, not only has “practice” column, after each section also has “exercises” column, “reading and thinking”, “exploration and discovery” and so on; And version directly introduce the concept of plural number inquiry, “the main knowledge” using different colors of bold font or squares are given, and the error-prone points set up under the “note”, after the end of each section also has “exercises”, set “practice” after each sample, sample corresponding to the corresponding knowledge point tailgating exercises, did not specifically set post-holiday “exercises”, through “research” to the text of the knowledge of the extension, and in the end of the chapter there are related exercises.Human education edition of the “review reference” is divided into review and consolidation, comprehensive application, expansion and exploration of three parts, the end of the chapter set up a “summary”, mainly this chapter of knowledge structure summary and content review and thinking; There are two types of “problem” and “practice problem” in the mathematical research edition, and the “practice problem” is divided into A and B groups. At the end of chapter ii of Mathematical Research Edition, the phenomenon explained by complex numbers extends complex numbers to life. In comparison, the column design of human education edition is more abundant and diverse, while the number research edition is relatively brief.

2.4 Setting of Sample Questions and Exercise Difficulty

Mainly referring to Bao Jiansheng's comprehensive difficulty model(Bao, 2002), this paper analyzes the comprehensive difficulty of example exercises with the same complex contents in Chinese and Japanese textbooks from the five difficulty factors of "background", "operation", "inquiry", "inference" and "knowledge content", and obtains Table 1.

Table 1 Weighted average of difficulty factors

	Explore	Background	Operation	Inference	Knowledge content	Range	Comprehensive difficulty
Chinese	1.877	1.074	2.605	1.704	1.938	1.531	1.8396
Japanese	1.686	1.043	2.957	2.042	2.043	1.914	1.9544

According to the table above, the comprehensive difficulty level of complex examples and exercises in Chinese and Japanese textbooks is obtained. The details are as follows: In terms of the "background" factor, the weighted average of the two editions of textbooks is relatively low, and there is no significant difference between them, showing a downward trend. The factor of "inquiry" in human education edition is 0.191 higher than that in mathematical research edition. In terms of "operation" and "reasoning" factors, the weighted average of the two editions is significantly different, and the weighted average of the number research edition is 0.352 and 0.338 higher than that of the human education edition respectively. The factor of "knowledge content" in the research edition is 0.105 higher than that in the human education edition. From comprehensive difficulty can be concluded that the research version of a number of case problem difficulty slightly higher than that of o 'clock, weighted average of the two versions example exercises are shown on the five difficulty factor is not balanced, is focused on the "operation" and the content of "knowledge", in the plural chapters mainly in the form of pure math, cases of difficult problem sets several research version slightly higher than the o 'clock, the mathematical research edition focuses on "operation" and "reasoning", while the human education edition focuses on "exploration".

3 Enlightenment

3.1 In the Preface, the Content of Mathematical History Can Be Appropriately Added to Reflect the Historical Value of Complex Numbers in the Field of Mathematics

Chapter theme page is the window to open the knowledge of this chapter, leading the content of the whole chapter, and an important platform to reflect the connection between mathematics and real life. Accurate guidance on chapter theme page content can stimulate students' interest in learning. Comparison, textbook chapter theme page style is different, the two countries are in the practical life cohesive, and the old and new knowledge and pay attention to the knowledge of the history of mathematics is introduced several research edition way, of course, are the content of the history of mathematics for complex related arrangement in the final summary of teaching materials, the introduction of the two different way will produce different effect on the students. O ‘clock version also can draw lessons from several research appropriate add content to the history of mathematics in the chapter introduction, help students learn the history of the plural before learning this chapter, the development, understand the plural of discovery and development process, feeling the mathematicians in the process of exploring the plural perseverance and excelsior spirit, understand the role of human rational thinking in number system expansion.

3.2 Optimize the Knowledge Structure of Complex Content in Teaching Materials, and Embody the Systematicness and Practicability of Complex Knowledge System

Knowledge structure is the core of students’ learning mathematics, and it is the concrete embodiment of curriculum objectives. Knowledge content comes from the abstraction of the world. The study found that there are great differences in the arrangement of knowledge structure in the two textbooks. The “complex content” of the digital research edition is still relatively rich, difficult and comprehensive, which means that students can practice at the same time, while the human education edition generally focuses on the logical order and the content is written in a spiraling form, which is conducive to students’ mastery and understanding of the “four bases”. O ‘clock for the writing of the basic knowledge of plural detailed, but still can be reference for r version of the appropriate adjustments to the plural breadth of content, the “reading and thinking”, “exploration and discovery,” expand the columns of the knowledge to appropriate text, several department expanded the integrity and consistency of performance, help students in learning the plural form a complete knowledge structure.

3.3 Strengthen the Diversity and Integrity of the Column, Reflect the Rich and Diverse Column Design

The column design is an indispensable part of the textbook, which shows the unique characteristics of the textbook, is the supplement and extension of the text content of the textbook, and makes the textbook system more complete. Compared with the digital research edition, the humanities edition is more complete and rich, focusing on cultivating students’ awareness of problems and innovation, boosting the development of students’ core literacy, which is worth inheriting and carrying forward. At the same time, through the comparative analysis of the two countries, we can continue to enrich and optimize the setting of the special columns of the teaching and research edition. Such as a number of research version at the end of the chapter extends the complex phenomenon of the specification, contact life have increased interest in math learning of mathematics, broke the rote learning of complex number, lets the student understand mathematics originates from life, and applied to the life, give play to the importance and practicability of the plural, not only embodies the plural courses in high school, also can strengthen the practicability of students on the plural.

3.4 Pay Attention to the Combination of Typicality and Comprehensiveness of Example Exercises, Reflect the Effectiveness of Example Exercises

Problem is the heart of mathematics, is the original driving force to promote the development of mathematics, example exercises occupy a large space in the textbook, can help students understand the content of knowledge, and the abstract mathematical knowledge to be used in practice, so the importance of setting typical example exercises is self-evident. Through the comparison of the difficulty of exercises in the two editions, it is found that the two editions do not maintain a relative balance between the five difficulty factors, the "background" factor is much smaller than other factors, and the "operation" factor is relatively high. Therefore, the selection of example exercises in any edition should pay attention to the balance between each difficulty. However, we should also consider the characteristics of the knowledge in this chapter, and pay attention to its practicality and typicality when weighing the five factors. As the characteristics of the complex content is mainly based on calculation and reasoning, calculation is the basis, which can improve students' operation skills and help cultivate their mathematical operation accomplishment. Reasoning is the guarantee, through reasoning can develop students rigorous and realistic scientific spirit, conducive to the cultivation of their logical reasoning accomplishment.

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Research on Teaching Resources Facilitating Prospective Mathematics Teacher of Designing Project-Based Learning

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Abstract. The application of Project-based Leaning (PBL) in mathematics has gradually become the focus of international mathematics education research since students benefited greatly from PBL. This also makes the teacher-educators who teach prospective teachers (PTs) about PBL in courses need to find abundant teaching resources for PTs' deeply understanding on PBL design. This research collected and analysed PTs' (n = 20) reflective journals and other transcript data of the course based on the theoretical framework of Activity Theory to answer what kind of teaching resources were needed and how they influence the teaching and learning process. There are two major findings: (1) The course enabled the teaching resources to contribute to PTs' understanding of PBL design; (2) the teaching resources varied from forms, topics of relevant knowledge, and developable resources influence PTs on PBL design in different ways.

1 Introduction

In recent years, the application of Project-based Learning (PBL) in mathematics has gradually become the focus of international mathematics education research since students benefited greatly from PBL in learning mathematics are intrinsically motivated, think more critically, and prefer to peer learning (Holmes and Hwang 2016; Özdemir et al. 2015). However, it is still limited to be applied in a small range in China that most disciplines are in the theoretical research and experimental stage, which requires educators focus more on the junction point of PBL and subject teaching (Huang and Zhao 2014), and the specific link of teaching and learning, with refining the research direction and spotlighting on PBL in the teaching process, and exploring some research results that can be extended.

Based on the new requirements, present situation and changed environment, teacher educators who teach prospective teachers (PTs) about PBL in a course need to find abundant materials which derive from different sources with distinguishing teaching purpose to facilitate PTs of learning how to design PBL. (In this case, the teacher educator is also referred to "the facilitator" in this research.) The products coming from the process of the design and modification of PBL are also teaching resources used by facilitators and PTs to discuss and inquire. Research on what type of materials is utilized by participants (facilitators and PTs) in course, and how the facilitator used these teaching resources

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to facilitate the PTs can help them to identify the major problems at the beginning stage, is aiming to integrate a more efficient teaching and learning process. By solving the problems found via analyzing with the framework of Activity Theory immediately, the teaching objectives can be better carried out, and both sides of the participants are capable to form clear goals and improve the professional growth.

Our study was guided by the following research questions:

1. What teaching resources did facilitator use to teach PTs designing PBL?
2. What impacts did teaching resources have on PTs designing PBL?

2 Literature Review

While curriculum resources have been examined in terms of the quality and support for student learning, less research has focused on teacher learning with educative curriculum resources, meanwhile, teachers are increasingly expected to design their own materials, particularly in times of curriculum change and profusion of digital materials on the web (Pepin 2018). To meet the need of this state, the education of PTs is becoming more and more necessary, which also proves the teaching resources of the courses that PTs participate in are playing a crucial part.

PBL design experience within teacher education provides opportunities for PTs to simultaneously acquire knowledge and develop professional skills deemed important for the future teachers. Some studies have applied the principles of PBL with PTs and claimed that PTs can become better problem-solvers (Mettas and Constantinou 2008), and gain benefits from formative assessment of the final outcomes of the project that are group and individual written reports, portfolios, multimedia presentations and physical models (Frank and Barzilai 2004). Therefore, it should be noticed that the preparation of PTs education is as important as in-service teacher education, or even should be paid more attention on this.

3 Framework

The theoretical framework of PTs designing PBL through teaching resources is based on Activity Theory which provides tools that we use in bringing to light and analyzing the tensions during PBL design activities.

Scholars such as Cole and Engeström applied and continued to develop through extensive practice until the third generation was established (Engeström 2001). The conflicts, disagreements, or contradictions in cases of teaching and learning how to design PBL can be deeply explored from the collaborative data by the framework of activity theory, and the strategies to deal with and resolve these problems can be analyzed in details. As a result, the perspective of activity theory can reveal the contradictions related to teaching resources existing in the activity system (Fig. 1) helping to understand the formation process of PBL design.

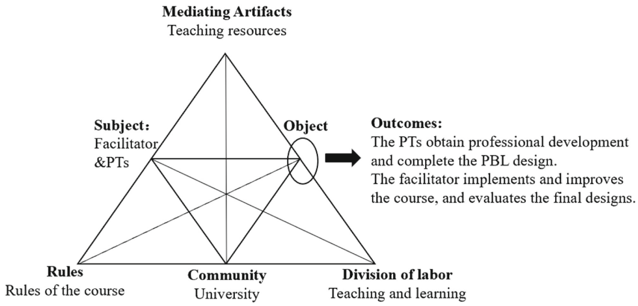

Fig. 1. The activity system of the course in the framework of the third generation activity theory

4 Methodology

4.1 Data Collection

This research studied a group of twenty PTs learning how to design PBL in the course named “Research on Mathematics Curriculum and Teaching Materials”. The research investigated the PTs’ learning before and after the course as well as integrated all the teaching resources used and developed by facilitator and PTs. Data collection involved analysis of video or audio data of interaction among the participants and text documents of PTs’ journals collected during the process in conjunction with individual interviews immediately after the course (Table 1).

Table 1. Data collection of each week

Week	1st	2nd	3rd	4th
Teaching records	1	1	1	×
Courseware (slideshows, videos, products of learning, etc.)	1	3	6	5
Group discussion records	5	5	5	5
Reflective journals	20	20	20	20
Interviews	3	3	3	3
Performance Feedbacks	×	1	1	×

4.2 Data Analysis

To answer the research questions, data analysis of the reflection journals and the post lesson interviews began by adapting the third generation of Activity Theory as coding scheme initially developed by Engeström (2001).

For the first step, the teaching resources should be collected and interpreted in details (Table 2). The two authors independently coded approximately 8 reflective journals (10% of the total) of the data and then met to discuss and clarify interpretations. After several iterations, the inter-rater consistency is above 90%. The coding work is completed with Nvivo11, the software for qualitative research.

Table 2. Teaching resources in PTs education of PBL design

Teaching resources	Interpretations
Slideshows and random discussion	The contents of the slideshows contain the basic knowledge of PBL, photos and designs of PBL lessons etc. The participants discussed randomly about the contents in slideshows
Definitions and standards of PBL from PTs	The facilitator asked PTs to discuss in groups about the definition and standards of PBL and hand in their results in class. All the participants had ample conversations with these documents
Shared books or literatures	The facilitator offered middle school mathematics textbooks and recommended some reading materials which are shared to all participants in the group chat online
Procedural and final version of PTs' PBL designs	The PTs handed in their PBL designs as products of groupworks in each week, which contains 3 drafts and 1 final version of PBL design
Videos or designs of existing PBL implementations	Facilitator showed 2 video clips of British PBL course. PTs shared the process of making the products of their PBL designs

5 Conclusion and Discussion

5.1 The ideological transformation of PTs

Along with learning from the teaching resources, PTs' mindsets transformed in the course in different aspects: (1) The promotion that PBL may lead to dose not aim to the outstanding performance, but to the students learning process; (2) The evaluation of PBL cannot focus only on students' products, but the whole procedure.

The contradiction between their previous and subsequent cognitions on PBL allow PTs to think deeply about the nature of PBL and its design, which also lead to some new ideas and a higher level of understanding on mathematics teaching and learning, which can be proved by a PT's reflection in the interview, he talked about his feelings and gains of this course:

> In the process of design, I realized that PBL has its special meaning for mathematics teaching. It is an effective way to cheer the students up and join into the class. Certainly, it also has problems on designing and promoting.

5.2 The Obstacles in the Designing Process of PBL

The troubles that PTs met while designing with present resources, and the issues that they realized while sharing designs in the class helped them reflect on their procedural outcomes. Firstly, it is difficult for them to balance all the six standards concluded by themselves when they design the details of PBL. Secondly, the traditional teaching design that they are familiar with always gives interferences on PBL design and causes that the PTs cannot keep their minds out of the traditional way of teaching, and always confused about whether their design is PBL or not. Thirdly, the PTs may not fully understand facilitator's amending advice and carry out them hundred percent.

These obstacles are neither positive nor negative for PTs working on the design. It depends on how they treat the troubles and how their attitudes are to the PBL's influence on mathematics teaching and learning. These phenomena also need facilitations from the facilitator to encourage PTs keep their enthusiasm of exploring a best design.

5.3 The Difficulties in the Future PBL Implementation

With the implementation of the course and in depth, the PTs considered not only the content of design, but the ultimate effects of putting into action on the PBL design in a real math class by using hypothesis. They gradually find the difficulties of in-service teacher implementing PBL by watching videos and learning from sophisticated cases and designs: the in-service teachers need plenty of time to learn about PBL, which may influence their daily teaching, in other words, they have too much regular work to take time for designing and implementing PBL. In the subsequent interviews, they said that they were worried about the future teaching works would not be available for carrying out PBL, as would bring the knowledge they have learned to nowhere in math teaching. Except for the worries, the PTs also mentioned that they had no opportunities to implement their designs in authentic mathematics class, which is unhelpful for them to answer guesses, solve doubts, and solute those practical problems.

As the analysis of PTs learning how to design PBL, the expansive learning process showed up at the same time. The expansive learning cycle goes upwards spirally to a new cycle, and operates together with the contradictions found in the activity system where the teaching resources function as mediating artifacts. The resources helped PTs completing the outcomes and inspiring their thoughts related to a broader scope of PBL and mathematics teaching and learning.

For the improvements of this course, several pieces of adjustments have been analyzed above already. The future arrangements hope to meet some opportunities for PTs to teach students with their own PBL designs, which must bring them deeper understanding and more profound effects.

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A Comparative Analysis on the "Arbitrary Angle and Radian System" Content of China and America

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Abstract. This paper analyzes the content of "Arbitrary angle and radian system" in Chinese and American high school mathematics textbooks from macro and micro levels. At the macro level, curriculum standards and content range are compared, while at the micro level, concept learning, exercises and mathematical history are compared. The results show that compared with American mathematics textbooks, Chinese mathematics textbooks pay more attention to the cultivation of students' "basic knowledge and ability" rather than inquiry ability. Chinese textbooks respect the logic of mathematical knowledge and emphasize the expression of mathematical language in concepts and exercises, while American textbooks are more reflective of real life. Chinese textbooks are more closely related to knowledge than American textbooks. Finally, some suggestions are put forward.

1 Introduction

Textbooks are the main tools for teachers and students to carry out teaching and learning, which directly affect the effect of teaching and the quality of school education (Schmidt 2001). "They represent an interpretation of policy in terms of concrete actions of teaching and learning." (Valverde et al. 2002). Textbooks play an important role in both theoretical research and teaching practice. Although there are many researches on mathematics textbooks in various countries, there are few researches on trigonometry content in middle school textbooks. Trigonometry is very important in mathematical system. It is a bridge linking geometry and algebra and a channel connecting elementary mathematics and higher mathematics. Many important mathematical knowledge such as function, vector and coordinate are related to trigonometry, which is used to solve a large number of practical problems. However, previous studies have shown that students and even on-the-job mathematics teachers have some difficulties in understanding trigonometry, especially when learning radian system. In addition, trigonometric functions in high school textbooks all start from learning arbitrary angle and radian system, so this section is the focus of this study.

In particular, we aimed to answer the following questions:

1. What are the characteristics of the compilation of "arbitrary angle and radian system" in Chinese and American textbooks?
2. What are the characteristics of the concept learning in Chinese and American textbooks?

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2 Theoretical Framework

Li et al. (2009) compared textbooks from macro and micro dimensions. Specially, the process of conceptualization is studied in detail. Yuan (2012) detailed the comparative study on the introduction of the concepts, examples and exercises of the four countries' trigonometric functions. Fu et al. (2018) focused on the similarities and differences between trigonometric exercises in Chinese and American textbooks. Therefore, we will refer to the framework of Li et.al (2009) to analyse the content of "radian system" in Chinese and American textbooks from macro and micro dimensions. At the micro level, we will discuss the commonalities and differences in the process of learning concepts, exercises and mathematical history in the two countries' textbooks. We will refer to the comprehensive difficulty model of Bao (2002) to analyse the difficulties of exercises. For mathematical history, the framework on the way of applying mathematical history in textbooks of Wang (2012) will be our reference for analysis.

3 Method

We used three textbooks in this study. Regular high school textbook series A (Referred as CH-PEP(A)) and the regular high school textbook series B (Referred as CH-PEP(B)) published by the people's education press of China, which are matched with China's new mathematics curriculum standard and used uniformly throughout the country, were selected to present Chinese textbooks. By analysing these two textbooks, we can find the overall characteristics of Chinese textbooks in the "Arbitrary angle and radian system". The algebra series (Referred as AM-GLE) published by McGraw hill Education, written strictly according to the California mathematics curriculum standards. We will select the content related to radian system in Chinese and American mathematics textbooks. We combine quantitative research and qualitative research, and analyze the textbooks from the macro and micro levels. The specific analysis framework of this study is shown in Table 1.

Table 1: Textbook analysis framework

Macro level	Micro level		
	Concepts	Exercise difficulty	History of mathematics
Requirements in curriculum standards	Introduction Presentation Example	Exploration Background Operation Reasoning Knowledge content	Type Use
Proportion of pages			
Number of concepts			
Number of formulas			
Number of examples			
Number of exercises			

4 Results & Discussion

4.1 Macro Level

(a) **Requirements in curriculum standards**

Chinese goal specifications are referred to The National Mathematics Curriculum Standards for Senior Middle Schools in China, and American goals are referred to Common Core State Standards (CCSS). Firstly, both China and America emphasize students' understanding of the concepts of arbitrary angle and radian system, as well as the mutual transformation between angle and radian. Secondly, the goals in Chinese textbook are focused more on the students' learning outcomes while these in CCSS are highlighting the learning process of students. Thirdly, the Chinese curriculum standard emphasizes that students should understand the necessity of introducing radian system, but CCSS has no requirement for this part, which obviously reflects the importance of mathematical history and mathematical culture in China.

(b) **Content coverage**

Seeing from the following table and graph, we can find some interesting phenomenon. On the one hand, three textbooks take roughly the same length in the section of radian system (see Table 2). However, relative to all trigonometric knowledge points, the proportion of "radians" in American textbooks (about 7%) is much smaller than that in Chinese textbooks (about 10%).

Table 2: Comparison of content coverage

	Proportion	Concepts	Formulas	Examples	Exercises
CH-PEP(A)	9.89%	9	5	9	55
CH-PEP(B)	12.50%	9	4	8	62
AM-GLE	6.67%	8	1	14	79

On the other hand, both CH-PEP(B) and AM-GLE stratify exercises, providing more choice space for students with different learning foundations. CH-PEP (A) focus more on knowledge while AM-GLE does more on practice which may reflect the differences of the educational concepts between the two countries (Fig. 1).

4.2 Micro Level

(a) **Concepts**

Firstly, there are only four common concepts in the textbooks of the two countries which are the core concepts of this section. The concepts and formulas involved in Chinese textbooks (such as quadrant angle) are more abstract, while those in AM-GLE are more subtle (such as unit circle, standard position).

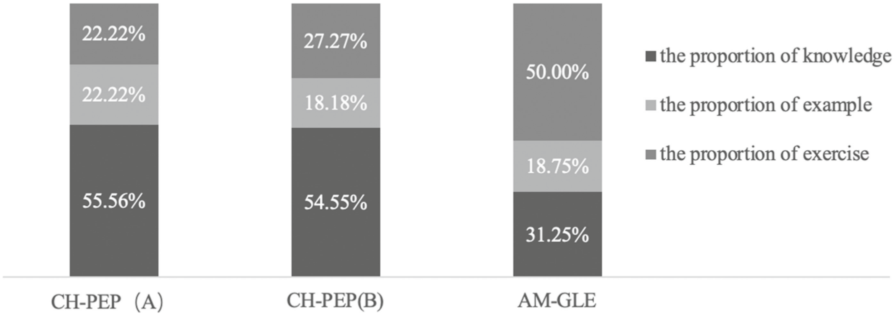

Fig. 1. Comparison of the proportion of knowledge, examples and exercises

Secondly, the introduction of concepts in the three textbooks are based on knowledge learning in the past and students' reality. But Chinese ones focus more on former while American ones concentrate more on later. Chinese textbooks focus on the expression of concepts and formulas in mathematical language, highlighting the importance of mathematical symbols. AM-GLE, tends to use graphs to describe intuitively and is less rigorous in mathematical expression of concepts. This point may reflects the difference in the logic of compiling mathematics textbooks in the two countries (Table 3).

Table 3: Comparison of concepts

	CH-PEP(A) & CH-PEP(B)	AM-GLE
Concept	**Positive angle, Negative angle,** Zero angle, Arbitrary angles, Quadrant angle, **Coterminal angle**, Degree measure, Radian measure, **One radian**	Initial side, Terminal side, Standard position, **Positive angle, Negative angle**, Unit circle, **One radian**, **Coterminal angle**
The presentation of concept	Abstract and highlighted in blue	Concise and with most illustrations to explain

Thirdly, in the three textbooks, there are some exercises accompanying after the concepts to help students consolidate their understanding of the definition. They all give detailed solutions to the examples, but each example in AM-GLE is accompanied by a short exercise with no solutions. Except for answering questions, there are some proof questions in Chinese textbooks, involving complex numerical calculation and simple abstract symbol operation. Some drawing questions are involved in AM-GLE, It is clear that the examples in Chinese textbooks are more focused on mathematical abstract thinking, logical reasoning and complex computing ability. In addition, in terms of the explanation of examples, only AM-GLE provides some explanations in the narration to help students understand better. The examples in AM-GLE take real life as the background, it will supplement some explanations for the situation.

Then we will take the concept of "radian system" as an example. In the three textbooks, Radian system is introduced by the angle system. The biggest difference between them is the specific derivation of 1rad. In the CH-PEP(A) and CH-EPE(B), the concept of 1 rad is based on the fact that the ratio of the arc length and radius is only related to angle, so that we can use the ratio of arc length and radius to measure the size of angle. Moreover, CH-PEP(A) adds the history of radian system which is consistent with the standard requirements in the form of a narration, making students understand the reasonableness of radian fabrication as a measurement tool. While AM-GLE directly gives the definition of angle of 1 rad without other explanation. In fact, it may be important for students to understand the necessity of introducing radians. Because after introducing radian system, the measure of angle changed to be decimal, establishing a one-to-one correspondence between the set of angles and the set of real numbers. Then, trigonometric functions can be calculated with other functions, expanding the application of trigonometric functions. For the examples following the concept, obviously compared with those in AM-GLE, the examples in Chinese textbooks involve more difficult and in-depth points.

(b) **Exercise**

In the section of "Arbitrary angles and radians", most of the exercises in the three textbooks are answer questions and a small number of them are drawing questions. There are more drawing questions in AM-GLE, maybe American courses pay more attention to the cultivation of students' visual imagination literacy. According to the results of the difficulty analysis model, we can find the following results:

From the comprehensive difficulty model (see Fig. 2), we can know that the exercises in Chinese textbooks are significantly higher in "Knowledge Content" than those in American textbooks (see Fig. 3), but there is little difference in the levels of "Exploration", "Background" and "Reasoning", both China and the United States focus on "Operation".

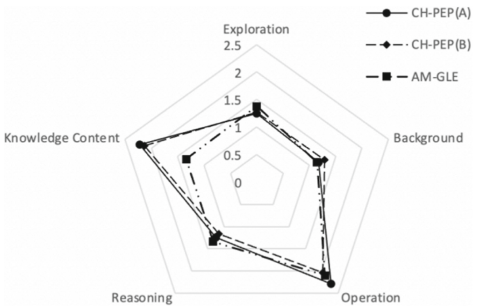

Fig. 2. The comprehensive comparison in the levels of Knowledge content.

There are fewer open questions and explorative questions as well as more memorization and comprehension questions in the three textbooks. Compared with American textbooks, Chinese textbooks have more memorization exercises and fewer comprehension questions. Exercises with numerical operations account for a higher proportion

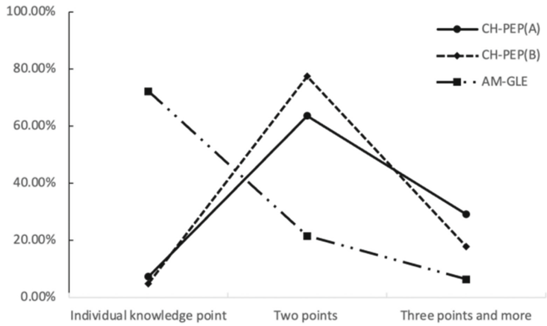

Fig. 3. The comparison of exercises of the difficulty of the exercises.

of all in the three textbooks. Comparatively speaking, Chinese textbooks attach more importance to the operation of abstract symbols. However, some exercises in American textbooks are a review of previous knowledge, so there are more simple reasoning and complex reasoning topics in them than these in Chinese textbooks.

(c) **History of Mathematics**

In the three textbooks, the history of mathematics only appears in CH-PEP(A), which is presented in the form of written reading materials.CH-PEP(A) applies the history of mathematics, which belongs to the adjunctive form, and briefly introduces the sinusoidal table made by Indians and the idea of radian system put forward by Euler. There is no relevant content in the other two textbooks.

5 Conclusion

Through the comparative analysis of "Arbitrary angle and radian system" in Chinese and American mathematics textbooks, the following conclusions can be drawn. Firstly, Chinese mathematics textbooks focus more on the cultivation of students' "basic knowledge and ability" than American one does on the exploring ability. Secondly, Chinese textbooks respect the logic of mathematical knowledge and emphasis the expression of mathematical language in concepts and exercises, while American textbooks present more about real life. Lastly, in Chinese textbooks knowledge is connected closer than that in American textbook. In addition, compared with the concepts in American textbook, those in Chinese textbooks are often introduced by prior knowledge, emphasizing abstract expression. The examples accompanied after them are more and deeper in knowledge and ability than those in American textbooks. Moreover, Chinese textbooks involve some part of the history of the radian system which American textbook doesn't do. The reason may need be further studied.

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Representations of Mathematicians in Lower Secondary Chinese Mathematics Textbooks

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Abstract. We conduct a study focused on representations of mathematicians in mathematics textbooks of lower secondary schools, adopting Foucauldian discourse analysis approach. The study shows that there is a stereotyped representation of mathematicians. The discourse of mathematicians is heterogeneity. The textbooks convey a cultural view of mathematics. The discourse of mathematicians in textbooks produces different subjectivities. The Foucauldian analysis of representations of mathematicians in textbooks can promote compilers of textbooks to reflect on the selection and presentation of mathematicians in textbooks, enhance teachers' ability of critical analysis and provide a new perspective for textbook evaluation.

1 Introduction

It's widely known that students possess several images associated with the nature of mathematics and mathematicians. Those images have been identified as critical for students to feel identified with, attracted to, and interested in the study of mathematics (Picker and Berry 2000; Rock and Shaw 2000). Lower Secondary is a critical period for the formation of students' role identity. Appropriate mathematician images can stimulate students' interest in the study of mathematics and enable them to better understand mathematics and mathematicians. If textbooks present an unfair image of mathematicians, it may hinder students' math learning. Some researchers suggest that the negative images of mathematics and mathematicians are a factor that inhibits students from pursuing an advanced degree in mathematics (Mendick 2005; Piatek-Jimenez 2008).

The study on representations of mathematicians, and especially scientists in general, has its origin in the pioneering work of Mead and Metraux (1957). One of the first collective efforts to investigate the popular image of mathematicians can be traced back to the late 1980s. Some studies explored the representation of mathematicians in mass media such as movies, books and popular songs (Furinghetti 1993; Wilson and Latterell 2001). Some focused on identifying representations of mathematicians in mathematics textbooks and found that mathematicians in textbooks were mainly dominated by men (McBride 1989; Aguilar and Castaneda 2020). In this research, we use a Foucauldian approach (Kollosche 2016) for the analysis of representations of mathematicians in lower secondary Chinese mathematics textbooks. We draw on theoretical notions from the French philosopher Michel Foucault (1926–1984) to frame our work, but we also use a form of discourse analysis based on his ideas and commonly identified as Foucauldian discourse analysis.

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2 Foucauldian Discourse Analysis of Representations of Mathematicians in Chinese Mathematics Textbooks

Our study is guided by the question:how the notion of "the mathematician" is constructed in lower secondary Chinese mathematics textbooks? To develop our study, we applied a Foucauldian analysis to the representative Chinese mathematics textbooks for lower secondary level. Lower secondary education in China is typically followed by students between 11 and 15 years old and it is divided into three grade. For this study, we selected 12 mathematics textbooks published by People's Education Press and Beijing Normal University Press since 2012 as research objects, all compiled by Chinese authors. A Foucauldian approach encourages the mathematics education researcher to question the taken-for-granted status of mathematical discourse (McBride 1989) and to relate these assumptions to issues of power, regimes of truth, and processes of subjectivation (Kollosche 2016). In our case, we break down our analysis into the following four stages: discursive constructions, discourses, positionings and subjectivity.

To determine how the notion of the mathematician was constructed in the selected textbooks, we focused on locating the representations of mathematicians contained in those textbooks. By analyzing the textbooks, it was noticed that the format of the representations of mathematicians is varied. For instance, there were images depicting mathematicians or representing groups of people performing some mathematical activity. There were also texts describing. Thus, with the aim of using a unit of analysis covering the range of representations that could be found in the selected textbooks, we decided to conduct the study of representation of a mathematician including not only text but also drawings, photographs, and illustrations in general.

It should be noted that the term mathematician was understood in a broad sense: a mathematician was considered to be any person (or a group of people) that in the textbook either (1) was explicitly referred to as a mathematician, (2) was credited with the development of a mathematical concept or tool, or (3) was displayed performing some sort of mathematical activity. Once a representation of a mathematician was located, we focused our attention on six aspects of such representation.

First is the individuals or groups of persons represented and their gender. We identified the most frequently represented mathematicians in the mathematics textbooks and the particular names of the people represented and their gender. Second is the period of time that the represented mathematician lived. We tried to determine the time in which the represented mathematicians lived. This allowed us to determine whether the representations included in the textbooks were of ancient or contemporary mathematicians. Third is the origin of the mathematician represented, including the determination of the country or cultural attribution of the mathematicians represented. Fourth is the role of representation, categorized the role that the representations had in the lessons where they appeared. Two categories were used: (1) the representation is used to illustrate a concept or to introduce a mathematical activity or (2) the representation only has anecdotal value. Fifth is the format of the representation which were classified as textual descriptions or illustrations. If there was a picture and also a statement about the same mathematician, two units were counted in such situation. Sixth is the knowledge domain in which the representation is located. What mathematicians appear in textbooks are mostly combined with the contents of mathematical knowledge contained in textbooks, and seldom introduced for the sake of introducing mathematicians. Focusing on the above six aspects was a means to produce a characterization of the representations of mathematicians that would provide us with an argumentative basis to answer how the notion of “the mathematician” is.

3 Results of the Application of the Foucauldian Discourse Analysis

According to Foucauldian discourse analysis, the presentation is organized into four stages. It permits us to answer the research question and allows us to delineate the effects that such discursive construction may have on students’ subjectivities.

3.1 Stage 1. How “the Mathematician” is Constructed in the Chinese Textbooks

The notion of mathematician is constructed through a discourse constituted by texts and images. In total, 144 representations of mathematicians were found in 12 textbooks: 60 in first-grade textbooks, 48 in second-grade and 36 in third-grade. 119 are textual representations of mathematicians and 25 are graphic illustrations. The format of the representation is important since it delineates possible forms of expression, and in the particular case of the combination of text and images, it favors that the represented objects are remembered by the reader (Mandl and Levin 1989). In most cases, the representations of mathematicians do not serve to introduce a mathematical concept or activity; rather, the representations have an anecdotal role and are used as isolated pieces of information (80 representations have an anecdotal value, while 64 are used to introduce an activity or a concept).

The individuals or groups of persons most often represented were ancient Egyptians, Pythagoras, the ancient Chinese, the ancient Greek, Euclid, Liu hui, Diophantus, Hua luo-geng and Al-Khwarizmi. In general, the textbooks mainly depict representations of mathematicians who lived in several centuries ago. In connection to the origin of the mathematicians represented in the textbook sample, they were mainly representations of individuals of Chinese and European origin. The analysis shows that the textbooks have an eminently male representation of mathematicians: out of 144 representations of mathematicians in the textbooks, only one female mathematician is identified.

3.2 Stage 2. Discursive Constructions of “the Mathematician”

In the case of the textbooks, it seems that the predominant discourse on mathematician is one that projects this individual as a man, primarily European and Chinese, who lived in ancient times. In all the textbooks, only ten representations of mathematicians belong to the 20th century, e.g. Hua luo-geng and Andrew Wiles. A discourse like this, where the representations of ancient mathematicians prevail, limits the opportunities for students to envisage mathematicians as people who have a role in contemporary societies. In terms of the knowledge domain: 60% of them were contained in the field of number and algebra, 34% in graphics and geometry, and 6% in probability and statistics. Although the discourses of representations in different fields of knowledge differ, the images of ancient mathematicians in textbooks are universal. Because most of the mathematical topics addressed at this educational level were developed by ancient mathematicians, like in the case of the Pythagorean theorem.

Yet often the representations of mathematicians in these textbooks are not used to introduce a mathematical concept nor an activity. Rather, the representations have an anecdotic role. This kind of representations with an anecdotic role represents an area of opportunity to broaden the diversity of representations of mathematicians and promote a richer image of them among students. We postulate that the discourse about mathematicians is non-homogeneous. This means that the dominant discourse is not unique. Through the in-depth analysis of mathematicians in Chinese textbooks, it is found that the discourse on mathematicians is diverse. The discourse about the mathematician that dominates in each region is determined "by its own specific set of social forces and power relations" (Walshaw 2001, p. 486).

3.3 Stage 3. Subject Positions

One of the effects of discourse on representations of mathematicians is the way in which they position students in relation to mathematics. Results show that lower secondary Chinese mathematics textbooks convey a cultural view of mathematics, that mathematics is a product of human effort. It also show a discourse that women are ignored, it presents mathematics as an endeavor for men. As the carrier of mathematics knowledge, textbook is an important tool for students to learn mathematics. By using textbooks, students will touch on the discourse about representation of mathematicians, and a similar situation could happen to the subjectification of Chinese students.

When using these textbooks, students will find representations of groups of people doing mathematics, which are likely to be representations of social groups, such as ancient Egyptians. They will also find representations related to Chinese mathematical culture and references to the development of mathematics in contemporary China. Such discourse favors dividing practices between social groups and classes.

3.4 Stage 4. Subjectivity

The discourse on mathematicians and the subject positions made available by this discourse, situates students in a vantage point from which certain ways of seeing the world and being in the world are descried. We refer to what students might feel, think, and experience from the subject positions offered by discourse. It is reasonable to assume that many Chinese students who are in contact with discourse transmitted through these textbooks adopt subject positions from which it is difficult to visualize themselves as part of mathematics community. If we consider for instance the case of female students who experience the discourse on mathematician that was identified in their textbooks, they are probably do not identify themselves with the mathematicians or as a potential member of the mathematics community. The problem is that this discourse sustains a regime of truth about what it means to be a woman in the world of mathematics, and in such regime of truth, women are definitely not its protagonists.

This study confirms that the existence of different discursive constructions about "mathematician". They can be different in each social group and determined by its own social dynamics and power structures. One consequence of this is that each "version" of the discourse creates different subjectivities in the students and manifests itself in different students' images of the mathematician. As can be noticed, our results on the representations of mathematicians are similar to those found in Western countries. All of them report the existence of stereotypical representations of mathematicians. The difference is that Chinese mathematics textbooks present more positive images of Chinese mathematicians.

4 Concluding Remarks

The study reported here identifies the presence of a stereotyped representation of mathematicians in the Chinese textbooks. The representations of mathematicians that most frequently appear in these textbooks are males, mainly Europeans and Chinese, who lived in ancient times, while mathematicians in other cultures mainly appear in the form of group mathematicians. The discourse of mathematicians is heterogeneity. The textbooks present a positive image of Chinese mathematicians, but a certain negative image of western mathematicians. The textbooks convey a cultural view of mathematics. The discourse of mathematicians in textbooks produces different subjectivities. However, from the perspective of educational equity, this is an imperfect mathematical cultural view, which is reflected in the discourse that women are practically exiled. This discourse tends to divide practice in which Chinese and European are classified as able to contribute to the development of mathematics, while groups such as ancient Egyptians and Ancient Indians are not. The discourse of mathematicians in textbooks produces different subjectivity. The discourse about mathematicians in lower secondary mathematics textbooks may lead many students, especially girls and minority students, to adopt subject positions that excludes them from the mathematics community.

However, Foucault's concept of resistance suggests that the power that is exercised through discourse can be questioned and contested. Indeed, we believe that both textbook authors and mathematics teachers could question and exert resistance to the knowledge and regimes of truth that a discourse creates. Teachers can follow textbook's assumptions and question the assumptions and structuring to generate mathematical discussion (Herbel-Eisenmann and Wagner 2007). This opens the possibility of using textbooks to help students interpret and improve the stereotype of the mathematicians. Teachers can work together with students to identify and analyze the stereotypical images of mathematicians contained in textbooks. However, exercise of this type of resistance by authors and teachers requires a certain level of awareness about the existence of the discourse, its consequences and the way in which it operates.

In fact, we believe that our study can contribute to nurturing awareness and knowledge about the discourse on the mathematician that prevails in China. Our analysis suggests that the processes of quality evaluation of mathematics textbooks in China should pay attention to how mathematicians are represented.

It is convenient to promote a more rich and inclusive representation of mathematicians in mathematics textbooks; a representation that discourages the perception that mathematics is a men's endeavor, destined for some social groups. The analysis presented shows that most representations of mathematicians have an anecdotal role. This could be an area of opportunity for nurturing a richer, more diverse, and updated image of mathematician among students. The set of representations of mathematicians with an anecdotal role could be enriched by including representations of contemporary mathematicians; mathematicians coming from racial, cultural, and social minority groups; and children and young people participating in mathematical competences.

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A Study on Learning Progression of Mathematics Teaching Material with Big Idea—Take Probability as an Example

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Abstract. The curriculum standard for general high school in China puts forward the concept of subject as the core to promote the cultivation of students' core literacy. The mathematics teaching material is developed with a "big idea", and learning progression is based on the "big perspective" of students' cognitive development to complete the teaching structure. Taking the probability theme of mathematics material of the PEP edition as an example, this paper analyzes the material thoroughly, extracts the conceptual hierarchy, and constructs the learning progression model based on Jones's theory, to accurately grasp the changes in students' learning to think.

1 Foreword

China's curriculum standard for ordinary high school pointed out that select the subject content, attach importance to the core role of the subject concept, realize the structure of course content, lead by the content theme, and realize the context of course content. This content provides an important basis for the compilation of mathematics materials, and it is also an action guide to teaching.

The new curriculum reform is based on the development of students and truly embodies the student-centered. In the mathematics curriculum standard of ordinary high school, the mathematics learning evaluation pays more attention to the students' skills and the formation of the mathematics core literacy. The standards of academic quality are divided into different levels, which requires teachers to carry out teaching on the principle of progressiveness. In September 2019, the People's Education Press High School Mathematics material adopted the subject-centered curriculum design orientation and paid attention to the learning progression of mathematics knowledge content. Learning progression is a typical description of the learning path of a subject, which is usually a series of concepts centered on the core concepts. Learning progression links with teaching material and teaching, teachers comb out the big idea according to the mathematics curriculum standards and teaching material, then according to the hierarchical relationship between concepts, establish the "Steps" of students learning. This article takes the high school probability topic as an example and unifies the big idea by analyzing teaching material, to explore the learning progression in mathematics teaching.

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2 Theoretical Framework

2.1 Summary of Big Idea

Many scholars have published their understanding of the big ideas. Although these views interpret the connotation of big idea differently, they still agreed on some characteristics of the big idea, such as the centrality and transferability of big idea.

Clarifying the type of big idea plays a crucial role in applying big idea in teaching. Formally, a big idea can be a concept or a theory, or it can be a theme or a point of view. In general, we can think of the big idea as a hierarchical structure composed of the big concepts, between the mathematics discipline units, within the mathematics discipline unit, and the mathematics subject class hours. The number of the more superior concepts is gradually reduced, but the abstractness and the transfer of knowledge are gradually strengthened. On the contrary, the number of the more inferior concepts is increased, but the abstractness and transfer of knowledge are gradually weakened.

The big idea has relativity. The big idea of mathematics can be further decomposed into several sub-concepts, similarly, sub-concepts can also be reduced to a larger concept. This mainly depends on the amount of mathematics knowledge and teaching content.

2.2 Summary of Learning Progression

Given the problem that the knowledge points in American education are not deep enough and the evaluation criteria can't satisfy the need for personnel training, the American educational circles have started educational reform. In 2004, Carol Smith and others reported to the National Research Council (NRC), that Learning Progression was officially introduced in the education field. This paper cites NRC's definition of learning progression as a process of enrichment, refinement and deepening of students' thinking and understanding of a learning topic over time.

Learning progression researchers in the United States discussed and organized learning progression elements as follows:

1. Big science ideas. That is the theory, concept, or way of thinking that a student is expected to learn over a period of time.
2. Starting points. That is the level of knowledge a student has before he or she enters the classroom.
3. The Upper anchors. That is the level that you expect the students to achieve.
4. Multiple pathways. There are Multiple intermediate levels between the beginning and end of a student's learning progression. If there is more than one advanced variable in the learning progression, there may be more than one up anchor point in the hierarchical learning progression.

3 Material Analysis with Big Idea

The People's education press of high school mathematics materials began used in September 2019, which formed mainly by compulsory, optional compulsory, and other components. Take the PEP-B edition as an example, the probability topic is mainly distributed in the mathematics compulsory second volume and the mathematics selective compulsory second volume. This paper mainly focuses on the compulsory content of the mathematics teaching material. The content of probability is distributed under the subject of "Statistics and probability" in Chapter 5 of Compulsory Volume 2 and is generally located in Chapter 5, Sect. 3, after the Statistics Section. The whole content is divided into five sub-chapters: 5.3.1 sample space and probability; 5.3.2 event relations and operations; 5.3.3 classical probability; 5.3.4 frequency and probability; 5.3.5 randomness of individual events.

Curriculum standards guide the preparation and review of teaching material, teaching material, and curriculum standards to maintain consistency. The curriculum standards of ordinary high schools are strict for students in the probabilistic section. The first is to be able to understand the meaning of the sample point and the sample space. The second is to grasp the properties of probability and calculate the probability of the occurrence of random events in the classical probability model. The third is to estimate the probability by frequency in the specific probability situation. Through the analysis of curriculum standards and material content, four basic concept levels are extracted: randomness; sample space; probability comparison; probability calculation. As shown in Fig. 1.

Students' understanding of randomness is the starting point of thinking development, and also the prerequisite for sample space and probability estimation. From the logical perspective of knowledge, the comparison and calculation of probability are based on the sample space. Starting with randomness, the cognitive level of students has been improved.

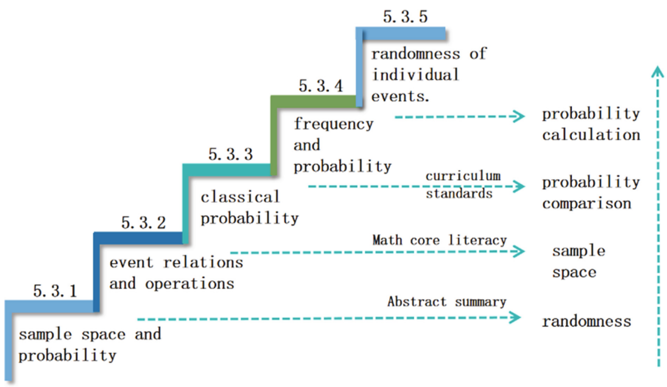

Fig. 1. Mathematical probability chapter distribution and its concept hierarchy

4 Learning Progression Study Based on Big Idea

From the above" randomness, sample space, probability comparison, probability calculation" hierarchical structure, combined with learning progression theory, design model diagram (Fig. 2) as follows.

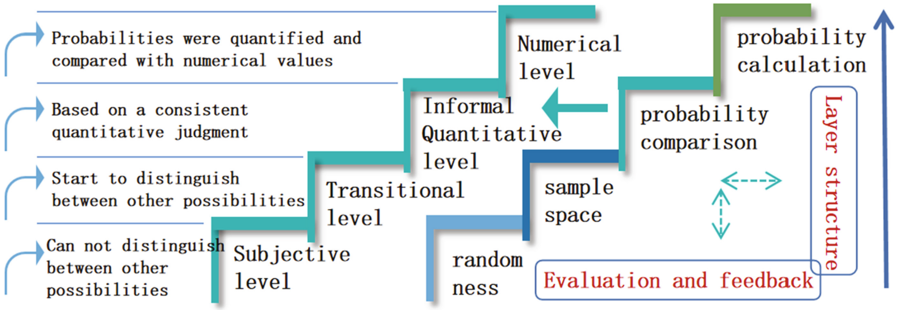

Fig. 2. Probability learning progression based on big idea

4.1 Determine the Starting Point of the Progression

Students' existing knowledge structure and learning ability are the basis for students' new learning. At the same time, pay attention to students thinking characteristics. After reviewing the research on the development of students' probability cognition, He Shengqing and many other teams involved the concept of "randomness", and the results showed that the students in high school performed well in the mastery of randomness. Therefore, this study takes sample space as the starting point and focuses on the levels of sample space, probability comparison, and probability calculation.

4.2 Determine the Ending Point of the Progression

The level at which the teaching is expected to reach. This is the higher level of understanding students need to achieve after learning a concept and is also the starting point for learning a higher level of concepts.

4.3 Divide Achievement Levels

It is helpful to accurately reflect the development of students' cognition to divide the cognitive level reasonably and to carry on the progressive description. Numerous educators and psychologists have focused on the development of individual cognition, and developed many generally accepted theories of cognitive development: Piaget's cognitive development stage theory throughout the individual age groups; combining the characteristics of mathematics, Dubinski studied the APOS theory of student concept learning; the Van Hill couple of the Netherlands proposed the Van Hill theory of student geometric learning. This lays a solid theoretical foundation for dividing the level of achievement of students at different levels.

This study takes the probabilistic content as an example and selects the cognitive division of the Jones team. Jones' research team explored student probabilistic thinking levels twice in 1997and 1999. Finally, it is divided into Subjective level, Transitional level, Informal Quantitative level, and Numerical level. The meaning of each level is stated as shown below (Table 1).

Table 1. A specific description of the probability level of Jones

Level	Name	Implications
1	Subjective level	Solving probability problems based on personal subjective preferences
2	Transition level	Personal thinking begins between subjective and quantitative, but more results return to subjective
3	Informal Quantitative level	Individuals have been able to think quantitatively, but have not yet formed an overall concept of quantity
4	Numerical level	The highest level of personal development can be expressed entirely in terms of quantitative concepts

4.4 Accurate Progression Description

This is a description of the student's expected performance. Taking the frame of Jones's thinking level as a theoretical guide, Table 2 gives a concrete progression description.

Table 2. The level of probability comparison

Level	Name	Probability comparison
1	Subjective level	Based on a subjective comparison of the probability of events occurring in two different sample spaces, and can not distinguish between equal possibility and unequal possibility
2	Transition level	Comparing the probability of events based on quantitative judgment, begin to distinguish between equal possibility and unequal possibility
3	Informal Quantitative level	Based on consistent quantitative judgment, but can not accurately quantify the probability of non-continuous events
4	Numerical level	Probabilities were quantified and compared with numerical values. Can give the same probability value for equally possible events

4.5 Accurate Assessment of Probability Levels

Researchers create an effective assessment tool, according to the results of the assessment tool to reflect and adjust the level of learning progress in time, to describe the changes in students thinking over time. Generally speaking, the most commonly used method in the form of the test questions, which can be drawn from the teaching examples provided in the course standards, the sample questions of the teaching materials, and the practice questions of the teaching aids. In the choice of questions to be in line with the cognitive habits of students, try to avoid interference from other factors.

The Opinions on Further Reducing the homework burden of Students in May 2021 raised the most prominent problems in compulsory education. First, the homework burden of students is heavy. Second, the after-school training is overheated. Accurate management of homework can help students to reduce the burden of homework. Taking probability as an example, students' probability levels can be accurately evaluated by reasonably compiling an assessment tool. To design the operation more accurately, to improve the sea-style homework mode from the source, to meet the needs of students' personalized development better.

5 Conclusion

The teaching based on big idea is operable, and learning progression provides a practical way for it, which can be carried out in an orderly way in the real teaching. Big idea and learning progression proceed from the perspectives of knowledge structure and thinking cognition. Mathematics teaching is developed with big idea, learning progression is based on the "big perspective" of students' cognitive development. It is helpful to accurately grasp the students' learning progress. Also, in line with China's double reduction education concept.

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A Comparative Study on the Presentation of Preparatory Knowledge in the New Mathematics Textbooks for Senior High School in China

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Abstract. As a special module added in the new curriculum standard of high school mathematics in China, the value of preparatory knowledge can't be underestimated. The special module are also added in the new high school mathematics textbooks published in 2019. This paper takes the preparatory knowledge in the new high school mathematics textbooks of People's Education version A, Beijing Normal University version & Hunan Education version as the research content, and analyzes the presentation of preparatory knowledge from two dimensions: the macro structure of the textbook and the main content. It is found that the three versions of textbooks have high consistency in the presentation of the main content; In the presentation of macro structure, they are distinctive. Then it puts forward three suggestions for textbook compilation, teachers' teaching and students' learning.

1 Research Background

In the past, many freshmen can not quickly change the junior high school mathematics learning methods, which lead to incoherence learning in junior and senior high school. In order to change this situation, the general high school mathematics curriculum standard (2017 Edition) (hereinafter referred to as the curriculum standard) pays attention to students' learning psychology and cognitive laws. It specially sets up the "preparatory knowledge" learning module, to help students complete the transition of junior high school mathematics learning in terms of learning psychology, learning methods, knowledge and skills. In 2019, the new version of high school mathematics textbooks were published in succession. This paper tries to explore the presentation methods of preparatory knowledge in different textbooks and compare their advantages and features, so as to provide some thoughts and inspiration for the improvement and use of new textbooks.

2 Research Contents and Methods

The research methods are text comparison and the qualitative & quantitative methods to carry out the research from two dimensions: the presentation of the macro structure of textbook and the presentation of the main content of textbooks. This paper takes

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the new high school mathematics textbooks of people's education version A(referred to as the version A), Beijing Normal University version(referred to as the version B) and Hunan Education version (referred to as the version H) published in 2019 as the research object to study the presentation of preparatory knowledge. The distribution of preparatory knowledge in each version of textbooks are as follows (Table 1).

Table 1: The distribution of preparatory knowledge in each version of textbooks

Textbook version	version A	version B	version H
Chapter division	Chapter 1: Collection and Common Logic Words;Chapter 2: Monary quadratic functions, equations, and inequalities	Chapter I: Preparatory Knowledge	Chapter 1: Collection and Logic;Chapter 2: Monary quadratic functions, equations, and inequalities
Number of pages	58	46	62

3 Research Results and Analysis

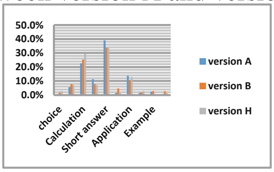
3.1 Comparison of Macro Structure of Textbooks

- Chapter introduction

Chapter introduction is the guidance of the learning contents at the beginning of the chapter. The analysis found that three versions of textbooks' chapter introductions are composed of words and images. The images are related to the learning contents of the chapter. For example, animal and grassland reflects the relationship between elements and collections. The speed limit marks of highway reflects unequal relationship in life. Readers can feel the connection between mathematics and life intuitively by the images.

- Knowledge generation process

There are two kinds of knowledge generation processes: "problem situation → abstract concept" & "concept → illustration". The version A shows a simple example through "observation" column, so that students can find out the mathematical methods. Further more, students can summarize the mathematical concepts through two columns: "thinking" and "exploring". The version B sets up three columns to present the teaching steps for teachers and students' reading. The content of the version H in the form of language description is not as clear as the version A, but the process is coherent. In terms of setting up examples, there are relatively large in version H, and little difference between version A and version B.



- Exercises

Exercises have the function of reviewing, consolidating and deepening understanding, which is an important part of textbooks. The types of exercises are varied, with different functions. Referring to the classification of textbook exercises by Peng, S. G. (2006), we can get the statistical chart after counting the type and the number of exercises in the three versions.

- Reading materials

Reading materials, with rich cultural attributes, supplement the contents of textbooks. Wang, J. P. et al. (2015) divided mathematical culture materials into four categories: mathematics history, mathematics & life, mathematics & technology, mathematics & humanities and arts. On this basis, Huang, H. (2020) added pure mathematics reading materials. Under this classification, this paper obtains Table 2 after analyzing the three versions of textbooks.

Table 2: The reading materials of the three versions of textbooks

Textbook version	version A	version B	version H
Material quantity	2	3	3
Material content	Sets, geometric propositions, sufficient conditions and necessary conditions	Cantor and set theory, String diagram, mathematical culture	Set - "white horse is not horse", logic and set, cross multiplication
Material type	pure mathematics	mathematics history	mathematics & humanities and arts, mathematics history, pure mathematics

- Chapter summary

Chapter summary is a distillation of learning content. It is not only a review of knowledge content, but also a refinement of mathematics thought. After comparing the chapter summary of the three versions of textbooks, Table 3 is obtained.

3.2 Comparison of the Main Content of Textbooks

The main content of textbooks is the process of knowledge generation, which includes three parts:scenario creation, concept presentation and example explanation.

- Reproduction of existing knowledge

The existing knowledge is a knowledge which is learned in compulsory education stage. When the textbook shows preparatory knowledge, it is bound to reproduce the existing knowledge. Combing the reproduced knowledge in textbooks, the following four ways of reproduction are summed up:

Table 3: The chapter summary of the three versions of textbooks

Textbook version	version A	version B	version H
Content	Knowledge structure chart, review and reflection	Knowledge structure chart, learning requirements, issues needing attention	Knowledge structure chart, review and reflection
Number of tasks	11	4	11
tasks characteristics	Numerous, specific	condensed	Numerous, specific

1. Recall: Only recall existing knowledge, then show the new knowledge.
2. Analogical: Learning similar knowledge by analogy with existing knowledge.
3. Fusion: The existing knowledge is part of new knowledge.
4. Sublimation: New knowledge is the further sublimation of existing knowledge.

For example, in the introduction part on page 10 of version A, the textbook presents the following contents: "We have learned the addition, subtraction, multiplication and division of real numbers. Does sets have similar operations?" This problem belongs to analogical reproduction. Its purpose is to enable students to learn analogically through the mastered real number operation knowledge when learning the new knowledge of operation between sets, so as to reduce learning difficulties.

Statistics show that the reproduction of existing knowledge in the three versions of textbooks is mainly based on fusion reproduction (version A 57%, version B 61%, version H 57%). In addition, recall reproduction is also more common in textbooks (version A 17%, version H 22%). Analogical reproduction is rare in the three versions of textbooks, and mainly appears in "univariate quadratic equation from the perspective of function" and "univariate quadratic inequality".

- Problem situation creation

Problem comes from the situation. Mathematical problems are inseparable from the creation of the problem situation. The textbooks always create corresponding problem situations before the presentation of new knowledge. Combing the literature and textbooks, the following three situations are summed up:

1. Real: From daily life and the real world.
2. Fictitious: Artificially constructed situations with certain practical basis.
3. Pure: Pure mathematical problem.

For example, in the activity task on page 39 of version A, the contents of the textbook are shown in the right figure. This task belongs to the real situation. It takes mathematics culture as the background, letting students feel the mathematics culture while thinking about math problems.

Statistics show that three versions of the textbook is dominated by pure mathematics situations (version A 50%, version B 64%, version H 42%). Real situation is helpful to improve students' interests in learning. Comparing three versions, the real situation in version H is the most. Although the version B has set up the example column, the pure mathematical problem situation accounts for the majority, and there are few real situations and fictitious situations, which increases the difficulty of the textbook.

- Textbook language expression

The language of textbooks are the carrier of knowledge transfer. From the perspective of expression, there are three forms: text, graphics and tables. From the perspective of sentence type, there are two forms: declarative sentence and interrogative sentence.

Comparing the language expression in three versions of textbooks, it is found that the frequency of using words, graphics and forms are basically the same. The text occupies the majority (70%–80%), graphics come second (20%–30%), and forms are few, less than 4%, which indicates that text is the mainly way. Declarative sentences have imperative mood, while interrogative sentences help motivate students to think actively. The version A is mainly composed of interrogative sentences, accounting for 65%, while the version B is mainly composed of declarative sentences, accounting for 81%, while the two kinds of sentences in the version H is basically the same.

4 Research Conclusion

- The three versions of textbooks are very similar in presenting the main content.

Firstly, when the textbooks reproduce existing knowledge, it is mainly based on fusion reproduction, presenting the knowledge of compulsory education as a part of new knowledge, so that students can learn new knowledge while they are familiar with existing knowledge. Analogical and sublimation reproduction are reflected in the textbook to a certain extent, but the proportion is not high. Sublimation reproduction focuses on re-understanding junior high school knowledge from the perspective of senior high school knowledge, and reflects the spiral process of knowledge learning. This form is more difficult for the study of preparatory knowledge.

Secondly, the problem situations, created before the introduction of new knowledge, are mostly pure mathematical situations and less real situations. Real situations help to stimulate students' interests in active thinking and improve their attention. However, problems in textbooks lack authenticity and interest, to a certain extent, which reduce students' interests in reading and are not conducive to completing the task of "connecting the preceding and following" in preparatory knowledge.

Thirdly, the language expression of textbooks is mainly written in words. Graphics and charts are used to assist. The language of words is not as vivid as video and voice, but, it is the most convenient language. Readers can get information quickly and directly by words, with the aid of graphics and forms. However, textbooks can also use video and audio language to present knowledge, such as inserting QR code or links, so that students can learn easily before class preparation or after class review.

- In the presentation of macro structure, the three versions of textbooks have their own characteristics.

Version A The content of the preparatory knowledge is quite detailed, no matter in the introduction of the chapter, the process of knowledge generation or the summary of the chapter. When presenting knowledge, textbooks set up many activity columns, and arouse students' thinking by interrogative sentences. The shortcoming is that although textbooks have reading materials, they are all pure mathematics content.

Version B The content of the textbook is clearly arranged and presented in a logical form. Chapter introduction gives famous quotes and sets up a column for mathematics culture. Chapter summary is echoed with the curriculum standard, which reflects its unique compilation features. The disadvantage is that there is little real situations and fictitious situations, many sublimation types, and the problem expression is mainly by declarative sentences, which makes the textbook difficult.

Version H This version of textbook combines much mathematical culture, which is embodied in expanding the types of reading materials, with a special mathematical culture column. It contains more exercise quantities and types than other two versions. The shortcoming is that there is no division of columns in content. A large number of texts are not good for students to read textbooks.

5 Inspiration

- Each version of textbooks should learn from each other to optimize themselves.

The three versions of textbooks have their own advantages. Therefore, when writing textbooks, different versions of textbooks should learn from each other and "extract the essence and discard the dross", so that they can serve the education better.

- Teachers should study textbooks earnestly and use multiple textbooks to teach.

Teachers' understanding of textbooks directly affects their teaching design ideas, teaching methods and teaching results. Textbooks is the main material of teaching. Teachers should thoroughly study textbooks and study examples, pictures and even punctuation marks in textbooks. Teachers should make full use of chapter introduction and chapter summary to do a good job of early guidance and later summary, so as to make the teaching of a chapter complete and smooth. In addition, only one textbook is used in teaching, but teachers can study other versions of textbooks and learn from excellent cases to make up for the lack of single textbook.

- Students should read textbooks independently to play the basic guiding role of textbooks.

In the process of learning, students often fail to use textbooks effectively, so that the basic functions of textbooks are lost. Therefore, students should develop the habit of reading textbooks and using textbooks. The learning objectives in the chapter introduction can guide students to study with aims. The knowledge structure diagram and problem thinking in the chapter summary can help students build a knowledge system. Most importantly, the knowledge generation process of the textbook shows problem situations, concepts and cases. It is a very detailed learning material, which plays a great role in both self-study and after-school review. Therefore, students should make full use of and make effective use of the textbook.

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The Potentially Implemented Spiral Curriculum: A 3-Feature Analysis of Geometry Textbook Chapters

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Abstract. Since the Philippine K-12 reform in 2012, textbooks now follow a Spiral Progression Approach (SPA) across grade levels. This content analysis investigated the manifestations of the 3 features of a spiral curriculum: (1) revisit of topics, (2) increase in difficulty level, and (3) relating to pre-requisite knowledge, in math textbooks. Combining frameworks, the study analysed the discussions, examples, and exercises of the geometry chapters of 4 textbooks (1 series for junior high school). It was found that topic discussions were more repetitive than spiralling. Majority of the examples and exercises were also focused on low difficulty levels, thus leading to underdeveloped competency. The study reinforces the integral role of textbooks and crafting resources with the SPA features in mind to help in the implementation of the K-12 curriculum.

1 Introduction

When Philippine congress declared the enactment of the K-12 basic education curriculum, also known as the "Enhanced Basic Education Act of 2013" (The Official Gazette, 2013), a myriad of new textbook resources were made. According to several international sources (Chang & Silalahi, 2017, Fan, Zhu, & Miao, 2013, O'Keeffe & O'Donoghue, 2011), one of the vital elements in implementing a curriculum is providing high-grade resources. More so when implementing a paradigm shift or curriculum reform, much like the K-12 spiral curriculum. According to Valverde et al., building upon the study of the Third International Mathematics and Science Study (TIMSS) in 2002, textbooks and other organized resource materials are regarded as the 'potentially implemented' curriculum, the bridge between intention, which contains the aims, intentions, and goals of the curriculum, and implementation. However, despite the undeniable need to look at textbooks and resources, there seems to be little to no local research that tie up the K-12 curriculum, its structures. Like the spiral progression approach and the resources that we have. Thus, this directed content analysis was formulated. A directed content analysis is done when an existing theory would be used to guide the analysis of a text. It may also be employed when data regarding a particular phenomenon is incomplete or would benefit from further expansion (Weber, 1990). The existing theory, in this case, is the spiral progression approach, and the quantifiable data would be the four features. Harden

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and Stamper (1999), summarizing Bruner's (1960) discussion on the spiral progression approach, identified four main features of a spiral curriculum: (1) topics are revisited; (2) difficulty level increases; (3) new learning is related to previous knowledge; and (4) competence of students increases. However, since the study is a textbook analysis, a students' competency does not fall within the scope of the research. The essence of a spiral curriculum is that topics are revisited with an increasing level of difficulty and complexity at each visit.

1.1 Research Questions

The aim of this study is to craft and utilize a 3-dimensional framework to show the manifestations of the features of the spiral curriculum in junior high school geometry textbook chapters. Specifically, it aimed to answer the following questions: How much are topics revisited and built upon themselves in the 4 levels of junior high school textbooks? To what extent do textbook examples and exercises offer the full range of difficulty levels and use prerequisite knowledge?

2 Methodology

The goal of the study was to determine whether the spiral curriculum is evident in a selected textbook series for junior high school geometry. To achieve this, the researcher employed a directed content analysis to a widely used textbooks series Multiple frameworks were used to establish a well-grounded coding system that articulates the objectives of the study. The coding system is composed of a modified listing of content strands and sub-strands from the DOST-SEI & MATHTED Framework for basic education incorporating learning competencies from DepEd Curriculum Guide for Mathematics and level descriptors for Van Hiele's model of geometric understanding. A particular coding system was followed to code different parts of the textbook depending on the feature being measured (Table 1).

Table 1. Instrument for textbook analysis

	Textbook Component	Framework Used	Classification/Coding
F1	Discussion	DepEd/DOST-SEI MATHTED	Occurrence (1); Distinct/Repetitive (2); Consecutive/Non-consecutive (3)
F2	Examples and Exercises	Van Hiele's Model of Geometric Understanding	Level 0: Visualization; Level 1: Analysis; Level 2: Informal Deduction; Level 3: Deduction; Level 4: Rigor
F3	Examples and Exercises	DepEd/DOST-SEI MATHTED	Prerequisites used in Grade 7, 8, 9, 10

Textbook discussions were analysed with the content they cover according to the DepEd Curriculum Guide for Mathematics. Revisits are regarded as encounters of a topic that belong to the same content domain in the DOST-SEI and MATHTED Framework. The content domain in the framework are as follows, Grade 7: undefined terms, angle relations, polygon properties, circles (basic concepts); Grade 8: inductive reasoning, triangle congruence, triangle inequalities, parallel lines (special angles); Grade 9: quadrilaterals, triangle similarity, Pythagorean theorem; Grade 10: circles (properties and construction) and coordinate geometry. Reiteration of information were also differentiated from successive deepening of knowledge in each encounter. These were labelled using progressive icons. If the discussions of the topic do not add any depth or additional information these were assigned the same symbol. Thus, dis tinct discussions were distinguished from mere repetitions of content. Examples and exercises were labelled from level 0 to 4 according to the corresponding Van Hiele's level of understanding. The following are the levels of geometric understanding summarized by Fuys et al. (1988): Level 0 (Visualization) where the student identifies, names, compare, and operates on geometric figures according to their appearance, Level 1 (Analysis) where the learner analyses shapes, their properties, and relationships, Level 2 (Informal deduction) where the learner uses logic to relate previously discovered properties/rule following informal arguments, Level 3 (Deduction) when the student establishes theorems deductively and established interrelationships among networks of hypotheses, and lastly, Level 4 (Rigor) where the student establishes theorems in different postulational systems, and analyses/compares these systems. It is imperative to note that since the coverage of the study is only for junior high school, the last Van Hiele level is not expected to be utilized in the textbooks. For the third feature, textbook examples and exercises were analysed to look at specific proficiencies (knowledge and skills) that are embedded in each worked example and exercise in the geometry chapter. Whenever necessary, the researcher provided a sample solution that could make use of knowledge and skills discussed in the previous textbooks from the same series. The problem, together with its corresponding solution, was deconstructed to identify specific pre-requisite skills, from previous textbooks, that were used in succeeding grade levels.

3 Results and Discussions

From the data collected for the textbook series, most topics only have low level of revisits relative to the grade level they were first encountered. This implies that a significant number of topic strands and sub-strands can only be found in 2 of the four textbooks in the series with distinct discussions. It was also found that majority of the revisits were assigned the same symbol in several instances. This implies that several encounters had the same content coverage and did not provide additional information to what was previously learned.

It was observed in Table 2 that grade 7 topics were revisited in various extents for the two series. For the series, 13 out of 28 of grade 7 topics had 3 out of 4 occurrences. 42% of grade 7 topics were found in three of the four textbooks of the series but only 1 of the 13 topics were labelled with distinct symbols.

This would mean that the other 12 topics had at least 2 encounters that did not give additional information. It was also found that 36% (10) of the grade 7 topics had 2 out of

Table 2. Results for feature 1: topics are revisited

Grade Level	7	7	7	7	8	8	8	9	9
Occurrence	4/4	3/4	2/4	1/4	3/3	2/3	1/3	2/2	1/2
Distinct	0	1	10	5	0	0	2	0	3
Repetition	0	12	0	0	0	1	0	0	0

4 encounters. Only 18% (5) topics were only found in grade 7. A possible weakness of mismanagement of a spiral curriculum is when 'revisits' to a topic become a repetition of the lesson. If textbooks are regarded as the 'potentially implemented' curriculum, repetitive content would imply missed opportunities for deeper learning (Table 3).

Table 3. Results for feature 2: difficulty level increases

	GRADE 7		GRADE 8		GRADE 9		GRADE 10	
	f	%	f	%	f	%	f	%
Level 0	66	17%	16	4%	13	5%	2	1%
Level 1	266	71%	152	38%	166	60%	176	74%
Level 2	44	12%	119	30%	70	25%	30	13%
Level 3	0	0%	110	28%	28	10%	28	12%
Level 4	0	0%	0	0%	0	0%	0	0%
TOTAL:	376		397		277		236	

Majority of the items across grade levels belong to level 1 of Van Hiele's model of geometric understanding. According Fuys et al. (1988), one of the indicators that a learner is at level 1 of Van Hieles' model is "the ability to solve geometric problems by using known properties of figures." This was the most apparent competency shown in several textbooks from the selected series. The grade 7 textbook poses several direct applications of properties or theorems being discussed. Examples and exercises were found to be focused more on solving and application rather than reasoning. This is contrary to the twin goals of the curriculum stated by the Department of Education, which are critical thinking and problem-solving. Also noteworthy is that textbooks with the highest percentage of levels 2 and 3 items were grade 8 textbooks, not grades 9 or 10. Levels 2 and 3 focuses on informal and formal deduction. Thus, it would not be far from possible for a student study definitions, theorems, and properties in grade 7 and jump to using these to form a proof in grade 8. One way of showing build on a previous lesson was seen on the discussion of triangle exterior angle theorem in grade 7 and grade 8. In the grade 7 text, only the concept of the measure of the exterior angle of a triangle is presented. It was given as a "Pop-up!" statement and not as a theorem, unlike the discussion in grade 8. In grade 7, there was also a direct application of the concept as an example. In grade 8, an analogous concept was found as a theorem. It was also given a formal proof which demands a higher Van Hiele level of geometric understanding. However, this can be more attributed to the nature of the lesson itself, i.e., inductive reasoning and triangle congruence, rather than the manner the textbooks were crafted.

Table 4. Results for feature 3: previous learning is related to new content

Integration succeeding textbooks	Topics in G7		Topics in G8		Topics in G9		Topics in G10	
	f	%	f	%	f	%	f	%
Grade 8	21	75%						
Grade 9	21	75%	22	100%				
Grade 10	24	86%	20	91%	17	100%		

Since topics such as proving are in grade 8, most competencies were found to be incorporated in textbook examples and exercises of this textbook. This was observed for the third feature. Definitions, theorems, and properties from grade 7–9 were used multiple times in succeeding textbooks as shown in Table 4 above.

4 Summary and Recommendations

One of the main implications and contributions of this study is the method of examining textbooks and other possible resource materials; and how the structure of the spiral curriculum comes into play. A concrete methodology shows that the features of the spiral curriculum can be shown, measured, and crafted more explicitly. When the spiral curriculum is re-evaluated, curriculum developers may consider that the curriculum guide for Mathematics can be further improved by showing a more explicit spiral. Considering the nature of the topic itself is also an important takeaway from the study. Textbook authors and publishers may consider that it was seen that seamless building on previous knowledge was achieved when general ideas/concepts were presented first followed by a more formal approach. A way to do this may be by dividing lessons in several encounters to lessen compartmentalization. Educators may also consider that due to limitations found in the textbooks, teachers may opt to have review sessions and revisit topics that may be prerequisites to their lessons. In addition, giving examples and exercises that are of higher Van Hiele levels may also help in increasing difficulty level and general student competency.

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Research Progress of Mathematic Textbooks for Primary and Secondary Schools in Recent 20 Years

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Abstract. The study of mathematic textbooks has always been the focus of mathematics education in primary and secondary schools at home and abroad. With 345 SSCI literatures in the research field of mathematic textbooks for primary and secondary schools from Web of Science Core Collection Database (2000–2022) as research objects, the research hot spots and development trends are determined by using the methods of co-word analysis, cluster analysis and multidimensional scale analysis. The future of the research on mathematic textbooks in primary and secondary schools abroad is prospected from the aspects of perfecting the research framework, constructing academic community and developing digital textbooks.

1 Introduction

Mathematics textbooks are compiled according to mathematics curriculum standards, which systematically reflect the learning content of mathematics. They are the most reliable and important witness in the development history of mathematics education in all countries in the world, and also the important carrier for the implementation of the reform idea of mathematics education in China (Baker, David, & Knipe, 2010). Mathematics textbooks are also important resources for students and teachers. Therefore, mathematics textbooks not only affect the implementation of the national mathematics curriculum plan, but also affect the effectiveness of teachers' teaching and students' mathematics learning achievements. To understand foreign advances in the research of primary and secondary school mathematics textbooks will be sorted out of the existing research literature, in the process of combing the literature, found that quite a number of domestic researchers abroad mainly tend to use content analysis to summarize the research trends in primary and middle school mathematics textbooks, of course, any study not only the exposed on the surface of the information, also need to dig in the field of inner link (Xu & Qi, 2018). Therefore, in this paper, 345 foreign SSCI literatures in the research field of mathematics textbooks for primary and secondary schools included in Web of Science core Collection database from 2000 to 2022 were measured and visualized by bibliometric method. Firstly, discourse matrix and co-occurrence matrix are generated by Bicomb2.0 bibliographic co-occurrence analysis software. Secondly,

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NetDraw tool of Ucinet6.0 was used to draw high-frequency keywords visualization map and analyze the hot spot distribution in this field. Finally, cluster analysis and multidimensional scale analysis were used to classify the hot topics.

2 Data Sources

Bradford's dispersion law states that most key papers are usually published in a small number of core journals. Therefore, the data in this study are from the Web of Science core Collection database. This paper uses "Mathematics textbook" and "Mathematical textbook" as the search terms to search the subject. The retrieval time is set as January 31, 2000 to February 1, 2022, and a total of 566 SSCI documents are retrieved. After three rounds of intensive reading, 221 non-research papers such as conference reviews and conference reports were excluded, and 345 valid research papers were confirmed (including 335 journals, 5 reviews, 4 conference papers and 1 monograph chapter).

3 Research Results and Analysis

3.1 Keyword Frequency Statistics and Analysis

Based on 345 literatures, there are 989 keywords in total. Keywords with word frequency greater than 4 were extracted by Bicomb2.0. To obtain 18 high-frequency keywords (Table 1).

Table 1: Ranking of the top 18 high-frequency keywords

Serial number	Keywords	Frequency	Serial number	Keywords	Frequency
1	Textbooks	42	10	Reasoning	5
2	Curriculum	25	11	Content analysis	5
3	Textbook analysis	24	12	Calculus	5
4	Mathematics education	22	13	Achievement	4
5	Mathematics	19	14	Geometry	4
6	Mathematics curriculum	13	15	China	4
7	Mathematics textbooks	10	16	Curriculum materials	4
8	Problem solving	9	17	Algebra	4
9	Proof	5	18	Teaching	4

From the ranking of high-frequency keywords, we can get a preliminary understanding of the hot spots and trends in the research field of mathematics textbooks for primary and secondary schools abroad in the past 20 years, and have an overall grasp of the hot spots in this research field.

The matrix of high-frequency keywords generated by Bicomb2.0 was imported into SPSS software to generate similarity coefficient matrix.

3.2 Social Network Analysis

The co-occurrence matrix was generated by Bicomb2.0, then it was imported into Ucinet6.0. The social network diagram was drawn by NetDraw tool (Fig. 1).

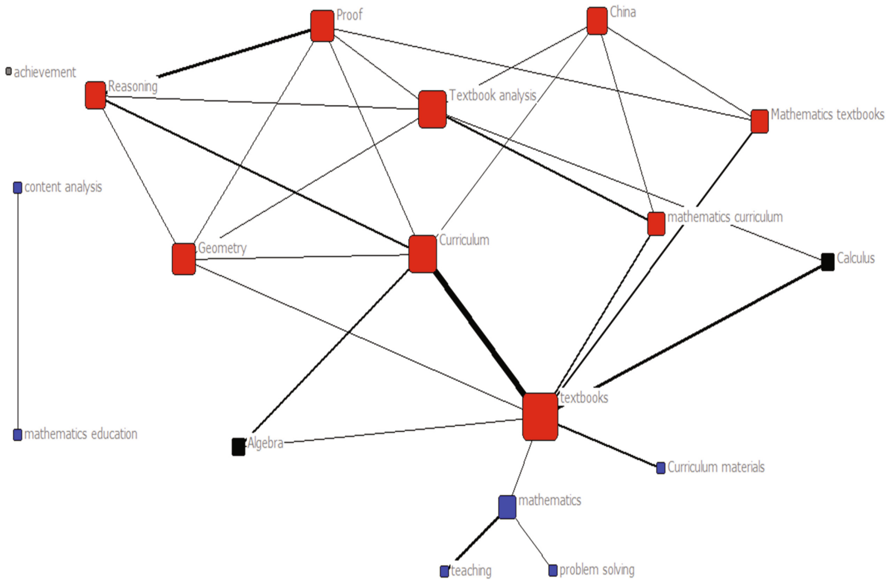

Fig. 1: Social network diagram of high-frequency keywords

In Fig. 1, the 18 high-frequency keywords form a crisscross network structure, which can reveal the hot spot distribution in the research field of mathematics textbooks for primary and secondary schools abroad.

3.3 Cluster Analysis Diagram of High-Frequency Keywords

SPSS was used for systematic clustering analysis of keyword similarity matrix, and the clustering results were shown (Fig. 2).

According to the results of cluster analysis, are divided into 4 groups according to the distance between the clusters, and the classification results are shown (Table 2).

Through systematic cluster analysis, we can find the main hot areas of foreign mathematic textbooks research, but do not know their internal relationship. We use the multidimensional scale analysis in SPSS for further exploration.

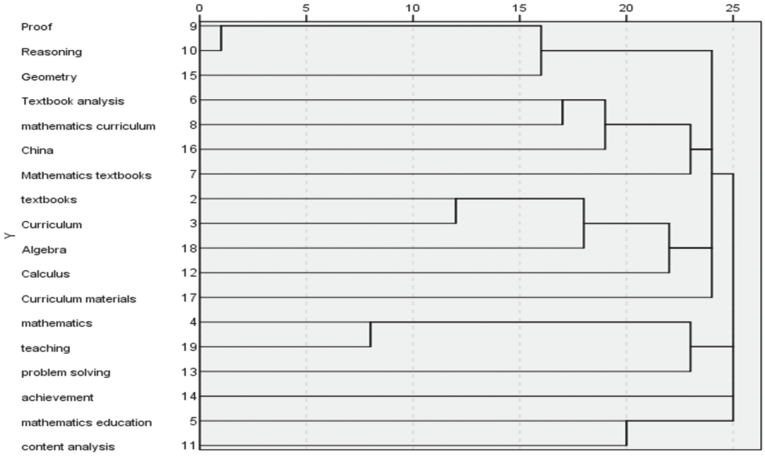

Fig. 2: Clustering analysis chart of keywords

Table 2: Cluster analysis of high-frequency keywords

Category 1 (Mathematic textbooks and textbook analysis)	Proof; reasoning; geometry; textbook analysis; mathematics curriculum; china; mathematics textbooks; textbooks; curriculum; algebra; calculus; curriculum materials
Category 2 (Mathematics teaching and problem solving)	Mathematics; teaching; problem solving
Category 3 (Achievement)	Achievement
Category 4 (Mathematics education and content analysis)	Mathematics education; content analysis

3.4 Multidimensional Scale Analysis

SPSS was used for multidimensional scale analysis, and Z score was selected as the standardization method. Among them, $Stress = 0.24687$, $RSQ = 0.70567$, indicates that the fitting effect is good. And the knowledge diagram of hot topics in the research of foreign primary and secondary school mathematics textbooks is drawn (Fig. 3).

3.5 Research Hotspot Analysis

By means of literature study, combined with statistical analysis of high frequency keywords frequency, social network diagram, clustering analysis and hot spot of knowledge map, review and analysis, the research samples again and again in the nearly 20 years abroad elementary and middle schools mathematic textbooks studies are summarized three major focus in the research field: the research of mathematic textbooks content, the

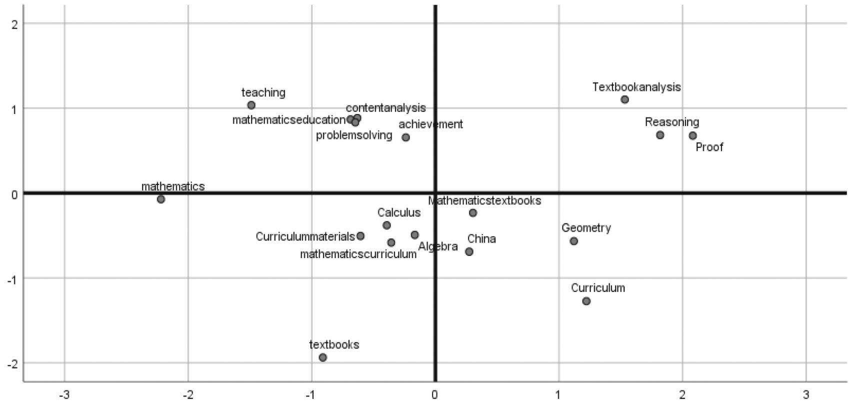

Fig. 3: Knowledge diagram of hot topics

comparative study of mathematic textbooks and mathematic textbooks and curriculum reform.

4 Future Research Prospects

4.1 Comprehensive Research, Enrich and Perfect the Mathematic Textbooks Research Framework, Promote the Mathematic Textbooks Research Depth Development

At present, the research theme of foreign primary and secondary school mathematics textbooks is still constantly updated, and many researchers carry out research on mathematic textbooks from different angles. However, from a macroscopic point of view, the research field of mathematic textbooks is not very extensive, involving only a few topics. For example, the content research and comparative research of mathematics textbooks. In the background of the international education, countries should strengthen the communication, enrich and perfect the mathematic textbooks research framework, like professor Fan Lianghuo have made point: textbook research is a scientific research on the relationship between itself and other factors (Lianghuo Fan, 2013).

4.2 Work Together to Build an Academic Community and Promote the Balanced Development of Mathematics Textbook Research

According to the review of the authors of the literature, it is found that the subject of the study of mathematics textbooks in foreign primary and secondary schools is not stable, and only limited to some scholars and experts. As we all know, the implementers of mathematic textbooks are front-line teachers, who know the needs of students and the content of mathematic textbooks best. However, front-line teachers have very little participation in the development and compilation of mathematic textbooks, which leads to the imbalance of research power on mathematic textbooks. Therefore, front-line teachers should be involved in the mathematic textbooks writing development, building a "researcher" identity, and experts and scholars to discuss mathematic textbooks deficiency, and the use of some problems in the mathematic textbooks in teaching, combined with the needs of students, develop truly benefit the student mathematic textbooks, will be closely integrated "teaching" and "research". And promote the balanced development of mathematics textbook research.

4.3 Innovation, the Development of Digital Mathematic Textbooks, Promote the Development of Mathematic Textbooks Research Information

The 21st century is an era full of science and technology, no one can be free from science and technology, the same for mathematic textbooks. In recent years, digital textbooks have been put into use in various countries. Compared with paper textbooks, digital textbooks do meet some needs of teachers and students, and enhance the richness and flexibility of the content of textbooks. However, according to the results of co-word analysis, "digital mathematic textbooks " does not appear as one of the high frequency keywords, indicating that there is less research on digital textbooks, because many theories are still in the stage of perfection. In future studies, therefore, to improve the theoretical framework of digital textbooks, students from the students' learning needs, the use of habits, teachers level of operating aspects of designing digital textbooks, condensing technology and innovation strength, eventually to develop high quality, multidimensional and interactive quality digital textbooks mathematical disciplines, promote education and the future study of learning and innovation in the future (Deng, Wu, & Cao, 2021).

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Examining Curriculum Alignment in Mathematics in a Transition Period: An Analysis of the New Elementary School Mathematics Textbooks

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Abstract. This study examines the alignment of school mathematics curriculum with a focus on the relationship between the intended and the implemented curriculum by analysing the elementary school mathematics textbooks based on the new national curriculum standards released in 2017. A theoretical framework of curriculum alignment in the earlier TIMSS curriculum analysis was used in this study. The results of analysis reveal that textbooks do not cover all types of mathematical activities emphasized in the new national curriculum standards and that there are certain limitations in the types of problem included. The results suggest that teachers need to supplement tasks and activities that are not explicitly provided in the textbooks for achieving the new competency-oriented goals intended by the national curriculum standards.

1 Introduction

School mathematics curriculum reforms have taken place in many countries around the world within contexts that vary significantly (Shimizu and Vithal 2018). The recent Mathematics Curriculum Document Analysis (MCDA) study by OECD reports a diversity in the extents to which issues related to the development of mathematics literacy are now being included in national curriculum standards and in the textbooks (Schmidt et al. 2022). In Japan, one of the participating countries in MCDA study, the revised national curriculum standards have been released (Ministry of Education, Culture, Sports, Science and Technology 2018) for the implementation of a "competency-based" curriculum. The current study examines the alignment of school mathematics curriculum with a focus on the relationship between the intended and implemented curriculum by analysing the new school mathematics textbooks authorized by the Ministry of Education, Culture, Sports, Science and Technology to be used from school year 2020 onward. While the Japanese

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textbooks analysed in MCDA study were those based on the previous curriculum standards, the current study explore to what extents "competency-based" curriculum can be implemented with the new textbooks as "potentially implemented curriculum". This paper discusses issues related to the development of mathematics textbooks to be used for implemented a "competency-based" curriculum.

A theoretical framework of curriculum alignment in the earlier TIMSS curriculum analysis (Valverde et al. 2002) was used to conceptualize the role of textbooks across three different aspects of curriculum as broadly defined: the intended, the implemented, and the attained curriculum. This study examines to what extents the new competency-oriented goals are reflected in all the six mathematics textbooks series currently available in elementary schools in Japan. This study particularly analysed the textbooks with a focus on topics related to proportions/inverse proportions in the "change and relationships" area. A coding system was developed for identifying in the textbooks the new emphases in the national curriculum standards: the role of mathematical perspectives/thinking used in the various phases of mathematical problem solving and introducing various types of problems for students' mathematical activities in the classrooms.

2 Alignment Between Intended and Implmented Curriculum

This study particularly focuses on the alignment between the intended and the implemented of the three aspects of the curriculum and on the role of textbooks. Textbooks are treated here as a potentially implemented curriculum that lies between intended and the implemented curriculum (Shimizu et al. 2021; Valverde et al. 2002).

A coding system was developed for the analysis of "proportion/inverse proportion" unit in the "change and relationships" area in elementary school mathematics and in the "function" area of junior high school mathematics. Referring to the National Course of Study and its supplementary materials as the intended curriculum, codes for "mathematical perspectives/thinking" and "mathematical activities" were specified (Shimizu et al. 2021).

Based on the premise that it is necessary to examine all textbooks to investigate the status of curriculum alignment, this study examined the textbooks, which are situated between intended and the implemented, focusing on the proportion/inverse proportion unit in the change and relationship area. The proportion/inverse proportion unit in the change and relationship area was chosen because, first, the area in question was newly introduced in this round of curriculum revisions; it is therefore likely that the intention of the course of study is explicitly reflected in this unit of new textbooks. Second, as the connection of the change and relationship area to the function area in junior high schools and high schools is clear, we thought we could compare curriculum alignment between different types of schools with an eye of longitudinal analysis.

3 Method

To examine curriculum alignment, we employed the method of textbook analysis as reported in the study of curriculum of Valverde et al. (2002). This method was developed for the Survey of Mathematics and Science Opportunities (SMSO) by a group of researchers from six countries in time for the Third International Mathematics and Science Study (TIMSS) carried out by The International Association for the Evaluation of Educational Achievement (IEA). Their analysis focused on the content in the textbooks that reflected the curriculum of the countries participating in the study.

As the current textbook analysis examined alignment between the intended and the implemented aspects in the school mathematics curriculum, codes were devised not to systematically cover all mathematical content of the textbook but rather to capture items related to the clearly stated objectives in the current version of the Course of Study. The coding system was formed of categories of sub-units of mathematical perspectives/thinking and mathematical activities, which are explicitly stated as subject objectives in the new national curriculum standards.

The coding of six textbooks series was carried out by the same pair of coders who worked separately and then checked each other's results. We use the proportion of agreement for each block and Cohen's Kappa coefficient as the reliability indicator to evaluate the degree of agreement of all coding. If there were discrepancies in coding, the coders consulted to reach a consensus decision. Both coders were graduate school students without actual teaching experience. These persons were chosen as coders because they were able to code the intention in the national curriculum standards and its supplementary materials published by the Ministry of Education, Culture, Sports, Science and Technology as reflected on the textbook without any prejudice that might derived from teaching experience. In carrying out the analysis, the number were recorded of codes assigned to the "blocks" in the textbooks of mathematical perspectives/thinking and mathematical activities.

4 A Coding System

Mathematics in the national curriculum standards aims to develop students' competencies by focusing on mathematical perspectives/thinking, which are employed in the problem identifying and problem-solving processes in mathematics, and by conducting planning/teaching processes in an intentional and planned manner.

Table 1 Codes for mathematical perspectives/thinking

Code	Items in mathematical perspectives /thinking
a1	When investigating a single quantity, determining another quantity that is deeply related to it
a2	Focusing on the relationship between two quantities
b1	Investigating the relationship between two quantities using tables, formulae and graphs
b2	Identifying characteristics of change or correspondence in a functional relationship
C	Using proportion/inverse proportion, to capture, examine, and express a concrete phenomenon

Consequently, from the national curriculum standards and its supplementary materials published in 2018, codes on mathematical perspectives/thinking and mathematical activities were extracted, as shown in Tables 1 and 2. The extraction method was informed by Shimizu et al. (2021).

Table 2 Codes for mathematical activities

Code	Items in mathematical activities
A01	Problems in daily life and social phenomena
A02	Problems from mathematical phenomena
A1	Expressing daily or social phenomena using mathematics
A2	Expressing mathematical phenomena using mathematics
B	Moving from a mathematically expressed problem to a focused problem
C	Solving a focused problem
D1	Using the solution to solve problems in daily and social phenomena
D2	Integrating/developing mathematical phenomena from the solution

With reference to the codes for mathematical activities, the difference between B and C was unclear. Elementary school textbooks frequently contain the invitation, "let's investigate," and it is not always clear if the mathematical activity in question falls into classification B or C. Consequently, when the method of investigation had already been clearly identified and only one result was expected from the investigation, this was considered an activity to solve a focused problem and C was assigned. In contrast, if the method of investigation was not clearly identified and various results were expected from the investigation depending on the views, this was considered an activity to generate a question and B was assigned.

5 Results and Discussion

The agreement between the coders was high, ranging between 78.7% and 93.3% for mathematical perspectives/thinking and between 78.7% and 85.7% for mathematical activities. Overall Cohen's Kappa coefficient of reliability was 0.754 for mathematical views/thinking and 0.413 for mathematical activities, indicating "substantial agreement" and "moderate agreement," respectively. The value was lower for mathematical activities probably because a high number of code "C" were assigned to each textbook for mathematical activities; thus, we cannot conclude that the reliability of the pair's coding was low.

In the process of coding, the two coders' agreement was high for both mathematical views/thinking and mathematical activities. Furthermore, Cohen's Kappa coefficient, an indicator of overall reliability, was lower for mathematical activities than for mathematical perspectives/thinking. The result suggests there were differences as to what kind of mathematical activities were suggested based on the description of questions and activities in the textbooks. This indicates it is difficult to read and interpret an actual process of mathematical activities from the textbooks. It can also be assumed that this type of difference in interpretation can occur among teachers.

With reference to mathematical perspectives/thinking, the proportion of items codes as b2 was particularly large for all textbooks, at between 46.8 and 71.1%. This is because at the level of elementary school, the two quantities of a concrete phenomenon are presented beforehand, and children are asked to consider the relationship with the assumption that it is either a proportion or inverse proportion. B1 was the second most numerous code (between 7.1 and 24.4% of the total codes).

The proportion of items coded as a1 or a2 was small (some textbooks did not contain a1). The results reveal that in many cases, the two quantities were given at the beginning of the problem, which means there were few questions and activities that identifies two quantities that varied together. That is, this reflects that the aim of teaching proportion/inverse proportion in elementary school is examining how two quantities in a proportional relationship change.

Furthermore, some textbooks did not contain code c. This means that even for material that used daily phenomena, the assumption was that the two quantities observed a proportional relationship and children would solve the problem based on this assumption. In this case, while the publisher might have posed questions and activities regarding concrete phenomena that required the use of proportion/inverse proportion to solve the problem, if the teacher cannot plan a lesson that uses proportion or inverse proportion to solve the problem, children may not be able to develop appropriate awareness. To address this, in addition to further thinking about how to present the problem in a textbook, it is necessary to intentionally build in a1 or a2 activities, which require the teacher to identify two quantities in order to solve a problem related to a concrete phenomenon, such as asking children what kind of quantity can be used for a given problem.

Regarding mathematical activities, the proportion of the number of codes was similar across the six textbooks. More specifically, no textbook contained A01 or A02, which highlights the difficulty of including the process of problem finding in the main text of a textbook. It can be argued as a limitation of the textbook as an potentially implemented curriculum. Furthermore, the proportion of D1 or D2 activities, which develop and integrate a result by returning it to daily or mathematical phenomena, was small for all textbooks. This is because as activities coded C define many basic activities regarding how to investigate in order to solve the problem or how to express data in tables, formulae, and graphs, rather than problem solving, there were few cases for which the result of problem solving was interpreted with reference to concrete phenomena.

The results of analysis reveal that textbooks do not cover all types of mathematical activities emphasized in the new national curriculum standards and that there are certain limitations in the types of problem included. The results suggest that teachers need to supplement tasks and activities that are not explicitly provided in the textbooks for achieving the new competency-oriented goals intended by the national curriculum standards.

6 Concluding Remarks

The current study analysed the proportion/inverse proportion unit in six textbooks for elementary school mathematics in Japan. The results suggest that teachers need to supplement tasks and activities. In the further study, the alignment between the intended and the implemented curriculum need to be examined with the data of what is actually taught in the classroom. Also, we intend to use the same coding system with the proportion/inverse proportion unit in all the seven textbooks for junior high school mathematics to explore alignment between the intended curriculum of junior high school mathematics and the textbook. In this way, we will be able to examine longitudinal alignment between elementary schools and junior high schools. We intend to continue developing a coding system for other areas to permit examination of alignment between the intention of a course and its textbook. Furthermore, it is important to determine how to capture activities coded c ("activities to explain and communicate using mathematical expression"), which were not codified in the types of mathematical activities, since these did not appear in writing in the textbooks.

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The Complexity of Mathematical Expressions in Mathematics Textbooks

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Abstract. Using a binary-lambda-calculus based calculation method for complexity, we analyze and discuss the mathematical expressions and their complexity in two editions of the Chinese high school mathematics textbooks published by the People's Education Press. There is no statistically significant difference in the complexity of mathematical expressions between the two editions, and the differences are mainly in the interpretation of conceptual knowledge using mathematical expressions, as well as in the types of examples and their solution processes. Among them, the new edition uses changes of mathematical expressions to change the type of examples without increasing the complexity.

1 Introduction

Discourse analysis is receiving increasing attention. Among the existing theories, Halliday's (1994) Systemic Functional Linguistics is a relatively complete one. O'Halloran (2005) studied mathematical discourse from the perspective of Halliday's systemic functional grammar, in which he conceptualized mathematics as a multisemiotic discourse involving language, visual images, and symbols, and constructed a grammatical framework for mathematical symbols and images. In the context of Chinese language, Chu *et al.* (2017) used Halliday's Systemic Functional Grammar to critically analyze the ideology of mathematics textbooks. Concerning images, Zhang *et al.* (2019) constructed an analytical framework for nonverbal elements in the context of specific mathematics textbooks. Mathematical symbols as an important part of mathematics textbooks, yet not much research has been done on mathematical symbols in textbooks. This paper aims to explore the characteristics of mathematical symbols by examining the complexity of mathematical expressions in mathematics textbooks. An influential study on the difficulty of mathematics textbooks in China is the theory of difficulty degree of course initiated by Shi et al. (2005). The number of mathematical expressions corresponds to the scope and length of the course, and the complexity of mathematical expressions corresponds to the depth of course in the difficulty degree. However, while Shi *et al.* focused on the course difficulty, the object of this paper is the complexity of mathematical expressions, and the analysis of mathematics textbooks from a more specific perspective. The research question of this paper is to compare how the number and complexity of mathematical expressions in the same topic in two different editions of mathematics textbooks, and how the specific presentation of mathematical expressions differs between the two editions.

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2 Research Methods

2.1 Complexity of Mathematical Expressions

Su (Su et al., 2021) proposed that there are three factors that affect the complexity of mathematical expressions: representation complexity, computation complexity, and intelligibility. In this paper, we adopt their method for calculating the representation complexity of mathematical expressions. In terms of syntax composition, a mathematical expression can consist of three elements: operator (function), operand, and combination rule. The complexity of a mathematical expression is calculated as the sum of the storage lengths of the operands (functions), operators and combination rules, using binary as the storage length unit. The complexity of an integer I is

$$C_r(I) = \begin{cases} \lfloor log_2(I) \rfloor + 1 : & I \neq 0, \\ 1 : & I = 0. \end{cases} \tag{1}$$

Let D be a decimal whose standard form is $\mathrm{D} = \mathrm{D}^{'} * 10^{\mathrm{t}}$ (the decimal point of $\mathrm{D}^{'}$ is on the left most), and assume that $\mathrm{D}^{''}$ is the decimal part of $\mathrm{D}^{'}$ (not including decimal point), then the complexity of the decimal D is

$$\mathrm{C_r(D)} = \begin{cases} \lfloor \log_2(\mathrm{D}'') + \log_2(\mathrm{t}) \rfloor + 1 : & t > 0, \\ \lfloor \log_2(\mathrm{D}'') \rfloor + 1 : & t = 0, \\ \lfloor \log_2(\mathrm{D}'') + \log_2(|\mathrm{t}|) \rfloor + 2 : & t < 0. \end{cases} \tag{2}$$

Let $F = \frac{F_n}{F_d}$ be a fraction. The complexity of F is

$$\mathrm{C_r(F)} = \lfloor \log_2(\mathrm{F_n}) \rfloor + \lfloor \log_2(\mathrm{F_d}) \rfloor + \lfloor \log_2(\lfloor \log_2(\mathrm{F_d}) \rfloor + 1) \rfloor + 2. \tag{3}$$

Set the binary length of all variables and symbolic constants as 8. Suppose O is a variable or a symbolic constant and O occurs i times in an expression, then the total complexity of all O's in that expression is defined as

$$\mathrm{C_r(O)} = \mathrm{C_r^{'}(O)} + (\mathrm{i} - 1) * 4 \tag{4}$$

Here $\mathrm{C_r^{'}(O)}$ denotes the complexity of a single O.

We take the expression $\sin\left(\mathrm{x} + \frac{\pi}{4}\right) + \mathrm{x}^4 - \frac{\pi}{3}$ as an example to calculate the length of the combination rule. The expression contains four operands (x, π, 4,3) and five operators (sin, +, /,· −), and a combination rule, as shown in Fig. 1(a).

The variables, constants, functions and sub-expressions in the syntax tree are replaced with type-free symbols shown as in Fig. 2(a), and can be written as

$$\lambda S_1.S_1(\lambda S_2.S_2(\lambda S_4.S_4(\lambda S_8.S_8 S_{11}(\lambda S_{13}S_{14}S_{12}.S_{12}S_{13}S_{14})))(\lambda S_9 S_{10} S_5.S_5 S_9 S_{10})) \\ (\lambda S_6 S_7 S_3.S_3 S_6 S_7).$$

The De Bruijn notation is used to represent the λ-calculus terms, and the names of the variables can be eliminated from the symbols. The De Bruijn notation for the combination rule of the expression is

$$\lambda 0(\lambda 0(\lambda 0(\lambda 02(\lambda 021)))(\lambda 021))(\lambda 021).$$

Using the encoding rules of De bruijn notation, its binary λ-calculus is obtained as

$$
\begin{gathered}
000101100001011000011000010110111000010110111 \\
01100001011011101100001011011110110.
\end{gathered}
$$

The complexity of the operands of the expression is 32 and the complexity of the combination rule is 79, so the complexity of this mathematical expression is 111.

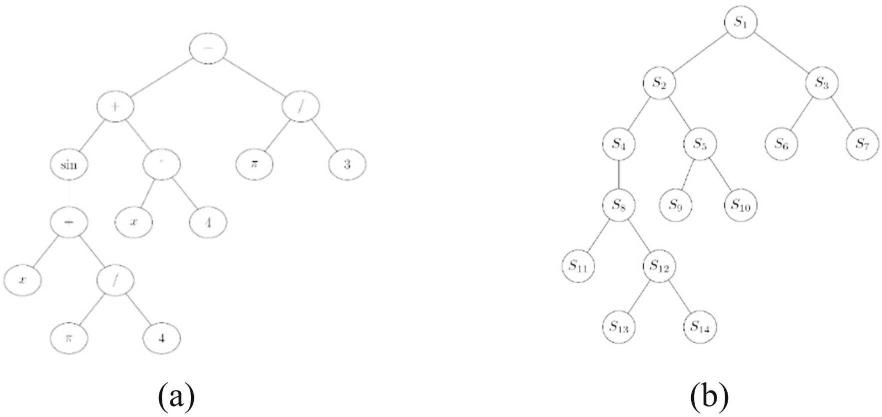

Fig. 1. (a) The syntax tree of the mathematical expression; (b)The tree structure for combination rule with type-free symbols.

2.2 Research Object

In this paper, two textbooks published by the People's Education Press are choosed for the study: the Experimental Textbook of General High School Curriculum Standards-Mathematics 4, Version A, published in 2007 (hereinafter referred to as "the 07 version"), and the Textbook of General High School-Mathematics Compulsory Book 1, Version A, published in 2019 (hereinafter referred to as "the 19 version".) The specific contents of the study are the mathematical expressions in the body of Chapter 1 of the 07 version and the mathematical expressions in the body of Chapter 5 of the 19 version. The contents are shown in Table 1.

The 19 version puts the "trigonometric identity transformation" in the Chapter 5. In order to present the mathematical expressions objectively, Section 5.5 of the 19 version is not considered. The mathematical expressions in this paper satisfy the following requirements: (1) they should contain at least one operator; (2) the formula on each side of the " =" is treated as a mathematical expression; (3) consecutive inequalities are split into simple inequalities; (4) intervals are not included.

Table 1. Contents of the two editions.

Textbooks	The 07 version	The 19 version
Title	Chapter 1 Trigonometric Functions	Chapter 5 Trigonometric Functions
Contents	1.1 Arbitrary angle and radian system	5.1 Arbitrary angle and radian system
	1.2 Trigonometric functions of arbitrary angles	5.2 The concept of trigonometric functions
	1.3 Induction formula for trigonometric functions	5.3 Induction formula
	1.4 The image and properties of trigonometric functions	5.4 The image and properties of trigonometric functions
	1.5 Image of the function y $=$ Asin$(\omega x + \varphi)$	5.5 Trigonometric constant transformation
	1.6 Simple applications of the trigonometric model	5.6 The function y $=$ Asin$(\omega x + \varphi)$
		5.7 Application of trigonometric functions

3 Result

3.1 Overall Situation

According to the above four rules, the mathematical expressions in the two textbooks were counted, of which there are 502 mathematical expressions in the 07 version and 577 mathematical expressions in the 19 version. Calculating the complexity of mathematical expressions in the two editions, it is found that the complexity of operands, combination rules, and mathematical expressions in the 19 version are greater than the 07 version. About the average, there is not much difference between the two editions, with the 19 version slightly more complex than the 07 version. This indicates that although the 19 version has more mathematical expressions than the 07 version, in average the complexity of an individual mathematical expression does not increase.

In order to compare the distribution of mathematical expressions in the two editions, the mathematical expressions in the example part and the non-example part are counted separately, and we call the non-example part as knowledge part. A qualitative analysis is conducted to analyze the reasons for the differences in mathematical expressions between the two editions.

3.2 The Number and Complexity of Mathematics Expressions in Knowledge Part and Example Part

The 07 version has 224 mathematical expressions in the knowledge part and 278 in the example part. In the 19 version, there are 235 mathematical expressions in the knowledge part and 342 in the example part. The complexity of mathematical expressions in the two editions is shown in Table 2. From Table 2, it can be seen that the complexity and the mean of the example parts of both editions are higher than the complexity of the knowledge part, and the complexity of the 19 version is higher than that of the 07 version in the knowledge and example part, but the mean of the complexity of the combination rules and mathematical expressions in the example part are lower than that of the 07 version.

Table 2. The knowledge and example parts of the two editions.

The 07 version				
	Classifications	Operands	Rules	Expressions
Complexity	Knowledge	3601	5186	8787
	Example	5029	10003	15032
Average	Knowledge	16.08	23.15	39.23
	Example	18.09	35.98	54.07
The 19 version				
	Classifications	Operands	Rules	Expressions
Complexity	Knowledge	3891	5720	9611
	Example	6247	12056	18303
Average	Knowledge	16.56	24.34	40.90
	Example	18.27	35.25	53.52

3.3 Differences Between the Mathematical Expressions of the Two Editions

The above results show that there is no significant difference in the overall complexity of mathematical expressions between the two editions of the textbook, and the distribution of the corresponding subsections is also similar. However, there are some differences in specific expressions, which are mainly reflected in the following situations: First, there are additions and deletions of knowledge content in both editions, which lead to different mathematical expressions; Second, the example part in the 19 version pays more attention to combining real-life situations and is more standardized in the use of mathematical expressions, while the process of solving the examples has increased the details, making the number of expressions increase; Third, the example part of the 19 version adds richer types of mathematical expressions without increasing the complexity of their representation. One specific example is explained below.

In Fig. 2 we can see a very interesting phenomenon that the two examples do not change in their complexity without being identical. Both of these examples examine the use of the induction formula. The example on the right from the 07 version involves only the sine and cosine functions, while the left from the 19 version adds the tangent function. In addition, the example on the left replaces $\cos(-180° - \alpha)$ with $\cos(-180° + \alpha)$, adding a transformation scenario. Calculating the complexity of the representation of these two formulas, we get a result of 178. The complexity of the mathematical expressions in the

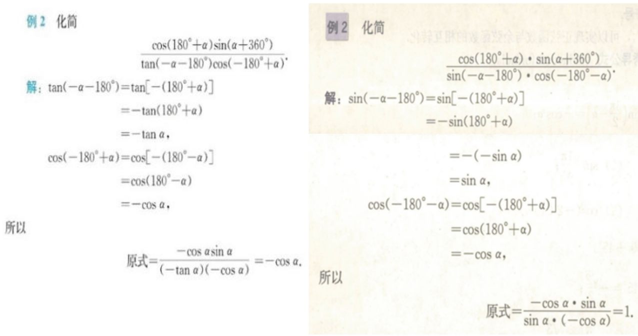

Fig. 2: Comparison of the same type of examples in the two editions.

solution process of the example in the 07 version is 489, and in the 19 version it is 498. It can be seen that the 19 version presents the use of different types of functions and induced formulas without increasing the complexity of the mathematical expressions in the example and with essentially no difference in the complexity of the computational process.

4 Conclusions and Suggestions

The study of mathematics textbooks using a linguistic approach has been an active topic in recent years. The complexity of mathematical expressions provides a new perspective for the study of mathematics textbooks and gives some insights into the compilation of mathematics textbooks. Mathematics textbooks should focus more on the essence of knowledge rather than skills that increases students' learning burden; the elaboration of concepts should be detailed in a way that both clarifies the concepts and inspires students; the demonstration of examples should make the solution process concise and easy to understand, and be more logically structured without making the examples more complicated.

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A Comparative Analysis of the Time Content in Textbooks from Eight Countries

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Abstract. In this paper, we report on a comparison of the treatment of the time content in primary mathematics textbooks used in eight countries, involved China, Japan, South Korea, Singapore, France, Germany, Russia, and the United States. For this purpose, we develop and apply a framework to analyse the similarities and differences of the time content in textbooks, especially the distribution characteristics of time concepts at different cognitive stages and the involved sub-concepts. We find both similarities and differences co-exist in the arrangement structure and grade distribution among these textbooks. We also draw on the results and our analysis to provide suggestions and directions for future textbook compiling about the time content.

1 Introduction

Time is an important structural element of society that has been incorporated as thematic content into the primary school curriculum in various countries (Mullis et al. 2004). In the face of such abstract and complex learning content, the learning and teaching of time content conjointly face challenges (Monroe et al. 2002). Therefore, most researchers explore primary school students' understanding of time from the perspective of cognition, and put forward suggestions from the perspective of teachers' teaching design and implementation. As an important physical quantity, time has become an indispensable content in the primary school mathematics textbooks of all countries. Many countries have explored the compilation of the time content in primary mathematics textbooks. Based on it, this study selects the time content of primary school textbooks in different countries to compare and analyze. The purpose of this study is not to judge the compilation quality but to describe the treatment of the time content in primary mathematics textbooks used in these countries. We are to answer this question: what are the differences and similarities in arrangement structure and grade distribution of the time content in primary mathematics textbooks used in these countries?

2 Perspectives and Methodology

This study makes a transnational comparative analysis of the time concept, and deeply understands how the time concept is presented and arranged in these textbooks. Content analysis is employed in this study, which mainly focuses on three aspects, the cognitive

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development of children's time concept, the connotation of the time concept, and the core content in the curriculum standards and textbooks of the countries. The cognitive development of children's time concept is divided into four aspects: comparing scales, establishing the quantity sense, establishing the equal quantity sense, and the calculation and application (Zhong 1998), which are taken as the main categories. The connotation of the time concept is divided into epoch and interval from the perspective of the number line (Goudsmit and Claiborne 1966). Based on the perception and connotation of the time concept, combined with the structure and content of the time content of the curriculum standards and textbooks of the countries, we further divide the time concept that primary school students should understand into seven sub-categories, including clock, unit, the relationship of units, order, period, epoch and interval, which can cover all the content of the time content in primary mathematics textbooks of the countries. We take tasks as the unit of analysis, involved examples, exercises, activities, and separately marked sections of textbooks. In order to ensure the validity of the study, reliability, and validity tests are carried out for the analysis categories. Inter-rater reliability is used to test the consistency degree of three raters independently coding the time content. The test result is 0.875, so the analysis categories of this research have high reliability. Inspection and revision are done to establish expert validity by researchers who have expertise in elementary mathematics education (Table 1).

Table 1: The analysis category of the time theme in primary mathematics textbooks

Main Categories	Sub-Categories
1 Comparing Scales	1–1 Clock
	1–2 Epoch
	1–3 Order
	1–4 Period
2 Establishing the Quantity Sense	2–1 Unit
3 Establishing the Equal Quantity Sense	3–1 The Relations of Units
4 Calculation and Application	4–1 Interval

We selected eight representative editions according to the range of use and the authority of publishing presses and other selection principles, involving Chinese, Japanese, South Korean, Singaporean, French, German, Russian, and American versions, coded RJ, DJ, TC, MC, IS, KL, RH, AC. The information about the primary school systems of eight countries is shown in the following table (Table 2).

Table 2: The information about the primary school systems of eight countries

Country	China	Japan	South Korea	Singapore	France	Germany	Russia	USA
Grade	6	6	6	6	5	4	4	5

3 Result

3.1 The Quantity Structure of the Categories About the Time Theme in Textbooks of Eight Editions

According to the statistical analysis, seven countries' mathematics textbooks cover all the analysis categories, but only the Japanese DJ does not write the content of "order". There are great differences in the amount of time content among the textbooks of eight editions. French IS has the largest number of problems with 126, and Japanese DJ has the least with 48. The number of problems in Chinese, Russian and American textbooks is relatively close (Table 3).

Table 3: Quantity distribution of the categories of time content in textbooks

Category		RJ	DJ	TC	MC	IS	KL	RH	AC
1	1–1	4	4	5	4	2	2	2	3
	1–2	12	17	28	22	18	12	7	25
	1–3	6	0	4	1	7	1	3	6
	1–4	7	3	3	11	14	9	13	2
2	2–1	12	3	6	3	2	5	2	2
3	3–1	10	4	6	10	39	5	25	15
4	4–1	28	17	11	38	44	22	30	24
Total		79	48	63	89	126	56	82	77

From the horizontal view, we find that the content distribution proportion in eight countries' textbooks is different. Compared with the average proportion of each stage (13%), there is no large difference in the distribution proportion of the stage of comparing scales in eight versions. However, there are large differences in the remaining three stages. In the stage of establishing the quantity sense, Chinese RJ has the highest proportion (34%). In the stage of establishing the equal quantity sense, French IS has the highest distribution proportion (34%). The two stages' lowest proportion is about 5%, and the low proportion is more widespread. In the calculation and application of time, Korean TC (5%) and Japanese DJ (8%) have the lowest proportion, while other versions are around or above the average proportion.

From the vertical view, we find that there are obvious differences in the compilation emphases of the main categories in each version, while similarities and differences coexist in the compilation emphases of the sub-categories. In terms of the main categories, in the Japanese, Korean and American versions, the proportion of the stage of comparing scales is significantly higher than in the other three stages. While in French IS and Russian RH, the proportion of the other three stages except for the stage of establishing quantity sense, is higher and balanced. In terms of sub-categories, the proportion of epoch (1–2) and interval (4–1) is the highest among all versions. But in French IS and Russian RH, the proportion of the relations of units (3–1) and interval (4–1) is the highest.

3.2 The Grade Distribution of the Categories About the Time-Themed Content in Textbooks of Eight Editions

From Fig. 1, there are obvious differences in the grade distribution of the time-themed content in eight editions. In five countries, time content is centralized and arranged in the first, second, and third grades of primary school. Chinese, Korean, Japanese, and American versions scatter the content in grades 1–3, while Russian RH arranges it in the third grade. French IS (grades 1–5) and German KL (grades 1–4) make time content throughout the entire primary school, and each grade is involved. Singaporean MC arranges it in grades 1–4.

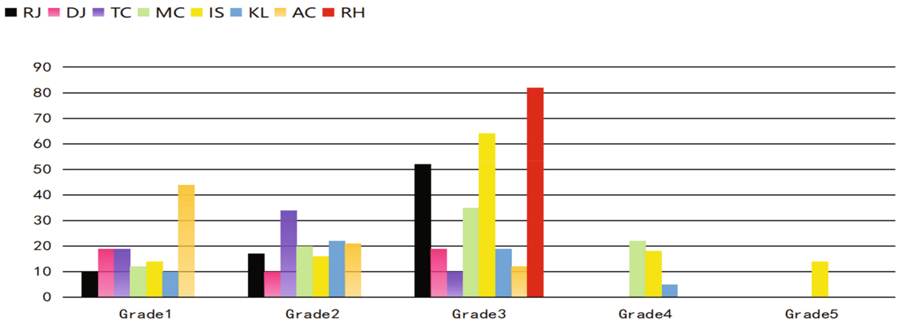

Fig. 1: Grade distribution of time-themed content in textbooks

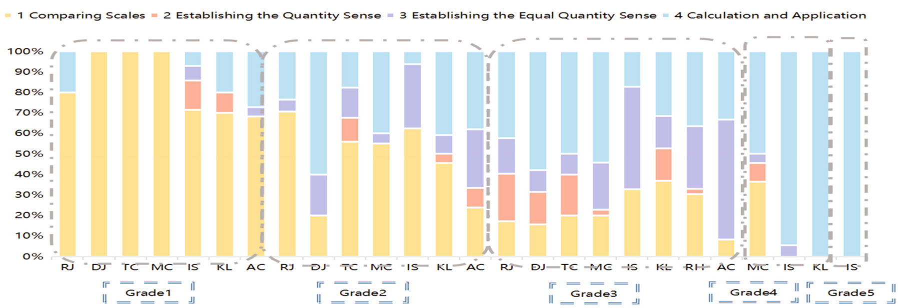

Fig. 2: Main category distribution of the time-themed content in textbooks

From Fig. 2, on the whole, the content distribution of the four main categories in different grades has the following characteristics. The first grade of primary school mainly focuses on the learning of the stage of comparing scales. With the raise of grade, the number of stages involved increases (except for IS and RH), and the calculation and application of time are highlighted. In terms of specific categories, the initial grade in which the concept of unit (the stage of establishing the quantity sense) is arranged differs greatly. Many versions arrange the concept of the unit in the third grade, while French IS and German KL compile it in the first grade, but the specific contents of the arrangement are different. As shown in Figs. 3 and 4, French IS has the content of year, month, and day, while German KL has the content of 1 h and 1 min in quantity sense of time.

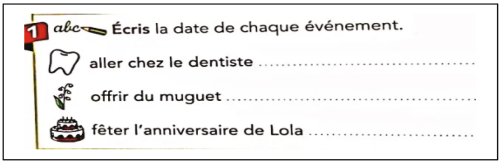

Fig. 3: The content of time units of IS (Erich Wittmann et al. 2017 (G1), P.134)

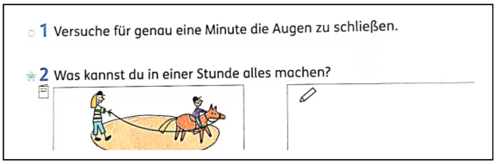

Fig. 4: The content of time units of KL (Monica Neagoy et al. 2018 (G1), P.79)

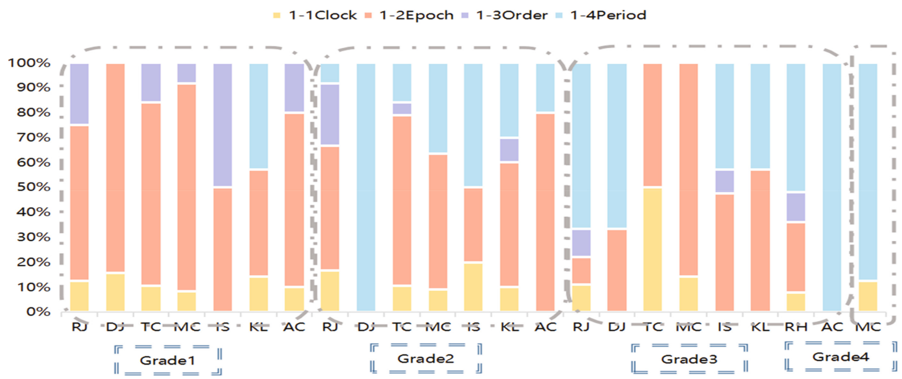

Fig. 5: Sub-category distribution of the time-themed content in textbooks

From Fig. 5, there are large differences in the grade arrangement of "epoch" in different editions of textbooks. In terms of grade distribution, the content is generally arranged consecutively in grades 1–3. American AC arranges it consecutively in grades

1–2, while Russian RH arranges the content all in the third grade. In contrast, the Japanese DJ breaks the continuity and distributes the content in the first and third grades. In terms of the correlation between epoch and other contents, it is found that in French “IS” and German KL, epoch (1–2) and period (1–4) not only have the same grade distribution but also have roughly the same content proportion, while other versions do not have such characteristics.

4 Discussion

4.1 The Arrangement Structure of Time-Themed Content

There are obvious differences in the content proportion of the stage of establishing the quantity sense, which may be related to the characteristics of time itself and the thinking of textbook design. The establishment of the correct representation of time units and the selection of appropriate time units for estimation are crucial to the formation of the quantity sense of time. However, the abstraction and complexity of time itself may make it difficult for students to establish quantity sense. For example, the ability of estimation, which refers to estimating time by unit iteration, is not formed until the sixth grade (Long and Kamii 2001). Therefore, textbooks may weaken or delete the elements, and generally appear the distribution characteristics of low proportion. However, Chinese RJ places more emphasis on the cultivation of time-related ability (Zhou et al. 2004), so the content of the stage of establishing the quantity sense has a high distribution proportion.

4.2 The Grade Distribution of Time-Themed Content

There are obvious differences in the grade arrangement of “epoch” in each version, and there are two reasons for our analysis. One is related to the relevant theoretical research. Early studies found children’s developmental stage of time reading: most of the children are able to identify whole-hour-times by age 6, 5-min-times by age 7 or 8, and 1-min-times by ages 8–10 (Siegler and McGilly 1989). Many versions’ grade distribution contradicts this conclusion. However, the exact age division that is suitable for the development of children’s reading ability has not been formed. Therefore, the differences in grade arrangement of each edition are significant. The second is related to the thinking of textbook design. Different from French IS and German KL that integrate “epoch” with “period”, Chinese RJ lays more emphasis on the arrangement order of the two contents. It may be because Chinese mathematics courses and teaching emphasize the solid foundation (Moy and Peverly 2005) and students’ understanding of each concept, thus the content is more sequential and less intersectional.

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Giving a Boost to Textbook Tasks

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Abstract. Despite the generally acknowledged relevance of providing students with rich, challenging tasks that require genuine problem solving, several studies have shown that these tasks are often lacking in textbooks. Bringing new textbooks to the market might feel as a logical solution for this, but maybe there is also another way of resolving this problem in which teachers can also play a pivotal role: Enriching textbook tasks. In this paper we report about a case study carried out in a Norwegian fifth-grade classroom in which we explored Denisse Thompson's suggestion (2012) to make textbooks richer by modifying existing textbook exercises. The focus in our study is on multi-digit algorithmic subtraction. The analysis of the students' worksheets shows some interesting differences between the original and the enriched tasks.

1 Background of the Study

Promoting deep learning—robust understanding of mathematics as Schoenfeld phrased it (Schoenfeld et al., 2016; 2020)—has been generally acknowledged as one of the key goals of mathematics education. In mathematics, deep learning specifically refers to integrated learning that leads to relational understanding, which contrasts with fragmented learning and instrumental understanding (Skemp, 1976). Relational understanding implies comprehending the underlying structure of mathematical concepts and being able to apply knowledge to new situations. The most important pre-requisite to achieve this goal is offering students the learning opportunities to develop this understanding (Kilpatrick et al., 2001). Textbooks play an important role in realizing these opportunities as is shown by several studies which have provided empirical findings about the relation between the mathematical content offered in textbooks and students' mathematics achievement (e.g. Espeland, 2017; Sievert et al., 2019; Törnroos, 2005; Wijaya et al., 2018). At the same time however, research has revealed that rich, challenging tasks that require genuine problem solving are often lacking in textbooks (e.g. Brehmer et al., 2016; Jäder et al., 2020; Van Zanten & Van den Heuvel-Panhuizen, 2018). Most of the textbook tasks are straightforward of nature. Within the domain of numbers and operations this means that the numbers and the operation of the tasks are given, implying that the students 'just' have to carry out a prescribed calculation to find the answer.

Note The lesson was video recorded, but unfortunately the quality of the audio recording of the group work was not sufficient to get useful information about the students reasoning.

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As Thompson (2012) has shown having textbooks with straightforward tasks does not mean the end of the story. She came with the suggestion to modify textbook exercises in order to incorporate reasoning and communication into the primary mathematics classroom. For example, a textbook task such as "calculate the problems $12 + 8 =$ and $12 \times 8 =$" can be modified as "Find two whole numbers whose sum is 20 and whose product is 96." In addition, the students can be asked to explain their approach to solve this modified task. Compared to the original task the revised task is more demanding. It is still about basic computations but now the students must figure out themselves a way to solve the problem and decide themselves which operations must be carried out. They must think in the reverse way and must make estimations.

In this paper we elaborated on the idea of enriching existing textbook tasks. Our aim was to get some experience in developing and using enriched tasks and to investigate what we can learn from these tasks about students' mathematical understanding. The focus in our study was on multi-digit algorithmic subtraction.

2 Method

The study was carried out by two university researchers, a classroom teacher and two teacher students and took place in a Norwegian fifth-grade classroom in a city in the north of Norway halfway the school year. The class consisted of 18 pupils, aged 10–11. In the mathematics lessons, the pupils worked on various tasks such as counting in chorus, and arithmetic and mathematics trails, in which the pupils can experience mathematics in the real world. The school year started with some activities related to the national tests.

When planning the study, we first discussed with the classroom teacher which chapter of the textbook she would teach in about six weeks and which types of tasks will then be the topic of her lessons. The textbook that then was in use was the Multi 5A textbook (edition 2006, published by Gyldendal). In addition, the teacher also used several mathematics materials from other textbooks. In the previous months the pupils had worked on whole numbers covering positive and negative numbers, the decimal system, and addition and subtraction with whole numbers.

It was decided to focus on addition and subtraction with multi-digit numbers. The teacher felt a need for deepening pupils' understanding in this topic. Her assessment was that the pupils still require some extra instruction on subtraction and borrowing. Therefore, she suggested to make Task 1.83 (Fig. 1) more challenging.

For the textbook tasks an enrichment was developed aimed at deepening the pupils' understanding of the subtraction algorithm by offering them an opportunity to delve into the underlying structure of the standard algorithm.

During the lesson the pupils had to work on two worksheets (Fig. 2). They were offered to the students after each other, each preceded by a short introduction. Worksheet 1 contains the three subtraction tasks from the textbook. Worksheet 2 holds the enrichment. On this worksheet, instead of repeating the instruction of how to borrow, the pupils are presented an alternative way of calculating with whole numbers rather than with digits. The three questions are meant to elicit discussion and argumentation among the students.

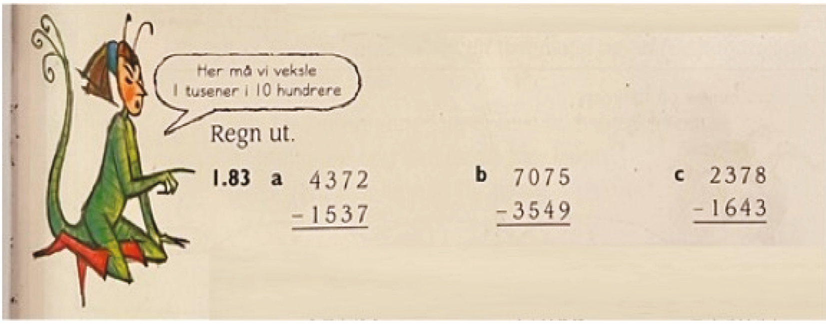

Fig. 1. Task 1.83 from the Multi 5A textbook, p. 27 (Translated text: "Here you have to exchange 1 thousand for 10 hundreds")

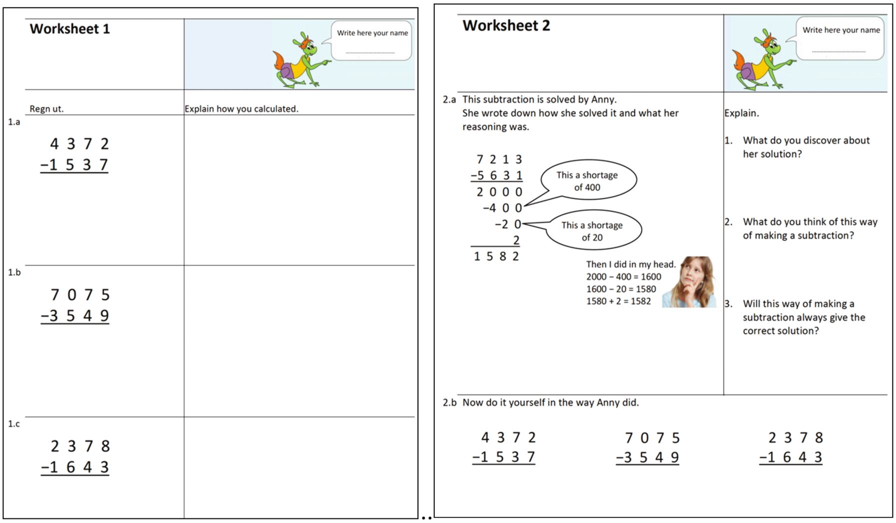

Fig. 2. Worksheet 1 (textbook tasks) and Worksheet 2 (enriched tasks)

3 Results

Although the pupils had to solve two times the same tasks, the proportion correct answers were not the same for the two worksheets.

The enriched tasks led more often to a correct answer. In each worksheet, 54 answers had to be given (18 pupils with each three tasks). In Worksheet 2, the proportion of correct answers was 51%, while in Worksheet 1 this was only the case for 44% of the answers (Table 1).

When comparing the correctness of the answers at an individual level it was found that in three quarters of the pair of tasks the correctness did not differ (Table 2). In 35% of the pairs, they were answered correctly on both worksheets and in 38% the answers were two times incorrect. In a quarter of the pairs of tasks there was a difference between the

Table 1. Overall results correctness of answers

	Worksheet 1					Worksheet 2				
	Task 1	Task 2	Task 3	Total		Task 1	Task 2	Task 3	Total	
	n	*n*	*n*	*n*	%	*n*	*n*	*n*	*n*	%
Correct	8	7	9	24	44	12	5	11	28	51
Incorrect	10	10	7	27	50	6	11	5	22	41
No answer	0	1	2	3	6	0	2	2	4	7
	18	18	18	54	100	18	18	18	54	99

two worksheets. In 9% the pupils' initially correct answer was incorrect in Worksheet 2. Yet, in 17% of the pairs of tasks the pupils improved their score.

Table 2. Correctness in Worksheet 1 and 2

Worksheet 1	Worksheet 2	Pair of tasks	
		n	%
Correct answer	Correct answer	19	35
Incorrect answer	Incorrect answer	21	38
Correct answer	Incorrect answer	5	9
Incorrect answer	Correct answer	9	17
Total		54	99

The influence of the enrichment was for example clearly visual in the work of Pupil D (Fig. 3).

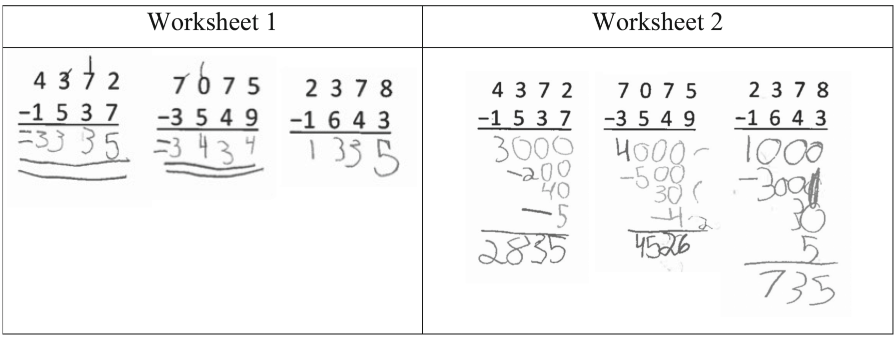

Fig. 3. Work of Pupil D

Another finding was that of the 8 pupils who had one or more tasks on both worksheets correct, 5 of them picked up completely or partly Anny's way of calculating with whole numbers.

However, the pupils' responses to the question whether Anny's method will always give a correct answer revealed that it was not automatically self-evident for all pupils that this indeed is the case (Table 3). In this class this applied both to the pupils with mostly correct answers and those with mostly incorrect answers, which offered a good starting point for a discussion in the next lesson.

Table 3. Pupils' responses to Question 3

Responses given to Question 3: Will it always give a correct solution?	Pupils with mostly	
	Correct answers	Incorrect answers
"Yes, it will always give the right answer!"		X
"Yes"	C, O, V	W, H, S
"Yes I think so"	J	
"Yes maybe"	Y	
"Maybe"	Z	
"Maybe sometimes, but not always"		R
"No"	T	
"I don't know"		A
"No answer"	B, D, F, P	E

4 Concluding Remarks

This was only a very limited study to get some experience in developing and using enriched tasks, but by going all the way through the process of designing the tasks, trying them out in classroom, analyzing the student data and jointly discussing the findings and experiences, we learned a lot about the potential of enriching textbook tasks.

Of course, pupils can also be offered challenging tasks by bringing new textbooks to the market with extra sections with problem solving tasks, but the advantage of adding extra challenging questions to existing straightforward textbook tasks may stimulate pupils also to think deeper when solving other tasks in the textbook. Working on the enriched tasks may contribute to building up a more mathematical attitude in pupils. Even in straightforward tasks there is something to think about.

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A Comparative Study on Mathematical Culture of My Country's New Version of High School Mathematical Textbooks—Take the Content of Functions in the Compulsory Textbooks of the People's Education a and Hunan Education Editions as Examples

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Abstract. The research takes the function topics in the compulsory textbooks of the People's Education Edition A and the Hunan Education Edition as examples, and compares and analyzes the column distribution, cultural type, application level and cultural function of their mathematics culture. Through the conclusion, it puts forward suggestions to improve the setting of mathematics culture in Chinese textbooks.

1 Preface

Mathematical culture is about the thought, spirit, language, method, and viewpoint of mathematics, as well as its formation and development, including the contribution and significance of mathematics in human life, science and technology, social development, and mathematics-related humanistic activities (Ministry of Education of the People's Republic of China 2018). Mathematical textbooks are not only an important carrier of mathematical knowledge, but also an important resource for achieving the goals of mathematics curriculum, which help develop students' core mathematical literacy and promote students' mathematical progress (Lv et al. 2015). In order to improve the compilation level of mathematics culture in mathematics textbooks in our country, help teachers better understand the content of mathematics culture in textbooks, and give better play to the educational function of mathematics culture in teaching practice, this study selects two versions of textbooks. A comparative study of the content of mathematics culture, analysis of its commonalities and characteristics.

2 Research Design

2.1 Research Objects

The A-version of the People's Education is the most widely used and influential textbook in China, with content related to mathematical culture in every chapter, with a wide range of material in various forms and rich content (Shao and Zhang 2019). The Hunan

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Education Edition textbook is the newest mathematics textbook used by ordinary high schools in Gansu Province, and a column of "mathematical culture" is also set up (Shen et al. 2013). The topic of function is one of the most important topics in high school mathematics curriculum and is the core content in high school mathematics. There are no research papers on the mathematical culture in these two editions of textbooks.

2.2 Research Framework

This study draws on the analytical framework of Wang Jianpan, Shen Chunhui, Tang Hengjun and other scholars (Tang et al. 2016; Wang 2011; Wang et al. 2015). The columns are divided into non-text, introduction, examples, and exercises; the types are divided into mathematics history, mathematics and life, mathematics and technology, Mathematics and Humanities and Arts; the application level is divided into external type, separable type and inseparable type; functions are divided into providing background context, providing applied context, expanding mathematical thinking and methods, promoting mathematics and culture integration and experiencing cultural practice.

2.3 Research Methods

The SPSS 21.0 statistical software was used to conduct a chi-square test for the number of mathematical cultures in the two editions, and the corresponding chi-square values and values were derived to determine the consistency of the distribution of mathematical cultures in the two editions.

3 Comparison of Mathematical Culture in the Contents of "Function" in Two Versions of High School Textbooks

3.1 Comparison of the Distribution of Mathematics Culture in Different Columns

The p-value of the chi-square test ($p = 0.001 < 0.05$) shows that the mathematical culture of the two editions of the textbook differs significantly in different columns. From the statistical results, it can be seen that the total amount ($162 > 115$) of the content of mathematics and culture in the textbooks of the People's Education Edition is higher than that of the Hunan Education Edition. From the specific columns, it can be found that the proportion of non-text in the PPE version (16%) is higher than that in the Hunan Education version (10%), and over 50 percent of the exercises (54%, 50%) in the two versions of the textbooks permeate the mathematical culture.

3.2 Comparison of Mathematical Culture Types

From the P value of the chi-square test ($P = 1.000 > 0.05$), it can be seen that the mathematical culture of the two editions of textbooks has a significant consistency in the type of mathematical culture. From the statistics of the percentages, it can be found that the two editions of textbooks attach great importance to the connection between mathematics and technology (59%, 57%), followed by the connection between mathematics and life, both of which are over 30%. In the category of mathematics and technology, the total amount of mathematics culture in the textbooks of the People's Education Edition is 30 more than that of the textbooks of the Hunan Education Edition. In general, these two editions of textbooks have paid less attention to other two types of mathematical culture, especially mathematics and humanities and arts (3%, 3%).

3.3 Comparison of the Application Level of Mathematical Culture

From the P value ($P = 0.993 > 0.05$), it can be seen that the mathematical culture of the two editions of textbooks has a significant consistency in the level of mathematical history application. The application level of mathematics history in the two editions of textbooks is higher, and the application level of additional type is the highest (22%, 17%).

From the P value ($P = 0.153 > 0.05$), it can be seen that the two editions of the textbook are basically consistent in the application level of other types of mathematical culture. Statistics show that the two editions of textbooks have relatively high levels of inseparable mathematical culture (41.2%, 36.7%), but the proportion of separable mathematics is still the highest (50.3%, 44%).

3.4 Comparison of Mathematical Cultural Functions

From the P value ($P = 0.335 > 0.05$), it can be seen that the functions of mathematical culture in the two editions of the textbook are basically the same. From the statistics, it can be seen that the function of mathematics culture in the PEP textbooks is mainly to provide applied situations (61.2%), followed by the provision of background situations (34.5%). Besides, the Hunan Education Edition textbooks provide the most contextual situations (53.9%). In addition, the function of mathematics culture in the two editions of textbooks involves promoting the integration of mathematics and culture (1.9%, 1.7%), and the PPE edition also involves experiential cultural practice (1.2%).

4 Conclusion

4.1 There Are Significant Differences in the Distribution of Mathematics Culture in Different Columns, and the Proportion of Exercise Columns Is Relatively High

In terms of the total amount of mathematical cultural content, the Human Education Edition contains more textbooks A than that of the Hunan Education Edition, indicating that it contains richer mathematical cultural content compared to the Hunan Education Edition of the textbook. Specifically to each column, the mathematics culture of the PEP A textbook is concentrated on the exercises and example columns, and there are few columns introduced in textbook A. Compared with the Human Education Edition, the mathematics culture of Hunan Education Edition textbooks is concentrated in the exercises and citations, with less non-text, which indicates that the they are more inclined to let students feel the charm of mathematics culture when exploring and solving problems. Besides, the Hunan Education textbook also has a special section on mathematics and culture, so that teachers and students can quickly orient themselves when preparing and studying.

4.2 The Distribution of Mathematical Culture Types Is Significantly Consistent, with Mathematics and Technology Types Accounting for the Highest Proportions

In terms of mathematical culture, these two editions of textbooks focus on the history of mathematics, mathematics and life, mathematics and science, Mathematics and Humanities. Above all, the two editions of the textbook have the largest proportion of the connection between mathematics and technology. Most of the history of mathematics is set in the non-text part, and the People's Education Edition A also involves the implicit history of mathematics. The two sections: mathematics and life, social life and economic life, account for the highest proportion in the two editions of textbooks, and entertainment and professional life are less in personal life and public life. Since this part of the function is more closely related to physics, chemistry, biology, and information technology, the two editions of the textbook have the highest proportion of the mathematics and science and technology type. The content and types of mathematics and humanities and arts types are set less in both editions.

4.3 The Application Level of Mathematical Culture Is Basically the Same, Mainly Based on the Additional and Separable Application Levels

The application level of the history of mathematics in the two editions of textbooks is mainly based on the supplementary style. The teaching material of the People's Education Edition involves the adaptive style, and the textbook of the Hunan Education edition involves the copy style, and the number is relatively small. When reading the textbooks, it could be found that the history of mathematics is usually unprocessed and direct. The difference is that the PEP textbook has a column of cultural practice, but it appears in the form of elective content. From the perspective of teaching practice, many teachers will use the method of "not reading or reading after class" in teaching, which makes it difficult for mathematics culture to exert its educational significance. The application level of other types of mathematical culture is mainly separable, mainly distributed in the examples and exercises; Besides, the proportion of inseparable types is also higher, mainly in the introduction, examples and exercises, as well as body part. From the data of the percentage of inseparable types, it can be seen that the two editions of the textbooks have a higher level of application in other types of mathematics culture.

4.4 The Functions of Mathematical Culture in Textbooks Are Basically the Same, and Are Concentrated in Background and Applied Situations

In terms of mathematical culture in the function of mathematical materials, the two editions of textbooks mainly provide the background and applied situations. The mathematics culture in the PEP textbook provides more than half of the applied situations, which is suitable for students to feel the charm of mathematics culture in the process of applying knowledge. More than half of the mathematics culture in the Hunan Education Edition textbooks provide background situations, which can attract students' interest in learning and improve knowledge understanding. The difference is that the PEP textbooks involve the experience of cultural practice, and the literature reading and mathematics writing parts distributed in the non-text columns A help teachers and students to fully appreciate the functions of mathematical culture and experience the occurrence and development of mathematical knowledge. It is conducive to changing mathematics culture from dessert to dinner, and it is truly integrated into the daily teaching of mathematics.

5 Suggestions

5.1 Enrich the Content of the History of Mathematics in Non-text Columns

Most of the mathematics culture content in the two editions of textbooks appears in the exercises column, followed by the mathematics culture in quotations and examples, and less mathematics culture is contained in the non-text. Most of the mathematics culture in the non-text column is presented in the history of mathematics. In fact, infiltrating the history of mathematics in the non-text column can make students understand the connection between the history of mathematics and real life, and broaden their learning horizons. Vivid and interesting math stories and the wisdom of mathematicians through time and space can stimulate students' interest in learning. The content of mathematics history in textbooks is usually deleted artificially, and students cannot fully understand the ins and outs of mathematics knowledge. Therefore, textbooks should enrich the content of mathematics history in non-text columns.

5.2 Increase the Number of Mathematics Culture in Mathematics and Humanities and Arts

Mathematical culture emphasizes that mathematics is a cultural practice of human beings, and the development of mathematics is historical, diverse, and has humanistic value. Therefore, the diversity of mathematical culture types presented in the textbook will help students to form a more comprehensive view of mathematics culture, and also broaden the horizons of teachers and students in mathematics culture. This paper finds that there are relatively few mathematical cultures of mathematics and humanities and arts in the content of functions, so it is necessary to increase the number of mathematical cultures of mathematics and humanities and arts in the content of functions in the textbook, so that students can not only appreciate the application value of mathematics, but also feel the scientific and aesthetic value of mathematics and promote their all-round development.

5.3 Adopt High-Level Mathematical Culture Application Methods Such as Adaptive and Inseparable Types

"High evaluation, low application" is a serious problem in the current mathematics culture and mathematics history (Zhang and Hu 2019). The reason is mainly because the application level of mathematical culture is not high. The history of mathematics in textbooks rarely appears in an adaptive form, but if it can be integrated into the history of mathematics in a high-level way, it can directly reflect the profound foundation of mathematics culture. The history of mathematics cannot be limited to appearing in an embellished or add-on manner, but should be used at a higher level so as to add color to mathematical knowledge. Most of the application levels of other mathematical cultures in textbooks are in the separable type. This phenomenon will lead to a misunderstanding that mathematical culture is imposed on the problem situation. On the contrary, it makes people feel that mathematical culture is the "hat" of mathematical knowledge, which needs to be improved greatly. The level of mathematical culture application can deeply integrate mathematical culture and mathematical knowledge, so that students can truly appreciate the importance of mathematical culture.

5.4 Give Full Play to the Educational Function of Mathematical Cultural Diversity

The integration of mathematics culture into mathematics textbooks must be based on the setting of its functions. The unclear or subordinate cultural functions are another important reason for the "high evaluation and low application" of mathematics culture. The reason is that, when teachers do not understand the textbook, and the textbook itself does not have a clear mathematical cultural function orientation, both teachers and students will feel that the integration of mathematical culture into teaching is relatively blunt, then its practical effect will inevitably be affected, and in teaching, it can even become a "dessert after the main meal". When compiling textbooks, it should fully reflect the functions of mathematics culture, and provide ideas for teachers' mathematics culture teaching, so as to promote the integration of mathematics culture into mathematics classrooms, and let students know mathematics while learning mathematics.

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Exploring How the Senior High School Textbooks Support Students' Learning of Mathematics

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Abstract. Mathematics textbooks are supporting materials for students' learning of mathematics, the compilation of which should assist students in mathematics learning. PEP2019 was selected to analyse the design feature of the content of trigonometric function, and then to compare with PEP2004 in this article. On the basis of the content comparison, it reveals how PEP2019 supports students' learning of trigonometric function. These insights also indicate the design of PEP2019 aligned with the ideas of mathematics curriculum reform.

1 Introduction

Mathematics textbooks are a main resource in mathematics education and have a decisive role in shaping learning opportunities for students (Rezat et al., 2019). This means that students' learning of topics and skills stems from the learning opportunities provided by the textbooks. The textbooks should be the student's guide that helps him, in different situations, to find the path leading to mathematics (Turnau, 1980). So the compilation of the textbooks should be beneficial to students' learning, that is, to provide corresponding support for students to learn mathematics effectively.

China is currently carrying out and deepening a new round of mathematics curriculum reform. The senior high school mathematics textbooks compiled based on the new version of the senior high school mathematics curriculum standards (hereafter "MCS2017") have been published by People's Education Press in 2019 (hereafter "PEP2019") and then put into practical use. MCS2017 states that the compilation of mathematics textbooks should be conductive to students' learning. Thus the following research question was raised: *What does PEP2019 provide to support students' learning of mathematics?* The present study was set up to find an explanation for PEP2019 indeed beneficial to students' learning of mathematics.

2 Methodology

The approach taken in this study was investigating what the mathematics textbooks provide to support students' mathematics learning. In TIMSS, textbook analysis focused on investigating the content profiles of textbooks (Schmidt et al., 1997). With the deepening of textbook research, the content of textbook analysis became more and more

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diversified and specific, including how the textbooks help students understand the content. Thus the method of text analysis was used to analyse PEP2019 from the aspects of physical characteristics and the organization of the mathematics content. Specifically, physical characteristics refer to the textbook structure on the chapter- and lesson-level. The chapter-level structure refers to the chapters and the corresponding sections set up in the textbooks when organizing a certain mathematical theme. The lesson-level is most adequately characterized in terms of blocks, such as worked examples (Rezat, 2013). And the organization of the mathematics content refers to how textbooks present and treat the content, offering an in-depth understanding of the mathematical content (Charalambous et al., 2010). The context in which the mathematics content is presented was also considered, but its type was not further divided. The focus is on real-word contexts, called "extra-mathematical contexts" in PISA. As a contrasting case, PEP2004 was included. PEP2019 is revised and compiled based on PEP2004 and then published. Table 1 provides the information about publishers and the publication time of two mathematics textbooks.

Table 1. Details of two mathematics textbooks

Textbooks	Descriptions
PEP2004	The senior high school textbooks- Chapter 1 & 3 (pp. 1–71, 122–147), published by People's Education Press in 2004
PEP2019	The senior high school textbooks- Chapter 5 (pp. 167–257), published by People's Education Press in 2019

The content of trigonometric function was chosen for analysis rather than all the mathematical content. As a typical periodic function, trigonometric function has become one of the core contents of senior high school mathematics curriculum in various countries. But there are relatively few studies on trigonometry content in the middle school mathematics textbooks research, which mainly focus on the educational materials prepared features, including the examples, exercises and the knowledge sequencing of trigonometric function. For example, Fu and Zhang (2018) compared the similarities and differences of the trigonometric exercises between Chinese and American textbooks. Similar studies analysed the design features of trigonometric content in textbooks (see, e.g., Liu & Dai 2015), but did not further address how the design of the content supports students' mathematics learning. As some researchers have pointed out, the mathematical content in the textbooks must be systematic and coherent, reflecting the integrity at the level of logical structure, and its presentation can guide students to learn in a mathematical way (Zhang, 2016). Therefore, the content of trigonometric function was analysed to explore what senior high school written curricular materials provide to support students' learning of mathematics.

3 Result

3.1 The Chapter- and Lesson-Level Structure

PEP2019 integrates the two chapters of "trigonometric function" and "trigonometric identity transformation" in PEP2004 into one chapter "trigonometric function", and presents sections according to the logical sequence of "background – object – concept – graph – property – application". The chapter-level structure of the two textbooks follows the order of "chapter introduction – text – summary – review". For the lesson-level structure, the number of the blocks of thinking and inquiry in PEP2019 is slightly more than that in PEP2004, providing students with more opportunities to think in the form of questions or chains of questions. The most obvious difference is that PEP2019 sets up the introductory blocks as the start of each section, so as to lead to the question to be solved. For instance, PEP2019 presents the following introductory block as the start of Sect. 5.2 "The Concept of Trigonometric Function":

> We have extended the range of angles to all real numbers based on radian measure. Let's use this knowledge to investigate the question posed at the beginning of the previous section. Without loss of generality, first study the changes of points on the unit circle. The task now is…

3.2 The Organization of the Content on Trigonometric Function

Compared with PEP2004, PEP2019 uses the unit circle as a link to connect the knowledge of trigonometric function so as to give full play to the auxiliary role of the unit circle in learning trigonometric function. Figure 1 shows the relationship between the content of trigonometric function and the unit circle in PEP2019.

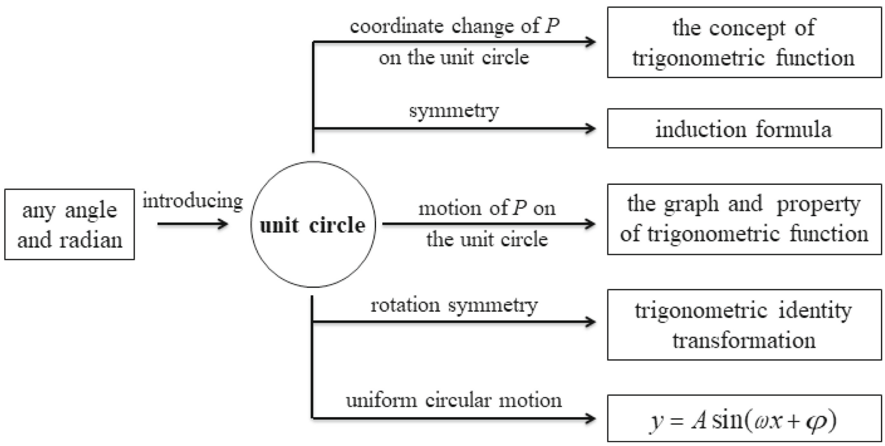

Fig. 1. The relationship between the unit circle and the content of trigonometric function in PEP2019

The following is the logical order in which the content of "the concept of trigonometric function" is presented in the PEP2004 and PEP2019:

> PEP2004: introduction and generalization of trigonometric function of acute angle — coordinate ratio of the point on the terminal side of the angle — define trigonometric function of any angle by the unit circle — calculate the trigonometric values for a given angle (worked example 1) — calculate the trigonometric values based on the coordinates of a point (worked example 2)
>
> PEP2019: explore the correspondence between the coordinates of P and the angle whose terminal side is OP — the abscissa or ordinate of P is a function of angle α — the definition of trigonometric function — explore the relationship between trigonometric function of any angle and that of acute angle — calculate the trigonometric values for a given angle (worked example 1) — analyse the definition of trigonometric functions based on coordinate ratio (worked example 2)

Different from PEP2004, PEP2019 simplifies the research object of trigonometric function from "periodic phenomena" to "the motion of a point on the unit circle", and leads students to learn with the task of "establishing a mathematical model to describe the position change of the point P on the unit circle". In the exploration task, PEP2019 guides students to explore whether the coordinates of the intersection of the terminal side of a given angle and the unit circle are uniquely determined, and then to consider whether it can be generalized to any angle.

Moreover, PEP2019 guides students to explore the definition of the coordinate ratio of trigonometric function through the following worked example 2, and thus think about the equivalence of the two definitions of trigonometric function.

> The worked example 2 in PEP2019: Suppose α is any angle, and the coordinates of any point P (not coincident with the origin O) on its terminal side are (x, y). The distance between P and O is r. Please prove: $\sin\alpha = y/r$, $\cos\alpha = x/r$, $\tan\alpha = y/x$.

Similarly analysing the logical order about the knowledge of "$y = A\sin(\omega x + \varphi)$ and its application" in the PEP2004 and PEP2019, it is found that PEP2019 helps students to establish the corresponding mathematical models by analysing the bucket motion, and then to use information technology to describe the influence of parameters on graphic transformation by analysing the actual meaning of parameters. Furthermore, PEP2019 designs a worked example about Ferris wheel, allowing students to experience the basic process of mathematical modeling again by establishing a functional model to solve the practical problem.

For the context used in the textbook, both two textbooks select natural phenomena (e.g., tide), physics (e.g., harmonic motion), physiology (e.g., blood sugar concentration) and history of mathematics (e.g., table of trigonometric function), etc. Compared with PEP2004, PEP2019 deletes the context of watch calibration, television transmission tower, etc., and meanwhile adds real-life phenomenon about periodic motion, such as pendulum swing, buoy floating up and down in the water, and string vibration, aiming to support students to deeply feel the close connection between trigonometric function and real life.

4 Discussion and Conclusion

PEP2019 provides support for students to master the content of trigonometric function as a whole and to make organic connections between different knowledge within the same mathematical topic. Compared with PEP2004, considering that trigonometric function is the mathematical model describing uniform circular motion, PEP2019 uses angle and radian as the preparatory knowledge describing circular motion, and then follows "any angle and radian – graph and basic properties – other properties – model application" as the logical order of the whole chapter compilation, so as to emphasize the systematicness of the content of trigonometric function. Also, PEP2019 provides opportunities for student's self-regulated learning and inquiry learning. On the one hand, PEP2019 sets up the introductory blocks that guide the direction of thinking in each section, so as to help students understand the learning task. On the other hand, PEP2019 provides a series of related inquiry tasks by the blocks of thinking and exploration, to help students experience the learning process of trigonometric function gradually. Besides, PEP2019 designs open-ended exercises for students with learning needs to choose, which is not available in PEP2004.

Compared with PEP2004, PEP2019 provides more instrumental support for students to understand trigonometric function intuitively, regarding the unit circle as an important carrier for learning trigonometric function. With the help of the geometric elements and properties of the unit circle, students could gain an intuitive perception of the knowledge of trigonometric function, such as the concepts of trigonometric function and induction formulas. PEP2019 presents the explorative questions visually by using the information technology, so as to give full play to the instrumental role of the unit circle. In addition, by carefully selecting the phenomenon of periodic motion variation in life as the context of knowledge, PEP2019 allows students to fully experience the connection between mathematics and real life, and deeply appreciate the applicability of trigonometric function as a model in solving practical problems.

Through the analysis and comparison of the content, it is revealed what support PEP2019 provides for students to effectively learn trigonometric function. As mentioned above, PEP2019 provides supportive resources for students to learn trigonometric function from the structural design of the chapter and sections and the presentation of specific knowledge content. It can be inferred that the compilation of PEP2019 is to abide by student-based educational principle, offering students much more chances to learn. As Xu (2014) proposed, the learning-centeredness education philosophy emphasizes the priority of students' thinking, understanding and ability improvement. PEP2019 is designed to help students gain basic experience of learning mathematics knowledge, so as to assist students in learning mathematically. Instead of directly showing students what the mathematical knowledge is, PEP2019 drives students mathematics learning with inquiry tasks or questions, thus to give full play to students' subjective initiative in learning.

In addition, China is undergoing a new round of competence-oriented mathematics curriculum reform currently, which puts forward corresponding requirements for the compilation of textbooks. The compilation of mathematics textbooks should underline scientific and psychological natures, that is, the content must be systematic and coherent (Zhang, 2016). The handling of content in PEP2019 shows consistency with the ideas of curriculum reform to a certain extent. For instance, PEP2019 supports students to learn trigonometric function according to the path of "clarifying the object – analysing the corresponding relationship – obtaining the definition – drawing the graph – studying the properties", which is in line with the basic experience of learning function proposed in MCS2017.

To sum up, the present study examines how mathematics textbooks support students' mathematics learning from a content analysis and comparison perspective. This may provide inspiration for the study of textbook content, that is, to analyse the compilation of textbooks from the perspective of helping students' mathematics learning. Also, it could be meaningful for students how to use textbooks effectively. It has to be admitted that, it may be more necessary to study how students use textbooks in practice, that is, to pay attention to students' use of textbooks, in order to better illustrate the support of textbooks on students' learning. The follow-up research needs to further analyse and summarize the types of support that mathematics textbooks provide for students' learning, and examine how students use these supports in the actual learning process.

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Math Textbooks Use: Explicit and Implicit Influence for Teachers and Students

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Abstract. As an important teaching resource in curriculum implementation, textbooks are one of the main mediums for teachers to teach and students to learn. Based on Rezat's tetrahedron model of the activity "textbook use" and the theme of this study, this paper selects "student—teacher—textbook", "teacher—textbook—mathematical knowledge" and "student—teacher—mathematical knowledge" as the three dimensions of the study. A questionnaire was compiled by conducting a survey and an interview of the junior high school teachers who use the mathematics textbooks of Beijing Normal University in more than a dozen provinces and cities in China. The paper analyzes the explicit influence of textbooks on teachers' teaching design and selection of teaching methods in the teaching process, the implicit influence of textbook writing concepts on enriching teachers' pedagogical knowledge. Based on the research results, relevant suggestions have been put forward from the aspects of textbook compilation concept, compilation content and teaching resources development which give full play to the explicit and implicit influence of textbooks in mathematics teaching.

1 Introduction

Textbooks are an important medium for teacher-student classroom interaction and it also serves as a bridge between teaching theory and practice. These years, the study on textbook use has been attached great importance by scholars, increasing years by years. However, mostly of the study focused on the simply frequency statistics on textbooks use. Many scholars ignored the analysis under the theoretical framework.

The main ideas of Activity Theory are the theory of mediation, internal and external forms of activity and their transformation, the activity structure and the triangular model (2007). Engstrom's elaboration of the third generation activity theory was applied by the German scholar Sebastian Rezat, analysing teachers and students' use of textbooks. Rezat also put forward a tetrahedron model, as shown in Fig. 1 (2006).

We can see that the textbook is a dominant but not sole element that determines "what" and "how" teachers teach'? What role does it play during teaching and learning procedure? How does it affect teachers' teaching behaviors and approaches? These are the questions that need to be considered.

In this paper, we have selected three of these dimensions to exploring teachers' use of textbooks and how the textbooks explicitly or implicitly influence on teaching and learning. Furthermore, we have developed a specific questionnaire after eliminating the "student-textbook-knowledge" dimension.

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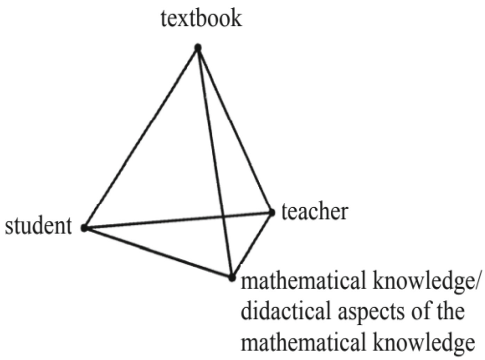

Fig. 1. 3-D-representation of the model of textbook use

2 Research Design

2.1 Research Framework

After reviewing the literature on teachers' use of textbooks at home and abroad, we chose to adopt the "textbook use activity model" proposed by Sebastian Rezat by useing three sides of it as three primary dimensions of this study, and we put forward the secondary dimensions, which are shown in Fig. 2. "Student-teacher-textbook" (STB) means that the student is the subject of the activity in this triangle while the textbook is the objection, and the teacher regulates the student's use of the textbook. Besides, "teacher-textbook-knowledge" (TBK) refers to the subsystem of the tetrahedral system model in which the teacher uses the textbook. In this system, teacher is the subject, while the mathematical knowledge and pedagogical knowledge is the object of the activity presented in the textbook. "Student-teacher-knowledge" (STK) is a traditional didactical triangle and the textbook is in an implicit position in this facet.

There are 27 questions in our questionnaire basing on Likert scale. And we used SPSS 26.0 software to process the survey data, obtaining a standardized Cronbach's alpha of 0.949 and a coefficient of 0.969 for the KMO (Kaiser-Meyer-Olkin) test for this questionnaire, which indicates that the questionnaire has good overall reliability and validity.

2.2 Research Methodology and Research Subjects

The method of non-random sampling was applied in this study.3766 junior high school mathematics teachers from nearly twenty provinces were selected for the study aiming at gaining a comprehensive and in-depth understanding of the teachers' use of mathematics textbooks of the Beijing Normal University's edition. The distribution of participants basically met the requirements of the sample survey, in which the gender survey results showed that the proportion of males was 38% and females was 62%. Thus, the results of this survey focused more on female teachers, in line with objectivity; Participants with more than 10 years teaching experience accounted for 70%, with bachelor's degree for 85%, in urban schools for 61%, who had participated in textbook training accounted for

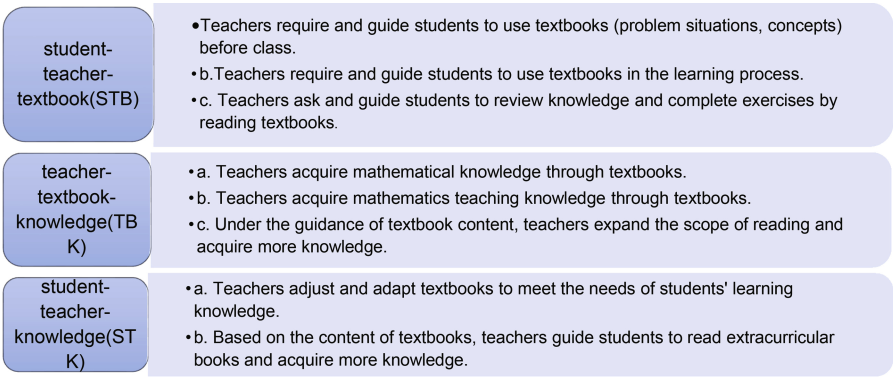

Fig. 2. Questionnaire dimension

61%. Last but not the least, the rest demensions of study subjects had a more balanced data distribution.

3 Results and Analysis

According to the data, the following characteristics can be summarized, regarding the overall distribution of teachers' use of textbooks.

- Teachers' use of textbooks was generally consistent and scored high.
- In the facet of TBK, the highest frequency of mean value between 4–5 means that most teachers highly agree that textbooks are supportive for them;
- In the facet of STB, the highest frequency of mean values is between 3–4. Showing that as a dominant presence, textbooks have a great impact on teachers' mediating behaviors.

3.1 Teachers Play an Obvious Mediating Role in Guiding Students' Use of Textbooks

Teachers act as mediators to guide students in using textbooks before and during class, with textbooks as a dominant presence. According to the data,it can be found that, the highest frequency of STB.b mean value between 4–5 shows that teachers are generally more actively in using textbooks' all column. They would ask students to do finger reading of concepts in class, and they would also guide students to complete the content of each section in the textbook comprehensively, ($M = 4.23$), indicating teachers' fidelity and satisfaction with textbooks.

The highest frequency of STB.a's mean value of 3–4 indicates that teachers would pay more attention to suggesting students using the textbook before class. And when guiding students with pre-reading, teachers would provide students with few questions to thinking before class ($M = 4.20$). Teachers are likely to pay more attention to students' ways of pre-reading practice, suggesting teachers' significant mediating role in guiding students in use of the textbooks.

In STB.c, the tendency of teachers asking and guiding students in review and completing exercises by reading textbooks after class was relatively average. 73% of teachers agreed that "asking students to review what they had learned in textbooks after class the same day" and asking students to finish the "Read" section on their own.

3.2 Textbooks Explicitly Help Teachers Grasp Knowledge

Teachers acquire mathematical theories and teaching knowledge, expand their own knowledge, and teach mathematics to students through textbooks in which textbooks as a dominant presence.

According to the data, it can be found that TBK.a had the highest frequency between 4–5, showing that teachers would acquire mathematical theories through textbooks, with which they agree that the "Have A Read" section could expand their knowledge ($M = 4.33$), indicating that extended reading is equally important for teachers and students.

Guided by the specific content of textbooks in TBK.c analysis, 43% of teachers would look up in journals, magazines as well as some online resources ($M = 4.18$), and some of them tend to get interdisciplinary knowledge ($M = 4.06$) in lesson planning.

The mean value of TBK.b is more evenly distributed, suggesting that textbooks are mid-level supportive to teaching. It is generally agreed that textbooks provide access to the mathematics teaching theories, that the questions in textbooks provide a dialogue environment for teachers and students ($M = 4.33$). And the problem situations in textbooks provide a context for teachers to guide students' survey, indicating that textbooks have some explicit influence on teachers' instructional design.

3.3 Textbooks Play an "Implicit Role" in Knowledge Imparting

Teachers act as mediators to guide students to acquire and expand their knowledge, with textbooks as implicit presence. According to the data, it can be found that the highest frequency of STK.a mean value was between 4–5, and the results showed that based on textbooks' content and students' characteristics teachers would generally add or subtract teaching activities and examples flexibly, and were also able to put forward questions for students before class ($M = 4.31$). 90% of teachers would use textbook-based Guidance case for teaching, showing textbooks' support for teachers to the course design to some extent. Therefore, even though textbooks are replaced by the Guidance case in class, they still play an important role in teaching as an implicit presence. In the STK.b, teachers' guidance in students' extensive reading based on textbooks showed that teachers would attach importance to guiding students in learning mathematical stories and other subjects before and after class ($M = 4.23$, $M = 4.17$), suggesting that textbooks play an implicit influence on teachers' mediating behaviors.

3.4 Differences in Textbooks Use of Each Teacher Group

Based on the results of the one-way ANOVA, the two dimensions STB and TBK differed in terms of teachers' teaching experience ($0.00, 0.01 < 0.05$) and professional titles

(0.001, 0.047 < 0,05). For example, the results of comparisons of different dimensions showed that teachers with about 5–20 years teaching experience were better at guiding students in using textbooks than teachers with 1–5 years teaching experience, and teachers with first-grade titles are better than second-grade and other grades. This result indicates that novice teachers are not sufficient in mastery of textbooks and are more likely to guide students to use textbooks only in class. Therefore, novice teachers are less proficient in textbook use than experienced teachers. However, teachers with more than 20 years experience are less able to expand knowledge or mastering knowledge based on textbooks than teachers with less than 20 years of teaching experience. Similarly, teachers with senior titles or above are less efficient than teachers with lower titles. Perhaps for some professors, textbook-use is no longer a difficulty, their experience can enable them to teach normally, and their titles are adequately in proving their teaching ability. So, novice and experienced teachers are more willing to expand their knowledge, improve their teaching theoretical knowledge referring to textbooks.

In addition, according to the results of the independent sample t-test, there is a difference between the three dimensions on whether or not attending textbook use training (0.000 < 0.05): teachers who have attended textbook use training and those who have not are different in terms of guiding students in using textbooks, expanding their own knowledge and imparting knowledge. What's more, according to the comparison of means, it is found that teachers who have attended textbook training can use textbooks to a higher extent than those who have not attended. The result suggests that textbook training has significance in guiding teachers with textbooks use.

4 Conclusions and Advices

4.1 Strengthen the Explicit Role of Textbooks and Provide Support for Teachers to Acquire Knowledge

Textbooks are used by both teachers and students. Therefore, teachers is not only a mediation to guide students in using textbooks, but also learners to gaining knowledge from textbooks themselves**.** Besides, the accuracy and thoroughness of textbooks help teachers accurately grasp knowledge and build up own logical structure between knowledge, and gradually shift from "follow textbooks" to "master textbooks". Considering the lack of teaching resources in some areas, textbooks are the only resources for teachers to prepare and teach, and the columns in textbooks are the foundation for teachers to teach. Thus, it is necessary to continuously enrich the content and improve the knowledge structure of textbooks, so that textbooks can provide adequate support for teachers.

4.2 Focus on the Implicit Role of Textbooks and Provide Support for Teachers' Creative Use of Textbooks

Some teachers choose to use Guidance case for teaching showing their overall awareness of the implicit role of textbooks is good. But it is still less effective than textbooks working as an explicit role. Therefore, textbooks should be presented with good problem situations and sections, and its content should be set in a standardized and scientific way, keeping open-minded in problem setting appropriately. So that textbooks can be accessible and achieving extensions by teachers, who can adapt textbook content according to students' levels and characteristics by choosing other matching teaching materials. In this way, the implicit role of textbooks can be applied and can provide teachers environment to using textbooks creatively.

4.3 Strengthen the Textbook Training for Teachers to Further Develop the Implicit Role of Textbooks

Textbook training for teachers is an important way to enhance teachers' teaching ability. For the training can not only strengthen teachers' recognition of textbooks, but also make teachers deeply understand the logic of textbooks, so that teachers would achieve improvement of their teaching design. However, any inaccurate and poor understanding of textbooks will only lead to bias when guiding students' use of textbooks. In summary, textbook training should be conducted to enable teachers to grasp the general direction of textbook writing, so that they can sort out textbook content based on their own experience and students' characteristics, and better utilize the explicit and implicit roles of textbooks.

4.4 Teachers Should Guide Students in Using Textbooks Comprehensively, Focus on Extensive Reading, and Make Reasonable Utilization of Online Resources

Teachers should make good use of the textbook sections, but also enhance the output of mathematical culture and history guiding students with "Have A Read" and other related sections, expanding reading materials of ancient and modern mathematical stories, interesting mathematics, etc. Teachers ought to give students sufficient independent learning opportunities and time to stimulate their interests development and confidence building in learning mathematics. At the same time, now that we are in an on the informationization times, it is high time that students should be guided to make good use of online resources to searching about mathematical culture and history, so that they can enlarge their knowledge scopes and develop a critical thinking finally.

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Research on the Consistency Between Contents of Probability and Statistics of Compulsory Mathematics Textbook for Senior High School of Hunan Version and Curriculum Standards

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Abstract. This paper uses the methods of content analysis, data statistics and comparative analysis, a "Survey of the Enacted Curriculum Model" is adopted to conduct consistency analysis from two dimensions: content subject and level (cognitive and process). The results show that body content of textbook and the Curriculum Standard have the strongest consistency in Comprehensive Level; textbook exercises and the Curriculum Standard have the strongest consistency in Mastery Level; the consistency between the body content of textbooks and the curriculum standard is basically the same at the level of Exploration and Experience.

1 Introduction

Whether the content of the textbook matches the curriculum standard brings huge effect on the implementation of curriculum standards, usage of textbooks and a smooth progress of the curriculum reform. How deep they match to each other also reflects the quality of textbooks. Therefore, it is necessary to bring on the research of the consistency between textbooks and curriculum standards.

2 Object of Study

This paper takes the Chapter Probability and Statistics in Compulsory Mathematics Textbook for Ordinary Senior High School published by Hunan Education Press in 2019 (hereinafter referred to as the textbook) and the *The Mathematics Curriculum Standard For Ordinary senior high school (2017 Edition)* (hereinafter referred to as the curriculum standard) as the research objects. The textbook is divided into two parts: body content and exercises.

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3 Method of Study

A "Survey of the Enacted Curriculum Model" is adopted to conduct consistency analysis from two dimensions: content subject and level (cognitive and process). The level of the body content can be divided into cognitive level and process level while the exercises part in text book is only cognitive level. The consistency between body content and the curriculum standard is analyzed from two aspects: content subject and cognitive level; content subject and process level. The consistency between exercises and the curriculum standard is analyzed from the aspects of content subject and cognitive level.

4 Process of Study

Through the overall analysis of the research object, the classification principles of the analysis dimension of this paper are formed.

4.1 Theme Divisions

By Integrating the two secondary subjects included in Chapter Probability and the four secondary subjects included in Chapter Statistics part of the Curriculum Standard, there are six content subjects: random events and probability, the independence of random events, the basic way to obtain data and related concepts, sampling, statistical charts, and estimating the overall with samples using A, B, C, D, E and F to represent these six subjects separately. The content subjects of the textbook text and exercises are also in accordance with the Curriculum Standard.

4.2 Level Divisions

The levels applied in this paper are divided into Cognitive level and Process level. According to the classification in the Ordinary Senior High School Mathematics Curriculum Standard (Experiment), the cognitive level of this paper is divided into three parts: understanding, comprehension and mastery; process level is divided into two parts: experience and exploration. The meanings and action verbs of each level are shown as follow.

The Level of Understanding: Could know or illustrate the characteristics of the object from specific examples according to the characteristics of the object, and identify or illustrate the object in special situations, could directly and simply express a shallow content without deep study. Action verbs could contain: know, identify, understand, etc.

The Level of the Comprehensive: Could describe the characteristics and origin of the object, explaining the difference and connection between certain object and related other objects, and could give further explanations by using examples. Action verbs could contain: representation, interpretation, calculation, etc.

The Level of the Mastery: Could apply objects to new situations based on comprehension, and could select or create appropriate solutions to problems. Action verbs could contain: usage, proof, discussion, etc.

The level of the Experience: Could know or validate different characteristics of the object forwardly by attending specific mathematical activities and get experience and perceptual knowledge. Action verbs could contain: experience, with the help of, participation, etc.

The level of the Exploration: Could participate in specific mathematical activities independently or cooperate with others, could understand or pose questions and seek solutions to them, discover the characteristics of objects and recognize differences and connections between specific objects and related other objects, obtain certain rational knowledge. Action verbs could contain: design, analysis, exploration, etc.

This paper sums up the different distribution of knowledge points of different subjects in different levels of Cognitive and Process Live, classifies different knowledge points with Cognitive and Process Level in text book and Curriculum Standard, the knowledge points that contains both Cognitive and Process Level would be duplicated summed up. For example, the knowledge point "The addition formula of Probability and simple extension", in Cognitive Level it could be divided as Mastery Level and Exploration Level in Process Level. Therefore, this knowledge point could be classified into both Cognitive and Process Level. The statistical results of curriculum standards and textbook are shown in Table 1 and Table 2.

For textbook exercises, the total number of exercises should be conformed at first. If a question contains several branch questions, then it should be counted according to the number of branch questions. Secondly, one should decide which cognitive level does each question lies in, exercises with obvious action verbs of cognitive level could be counted directly. For example, action verb "prove" in the exercise is judged as Mastery Level. If there are no obvious action verbs in exercises, it is necessary to estimate the specific requirements of the question, reassure the action verbs, then classify them into different levels. The statistical results of textbook exercises are shown in Table 3.

Table 1. Statistical Table of Curriculum Standards Data

Content subjects	Cognitive level			Subtotal	Process level		Subtotal
	Understanding	Comprehensive	Mastery		Experience	Exploration	
A	0.0652 (3)	0.1957 (9)	0.0217 (1)	0.2826 (13)	0.3125 (5)	0.0000 (0)	0.3125 (5)
B	0.0217 (1)	0.0218 (1)	0.0000 (0)	0.0435 (2)	0.1250 (2)	0.0000 (0)	0.1250 (2)
C	0.1087 (5)	0.0000 (0)	0.0000 (0)	0.1087 (5)	0.0000 (0)	0.0000 (0)	0.0000 (0)
D	0.1087 (5)	0.0652 (3)	0.1087 (5)	0.2826 (13)	0.1875 (3)	0.0625 (1)	0.2500 (4)
E	0.0000 (0)	0.0218 (1)	0.0217 (1)	0.0435 (2)	0.0625 (1)	0.0000 (0)	0.0625 (1)
F	0.0000 (0)	0.2391 (11)	0.0000 (0)	0.2391 (11)	0.2500 (4)	0.0000 (0)	0.2500 (4)
Subtotal	0.3043 (14)	0.5435 (25)	0.1522 (7)	1.0000 (46)	0.9375 (15)	0.0625 (1)	1.0000 (16)

Table 2. Statistical table of text data of textbooks

Content subjects	Cognitive level			Subtotal	Process level		Subtotal
	Understanding	Comprehensive	Mastery		Experience	Exploration	
A	0.0822 (6)	0.2329 (17)	0.0959 (7)	0.4110 (30)	0.1111 (1)	0.1111 (1)	0.2222 (2)
B	0.0000 (0)	0.0274 (2)	0.0136 (1)	0.0410 (3)	0.2222 (2)	0.0000 (0)	0.2222 (2)
C	0.0685 (5)	0.0137 (1)	0.0000 (0)	0.0822 (6)	0.0000 (0)	0.0000 (0)	0.0000 (0)
D	0.0411 (3)	0.0411 (3)	0.0274 (2)	0.1096 (8)	0.2222 (2)	0.0000 (0)	0.2222 (2)
E	0.0000 (0)	0.0274 (2)	0.0274 (2)	0.0548 (4)	0.1111 (1)	0.0000 (0)	0.1111 (1)
F	0.0274 (2)	0.2192 (16)	0.0548 (4)	0.3014 (22)	0.2223 (2)	0.0000 (0)	0.2223 (2)
Subtotal	0.2192 (16)	0.5616 (41)	0.2192 (16)	1.0000 (73)	0.8889 (8)	0.1111 (1)	1.0000 (9)

Table 3. Statistical table of textbook exercises

Content subjects	Cognitive level			Subtotal	Cognitive level			Subtotal
	Understanding	Comprehensive	Mastery		Understanding	Comprehension	Mastery	
A	12	72	3	87	0.0628	0.3770	0.0157	0.4555
B	0	19	6	25	0.0000	0.0995	0.0314	0.1309
C	9	0	0	9	0.0471	0.0000	0.0000	0.0471
D	2	5	7	14	0.0105	0.0262	0.0366	0.0733
E	0	15	1	16	0.0000	0.0785	0.0052	0.0838
F	0	40	0	40	0.0000	0.2094	0.0000	0.2094
Subtotal	23	151	17	191	0.1204	0.7906	0.0890	1.0000

5 Results and Analysis

The Porter Consistency Coefficient Calculation Formula using "Survey of the Enacted Curriculum Model" is shown as below:

$$p = 1 - \frac{\sum_{i=1}^{n} |X_i - y_i|}{2}$$

(Where, n represents the number of cells in the two-dimensional matrix, Xi represents the ratio value of the i th cell in the curriculum standard analysis matrix; Yi represents the ratio value of the i th cell in the research content matrix.)

The consistency coefficients of body content of textbooks and curriculum standards can be obtained using the above formula: the consistency coefficient of text and curriculum standards at the Cognitive Level is 0.7452, while the consistency coefficient at the

Process Level is 0.7084; the coefficient of consistency between textbook exercises and the curriculum standard at the Cognitive Level is 0.6529.

In this study, only part of the main contents of one edition of textbooks are selected, and the number of samples has a large deviation influence on the critical value. Therefore, this paper does not use the critical value calculation method of the American scholar Gavin, but uses the method of Chinese scholars to define the scope standard, that is, the consistency coefficient is equal to zero, indicating that the two are completely inconsistent; between zero and zero point three, indicates that they are weakly consistent; between zero point three and zero point eight, indicating that they are consistent to a certain extent; between zero point eight and one, indicating strong consistency; the consistency coefficient is equal to one, indicating that they are completely consistent.

By comparing the consistency coefficient of calculation with the critical value, it is found that the consistency coefficient between the body content of textbooks and the curriculum standard, the consistency coefficient between the textbook exercises and the Curriculum Standard are between zero point three and zero point eight, indicating that the degree of consistency has reached a certain degree of consistency.

Further analysis shows that body content of textbook and the Curriculum Standard have the strongest consistency in Comprehensive Level; textbook exercises and the Curriculum Standard have the strongest consistency in Mastery Level; the consistency between the body content of textbooks and the curriculum standard is basically the same at the level of Exploration and Experience. Hunan Edition textbook probability and statistical content as a whole consistent with the curriculum standard; the probability and statistical content of textbooks are consistent with the requirements of the curriculum standard on the whole, but there are differences at different levels.

Based on the research results, the following suggestions are put forward for the textbook writers: adjust the exercises of some topics evenly, and appropriately allocate the exercises of each cognitive level.

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A Comparative Study on the Content of "Number and Algebra" in Chinese and British Junior Middle School Mathematics Textbooks

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Abstract. The content of "Number and Algebra" in Chinese and British junior middle school mathematics textbooks has been studied in terms of overall arrangement and content difficulty based on the framework of the study, and the following suggestions for the compilation of mathematics textbooks in China through the comparative study have been given: focus on the spiral of knowledge and show the system of mathematics; set up a variety of problem situations and improve the quality of illustration and narration; optimize the background of examples and exercises, and pay attention to quantity and stratification; emphasize comprehension and expression, and give full play to the function of "reading, doing and learning".

1 Question Raising

To date, elementary mathematics education in China is generally recognized internationally. A large number of textbooks have also been translated into English for use abroad. The textbooks of the UK, as a country with a strong international presence and strength in mathematics education, should be valued. It is also easier to obtain typical results by selecting one of the four areas of mathematics curriculum content, "Number and Algebra", for comparative study. Therefore, a comparative study of the "Number and Algebra" content of Chinese and British junior middle school mathematics textbooks is needed to gain valuable experience in writing and to gradually enhance the international influence of mathematics education in China.

2 Research Design

2.1 Selection of Mathematics Textbook Version

The Chinese version is selected from the most widely used edition of Mathematics for Junior Middle School, Grades 7–9, published by People's Education Press (2012–2013, abbreviated as RJ). The British version is selected from Cambridge Checkpoint Mathematics, Grades 7–9, published by Cambridge University Press (2012–2013, abbreviated as CCM).

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2.2 Research Questions

The research questions are determined as follows:

Question 1: What are the characteristics of the overall arrangement of “Number and Algebra” in RJ and CCM junior middle school mathematics textbooks?
Question 2: What are the characteristics of the content difficulty of “Number and Algebra” in RJ and CCM junior middle school mathematics textbooks?
Question 3: What are the implications for the compilation of junior middle school mathematics textbooks and the content of “Number and Algebra” in RJ and CCM?

2.3 Research Methods and Framework

The theoretical basis is “Webb Model” and “Cognitive Load Theory” (Sweller 1998). The four main research methods are as follows: literature research method, comparative research method, case study method and statistical analysis method. The difficulty comparison model of mathematics textbook improved by Cao Yiming has been adopted:

$$N = a \times G + b \times S + c \times L + d \times X$$

This model is used to study the overall difficulty of “Number and Algebra” in Chinese and English junior middle school mathematics textbooks. For the weight of each dimension involved in the difficulty model, the weight value of the junior middle school mathematics textbook difficulty model obtained by Wu Libao after using the expert evaluation method is used here, where N represents the textbook difficulty, $a = 0.28$, $b = 0.30$, $c = 0.20$, $d = 0.22$. The meanings of each letter in the formula are shown in the following Table 1:

Table 1. Explanation of the meaning of the letters in the difficulty model

The letter	G	S	L	X
Meaning	Content breadth	Content depth	The difficulty of examples	The difficulty of exercises

A comparative research framework is established as shown in the diagram below, and the content of the framework is analyzed on a case-by-case basis. The text-specific study compares the overall layout of the textbook; the corresponding content difficulty formula is used to compare the content difficulty of the textbook.

3 Research Results

3.1 System and Design Structure

In terms of stylistic columns, RJ has more columns than CCM. In terms of the introduction of concepts, RJ draws mainly on practical problems. CCM is mostly in the form of introductory stories. In terms of concept presentation, CCM gradually deepens the

Table 2. Comparison framework index of "Number and Algebra" in junior middle school mathematics textbook

Comparison of dimensions	Comparative indicators	Detailed indicators	Definition of Indicators
Layout structure	System structure	Style column	Chapter organization, including pre-chapter columns, section columns and end-of-chapter summaries
		Knowledge Representation	The introduction way, concept presentation way and examples and exercises setting
	Design structure	Narrator design	Number and type of narration
		Illustration design	Number and type of illustration
	Content structure	Content construction	The specific distribution of chapter and section content and the proportion of content in each field
		Content distribution	Number of statistical knowledge points
Content difficulty	The content difficulty	Content breadth	The extensive degree of the scope and field and the number of knowledge points
		The depth of content	The depth of thinking and the presentation of concepts involved in knowledge points
	The difficulty of examples and exercises	The requirement of examples and exercises	knowledge, comprehension, application and enquiry

(*continued*)

content, focusing on students' understanding of the concepts; CCM mainly condenses the concepts and uses concise language to explain the process of applying knowledge, emphasizing the value of knowledge and promoting students' active understanding. In terms of narration design, RJ has slightly more narration than CCM, and the content is more diverse. In terms of illustration design, RJ focuses more on mathematical illustrations. CCM places more emphasis on illustrations related to the history of mathematics, science and technology.

Table 2. (*continued*)

Comparison of dimensions	Comparative indicators	Detailed indicators	Definition of Indicators
		The background of the examples and exercises	No background, personal life background, public life background and scientific background
		Quantity of knowledge	1 knowledge point, 2 knowledge points, 3 knowledge points and 4 and more knowledge points

3.2 Content Structure

Both the Chinese and English editions attach great importance to the setting of the content of "Number and Algebra". The proportion of the content of "Number and Algebra" of RJ in Grade 7 is significantly higher than that in Grade 8 and Grade 9, and the content is systematic and coherent. The content of "Number and Algebra" in CCM is evenly distributed among the three grades, with the corresponding content being repeated at different grades and the knowledge required being expanded and deepened, with the "spiral" arrangement being prominent.

3.3 Content Difficulty

The breadth of CCM "Number and Algebra" is higher than that of RJ, but the depth of content is significantly lower than that of RJ.

3.4 The Difficulty of Examples and Exercises

The number of examples and exercises in CCM "Number and Algebra" is higher than these in RJ. In terms of the types of examples, both CCM and CCM "Number and Algebra" do not contain multiple-choice questions and proof questions. RJ focuses on developing students' arithmetic skills, while CCM focuses more on presenting guided problems. In terms of the types of exercises, RJ has more judgement and calculation questions, while CCM has a slightly higher number of fill-in-the-blank and answer questions. In terms of the difficulty of the examples and exercises, RJ is generally higher than CCM. It is advisable to increase the amount of exercises and enhance the openness of exercise conclusions in Chinese mathematics textbooks (Cao and Wu 2021).

3.5 Overall Difficulty

CCM is slightly higher than RJ, mainly because CCM has set a large proportion of mathematics content which is originally in Chinese primary school at the junior secondary level, thus increasing the overall difficulty.

4 Suggestions and Reflections

4.1 Pay Attention to the Spiral Arrangement of Knowledge and Give Play to the Advantages of Mathematical System

RJ focuses on the systematic and coherent nature of knowledge in “Number and Algebra”, but needs to emphasise key elements and modularity, for example, in organizing units on functions, equations and inequalities, with attention to concentration and avoiding fragmentation. There is also an emphasis on “spiral”, but this is a far cry from CCM. However, there are some shortcomings in this layout. There is often a time lag in the implementation of knowledge, which can lead to forgetfulness and thus affecting the accuracy and systematicity of knowledge acquisition. Therefore, a balance should be struck between extending the knowledge structure and the possibility of infusing the reading material or exercises that follow the chapter or sub-section with the knowledge that will be studied in the future.

At this stage of our education, the “linear” structure should not be abandoned completely, but should be combined with the “spiral” structure, and gradually enlarge the proportion of the spiral structure.

4.2 Create Multiple Problem Situations and Improve the Quality of Illustration and Narration

There is a need to be more diverse in introducing concepts and topic contexts, such as drawing on the economic, social science and life ecology contexts covered by CCM. It is also important to continue to maintain the cultural characteristics of our country by including topics such as food, paper-cutting and so on, so that students can appreciate excellent culture and the broad value of mathematical applications. Knowledge of current news can also be added, such as the various ice and snow sports in the context of the 2022 Beijing Winter Olympics. In terms of teaching design, an introduction to the history and development of “Number and Algebra” could be enhanced, with open-ended questions to encourage students to study more deeply and continuously.

In terms of narration, Chinese textbooks can increase the number of narrations and make the content more interesting. Incorporate life or the humanities to match students’ real-life and cognitive levels. The proportion of annotations should also be increased to take into account the nature of “Number and Algebra” knowledge and to stimulate students’ desire to explore. In addition, a more lively presentation of the narration will help students to notice the content of the narration and gain enlightenment.

4.3 Optimize the Background of Examples and Exercises and Consider Both Quantity and Stratification Gradient

A rich background of examples and exercises should be created. The number of topics involving “public life context” and “science context” is relatively low and needs to be increased. In addition to adding interdisciplinary content, it is advisable to enhance the depth of content and to integrate mathematical ideas and methods.

RJ is leading the way in terms of the number of knowledge points contained in the examples and exercises. A topic containing many knowledge points, while comprehensive, can be demotivating and lead to students being intimidated by practice questions and less effective testing. More attention could be paid to examining individual points of knowledge and deepening understanding. On the other hand, RJ could increase the number of exercises, but reduce their difficulty by trying to add signs such as "problem solved" before the question number. In addition, students need to have more choices so that they can choose topics they are interested in for proper practice.

4.4 Pay Attention to Students' Understanding and Expression and Play the Function of "Reading, Doing and Learning"

Our latest curriculum standards emphasise that students "express the world in mathematical terms" and encourage "mathematical expression" (Liu 2011), which helps teachers understand students' real thinking processes and improve their teaching. Analysis reveals that some of CCM exercises require an explanation of the reasons for using a particular mathematical method, which can help students reflect on the problem-solving process and understand the nature of knowledge. In addition, examples of typical student errors are set, which in turn lead to finding the reasons for the errors and correcting them, which places a higher demand on students' ability to understand and express mathematics.

In contrast, our textbooks set high level learning tasks, which also make reading and learning more difficult. Therefore, the difficulty should be reduced and practiced at different levels to give students a greater sense of achievement. In addition, there are fewer learning activities such as project activities, experimental activities and writing activities. For example, mathematical writing activities, such as a reflective journal, which encourage the development of various aspects of mathematical expression and writing, can also develop students' aesthetic sensibilities (Sinclair 2008). In addition to satisfying the "reading" function, there is also a need to focus on "manipulative". For example, in the case of "properties of equations", students can build a balance from objects around them and try out the steps according to the task, "plan and operate" on their own, and discover and understand the properties of equations. In the process of "planning and manipulating", students can discover and understand the properties of equations. In addition, digital mathematics textbooks evoke interactive experience and enhance student engagement in the classroom (Lew 2020).

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An Empirical Study on Fifth Grade Students Using Statistical Model to Solve Practical Problems

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Abstract. Statistical model is one of the mathematical models. Knowing which statistical models students will use in solving problems and their level is helpful to cultivate students' statistical modelling capability. This study is based on the New Century Mathematics (NCM), one of the mainstream textbooks in China. Taking the unit "Fallen Leaves on Campus" in the field of synthesis and practical teaching in Grade 5 as an example, by means of the method of personal interview, it is found that students will utilize descriptive models and inferential models when solving the tasks of "calculating the area of a fallen leaf" and "counting the number of leaves of a tree". And it is also found that students have shortage in the awareness of selecting representative data and difficulties in solving the problem of "counting the number of leaves of a tree". Therefore, this paper put forward two suggestions for the compilation of the content of "Synthesis and Practice" in mathematics teaching materials: first, more activities and opportunities for students to choose appropriate data can be added in the compilation of teaching materials; second, it is required to consider whether students can really complete the tasks in the textbook.

1 Introduction

Mathematical modelling, as a subject in different fields of universities or in classrooms of primary and secondary schools, as a way to connect mathematics with reality, has attracted the attention of school education. Studies have shown that using mathematical models in teaching can promote the success of teachers and students (Biembengut and Hein 2013). When discussing models, teachers no longer evaluate students' models, but refine and modify their models. Students, as evaluators of models, take the initiative to reflect on them (Doerr 2007). When students face problems in real situations, they are more likely to analyze data and build models to solve problems. Studies have shown that it is more complicated to train students to reason about data than expected (Garfield and Ben-Zvi 2007). China's "Mathematics Curriculum Standards for Compulsory Education (2022)" also mentions that students should pay attention to developing their model consciousness in mathematics curriculum, which is the basic way for students to experience and understand the connection between mathematics and the outside world, and helps to improve their application consciousness (Ministry of Education 2022).

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2 Literature review

2.1 Statistical Models and Modelling

With the development of technology, statistical modelling and statistical models are gradually studied in mathematics education. On the one hand, researchers know that primary school students will use descriptive and predictive static models and dynamic models to solve problems in informal statistical inference (Braham and Ben-Zvi 2017). It is found that teachers think that the statistical model can support decision-making and reasoning, and copy the data collection process (Justice et al. 2018). Studies have also shown that the core statistical feature of informal statistical modelling process is the simultaneous existence of two independent, sometimes similar and sometimes very different data and conjecture models (Dvir and Ben-Zvi 2018). On the other hand, many researches study the students' modelling process and develop a framework, such as developing a statistical modelling framework through students' activities to clarify how students reason (Pfannkuch et al. 2018).

2.2 The Role of Statistical Modelling

During recent years, researchers have done a lot of research on the effect of statistical modelling on students, mainly on students' statistical reasoning. Studies have shown that modelling activities can support the development of aggregation reasoning, which is the core component of reasoning by using data and opportunities (Aridor and Ben-Zvi 2018). Other studies have shown that statistical modelling will develop young students' statistical reasoning about distribution, center and variation in the context of real problems (Fielding-Wells 2018). There are also some researchers who study the effects of one aspect of statistical modelling on students. For example, the variability of statistical modelling can promote the informal reasoning of children in the upper grades of primary schools (Lehrer and English 2018).

2.3 Summary

Mathematics textbook is an important carrier of students' learning and teachers' teaching, especially the content of "synthesis and practice" is the process that students really solve practical problems and build models. At present, there are few studies on which statistical models and levels students will use to solve problems in textbooks, so we want to study this aspect to provide more reference for the compilation of mathematics textbooks.

3 Research Questions

At present, there is little concern about which statistical models students will use to solve practical problems in mathematics textbooks and at what level. Therefore, combined with the NCM, the two tasks in the "Campus Fallen Leaves" unit of "Synthesis and Practice" are "Finding the area of a fallen leaf" and "Counting the number of leaves in a tree", and the following research questions are put forward: (1) What statistical model will students choose? (2) What level students are at when using statistical models?

4 Methods

4.1 Theoretical Framework

If the model has the purpose of statistical description, explanation or prediction, it can be regarded as a statistical model (Garfield et al. 2008). Researchers know that primary school students will use descriptive and predictive static models and dynamic models to solve problems in informal statistical inference (Braham 2017). The above research may tend to divide statistical models into descriptive models, explanatory models and predictive models. Combined with the description of the related curriculum objectives of "statistics" in China's compulsory education mathematics curriculum standards, although the statistical model is not mentioned directly, the related knowledge of statistics can be found that it is mainly described by statistics, explained by various charts, and inferred by sample data (Ministry of Education 2022). In this study, statistical models are divided into three categories: descriptive model, explanatory model and inferential model. The specific performance and level are as follows.

The concrete expression of descriptive model is to select appropriate data, analyze the data correctly, and select appropriate statistics to describe according to the actual problems to be solved. The specific levels are as follows: Level 1 will not choose the appropriate data; Level 2 will choose the appropriate data without realizing that it needs to be selected first, and will not choose the appropriate statistical description; Level 3 will choose the appropriate data and use the appropriate statistical description.

The concrete expression of the explanatory model is to analyze the collected data and choose the appropriate statistical chart to explain according to the actual problems to be solved. The specific levels are as follows: Level 1 will not analyze the data; Level 2 will analyze the data, but will not choose the appropriate statistical chart to explain it; Level 3 will analyze the data and explain it with the appropriate statistical chart.

The concrete expression of the inferential model is to select a suitable sample, calculate a certain amount of sample data by mathematical methods, and then infer the overall characteristics according to the sample data. The specific levels are as follows: Level 1 will not choose the appropriate sample, Level 2 will choose the appropriate sample and calculation, but will not infer, Level 3 will choose the appropriate sample and calculation, and can infer the overall characteristics.

4.2 Participants

Six fifth-grade students are divided into three groups. The first group is excellent students in mathematics, the second group is moderate students in mathematics, and the third group is weak students in mathematics. Each group consists of a boy and a girl.

4.3 Data Collection and Analysis

Record the interview process with a recording pen, and collect the works of each participant. Then, combined with interviews, the students' works are analyzed according to the theoretical framework.

5 Results

The analysis of the data collected in the first task shows that participants will choose descriptive models, including those students who have difficulty in solving problems under the guidance of teachers. In the second task, participants will choose inferential models, including students who have difficulty solving problems after being instructed by teachers. Although students choose the same model to solve the same practical problem, the participants are at different levels. Below we provide more explanations and evidence to support these findings.

5.1 Activity 1: Calculate the Area of a Fallen Leaf

1 case: Through the interview, student A knew how to find the area of leaves, but didn't know which leaf to choose, only said to choose the tender one. In addition, students didn't have the consciousness of choosing leaves at first, and teachers also had guidance. This student can't choose the right leaves and is in Level 1.

2 case: Through the interview, student B knew to choose four leaves of different sizes, but she didn't know why she chose these four leaves. She just said she couldn't choose them casually. In addition, the student didn't have the consciousness of choosing leaves at first, and teachers also had guidance. The student will choose the appropriate leaves, but she will not use the appropriate statistics to describe them, so it is level 2.

3 case: Through the interview, student C first selected several buttonwood leaves of different sizes, calculated their areas respectively, and then calculated the average of these leaf areas as the area of a leaf. This student will choose appropriate leaves and describe them with the statistic of average, so he is at level 3.

5.2 Activity 2: Count the Number of Leaves of a Tree

Students should choose the appropriate number of samples and calculate the number of leaves, and then infer the total number of leaves according to the number of samples.

1 case: With the teacher's support twice, student B still doesn't know how to choose a certain number of leaves to infer the total number of leaves. The student can't choose the correct sample and is at level 1.

2 case: Through the interview, student A intends to tie up 10 branches with a rope as a bundle, then count the number of leaves in a bundle, and then count how many bundles a tree has. But when actually counting, students finally infer the total number of leaves with the help of teachers. This student will choose a suitable sample for calculation, but can't deduce the total amount, which is at level 2.

3 case: Through interviews, student C took the number of leaves on a twig as a sample, then counted out several medium and thick twigs, and finally deduced the total number of leaves. This student will choose the appropriate sample to calculate, and can deduce the total number of leaves, so it is at level 3.

6 Discussion and Conclusions

The focus of this study is to find out which statistical models fifth graders will choose when solving different problems and at what level. The results of this study show that through such tasks, students' statistical models for solving different task choices are different; Even if it is the same task, different students use statistical models at different levels.

First of all, this study shows that with the partial support of teachers, students in Grade 5 can solve the problem of "how big is the area of a fallen leaf", but they lack consciousness or have some difficulties in choosing suitable leaves. Through the interview, most students didn't have the consciousness of choosing the representative leaves first when seeking the area of a fallen leaf. But when faced with real problems, there are all kinds of data, so we need to choose the appropriate data first. Therefore, this is something that our students lack, and it is also something that needs to be considered when compiling mathematics textbooks. The "Falling Leaves on Campus" unit fully considered this point, and revised the contents of the textbook, so that students can fully appreciate the randomness of data and increase the activities of selecting representative data.

Secondly, this study also shows that most students have some difficulties in the task of "counting the number of leaves in a tree". Through interviews, most students, with the support of teachers' study, have strategies of how to infer, but actually they will feel that there are many branches when counting, which makes it difficult to count them, and the error is large. It shows that the above students can solve this problem in practice, that is, roughly estimate how many leaves there are. Studies have shown that estimation can promote the generation of mathematical models (Albarracín 2021). But it is difficult for students to solve such a problem, and there is a big error. After discussion, we adjusted this task.

Finally, based on the above two findings, the design and compilation of primary school mathematics textbooks, especially the content of "synthesis and practice", should consider two aspects. On the one hand, more activities and opportunities for students to choose appropriate data can be added in textbook compilation. On the other hand, the compilation of mathematics textbooks should consider whether students can really solve the tasks in the textbooks.

To sum up, this study relies on the content of "Synthesis and Practice" in the NCM, and understands that students will choose different statistical models when solving different practical problems, and the level of students is different. It is found that students' awareness of choosing representatives is relatively lacking when solving the task of "How big is the area of a leaf", and it is difficult to estimate the total number by inference when solving the task of "How many leaves are in a tree". Based on these two points, this paper puts forward two suggestions on the compilation of the content of "Synthesis and Practice" in primary school mathematics textbooks.

7 Future Work

We hope that through further research, we can find out what kind of learning support can promote the cultivation of students' statistical modelling ability.

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A Comparative Analysis of the Linear Equation Unit in Secondary Mathematics Textbooks in Chinese-English Bilingual Schools

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Abstract. Chinese–English bilingual programs in China advocate using both English and Chinese in instruction. Mathematics teachers in these schools need to use English as the medium of instruction to achieve the learning objectives of the school mathematics curriculum and also meet the English proficiency goals. Mathematics textbooks play a crucial role in teaching and learning within such a learning environment. This study presents a comparative study aiming to examine how linear equation unit is arranged in secondary school mathematics textbooks (i.e., IB, IGCSE, and Chinese textbooks) in Chinese-English bilingual schools. It explores the similarities and differences in these three series of textbooks. Four aspects of linear equations in different textbooks are compared: the number of examples, the distribution of contents, the learning activities, chapter review exercises. The results show that the three textbooks have different approaches and priorities that emphasize the positions and weights of the linear equation unit. The textbooks in China contain the highest percentage of linear equation contents and pay the most attention to explaining and analyzing how to set the unknown variable and using the concept of the linear equation to solve real-life problems. The findings can enrich teachers' and researchers' knowledge of mathematics textbooks in international schools in China and put forward some suggestions to improve the effectiveness of using textbooks in such schools.

1 Introduction

During the past twenty years, many Chinese secondary students have been looking for internationalized educational opportunities in China. There has an increasing number of private stand-alone international schools and international departments within public schools in China (Hayden, 2011). Those schools are either affiliated with the international baccalaureate (IB) program or IGCSE program that emphasizes subject-matter knowledge, academic achievement, and global skills. As for school mathematics, in these international programs, mathematics textbooks are English, and the medium of instruction is also English. Meanwhile, as required by China's nine-year compulsory education policy, all should learn the Chinese textbooks to meet the national curriculum

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syllabus. How to balance the mathematical content and language requirement may be challenging for teachers and students in international programs.

Textbooks are essential in mathematics teaching and learning. They "have a strong influence on mathematics teaching and learning" (Ponte, 2007). Fan and Kaeley (2000) proposed that mathematics textbooks play a role in mathematics teachers' pedagogy by conveying pedagogical messages and providing an encouraging or discouraging curricular environment for them to employ different teaching strategies. Different mathematics textbooks can vary different examples and exercises, different sequences of contents, and different teaching strategies. For example, Pepin and Haggarty (2002) analyzed one of the best-selling mathematics textbook series in English, French, and German. They found that the structures of mathematics textbooks in these countries were quite different. Even in the same region and on the same topic, the arrangements of mathematics textbooks could also be different.

Currently, there are no systematic bilingual mathematics textbooks for international programs in China. Bilingual teaching in mathematics is still in the exploratory stage. By focusing on the content topic of algebra, this study compares a few different aspects in selected textbooks from IB, IGCSE, and Chinese textbooks, in which linear equation is chosen for textbook analysis. That is a pivotal topic at the secondary level tightly relating the primary level algebra to the abstract algebra in higher education. The following questions are addressed: 1) What are the specific contents of IB, IGCSE, and Chinese textbooks on the unit of linear equations? 2) What are the differences and commonalities among IB, IGCSE, and Chinese textbooks?

2 Methodology

All three textbooks were selected from the different programs in the same bilingual secondary school in China. This paper compares three series of mathematics textbooks: the Chinese textbook, the IB textbook, and The IGCSE textbook. The Chinese textbook was published by Beijing Normal University (BNUP) published for grade 7 in 2012. The IB textbook named the MPY4 was published by Hase and Harris for grade 9 in 2008. The IGCSE textbook named Mathematics for Cambridge IGCSE (fifth edition) was published by Oxford University Press for grade middle school grades in 2018.

2.1 Content Analysis

To better understand the content and structure of these textbooks, the content analysis method is applied in this study, as it can quantify and analyze the text into some specific aspects. As Riff and Lacy (2013) mentioned, that content analysis paid more attention to the measurement and observation of communication symbols and assigns values to relevant symbols. This study analyzes the different levels and chapter overviews of the selected textbooks, focusing on the few aspects of linear equation content. We explore the distribution of internal content of the linear equation. The differences and similarities among three sets of textbooks are examined.

2.2 Result

Through the analysis, it can be found that different textbooks differ in several aspects. They have different approaches and priorities that emphasize the positions and weights of the linear equation unit. These differences are reflected through the number of examples, the learning activities, chapter review exercises, the distribution of contents and learning outcomes.

2.3 Distribution of the Content of Linear Equation

When further researching the difference in the distribution, the chapter distribution and section distribution of the selected content in the textbooks vary greatly. For example, the BNUP seems to have more sections related to real-life problems, while the IB and IGCSE are more focused on the traditional mathematics topic to deliver the knowledge (see Table 1). One interesting thing is that IB and IGCSE textbooks don't have a clear teaching sequence, that the linear equation chapter is mixed with different sections into different chapters. Besides, each section of the IB textbook has more subtitles and exercises for the chapter.

Table 1. The distribution of contents in the three textbooks

	IB	IGCSE	Chinese (BNUP)
Total sections	4	3	6
Chapter content distribution	1. Chapter Algebra section C Linear equations 2. Chapter Mensuration section B Length and perimeter 3. Chapter Algebraic fraction section E more complicated fractions 4. Chapter formula section B Rearranging formulae	1. Chapter Algebra1 section Linear equations 2. Chapter Algebra 1 section problems solved by linear equations 3. Chapter Algebra 2 section linear programming	1). How old are you? 2). Solve the equation 3). I got taller 4). Discount and sales 5). "Project Hope" Performance 6). Can you catch up with Xiao Ming?
Each section distribution (Flow)	1. Introduce the definition and concept 2. Investigation (solving equations) 3. Solution steps 4. Examples 5. Exercise	1. Each example with the explanation 2. Exercises	1). Introduction 2). Examples 3). Classroom exercises 4). Exercises. Knowledge skills, mathematical understanding, problem-solving)

Even for the same topic, three textbooks aim to meet different requirements and objectives. The IB does not contain specific details about the chapter teaching objectives, while IGCSE and BNUP have precise requirements for the linear equation chapter. Besides, the teaching objectives in IGCSE and BNUP are also different. For example, the IGCSE and BNUP require students to master the basic concepts of linear equations in one unknown and the method of solving equations, but IGCSE also teaches students how to derive and solve simple linear equations in two unknowns. The IGCSE syllabus mentions that there is no need to follow the textbook's order. It means that teachers may combine or separate teaching of topics based on their students' learning progress and their understanding of the knowledge.

2.4 The Contents of the Linear Equation

Generally, it is worth noticing that the three books with different chapters are used by students of different grades. BNUP has only one chapter (Chapter 5) on the linear equation with one unknown at grade 7. However, in IB and IGCSE textbooks, the content of linear equations is spread across different chapters of the books. They are covered in the interception of other chapters when it is relevant. They have some differences in the chapter and distribution (see Table 2). For example, there are different numbers of examples and distribution sections in the textbook. For IB, it has 13 examples, compared with 11 for IGCSE and 10 for BNUP.

Table 2. Aspects of analysis in the three textbooks

Books	Grade	Aspects of analysis				
		Examples	Learning activities	Chapter review exercises	Content of distribution of the Chapter	Chapter
IB	9	4+3(M)+3(af)+3(f)	1+2(M) (Investigation)	16 questions	4 sections; 1 +1 (M)+1(af)+1(f)	Divided into 4 chapters (Algebra notation and equations, Measurement, Algebraic fraction, Formulae)
IGCSE	9&10	11	No	34 questions	3 sections	2 Chapters (Algebra 1 Linear equations, Algebra 2 linear programming)
BNUP	7	10	5 (Discussion)	22 questions	6 sections	Linear equation with one unknown

Among three sets of books, The BNUP textbook contains the highest percentage of contents in linear equations. It also pays the most attention to explaining and analyzing how to set the unknown variable and use the concept of linear equations to solve real-life problems. While, the other two books are more diffuse and contain relatively little linear equations content.

2.5 Exercises and Learning Activities in the Textbooks

Learning activities and exercises can activate students to engage in learning. The sample of exercises and questions in these three sets of textbooks questions represented in Fig. 1.

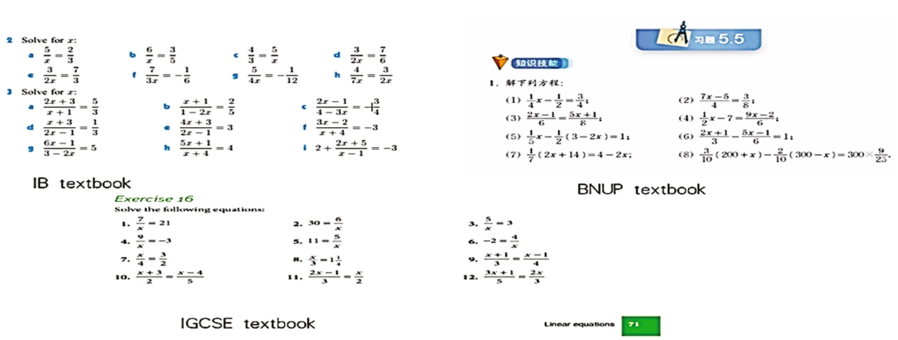

Fig. 1. Exercises of three sets of textbooks

It is found that IGCSE has 34 exercises, compared to 22 questions in BNUP (MCT, 2018) and 16 questions in IB (Vollmar, 2008), which means IGCSE focuses more on honing students' practical ability.

Textbook exercises and activities that invite students to draw relationships and stimulate students to use new knowledge or concept to produce new ideas are more likely to direct students in a deep approach to learning (Kahveci, 2010). For example, this type could be: 'Think about the questions and summarize the solve problem steps' (See Fig. 2). It seems that IB and BNUP textbooks did better on this part.

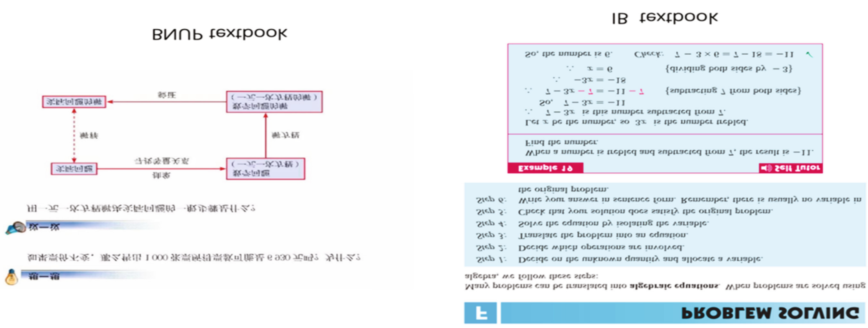

Fig. 2. Problem-solving section of the unit on the IB and BNUP textbook

However, the IGCSE textbook (R, 2018) does not contain any activities or other investigations, while the Chinese one has five discussion activities, and IB has multiple investigation parts.

3 Conclusion

By comparing their sets of mathematics textbooks in international schools in China, it is found that the Chinese textbook has more explanations and guidance than the other two textbooks. IGCSE and IB textbooks construct more intersections between the linear equation unit and other topics than BNUP. Among these textbooks, the unit of linear algebra is designed for students of different grades. Compared to BNUP, IB and IGCSE textbooks always emphasize that the students should learn to use the calculator with the function of linear equations. This may reflect those English textbooks focusing on analyzing and solving problems with tools. Chinese textbook emphasizes the procedural skills to solve problems and practice questions step by step. Regarding the characteristics of the bilingual teaching system, in future work, we can analyze how international school teachers combine Chinese and international textbooks to teach and how international school students view the role of curriculum material in their mathematics learning.

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A Study on Junior High School Teachers' Use of Mathematics Textbook: A Case Study in China

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Abstract. One of the focuses on the current Chinese mathematics curriculum reform is shifting from "teaching with textbook" to "using the textbook for teaching". Based on this background, this study aims to analyse the use of mathematics textbooks by an excellent Junior High School mathematics teacher in mathematics classroom. The content analysis method is used to compare the textbooks and the teaching materials actually used by teachers from teaching content and design sequence. In terms of teaching content, teachers have a higher degree of recognition of the mathematics textbooks contents and use most contents in the textbooks. However, the content of the textbook will be split, reorganized, supplemented and deleted when used. Moreover, teachers adjusted the order of teaching based on textbooks. It can be seen that teachers have redesigned textbooks when using them. This research helps inform that teachers should not use math textbooks directly, but should modify them according to their respective schools.

1 Introduction

Textbooks are the most basic and popular medium of instruction in schools (Fan & Zhuo, 2007; Plianram & Inprasitha, 2012). In the mathematics classroom, textbooks are a complex component of the school mathematics curriculum, providing a framework for what to teach and how to teach it (including the order in which it is taught) (Nicol, C. C., & Crespo, S. M., 2006). According to a 2011 TIMSS survey of 45 countries and regions, 70% of fourth-grade teachers and 74% of eighth-grade teachers use textbooks as a basic teaching resource (Mullis, I. V et al., 2012). It can be seen that textbooks play an important role in teaching. Therefore, teachers' research on textbooks has also attracted the attention of scholars (Fan, 2013). However, people still care less about how textbooks are used in the classroom (Plianram & Inprasitha, 2012).

Remillard (2000) found that experienced teachers use textbook materials thoughtfully, and they use, understand, and adapt tasks in textbooks and practice them in their teaching. The findings of Qi et al. (2018) indicate that Chinese junior high school mathematics teachers' use of textbooks has reached the level of elaborating and creating, but how mathematics teachers use textbooks has not yet been answered.

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The concept put forward by Chinese mathematics curriculum reform has changed the authority of textbooks, which requires teachers to change from the concept of "teaching textbooks" to "teaching with textbooks" (Li, 2008). Therefore, the role of textbooks in China has changed from "authoritative course" gradually transformed into "supporting material". This provides a lot of freedom for Chinese mathematics teachers to use textbooks, but it also brings certain challenges. Therefore, this research mainly focuses on how Chinese mathematics teachers use textbooks. This research provides reference materials for textbook writers and teacher training. The specific research questions are as follows:

1. How well do mathematics teachers use the content of the textbook?
2. Is the order in which mathematics teachers use materials consistent with the order in which they are arranged in textbooks?

2 Method

2.1 Participant

This study explores the use of textbooks through a case study of a math teacher. The teacher is a junior high school math teacher named Chen in an ordinary full-time public school in Beijing, China. She has 10 years of teaching experience and once won the second prize in a teaching competition in Beijing.

2.2 Research Design

The researchers conducted a four-month (a full semester) follow-up study of Chen in a junior high school, with 79 new classes of 40 min each. We observed, recorded and collected the teaching materials (such as PPT, etc.) used by Chen in the actual teaching of a class in a natural state, then compared with the textbooks to study the situation of Chen's use of textbooks. In a four-month follow-up study, Chen had formed a stable textbook use pattern in ten years of teaching practice, so only a part of it could represent Chen's textbook use. This study selected the Addition and Subtraction of Integer Formulas as the research unit. The textbooks of it are compared and analysed with the actual teaching materials.

This study compares the content and design order of textbooks and teaching materials, respectively. Specifically, the teaching materials and corresponding textbooks used in the 6 new lessons in the Addition a*nd Subtraction of Integer Formulas* were compared in content and design order.

3 Results

3.1 Math Teachers' Use of Textbook Materials

Overall analyse. *Addition and Subtraction of Integer Formulas* mainly covers the concept of integers and addition and subtraction of integers. Table 1 is the overall arrangement of the content of textbooks and teaching materials used by Chen. Obviously, the main content of *Addition and Subtraction of Integer Formulas in* the instructional materials have not changed when compared with the textbook. However, there are still differences in the specific content. For example, on the basis of the content of the textbook, Chen added the content of algebraic expressions and parentheses, and deleted the content of math activities.

Added Algebraic Expressions contains definitions for algebraic and represent numbers by symbols. Corresponding to the content of removing parentheses, Chen added the teaching content of adding parentheses, including the rules, the function and the relationship between adding and removing parentheses. The *algebraic expressions* is mainly a macro introduction to the content of expressions that students in junior high school have learned. It is helpful for students to learn the overall understanding of integral expressions, fractions and radical expressions. The adding parentheses can cultivate students' reasoning ability and problem-solving skills.

Table 1. Content of textbooks and teaching materials

Content area	Textbook	Instructional Materials
–	Represent numbers by symbols	Algebraic
integral expression	Monomial	Monomial
	Polynomial	Polynomial
	Integral expression	Integral expression
Integer addition and subtraction	Merger of similar items	Merger of similar items
	Remove parentheses	Remove parentheses and add parentheses
	Applications of Integer Addition and Subtraction	Applications of Integer Addition and Subtraction
–	Math activities	–

As for the reduced math activities, there are exploration problems similar to those in the teaching materials of the last lesson. For example, the textbook is a square while the material used by Chen is a "house" (see Fig. 1). In addition, the researchers observed that relevant content is designed in students' daily homework, Chen will use unified self-study class time to teach it. There are similar problems for direct use by students and teachers in exercise books. It can be seen that when Chen chooses teaching materials, she does not simply choose randomly, but has a plan and deep thinking.

Specific content analysis. Take the integral expression as an example for analysis (see Table 2). The main content in the textbook is represent numbers by symbols, monomials, polynomials and integral expression while teaching materials used by Chen is monomials, polynomials, descending powers, ascending powers, integral expression and review. Taking monomials as an example, the instructional design is exercises, examples and concepts while the materials used by Chen is introduction, concepts and examples. Judging from the exercise, the textbook is mainly used for judging the degree and coefficient of monomials, while the materials used by Chen involved the judgment of

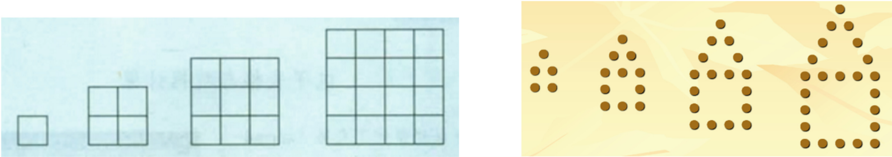

Fig. 1. Materials for math activities in textbooks (left) and teaching (right)

monomials and polynomials many times, and the content of descending and ascending power arrangement is also added.

In the instructional design process, Chen used a lot of materials in textbooks, but these materials are split and reorganized. For example, 8 problems in example 1 and 2 of the content of represent numbers by symbols in the textbook are all used by Chen. But it is divided into two sections. Specifically, Example 1 is incorporated into the monomials as the introduction content, and Example 2 is incorporated into the polynomials as the introduction content. Besides, new materials are added in the concepts when teaching.

Table 2. Content and order of textbooks and teaching materials

Order	Main content	Textbook	Main content	Teaching materials
1	Represent numbers by symbols	**Introduction** A travel problem in the middle of a train **Example** 1. Express quantitative relations in algebraic form (Monomial: consistent with the type and difficulty in the introduction) 2. Express quantitative relations in algebraic terms (polynomials)	Monomial	**Introduction** Four mathematical problems to solve: the train's travel problem, and the problem in Example 1 of the three textbooks **Concept** Coefficient of Monomial and Polynomial; degree of a Monomial and Polynomial **Example** Identify monomials, name their degrees and coefficients; and summarize error-prone points based on examples

(*continued*)

In addition, it is easy to find that Chen has used a lot of practice content (math problems) in the addition and subtraction of integral expression. The addition and subtraction of integral expression is the main content of this chapter. Chen used a total of

Table 2. (*continued*)

Order	Main content	Textbook	Main content	Teaching materials
2	Monomial	**Practice** Use algebraic expressions to express quantitative relationships (2 monomials + 2 polynomials) **Concept** Coefficient of Monomial and Polynomial; degree of a Monomial and Polynomial **Example** Express quantitative relationships in algebraic terms and state the degree and coefficient of monomials **Practice** 1. Name the degree and coefficient of the monomial 2. Use algebraic expressions to express quantitative relationships	Polynomial	**Introduction** 1. Use algebraic expressions to express quantitative relationships (using example 2 from the textbook) 2. Variation of example 2 (2) in the textbook **Concept** Polynomial; constant term; degree of a Polynomial **Example** Name the terms of the following polynomials, the number of terms, the degree, the highest-order term, the constant term

(*continued*)

71 problems including examples and exercises, nearly 20 more than the textbook. Chen summarized three types of problems: finding the value of algebraic expressions, application problems and exploration in the last lesson, and exercises carried out one by one. In addition, in the content of addition and subtraction of integral expression, Chen also added a lot of exercises that are more difficult than textbooks, such as "The polynomial of x, y is $(2x^2 + ax - y + 6) - (2bx^2 - 3x + 5y - 1)$, if the value of the polynomial has nothing to do with the value of the letter x, find the value of a, b" similar exercise. It is worth noting that Chen only used examples or practices in the text of the textbook, and did not use the exercises in the textbook at all.

Table 2. (*continued*)

Order	Main content	Textbook	Main content	Teaching materials
3	Polynomial and integral expression	**Concept** Polynomial; constant term; degree of a Polynomial; integral expression **Example** Use algebraic expressions to express quantitative relationships (polynomials), algebraic values, and calculations **Practice** 1. Use algebraic expressions to express quantitative relationships (polynomials), algebraic values, and calculate 2. Use integer expressions to express quantitative relationships, indicating the degree of monomials and polynomials	Descending powers, ascending powers	**Concept** descending powers, ascending powers **Example** Polynomials arranged in descending or ascending powers of a letter **Practice** Polynomials arranged in descending or ascending powers of a letter

(*continued*)

3.2 The Order in Which Teaching Materials and Textbook Materials Are Arranged

We divide the teaching process into four aspects: introduction, concept explanation, example displays and practice.

Explicit order. The overall explicit order of textbooks and instructional design is more consistent. Taking the content of the *integral expression* as an example, from the perspective of the overall teaching sequence, it is designed in the order of monomial and polynomial first, then integral expression, and follows the teaching rules from easy to difficult, from simple to complex.

Table 2. (*continued*)

Order	Main content	Textbook	Main content	Teaching materials
4	–	–	Integral expression	**Concept** integral expression
5	–	–	Review	**Practice** 1. Coefficients and Degrees of Monomials 2. Identify monomials, polynomials, and integers 3. The number of terms, degree, highest degree term, constant term of polynomial 4. Polynomials are arranged in descending or ascending powers of a certain letter

The specific teaching links follow the textbook design. From the perspective of instructional design, the learning of monomials and polynomials in textbooks is carried out in the order of concept, example and practice, but the teacher's instructional design sequence is introduction, concept and example. The teacher's design has added an introduction link before the concept, and reduced practice content. In fact, the concept content in the textbook is also introduced, which are all in the content of *represent numbers by symbols*. Besides, examples in the classroom learning materials also included lots of practice. It can be seen that the teaching design order in the teaching materials and the textbooks is not much difference in reality. The materials used by Chen add descending and ascending power which is closely integrated with the polynomial sorting after the polynomial, and use this content to separate the two parts of the polynomial and the integral expression. It can be seen that Chen's teaching design has a modules way of thinking.

Invisible teaching logic. Teaching logic is the causal relationship of teaching activities and its unfolding sequence. The concepts in the textbook are not directly presented, but closely integrated with the actual situation, helping students review the represent numbers by symbols learned in elementary school, and using examples to let students experience the simplicity of expressing the quantitative relationship with formulas containing letters. Then summarize the common points of these formulas, obtaining the concept of the monomial and its coefficients and degrees, further using case questions to consolidate monomial concept, familiar with the expression of quantitative relations. The treatment of polynomials is similar to monomials.

Mathematics teachers do not directly present concepts. The design starts from problem solving to cultivate students' ability of analysing problems and using mathematical methods to find and express quantitative relationships. These are the forerunners of the next chapter: Linear Equation in One Unknown. Chen also draws on the design ideas in textbooks, analogizes the formula obtained after solving the problem, and induces the concept of monomial. In the example section, mathematics teachers mainly focus on understanding the concept of monomial. Polynomials are similar in design to monomials.

Judging from the above, textbooks focus on thought with profundity and an easy-to-understand approach. Put problem solving after the concepts. Chen uses the different teaching logic. She is more inclined to problem solving and reasoning.

4 Conclusion

There is a real need to consider using mathematics textbooks as a learning medium to provide knowledge to students. The content and exercises in the textbook may not be appropriate for factual situations. This research helps inform that teachers should not use math textbooks directly, but should modify them according to their respective schools. Our qualitative research provides an example of a teacher with more than 10 years of teaching experience who is used to modifying content to improve the quality of classroom learning. In addition, the practices and exercises in the mathematics textbooks should also be revised to improve the students' critical mathematical abilities truly. We support other researchers researching the use of mathematics textbooks elsewhere, both qualitatively and quantitatively, to better understand how math teachers use mathematics textbooks to teach.

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Comparative Study of the "Vectors in the Plane" in Senior High School Mathematics Textbooks of the Three Editions

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Abstract. Using the method of combining qualitative and quantitative methods, the content of the "vectors in the plane" in the compulsory mathematics textbook of ordinary senior high school published by People' s Education Press, Beijing Normal University Press and Hunan Education Press in 2019 is compared and analyzed from two aspects of content structure and content difficulty. In terms of content structure: the overall structure of the "vectors in the plane and its application" in the textbook of the three editions is basically consistent; The presentation mode in the PEP focuses on considering the existing knowledge structure of students, the BNUP pays attention to the unity of the internal logic and the physical background of the vector, and the HEP pays attention to the rigor of mathematical; the BNUP has the largest number of examples and exercises, the HEP is in the middle, and the PEP is the least. In terms of the content difficulty: the content difficulty of the "vectors in the plane" in the HEP is the most difficult, the BNUP is in the middle, and the PEP is the smallest.

1 Introduction

The vectors is the bridge to link algebra and geometry, with rich physical background. Learning vector in senior high school is helpful to establish the relationship of between mathematics and real life and between mathematics and the other disciplines, experience the combination of numeral-form thinking method and develop the key competence of mathematics. In 2019, China published several versions of ordinary senior high school mathematics textbooks. Among them, the ordinary senior high school mathematics textbooks published by People's Education Press, Beijing Normal University Press and Hunan Education Press respectively, simply referred to as "PEP", "BNUP" and "HEP", are widely used in China. An analysis of the layout characteristics of the "vectors in the plane" in senior high school mathematics textbooks of these three versions will help to better grasp the teaching. This study focuses on the following two questions:

(1) How is the content structure of the "vectors in the plane" in the ordinary senior high school mathematics textbook of the three editions?
(2) How is the content difficulty of the "vectors in the plane" in the ordinary senior high school mathematics textbook of the three editions?

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2 Methodology

Professor Jianpan Wang (Wang, 2011) in East China Normal University put forward theoretical models and analytical tools for comparative research of mathematics textbooks in the "Comparative Study of Mathematics Textbooks in Major National High Schools" in the "Eleventh Five-Year Plan" annual pedagogical key project. The research group proposed a framework for the core content structure of mathematics textbooks (Wang & Zhang, 2014). In addition, Yiming Cao proposed a model for comparing the difficulty of mathematics textbooks of the different versions (Cao & Wu, 2015). That provides a theoretical basis for this study to analyze the characteristics of the arrangement of the "vectors in the plane" in the senior high school mathematics textbooks of the three editions. The "vectors in the plane" in the "PEP", "BNUP" and "HEP" is analyzed from the two aspects of the content structure and content difficulty.

2.1 The Comparative Framework for Content Structure

The structure of the "vectors in the plane" in the different textbooks is qualitatively analyzed from four dimensions: style and structure, the distribution of content, the way of presentation, the arrangement of examples and exercises. The style and structure is studied by analyzing the characteristics of the organization of the vectors in the plane. The distribution of content is studied by analyzing the selection and arrangement of vectors in the plane. The way of presentation is studied by analyzing the intention of editors' designs of the vectors in the plane mainly from the aspects of the concept of vector, the operation of vector, the basic theorem and coordinate representation of vector and so on. "Examples and exercises" is examples and exercises which are numbered. Some exercises are used in the classroom learning and some arranged in the section or chapter. They are collectively referred to as "exercises". "Examples and exercises" mainly are analyzed from two aspects of the distribution of quantity and the arrangement of hierarchy.

2.2 The Comparative Framework for Content Difficulty

The content difficulty is quantitatively analyzed from four dimensions: the breadth of content, the depth of content, the difficulty of examples and exercises. The breadth of content is depicted by the number of knowledge points that refer to the concepts, theorems and properties in mathematics. In this study, the vectors in the plane is divided into four themes: the concept of the vector, the operation of the vector, the basic theorem of the vector and the representation of the coordinate, the application of the vector and the solving triangles. Take the PEP as an example:

Step one: Calculate the number of knowledge points in the theme contained in the vectors in the plane of the PEP, recorded as a_j $(j = 1, 2, 3, 4)$;

Step two: Determine the relative breadth of the content of each theme, recorded as $G_j = \frac{a_j}{b_j}$ $(j = 1, 2, 3, 4)$. b_j $(j = 1, 2, 3, 4)$ is the maximum number of knowledge points of the theme of the j among three edition of the textbook;

Step three: The breadth of the content of the "vectors in the plane" in the PEP is $\mathrm{G} = \sum_{\mathrm{j}=1}^{4} \mathrm{A_j G_j}$ (j = 1, 2, 3, 4). $\mathrm{A_j}$ is the weight of each theme in the vectors in the plane of the PEP, which is obtained from the ratio of the capacity (page) of the corresponding theme and the total capacity of the four themes.

The depth of content is mainly depicted from the way of the presentation of mathematical knowledge points in the textbook. The way of presentation are divided into intuitive description, induction and deduction, assigned 1, 2 and 3, respectively. The depth of the content of each themes is as follows:

$$\mathrm{c_j} = \frac{1 \times \mathrm{A} + 2 \times \mathrm{B} + 3 \times \mathrm{C}}{\mathrm{A} + \mathrm{B} + \mathrm{C}} \ (\mathrm{j} = 1, 2, 3, 4);$$

A, B and C represents the number of knowledge points that are presented by the way of intuitive description, induction and deduction, respectively. First, normalize the depth of the content as we normalize the breadth of content, and obtain the relative depth of content in the four themes of the PEP. Second, calculate the depth of the vectors in the plane, and the method of calculation is consistent with the breadth of the content.

The difficulty of examples and exercises is described from three difficulty factors: cognition, knowledge content and background. The division and assignment of the level in the three difficulty factors is shown in Table 1:

Table 1. Table of level division and assignment of the difficulty factors of examples and exercises

Difficulty factors	The division of level			
Cognition	Memory	Understanding	Exploration	-
Knowledge content	One knowledge point	Two knowledge points	Three knowledge points	Four or more knowledge points
Background	No actual background	The background of life and production	Scientific situation	-
The assignment of level	1	2	3	4

The value of the difficulty factors in the examples and exercises can be calculated by the following formula:

$$d_i = \frac{\sum\limits_{m=1} n_{im} d_{im}}{n} \ (i = 1, 2, 3;\ m = 1, 2, \ldots)$$

$\mathrm{d_i}$ (i = 1, 2, 3) successively indicates the value of three difficulty factors: "cognition" "knowledge content" "background"; $\mathrm{d_{im}}$ is the assignment of the level of the m in the difficulty factor of thei; $\mathrm{n_{im}}$ represents the number of examples or exercises in the difficulty level of the m belonging to the difficulty factor of the i and the sum is equal to the total number of examples or exercises in the group.

The comprehensive difficulty of the examples and exercises in the theme of the j is $h_j = \sum_{i=1}^{3} k_i d_i$ $(i = 1, 2, 3; j = 1, 2, 3, 4)$. k_i is the weight corresponding to the three difficulty factors: "cognitive" "knowledge content" and "background", assigned $k_1 = 0.38$, $k_2 = 0.36$, $k_3 = 0.26$, respectively. First, normalize the comprehensive difficulty of the examples and exercises as we normalize the breadth of content, and obtain the relative difficulty of the examples and exercises in the four themes of the PEP. Second, calculate the difficulty of the examples and exercises in the vectors in the plane, and the method of calculation is consistent with the breadth.

Use N_j, G_j, S_j, L_j and X_j $(j = 1, 2, 3, 4)$ to indicate the difficulty, the relative breadth of content, the relative depth of content, the relative difficulty of examples and exercises in the theme of the j. Use α, β, γ and ω to indicate the weight of the breadth of content, the depth of content, the difficulty of examples and the difficulty of exercises, taking the values of 0.28, 0.30, 0.20 and 0.22, respectively.

The formula of calculating difficulty of theme the j is $N_j = \alpha G_j + \beta S_j + \gamma L_j + \omega X_j$; The difficulty of the vectors in the plane in the PEP is $N = A_1N_1 + A_2N_2 + A_3N_3 + A_4N_4$. By the above, we have obtained the value of difficulty of the vectors in the plane in the BNUP and the HEP successively.

3 Finding

3.1 The Characteristics of the Content Structure

In terms of style and structure, the vectors in the plane is mainly arranged in the chapter of the "vectors in the plane and its application" in the three editions of the textbooks. The overall style and structure is the same basically, but there are some differences in the column. The PEP designs a large number of the thinking and exploring activities in the process of presenting the core knowledge of the vectors in the plane. The editors attach more importance to the thought and exploration in the process of presenting content. The BNUP designs the activities such as asking questions, analysis and abstract in the process of presenting its core knowledge, indicating that the editors attach more importance to the process of generating knowledge. There are few activities in the HEP, and the presentation of core knowledge is mainly driven by the internal logic of mathematics.

In terms of the distribution of content, both the PEP and the BNUP pay more attention to the content beyond the core knowledge, and at the same time, the PEP also pay more attention to the content of the exploration of math. For example, the PEP and the BNUP designs the content of "the origin of vector and vector symbol", and the PEP also designs the exploration of math: "study the nature of the triangle by vector method", while the HEP doesn't design the extended content.

In terms of the way of presentation, the PEP focuses on the recent area in the development of students, and pays attention to the previous knowledge and experience of students in learning algebra. The textbook always emphasizes the analogy number and its operation to learn vector and its operation in the presentation of the vectors in the plane. The BNUP focuses on the physical background of the vector. In the process of the title of chapter and the presentation of new knowledge, it highlights the connection between the actual background and the concepts and operations of vector. The HEP from the perspective of algebra considers the important role of the basic concepts of vector

(vector, unit vector, etc.) in the presentation of the operation and the basic theorem of the vectors in the plane, and it attaches great importance to the logical system in the process of the presentation vector.

In terms of the examples and exercise, the total number of examples and exercise in the BNUP is the most (196), followed by the HEP (183), and the PEP is the least (177). The HEP designs three levels for the exercises arranged in the chapter and section. The BNUP only designs three levels for the exercises arranged in the chapter and two levels for the exercises arranged in the section.

3.2 The Characteristics of the Content Difficulty

The content difficulty of the vectors in the plane in the PEP, BNUP and HEP is 0.915, 0.921 and 0.929, respectively. The HEP is the most difficult, followed by the BNUP, and the PEP is the easiest. Specifically:

In terms of the breadth of content, the value of the breadth in the HEP is the largest (0.944), followed by the PEP (0.913), and the BNUP is the least (0.881).

In terms of the depth of content, the value of the depth in the PEP is the largest (0.939), followed by the HEP (0.916), and the BNUP is the least (0.880).

In terms of the difficulty of examples, the value of the difficulty of examples in the BNUP is the largest (0.985), followed by the PEP (0.937), and the HEP is the least (0.901).

In terms of the difficulty of exercises, the value of the difficulty of examples in the BNUP is the largest (0.970), followed by the HEP(0.954), and the PEP is the least (0.865).

4 Summary and Conclusions

To sum up, it can be seen that the three textbooks treating the difficulty factors of the vectors in the plane is different. For example, the layout of the core knowledge of the vectors in the plane in the PEP is “breadth and depth”, while the BNUP is “narrow and shallow”, but the difficulty of examples and exercises are the most difficult among the three textbooks. Therefore, the interpretation of the high school mathematics teaching materials should be comprehensively analyzed from many aspects, which provides a more comprehensive and operational method for teachers to analyze the mathematics textbooks.

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Study on the Types of Problems in Junior High School Mathematics Textbooks

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Abstract. As the most important resource for mathematics learning, mathematics textbooks directly affect teachers' teaching activities and students' learning, the presentation of its content has always been one of the focuses of textbook attention and research. The content of textbooks should not merely be the presentation of concepts, theorems and other mathematical facts. As students' learning materials, mathematical problems should be an important part of mathematics textbooks. As P. R. Halmos said, "Problem is the heart of mathematics", the real component of mathematics is the raising and solving of problems. The textbook of version of Beijing Normal University Press is based on the model of "problem situation-mathematical modeling-explanation, application and expansion", which builds a platform for students to learn mathematics by trying to solve various problems. Based on the existing researches on the types of questions, classroom questions, textbook questions and Bloom's target taxonomy, this research statistically collation the problems in the textbooks, divides them into 5 types according to their characteristics, analyzes and studies their specific distribution, some of their characteristics revealed in textbooks and how textbooks provide students in different fields and grades with different types of problems.

1 Introduction

The famous mathematician Hilbert once said: "A branch of science is full of life as long as it is filled with a large number of problems; the absence of problems means death or the termination of independent development". The "Mathematics Curriculum Standard for Full-time Compulsory Education" states that the teaching of the third learning stage should be carried out in the model of "problem situation-mathematical modeling-explanation, application and expansion" in accordance with the specific teaching content, so that students can experience the process of knowledge formation and application…Based on this, the textbook of Beijing Normal University (BNUP) builds a learning platform for students along with the mathematics problems. Problems have become an important "connection point" in junior middle school mathematics textbooks with the pace of curriculum reform. It can be seen that problems are of great significance to the development of mathematics and mathematics textbooks.

Before the 1990s, the knowledge was directly presented in textbooks on the whole, and the problems of textbooks were exercises and examples, hence a stereotype about a

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certain type of problem and students' limited thinking. Therefore, the distribution and focus of problems in textbooks have been constantly changing over the past 30 years. Until today, problems have been in textbooks everywhere. The appearance of problems in textbooks triggers a "conscious thinking activity" aimed at cultivating students' mathematical thinking, and also makes educators realize that acquiring knowledge is as important as experiencing processes of knowledge production. Given the above, it is worthwhile to study how to better clarify the different types of problems in textbooks and build a benign platform for students to learn mathematics.

2 AnalyticaI Framework

The problems of textbooks are mainly shown in following two parts: the process of explaining knowledge and the presentation of the topic. This study is a statistical collation of all the problems in the knowledge explanation part of the textbook of version of Beijing Normal University Press (BAUP). Based on Bloom's taxonomy of goals and the thinking activities behind the problems, synthesizing the relevant previous studies, the problem types are divided into two levels from the perspective of cognition: Knowledge level K ("Let students master knowledge") and Thinking level T ("Let students experience changes in thinking"), and the two levels are divided into 5 types, forming and constructing the "Analysis Framework of Problem Types in Mathematics Textbooks" (TP-KT), which is shown as follows.

The related work was carried out by two researchers respectively, and the differences were discussed with expert to ensure the consistency and reliability of coding and data (Table 1).

3 Result and Analysis

Based on the above classification method, we focus on the statistics and analysis of problems from the distribution of grades and fields of problem types.

3.1 Distribution of the Total Number of Problems

From Table 2, we can see that the number of problems in grade 7 and grade 8 is more than that in grade 9. The main reason is that grade 9 is the graduation grade, which needs to set aside time for students to have a systematic review of what they have learned in the past. As page decrease, the content and problem content will be relatively reduced, which matches the characteristics of grades and also reflects the homogen of question distribution (Table 3).

3.2 Distribution of Problem Types

From the horizontal perspective, K-C have the highest proportion in grades 7 and 8, followed by T-AE, and T-AE in grade 9 have the highest proportion, followed by T-JE, The proportion of K-M, T-AG have the smallest in three grades. From a longitudinal

Table 1. Analysis framework of problem types in mathematics textbooks (TP-KT)

Level	Type	Definition
Knowledge level (K)	Memory problem (K-M)	Let students retrieve or extract relevant information from memory, including recognition and recall of knowledge
	Comprehension problem (K-C)	Let the students try to understand the meaning of the materials through problems, including the explanation and examples of knowledge, analyze and compare, and carry out relevant operations to help understanding
Thinking level (T)	Abstraction and generalization (T-AG)	Students extract the common and essential characteristics of similar things by observing specific things or from specific situations, or extend some attributes of individual things to similar things in the thinking process
	Application and expansion (T-AE)	Students are required to make connections between new and old knowledge or different fields in learning, or to think in a larger scope on the basis of existing problems, and extend existing problems
	Judgment and evaluation (T-JE)	Students are required to make reasonable judgments and evaluations on issues based on existing facts or conclusions, so as to promote the understanding and internalization of new content

perspective, the proportion of T-AE, T-JE gradually increases with the increase of grades; the proportion of K-C gradually decreases with the increase of grades.

From Table 4, it can be seen that the proportion of K-M in the three fields is the lowest. The number and algebra domain have the highest percentage of K-C, followed by T-AE; the T-AE in the field of graphics and geometry also accounted for the highest proportion, followed by the T-JE; and the statistics and probability domain have the highest percentage of A-JE, followed by T-AE.

3.3 Example Analysis of Problem Types

The following are examples of problems selected for coding from six textbooks of junior high school of Beijing Normal University Press and specific reasons for coding.

Table 2. Grade distribution of the totol number of problems

Grade	Volume/number	The total number of problems	Total
Seven	First volume/201	207	480
	Second volume/170	273	
Eight	First volume/199	270	482
	Second volume/173	212	
Nine	First volume/179	249	414
	Second volume/123	165	

Table 3. Distribution of different problem types in grade levels

	K-M	K-C	T-AG	T-AE	T-JE
Seven	14(2.92%)	185(38.54%)	83(17.29%)	101(21.04%)	97(20.21%)
Eight	22(4.56%)	167(34.65%)	60(12.45%)	126(26.14%)	107(22.20%)
Nine	15(3.62%)	109(26.33%)	56(13.52%)	120(28.99%)	114(27.54%)

Table 4. Distribution of different problem types in different fields

	K-M	K-C	T-AG	T-AE	T-JE
Numbers and Algebra	25(4.26%)	199(33.90%)	82(13.97%)	1(30.66%)	101(17.21%)
Graphics and Geometry	25(3.86%)	241(37.19%)	112(17.28%)	12(19.29%)	145(22.38%)
statistics and probability	1(0.70%)	36(25.53%)	5(3.55%)	42(29.79%)	57(40.43%)

Example 1: Have you ever ridden a Ferris wheel? Think about how your height above the ground changes over time if you are sitting on a Ferris wheel? (Introduction on page 75, Volume I, Grade 8).

Example 2: what properties and judgment methods of right triangles have we explored? (Introduction on page 14, Volume II, Grade 8).

Example 1 is the memory extraction of life experience, and Example 2 is the recall of previously learned mathematical knowledge. The problems have no higher-level expression other than memory retrieval, so they belong to the memory problem of knowledge level (K-M).

Example 3: The paper knife forms angles of different sizes during opening and closing. Can you cite other examples like this? (Discuss on page 115, Volume 1, Grade 7).

Example 3 is after learning the concept of angles, students are expected to deepen their self-construction of angle by giving real-life examples, and also feel the connection between mathematics and life, so it belongs to the comprehension problem of knowledge level.

Example 4: observe the polygons in the figure below. what are the characteristics of their sides and angles? (Discuss on page 123, Volume 1, Grade 7).

Example 4 requires students to abstract the essence and generalize common features by observing specific graphics or images. Therefore, it belongs to the abstraction and generalization problem of thinking level (T-AG).

Example 5: If ΔABC~ΔA'B'C', the similarity ratio is 2, what is the perimeter ratio of ΔABC to ΔA'B'C'? The area ratio? (Introduction on page 109, Volume 1, Grade 9).

Example 5 is students are required to apply the new knowledge they have learned after learning about similar triangles and quadratic functions, and expand and extend it at the same time, connect the old and new knowledge to solve new problems. Therefore, it belongs to the application and extension problem at the thinking level (T-AE).

Example 6: Given the line segment AC, can you draw a rhombus ABCD with ruler and compass so that AC is a diagonal line of the rhombus? Do you think Xiao Gang is right? (Do it on page 6, Volume 1, Grade 9).

Example 6 is the practice of asking students to judge and evaluate Xiaogang's approach after he has performed a certain operation, so it belongs to the judgment and evaluation problem at the thinking level (T-JE).

4 Conclusion and Discussion

It can be seen that problems really play an important role in the compilation of textbooks, running through every step of the textbook setting, reflecting the efforts made by textbook writers in cultivating students' problem-solving ability.

4.1 Conclusion

1. Textbooks "strung together" with problems

In each book, the number of problems (not including exercises and examples) in the textbook is staggering, which averages out to 2–3 problems per page. Textbooks with various problems "strung together" make students experience changes in thinking in the process of acquiring knowledge. Regardless of whether it is a knowledge problem or a thinking problem, it points to the content itself and stimulates students' thinking in step-by-step problem inspiration and problem guidance.

2. Grade discussion and field discussion of various problem types

After entering junior high school, students' learning content has changed from intuitive and fragmented knowledge points to a more complete and systematic knowledge. Teaching lays more emphasis on students' understanding of knowledge and requires students to learn to analyze knowledge with thinking. The proportion of relevant problems gradually increases as the grade level grows and the problems become more broad and comprehensive, focusing more on the development of students' depth thinking, problems in various fields are basically compatible with the learning focus. For example, the focus of learning of number and algebra is to develop students' operational capability, symbolic awareness and modeling ideas based on knowledge, and apply them to solve practical problems. Problems more stress on understanding, application and expansion.

3. The diversity of forms of the same type of problem

Although the problems are only divided into 5 categories, each type of problem has different forms of expressing and questioning, such as abstract and general problems, which mainly include: abstract from specific graphics or situations; generalize commonalities from specific formulas; extend the existing conclusions to other similar forms. At this level, the problems become more vivid and rich, giving "new vitality" to the textbook.

4.2 Discussion

Although the "problems trigger students' learning" has become a compile characteristic of the textbook of version of Beijing Normal University Press (BNUP), and the writers can also raise questions with focus according to the characteristics of grades and fields, some problems seem to exist only as problems themselves, which does not play a role in stimulating students' interest and guiding their thinking.

1. Problem setting itself—Improve the guidance and effectiveness of the problem

Some problems in the textbook seem to be just rigid, plain and straightforward questions, and have not achieved the objective of guiding students. For example, in "Rational Numbers and Their Operations", after introducing the concept of the number axis, then put forward "Which point on the number line? 1.5?" After this question is "any rational number can be represented by a point on the number axis". Although students can think quickly, the representation of the two numbers given is not enough to promote students' better understanding of the conclusion. And since students have learned the classification of rational numbers earlier in this section, it is perfectly acceptable to give an example of each of the several types here to help students understand them better.

2. Distribution of problem types—as perfect as possible planning

For different content areas, textbooks should be planned specifically according to their specific characteristics, so that the problems of each part can play the maximum role. Although textbooks have shown it so far, it is still not perfect and prominent enough. Judgment and evaluation problems, for example, are very important in every field, which can reflect students' understanding, direction of thinking, and expression of thinking on a problem, but it is underrepresented in some domains. Therefore, textbook writers should make macro-consideration and micro-breakthrough to achieve the optimal ratio of different problem types while having a bias, so as to promote the development and improvement of students' corresponding abilities in different academic years and fields.

3. Practical application of problem—strengthen the development of problem-based teaching tasks

While the problem has become a "dark line" in textbooks, there are certain challenges in how to make good use of it. The most important is how to use problems to teach, which requires teachers to complete problem presupposition and judgment, and master the meaning of the problem skillfully. Teachers should first increase their academic nature and understand the practical utility of each type of problem in teaching. Then transfer the power of the classroom to students, giving them the opportunity to feel, think and explore the problems in textbooks. Finally, appropriate guidance can make the problem no longer a mere formality and achieve a meaningful addition.

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Evolution and Prospect of the Textbook Policy for Elementary Education of China in the Past Seven Decades

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Abstract. Textbooks play a pivotal role in school education, and textbook policies provide the "regulatory" basis for the development and publication of textbooks. In this study, a total of 81 textbook policies were searched and analyzed using the NVivo analysis tool, taking textbook policies in China from 1949 to 2021 as the object of study. By collecting, organizing, classifying, and analyzing the relevant literature, we divided the evolution into four stages by key points, and reviewed the evolution of textbook policies in China's elementary education stage, with the aim of exploring the laws of the development of China's textbook policies, and hoping to provide direct or indirect reference suggestions and materials for the progress of China's textbooks, the development of policies, and the research of related scholars.

As a special kind of text, the content of textbooks is a concentrated expression of the will of the state, social progress, cultural traditions, and scientific development. Since1949, China has issued a series of textbook policies. Looking back at history and examining the history of policy development and changes is of great theoretical value and practical significance for us to reflect on our experience and examine reality.

1 Selection of Policy Texts and Text Features

1.1 Selection of Policy Texts

This study takes the textbook policy texts in China during the period 1949–2021 as the object of study. In accordance with the three criteria of authority, completeness, and relevance and upholding the principle of being as complete as possible, the range of texts searched includes selected literature and databases, and a total of 81 policy texts were searched.

2 Text Features

2.1 Annual Characteristics of Policy Text

The earliest year of text release on elementary education textbook policy in China was collected was 1952, and the number of published policy texts was counted according to the year of policy text release, and a line graph was drawn as shown in Fig. 1.

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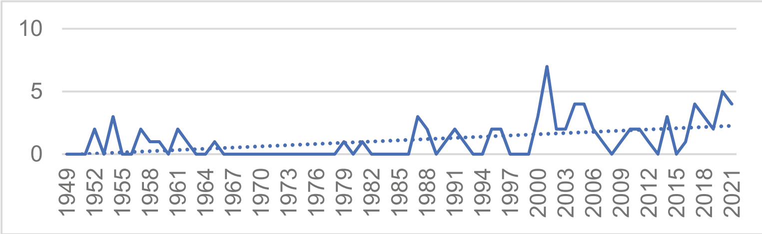

Fig. 1. Annual changes in the number of policy texts: 1949–2021

It can be seen that the number of texts has fluctuated up and down in some years since 1949, but the overall trend is up. From the 1950s to the early years of reform and opening up, the writing and publishing of primary and secondary school textbooks were highly concentrated, and between 1967 and 1977, textbook policy releases were in a freezing period due to the devastation of the Cultural Revolution on education. From the Third Plenary Session of the Eleventh Central Committee of the Party to the end of the 20th century, the situation of using one textbook nationwide began to break down, and a diversified approach to the development of textbooks began to be implemented, during which time a series of new policies were promulgated by the state. From 21st century, new policies was also significantly more than that of the previous stage, especially from 2000, due to the further reform of the textbook management system and the implementation of a diversified approach to textbooks, with a maximum of seven policies in some years and a shorter interval of years, the number of new policies increased in general and the growth rate was also significantly higher. This indicates that the construction of textbooks in China has gradually developed, and the state has actively introduced new policies to ensure their development and to guide and regulate them.

2.2 Features of the Issuing Subject

Through the text analysis, the main issuing subjects mainly include: the Ministry of Education, the State Council of the CPC Central Committee, the Office of the National Textbook Commission, and joint issuing. Joint issuance mainly refers to multi-agency joint issuance, and the main issuing agencies of joint issuance in the policy texts are mostly the Ministry of Education, the State Council, etc., and joint issuance by other departments, such as the Ministry of Finance, the General Administration of Press and Publication, the National Development and Reform Commission, etc.

In general, the subjects of promulgating policies on elementary education textbooks in China show diversified characteristics, and the Ministry of Education occupies a central position among the issuing subjects, which can reflect that the construction of textbooks has a certain complexity, but the work of the Ministry of Education alone is not enough; it also requires the joint participation of multiple departments, such as the highest state power organs like the Central Committee of the Communist Party of China and the State Council to provide legal protection and institutional support, and other administrative departments to provide assistance and cooperation, etc.

2.3 Policy Type Characteristics of Policy Texts

The policy type of policy text refers to the type of policy text, which refers to the form adopted when the policy is promulgated. Taking into account the actual situation of the policy on elementary education materials promulgated in China, the policy text type is divided into opinions, notices, methods, standards, outlines and programs, etc.

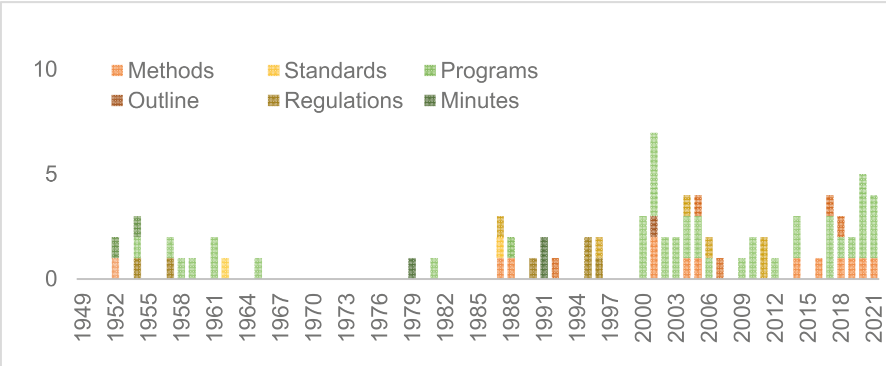

Fig. 2. Annual change in the text type, 1949–2021

As shown in Fig. 2, since 1949, the number of textbook policies has gradually increased in different periods, and especially after the end of the 20th century, multiple types of policies have regulated and guided the construction of elementary education textbooks in China in different degrees. In terms of policy style, textbook policies such as "notices," "instructions," and "opinions" on the one hand state the reality of the current year, point out the crux of the problem, and are a reflection of the reality of the situation. On the other hand, they also provide guidance and planning for the problems that arise and propose some specific standards and measures, which reflect the high importance and support of the state for the construction of elementary education materials. Secondly, a series of "measures" have been issued to strengthen the regulation of various work, which has increased the normative nature of the overall work of textbook construction. In contrast, textbook policies such as "regulations" and "decisions" are mandatory in the legal sense, but are few in number.

3 Evolution in Textbook Policy Themes

3.1 Stages of Policy Change

Since 1949, China's elementary education curriculum policy has undergone several significant changes, and the textbook policy has been largely influenced by the curriculum policy. Based on this, this study takes the establishment of the People's Republic of China (1949), the budding of textbook diversification in the education system reform (1985), the explicit implementation of textbook diversification policy in the elementary education curriculum reform (2001), and the convening of the "18th National Congress" to "unify" and “diversify” textbooks (2012) as key points. The development of China’s elementary education textbook policy is divided into four stages, with the number of policy texts being 15, 16, 27, and 23.

3.2 Characteristics and Patterns of Textbook Policy Evolution

Since 1949, the state has issued a large number of textbook policies. In order to further understand the key issues of policies in each stage, NVivo is used to search word frequency of policy texts in the four stages and cluster analysis the high-frequency words.

It is found that the policy contents of each stage focus on: (1) In the era of “textbook”, which was uniformly supplied by the state, the focus was mainly on the reorganization of textbooks. During this period, a series of policies were introduced to emphasize the compilation, publication and supply of textbooks; (2) In the “budding” era of the decentralization of the diversification of textbook. The versions and presentation forms of textbook began to be diversified; (3) From 21st century, the three-level textbook management system has been formed, and the diversification of textbook has become prosperous. With the introduction of various standards, systems and methods, a relatively stable textbook management situation has been formed. (4) In the era of “unified compilation” and diversified development, the construction of textbook is emphasized as the state’s right. The National Textbook Committee, Teaching Materials Bureau and Curriculum Textbook Research Institute have been established.

In general, there are three characteristics in the course of textbook policy development since 1949: Firstly, the content of textbooks has changed from “experience” to “science”. The emphasis on “publication” and “use” in the early stage has changed to research. Secondly, textbook institutions have changed from “isolated” to “jointly managed”. In the early days, editors were extremely scarce. After 2010, the specialization of institutions has been continuously improved. Thirdly, the textbook system has changed from “general” to “specific”. After 1949, the textbook system has been “customized by the state” for a long time. Then, the state began to implement the policy of diversification.

4 The Future Prospect

Textbooks stipulate the main content of teaching, which is the main basis for teachers to teach and the main channel for students to acquire knowledge and develop ability. The standardization and authority of teaching materials need to be guaranteed by policies.

4.1 Building a Strong Sense of Community of the Chinese Nation: The Construction of Textbook is the Power of the State

Textbooks reflect the will of the state, are an important basis for education.Therefore, in the whole process of the construction of textbook, the party’s leadership is important. The popularization of textbooks compiled by the three subjects is the protection of political positions. Since the 18th National Congress of the CPC, effective measures have been taken in promoting Xi Jinping Thought on Socialism with Chinese Characteristics for a New Era, Excellent Traditional Chinese culture and revolutionary tradition into curriculum materials. In particular, in 2021, the Ministry of Education and the Textbook Committee issued a series of documents to guide the compilation of textbook.

4.2 Clarify the Strategic Objectives: To Promote the High-Quality Development of Textbook Construction

The Fifth Plenary Session of the 19th CPC Central Committee held in 2020 defined the requirements of "building a high-quality education system". The construction of textbooks is an important part of it. The key is to promote the high quality development of textbook, establish the evaluation mechanism of textbook and improve the evaluation standard of textbook.

1. Improve the subject-specific textbook evaluation criteria: There are few studies on textbook evaluation standards that truly combine the subject characteristics. It is insufficient for the study of textbook evaluation standards to ignore the dominant position of textbook subject attributes and to evaluate other attributes of textbook quality. In the future, the research on improving textbook evaluation criteria should pay attention to the unique value of the subject and improve the development of textbook evaluation standards in combination with the subject characteristics.
2. Strengthen the specificity and operability of textbook evaluation criteria: The research of textbook evaluation criteria in China is still relatively weak. From the perspective of mathematics textbook evaluation criteria, the current research focuses on the construction of macro textbook evaluation index system. There are great differences in evaluation dimensions and specific indicators. The existing evaluation index systems are difficult to unify and have weak operability. In the future, it is urgent to establish a specific, operable and highly recognized evaluation standard for mathematics textbooks, which are conducive to the operation of the evaluators.
3. Pay attention to the evaluation of core literacy and key abilities: The curriculum standard is one of the important bases to study the evaluation criteria of mathematics textbook. The new curriculum standards are often emphasized in the form of some "key changes" to show its progressiveness compared with the previous. The 2021 edition of curriculum standard will be promulgated soon. From the perspective of mathematics discipline, mathematics core literacy and mathematics key ability will become a new key changes. The evaluation criteria of textbook should be changed. In the standard, we should pay attention to the evaluation of mathematics core literacy and mathematics key ability in Mathematics textbook.

4.3 Realize Institutional Norms: Make up for the System Gap and Form a Long-Term Mechanism

From the "conception" of a textbook, to what is written, designed, published, used by students, and eventually "learned" by students, each step will influence the outcome. Our country is in urgent need of systematic textbook evaluation criteria. This standard should be multi perspective and multi-user participation, integrate and cooperate with the feedback and revision process. The formulation and improvement of textbook quality evaluation criteria should be based on the process of combining static evaluation with dynamic evaluation. The formulation of textbook evaluation criteria should not only be based on the static evaluation of the content characteristics, organization, arrangement and text style, etc. In addition, it should also consider the dynamic evaluation of the users of textbooks, such as the teachers, students and education administrative departments.

The actual use effect is one of the important standards to measure the quality of textbook, especially for mathematics textbook. The presentation of mathematical knowledge have a variety of forms. In the process of dynamic evaluation of textbook, collect and analyze the feedback information of users', and adjust the weight of each evaluation index in the standards. Make the textbook evaluation criteria more objective, comprehensive and scientific.

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Textbook Analysis Framework and Case Analysis Pointing to Mathematical Modeling Literacy—Taking the Version of Hunan Education Press' "Mathematical Modeling" as an Example

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Abstract. Mathematical modeling is the basic means to solve practical problems, and also an important component of the core quality of mathematics. High school mathematics textbooks pay close attention to the implementation of mathematical modeling activities and mathematical inquiry activities. The article makes an in-depth analysis of the content of the teaching material from two aspects of mathematical modeling knowledge and mathematical modeling topics, and proposes that the arrangement of mathematical modeling content can effectively implement the requirements of the requirements of the new high school curriculum standard for mathematical modeling activities, and suggests that the "1+1" mode should be adopted to coordinate with the scientific evaluation, and the implementation of the normal course and special course teaching of mathematical modeling.

1 Introduction

With the promulgation of the *Mathematics Curriculum Standards for Ordinary High Schools (2017 Edition 2020 Revision)* (hereinafter referred to as the *New Curriculum Standards for High School Mathematics*) and the *Mathematics Curriculum Standards for Compulsory Education (2022 edition)* successively, the curriculum reform of basic mathematics education has entered a stage of comprehensive deepening, based on the core quality of the discipline. Mathematical modeling is one of the core qualities of mathematics discipline. The *New Curriculum Standard of High School Mathematics* sets up mathematical modeling and mathematical inquiry activities in both the compulsory courses and the optional compulsory courses, and lists it together with function, geometry and algebra, probability and statistics as the four main lines that run through the high school mathematics curriculum. Thus, mathematical modeling plays an important role in high school mathematics teaching.

As the link between the curriculum standards and the classes, mathematics teaching materials are an important carrier to realize the goal of mathematics curriculum and develop the students' core literacy of mathematics discipline. The senior high school

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mathematics textbook is an important member of the mathematics textbook system in China, which pays great attention to the implementation of mathematical modeling activities and mathematical inquiry activities. In the functional application part of Chapter 4 and Chapter 5, the first part of the textbook emphasizes the use of knowledge to solve practical problems and allow students to experience mathematical modeling. Chapter 6, “mathematical modeling” introduces the meaning and guides the whole process of mathematical modeling in the compulsory part, helping students to gradually form and develop the core mathematical literacy, and develop students’ problem consciousness and innovation consciousness in the subtle way. Based on the high school mathematics textbook of Hunan Education edition, this paper analyzes the content of mathematical modeling and puts forward teaching suggestions, in order to inspire the majority of mathematics teachers to better use the teaching materials and cultivate students’ mathematical modeling literacy.

2 Textbook Analysis

The general goal of mathematics curriculum in the basic education stage can be summarized as “three abilities”, that is, “one can observe the real world with the perspective of mathematics, think about the real world with mathematical thinking, and express the real world with the language of mathematics”. In the *New Curriculum Standards for High School Mathematics*, it is clear that “ mathematical modeling is the quality of conducting mathematical abstraction of practical problems, expressing problems in mathematical language, and building models to solve problems with mathematical methods.” In my opinion, this is exactly another expression of the “three abilities”. Comprehensive view of the core qualities of the six mathematics subjects, mathematical abstract and intuitive imagination pay attention to mathematical vision, logical reasoning and mathematical operation (mathematical operation can be regarded as special logical reasoning) pay attention to mathematical thinking, data analysis pays attention to mathematical language. The process of mathematical modeling is the synthesis of mathematical vision, thinking and language.

2.1 Analysis of Mathematical Modeling Knowledge

In the high school mathematics textbook of Hunan Education edition, mathematical modeling as knowledge is involved in various relevant chapters, mainly distributed in the field of function and algebra and geometry. The content in the field of probability and statistics focuses more on cultivating students’ data analysis literacy, and statistics are not made here. In the textbook, the mathematical modeling knowledge are mainly presented in the form of problem introduction, examples, exercises, etc. The author sorted out the mathematical modeling content in the high school mathematics textbook of Hunan Education edition, as shown in Tables 1 and 2.

From the perspective of students’ problem solving, a complete mathematical modeling process needs to go through two transformations. Blum, a scholar, pointed out the transformation relationship between practical problems and mathematical models. Blum focused on the changes in the mental state of participants in the process of mathematical

Table 1. Statistics of mathematical modeling content in the function section of high school mathematics textbook of Hunan Education Edition

	chapters and sections		**Mathematical modeling content**	**quantity**
Compulsory volume 1	**chapter 2**	**2.1**	P32 question 2 and Example 2, P40 9,10 and P41 practice 1, 2,Exercise questions 8, 9 ,12 and 13	24
		2.3	P49 questions, P54 cases 7, cases 8, and cases 9, P56 exercises 1, 2, exercises 6, 9 and 10,P60 exercises 7, 13, 14,15 and 17	
	chapter 3	**3.1**	P68 cases 3, P71 exercises 1, 2, P73 cases 5, P76 exercises 10	8
		3.2	P85 exercises 8, P89 exercises 11, 13	
	chapter 4	**4.2**	P105 cases 1, case 2, exercises 1, 2, P110 exercises 3, 4	25
		4.3	P117 cases 9, practice 4, P122 exercises 7, 8, 9 and 21	
		4.5	P139 exercises 1, example 3, P142 exercises, exercises 4, 5, 7, 8, P147 exercises 4, 71, 18, 22, and 26	
	chapter 5	**5.5**	P194 cases 1, case 2, P196 practice 2, exercises 3, 4, and 5,P202 exercises 12, 13	8
Compulsory volume 2	**chapter 2**	**2.1**	P73 cases 9	5
		2.2	P81 exercises 10	
		2.3	P89 7, P91, 8, P97, 16	
Optional Compulsory Volume 1	**chapter 1**	**1.1**	P6 cases 3, P10 exercises 11, 12	13
		1.2	P18 cases 7, P20 problem 8	

(*continued*)

Table 1. (*continued*)

	chapters and sections		**Mathematical modeling content**	**quantity**
		1.3	P24 cases 3, practice 4, P30 cases 3, and P32 practice 4	
		1.4	P49 exercises 12, 20, 21 and 23	
Optional Compulsory Volume 2	**chapter 1**	**1.1**	P4 cases 4, P7 cases 5, and cases 7, P10 exercises 2, and 3, and P14 exercises problem 8	16
		1.2	P17 cases 1	
		1.3	P38 8, 9, 10, P40 exercises 1, 2, 3, exercises 7, P47 exercises 8, 19	
Total: 99				

modeling, and found that there is also a key state —— realistic model between practical problems and mathematical models, which produced the five-stage mathematical modeling process shown in Fig. 1 below. The first time is the transformation from the real world to the mathematical world. To experience the process of "mathematics", students need to have an abstract and intuitive mathematical vision. In Hunan Education edition of high school mathematics, the "problem introduction column" in the textbook aims to guide students to observe the real world with the perspective of mathematics, and transform the real problems into mathematical problems. For example, in the compulsory volume one' page 39 of "problem introduction column", have the following content: "in daily life and production we often encounter how to make the material, the province, the highest profit and lowest cost, these problems can usually be solved by basic inequality, through the discussion of the following examples, we will experience the application of basic inequality."

In the mathematical world, students must analyze mathematical problems through logical and reasoning mathematical thinking, choose and use the mathematical language of models and data to express solutions, and then look back to the real world. At this point, students need to undergo the second transformation, that is, from the mathematical world to the real world. The columns of examples, exercises and exercises in the senior high school mathematics textbook can help students improve their ability to analyze and solve problems, cultivate students' literacy of mathematical modeling and data analysis, and finally complete the second transformation of mathematical modeling. For example, in Exercise 2, page 56 of compulsory volume 1, there are following examples: When the car is driving, due to the effect of inertia, it has to slide forward for a distance to stop, this distance is called "braking distance". Braking distance is an important indicator for the analysis of traffic accidents, in a speed limit for the curve, A, B two cars opposite to each

Table 2. Statistics of mathematical modeling content of algebra and geometry in high school mathematics textbooks of Hunan Education Edition

	chapters and sections	**Mathematical modeling content**		**quantity**
Compulsory volume 2				
chapter 1	**1.3**	P17 cases 3		21
		1.4	P30 exercises 11	
		1.6	P49 cases 9, cases 10, cases 11, P51 exercises 1, 2, and exercises 8, 9	
		1.7	P55 cases 3 and 4, P58 exercises 2 and 3, and exercises 2, 3, 5 and 6, P63 exercises 12, 13, and 23	
	chapter 4	**4.3**	P155 8 and P157 9	14
		4.5	P183 patients 3, exercises 1 and 3, P187 patients 5, and P189 patients 6, exercises 1, 2, and 3	
Exercise 4, 6, P200 exercises 19, 20				
Optional Compulsory Volume 1				
chapter 2	**2.3**	P79 exercises 31		13
		2.5	P90 exercises 9	
		2.6	P97 exercises 17	
chapter 3	**3.1**	P120 exercise problem 3		17
		3.2	P130 exercise problem 6	
		3.5	P149 cases 1 and 2, P152 cases 3, P154 practice 1 and 2, exercises 1, 2, 3 and 4, 5, 6, 7, 8, P163 exercises 11, P165 exercises 17	
Optional Compulsory Volume 2				
chapter 2	**2.4**	P105 exercises 11	1	
Total: 66				

other, found that the situation is wrong, at the same time brake, but still touch. 40 km/h After the scene investigation, the braking distance of A slightly exceeded 12 m, and the braking distance of B slightly exceeded 10 m. It is known that the braking distance s(m)

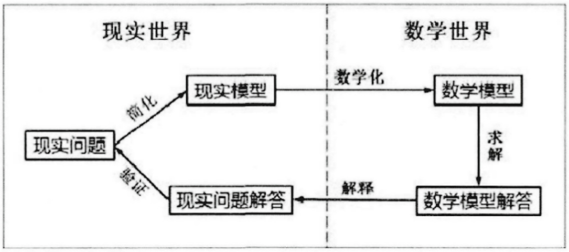


Fig. 1. Five-stage mathematical modeling

and speed of A and B x(km/h) respectively have the following relationships:

$$s_A = 0.1v + 0.01v^2, \quad s_B = 0.05v + 0.005v^2$$

Q: Do A and B cars have speeding phenomenon?

In the mathematical world, the students use the model of braking distance and speed to calculate the speed of the two cars A and B, and then judge whether the speed limit of the two cars is 40km/h speeding in the real world, thus completing the transformation from the mathematical world to the real world.

Such arrangement can effectively implement the new high school mathematics curriculum for the requirement of mathematical modeling activities: "the process of mathematical modeling mainly includes: in the actual situation from the perspective of mathematics, problems, analysis, build model, determine the parameters, calculation, test results, improve the model, finally solve the practical problems."

2.2 Content Analysis of Mathematical Modeling Topics

The mathematical modeling topics in the senior high school mathematics textbook appear in the compulsory volume 2, Optional Compulsory Volume 1 and Optional Compulsory Volume 2, as shown in Table 3.

The problem is the heart of mathematics, and the textbook contains a large number of cases of solving problems. These cases help students learn to analyze and solve problems. However, the problems in the real world, complex relationship, intertwined, the mathematical abstraction can greatly simplify the relationship between the real world, simplify practical conditions, the real problems mathematical and abstract a number of important variables to fit, with the help of function model, probability model, equation model, inequality model and vector model to explain the problem, and then back to the real world validation, to realize the application value of mathematical knowledge.

The cultivation of mathematical modeling literacy is carried out in project learning as an important practical form in the compulsory education mathematics curriculum. Scholar Xia Xuemei, through a review of mathematical literacy, The cultivation of mathematical literacy has specific requirements for the problem situations created by the driving problems, And from the authenticity of the driving problem, the diversity of the driving problem situation, the degree of knowledge synthesis or conceptualization of students causing the driving problem, the challenge of the driving problem and the setting

Table 3. The mathematical modeling topics of high school mathematics textbook in Hunan Education edition

principal line	edition	chapters and sections	special subject
Geometry and algebra	**Compulsory Volume 2**	chapter 6	6.1 Enter the colorful world of mathematical modeling
			6.2 Mathematical modeling —— From the natural to the ideal road
			6.3 Mathematical Modeling Case (I): the best perspective
			6.4 Mathematical Modeling Case (2): Manhattan Distance
	Optional Compulsory Volume 1	chapter 3	Glacier melting model
function		chapter 1	Music frequency and equal ratio sequence
	Optional Compulsory Volume 2	chapter 1	Optimized design of the cans
Probability and statistics		chapter 4	Model fitting of the data for body weight and pulse
	Compulsory Volume 2	chapter 6	6.5 Mathematical Modeling Case (III): Number estimation

problem students need to simultaneously call interdisciplinary literacy five dimensions for coding, Considering that the "mathematical modeling" topic in senior high school and the project learning in junior high school are the practical patterns of mathematical modeling literacy in the student section, And the driving problems in project learning are the same as those in the "mathematical modeling" topics, Built a bridge between "doing mathematics" and "mathematical conceptual knowledge" understanding. Here, the author refers to Xia Xuemei's analytical framework of project learning driven problem, and analyzes the 23 problems involved in the introduction of "mathematical modeling" in the Hunan senior high school mathematics textbook, in order to provide useful enlightenment for high school teachers to better develop the teaching of "mathematical modeling" (Table 4).

In the "**authenticity of the problem**" dimension, "False truth" refers to a false problem, artificially constructed for a certain mathematical knowledge point, The situation is not consistent with the real life; "Purifying truth" means simplified mathematical problems based on real situations, Set and simplify specific conditions for the acquisition of knowledge; "Simulated reality" means questions posed by simulating various factors and

Table 4. Coding analysis of mathematical modeling problems in High school mathematics textbooks of Hunan Education Edition

dimension	Index (number of cases)			
problematic facticity	False truth (0)	Purify the real world (10)	Simulated real world (0)	Real Reality (13)
The situation of diversity	Personal situation (6)	Career situation (4)	Social situation (10)	Scientific situation (3)
Knowledge synthesis or conceptualization	Shallow level (3)	Middle level (13)	Deep level (7)	
problematic challenging	Low order level (0)	Middle-order level (10)	Advanced level (13)	
Interdisciplinary accomplishment	No (0)	Less (7)	Medium (4)	High (12)

constraints in the realistic situation of the past, present or future, Put forward the results of the simulation problem solving; "Real reality" means problems that arise from the real world, Can be applied and operated in real-world situations, Facing a real audience, Forming designs or schemes that can be applied to the real world, And be tested by a real-world audience or experts. For the cultivation of "mathematical modeling" literacy, the problem of "false truth" is divorced from the real situation and deviates from the real meaning of mathematical modeling. The problem of "simulated truth" takes into account various limitations in the real world, and fails to simplify the real problems, which is not conducive to the solution of the problem. There is no false problem, nor is no real problem in the topic of "mathematical modeling" in the high school mathematics textbook. And 10 real purification problems. For example, the relationship between walking speed and rain volume when walking in the rain.13 real-life problems. For example: the case of the number of people estimates. This comes from the positioning of mathematical modeling in the textbook, that is, the mathematical model is a mathematical structure obtained by making a specific object in the real world, simplifying some necessary assumptions according to its specific internal laws for a specific purpose, and using appropriate mathematical tools. Such an arrangement is also scientific for the cultivation of mathematical modeling literacy.

In the dimension of "**diversity of situations**", "individual situation" refers to the model focusing on individual activities, individual families and individual peer groups, such as the data fitting model of weight and pulse. A "professional situation" refers to the problem of focusing on work in the real world, such as determining the maximum Angle from the goal when shooting in a football game. "Social context" refers to the focus on human groups (whether local, national, or worldwide), such as the Malthusian population model. "Scientific context" refers to the mathematical applications in nature and the problems and topics related to science and technology, such as the principle that the honeycomb structure is a hexagonal column when building nests. The topic of high school mathematics textbook "mathematical modeling" involves all the above situations, and there are many social situations. Developing core literacy means mastering the ability to use knowledge to solve problems, especially the ability to effectively solve problems in real situations. It also means that mathematics education based on core literacy pursues the original knowledge acquisition at the cognitive level to the application of knowledge in the situation. The senior high school mathematics textbook fully considers the extensive application of mathematical knowledge in the real world, which plays a helpful role in cultivating students' awareness of application and innovation.

In the dimension of "**knowledge synthesis or conceptualization**", the 13 cases contain at least 3–5 cross-field knowledge points, and there are 7 cases of coordinating large concepts or integrating multiple fields, such as: the discovery process of the gravity theorem. Thus it can be seen that the thematic content of "mathematical modeling" in the Hunan Education edition of senior high school mathematics textbook contains a wide range of knowledge, which makes students deeply understand the complexity of practical problems, gradually weaken the separation between the mathematical world and the real world, and improve their ability to analyze and solve problems in the real world.

In the dimension of "**challenging problem**", there is no problem that depends only on the low order level of memory and calculation. There are 10 cases that need students to understand and apply, and 13 cases that need students to analyze, evaluate and create. For example, the optimized design of cans. It can be seen that the case design of "mathematical modeling" in the Hunan Education edition textbook is in line with the recent development area of students, which enables students to jump enough, stimulate students 'learning motivation and activate students' mathematical thinking.

In the dimension of "Interdisciplinary literacy", the interdisciplinary literacy defined in this paper refers to 1. Cooperation, 2. Communication and communication, 3. Creative thinking, and 4. Critical thinking, the less means including the above one indicator, the medium means including the above two indicators, and high means including the above two indicators. The topic of "mathematical modeling" in the Hunan education edition includes at least one of cooperation, communication and communication, creative thinking and critical thinking, and there are 12 cases of more than two kinds. For example: a glacier-melting model. It can be seen that the textbooks play a positive role in cultivating students' cooperation ability, communication and communication skills, creative thinking and critical thinking.

Teachers should skillfully design the teaching and guide the students to master the basic process of mathematical modeling. In this process, students can not only review the various mathematical models they have learned, but also learn the flexible use of mathematical knowledge. Students' mathematical modeling literacy is also gradually cultivated in the process of repeating the above process. To be clear, the mathematical model and we usually say mathematical problems is different, the general mathematical problems require rigorous, clear, only answer, and according to the actual problem of the mathematical model and the answer is usually not unique, judge the merits of the mathematical model to conform to the actual standard.

3 Discussion and Conclusion

At present, some teachers are afraid of the teaching difficulties of mathematical modeling content, which affects the enthusiasm of teachers to implement mathematical modeling activities to some extent. Zhang pointed out that "mathematical modeling in middle school, it can be understood as a special curriculum form implemented in middle school, which is an active curriculum characterized by 'problem lead, operation practice' "Based on this, the author suggests that high school mathematical modeling teaching can be used" 1+1 mode" (as shown in Fig. 2 below). Specifically, the first "1" in "1+1" refers to the normal course of mathematical modeling, mainly for mathematical modeling as knowledge points. In the classroom teaching in the field of function, algebra and geometry, the models that the students have just learned are interspersed into the classroom, and the problems in the textbook are introduced, examples, exercises and exercises to let the students learn and use the model knowledge effectively. In normal teaching, teachers can decompose the complete process of modeling, let the students for "discovery and questions, establish and solve the model, inspection and improve the model, analysis and problem solving" one of the step for special training, whole, accumulate experience in basic mathematical modeling activities, improve students' mathematical modeling literacy. The latter "1" refers to mathematical modeling. From the above analysis of the topic of "mathematical modeling", mathematical knowledge comes not only from the internal mathematical system, but also from the actual social life; the application of mathematical knowledge is not only from the internal application of mathematics, but also to explain or solve practical problems in the broad life practice.

In view of the richness of the thematic content of mathematical modeling, the thematic courses of mathematical modeling cannot be taught by the traditional teaching method. We can use the following four teaching methods, namely, modeling learning, mathematical experiment, inquiry learning and thematic reading.

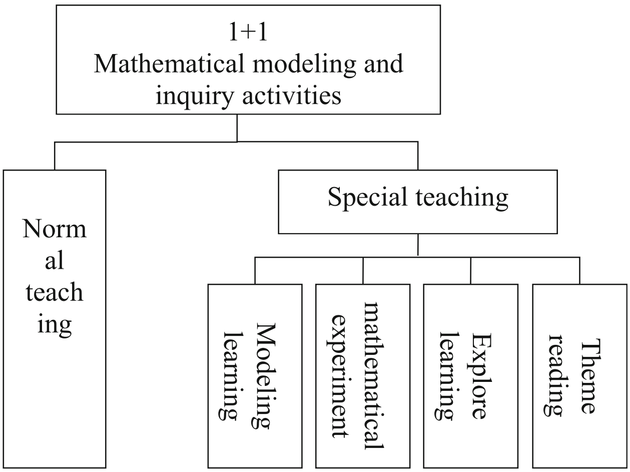


Fig. 2. Mathematical modeling of the "1+1" model

4 Modeling Learning

Modeling learning. It is a new way for students to solve their problems, which provides the space for students to study independently. Through the complete process of mathematical modeling, students will complete the accumulation of basic activity experience, experience the whole process of solving practical problems, experience the connection between mathematics and daily life and other disciplines, feel the practical value of mathematics, enhance their application awareness, and improve their practical ability. In the process of solving problems, students, teachers and students cooperate to cultivate their team spirit and develop their sense of innovation. For example, the author once tried to introduce the "explosion" of the Australian rabbit number as a scenario, and took a mathematical modeling class on the application of functional models. The content of this section is designed in 2018, after the publication of "High School Curriculum Standards" was issued, but the corresponding textbook has not been published yet. Therefore, there is no topic of "mathematical modeling" in the previous textbook. Therefore, the author integrates the application examples of Function Model in HHP Mathematics 1 (2007 edition). First, select the scene of the front map "rabbit explosion" to introduce the "modeling", then, explore the mathematical modeling process, then review and summarize the "modeling six steps": look back to see the month, summarize the law; finally, explore the questions, review the modeling process, namely, ride the wind and waves, strengthen the application. In classroom teaching, we should pay attention to life, life, generation and ecology, so that we hope to inspire readers.

The biggest characteristic of modeling learning that is different from other mathematical knowledge modules is its flexibility, the difficulty can be large or small, the items can be difficult or easy, and the evaluation should be more flexible. It is suggested to evaluate the students' mathematical modeling level in the form of experimental reports or small papers. The math paper is similar to the composition in Chinese, and the corresponding standard should be established when evaluation. We can establish the evaluation dimensions according to the four performance levels of mathematical modeling (see Table 5 below), and evaluate the mathematical modeling essay.

5 Mathematical Experiment

Mathematical experiment. In order to solve a certain kind of mathematical problem, students use measuring tools, manual materials, computers and geometric sketchboard and other related software to participate in the practice, explore the essence of mathematics, highlight the subject position of students, and accumulate the experience of mathematical activities. For example, after learning the knowledge of trigonometry function, the students designed the mathematical experiment activity "Measuring the height of the flagpole" (see Table 6 below). The students used the self-made angiometer and steel tape measure, using the knowledge of trigonometry function to solve the problem of the flagpole height, and realized the use of mathematical knowledge to solve practical problems.

Evaluation is the baton of teaching. The deepening stage of the new curriculum reform emphasizes the consistency of "teaching-learning-evaluation". The teaching of

Table 5. Evaluation indexes of mathematical modeling papers

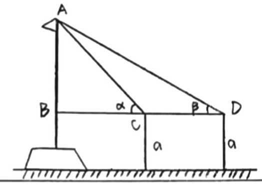
evaluative dimension	evaluating indicator
Discover and ask questions	1. Whether the problem situation is simple or complex, 2. Whether the students can clearly describe the question, 3. Whether students can use mathematical thinking to analyze, 4. Can students discover the mathematical relationships in the context, 5. Whether students can transform practical problems into mathematical problems and ask for mathematical problems;
Build and solve the model	1. Can students choose appropriate mathematical models to express the mathematical problems to be solved, 2. Can students understand the meaning of the parameters in the model and know how to determine the parameters, 3. Can students creatively build and solve models;
Test and refine the model	1. Can students test the results, improve the model and solve the problem according to the practical meaning of the problem, 2. Whether students can use mathematical language to describe the problems in the mathematical modeling process and the process and results of solving problems, 3. Whether the students can form the research reports and show the research results;
Analyze and solve the problems	Can students explain scientific laws and social phenomena through the conclusions and ideas of mathematical modeling in the process of communication

mathematics experiment is different from the traditional teaching of knowledge transmission, so its evaluation method can not be simply judged by whether to master the knowledge. The evaluation should pay attention to the students' experimental process, the division of labor in the group and the final experimental results.

6 Inquiry Learning

Explore learning. Is different from modeling learning and mathematical experiments. It generated around the learning process of a specific mathematical problems, guess reasonable mathematical conclusion, independent inquiry, autonomous learning, can consult information, can discuss with classmates, found some conclusions and properties, after research, explain to the students, this itself also promotes the students 'learning, cultivate the students' scientific research quality. For example, when learning the content of the graphics and properties of Sine Functions, we can use the geometric sketchboard to change the size of A, ϕ, ω and ask the students to observe the changes of the image and summarize the rules (see Fig. 3 below). Through active exploration, students observed

Table 6. Mathematical experiment case: Measuring the height of the flagpole

测量项目	测量旗杆的高度	
附图		
测量工具	自制测角仪　钢卷尺	
测量数据	① 测角仪的高度 a　④ CD 间距离 ② 在 C 处测得 A 的仰角记作 α ③ 在 D 处测得 A 的仰角记作 β	
计算步骤及结果	Step1. 在 △ABC 中 $AB=\tan\alpha\cdot BC$ Step2. 在 △ABD 中 $AB=\tan\beta\,(BC+CD)$ Step3. 由上两式得 $BC=\frac{CD\cdot\tan\beta}{\tan\alpha-\tan\beta}$ Step4. 由 $AB=\tan\alpha\,BC$ 得 $AB=\tan\alpha\frac{CD\cdot\tan\beta}{\tan\alpha-\tan\beta}$ Step5. 综上所述，旗杆高度为 $AB+a$ 即 $\tan\alpha\frac{CD\cdot\tan\beta}{\tan\alpha-\tan\beta}+a$	$a=1.6$ m $CD=15$ m $\tan\alpha=\tan 30^\circ$ $=0.5774$ $\tan\beta=\tan 17^\circ$ $=0.3057$ $AB=19.0232$ m $a+AB=20.6232$ m ∴ 旗杆的高度为 20.62 m.

the intuitive process of function image changes, and the resulting research conclusions will be deeply remembered.

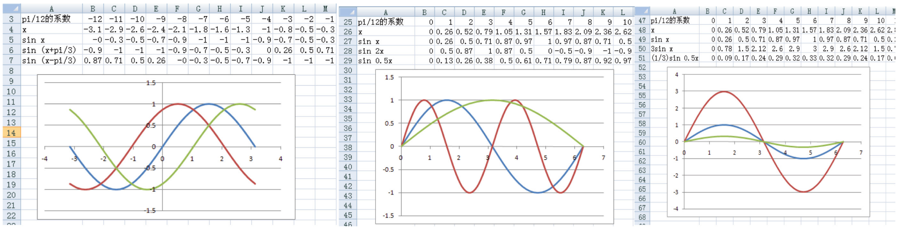


Fig. 3. Mathematical inquiry case: The A, ϕ, ω influence on the image of the function $y = A\sin(\omega x + \phi)$

Inquiry learning, we should pay attention to process evaluation and differences between students. In different stages of inquiry learning, different evaluation standards and methods should be adopted. The evaluation process is phased out, so that different students can have different gains. We can observe and evaluate students from the following perspectives: whether the questions raised are new, whether the operation solution is creative, whether the cooperative learning is efficient, whether the results are distinctive, whether there is a vision, whether the self-feeling are gained, whether the interest motivation has increased, and whether the mathematical literacy has been improved.

7 Theme Reading

Theme reading. Students determine the theme independently or the teachers determine the theme. Students collect information, select information, study together, and share the results. Through thematic reading, students learn how to collect information, how to process it, and how to express it. At the same time, but also let the students know more cutting-edge knowledge, broaden their horizons. For example, after learning the concept and representation method of series, students found that Fibonacci series has a wide application value through extracurricular reading, and put forward the reading theme of The Magic Fibonacci Series (see Table 7 below), in order to improve the understanding of the scientific value, application value and cultural value of mathematics. By completing this topic reading, we further discover the nature of Fibonacci series and the close connection with nature; realize the mathematical view that mathematics comes from life and serves life; understand the interaction between mathematical science and human social development, understand the scientific value and application value of mathematics, and expand the vision of students' vision.

Theme reading requires various ability, the process of reading literature requires students to learn in the real world complex the abstract, generalization, extract mathematical problems, with the aid of appropriate model analysis and solve the problem, and solve the mathematical model, mathematical results and promotion and migration mathematical modeling method, this process needs to read comprehension, information processing ability, mathematical ability, modeling ability, mathematical problem processing ability, reflection and critical ability, knowledge transfer ability and self-monitoring ability, etc. And because, the theme reading activity is a process of cooperation, some ability is not a paper test can measure, so the subject of reading evaluation should take diversified subject evaluation, students, parents and teachers participate in evaluation, each of the students have progress, positive evaluation can encourage students' learning enthusiasm.

We should encourage "promotion" rather than blindly "change short". The "1+1" mode of mathematical modeling teaching not only enables students to integrate the knowledge of function field, geometry and algebra field and probability with statistics field, and to realize the mutual connection and mutual support between knowledge. It can also stimulate students' interest in learning, promote students to actively participate in mathematics learning; can inspire thinking, guide students to explore ways to solve problems; can help students to sum up experience, promote reflection and critical thinking; and finally make students deeply feel the important role of mathematics in their own life and future career.

Table 7. The magical Fibonacci series

<table>
<tr><td>Subject name</td><td colspan="2">The magical Fibonacci sequence</td><td>classes and grades in school</td><td>Class 1, Grade 2</td><td>Leading discipline</td><td>mathematics</td></tr>
<tr><td rowspan="2">research group member</td><td>headman</td><td>Deputy team leader</td><td colspan="4">member</td></tr>
<tr><td>**</td><td>**</td><td colspan="4">***</td></tr>
<tr><td>Stage results</td><td colspan="6">stage I In August, 2021,The subject was determined and the research group was established
stage II From August-September, 2021, Collect data and integrate resources
phase III In October-November, 2021, Information processing, the initial formation of the results
Phase IV In December, 2021, Complete the report and make a reflective evaluation</td></tr>
<tr><td>Brief introduction of the project results</td><td colspan="6">Through group cooperation and exploration, we have explained the following contents:
1. Fibonacci Series and its history;
2. Fibonacci number column general term formula, first n term and formula;
3. Explore the Fibonacci series using Excel;
4. Fibonacci number and the growth of plants and animals;
5. Fibonacci columns and screws;
6. The Fibonacci Series and the Golden Section;
7. The relationship between the items in the Fibonacci sequence</td></tr>
<tr><td>Research group for self-evaluation</td><td colspan="6">Under the joint efforts of our research group, we have initially achieved the expected goal, realized the scientific value and application value of mathematics, broadened our horizon, and stimulated our interest in learning mathematics
Signature of the project leader: * *
On December 13, 2021</td></tr>
</table>

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The Centennial Changes of the Radian System in Chinese Mathematics Textbooks

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Abstract. As one of the two major units of measuring angles in trigonometric functions, the radian system has irreplaceable advantages and functions, and is widely used in mathematics and other scientific fields. This paper summarizes the history of the radian system for more than 300 years, and through the changes of the radian system in Chinese textbooks for more than 120 years from the end of the Qing Dynasty to the present, it is concluded that the radian system can make mathematical thinking economical and show the beauty of mathematics. At the same time, I give advice: I. Constantly improve the content setting of the radian system in the textbook; II. Further improve the teaching method of the radian system.

As one of the two major units of measuring angles in trigonometric functions, the radian system has irreplaceable advantages and functions, and is widely used in mathematics and other scientific fields. This paper summarizes the history of the radian system for more than 300 years, and through the changes of the radian system in Chinese textbooks for more than 120 years from the end of the Qing Dynasty to the present, it is concluded that the radian system can make mathematical thinking economical and show the beauty of mathematics. At the same time, I give advice: I. Constantly improve the content setting of the radian system in the textbook; II. Further improve the teaching method of the radian system.

As an important way to measure angle, the radian system has far-reaching ideological origin and extensive application value. The idea of the radian system germinated in the string watch made by the ancient Greek astronomers. After the Renaissance, with the development of mathematical branches such as trigonometry and calculus, the radian system emerged as an indispensable tool. Its application reflects the natural beauty and concise beauty of mathematics, and has played an important role in the development of mathematics. Since the British mathematician Roger Cotes (1682–1716) proposed the radian measurement in the early 18th century, it has a history of more than 300 years. Since its creation, the radian system has been continuously spread and applied, which has greatly facilitated the development of calculus, practical geometry, and other branches of mathematics.

Since the introduction of the second Western learning in the late Qing Dynasty, the radian system has a clear concept. It was quickly accepted by Chinese mathematicians, and it was immediately used in mathematics, astronomy and other fields, playing its important value.

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Throughout the history of the development of trigonometry textbooks in China, the first textbook that systematically introduced trigonometry was “*Sanjiaoshuli”*, published by Shanghai Zhuyitang in 1896. The radian system appeared in this book’s first volume. Since then, it has been arranged in textbooks as the basic content of trigonometric functions. It can be seen that the radian system has always been a key and classic content in mathematics education, and has an educational value that cannot be underestimated. It is reflected in its historical development and the changes in Chinese mathematics education textbooks.

1 The Historical Overview of the Radian System

Greek astronomers often used arcs rather than angles for trigonometric calculations, and true trigonometry begins with the origin of the Claudius Ptolemaeus geocentric system. Claudius Ptolemaeus divided the entire circumference into 360 equal parts. Since the arc length and the chord length are measured in the same unit of length, he approximated r to 3 to obtain an integer unit of 60 for the radius. The chord table was made in this unit, which is to measure the chord length with the unit of arc length, and the radiance system idea was born.

In medieval India, at the beginning of the 6th century AD, people used the chord length of an arc as a trigonometric function. Aryabhata made a sine table spaced $3°45'$ apart in the first quadrant. According to the habits of the Babylonians and the Greeks, he divided the circumference into 360 degrees, each of which is 60 min, in which the radius and circumference were measured in the same unit, which gave birth to the earliest concept of the radian system.

After Newton and Leibniz invented calculus in the 17th century, trigonometry also began to modernize. The rise of derivatives and integrals made degree measurement impractical and clumsy. The development of trigonometry toward fields from geometry to algebra prompted Roger Coates to invent the measurement of radians in 1714 (Glen Van Brummelen, 2009). For the first time he gave a value in radians, describing all but the name things, and recognize the naturalness of the radian as an angular unit (Roger Cotes, Robert Smith, 1722). Before the term radian became common, the unit was often referred to as the circular measure of angle.

In 1748, Leonhard Euler defined the concept of the radian system in “*Infinite Analysis Introduction*”, which made the radian widely used, and trigonometry was greatly simplified. It marks the further transformation of trigonometry from the study of triangular solutions to the study of trigonometric functions and their applications. The radian was quickly adopted in the mathematics community and spread rapidly at the end of the 18th century.

The first textbook to set the radian system was “*Analytische Trigonometrie*” (Fig. 1) written by Klügel, Georg Simon (1739–1812) in Germany in 1770, which pointed out that it is very convenient to use radians to calculate angles. In the textbook *Tiraité Élémentaire de Trigonométrie Rectigne et Sphérique*, popular in 1798 by Sylvestre François Lacroix (1765–1843), an alternative unit equal to twice the radian appeared.

From the correspondence between the Irish engineer J. Thomson (1822–1892) and the Scottish mathematician T. Muir (1844–1934) on who first invented the word “radian”

Fig. 1. *Analytische Trigonometrie* (Klügel Georg Simon 1770, cover)

(MUIR, 1910), we can see the process of determining the radian system word. The term "radian" was coined by J. Thomson in 1871 and first printed on Queen's College Belfast exam papers in 1873. The name used by T. Muir changes repeatedly between rad, radial, radial. In 1874, after discussing with J. Thomson, it was decided to use the word radial, think of it as short for "radial angle".

2 The Radian System in Chinese Mathematics Textbooks

2.1 Brief Description of Teaching Syllabus (Curriculum Standards) Requirements

Throughout the history of the development of Chinese trigonometry textbooks, in 1904, the first formal implementation of the Guimao school system in China, the "*Zouding Middle School Regulations*" included trigonometry as the teaching content. The radian system first appeared in the "Measurement of Angles" in the "*Elementary school numeracy syllabus*" in 1923. Followed by the "*Draft of Mathematics Curriculum Standards for Six-Year Middle Schools*" in 1941, the "*Condensed Outline of Plane Trigonometry for Senior High Schools (Draft)*" in 1950, and "*General High School Mathematics Curriculum Standards (2017 Edition)*" in 2017, etc. It is found that in the mathematics Teaching syllabus (curriculum standard) from the late Qing Dynasty to the present, the requirements of the radian system have been continuously refined, from the connection with real life, to the emphasis on the meaning of the radian system, and then to exploring the reasons for the introduction of the radian system. It not only plays a positive role in the compilation of textbooks, but also plays a positive role in the teaching of teachers in this content, and also prepares students to break through the difficulty of the the radian system.

2.2 The Setting of the Radian System in Chinese Mathematics Textbooks

The first modern trigonometry textbook introduced in China, "*Sanjiaoshuli*" (Twelve Volumes), as shown in Fig. 2 (J. Hymers, 1896). Explain the advantages of the radian system and its general application in algebra. According to the principle of the thirty-three questions in the sixth volume of "*Euclid's Elements*", the true radian is defined, and the reciprocal relationship between the constant degree and the true radian is discussed.

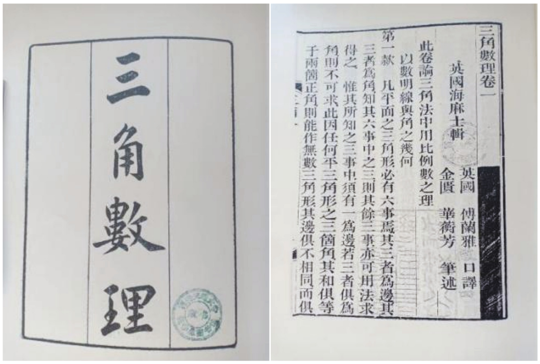

Fig. 2. Sanjiaoshuli (J. Hymers 1896, cover and p. 4)

The "*Latest Textbook"* is the earliest and most complete textbook in the country compiled according to the requirements of the academic system after the promulgation of the new academic system, and it has constructed the basic framework of modern Chinese mathematics textbooks. In 1907, "*The Trigonometry of the Latest Middle School Textbook*" (Frederick, Strang, 1907). The true radian arrangement adopts the method of directly defining the auxiliary angle, and indicate that its unit can be omitted.

While translating European and American textbooks, Chinese self-edited textbooks also began to develop.In "*Trigonometry of Senior Middle School Textbooks*" edited by Zhao Xiuqian in 1937 (Zhao Xiuqian, 1937), the radian system is derived from the proof that "if the angle is constant, the ratio of the arc to the radius is also constant", similar to today's textbooks.

Wentworth·George·Albert (1835–1906) mathematics textbooks were introduced into China with the most suitable teaching materials. "*Wendworth's Plane Trigonometry*" gives the concept of true radian standard arc, and also briefly introduces the history of the radian system: "True radian method. Used at the beginning of the eighteenth century. This method is used in advanced mathematics. The most Convenience" (Wende Fahrenheit, 1913).

Since the beginning of the 21st century, with the progress of the curriculum reform, the setting of the radian system in the textbooks has been continuously improved. At the same time, some introductions to the history of mathematics are given in the middle of the knowledge points to broaden the students' horizons. It also shows the calculation process of the calculator and the algorithm program of the mutual conversion of the measurement units of the two angles in the example, which provides a useful help to improve the integration of mathematics, modernization and information technology.

In the new version of the high school mathematics textbook, the first volume of the 2019 "*General High School Textbook Mathematics A Edition Compulsory*", the content settings of the radian system have been further optimized. It is not only easier for students to accept, but also promotes the formation and assimilation of the concept of radian system in their minds. Part of the history of Euler's proposed radian system is explained in the sidebar. The third volume of "*General High School Textbook Mathematics B Edition Compulsory"* also uses a life example - the introduction of fan folding, and the situation is created more vividly.

3 The Significance and Enlightenment of the Radian System

3.1 Simplify Mathematical Thinking and Show the Beauty of Mathematics

The radian measurement just right shows the beauty of simplicity and natural beauty of mathematics, and reflects the economical spirit of thinking pursued by mathematics. The radian system embodies the beauty of simplicity in mathematics. It unifies the unit of length and the unit of real number. Since the angle system is sexagesimal, when drawing a trigonometric function graph, unify the units of the horizontal axis (angle) and the vertical axis (trigonometric function value), and the graph will become appropriate and natural. The radian system plays an important role in simplifying formulas, making operations more concise. The radian system embodies the natural beauty of mathematics, where the sine of a smaller angle is numerically approximately equal to itself. Most textbooks in China set up a one-to-one relationship between radian values of different angles and real numbers, which facilitates trigonometric functions to form other elementary functions through algorithms, and promotes the development of calculus, advanced mathematics and other mathematics branches.

3.2 Complete the Content Settings of the Radian System

As a carrier of education and a basic tool for students to learn, textbooks should pay attention to psychological principles on the basis of clear rationale, stand on the students' standpoint, and educate students in line with their psychological development, in order to satisfy their sense of reality, reflects the value of education. E. Mach (1838–1916) once said, "Without any science education, we can ignore the history of science." The author believes that for the relatively unfamiliar content of the radian system, if the introduction of the meaning of the radian system is introduced, it will have a multiplier effect for students to accept the knowledge of this lesson. We know through history that in the difficulties of calculus and trigonometry, the superiority of the radian system is reflected. It is a good idea to write the history of the radian system in the introduction part of the textbook, which not only stimulates students' interest in learning, but also solves the "what" and "why" confusion in their minds. To enable students to better understand the radian system, enhance the awareness of use, and effectively realize the requirements for the meaning and necessity of radians in the curriculum standards.

3.3 Improve the Teaching Method of the Radian System

There are plenty of students in high school who don't understand the arc system, let alone its meaning and history. Therefore, teachers can infiltrate the combination of numbers and shapes of trigonometric functions in the radian system classroom, and strengthen the combination of theory and practice. As an activity for visual observation and exploration, have students notice that arcs are measured along the outside of a circle using the radius as a unit of length. The historical development of the radian system and the application of the radian system in other sub-disciplines are shown to students in the form of educational videos, so that they realize the importance of the radian system not only in the angle but also in the construction of the entire mathematics building and science palace. Of course, education is not achieved overnight. In the subsequent education and teaching, let students feel the convenience of the radian system in a timely manner, and discover the mystery implied by the simplicity and naturalness of using the radian system, which can be realized both cognitively and psychologically. To link the past and the future, the role of continuous improvement.

By understanding the changes in the content of the radian system, it is helpful to clarify the development process of the radian system and even the trigonometric function textbooks, and to have a glimpse of the modernization process of Chinese mathematics education. Although the history of the arc system is short, it has far-reaching uses. It should be implemented in education and teaching. On this basis, some superficial opinions are drawn, in order to provide reference for the compilation of current mathematics textbooks.

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A Comparative Study on the Difficulty of the Exercises in the Textbooks of the Distributive Law of Multiplication

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Abstract. Four levels of the difficulty of the distributive law of multiplication are established (including the operational memory level, the conceptual memory level, the explanatory understanding level and the exploratory understanding level). Based on this, this paper makes a comparative analysis of the exercises of the distributive law of multiplication in the six editions of the textbooks. The conclusions are as follows: The total amount of exercises in different textbooks varies greatly, the amount and proportion of exercises at the same level vary greatly. And in light of that, it is necessary to establish difficulty classification standards for exercises research and carefully consider the total quantity and the number of exercises of different levels. Meanwhile, teachers should add and subtract exercises of different levels according to the learning conditions.

1 Introduction

Exercises are an important part of teaching materials, which play a great role in consolidating knowledge, improving ability and improving quality for students. The distributive law of multiplication is an important operation law, and for the related exercises, most teachers can judge the difficulty by experience, but it is difficult to make meticulous description of difficulty. In this paper, according to the difficulty level of the exercises of the distributive law of multiplication, the difficulty of the multiplication assignment law in the current six editions of textbooks of the People's Education Press edition, the Beijing Normal University edition, the Zhejiang Education edition, the Shanghai Education edition, the Southwest Normal University edition and the Qingdao Edition are compared, with a view to better analyse the textbook exercise difficulty for textbook exercise writing and teachers provide a reference.

1.1 The Standard of Dividing the Difficulty of the Exercises of "the Distributive Law of Multilication"

What we usually call the difficulty of exercises refers to the students' reaction to the difficulty of exercises, and the ability of solving exercises reflects the students' the level of mathematical cognition. Gu Ling-yuan and others put forward four levels of

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mathematical cognition: Level 1: Calculation -- operational memory; Level 2: Concept -- conceptual memory; Level 3: Comprehension --illustrative understanding; Level 4: Analysis --exploratory understanding (Yang Yudong and He Zhenzhen 2007). Among them, Level 1 and Level 2 are memory levels, which belong to the low cognitive levels. Levels 3 and 4 are understanding. Levels, which belong to higher cognitive levels. On this basis, this paper gives the standard of dividing the difficulty of the exercises of "the distributive law of multiplication":

Level 1: Calculation -- operational memory. The exercises at this level require students to memorize the form of the distributive law of multiplication and to be able to imitate it under the condition of "auxiliary". There are two types of exercises:

(1) Identify whether a formula uses the distributive law of multiplication and judge whether the equation splitting process is correct.
(2) Provide partial information in the equation and ask the student to complete the other part in order to form an equation that conforms to the distributive law of multiplication, as shown in Fig. 1.

Level 2: Concept -- conceptual memory. The exercises at this level mainly refer to the ability to initially apply the distributive law of multiplication for simple calculations without "auxiliary"; and under the condition of "auxiliary", the distributive law of multiplication to addition is analogized to that of multiplication to subtraction. There are two types of questions:

(1) Directly applicate the distributive law of multiplication to do simple calculations. Direct application means students only need to use the distributive law of multiplication to make simple calculations according to the numbers given by the problems, without splitting the numbers first, such as "$125 \times (80 + 8)$". Indirect application refers to the application after splitting the numbers, like "125×88" needs to split "88" into "$88 + 8$".
(2) Under the condition of "auxiliary", the distributive law of multiplication to addition is analogized to that of multiplication to subtraction, and the forms of symbol and letter can be used to fill in the blanks. Such as "$() \times (85–13) = 29 \times ()-29 \times ()$".

Level 3: Comprehension --illustrative understanding. The exercises at this level mean that the distributive law of multiplication can be indirectly applied for simple calculations; or after listing the formula according to the exercises, the distributive law of multiplication can be flexibly used for simple calculations, such as "103×12" or "Team A and Team B dig a tunnel from both ends at the same time. Team A chisels 9 m one day, team B chisels 11 m one day, which will be finished after 120 days. Find the length of this tunnel."

Level 4: Analysis --exploratory understanding. The exercises at this level mean that the distributive law of multiplication to addition can be shown in combination with illustrations or other forms; the distributive law of multiplication to addition and division to addition and subtraction are analogized to that of multiplication to addition, and use different representations to explain whether the distributive law is valid; use the distributive law to solve the problems. Such as "Explain the Eq. $3 \times 6 + 4 \times 3 = (6 + 4) \times 3$ according to the Fig. 2".

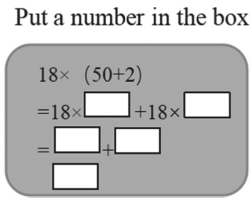

Fig. 1. xx

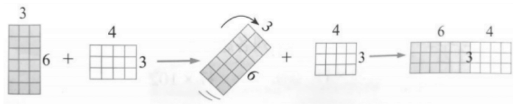

Fig. 2. xx

1.2 Comparison and Analysis of the Difficulty of the Exercises in Six Editions of Textbooks

According to the difficulty level of the exercises of the distributive law of multiplication, we divide and analyse the exercises of six editions of textbooks, which include the new lesson and the exercises that are followed closely after the new lesson, but not include the exercises in the general revision, and are counted according to the number of small questions in the exercises. Table 1 is the difficulty level of the exercises in the six editions of textbooks.

Table 1. Level table of the distributive law of multiplication exercises in the six editions of textbooks

	Level 1	Level 2	Level 3	Level 4	Total
People's Education Press edition	9 (33.3%)	4 (14.8%)	13 (48.1%)	1 (3.7%)	27
Beijing Normal University edition	0	9 (50%)	7 (38.9%)	2 (11.1%)	18
Zhejiang Education edition	7 (26.9%)	13 (50%)	6 (23.1%)	0	26
Shanghai Education edition	2 (12.5%)	9 (56.3%)	5 (31.3%)	0	16
Southwest Normal University edition	6 (23.1%)	6 (23.1%)	14 (53.8%)	0	26
Qingdao edition	7 (23.3%)	11 (36.7%)	12 (40%)	0	30

The following conclusions can be obtained from Table 1 above:

1. The total number of the problems varies greatly. Among the six editions of textbooks, the Qingdao edition which has the most number (30 questions), almost twice as many as the Shanghai edition, which has the least number (16 questions). Figure 3 shows the total number of the exercises in the six editions of textbooks.

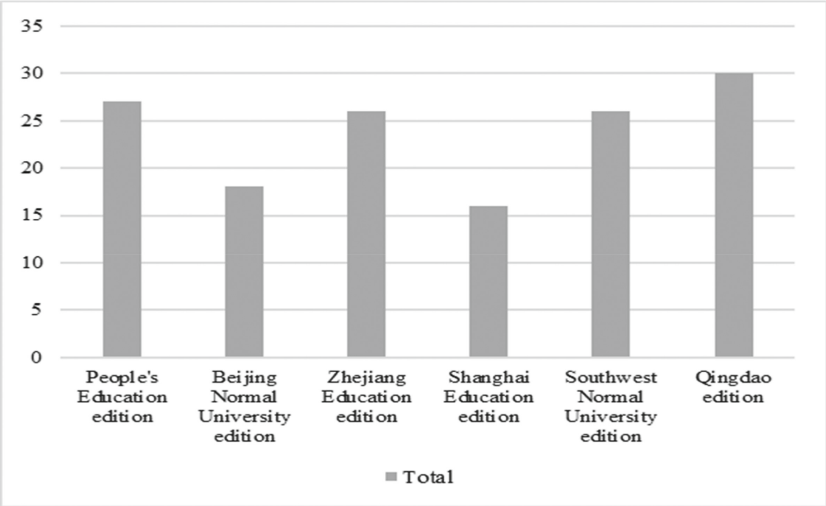

Fig. 3. Statistical chart of the total number of exercises in the six editions of textbooks

How many problems will be designed to enable students to meet the requirements of the curriculum standard is a great question worth studying.

2. There are four editions of textbooks do not have the exercises of Level 4. The exercises of Level 4 require students to use the distributive law to solve the problems, which will play a positive role in the cultivation of innovative thinking and higher-order thinking. There should be an appropriate number of the problems.
3. In each edition of the textbook, the quantity of exercises at each varies greatly. For example, in terms of the total number, the Beijing Normal University edition have no Level 1, the Shanghai edition has only two, and nine in the PEP edition, which is the most. For Level 2, the Zhejiang edition has 13 but only 4 in the PEP edition. Figure 4 shows the number of the exercises in the six editions of textbooks at each level.
4. The number of exercises at the same level as part of the percentage of the total number of exercises varied greatly. For example, for Level 1 is concerned, the number of exercises in Level 1 of the PEP edition accounts for 33.3% of the total, while the number of exercises in the Shanghai edition accounts for only 12.5%, a difference of 20.8%. For Level 2, the number of exercises in Level 2 of the PEP edition accounts for 14.8%, but it accounts for 56.3% in the Shanghai edition, a difference of 41.5%. This shows that the compilers of various sets of textbooks have a great difference in the understanding of “a certain level of exercises should account for a few percent of the total number”.

The following two fan-shaped statistical charts (Fig. 5) reflect the relationship between the number of exercises at each level in the textbooks of the PEP edition and the Beijing Normal University edition, we can clearly see the difference in the proportion of exercises at each level of the two editions of the teaching materials.

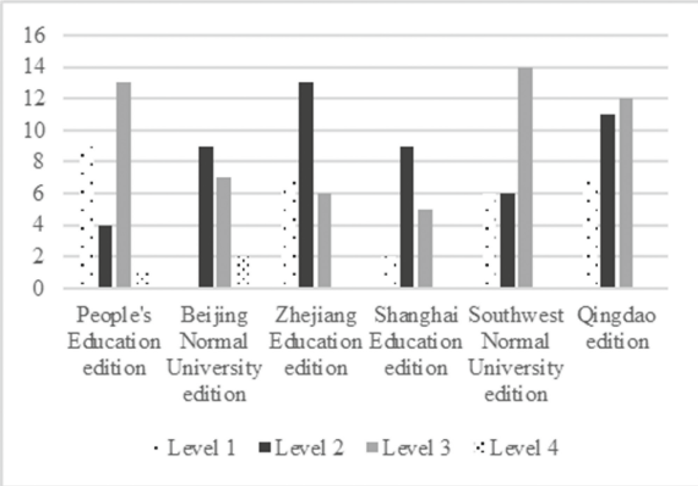

Fig. 4. The number of the exercises in the six editions of textbooks at each level

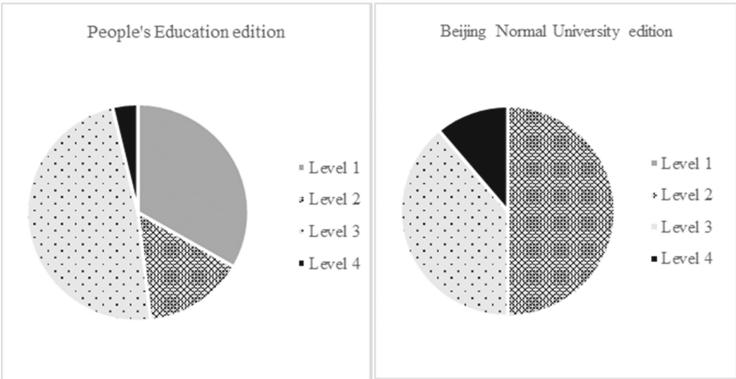

Fig. 5. Statistical chart of exercise

2 Research Conclusion

Through the establishment of the standard of dividing the difficulty of the exercises and the statistics and analysis of the number of exercises at different levels in each edition of the textbook, the following enlightenment and suggestions are obtained.

2.1 It Is Necessary to Establish the Standard of Dividing the Difficulty of the Exercises

For teachers with a long teaching age, the difficulty of exercises can be judged by experience. However, when establishing the standard of dividing the difficulty of the exercises, it is far from enough to rely on experience alone, which requires more rational thinking and judgment, and need to learn the cognitive level theory by Gu Ling-yuan and others or The SOLO classification and evaluation theory. Since then, teachers can move from vague judgment by experience to rational judgment based on the standard.

2.2 The Textbook Editor Should Consider the Total Number of Exercises and the Number of Exercises at Different Levels

We can know that among the six editions of the teaching materials, the number and level of exercises vary greatly through the above comparison. In order to better compile the teaching materials, the editor should carefully consider the level of exercises, as far as possible to each level of exercises are involved with a suitable proportion. Combined with practice, the four levels of 30%, 30%, 25%, 15% may be more appropriate.

2.3 Teachers Need to Add and Subtract Exercises at Different Levels According to the Teaching Materials and Learning Situations

Front-line teachers need to have the awareness of “the level of exercises” when using teaching materials, and have the ability to distinguish the level of different exercises in the teaching materials, and then combine with the actual situation of the students to process the exercises in the teaching materials, like increase or reduces the different levels of exercises.

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Implement Suggestions for Literature Reading and Mathematical Writing in Senior High School Mathematics Textbooks

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1 Introduction

Mathematical textbooks are an important resource for teaching and learning mathematics (Cai, Nie, and Moyer, 2010; Fan, Zhu, and Miao, 2013). Many researchers have compared textbooks in mathematics and discussed the impact of textbook teaching and learning in mathematics (Senk, Thompson, and Wernet, 2014). In particular, the 1st to 3rd International Conference on the Research and Development of Mathematics Textbooks (ICMT) is devoted to the study. Literature Reading and Mathematics Writing (LRMW) is a novel content in mathematics textbooks and is conducive to developing students' ability to express the world in mathematical language. The purpose of this discussion is to investigate Chinese high school mathematics textbooks on the material of LRMW. Therefore, we have two open questions: (1) the distribution, subject, and content structure in what way has the LRMW been introduced in the textbooks? (2) How can teachers use the LRMW to guide students in reading literature and writing mathematics papers?

2 Method

This study adopts a content analysis method to analyze selected mathematics textbooks in three dimensions: distribution, subject, and content structure, then propose pedagogical suggestions based on the analytic results of LRMW and the teaching experience of the authors. Ordinary high school textbooks: Mathematics (New Curriculum Standard) (PEP et al., 2019) were selected as representative textbooks.

3 Results

There are two compulsory textbooks and three selective compulsory books with five LRMW in them. The distribution, title, subject, and content structure of LRMW are shown in Table 1. Three LRMW are located in compulsory books and two are located in selective compulsory books 1 and 2. None LRMW in selective compulsory book 3. Comprehensive considering the analytical results on LRMW, the curriculum standards, the students' mathematical knowledge, and the authors' teaching experience, we would

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like to give the following useful implementation suggestions. (1) Teachers help students study the mathematics knowledge related to LRMW well. (2) Introduce to students the structure of a general mathematics paper and how to write a short mathematics paper. (3) Introduce some basic knowledge of how to find the wanted literature or reference books, e.g., literature on the history of mathematics. (4) Students can adapt the suggested topics given in the LRMW or can also make their own. (5) Experience the entire process of mathematics writing: determine the topic; collect data and material; analyze data and materials; write a paper in the form of group cooperation or individual; exchange, discuss, and demonstrate the paper. (6) Combine LRMW with mathematics practice activity and mathematical modelling.

Table 1. The information on LRMW

No	Book	Topic	Subject	Content structure
1	Compulsory 1 (pp.97–98)	Formation and rules of functions	Analysis	Target, Implement Suggestions, Reference Topics, Structure of the literature review
2	Compulsory 1 (pp.157)	The formation and development of the concept of the logarithm	Analysis	Topics, Implement Suggestions, Reference Topics
3	Compulsory 2 (p.166)	Development of geometry	Geometry	Target, requirements Process, communication
4	Selective compulsory 1 (p.142)	The formation and development of analytic geometry	Geometry	Topic, Implement Suggestions Reference Topics
5	Selective compulsory 3 (pp.101)	The creation and development of calculus	Analysis	Topic, Implement Suggestions, Reference Topics

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Tasks in Measuring in the Textbook Used in Teaching Indigenous Children

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1 Rationale and Theoretical Perspective

Mathematics textbooks are essential resources of mathematics learning and teaching (e.g., Fan et al., 2013). Examining their content, structure, and their classroom implementation is important (ibid). This paper reports about a study, which examines tasks of mathematics textbook used by children of the Sámi people, the Indigenous people of Norway. The textbook is a translation from the Norwegian one. Studies show that measuring can help children at upper primary to grasp mathematics' relevance to real-life (e.g., Gravemeijer et al., 2016), especially by using Indigenous culture as a starting point (e.g., Lipka et al., 2019). Measuring is a significant topic in connecting other topics (ibid). Tasks have significant role in mathematics learning (Sullivan et al., 2015). The central role of contextualized tasks to enhance mathematics learning and taking realistic phenomena as a starting point for teaching are often emphasized (e.g., Gravemeijer et al., 2016; Treffers, 1987). Lipka et al. (2019) suggests tasks, which are based on Indigenous cultural contexts. Studies focusing on Sámi contexts, have also suggested tasks about measuring (e.g., Siri & Hermansen, 2018) or provided an exposition of the cultural context of using measuring (e.g., Fyhn et al., 2021). This study draws on the reality principle of realistic mathematics education (RME), which, has two aspects: connection to real-life and being meaningful (Treffers, 1987). Drawing on the first aspect, the study attempts to answer the question: How relevant are the tasks about the topic measuring to North Sámi cultural context and way of life?

2 Method

Document analysis method is used. Following Bowen (2009), three stages were executed, which are skimming, thoroughly examining and interpreting. The stages are used to extract and analyze data from the textbook. Tasks are further examined using the reality principle. Tasks which are about real-life are coded as relevant. At a later stage such tasks are recoded as "relevant to Sámi" and "may not be relevant to Sámi".

3 Data Presentation

The chapter about measuring presents about volume, weight and time, with ample tasks. It has tasks with real-life context such as the following, "The train leaves Trondheim at 07.40. How long does it take to arrive at Værnes." (Alseth et al., 2011, p. 16). This task

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may not be relevant to most North Sámi children as there is limited train transport in the North Sámi inhibited areas.

4 Results and Final Remarks

Preliminary results indicate that the tasks about measuring are partly realistic to the students, but not relevant to the Sámi cultural context and way of life. On the other hand, the suggested tasks and contexts in earlier studies are opportunities to make the textbook relevant and it is vital to explore ways of incorporating those tasks, instead of mere translation from Norwegian. More studies are needed with a focus on such tasks. Adopting tasks from other similar Indigenous contexts is also an option. The current study focused on content. It remains to examine its implementation in the classroom.

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