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Dissipative Solitons in Fibre Lasers

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Abstract

Interdisciplinary concept of dissipative soliton is unfolded in connection with ultrafast fibre lasers. The different mode-locking techniques as well as experimental realizations of dissipative soliton fibre lasers are surveyed briefly with an emphasis on their energy scalability. Basic topics of the dissipative soliton theory are elucidated in connection with concepts of energy scalability and stability. It is shown that the parametric space of dissipative soliton has reduced dimension and comparatively simple structure that simplifies the analysis and optimization of ultrafast fibre lasers. The main destabilization scenarios are described and the limits of energy scalability are connected with impact of optical turbulence and stimulated Raman scattering. The fast and slow dynamics of vector dissipative solitons are exposed.

Keywords: Ultrafast fibre laser, mode-locking, dissipative soliton, non-linear dynamics, vector solitons, optical turbulence, stimulated Raman scattering

1. Introduction

Over the last decades, ultrafast fibre laser technologies have demonstrated a remarkable progress. By definition [1–4], these technologies concern generation, manipulation and application of optical pulses from a fibre laser or a laser-amplifier system with (i) peak power P_0 exceeding substantially an average laser power P_{ave} and (ii) pulse widths τ which are much lesser than a laser round-trip period T. Such a definition can be re-interpreted in terms of a laser *mode-locking*, which means a phase-locked interference of the M laser eigenmodes producing an equidistant train of ultrashort pulses. Then, $\tau \propto 1/M\delta\omega$ (M is a number of locked eigenmodes and $\delta\omega$ is an inter-mode frequency interval defined by $T \propto 1/\delta\omega$) and $P_0 \propto MP_{ave}$ [5,6]. It means that a mode-locked laser generates a comb of equidistant optical frequencies comprising the broad spectral range $\Delta \propto M\delta\omega$. It is clear that the substantial enhancement of P_0 (by the factor of $M \propto 1/\tau\delta\omega \sim 10^5 \div 10^6$, i.e. upto over-MW level [7,8]) and τ–reduction (~T/M,

i.e. down to sub-100 fs level [9]) promise an outlook for different applications [10] including non-linear and ultrasensitive laser spectroscopy [11–14], biomedical applications [15–19], micromachining [20,21], high-speed communication systems [22], metrology [23] and many others [24,25]. The extraordinary peak powers in combination with the drastic pulse width decrease bring the high-field physics on tabletops of a mid-level university lab [26–29]. Moreover, the over-MHz pulse repetition rates $\delta\omega$ provide the signal rate improvement factor of $10^3 \div 10^4$ in comparison with that of classical chirped-pulse amplifiers [26]. As a result, the signal-to-noise ratio enhances substantially, as well.

Another aspect of the ultrafast laser applications is connected with studying non-linear phenomena [30]. Ultrafast lasers became an effective platform for investigation of general non-linear processes such as instabilities and rogue waves [31,32], self-similarity [33] and turbulence [34]. A coherent self-organization in such non-linear systems [35,36] is the keystone of this review, and it will be considered below in detail. But here, we have to point at the multidisciplinary context of our topic. The ultrafast fibre lasers can be treated as an ideal playground for exploring of non-linear system phenomenology as a whole [37]. Such a playground spans gravity and cosmology [38], condensed-matter physics and quantum field theory [39–41], biology, neurosciences and informatics [42,43]. The advance of ultrafast laser technology is that the theoretical insights promise to become directly testable, controllable and, on the other part, the theory can be urged by new precise measurable experimental challenges.

To date, the solid-state lasers allowed generating shortest pulses with highest peak powers directly from an oscillator with high repetition rates ($\delta\omega > 1$ MHz) [44–49]. The main advantages of solid-state laser systems are (i) broad gain bands (i.e. very large M) allowing generation of extremely short pulses (τ approaches one optical cycle for such media as Ti:Sp or Cr:chalcogenides) and (ii) covering the spectral range from visible (Ti:Sp) through infra-red (Ti:Sp, Cr:forsterite and Cr:YAG) to mid-infrared (Cr:chalcogenides) wavelengths, as well as (iii) possibility of independent and precise dispersion [50] and non-linearity [46,48] control. Nevertheless, fibre lasers have unprecedented prospects [51] due to (i) possibility of mean power scaling provided by large gain, (ii) high quality of laser mode, (iii) reduced thermo- and environment-sensitivity, (iv) compactness and integrity of laser setup. Additionally, one has to point at broader gain bands of fibre media in comparison with the energy-scalable, thin-disk, solid-state oscillators operating within analogous wavelength ranges [9,52][1] and possibility to break into deep-UV and mid-IR optical spectral ranges [55,56].

In this review, we will concern the concepts of a mode-locking and a dissipative soliton in a nutshell.

2. Mode-Locking

The concept of mode-locking is universal and closely connected with a principle of synchronisation of coupled oscillators [57–64]. A laser is, in fact, the interferometer which possesses a

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1 The width of a gain band is not a decisive factor per se because both pulse width and its spectrum are affected by various factors including higher-order dispersions, non-linearity, etc. [53,54].

set of eigenmodes (longitudinal modes) separated by $\delta\omega = 2\pi/T$. Simultaneously, it is an *active* resonator, which means an amplification ΔA of mode amplitude A during the resonator round-trip in the vicinity of the maximum gain frequency ω_0 as $\Delta A(\omega) \approx (g(\omega_0) - \ell)A(\omega) - \alpha(\omega - \omega_0)^2 A(\omega)$, where $g(\omega_0)$ is a gain at the frequency ω_0, ℓ is a net-loss coefficient and $\alpha \propto g(\omega_0)/\delta\Omega^2$ takes into account a frequency-dependence of gain coefficient in the vicinity of ω_0 defined by a gain bandwidth $\delta\Omega$. In such an oversimplified model, only *one* mode with the maximum net-gain $\sigma \equiv g(\omega \to \omega_0) - \ell = 0$ is generated because the gain coefficient is energy-dependent, that is, it decreases with A (i.e. a gain is *saturable* that results in a mode competition or *mode selection*, Figure 1).
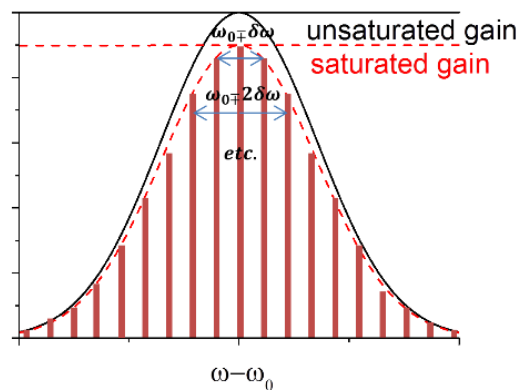

Figure 1. Comb of frequencies generated by a laser with the repetition rate $\delta\omega$. The gain band is centred at ω_0. Mode-locking consists in excitation and synchronisation of $\omega_0 \pm n\delta\omega$ sidebands.

In the case of *active mode-locking*, a periodic external modulation with the frequency of $\delta\omega$ excites the $\pm\delta\omega$ sidebands for each mode in the comb so that the modes $A(\omega)$, $A(\omega \pm \delta\omega)$ become coupled. In the framework of our oversimplified model, a steady-state regime $\Delta A = 0$ is described by the equation in time domain [59–61]:

$$\left(g - \ell\right)A(t) + \alpha\frac{\partial^2 A(t)}{\partial t^2} - \nu t^2 A(t) = 0, \tag{1}$$

which is the classical equation for an oscillator in the potential defined by $\nu \propto \delta\omega^2$. This equation has a trivial solution in the form of a Gaussian pulse [59–63]: $A(t) = A_0 \exp\left(-2\ln(2)\, t^2/\tau^2\right)$, where the pulse amplitude A_0 is defined by the condition of energy balance of $\sigma \propto \nu\tau^2$ (the saturable gain coefficient is energy, i.e. A_0^2-dependent) and the pulse width $\tau \propto \sqrt[4]{\alpha/\nu}$. In the general case, the excitation of $A(t)$ in the form of Hermitian–Gaussian solutions of Equation (1) is possible. Since the pulse width is defined by ν so that $\tau \propto 1/\sqrt{\delta\Omega\delta\omega}$, the minimum pulse widths of $\sim 1/\delta\Omega$ are hardly reachable due to limitation imposed by $\delta\omega$-value. The situation can be changed in the presence of the self-phase modulation (SPM) [65] and the dynamic gain saturation. Then $\tau \propto 1/\delta\Omega\sqrt{\sigma}$ [66].

Using the non-linear processes such as SPM, loss and gain saturation allows generating the ultrashort pulses due to mechanism of the so-called *passive mode-locking* [60]. Periodical perturbations caused by transitions through non-linear laser elements such as saturable absorber or gain medium enrich the spectrum with new components $\omega_0 \pm m\delta\omega$ ($m = 1, 2,..., M$), which becomes locked through non-linear interaction [64]:

$$\frac{\partial A(\omega_1,z)}{\partial z} = -\frac{i\pi\omega_1}{2cn(\omega_1)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega_2 d\omega_3 d\omega_4\, \chi^3(\omega_1,\omega_2,\omega_3,\omega_4)\times$$
$$\times A(\omega_2,z)A(\omega_3,z)A(\omega_4,z)e^{i\Delta kz}\delta(\omega_1-\omega_2-\omega_3-\omega_4),$$

(2)

where a four-wave non-linear process defined by non-linear susceptibility χ^3 mixes the frequencies ω_1, ω_2, ω_3 and ω_4 during the propagation through a non-linear medium along the z-coordinate ($\Delta k = k(\omega_1) - k(\omega_2) - k(\omega_3) - k(\omega_4)$ is the difference of wavenumbers, c is the speed of light, n is the frequency-dependent refractive index).

Both active and passive mode-locking concepts can be easily united from the point of view of spice-time duality [64,67–69]. For instance, let's consider *heat diffusion* equation:

$$\frac{\partial u}{\partial t} = \sigma u + \zeta \frac{\partial^2 u}{\partial x^2} - vx^2 u,$$

(3)

where a heat radiated at $x = 0$ by the point source σ diffuses along x-axis and is absorbed by cooler with the parabolic 'cooling potential'. The replacements $t \to z$ and $x \to t$ result in an equation for 'diffusion' of light describing an *active amplitude mode-locking* (see Eq. (1)):

$$\frac{\partial A}{\partial z} = \sigma A + \alpha \frac{\partial^2 A}{\partial t^2} - vt^2 A.$$

(4)

Eq. (4) is clearly understandable in the Fourier domain: $\frac{\partial \widetilde{A}}{\partial z} = \sigma \widetilde{A} + v \frac{\partial^2 \widetilde{A}}{\partial \omega^2} - \alpha\omega^2\widetilde{A},$ where an external (i.e. active) modulation defined by the v-coefficient 'diffuses' (i.e. broadens) a field spectrum $\widetilde{A}$ and such diffusion is compensated by spectral dissipation defined by the α-coefficient. In the time-domain, the spectrum broadening corresponds to a light pulse shortening due to parabolic potential action which is balanced by spectral dissipation causing a pulse widening.

The space-time duality can be extended further with the help of *diffraction-dispersion* duality:

$$\frac{\partial A}{\partial z} = -\frac{i}{2k}\frac{\partial^2 A}{\partial x^2} \;\leftrightarrow\; \frac{\partial A}{\partial z} = \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial t^2},$$

(5)

where k and β_2 are the wave number and group-delay dispersion coefficients, respectively. Both processes describe the beam/pulse spreading with propagation which is accompanied by phase ϕ profile distortion, i.e. by appearance of the *chirp* $Q \propto d^2\phi/dx^2$ (or $d^2\phi/dt^2$). The *active phase modulation* from this point of view

$$\frac{\partial A}{\partial z} = \sigma A + \alpha \frac{\partial^2 A}{\partial t^2} - iv t^2 A \tag{6}$$

(compare with (4)) looking as $\dfrac{\partial \tilde{A}}{\partial z} = \sigma \tilde{A} + iv \dfrac{\partial^2 \tilde{A}}{\partial \omega^2} - \alpha \omega^2 \tilde{A}$ in the Fourier domain describes a 'diffraction' (dispersion) in the frequency domain inspired by phase modulator which is balanced by spectral dissipation. The main difference from (4) is that the phase modulation in (6) distorts the phase and thereby produces chirp like the action of thin lens but in the time domain. In other words, the phase modulation in (6) pushes the spectral components out of the point of stationary phase $\partial\phi/\partial t = 0$, adding the frequency shift $\propto vt$ (Doppler shift) which enhances the spectral dissipation on the pulse wings and thereby forms a pulse like the active amplitude modulator. But the phase profile $\phi(t)$ is parabolic in this case.

The transition to a *passive mode-locking* looks straightforward, but one has to be careful in this case. The spice-time duality suggests a simple way to realize the temporal focusing like that in space domain: combination of phase modulation ('time lens') from Eq. (6) with dispersion ('time diffraction') from Eq. (5) allows compressing a pulse. Therefore, a replacement of time focusing by a time self-focusing (SPM) would provide a laser pulse self-trapping like the effect of laser beam self-trapping:

$$\frac{\partial A}{\partial z} = \frac{i}{2}\beta_2 \frac{\partial^2 A}{\partial t^2} - i\gamma |A|^2 A, \tag{7}$$

which is the famous *non-linear Schrödinger equation* describing propagation of optical solitons in a fiber ($\beta_2 < 0$ corresponds to an anomalous dispersion, γ is a SPM-coefficient) [35,70,71].

It is appropriate to mention here that the space-time duality $x \to t$ allows extending the physical context of consideration beyond scopes of optics. For instance, A, $E = \int dx\, |A|^2$, and ϕ can be related for a mean-field amplitude, number of particle (mass of condensate) and momentum (wave number) for a Bose–Einstein condensate [39]. Then, it is clear that the dispersion and SPM terms in Eq. (7) describe the kinetic energy and four-particle interaction potential for gas of bosons. Such an interpretation opens a road to a quantum theory of solitons [72–74].

Following the same procedure for Eq. (4), describing the active amplitude mode-locking results in the simplest version of equation for a passive mode-locking, so-called *cubic non-linear Ginzburg–Landau equation* [35,36,75]:

$$\frac{\partial A}{\partial z} = \sigma A + \alpha \frac{\partial^2 A}{\partial t^2} + \kappa |A|^2 A. \tag{8}$$

This equation describes a combined action of saturated net-gain (σ), spectral dissipation (α) and non-linear gain (κ). The last term results from loss saturation in a non-linear absorber with the response time much lesser than the pulse width. As will be shown below, such an assumption is valid for a broad class of fibre mode-locking mechanisms. Physics of passive mode-locking resembles that of active one: self-focusing in time domain causes a spectrum broadening which is balanced by spectral dissipation. Loss and energy-dependent gain are required for developing and stabilizing the mode-locking (all these factors are included in σ-term which is < 0 for a steady-state pulse). Eqs. (7) and (8) have a similar solution $A(t) \propto \mathrm{sech}(t / \tau)$ but the mathematical structures of these equations differ substantially that created discrepancies between concepts of the 'true' [76] and *dissipative solitons* (DSs, see next section) [36]. Combining Eqs. (7) and (8) gives the famous complex cubic non-linear Ginzburg–Landau equation (cubic CNGLE) [35–37,42]:

$$\frac{\partial A}{\partial z} = i \left(\frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - \gamma |A|^2 A \right) + \sigma A + \alpha \frac{\partial^2 A}{\partial t^2} + \kappa |A|^2 A, \tag{9}$$

which is a playground for study of DSs. Equation (9) allows a number of further generalizations such as: (i) description of non-distributed evolution due to dependence of the equation coefficients on z [77]; (ii) generalization of non-linearity type aimed first of all to adequate description of different mode-locking mechanisms (see below); (iii) taking into account the higher-order dispersions, i.e. ω-dependence of β_2 [68]; (iv) taking into account the vector nature of light, i.e. transition to a system of coupled two-component CNGLEs [68,78–81], etc.

Now let us consider the mode-locking mechanisms for fibre lasers in more detail. Active mode-locking can be utilized for DS generation from a fibre laser [82–84], but the widespread mechanism is based on the *non-linear polarization rotation* (NPR) which uses the effect of intensity-dependent polarization mode coupling in a fibre [85–88]. There is voluminous literature concerning the experimental realization of NPR mode-locking in fibre lasers; therefore, our selection of references is rather subjective and concerns the DS context [95–111].

It is known [70] that an ideal single-mode fibre supports two degenerate orthogonally polarized modes. However, a real fibre has inherent birefringence caused by core asymmetry or mechanical stress (Figure 2).

Since SPM as well as cross-phase modulation (XPM) contribute to refractivity index with the strength defined by field intensity, such a contribution will change the state of polarization (SOP, Figure 3) [60,70,89] that can be described by coupled equations for two orthogonal (x and y) polarization components [70]:

Noncircular core   Mechanical stress

Figure 2. Fibre cross-sections with cylindrical symmetry broken by manufacturing process that leads to fibre birefringence.
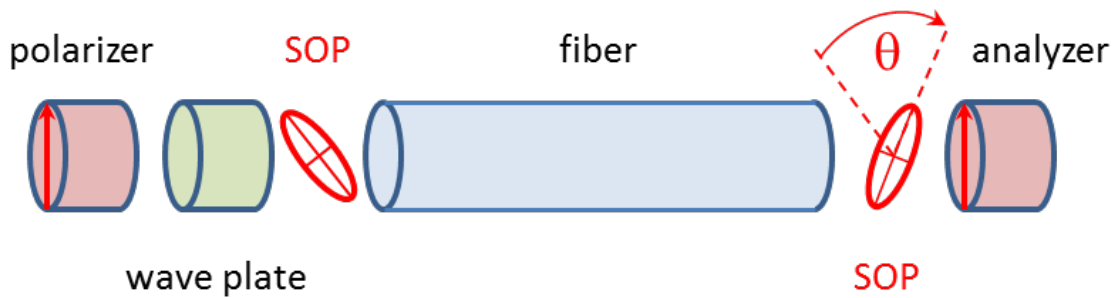

Figure 3. Block scheme of a self-amplitude modulator (SAM) based on NPR. Both linear birefringence and NPR contribute to the change of state of polarization (SOP, rotation angle θ) that leads to intensity-dependent transmission of this scheme.

$$
\frac{\partial A_x}{\partial z} + \beta_{1x} \frac{\partial A_x}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 A_x}{\partial t^2} - \sigma A_x - \alpha \frac{\partial^2 A_x}{\partial t^2} =
$$
$$
= -i\gamma \left(\left| A_x \right|^2 + \frac{2}{3} \left| A_y \right|^2 \right) A_x - \frac{i\gamma}{3} A_x^* A_y^2 e^{-2i\Delta\beta z},
$$
$$
\frac{\partial A_y}{\partial z} + \beta_{1y} \frac{\partial A_y}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 A_y}{\partial t^2} - \sigma A_y - \alpha \frac{\partial^2 A_y}{\partial t^2} =
$$
$$
= -i\gamma \left(\left| A_y \right|^2 + \frac{2}{3} \left| A_x \right|^2 \right) A_y - \frac{i\gamma}{3} A_y^* A_x^2 e^{2i\Delta\beta z},
$$

(10)

where the dissipative factors from Eq. (9) are taken into account and $\Delta\beta = 2\pi / L_b$ describes a 'strength' of linear birefringence (L_b is a beat-length). As was shown in [81,91–94,179], the multiscale averaging technique allows reducing Eq. (10) to the modified scalar non-linear Ginzburg–Landau equation (*so-called sinusoidal Ginzburg–Landau equation*) in which the self-amplitude modulation term (SAM, last term in Eq. (9)) is replaced by $\propto \log Q(|A|^2)A$, where Q is a complex function defined by birefringence and settings of laser wave plates and polarizer.

Such an approach opens a way to multi-parametrical optimization of fibre lasers mode-locked by NPR.

Despite its relative simplicity in principle as well as possibility of all-fibre-integrity of a laser, NPR in the form presented in Figure 3 is too sensitive to laser setup, uncontrollable perturbations and requires a precise manual tuning. The modified SAM setup, which can utilize both NPR and scalar SPM, is shown in Figure 4. It is the so-called *non-linear optical loop mirror* or *figure eight laser* (Figure 4) [112,113,265]. In principle, this setup is an all-fibre realization of additive-mode-locking [82,114] with inherently adjusted linear optical propagation lengths for counter-propagating beams. The main control parameter here is the beam splitting ratio ρ controlling the mutual intensities of counter-propagating beams.

The unique property of this SAM setup is its ability to utilize different types of non-linearities for mode-locking (e.g. see [115–118]). Different modifications of this mode-locking mechanism have been used in DS fibre lasers [119–125]. Nevertheless, a fibre loop defining SAM remains environment- and tuning-sensitive.
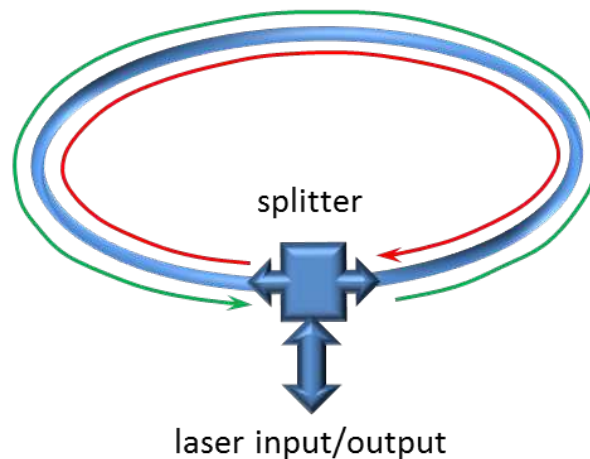

Figure 4. Block-scheme of SAM based on a non-linear optical loop mirror. Splitter splits input laser beam into two counter-propagating ones (red and green arrows) with some splitting ratio ρ. The beams interfere after round-trip and partially return into a laser. The result of interference is intensity-dependent due to NPR or/and SPM within a loop.

There is a class of alternative approaches utilizing non-fibre well-controllable non-linearities for mode-locking by the cost of broken fibre-integrity of a laser. Such an alternative was provided by development of high-non-linear *semiconductor saturable absorber mirror* (SESAM) [126–135]. The point is to put a semiconductor layer into a composed multi-layer mirror with the well-controllable spectral characteristics as well as with the adjustable intensity concertation of penetrating field within a semiconductor layer. In fact, it is an advanced non-linear Fabri–Perot interferometer with the reflectivity coefficient depending on the incident intensity (or energy) [136]. Interaction of light with a semiconductor layer can be characterized roughly as excitation of carriers from a valence band of semiconductor to its conduction band. Excited carriers thermalize inside a conduction band with the character time ~100 femtoseconds. This

time defines a fastest response of SESAM to a laser radiation. Then, the thermalized carriers can relax into valence band or intra-band trapping states with the characteristic times from picoseconds to nanoseconds. Thus, SAM due to SESAM is *slow* in comparison with that due to NPR or SPM because the response times of the lasts are defined by intra-atom polarization dynamics, i.e. these times belong to femtosecond diapason. Additionally, the *spectral diapason* of SESAM response is substantially squeezed in comparison with that of pure electronic non-linearities due to resonant character of SESAM non-linearity. This can trouble the mode-locking within a spectral range exceeding the SESAM bandwidth. But the reverse side of the SESAM-band squeezing is that a non-linear response of SESAM becomes resonantly enhanced. This means that SESAM can provide more easily starting, stable and controllable mode-locking. The key characteristics of SESAM are [127] *loss saturation fluence* $E_s = h\nu / 2\sigma_a$ ($h\nu$ is a photon energy, σ_a is an absorption cross section), *modulation depth* $\mu_0 = \sigma_a N$ (N is a density of states in semiconductor), *relaxation (recovery) time* T_r, *unsaturable loss*, *saturable loss bandwidth* and level of *two-photon absorption*.

Akin mode-locking methods providing full fibre-integrity, broadband absorption, sub-picosecond response time and avoiding a complex multi-layer mirror weaving use nanotube and graphene saturable absorbers [30,137–143] and other low-dimensional structures [144].

From the theoretical point of view, the response of saturable absorber (SESAM or other quantum-size structures) to a laser field can be very complicate. In principle, one has to take into account finite loss bandwidth, its dispersion, dependence of refractive index on carrier's (or exciton's) density (so-called linewidth enhancement), complex kinetics of excitation and relaxation, etc. However, the praxis demonstrated that a simple model of two-level absorber is well working [145]:

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} = -\frac{\mu - \mu_0}{T_r} - \mu \frac{\left|A\right|^2}{E_s} \tag{11}$$

with some possible modifications (e.g. see [146]). Since DSs, as a rule, have over-picosecond widths (see next section), one may use an adiabatic approximation for (11) so that the expression for SAM coefficient in the last term in Eq. (9) has to be replaced:

$$\kappa \leftrightarrow (-1) \times \mu = \frac{\mu_0}{1 + \left|A\right|^2 / P_s}, \tag{12}$$

where $P_s = E_s / T_r$ is a loss saturation power.

One may propose a hypothesis that an analogue of Kerr-lens mode-locking, which is a basic mechanism for generation of femtosecond pulses from solid-state lasers [60,85,147], can be realized in a fibre laser as well. Such an insight is based on possible enhancement of the laser beam spatial-trapping induced by non-linearity in a medium with spatially inhomogeneous

gain/loss or refractivity [148–152]. The model for analysis of such phenomena can be based on extension of dimensionality of Eq. (9), with taking into account the diffraction and transverse inhomogeneity of gain, loss or/and refractive index (the last can work as SAM due to the waveguide leaking loss) [153]:

$$\frac{\partial A}{\partial z} = i\left(\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{1}{2k}\frac{\partial^2 A}{\partial x^2} - \gamma|A|^2 A\right) + \sigma A + \alpha\frac{\partial^2 A}{\partial t^2} - \kappa x^2,$$
(13)

where cylindrical symmetry is assumed, x is a radial coordinate and κ is a coefficient (complex in general case) which describes a transverse inhomogeneity of a fibre. Figure 5 shows the net-gain profiles (*a*) and the intra-laser pulse energies (*b*) as function of an effective aperture size obtained on the basis of variational approach for Eq. (13) [153]. The results demonstrate a principal feasibility of the Kerr-lens mode-locking regime for a DS fibre laser.
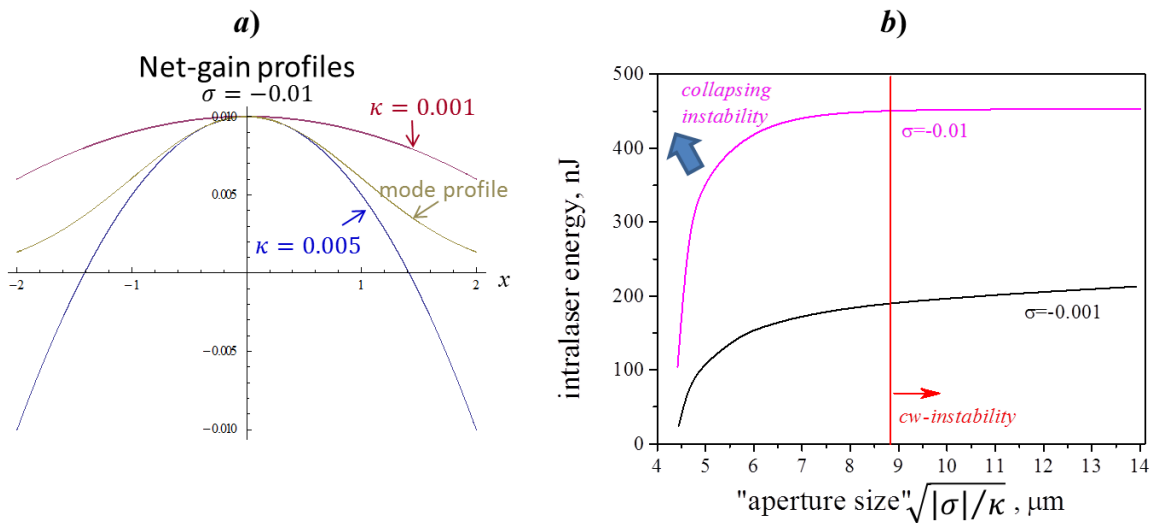

Figure 5. (a) Transverse net-gain profiles for different transverse parabolic distributions of net-gain coefficient which can be realized by inhomogeneous doping of fiber or by impact of waveguide leaking loss. (b) The dependence of in-tra-laser DS energy for a DS Yb-fibre laser with the 14 nm filter bandwidth and the average GDD of 330 fs^2/cm [153]. DS collapses for large energies and cannot start for large aperture sizes (here SAM has inverse sign and the continu-ous-wave generation prevails).

All these mode-locking techniques are realizable for both soliton proper and DS fibre lasers (excluding the Kerr-lens mode-locking which requires sufficiently high pulse energies provided by only a DS laser). Now let's consider the DSs fibre lasers proper.

3. DS concept: Theory and experiment

A 'classical' soliton can be formally defined as a solution of non-linear evolution equation belonging to discrete spectrum of the inverse scattering transform [71,76,154]. The non-linear

equations, which can be solved by inverse scattering transform, are 'exactly integrable'. This means that they are akin to linear equations in some sense. In particular, they obey the superposition principle and, as a result, can be canonically quantized [155,156]. One has to note that integrability of a non-linear evolution equation and non-dissipative (non-Hamiltonian) character of the latter are not equivalent because there are both non-integrable Hamiltonian systems and integrable dissipative ones [36]. The point is that the DS concept is not connected with 'integrability'; therefore, DSs are not 'true' solitons in a mathematical sense. However, many properties of DSs, in particular, their stable localization, robustness in the processes of scattering and interaction, well-organized internal structure, etc., resemble the properties of 'true' solitons. Formally, one may define *DS* as a *localized and stable structure emergent in a non-linear dissipative system far from the thermodynamic equilibrium* [36]. DSs are abundant in the different natural systems ranging from optics and condensed-matter physics to biology and medicine. In this sense, one may paraphrase that DSs "are around us. In the true sense of the word they are absolutely everywhere" [157]. Therefore, the concept of DS became well established in the last decade [36,37,42,158].

Stability of a DS under condition of strong non-equilibrium can be achieved only due to well-organized energy exchange with environment and subsequent energy redistribution within a DS. It results in energy flux inside a DS and, thereby, in DS phase inhomogeneity [36]. For a simplest case of Eq. (9), which has a DS solution in the form of $A(t) = A_0 \mathrm{sech}(t/\tau)^{1+i\psi}$ ($\psi \propto Q\tau^2 = \tau^2 \partial^2 \phi / \partial t^2$ is a dimensionless chirp parameter) [85,159,160], the DS energy generation [36]

$$\mathrm{E} = 2\sigma |A|^2 + 2\kappa |A|^4 - 2\alpha \left| \frac{\partial A}{\partial t} \right|^2 + \alpha \frac{\partial^2 |A|^2}{\partial t^2} \qquad (14)$$

as well as the spectrum $|\tilde{A}(\omega)|^2$ are shown in Figure 6 in dependence of β_2 (the data are based on an approach of [160]). One can see that the spectrum broadening transfers the action of spectral dissipation on a pulse 'in whole' into well-structured energy exchange: inflow at pulse centrum and outflow on its wings. The key characteristic of a dissipation inhomogeneity is a *chirp*, i.e. an inhomogeneity of phase. In absence of the chirp, the spectral dissipation acts on a pulse in whole that, in particular, induces a multi-pulse instability [161]. However, a power-dependent chirp causes inhomogeneity of energy transfer (Figure 7). Energy flows in the region closer to central wavelength where the gain is maximal. This region is located in the vicinity of pulse maximum. Energy flows out from the spectrum wings which are located on the wings of pulse, that is, the pulse localization is supported by spectral dissipation through non-linear mechanism of chirping [42,160,162]. One has to note that a direction of energy fluxes inside a DS depends on parameters and can be inversely related to the direction shown in Figures 6,7 (i.e. energy can flow from wings to centre). The corresponding structure was named *dissipative anti-soliton* [210].

Thus, an additional mechanism of SAM (in addition to mechanisms considered in the previous section) appears, which provides unique robustness of DSs (i.e. DS exists within a broad range of laser parameters [163,164]).
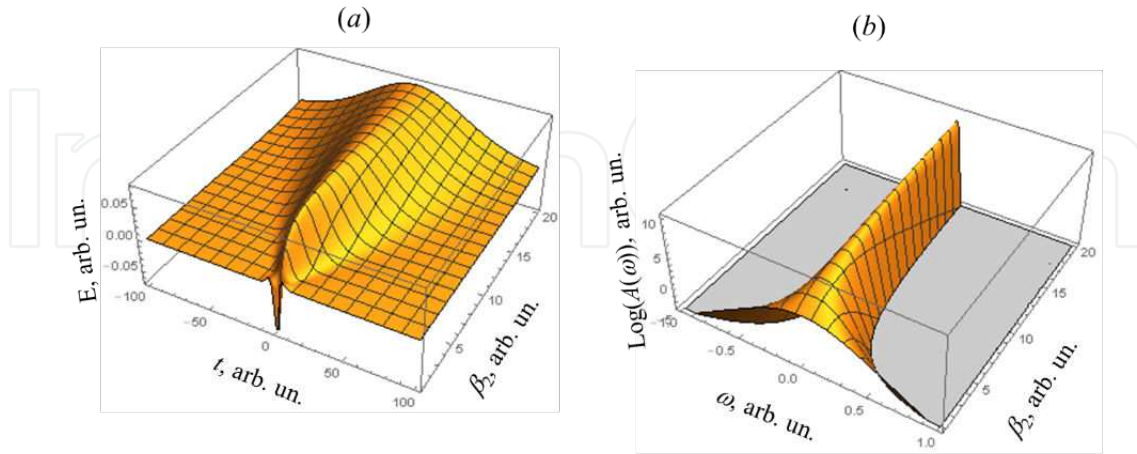

Figure 6. (a) Profile of energy generation and (b) logarithm of spectral power in dependence on GDD (normal dispersion range) for a DS of [160].

Below, we will consider a chirp as the essential characteristic of DS [210]. One of the reasons is that the chirp allows DS to accumulate energy $E \propto \psi$, which means that DS is *energy-scalable* [37,164–167]. The last statement does not mean that a chirp-free pulse is not energy-scalable. However, the energy-scalability of such pulses can be provided by only fine-tuned and separated control of SPM and GDD that can be achieved in solid-state oscillators [26,168] or in large mode area (LMA) fibre lasers [169]. For fibre lasers such an approach entails issues of full-fibre integrability, higher-order mode control [170,171][2] and thermo-effects impact [172].
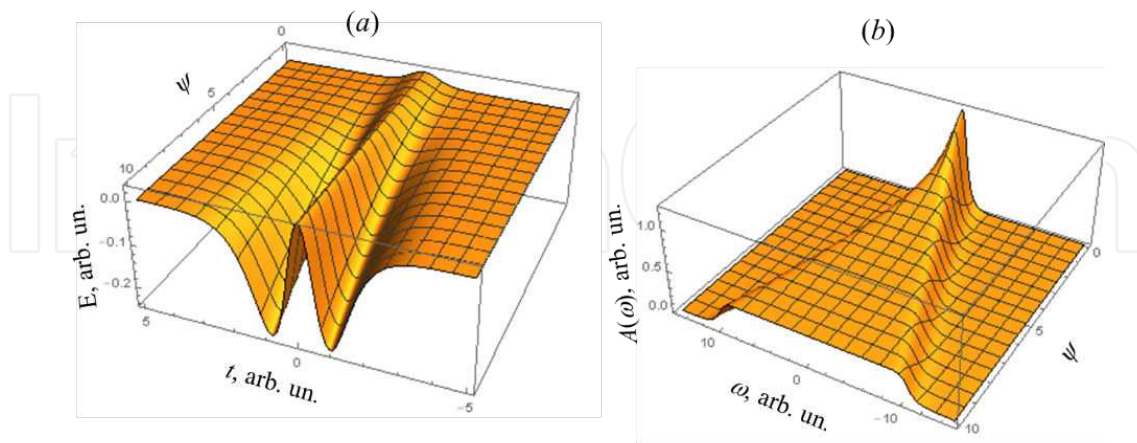

Figure 7. (a) Profile of energy generation and (b) power in dependence on chirp for a DS of Eq. (9): $\sigma = -0.01$, $\kappa = 0.025$, $\alpha = 0.05$, $A_a = 1$, $\tau = 1$.

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2 However, namely LMA and photonic-crystal fibres could realize a Kerr-lens mode-locking in a fibre laser [152,153].

In the terms of space-time duality (see above), the mechanism of formation of time window, within which a DS is localized, resembles a phenomenon of total internal reflection from some 'borders' created by phase discontinuity. Such borders are formally defined by the equivalence of the wave number of out-/in-going radiation $k(\omega)=\beta_2\omega^2/2$ (wave number of dispersive linear wave) and the DS wave number $q=\gamma P_0 : k(\pm\Delta)=q$, where $P_0=|A_{max}|^2$ is a DS peak power and a DS spectral width is $\Delta=\sqrt{2\gamma P_0/\beta_2}$ [173]. Since a system is dissipative, the above phase equilibrium has to be supplemented by loss compensation condition: spectral loss $\propto \alpha\Delta^2$ has to be compensated by non-linear gain κP_0. Combination of above criteria gives a definition of the parametric limits for DS [44,173]:

$$\frac{2\alpha\gamma}{\beta_2\kappa} \le \begin{cases} 2/3, \, E\to\infty \\ 2, \, E\to 0 \end{cases}, \tag{15}$$

where E is a DS energy. Eq. (15) is valid for the *cubic-quintic CNGLE* in which SAM has a form of $(\kappa|A|^2-\kappa\zeta|A|^4)A$, that is, a non-linear gain is saturable (despite the unsaturable SAM in Eq. (9)). A SAM saturability is necessary for DS stabilization [174]. Equality in Eq. (15) corresponds to the DS stability border where $\sigma = 0$ (see Eq. (9)). The asymptotic $E\to\infty$ corresponds to a *perfectly energy-scalable* DS or to a phenomenon of *dissipative soliton resonance* (DSR) [37,44,165–167,175–183], which is sufficiently robust, exists in different SAM environments and even within the anomalous GDD range [178,182]. Important property of DSR is that the DS energy E can be scaled without loss of stability by plain scaling of laser average power or/and its length L [44,180,184]. The chirp scales with length as well. As a result, the DS peak power and spectrum width tend to a constant for fixed parameters of Eq. (9) (i.e. fixed α, β_2, σ and κ) and the energy scaling is provided by DS stretching in time domain.

This ideology of energy scaling by the pulse stretching goes back to the so-called wave-breaking-free or stretched pulse fibre lasers where the propagation within the anomalous-dispersion fibre sectors alternates with the propagation under normal GDD action [96,101,185–187]. As a result of pulse stretching, the non-linear effects in such systems are reduced, which allows increasing an energy and suppressing a noise. As an alternative approach, one can exclude an anomalous GDD at all and to realize a so-called *similariton* regime, when a pulse accumulates an extremely large chirp and, thereby, an energy [33,103,188]. However, a self-similar regime is not soliton-like one in nature; therefore, we will focus on the all-normal-dispersion fibre lasers (ANDi) which produces DSs possessing a high stability within a broad range of laser parameters [97,101,185,186,189,190]. In Figure 8, the energy-scalable DS lasers are sub-divided into three main types: (1) *all-fibre*, (2) *fibre with a free-space sector* and (2) LMA including rod and photonic-crystal fibre PCF.

The advantage of the *first type* of lasers is their integrity, which does not require an operational alignment, includes potentially compressing and delivering sections, and provides environment insensitivity and easy integrability with fibre-amplifier cascades [200]. The last advantage is especially attractive because it allows a direct seeding of DS into chirp-pulse amplifier

without preliminary pulse stretching. The *second type* of the DS fibre lasers can be considered as a testbed for development of the first type. No wonder that the results achieved here are more impressive (Figure 8). At last, the *third type* of the DS fibre lasers is most akin to the thin-disk solid-state ones with simultaneous advantage of the broad gainbands. Such lasers provide the DS energy scalability by scaling of laser beam area in combination with the scaling of laser period and average power. Nevertheless, one has to keep in mind that both LMA and PCF technologies have some disadvantages (see above) which make them similar to solid-state lasers.
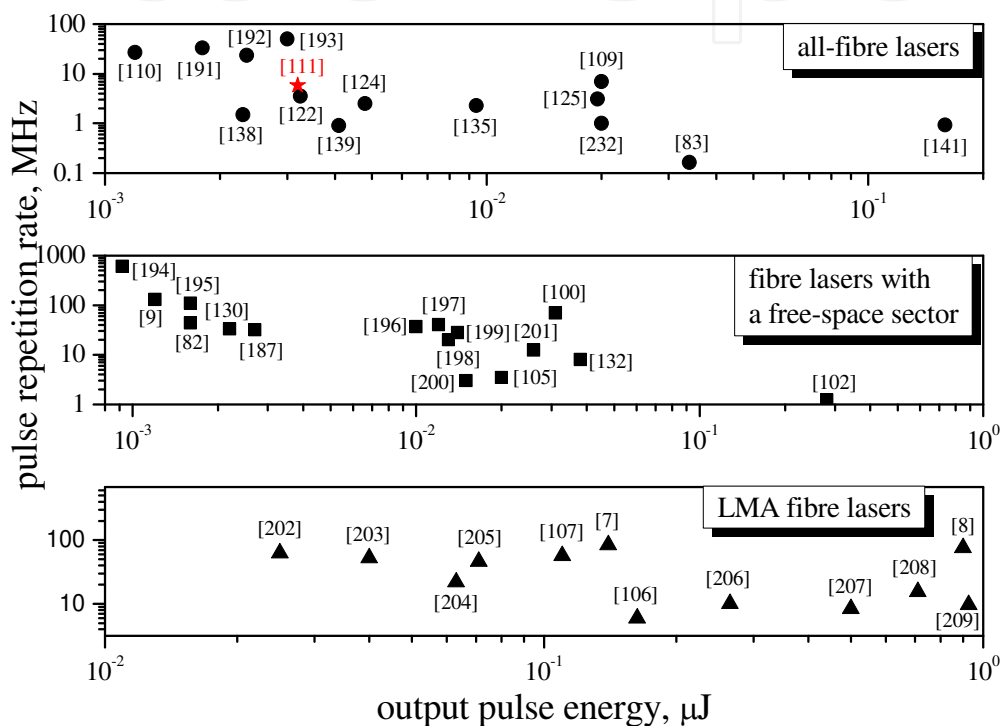

Figure 8. Experimental realizations of DSs in fibre lasers (Refs. [7-9,82,83,100,102,105-107,109-111,122,124,125,130,132,135,138,139,140,141,187,191-209,232]; Ref. [111] corresponds to a DS in the anomalous GDD region).

The diversity of the results obtained (Figure 8) needs a comprehension from a *unified view-point*; therefore, let's survey briefly some theoretical aspects relevant to the DS fibre lasers. There is vast literature regarding the theory of DSs. Some preliminary systematization can be found in [44]. However, it is necessary at first to declare the stumbling block of this theory: *absence of a unified viewpoint*. There exist unbroken walls between the circles of scientific community exploiting and exploring the DS concept: walls between the solid-state and fibre laser representations of the theory, condensed-matter one, numerical and analytical approaches, etc. Briefly and conditionally, the relevant theoretical approaches can be divided into (1) numerical, (2) exact analytical and (3) approximated analytical. The last includes the models based on (3.1) perturbative, (3.2) adiabatic models (AM) as well as those based on (3.3) phase-space truncation (i.e. variational approximation (VA) and method of moments (MM)).

As was emphasized repeatedly, both linear and non-linear dissipations are crucial for the DS formation. The simplest and most studied models for such a type of phenomena are based on the different versions of CNGLE (e.g. Eq. (9)).

Extensive *numerical study* of DSs of the cubic-quintic CNGLE has been carried out by N. N. Akhmediev with co-authors [35–37,42,157,166,175,176,178,182]. The simulations have allowed finding the DS stability regions for some two-dimensional projections of CNGLE parametrical space. The summarizing description of the results obtained is presented in [44]. Most impressive results are: (i) parametric space of DS has a reduced dimensionality resulting, in particular, in the appearance of DSR; (ii) DSR remains in a model with lumped evolution that is typical for the most of fibre lasers; (iii) DSR and, correspondingly, DS exist within the anomalous GDD region as well. However, the main shortcomings of the numerical approaches are: the parametrical space under consideration is not physically relevant, and the true dimensionality of DS parametric space is not identified. It is clear that the only advanced and self-consistent *analytical* theory of DS would provide, in particular, a true representation of DS parametric space and DSR conditions.

As was mentioned above, the evolution equations describing DSs are not-integrable. The efforts based on the algebraic techniques [62,213,214] and aimed to finding the generalized DS solutions of CNGLE were not successful to date. Nevertheless, few exact partial DS-solutions are known. For instance, *sole known exact analytical DS-solution* of cubic-quintic CNGLE is [110,166,176,182,189,211,212]:

$$A = \sqrt{\frac{A_0}{\cosh(t/\tau) + B}} \exp\left[\frac{i\psi}{2}\ln\left(\cosh(t/\tau) + B\right) + i\phi z\right],$$ (16)

where A_0, B, τ, ψ and ϕ are real constants [189]. This solution belongs to a fixed-point solution class, which means that it exists only if some constraints are imposed on the cubic-quintic CNGLE parameters. Solution (16) provides with important insights into properties of DSs. In particular, the systematical classification of DS spectra (truncated concave, convex, Lorentzian and structured spectra) and DS temporal profiles (from *sech*-shaped $A(t)$ to tabletop one) is possible in the framework of analytical approach. The transition to a DSR-regime reveals itself in the 'time-spectral' duality shown in Figure 9 [141,189]. The sense of this 'duality' will be explained below from the point of view of adiabatic theory of DSs.

The crucial shortcoming of the approach based on few exact DS solutions of evolution equations is that the strict restrictions are imposed on the equation parameters. As a result, the DS cannot be traced within a broad multidimensional parametric range and the picture obtained is rather sporadic and is of interest only in the close relation with the numerical results and experiment. Some additional information can be obtained on the basis of perturbation theory which provides with a quite accurate approximation for a low-energy DS [215–217].

Most powerful approaches to the theory of DSs have been developed in the framework of *approximated techniques* (for review see [44]): AM [165,167,217–221], VA [77,177,222–224,225]
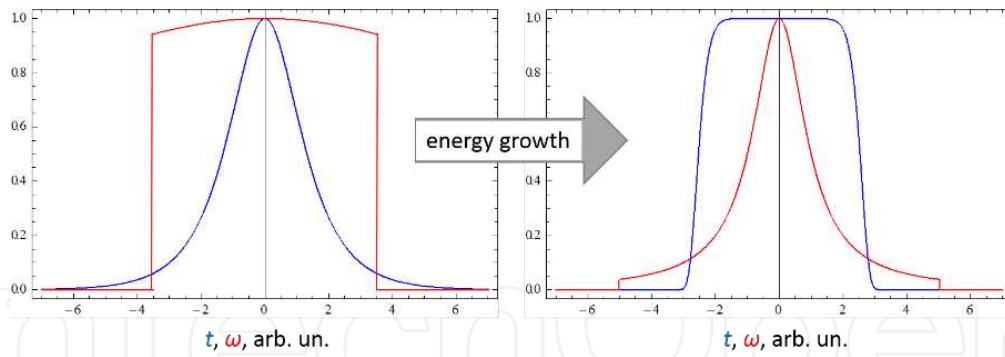
Figure 9. 'Time-spectral duality' for an energy-scalable DS: bell-like time-profile (blue curve on the left picture) transforms into tabletop one with truncated edges (right) and, inversely, truncated tabletop spectrum (red curve on the left picture) transforms into bell-like one (right) with the DS energy growth.

and MM [175,210,222,226]. The most impressive results obtained are: (i) physically relevant representation of DS parametric space was revealed (it is a so-called *master diagram*, see [44] for review and Figure 10); (ii) such a representation allows understanding the structural properties of DS and its energy-scaling laws (i.e. DSR conditions) for different mode-locking techniques; (iii) DS dynamics and an issue of optimal arrangement of laser elements providing the maximum DS stability and energy have been explored [224,225,227,228]; (iv) vectorial extension of VA concerning a vector DS (VDS) was endeavoured [229].
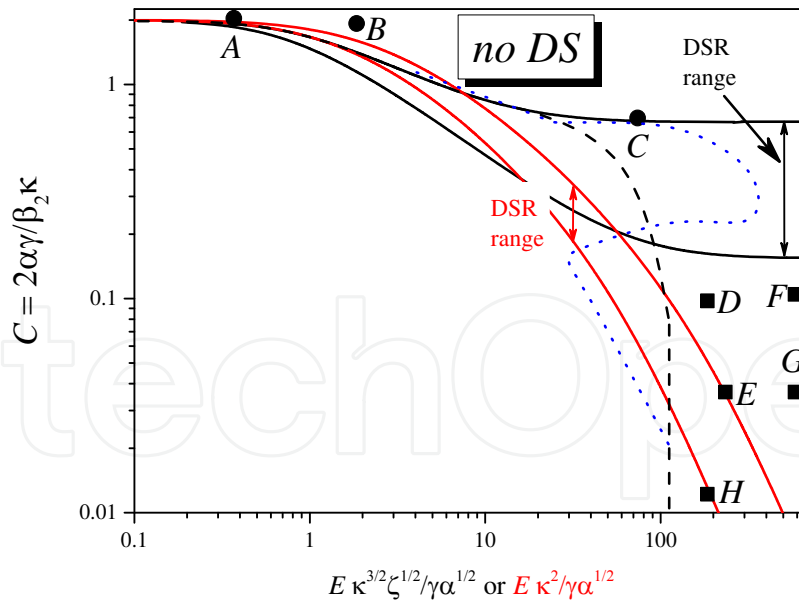

Figure 10. DS master diagrams for the cubic-quintic CNGLE (black) and the CNGLE with a SAM defined by Eq. (12) (red) (in the last case $\kappa = 1/\gamma P_s$). DSR ranges correspond to a so-called *positive branch* of DS [44,167], which has a highest DS energy-scalability and stability [221]. The dashed curve corresponds to a DS stability border obtained from numerical simulations of cubic-quintic CNGLE taking into account a quantum noise [243]. Dot blue curve shows the DS border under effect of SRS [260]. The parametrical space shown is the *physically relevant* parametrical space of DS. Points correspond to different scenarios of DS destabilization (see text).

Both AM and VA demonstrate two-dimensional representation of DS parametric space in the form of master diagram. Dimensionality can grow with complication of CNGLE non-linearity when SPM becomes saturable so that the cubic non-linear term in Eq. (9) has to be replaced by $[(\kappa - \kappa\zeta \mid A \mid^2) - i(\gamma + \chi \mid A \mid^2)] \mid A \mid^2 A$ [217]. This effect can appear in a fibre laser with NPR (e.g. see [179] where such a completely cubic-quintic CNGLE is connected with the NPR mode-locking technique). In this case, DS soliton exists in both normal and anomalous GDD regions [175,182,233].

The master diagram is a manifold of isogains (i.e. curves with $\sigma = const \leq 0$). Figure 10 demonstrates the zero-isogains ($\sigma = 0$) corresponding to the DS stability limit (upper curves) as well as the borders between 'energy-scalable' and 'energy-non-scalabe' branches of DS (lower curves, see [44,167] for a formal definition[3]). Figure 10 demonstrates that the saturation of SAM (so-called reverse saturable absorber provided, for instance, by NPR or graphene [141]; black curves) enhances the DS stability in comparison with an unsaturable SAM (red curves). Since $\lim\limits_{C \to const} E = \infty$ ($const \in$ the DSR range and its maximum value are of 2/3 for a saturable SAM (cubic-quintic CNGLE, black curves in Figure 10). Such a property of isogain curves corresponds to the DSR phenomenon. Since the stability threshold is defined by the condition of $C \equiv 2\alpha\gamma / \beta_2\kappa \leq 2/3$, one may conclude that the broadening of spectral filter band (or gainband) enhances stability against multi-pulsing ($\alpha \propto 1/\delta\Omega^2$, see above) [107,108]. Simultaneously, SPM has to be balanced by GDD ($C \propto \gamma / \beta_2$) that, in combination with normalization of energy, gives the energy-scaling law $E \propto L$ along a DSR curve. VA predicts [230]:

$$E \propto \beta_2 / \sqrt{\alpha} \tag{17}$$

which agrees with experimental observations of linear growth of DS energy with bandwidth [107,108] as well with a rule $E \propto L$ since $\beta_2 \propto L$ in the frameworks of distributed CNGLE.

In the case of unsaturable SAM corresponding to SESAM, some nanotube and graphene absorbers, Kerr-lensing, etc. (see Eq. (12)), the energy scaling requires scaling of the control parameter C. In this case, the asymptotic *energy scaling law* for $E\kappa^2 / \gamma\sqrt{\alpha} \gg 1$ becomes [44]:

$$E \propto \beta_2 / C\sqrt{\alpha}. \tag{18}$$

The spectral properties of DS are described clearly in the frameworks of AM [44,167]. In the simplest case of cubic-quintic CNGLE, the DS spectrum $\tilde{A}(\omega)$ in the limit of $\mid \psi \mid \gg 1$ is a Lorentzian profile which has a characteristic width Ω_L and is truncated at frequencies $\pm\Delta$ [44,167,218]:

$$\tilde{A}(\omega) \propto \frac{H(\Delta^2 - \omega^2)}{\Omega_L^2 + \omega^2}, \tag{19}$$

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3 Energy-non-scalable branch has two distinguishing characteristics: it turns into solution of Eq. (9) with ζ, $\chi \to 0$ ('Schrödinger limit' [218]) and is unstable in absence of dynamic gain saturation, i.e. if σ is not energy-dependent [221].

where H is a Heaviside function. The DS energy is

$$E \propto \arctan\left(\Delta/\Omega_L\right)/\Omega_L , \tag{20}$$

and

$$\gamma\left|A(t)\right|^2 = q - \frac{\beta_2}{2}\left(\frac{d\phi(t)}{dt}\right)^2,$$
$$q = \gamma P_0 = \frac{\beta_2}{2}\Delta^2 = \frac{3\gamma}{2\zeta}\left(1 - \frac{C}{2}\right), \tag{21}$$
$$\frac{\beta_2}{2}\Omega_L^2 = \frac{\gamma}{\zeta}\left(1+C\right) - \frac{5}{3}\gamma P_0,$$

Here, we trace the zero-isogain $\sigma = 0$. The DS time-profile is defined by an implicit expression:

$$t = \tau\left[\text{arctanh}\left(\frac{d\phi/dt}{\Delta}\right) + \frac{\Delta}{\Omega_L}\arctan\left(\frac{d\phi/dt}{\Delta}\right)\right], \tag{22}$$

with the DS width of $\tau \propto \Delta^{-1}\left(\Delta^2 + \Omega_L^2\right)^{-1}$. Now, there are the following limiting cases:

$$\lim_{C \to 2}\begin{cases} E \to 0 \\ P_0 \to 0 \\ \Delta \to 0 \\ \Omega_L \to const \\ \tau \to 0 \\ \gamma\left|A\right|^2 = q - \beta\Delta^2 \tanh^2\left(t/\tau\right) \end{cases}. \tag{23}$$

It is clear that in this 'low-energy' sector the DS time-profile is bell-like and its spectrum has tabletop form ($\Omega_L \gg \Delta$). In the DSR limit, one has:

$$\lim_{C \to 2}\begin{cases} E \to \infty \\ P_0 \to 1/\zeta \\ \Delta \to \sqrt{\gamma/\beta\zeta} \\ \Omega_L \to 0 \\ \tau \to const \\ \gamma\left|A\right|^2 = q - \beta\Omega_L^2 \tan^2\left(t\Omega_L/\tau\Delta\right) \end{cases}, \tag{24}$$

that is, a DS in the DSR sector has a flattop temporal profile and a Lorenzian spectrum ($\Omega_L \ll \Delta$). Eqs. (24) demonstrate that asymptotical growth of DS energy leads to a spectral condensation ($\Omega_L \to 0$) without a parallel temporal thermalization ($\tau \nrightarrow \infty$), which means an inevitable destabilization of a plain energy-scalability [180]. This conclusion does not mean a participial impossibility of DS energy-scaling in the frameworks of cubic-quintic CNGLE model. For instance, a saturable SPM allows DSs with tabletop profiles and $|d\phi / dt| \to \infty$ on the pulse edges. Such a DS possesses enhanced energy scalability and was observed experimentally [231].

4. DS spectrum and stability

As was explained, the dual balances in frequency domain:

$$\begin{cases} \beta_2 \Delta^2 \leftrightarrow \gamma P_0 \\ \alpha \Delta^2 \leftrightarrow \kappa P_0 \end{cases} \tag{25}$$

are formative for DS existence and stabilization. No wonder that the spectrum of DS is benchmark of its inherent properties.
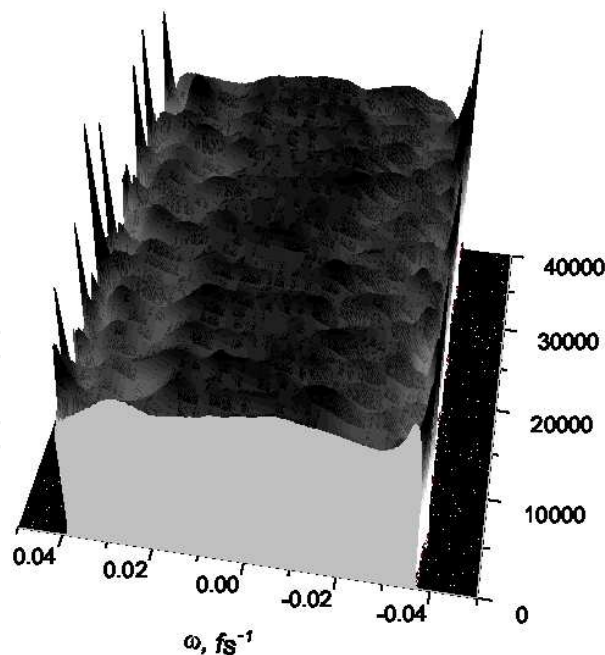

Figure 11. Perturbed DS spectrum [241].

Prior to consider the aspects of interweaving of spectral and stability properties of DSs, one has to point to a possibility of multi-wavelength multi-pulsing DSs provided by DS robustness.

As was demonstrated in [234] theoretically, the multi-DSs compounds in a mode-locked laser can be stabilized at multiple frequencies. Experimentally, such multi-frequency DS compounds can be realized by birefringence filters with a periodical (interference-like) dependence of transmission on wavelength under conditions of sufficiently broad gainband and powerful pump [235–239][4].
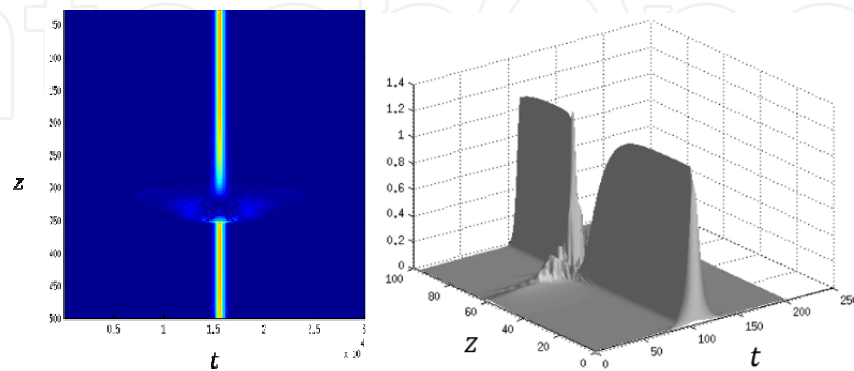

Figure 12. Exploding DS corresponding to parameters of point *A* in Figure 10. Left: contour-plot of instant power, right: 3D-graph of instant power in dependence on local time *t* and propagation distance *z* (arbitrary units) [243].

As was demonstrated in previous section, DS has non-trivial internal structure due to energy fluxes inside it. The elements of this structure (*internal modes*) can be excited that causes pulsating or chaotic dynamics of DS with preservation of its temporal and spectral localization [241]. The spectral envelope acquires a shape of 'glass with boiling water' (Figure 11). Appearance of such perturbations is understandable in frameworks of the DS perturbation theory in spectral domain [242]. One has to note that such perturbations take a place inside the DS stability region where $\sigma < 0$ (below the corresponding upper curves in Figure 10). Above the stability boarder ('no DS' region in Figure 10), there are three main destabilization scenarios [243]. For small energies and in the vicinity of stability border (point *A* in Figure 10), DS is *exploding* (Figure 12), which means its aperiodic disappearance with excitation of continuous waves and subsequent DS recreation [37,244–248]. With the energy growth (point *B* in Figure 10), the *rogue* DSs develop (Figure 13) [37,249–253]. Such a regime can be interpreted as DS structural chaotization, that is, generation of multiple DSs with strong interactions causing extreme dynamics.

For sufficiently large energies in the vicinity of stability border (point *C* in Figure 10), the typical destabilization scenario is the generation of multiple DSs (Figure 14). The source of this destabilization is the growth of spectral dissipation caused by DS spectral broadening with approaching to stability border so that the DS splitting becomes more energy advantageous [161]. Moreover, the DS splitting can be enhanced by its phase inhomogeneity because the gain (energy in-flow) is maximum at the points of stationary phase $d\phi/dt = 0$ [254]. Such a splitting

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4 A multi-porting configuration of a DS laser supports even simultaneous generation of conventional and dissipative wavelength-separated solitons [240].
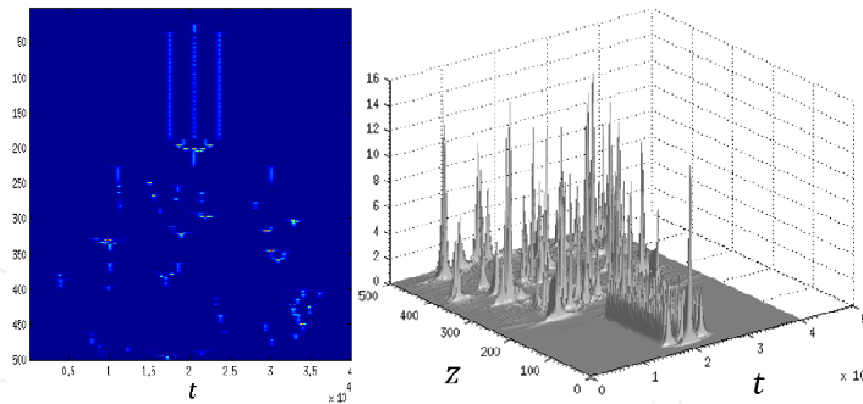
Figure 13. Rogue DSs corresponding to parameters of point *B* in Figure 10. Left: contour-plot of instant power, right: 3D-graph of instant power in dependence on local time *t* and propagation distance *z* (arbitrary units) [243].
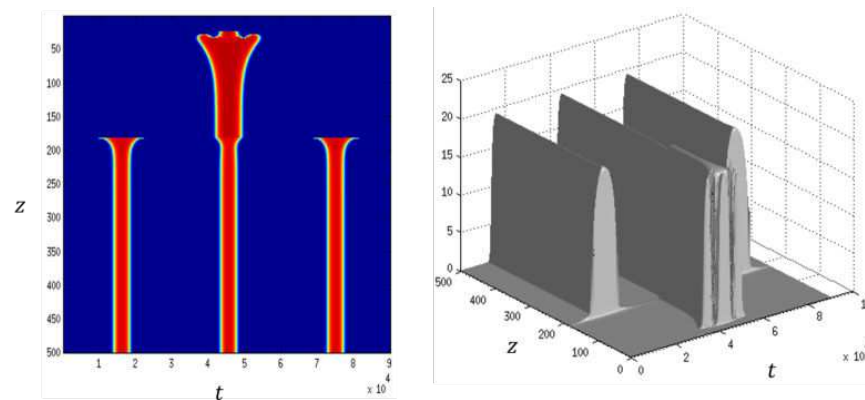

Figure 14. DS molecule corresponding to parameters of point *C* in Figure 10. Left: contour-plot of instant power, right: 3D-graph of instant power in dependence on local time *t* and propagation distance *z* (arbitrary units) [243].

can result in an extreme dynamics like solitonic turbulence (Figure 13) [255] or regular multi-DS complexes (Figure 14, so-called DS molecules) which can have non-extreme internal dynamics (soliton gas) and interact with a background (soliton liquid) [256-259]. If the dynamic gain saturation (see [44]) contributes, the multi-DS complexes can evolve slowly into a set of equidistant DSs with repetition rate multiple of laser one (harmonic mode-locking) [158,159].

The numerical simulations of cubic-quintic CNGLE with taking into account a quantum noise validated the fact of inconsistency of spectral condensation and absence of temporal thermolization that breaks the DS energy scalability (see previous section) [243]. As a result, the DS stability region breaks abruptly with energy growth (dashed curve in Figure 10) and multitude of turbulent scenarios of DSs evolution develops (Figure 15) [34,243].

Serious limitations on power and energy scalability of DSs in fibre lasers arise from stimulated Raman scattering (SRS) [2,109]. The stability border of DS under action of SRS is shown in Figure 10 by dot blue curve (DS is stable on the left of this curve) [260]. As was found, SRS enhances the tendency to multi-pulsing with energy growth caused by enhancement of
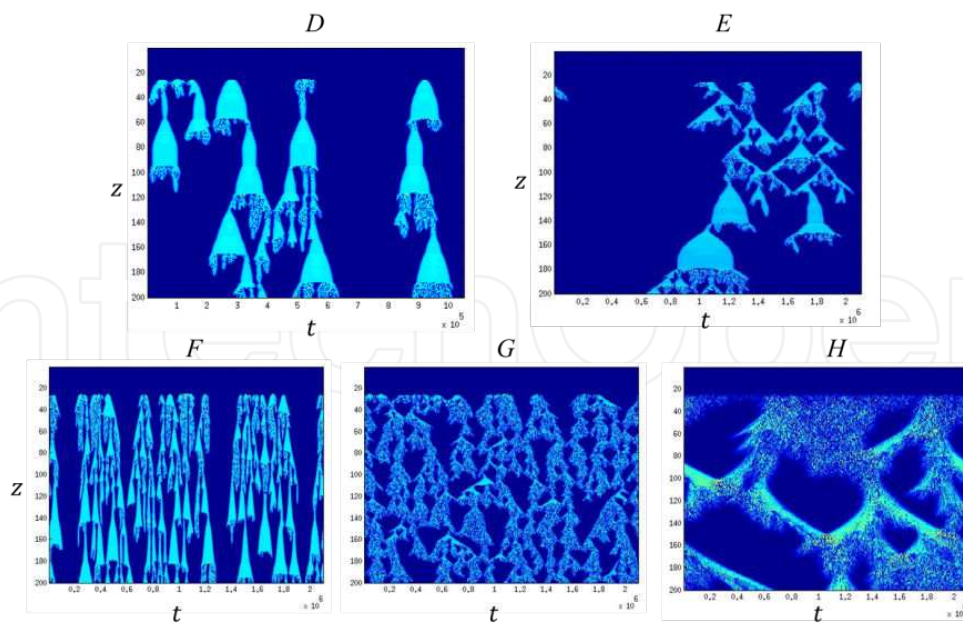
Figure 15. DS turbulent regimes (contour-plot of instant power, arbitrary units) corresponding to the parameters of points *D*, *E*, *F*, *G* and *H* in Figure 10 [243].

spectral dissipation due to SRS [260]. Simultaneously, generation of anti-Stokes radiation causes chaotization of DS dynamics and irregular modulation of DS temporal and spectral profiles [261] (Figure 16). DS profile remains localized, but it is strongly cut by colliding dark and grey soliton-like structures [34].

As was shown, the DS dynamics can be regularized by formation of *dissipative Raman soliton* (DRS). DRS can exist in the form of DS which is Stokes-shifted due to self-Raman scattering (Figure 17, *a*) [260] or as bound DS–DRS complex (Figure 17, *b*) [262]. In the last case, stabilization is achieved by feedback, i.e. reinjection of Stokes signal through a delay line [263]. In the absence of a feedback, the Raman pulse is noisy [264] (see dash line spectrum in Figure 16, *b*).
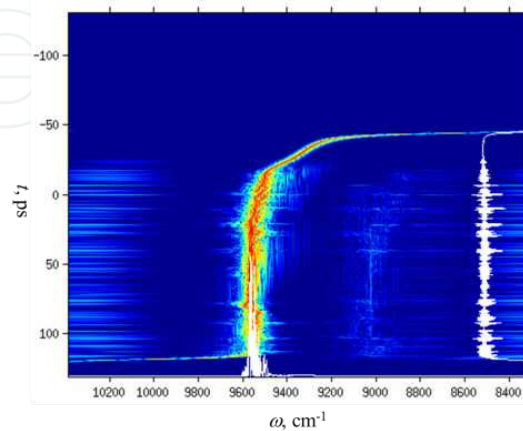

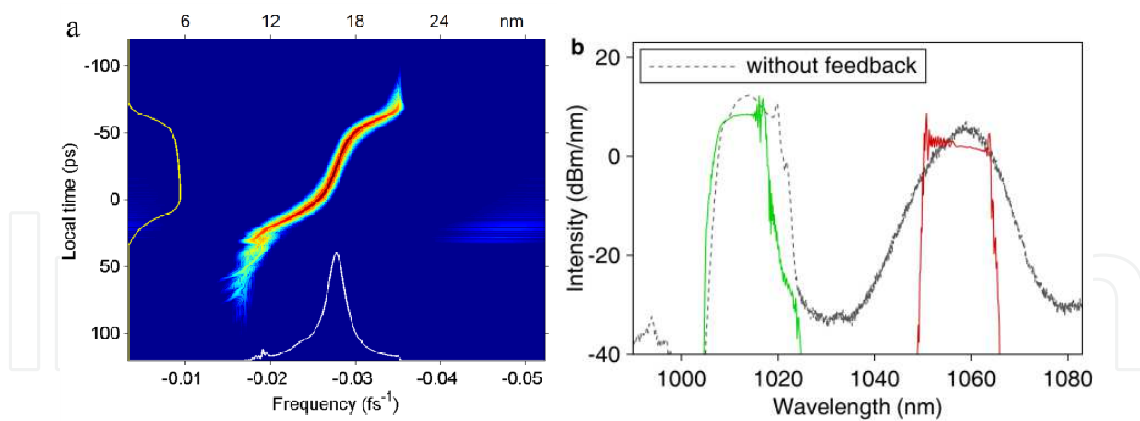**Figure 16.** Wigner function of turbulent DS in the presence of SRS.

Figure 17. (a) Wigner function of single DRS [260] and (b) spectrum of bound DS–DRS complex [262].

5. Vector DSs

As was pointed above, SOP can play leading role in a fibre laser dynamics. In particular, it can contribute to mode-locking or/and spectral filtering. However, diapason of polarization phenomena in a DS fibre laser spreads essentially broader. As was found, intrinsic fibre birefringence (Figure 2) can lead to DS splitting into two independent SOPs [78]. This phenomenon is used to realize the NPR mode-locking mechanism where a DS SOP evolves (or remains locked) as a whole during propagation [265-269]. The polarization dynamics can be fast ($\leq T$) or slow ($\gg T$) and vary from regular (with possible period multiplication or harmonic mode-locking) to chaotic one [270,271]. There are evidences of ultrafast SOP evolution when SOP changes across a DS profile [272].

The specific multiple pulse instability of vector dissipative solitons (VDSs) leads to generation of the bound states of DSs with different SOPs (*vector soliton molecules*) which are locked by a non-linear coupling [273,274] or by a group-velocity locking produced by spectrum shift between DSs with different SOP [275]. As was shown experimentally (Figure 18) [276], the dynamics of VDS molecules can be highly non-trivial and demonstrate both fast and slow periodic switching between fixed SOPs as well as SOP procession, which is especially interesting for fibre laser telecommunications based on polarization multiplexing.

The important breakthrough in the recent theory of VDSs is the demonstration of insufficiency of approaches based on the coupled CNGLEs (like (10)) for adequate description of DS polarization dynamics. It was demonstrated that an active medium polarizability contributes to DS dynamics substantially [277]. As was shown, the SOP-sensitive interaction between DS and a slowly relaxing active medium with taking into account the birefringence of fibre laser elements and light-induced anisotropy caused by elliptically polarised pump field change the SOP at a long time scale that results in fast and slowly evolving SOPs of VDSs (Figure 19).

The non-trivial contribution of active medium kinetics and polarizability with taking into account the pump SOP and SPM demonstrates a complex dynamics including spiral attractors
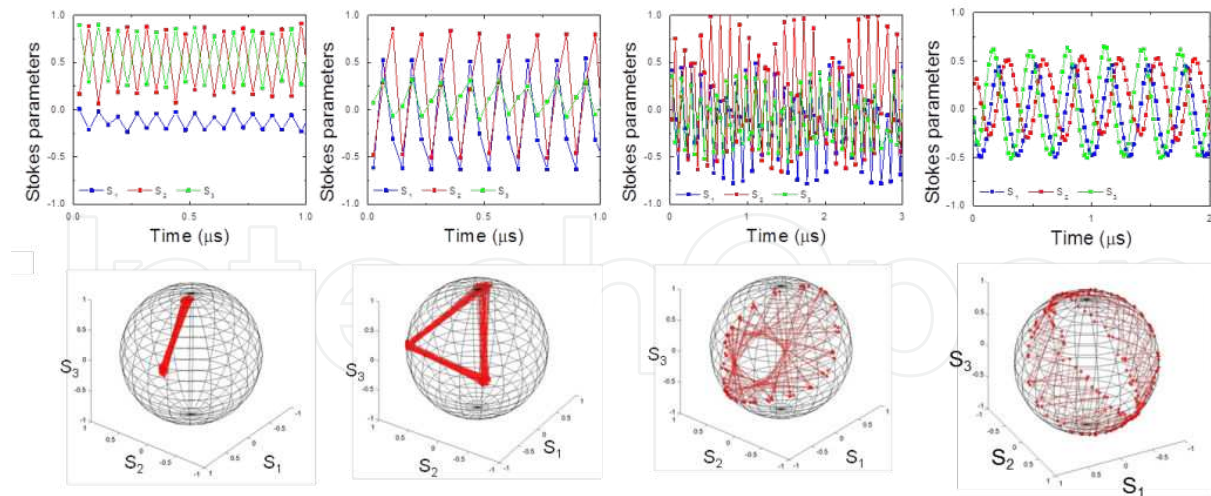
Figure 18. Polarization dynamics of VDS molecules in an Er-fibre laser mode-locked by carbon nanotubes [276]. Top row demonstrates the evolution of Stokes parameters. The bottom row reproduces this evolution on the Poincaré sphere (each point corresponds to SOP after one laser round-trip).

and dynamic chaos (Figure 20) [278]. One may assume that such a non-trivial polarization dynamics is of great importance for DS energy scaling, in particular, due to vector nature of SRS [279]. These topics remain unexplored to date.
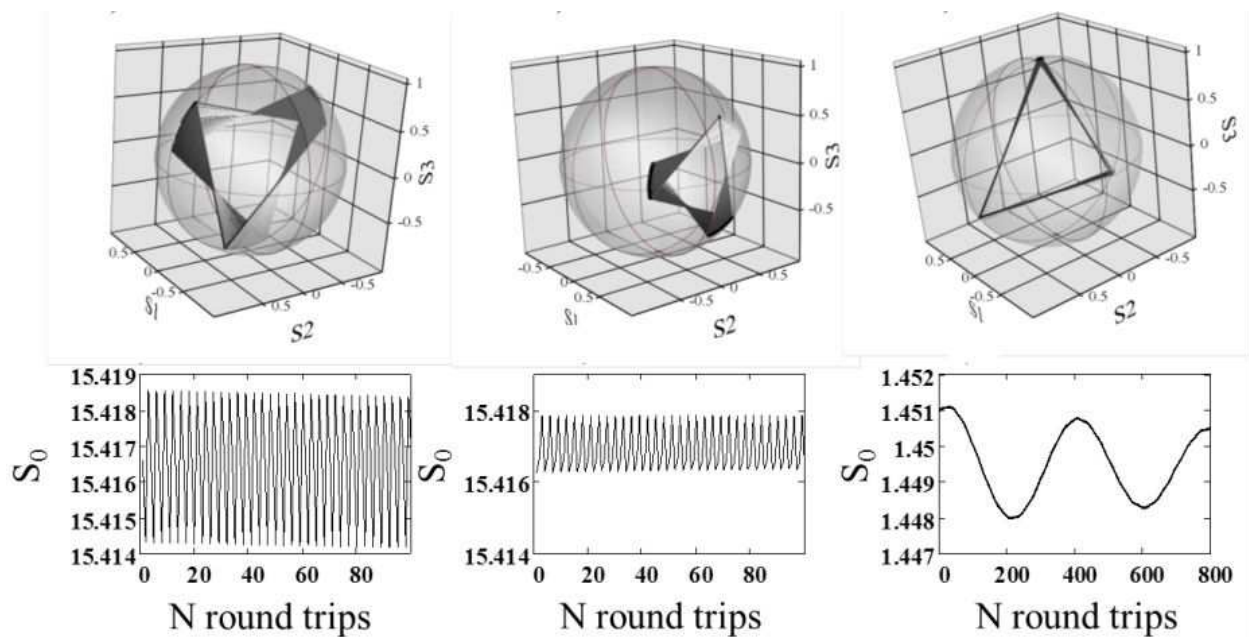

Figure 19. Polarization dynamics of VDS with taking into account an Er-fibre polarizability [277]. Both fast and slow SOP dynamics exist in dependence on fiber laser birefringence strength.
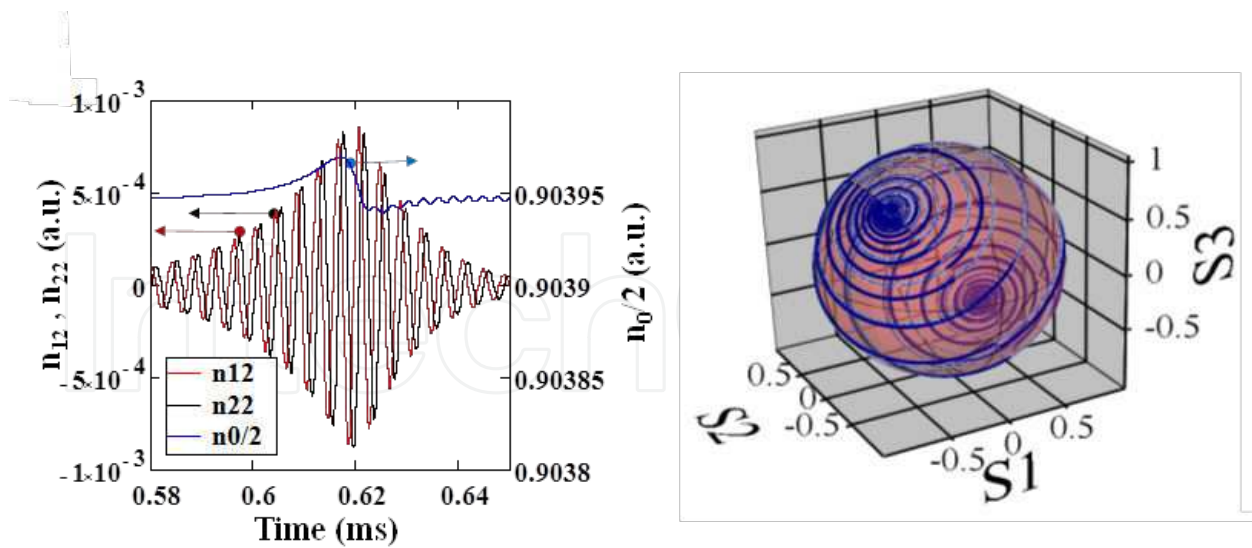
Figure 20. Dynamics of active fibre inversion components, which are polarization-sensitive (n_{12} and n_{22}) and non-sensitive (n_0), with the corresponding slow SOP evolution of VDS [278].

6. Conclusion

The recent progress in development of ultrafast fibre lasers and advances in exploring of DS are interrelated. DSs allowed scoring a great success in ultrashort pulse energy scalability that is defined by unprecedented stability and robustness of DS. At this moment, it is possible to achieve over-MW peak powers for sub-100 fs pulses directly from a fibre laser at over-MHz repetition rates. New spectral diapasons became reachable owing to development of mid-IR active fibres and using the frequency-conversion directly in a laser. Development of new mode-locking techniques, especially based on using of SESAMs, graphene and another quantum-sized structure allowed improving a laser stability, integrity and environment insensitivity. A great advance has been achieved in the theory of DSs. New powerful analytical techniques based on extensive numerical simulations and experimental advances extended understanding of the DS fundamental properties and revealed new prospects in improvement of characteristics of ultrafast fibre lasers. Based on achieved results, one may outline some unresolved problems. As was found, there are stability limits for a DS energy scaling imposed by optical turbulence and SRS. Deeper insight into the nature of these phenomena could allow to overcome these limits without substantial complication of laser setup. Simultaneously, control of intra-laser spectral conversion is a direct way to broadening of spectral range. Then, the dynamics and properties of VDSs remain scantily explored. Recent studies demonstrated a multitude of polarization phenomena, which cannot be grasped in frameworks of existing models. In particular, polarizability and kinetics of an active fibre in combination with birefringence of a laser in a whole can contribute non-trivially to a laser dynamics. As an additional aspect of further development, one may point at the development of new mode-locking techniques, which could improve DS stability and integrity of a fibre laser, decrease

pulse width and extend a diapason of pulse repetition rates. At last, one has to remember that a fibre laser is an ideal playground for study of complex non-linear phenomena and, undoubtedly, new bridges between different fields of science will be built with a further progress of ultrafast fibre lasers.

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